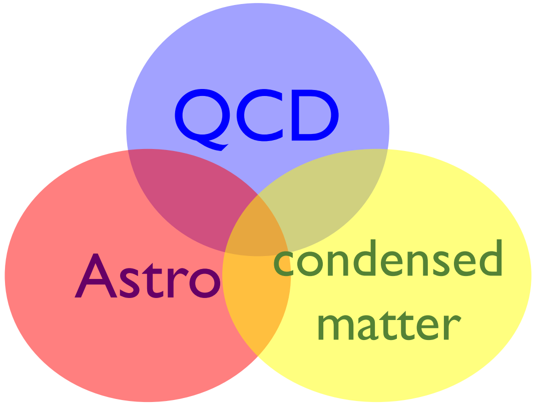
"Physics at high baryon density (PHD2025)," CCNU, Oct. 16-18, 2025

A quarkyonic matter model with strangeness

--- how to mitigate the hyperon puzzle ---



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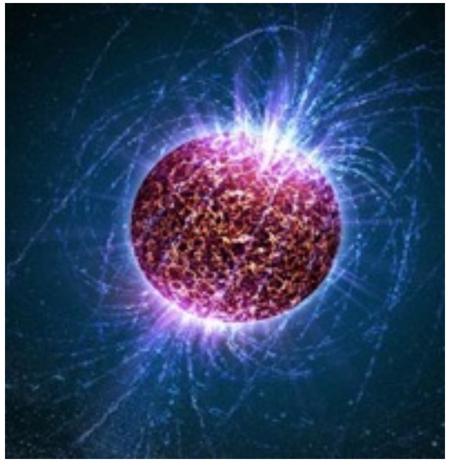


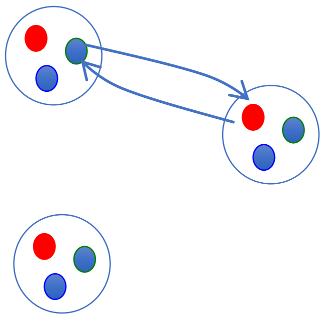
Refs:

Fujimoto, TK, McLerran, 2410.22758, PRL'24

TK, a mini-review, 2412.20442

Neutron Star matter ($n_0 = 0.16\ \mathrm{fm}^{-3}$) [Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

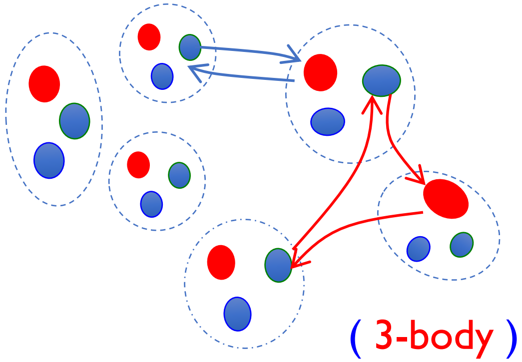


ab-initio nuclear cal.

laboratory experiments

steady progress

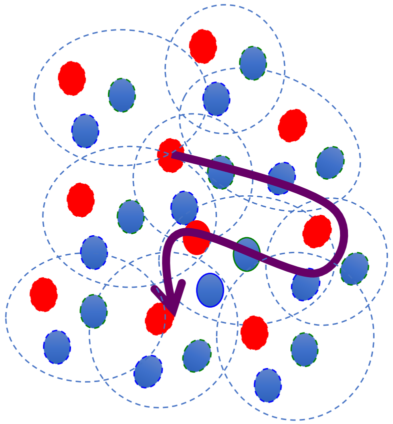
- many-quark exchange
- structural change,...
- hyperons, Δ, ...



most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

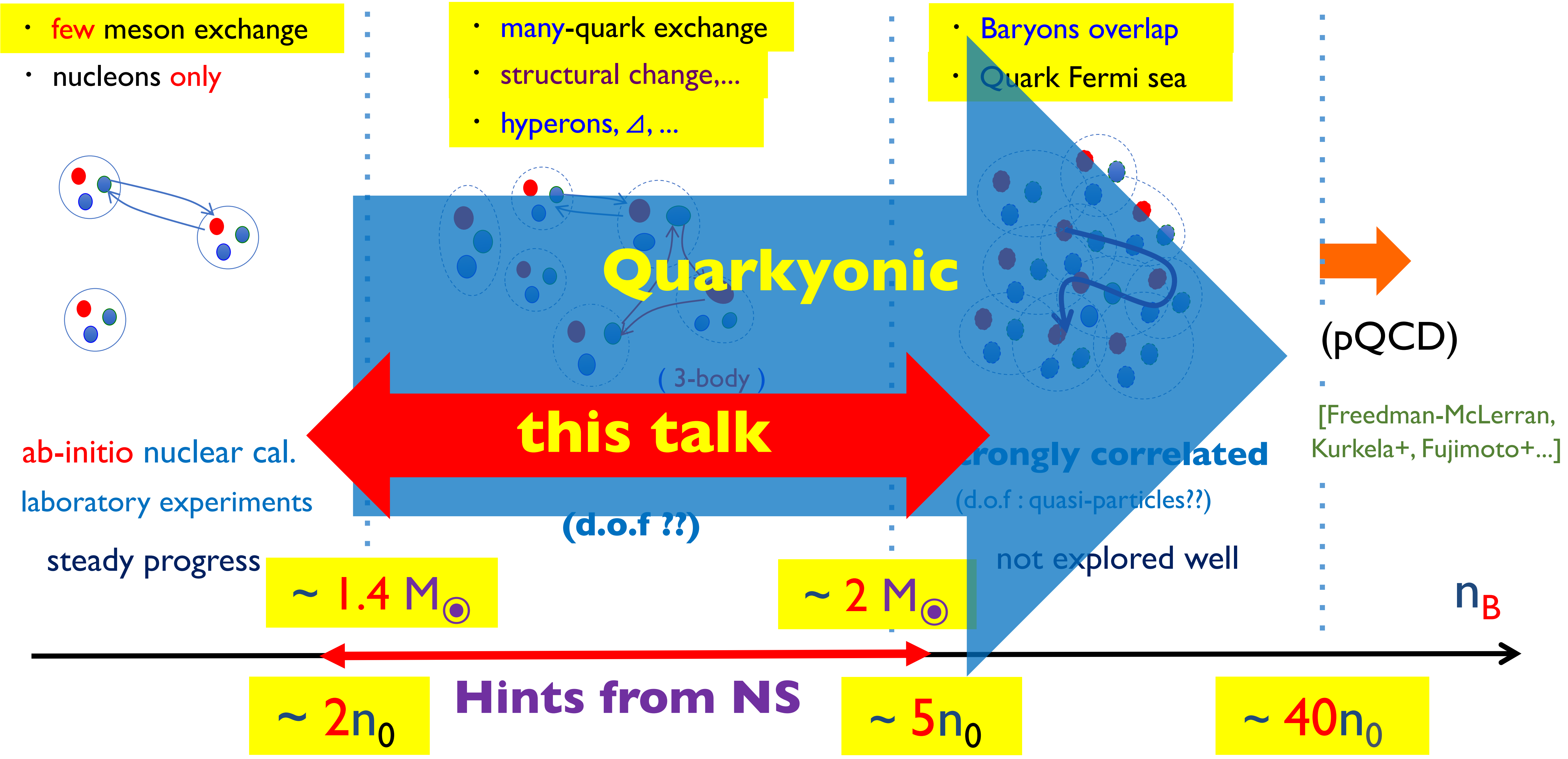
[Freedman-McLerran, Kurkela+, Fujimoto+...]

$\sim 1.4\, M_\odot$ $\sim 2\, M_\odot$

n_B

$\sim 2n_0$ **Hints from NS** $\sim 5n_0$ $\sim 40n_0$

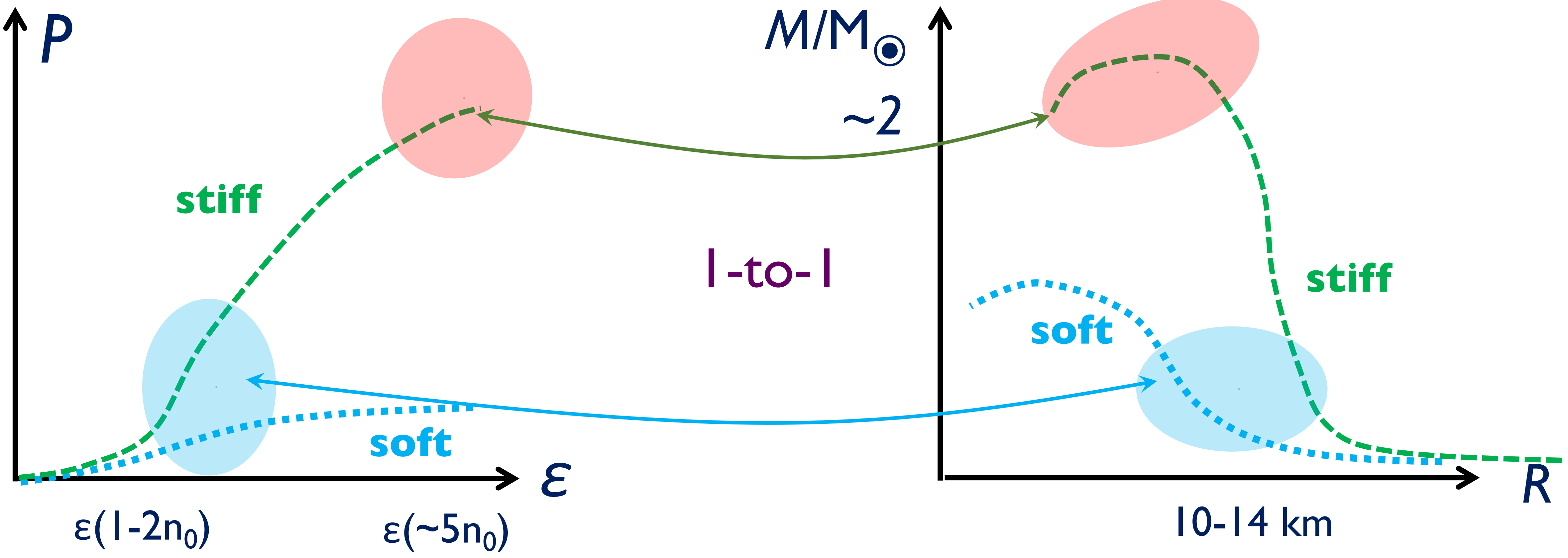
Neutron Star matter (n_0 = 0.16 fm^{-3}) [Masuda+ '12; TK+ '14]



EoS stiffness & M-R

Ref) Lattimer & Prakash (2001)

measure: P vs ε

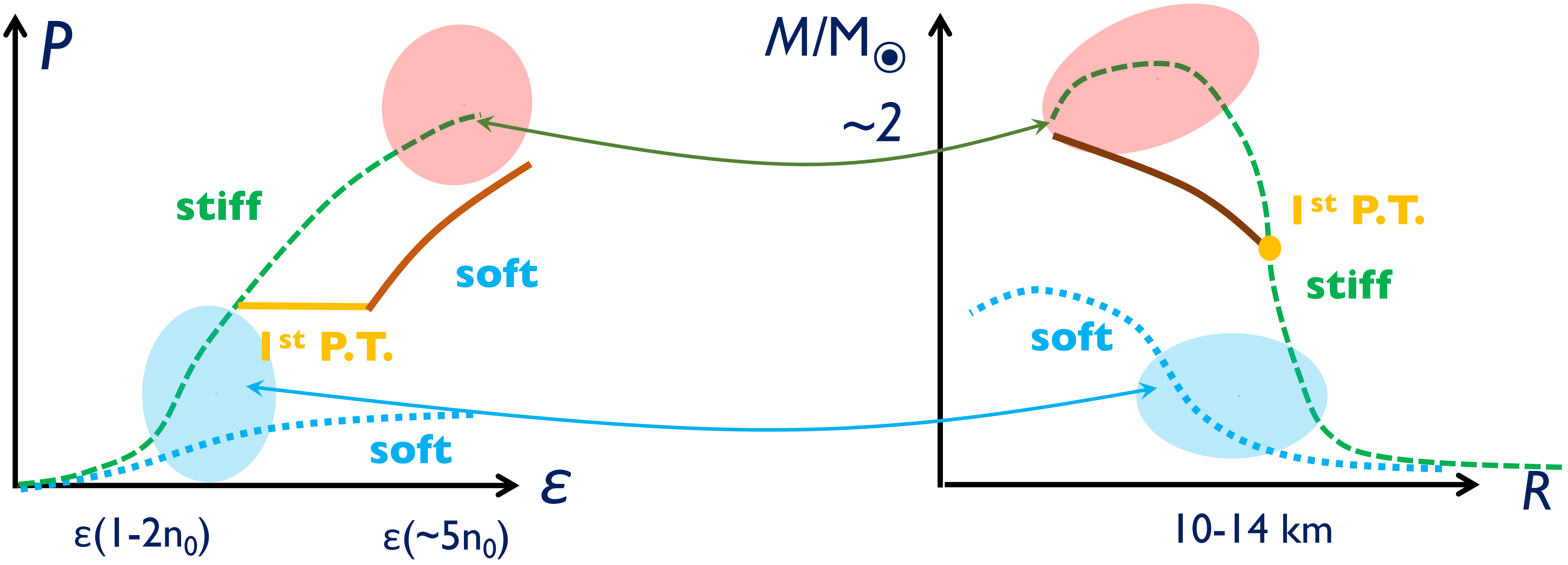


EoS stiffness & M-R

Ref) Lattimer & Prakash (2001)

stiff-to-soft EOS

e.g.） hadron-to-quark phase transition (1st order)



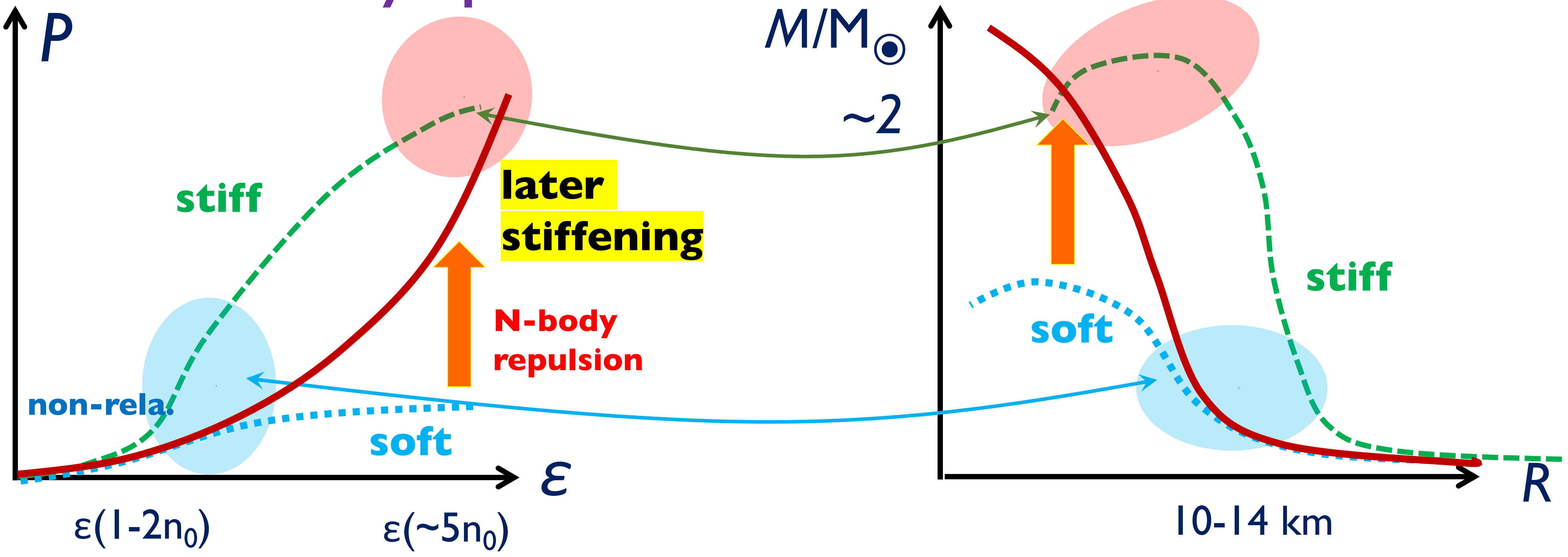
EoS stiffness & M-R
Ref) Lattimer & Prakash (2001)

soft-to-stiff EOS

e.g.) nuclear matter with N-body repulsion

$$\varepsilon = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^N$$

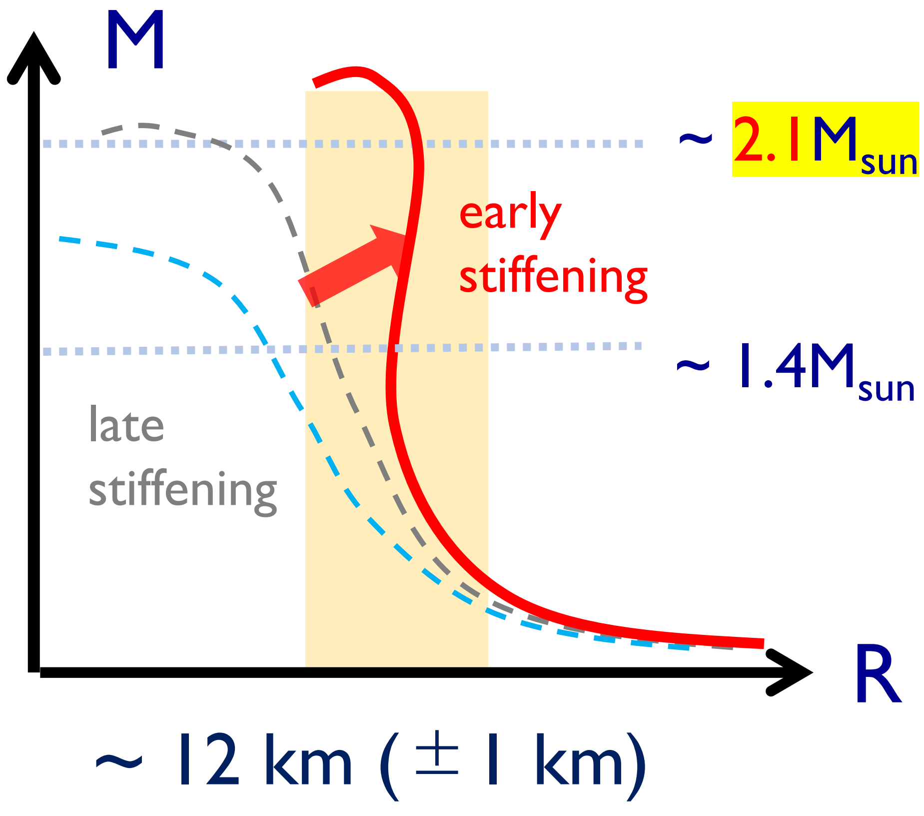
mass E　kin. E　N(>2)-body repulsion

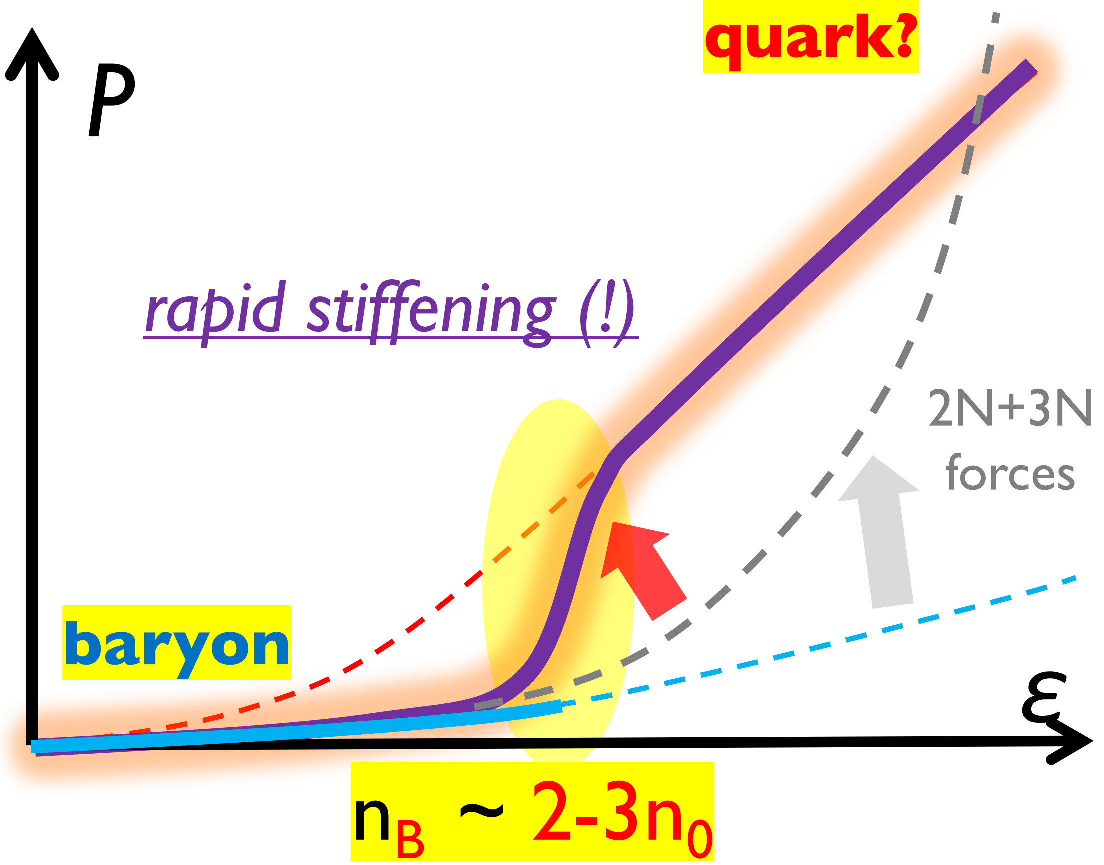


Implications from NS

NICER for 1.4 & 2.1 M_{sun} + **GW** + **nuclear** (< ~$1.5n_0$)

→ $R_{1.4} \sim R_{2.1}$ (!)





