Baryon Density-Driven Novel Phases in QCD: Resonant Screening, Rotation-Induced Confinement, and Quantum Criticality

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Outline

Quantum Color Screening in External Magnetic Field

the $\hat{\mathcal{H}}$'s positive definite Schwinger propagator high temperatures require weak-field treatment of magnetism

Fermi-Landau Matching Triggers Debye Mass Divergence

the $\hat{\mathcal{H}}$'s positive definite break in Schwinger propagator analytic continuation to introduce density effect comparison with the weak and strong magnetic field approximations connection with Shubnikov-de Haas effect (SdH)

Rotation-induced confinement

represent rotation by gravity cube model: addressing faster-than-light issue at boundaries

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Quantum Color Screening in External Magnetic Field: the $\hat{\mathcal{H}}$'s positive definite

Minimum coupling principle + external (classical) magnetic field $B = Be_z$: EOM

$$(i\gamma \cdot \partial + q\gamma \cdot A - m) G(x, x') = \delta(x - x'). \tag{1}$$

Introduce the kinematical momentum operator $\hat{\Pi}_{\mu} = \hat{p}_{\mu} + qA_{\mu}$, and solving the Dirac equation, the eigenstate equation of $\hat{\mathcal{H}} = -(\gamma \cdot \hat{\Pi})^2 = -\hat{\Pi}^2 - (q/2)\sigma^{\mu\nu}F_{\mu\nu}$ is

$$\hat{\mathcal{H}}|p\rangle = (-p_0^2 + p_z^2 + \epsilon_{lm_l m_s}^2)|p\rangle \tag{2}$$

with the Landau energy levels $\epsilon_{lm_lm_s}^2=2l|qB|+[1-|m_l|-\mathrm{sgn}(q)\ (m_l+2m_s)]|qB|$ $(l=0,1,\cdots,\infty,m_l=-l,\cdots,0,\cdots,l$ and $m_s=-1/2,1/2$ and the four-momentum eigen state $|p\rangle=|p_0,p_z,l,m_l,m_s\rangle$).

The Landau energy levels $\epsilon_{lm_lm_s}^2 \geq 0$ and the constraint $-p_0^2 \geq 0$ (in the imaginary time formalism of finite temperature field theory) lead to a positive definite $\hat{\mathcal{H}}$.

$$G(x,x') = \langle x | \frac{1}{\gamma \cdot \hat{\Pi} - m} | x' \rangle = -\int_0^\infty ds \, \langle x | \, (\gamma \cdot \hat{\Pi} + m) e^{-(m^2 + \hat{\mathcal{H}})s} | x' \rangle. \tag{3}$$

Schwinger propagator

Denote $\hat{\Pi}(s) = \hat{U}(-s)\hat{\Pi}\hat{U}(s)$ through $\hat{U}(s) = e^{-