Quark matter can be naturally stiff

For ideal gas picture

quarks baryons

NR kinetic energy: $\varepsilon_Q^{\text{kin}} \sim \underline{N_{\text{c}}} \dfrac{n_B^{5/3}}{M_q}$ >> $\varepsilon_B^{\text{kin}} \sim \dfrac{n_B^{5/3}}{\underline{N_{\text{c}}} M_q}$

$$\Rightarrow \quad P_Q^{\text{ideal}} \sim \underline{N_{\text{c}}^2} \times P_B^{\text{ideal}}$$

quark matter is stiff already at LO (i.e., without interactions)

(baryonic matter is soft at LO ➔ N-body forces dominate EOS)

Role of confinement

Quarks do contribute to the **energy density** through the baryon mass

$$E_B(P_B) \sim N_c \int_{\vec{q}} E_Q(\vec{q}) f_Q\left(\vec{q} - \frac{\vec{P}_B}{N_c}\right) \equiv N_c \langle E_Q \rangle_{P_B}$$

large !

But quarks in a baryon **hardly** contribute to the **pressure** => **soft** EOS

$$E_B(P_B) \sim \underbrace{N_c \langle E_Q \rangle_{P_B=0}}_{\text{large !}} + \underbrace{\frac{P_B^2}{N_c}}_{\text{small !}} \left\langle \frac{\partial^2 E_Q}{(\partial q)^2} \right\rangle + \cdots$$

- mechanical forces of quarks are largely cancelled by the other quarks
- pressure is large inside of a baryon, but it does not go out

When do quarks begin to contribute to the thermodynamic pressure ?

What to be explained in this talk

1) how quarks contribute to the ***thermodynamic*** pressure,
without *invoking baryon-to-quark* ***phase*** *transitions*

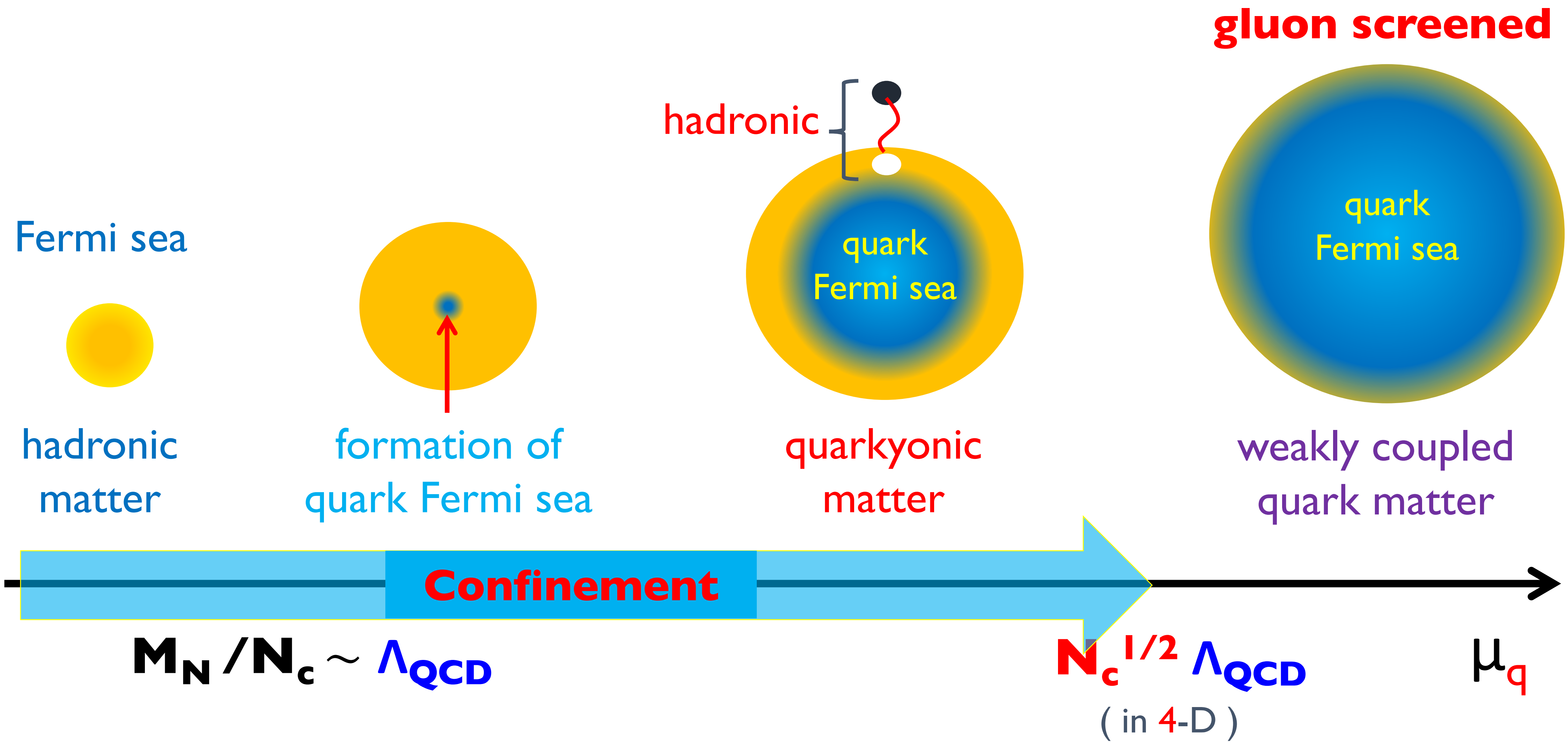
crossover descriptions

2) how quark constraints mitigate ***hyperon softening problem***

statistical repulsion

Evolution from nuclear to quark matter

Quarkyonic matter := quark matter with confining gluons



IdylliQ model

= **Ideal dual** Quarkyonic model

Describe **single** physics in **two** languages (baryon/quark)

Powerful in transient regimes (2-5n_0)

Sum rules for occupation probabilities

cf) [TK '21]

occupation **probability** of **quark** state with p

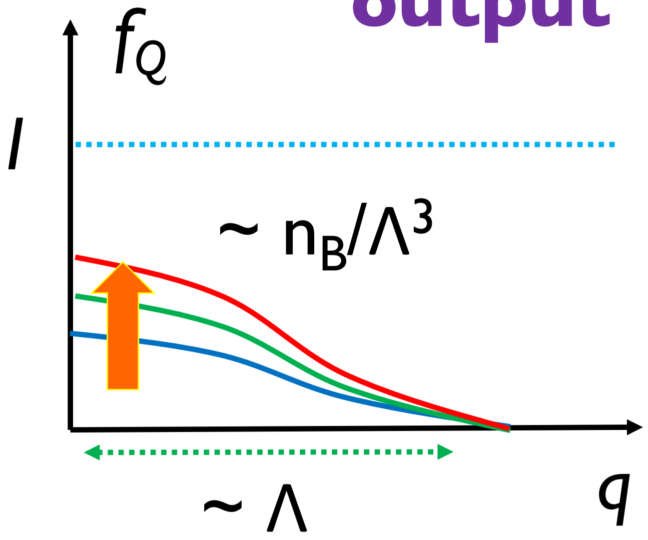
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_Q(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B)\,\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_c)$$

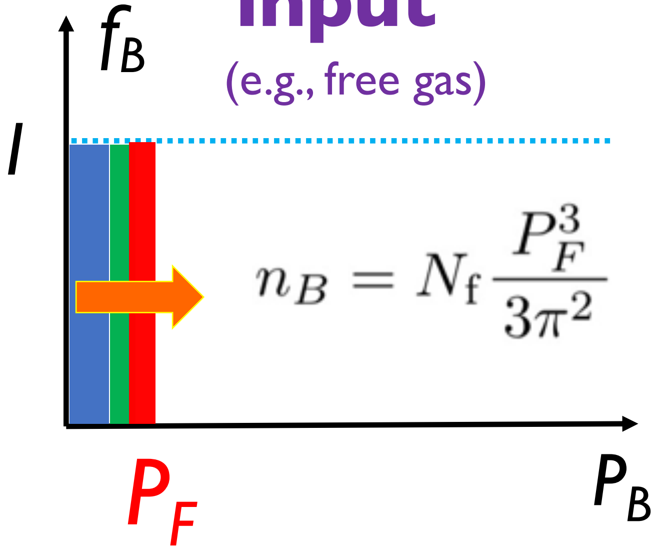
e.g.) in **ideal** baryonic matter

output



input

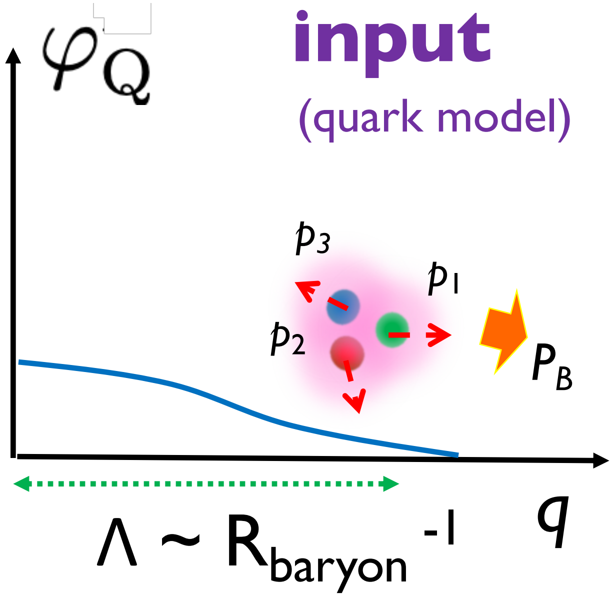
(e.g., free gas)



$\otimes$

input

(quark model)



An ideal model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\rm B}[f_{\rm B}] = 4\int_k E_{\rm B}(k) f_{\rm B}(k)$$

2) keep using the same $\varphi_{\rm Q}$ (quarkyonic)

3) use a special quark distribution → sum rules analytically *invertible*

$$\varphi_{\rm 3d}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2}\delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

*natural at **low** density*

⇄

nontrivial output

$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3}\hat{L}[f_{\rm Q}(\boldsymbol{q})]$$

*natural at **high** density*

An ideal model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $\varepsilon_{\mathrm{B}}[f_{\mathrm{B}}] = 4 \int_k E_{\mathrm{B}}(k) f_{\mathrm{B}}(k)$

2) keep using the ... (quarkyonic)

3)

<u>To Do:</u> solve variational problem of $f_B(k)$

with *quark substructure constraints* !

nontrivial output

$$f_{\mathrm{Q}}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\mathrm{B}}(\boldsymbol{P}_B) \varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\mathrm{c}})$$

*natural at **low** density*

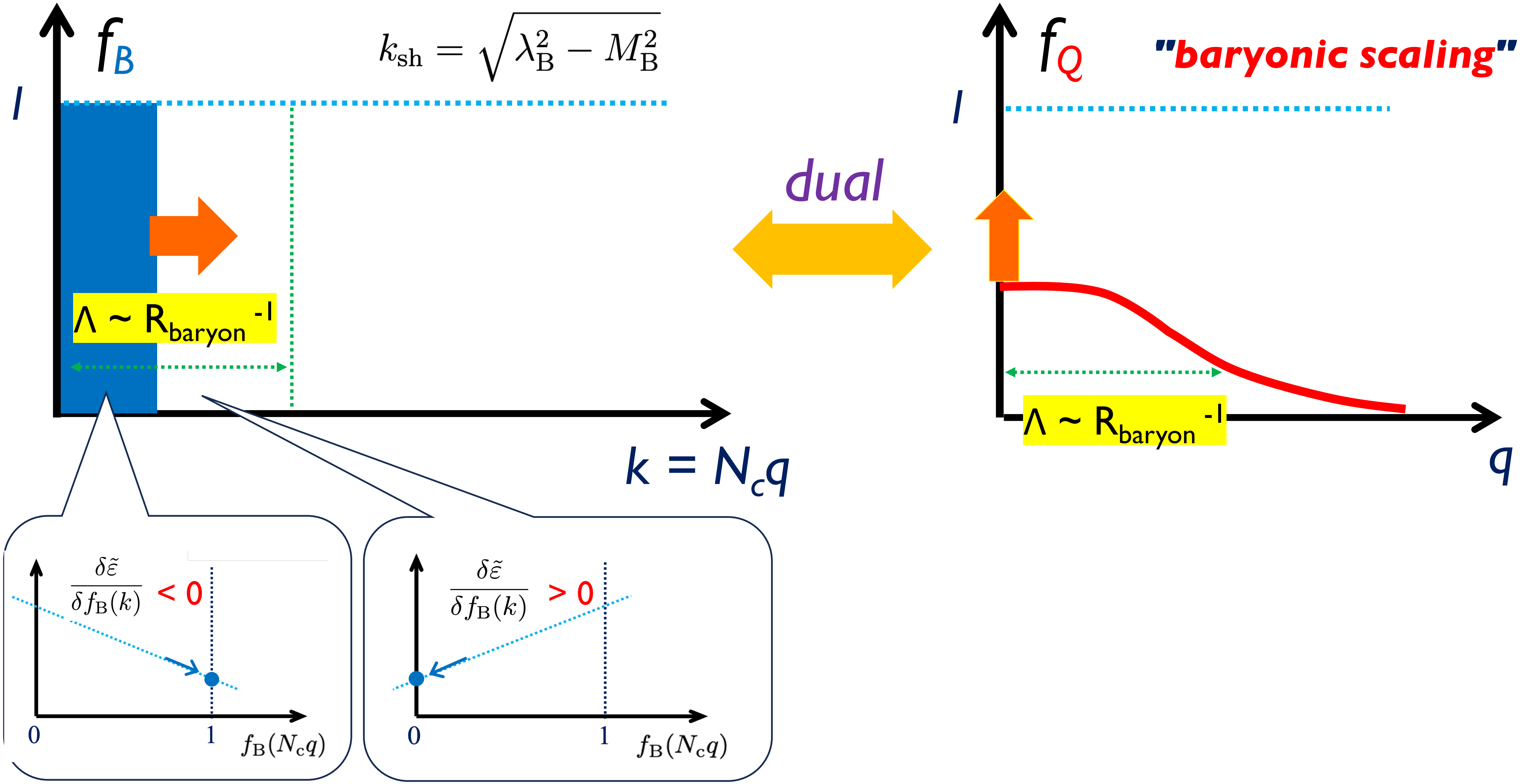
nontrivial output

$$f_{\mathrm{B}}(N_{\mathrm{c}}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\mathrm{c}}^3} \hat{L}[f_{\mathrm{Q}}(\boldsymbol{q})]$$

*natural at **high** density*

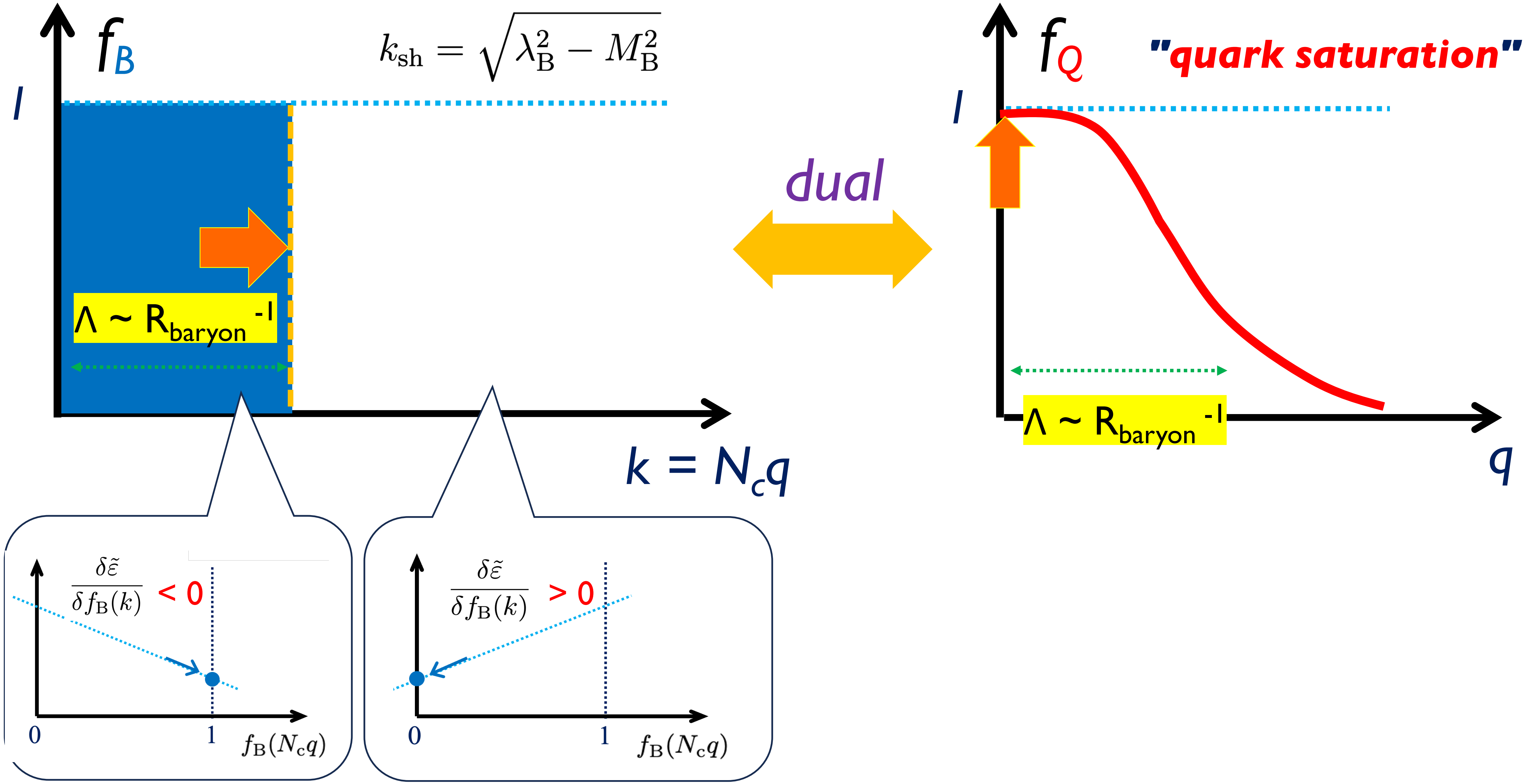
Solution (dilute regime)

[Fujimoto-TK-McLerran, PRL'24]



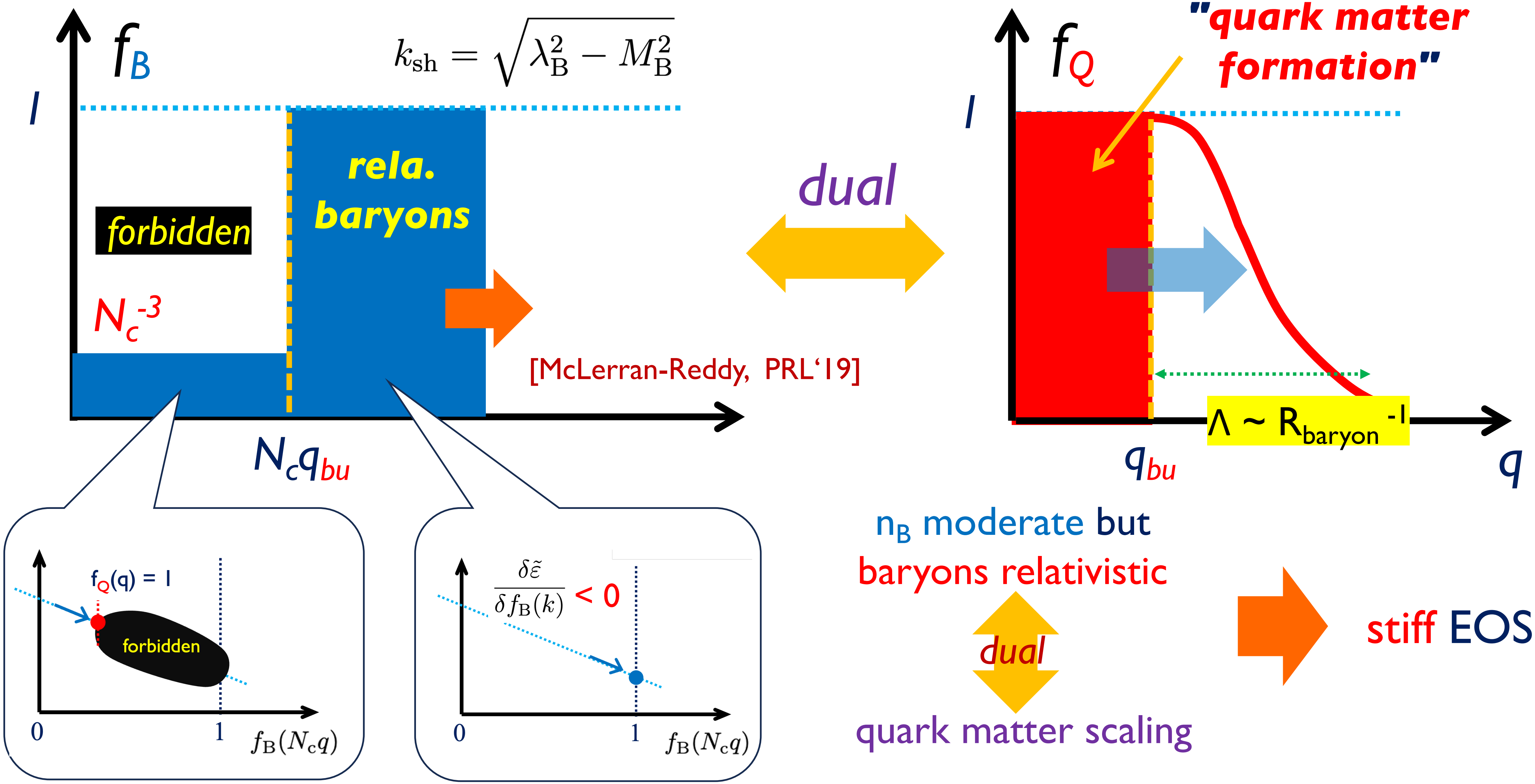
Solution (at saturation)

[Fujimoto-TK-McLerran, PRL'24]



Solution (post saturation)

[Fujimoto-TK-McLerran, PRL'24]



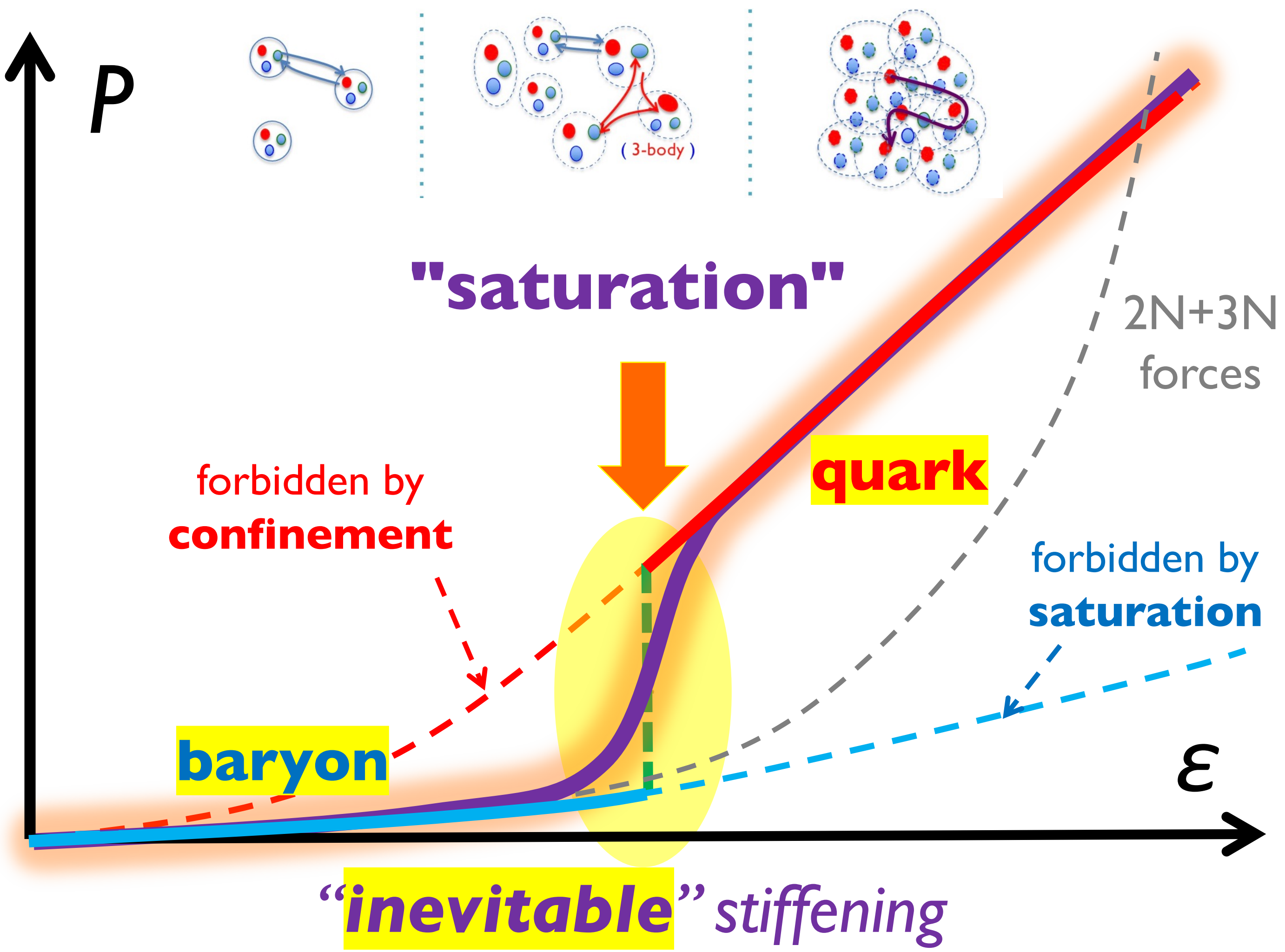
n_B moderate but baryons relativistic

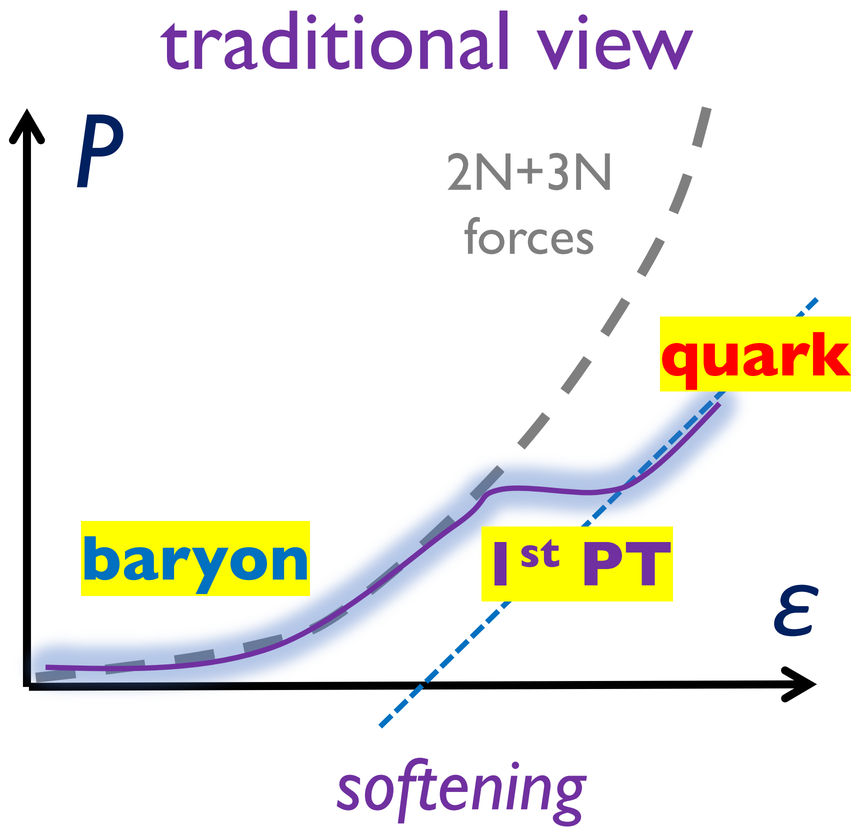
dual

quark matter scaling

stiff EOS

Inevitable stiffening



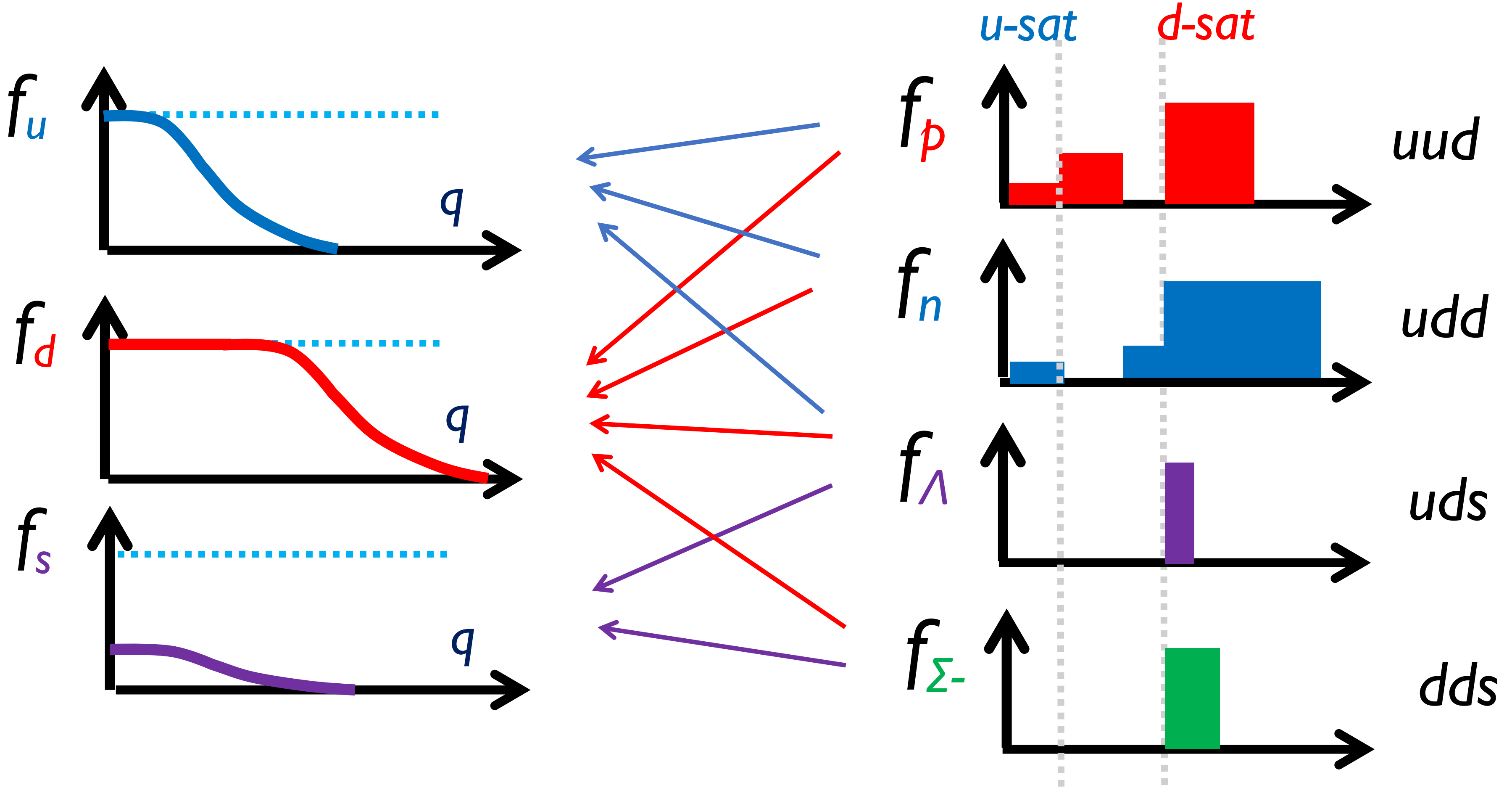


$n_{Q\text{-sat}} \sim 0.5\, n_{overlap}$ ➔ $n_{Q\text{-sat}} \sim 2\text{-}3 n_0$

Multi-flavor extension

$$f_{\mathrm{Q}}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\mathrm{Q}}^{\mathrm{B}} \int_{\boldsymbol{k}} f_{\mathrm{B}}(\boldsymbol{k})\varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right)$$

Q = u, d, s

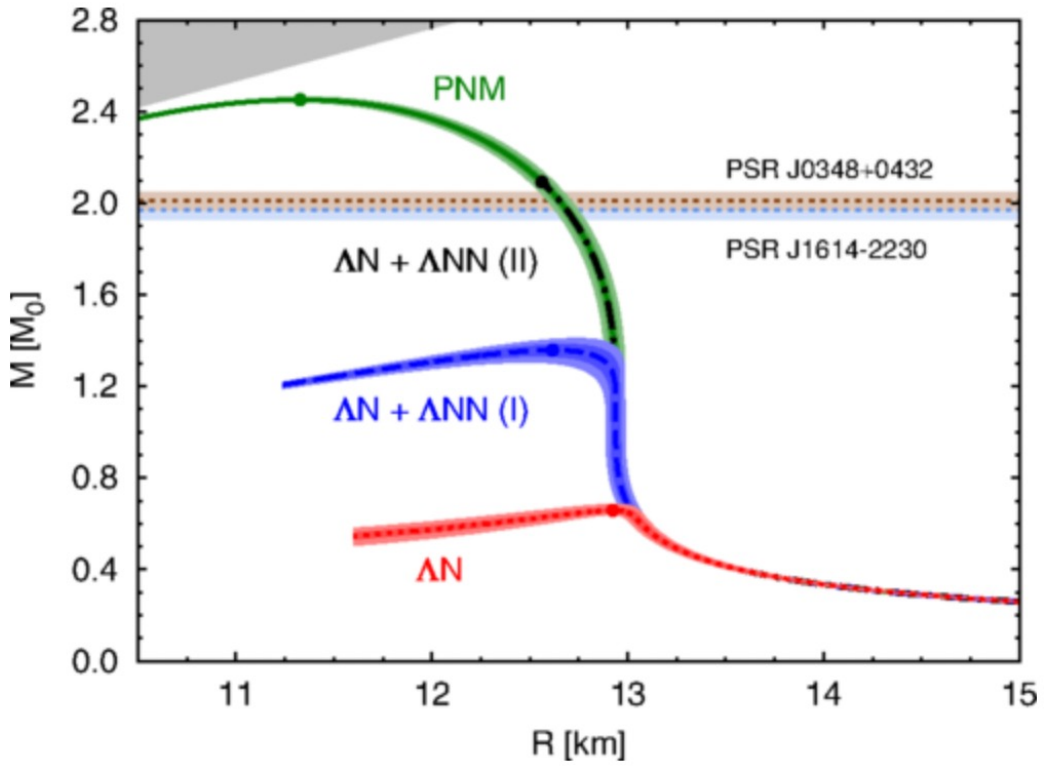


and so on...

saturation constraints

flavor correlations

Hyperon Puzzle ?



<u>the emergence of hyperons (or Δ,...):</u>

- adds large energy but small pressure
- many species; 3Σ, 2Ξ, Λ (octet),...

➔ softening of EOS (at 2-3n_0)

<u>often used approach to pass 2Msun:</u>

introduce strong YN, YY, YNN,.. repulsion

... convergence or regularity??

<u>alternative idea:</u>

quark saturation ➔

1) statistical repulsion (Pauli blocking)

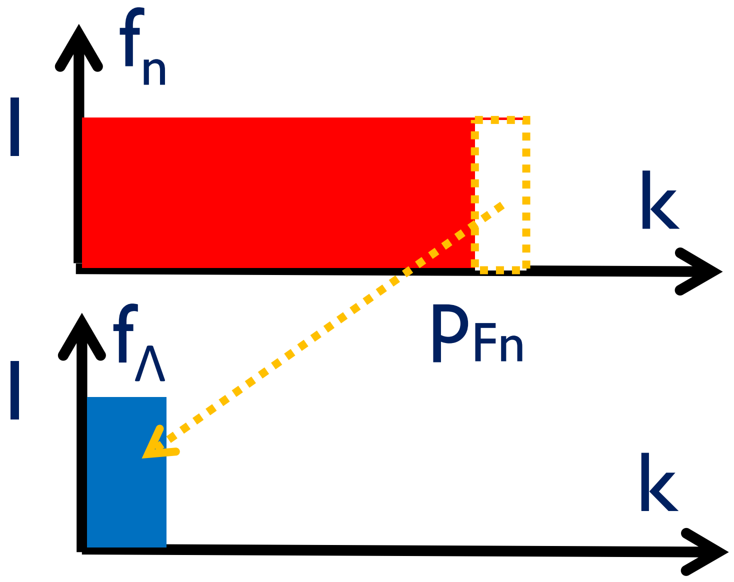
2) stronger repulsion at higher density

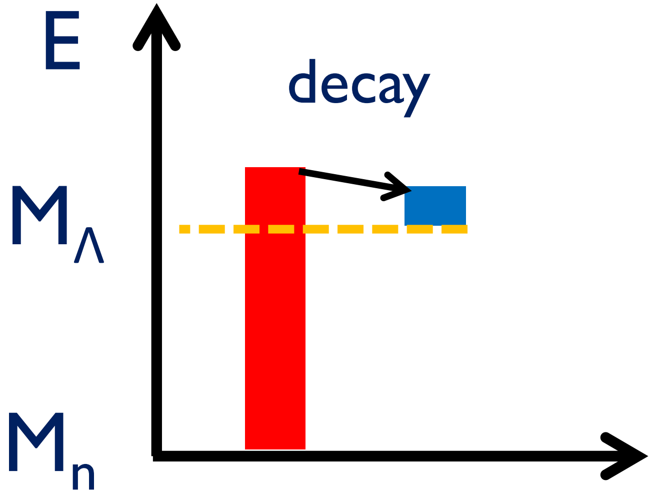
3) no double counting of quarks

n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

· **conventional**





at $n_B \sim 2\text{-}3n_0$

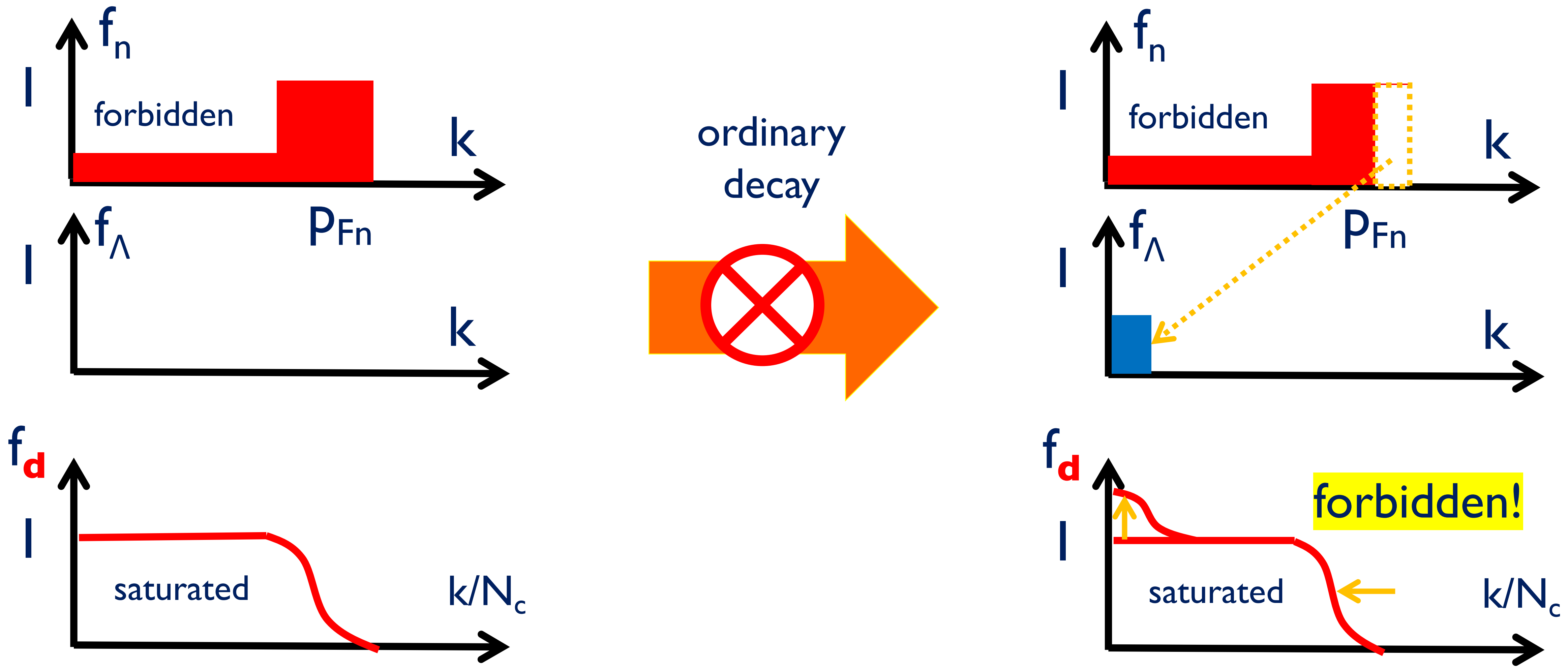
neutrons at high momenta → *non-relativistic* Λ

ε increases much but pressure does not (**softening**)

n-Λ_0 matter

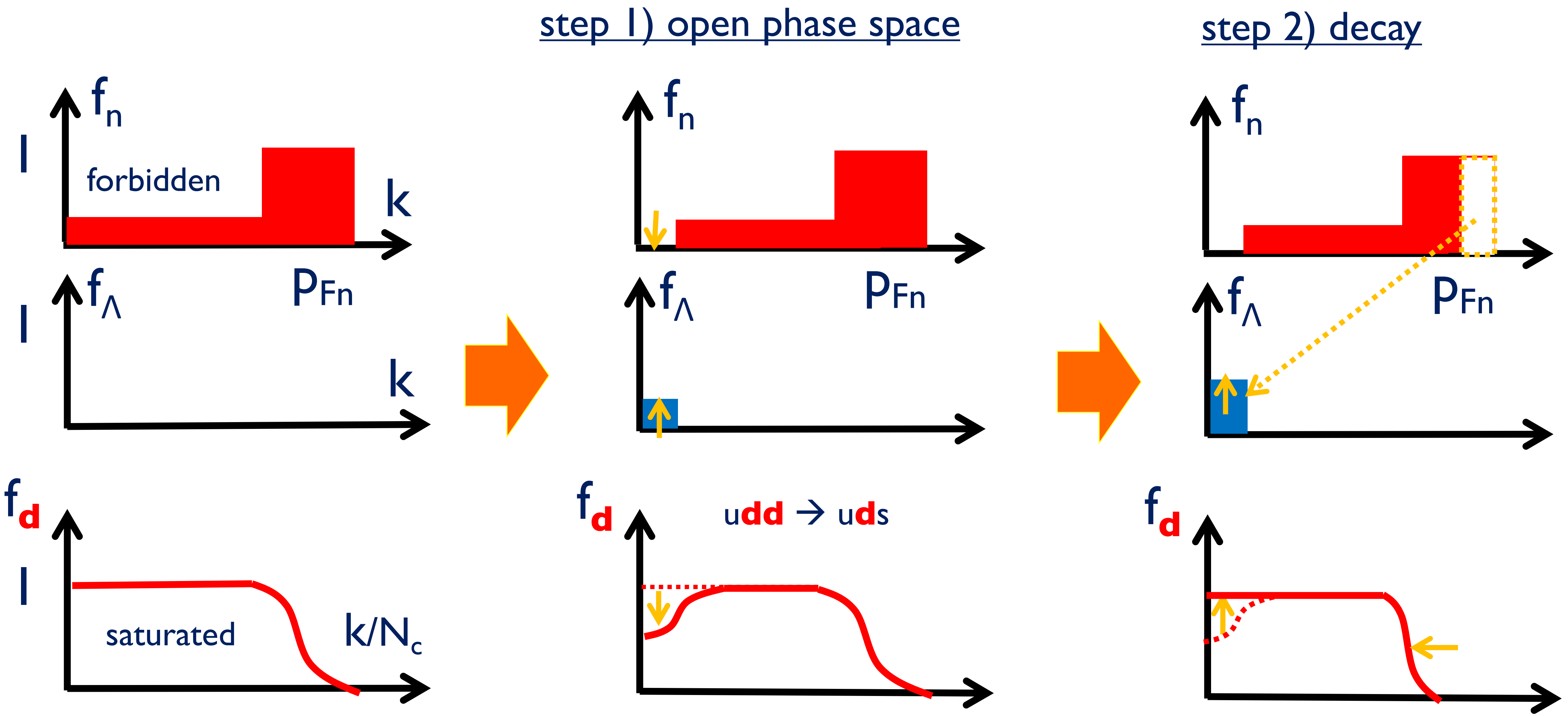
[Fujimoto-TK-McLerran, '24]

· **with d-quark saturation**



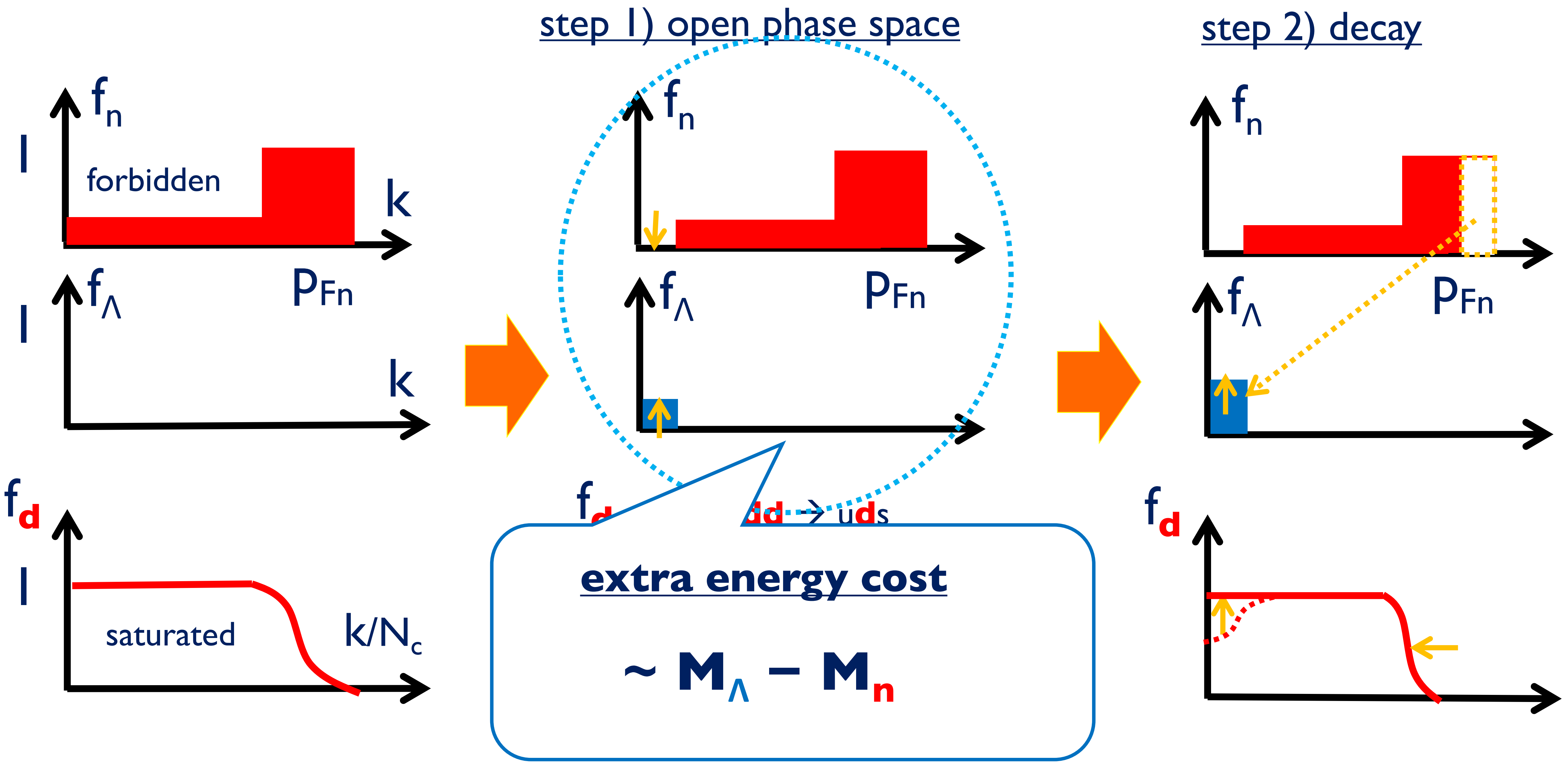
n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]



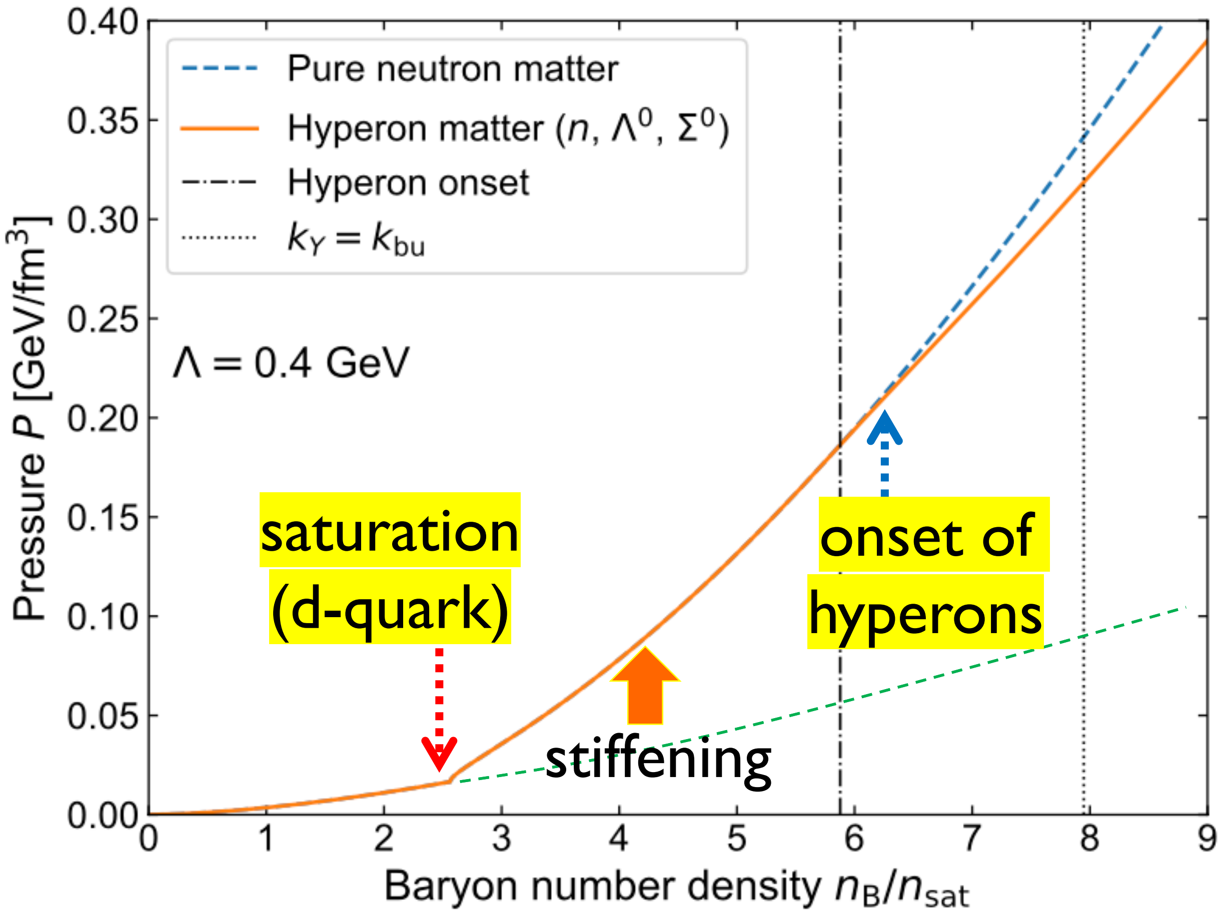
n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]



EOS: n-Λ matter

[Fujimoto-TK-McLerran, '24]



$n_B^{\Lambda\text{-onset}} \sim 2n_0 \rightarrow$ 5-6n_0

conventional   with saturation

Summary & Outlook

- revolutionary NS observations in the last decade, more will come
- interplay btw nuclear & quark physics; more important than ever
- quark substructures of hadrons directly impact NS physics
- interactions (not discussed today)

baryon-baryon int. **in vac.** => lattice QCD, scattering exp. (J-PARC, ALICE)

baryon-baryon int. **in medium** => hypernuclear exp. (J-PARC, J-Lab, Mainz,...)

A lot to be done

Back Up

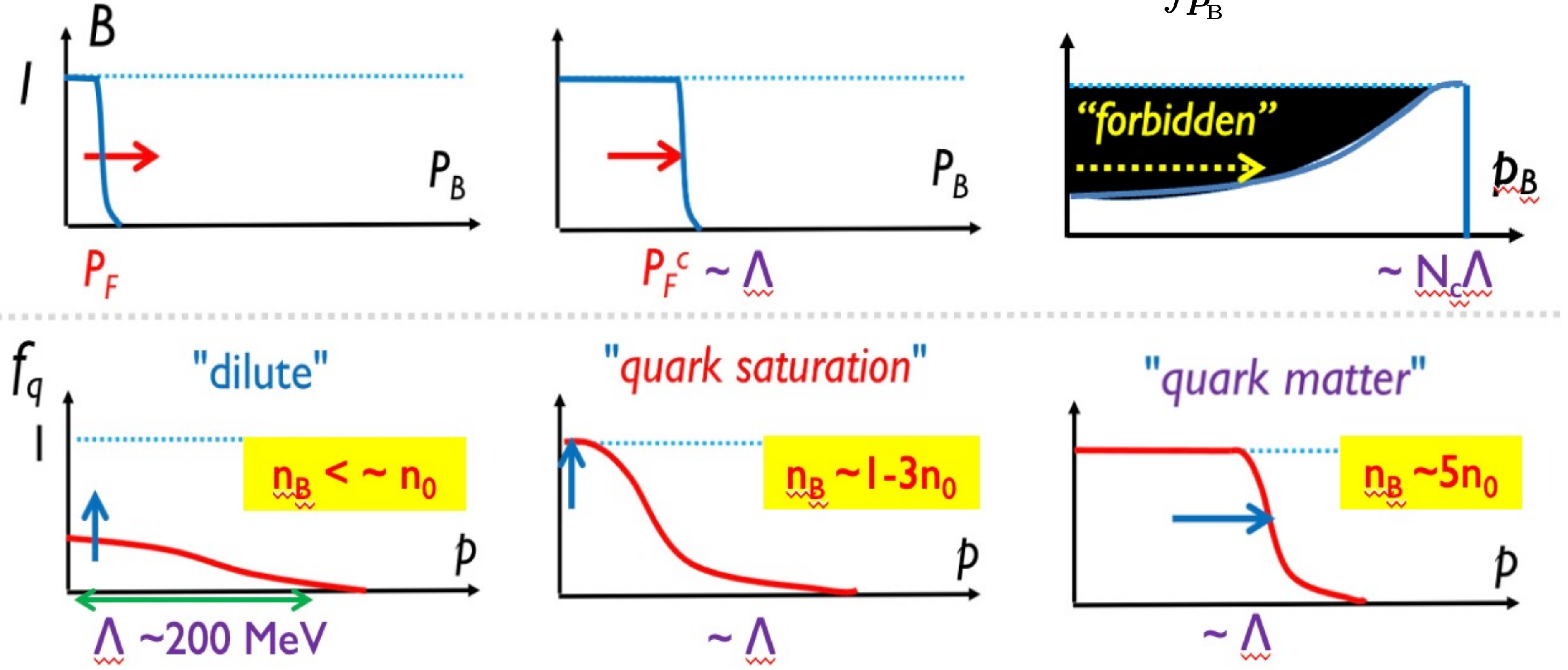
Evolution of occ. probabilities

$$f_{\mathrm{Q}}(\boldsymbol{q}; n_{\mathrm{B}}) = \int_{\boldsymbol{P}_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{P}_{\mathrm{B}}; n_{\mathrm{B}}) \varphi_{\mathrm{Q}}^{\mathrm{B}}(\boldsymbol{q}; \boldsymbol{P}_{\mathrm{B}})$$

baryon bases

dual

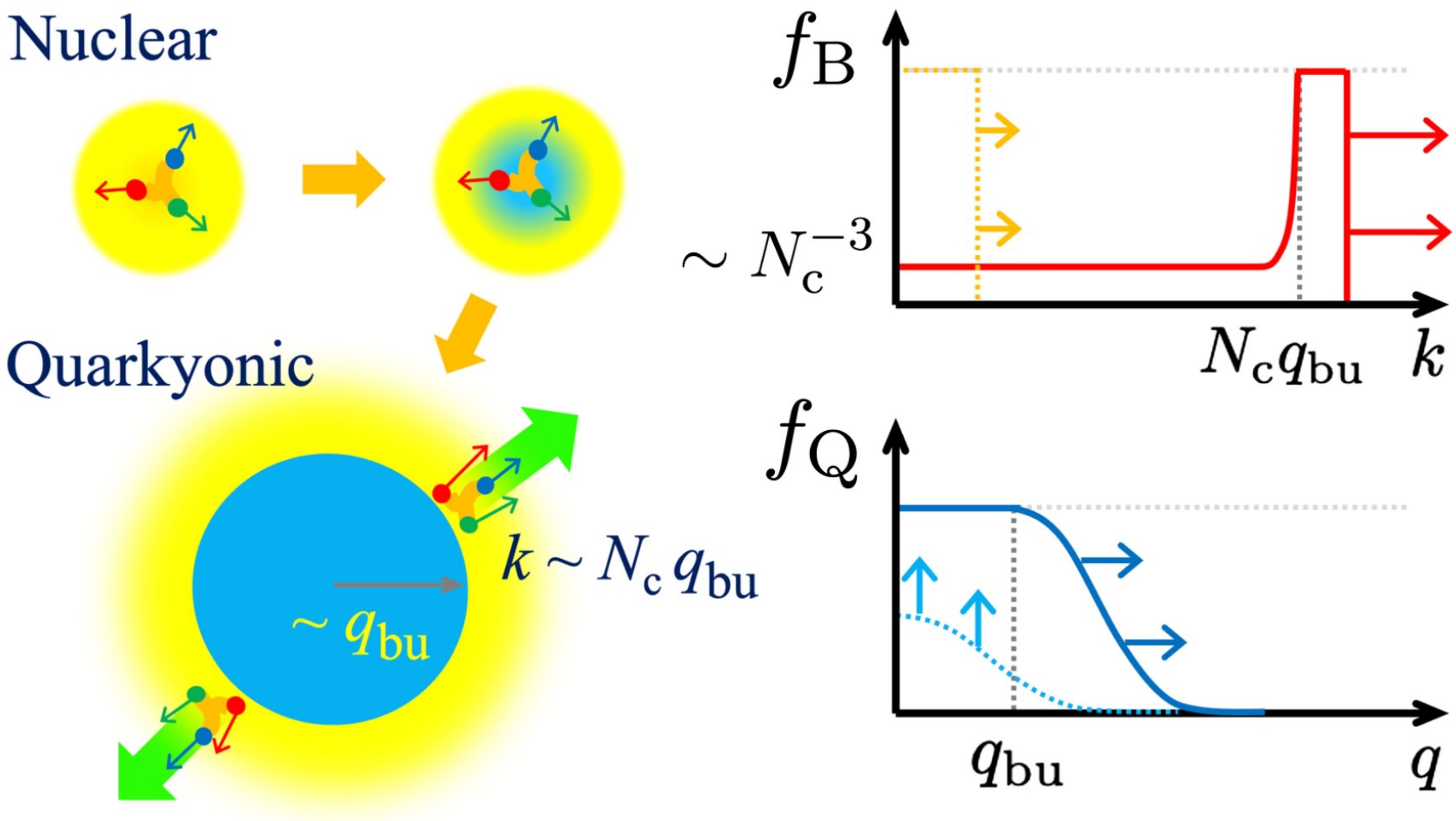
quark bases



"**quark saturation**" constraint

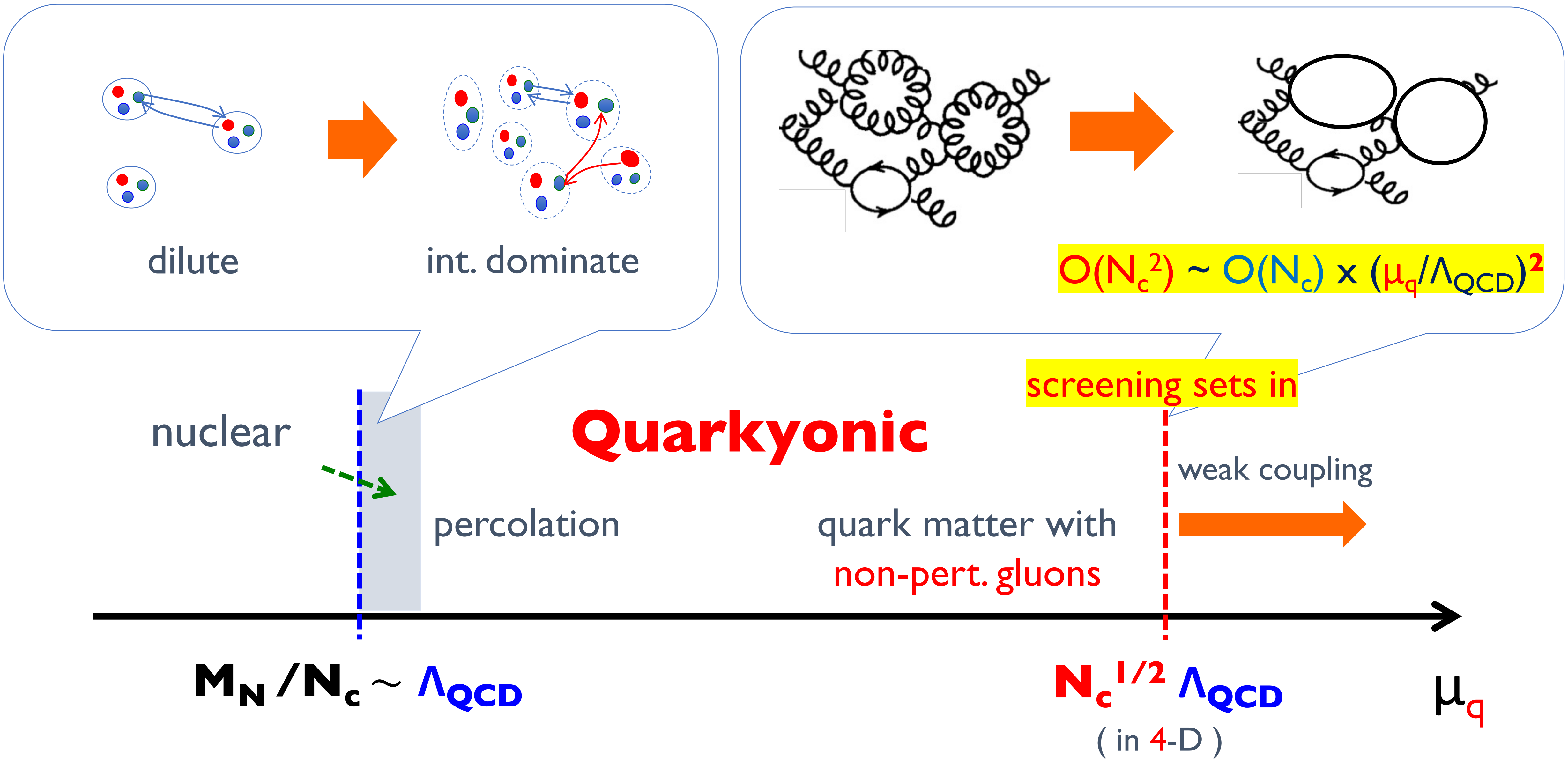
→ **relativistic baryons at low density, $n_B \sim 1\text{-}3n_0$!**

cf) McLerran-Reddy model ('19); microscopic description, TK ('21), Fujimoto+ ('24)



Nuclear
Quarkyonic
$\sim q_{\rm bu}$
$k \sim N_{\rm c}\, q_{\rm bu}$
$f_{\rm B}$
$\sim N_{\rm c}^{-3}$
$N_{\rm c} q_{\rm bu}$
k
$f_{\rm Q}$
$q_{\rm bu}$
q

McLerran-Pisarski's "two-scale picture" [McLerran-Pisarski '07]

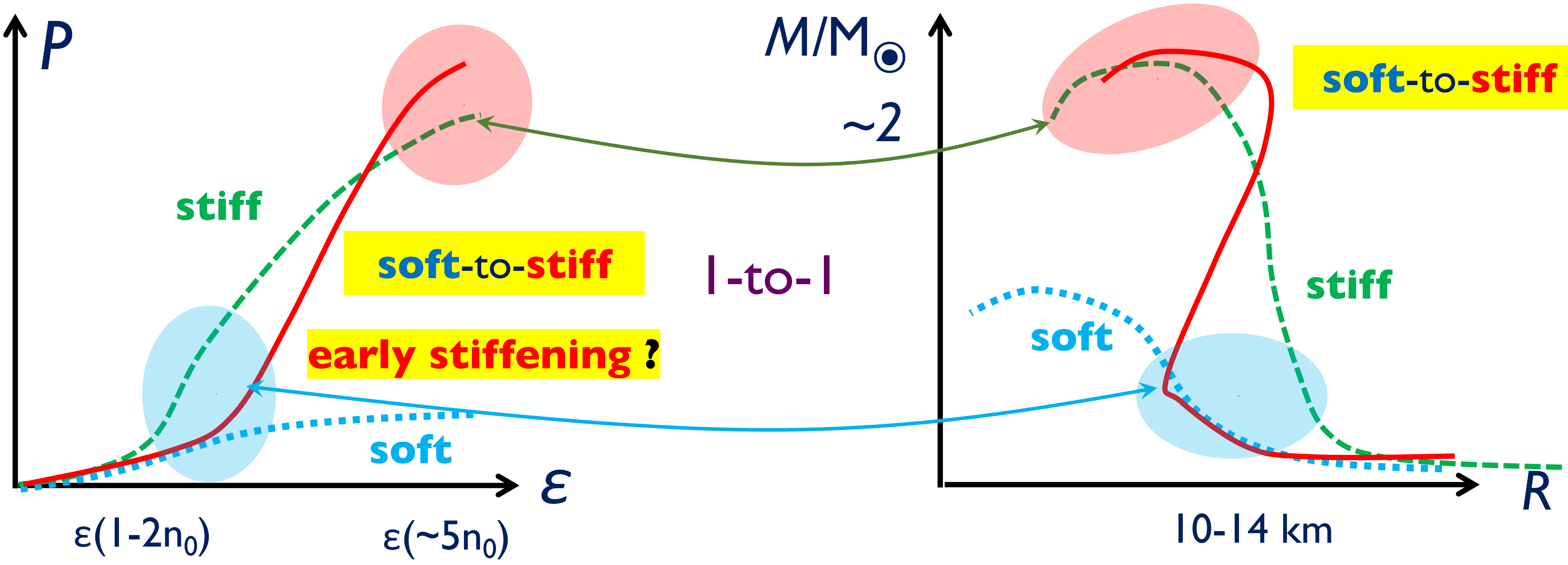


EoS stiffness & M-R

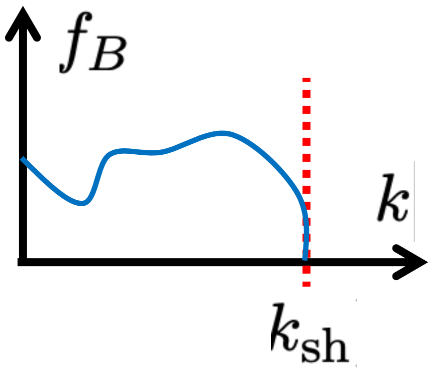
Ref) Lattimer & Prakash (2001)

soft-to-stiff EOS

e.g.) quark-hadron-crossover (QHC)



Number-conserving energy variation



$$\delta\varepsilon_{\rm B} = E_B(\mathbf{k})\delta f_B(\mathbf{k}) + E_B(\mathbf{k}_{\rm sh})\underline{\delta f_B(\mathbf{k}_{\rm sh})}$$

$$= \left[E_B(\mathbf{k}) - \underbrace{E_B(\mathbf{k}_{\rm sh})}_{\equiv \lambda_{\rm B}}\right]\delta f_B(\mathbf{k})$$

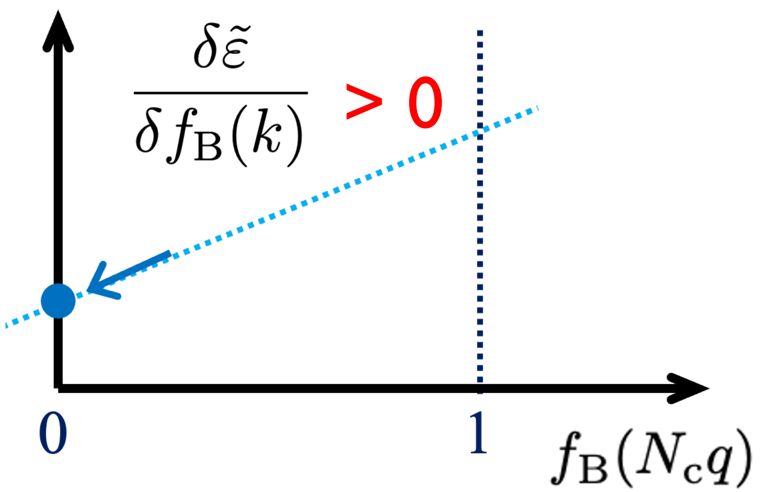
$$\delta f_B(\mathbf{k}) + \underline{\delta f_B(\mathbf{k}_{\rm sh})} = 0$$

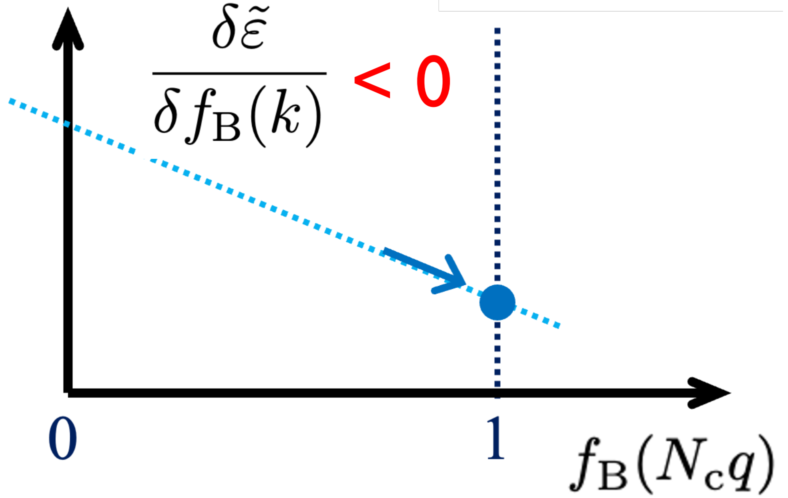
$E_B(k) > \lambda_B$

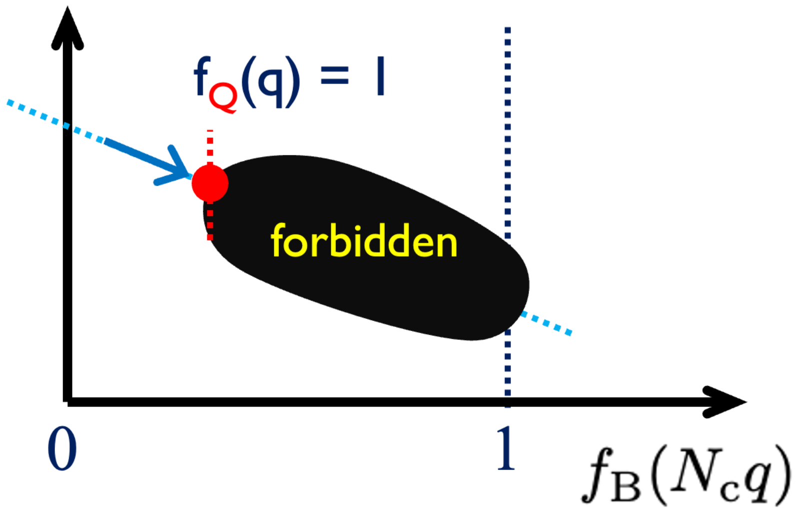
$E_B(k) < \lambda_B$

w.o. saturation

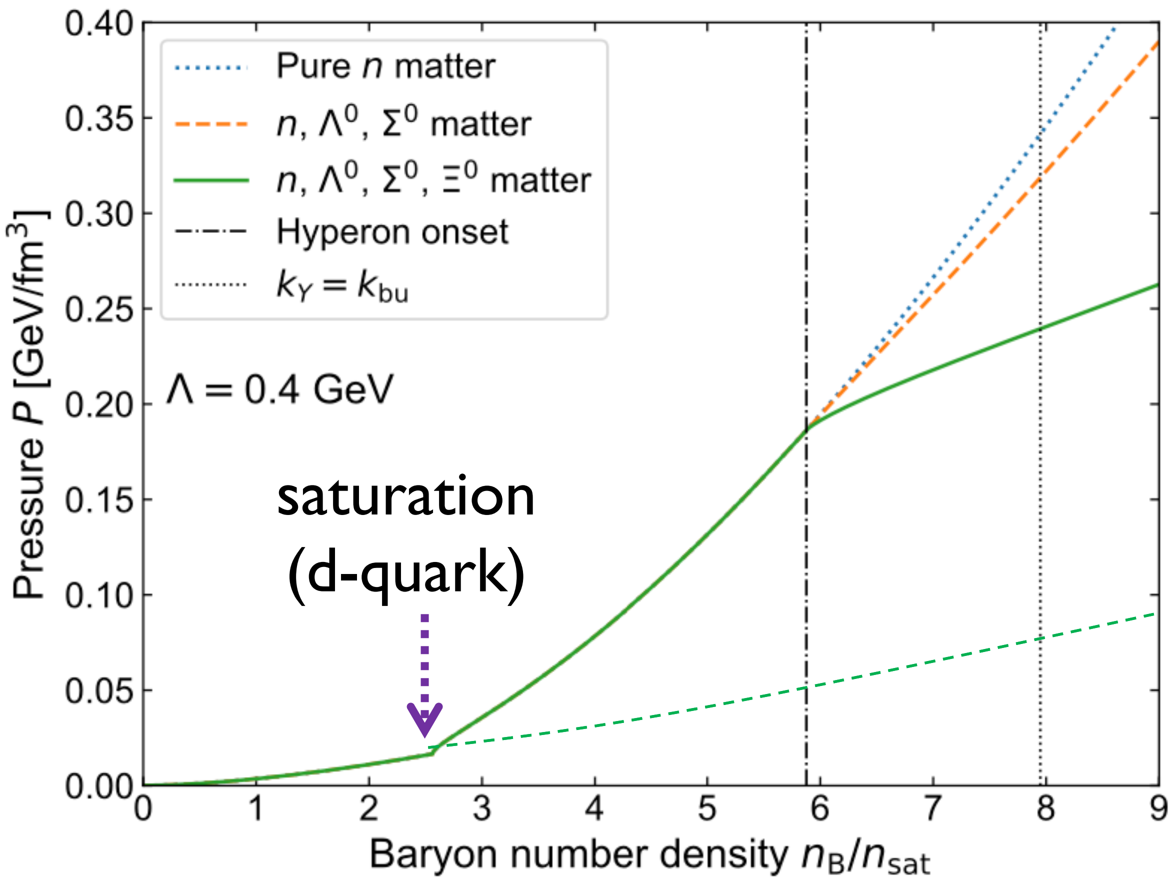
with saturation







EOS: n-Σ_0-Λ + Ξ_0 matter



Ξ_0 (uss) → free from d-quark sat.

$\mu_B^{\Xi\text{-onset}} = M_\Xi$ (as usual)

softening occurs,
but only at high density

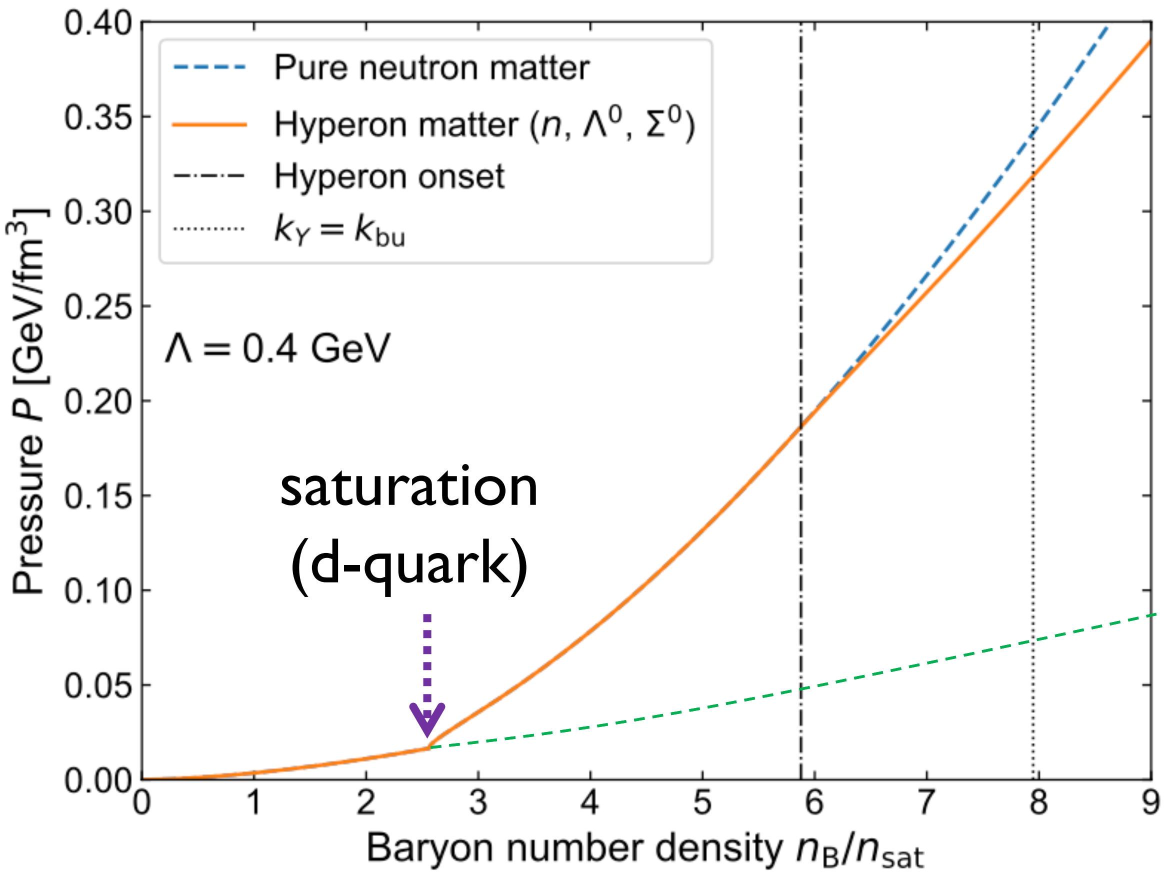
perhaps soon goes back to
quark scaling through u- or s- sat

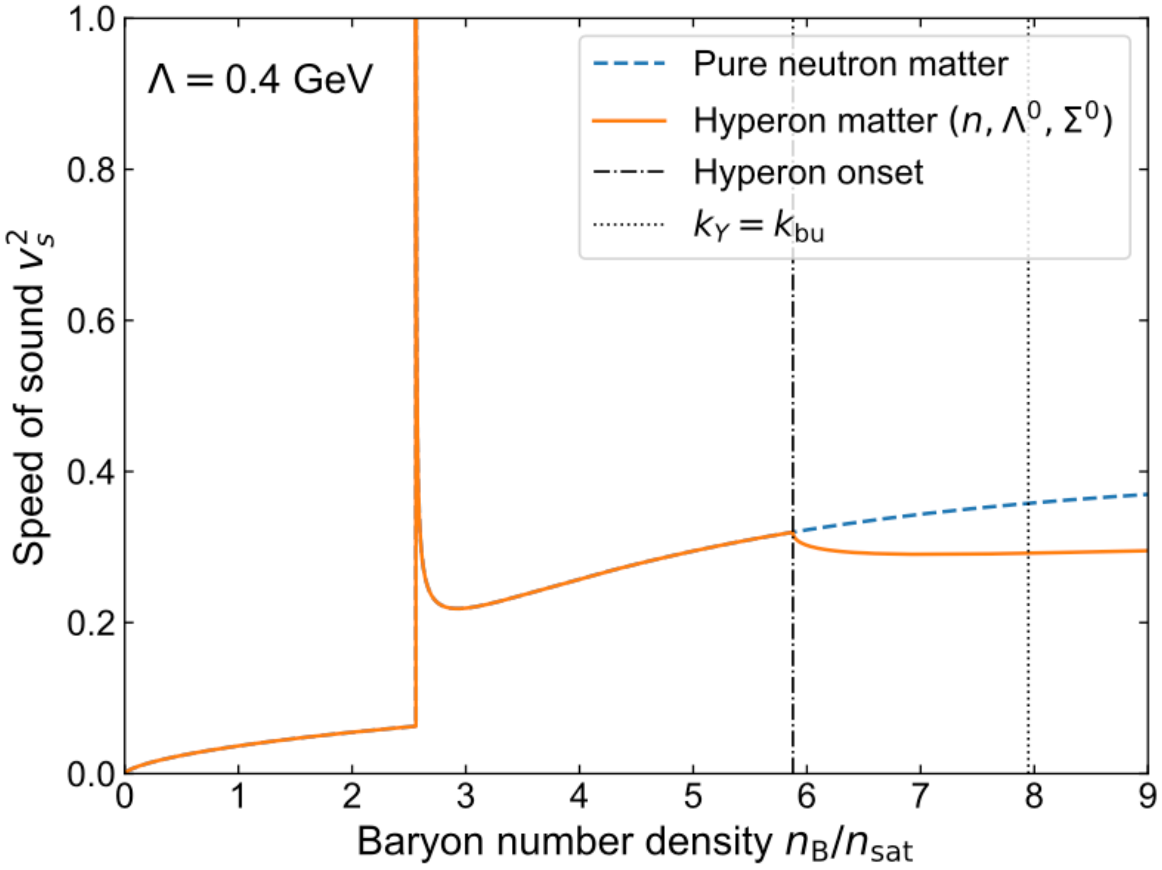
(to be studied)

$n_B^{Y,\,\Xi\text{-onset}} \sim 6n_0$

EOS: n-Σ_0-Λ matter

[Fujimoto-TK-McLerran, '24]





$n_B^{\text{Y-onset}} \sim 6n_0$

What physics can cause such rapid stiffening?

baryonic matter scenario

mass E | kin. E | N-body forces

$$\varepsilon_B \sim M_B n_B + \frac{n_B^{5/3}}{M_B} + V_0 n_B^N$$

large | small

kin. E | N-body forces

$$P \sim \frac{n_B^{5/3}}{M_B} + (N-1) V_0 n_B^N$$

mass E does not contribute | small

baryonic pressure entirely comes from N-body forces...

Q) the growth of stiffening is too slow ???

many-body expansion converges ???

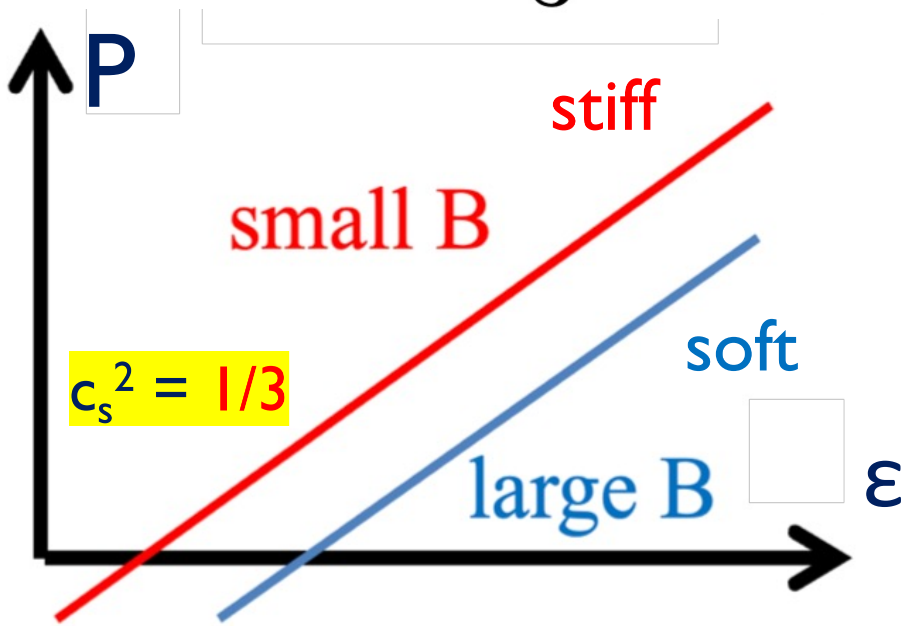
interactions dominate --- proper d.o.f ???

stiff quark matter EOS?

e.g.) free massless quarks

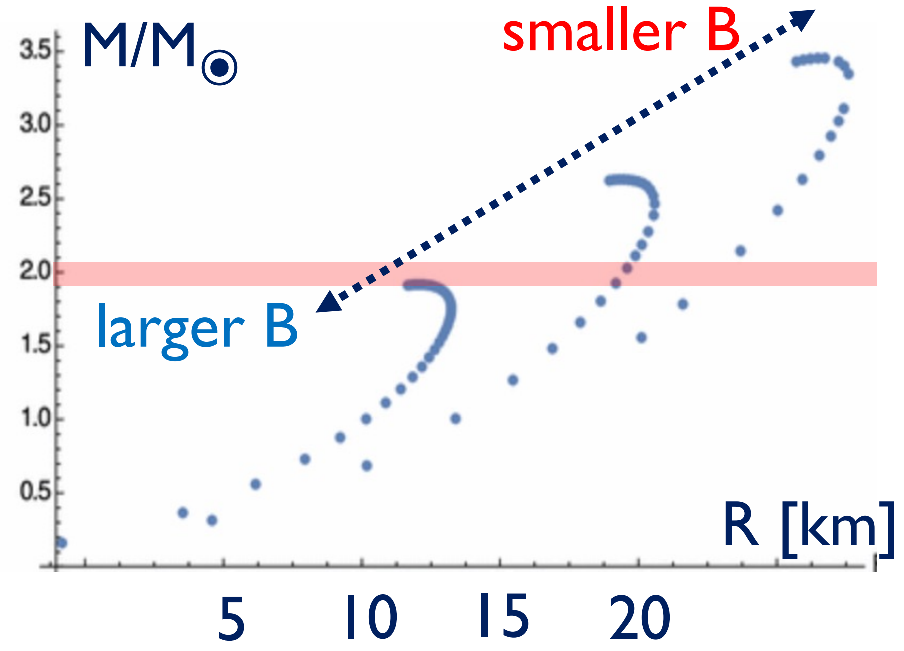
normalization

$$P = \frac{\varepsilon}{3} - B'$$



quark kin. E >> baryon kin. E

O(**N$_c$**) O(1/**N$_c$**)



sensible starting point (?!)

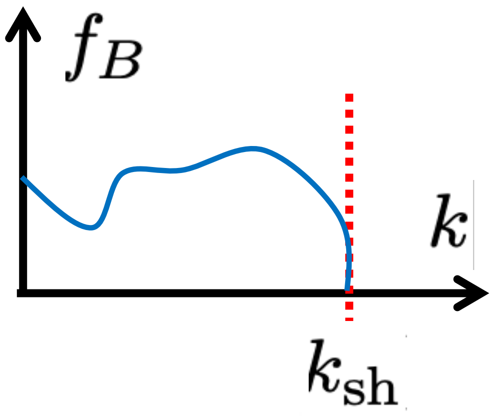
But confinement is totally neglected at low n_B ....

All we have to do

minimize

$$\varepsilon_{\rm B}[f_{\rm B}] = 4 \int_k E_{\rm B}(k) \underline{f_{\rm B}(k)}$$

variables



with constraints

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B) \varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

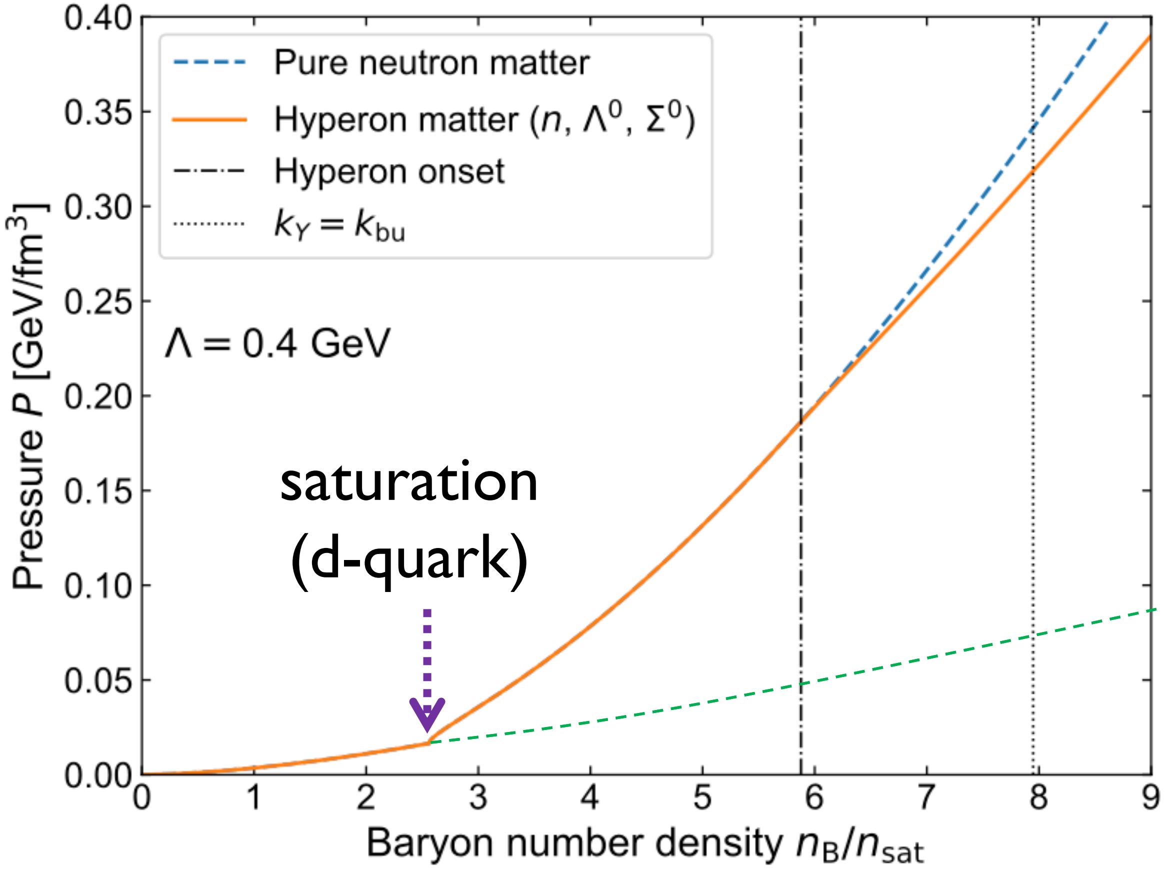
$$0 \leq f_{\rm B}(k) \leq 1 \qquad 0 \leq f_{\rm Q}(q) \leq 1$$

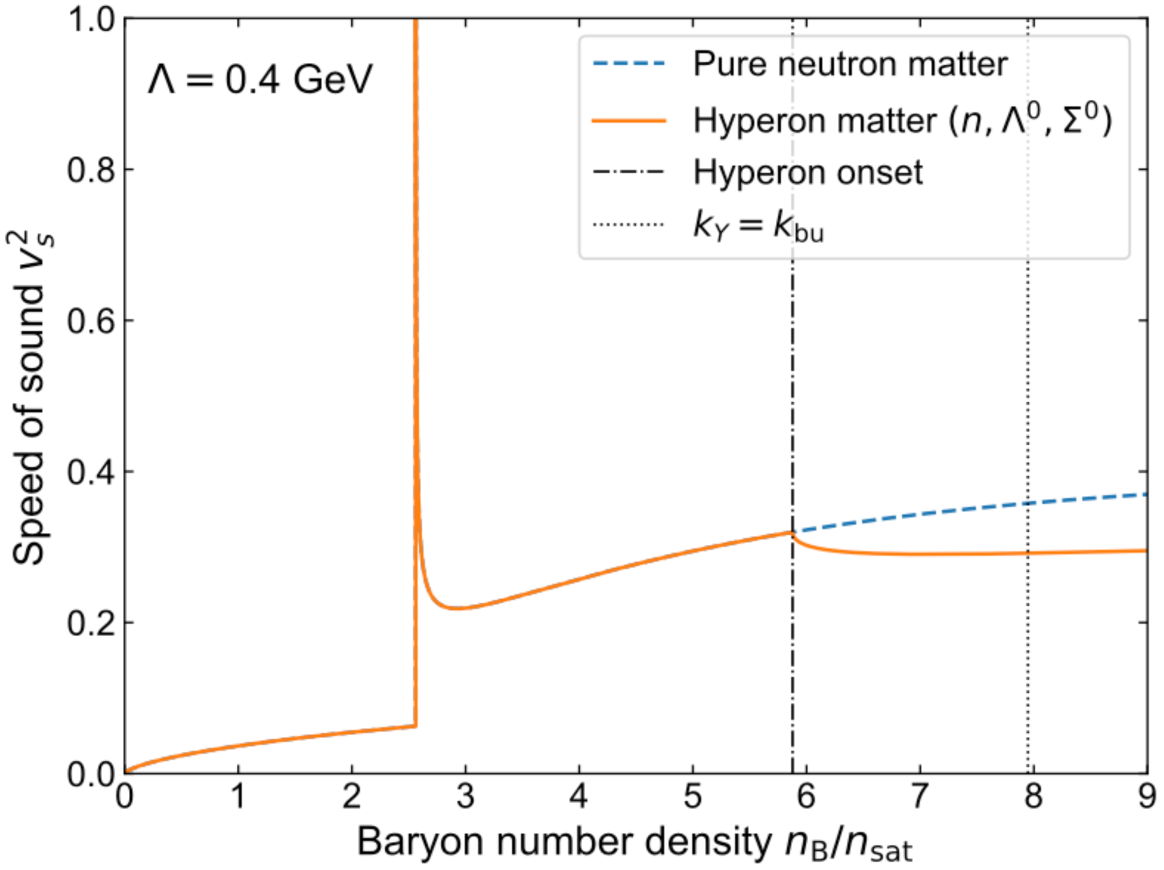
at fixed n_B

simple, but highly nontrivial problem

EOS: n-Σ_0-Λ matter

[Fujimoto-TK-McLerran, '24]





$n_B^{\text{Y-onset}} \sim 6n_0$

The condition for n(k) → Y(k) conversion

$n(k) \to Y(k)$: $E_Y(k) - E_N(k)$ energy **cost**

$n(k_{sh}) \to Y(k)$: $E_Y(k) - E_N(k_{sh})$ energy **gain**

update k_{sh}, k_{bu} : ΔE_{shell} energy **gain**

Total change in energy : $2E_Y(k) - E_N(k) \underbrace{- E_N(k_{sh}) + \Delta E_{shell}}_{(= -\mu_n = -\mu_B)}$

If negative, the domain of momentum k is saturated by Y

Onset of hyperons

$$E_{conversion}(k) = 2E_Y(k) - E_N(k) - \mu_B$$

The $n \to Y$ conversion at k is favored for $E_{conversion}(k) < 0$.

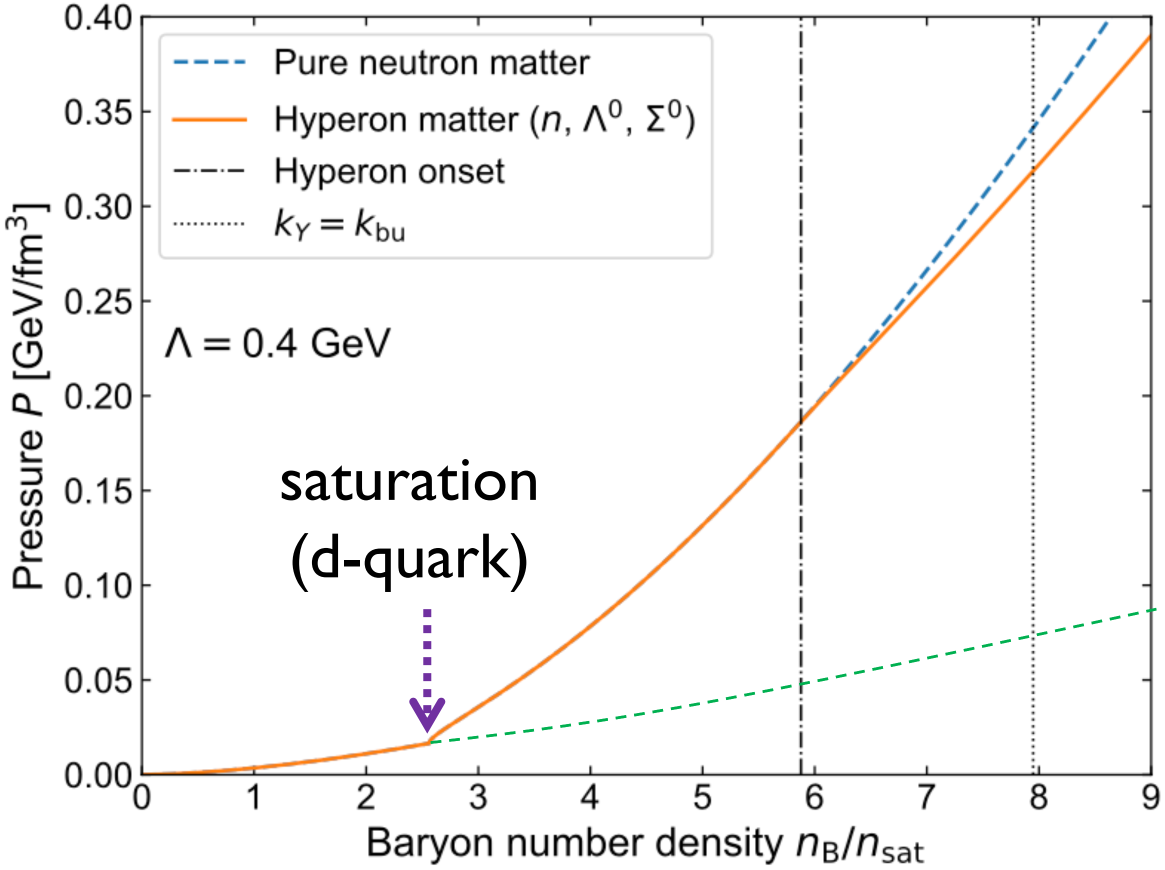
Hyperons begin to appear at $k = 0$,
so the minimal μ_B for the conversion is

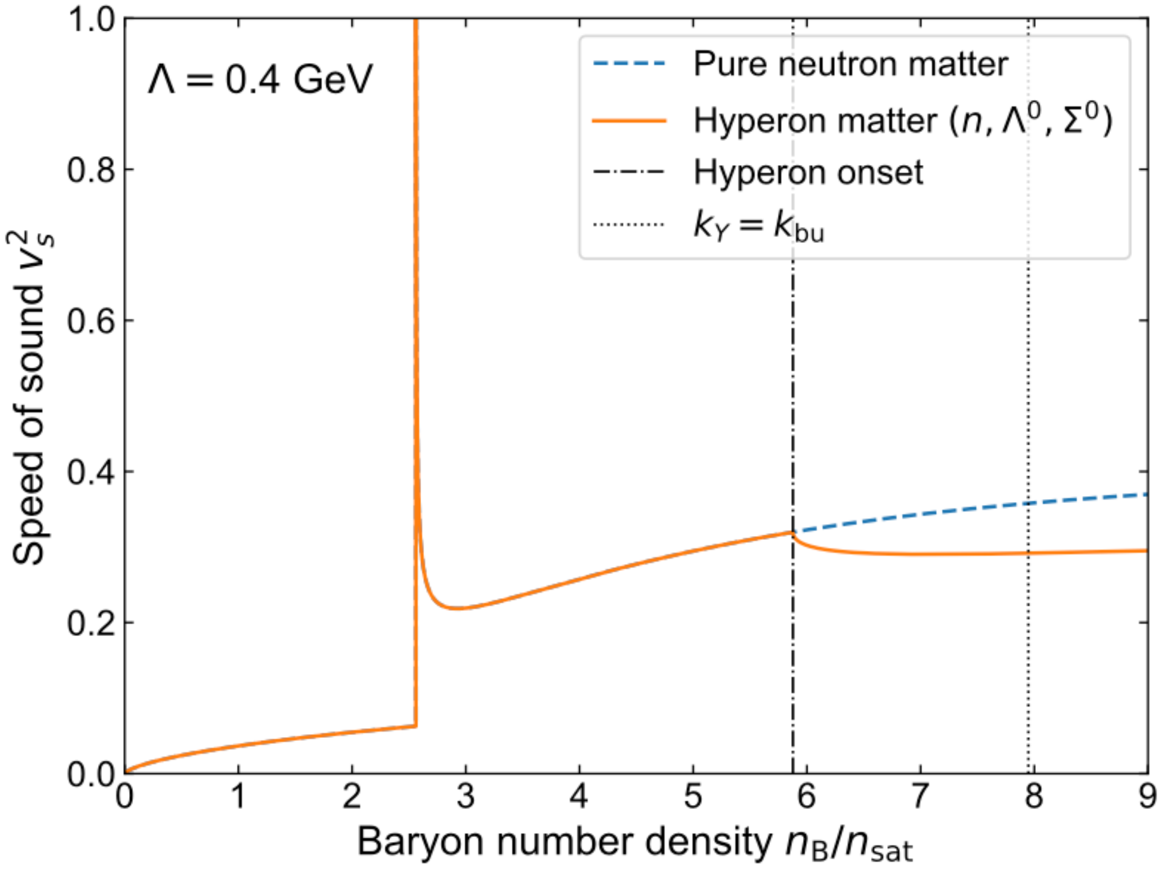
$$\mu_B^{onset} = 2M_Y - M_N$$

bigger than
the conventional: $\sim M_Y$

$$\sim 2(2M_{u,d} + M_s) - 3M_{u,d} = M_{u,d} + 2M_s \sim M_{\Xi\,(uss)}$$

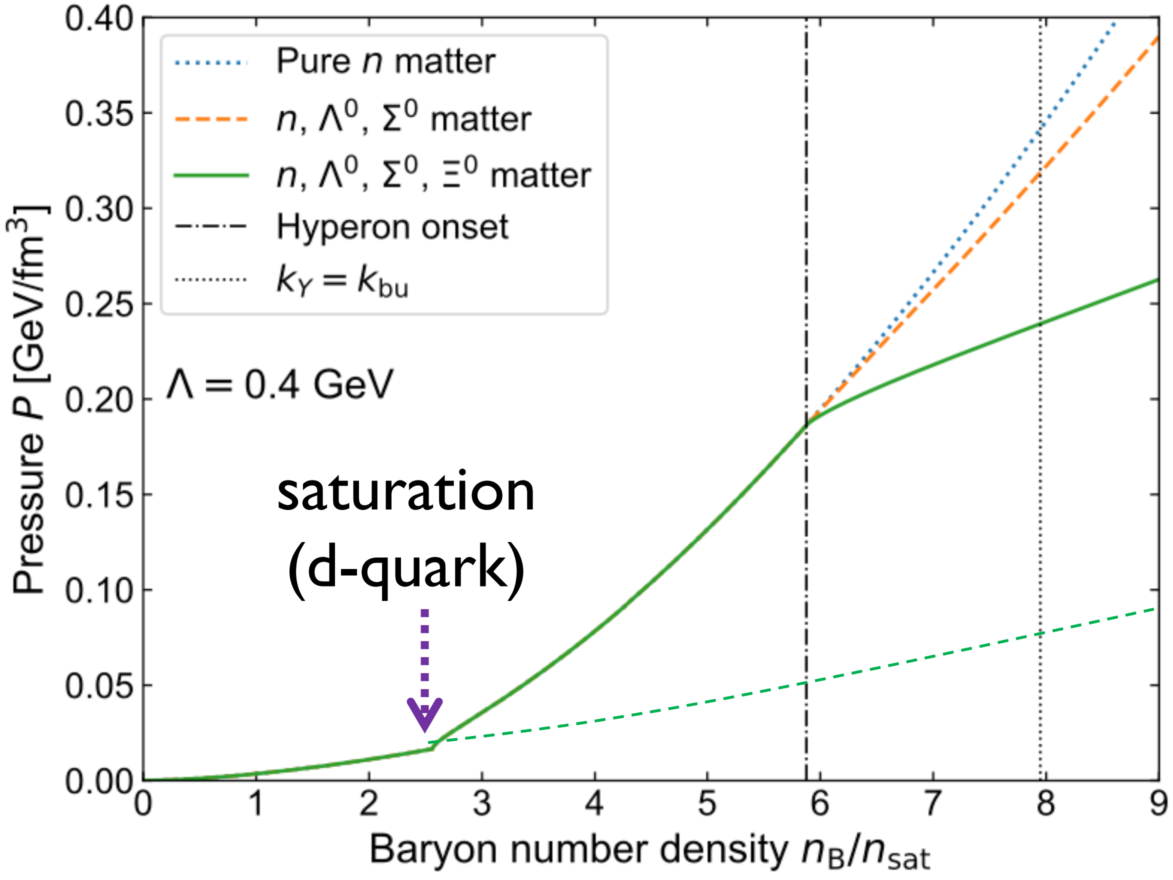
EOS: n-Σ_0-Λ matter





$n_B^{\text{Y-onset}} \sim 6n_0$

EOS: n-Σ_0-Λ + Ξ_0 matter



Ξ_0 (uss) → free from d-quark sat.

$\mu_B^{\Xi\text{-onset}} = M_\Xi$ (as usual)

softening occurs,
but only at high density

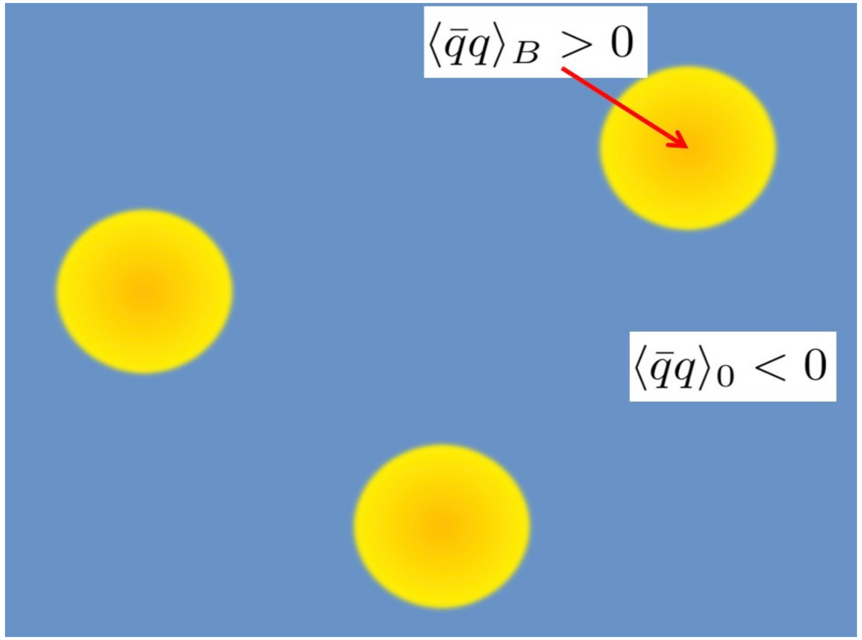
perhaps soon goes back to
quark scaling through u- or s- sat

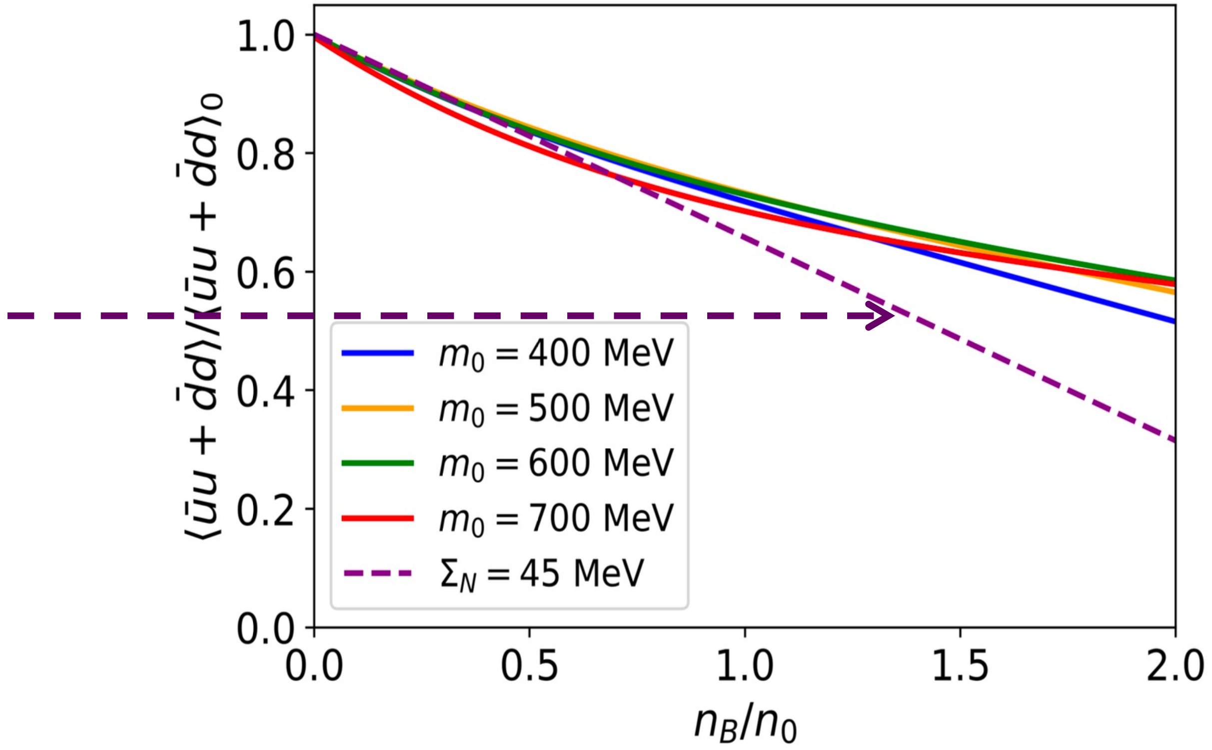
(to be studied)

$n_B^{Y,\,\Xi\text{-onset}} \sim 6n_0$

Chiral symmetry restoration in dense matter

reduction of condensates is inevitable





$M_N \propto g_Y \sigma$??

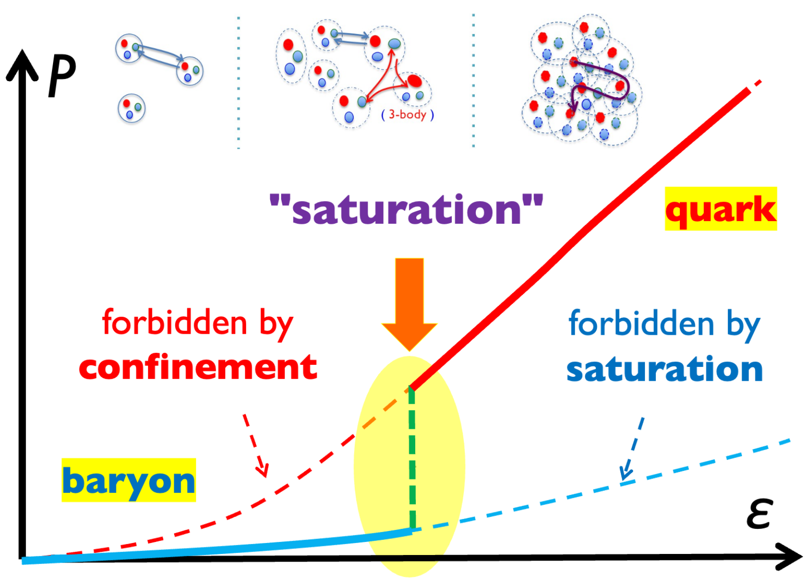
reduction of M_N by ~30% at n_0?

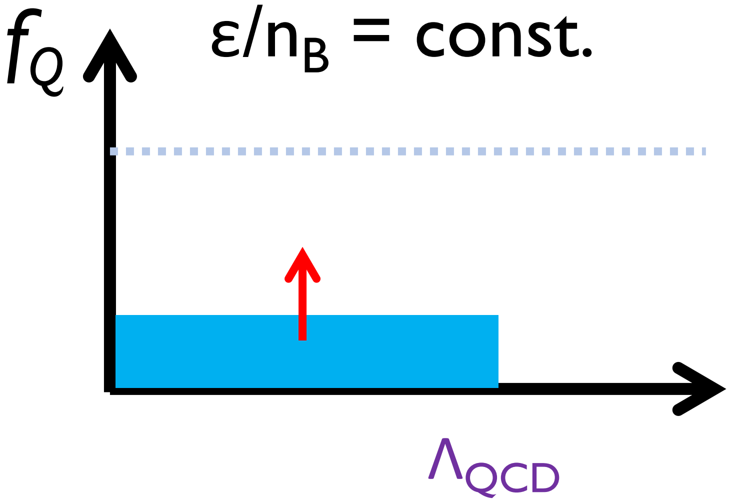
Stiffening in quark picture

(very schematic)

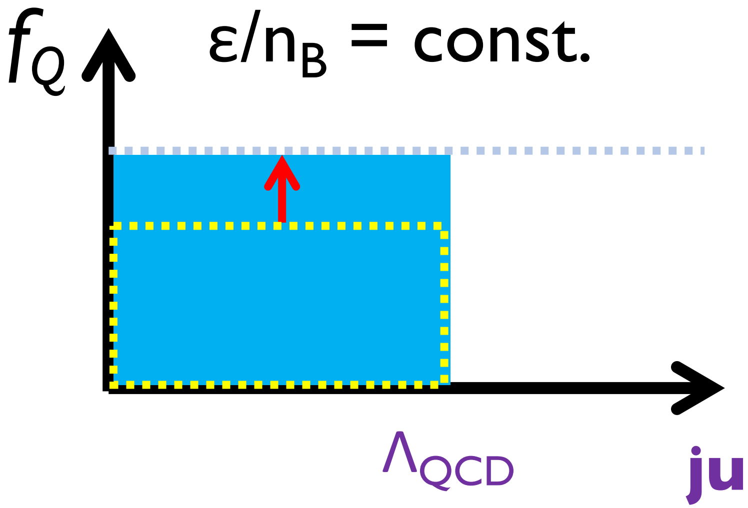
$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle



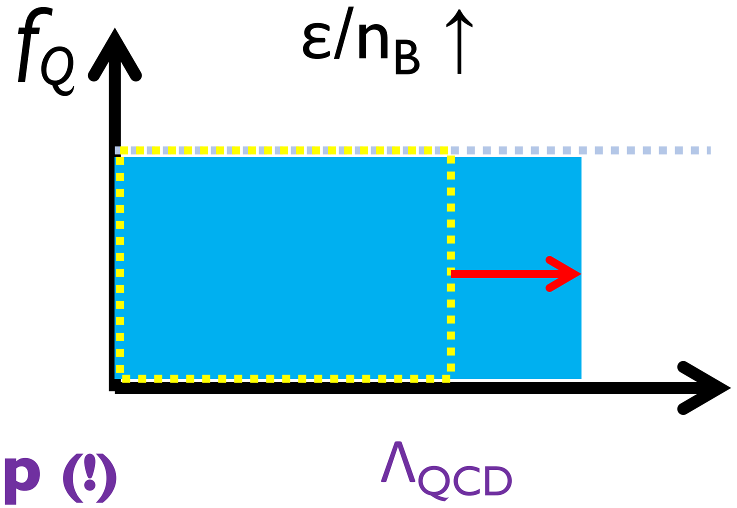


P = **0**



P = **0**

jump (!)

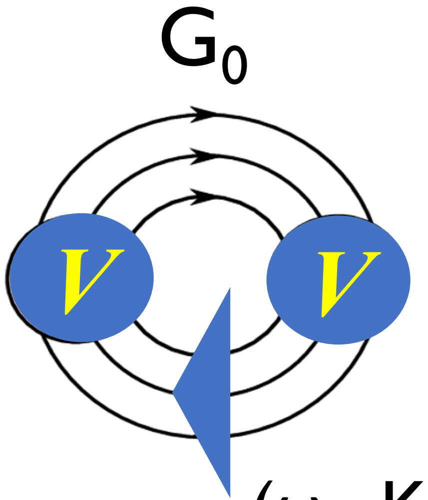


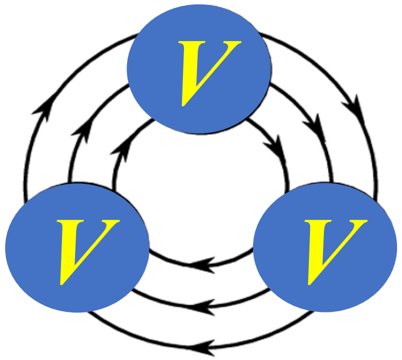
P = **finite**

Phase shift representation of thermodynamics and baryon- & quark-distributions

Tajima-Iida-TK-Liang, arXiv: 2412.04971

Thermodynamics of composite particles

$\Omega_{3\text{-body}}$ = 1/2 

+ 1/3 

+ ....

total frequency & momentum (ω$_n$, K)

$$= -T \sum_{\omega_n,\mathbf{K}} \mathrm{tr}_N \left[\mathrm{Ln}(1 - G_0 V) + G_0 V \right]$$

$$-V = G^{-1} - G_0^{-1}$$

$$= T \sum_{\omega_n,\mathbf{K}} \underline{\mathrm{tr}_N} \left[\mathrm{Ln}(G/G_0) - G_0 G^{-1} \right] + \mathrm{const.}$$

"trace" over all possible states with (ω$_n$, K)

Thermodynamics of composite particles

$$\Omega_{\text{3-body}} = T \sum_{\omega_n, \mathbf{K}} \mathrm{tr}_N \left[\mathrm{Ln}(G/G_0) - G_0 G^{-1} \right]$$

contour deformation & integrate by part

$$= -T \sum_{\mathbf{K}} \int \frac{\mathrm{d}\omega}{\pi} \ln\left(1 + \mathrm{e}^{-\beta\omega}\right) \frac{\partial}{\partial\omega} \mathrm{tr}_N \left(\mathrm{Im}\left[\mathrm{Ln}(G/G_0) - G_0 G^{-1} \right] \right)$$

define: $G/G_0 = \left| G/G_0 \right| \mathrm{e}^{\mathrm{i}\varphi}$

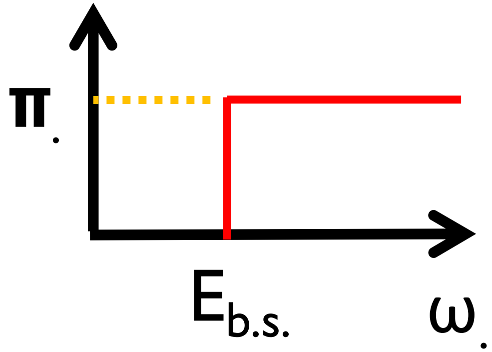
$$= -T \sum_{\mathbf{K}} \int \frac{\mathrm{d}\omega}{\pi} \ln\left(1 + \mathrm{e}^{-\beta\omega}\right) \frac{\partial}{\partial\omega} \mathrm{tr}_N \left[\varphi + |G_0/G| \sin\varphi \right]$$

phase shift representation of thermo. potential

Hadron Resonance Gas

$$\Omega_{\text{N-body}} = -T\sum_{\mathbf{K}}\int\frac{\mathrm{d}\omega}{\pi}\ln\left(1+\mathrm{e}^{-\beta\omega}\right)\underline{\frac{\partial}{\partial\omega}\mathrm{tr}_N\left[\,\varphi+|G_0/G|\sin\varphi\,\right]}$$

e.g.) stable bound particles $\quad \varphi = \pi\Theta\big(\omega - E_{\text{b.s.}}(K)\big)$



$$= \pi\delta\left(\omega - E_{\text{b.s.}}\right)\left[\,1+\cancel{|G_0/G|}\cos\varphi\,\right]+\cancel{\sin\varphi}\,\frac{\partial|G_0/G|}{\partial\omega}$$

→ 0 (G→∞) $\qquad$ = 0

➔ HRG model: $\quad \Omega_{\text{N-body}} = -T\sum_{\mathbf{K}}\ln\left(1+\mathrm{e}^{-\beta E_{\text{b.s.}}}\right)$

Constraints on general theory

num. of states:

(int. cannot modify)

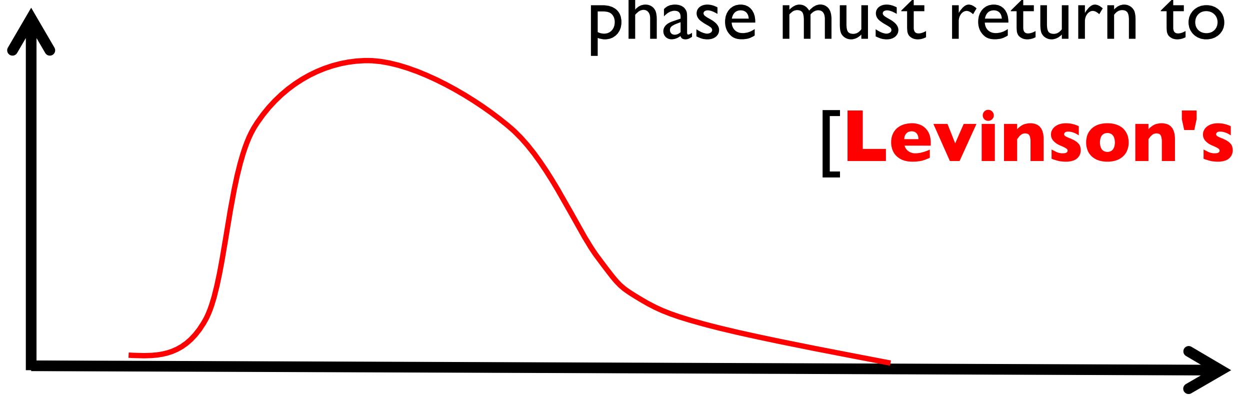
$$\int_\omega \mathrm{Im}\mathrm{Tr}G = \int_\omega \mathrm{Im}\mathrm{Tr}G_0$$

$$G = \frac{1}{\omega - H + \mathrm{i}\delta}$$

$$G_0 = \frac{1}{\omega - H_0 + \mathrm{i}\delta}$$

$$G = G\partial_\omega G^{-1} = \partial_\omega \ln G^{-1}$$

$$\rightarrow \quad 0 = -\int_\omega \mathrm{Im}\mathrm{Tr}\partial_\omega \ln\left(G/G_0\right) = \int_\omega \partial_\omega \mathrm{Tr}\varphi = \mathrm{Tr}\varphi(\infty) - \mathrm{Tr}\varphi(-\infty)$$

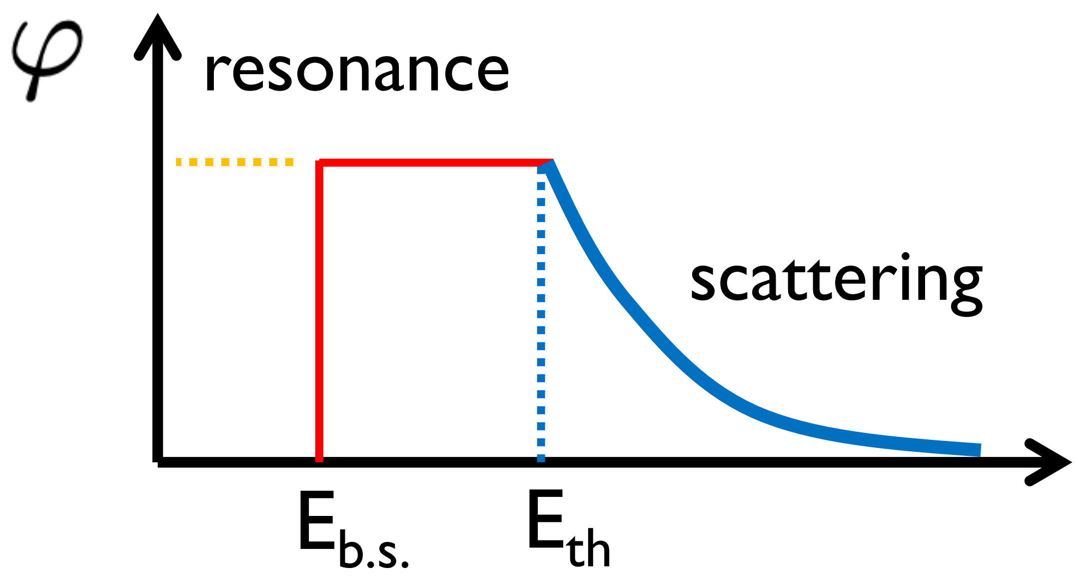


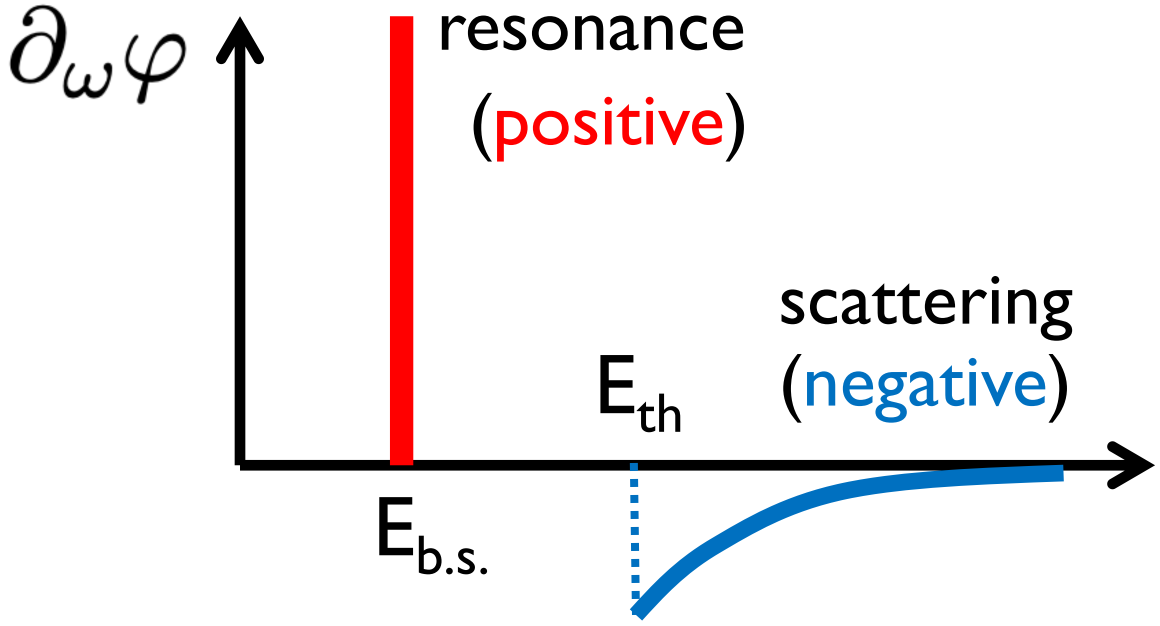
phase must return to the original value

[**Levinson's theorem**]

Hadron Resonance Gas with the decay

$$\Omega_{\text{N-body}} = -T \sum_{\mathbf{K}} \int \frac{d\omega}{\pi} \ln\left(1 + e^{-\beta\omega}\right) \frac{\partial}{\partial\omega} \mathrm{tr}_N \left[\varphi + |G_0/G| \sin\varphi \right]$$





resonance and scattering contributions tend to cancel

Saturating Ω by elementary particles

$$\Omega_{\text{N-body}} = -T \sum_{\mathbf{K}} \int \frac{\mathrm{d}\omega}{\pi} \ln\left(1 + \underbrace{\mathrm{e}^{-\beta\omega}}_{\text{thermal cutoff}}\right) \frac{\partial}{\partial\omega} \mathrm{tr}_N \left[\varphi + |G_0/G| \sin\varphi \right]$$

$$\Omega = \Omega_{q,g} + \Omega_{\text{meson}} + \Omega_{\text{baryon}} + \cdots \xrightarrow[\text{large T or } \mu]{} \Omega_{q,g}$$

At low T (or μ), the thermal factor **preferentially** pick up bound states.

→ **HRG**

At high T (or μ), the thermal factor **also** pick up scattering states,

then bound and scattering states **cancel**, leaving only small contributions.

Model study (1+1D)

$$H = \sum_{p,\alpha} \xi_p c^\dagger_{p,\alpha} c_{p,\alpha} + V \sum_P B^\dagger(P) B(P)$$ (B ~ qqq)

3-to-3 vertex yields a 3-body bound state

$$n_B = -\frac{\partial \Omega}{\partial \mu}$$

1-body 3-body

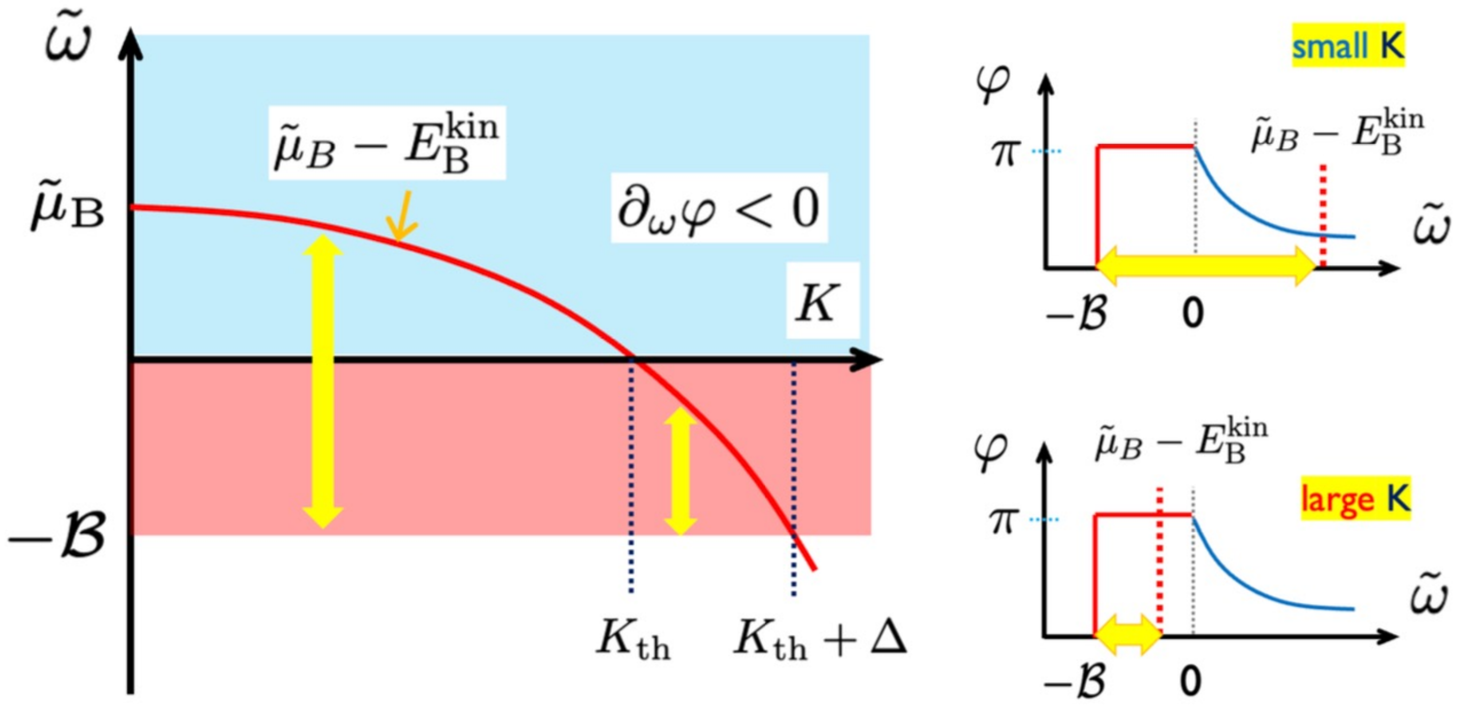
$$\Omega = \Omega_{\mathrm{q}} + \Omega_{\mathrm{b}}$$

$$n_B^{\mathrm{q}} \equiv \sum_{\mathbf{p}} f_q(\mathbf{p})$$

$$n_B^{\mathrm{b}} \equiv \sum_{\mathbf{K}} f_b(\mathbf{K})$$

Phase shift at each momentum

$$\Omega_{\text{N-body}} = -T\sum_{\mathbf{K}}\int\frac{d\omega}{\pi}\ln\left(1+e^{-\beta\omega}\right)\frac{\partial}{\partial\omega}\mathrm{tr}_N\left[\varphi+|G_0/G|\sin\varphi\right]$$

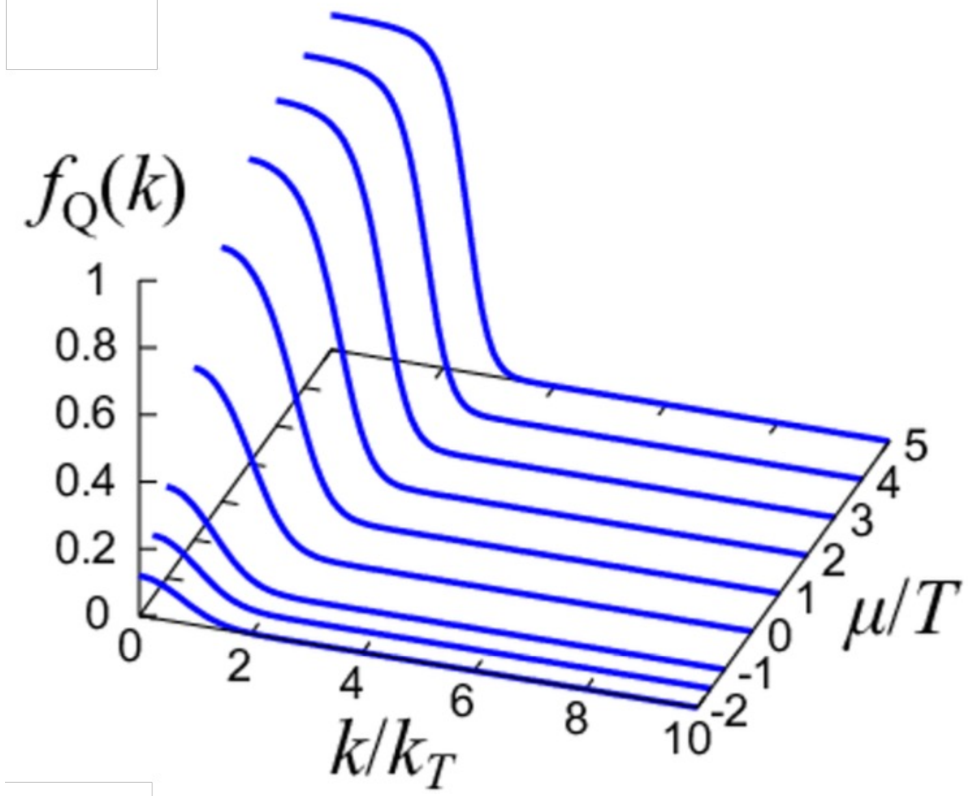


both bound & scattering states are picked up; **they cancel**

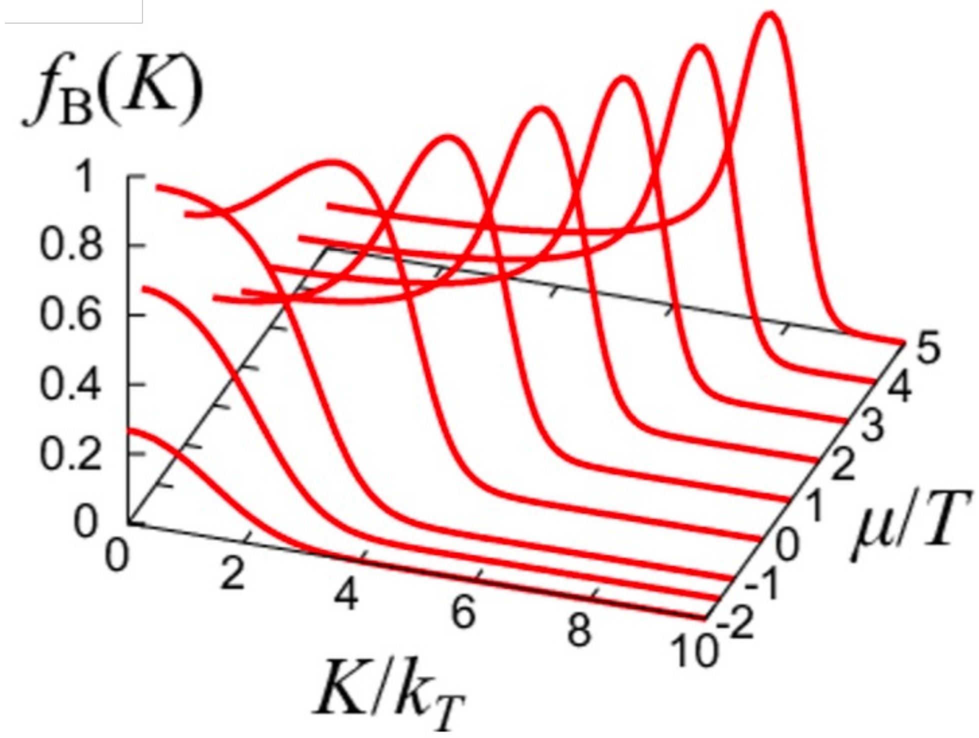
preferentially pick up bound states
➔ **baryons survive**

quark & **baryon** **distributions**

quark Fermi sea



suppression of bulk in f_B



quark Fermi sea + baryonic Fermi surface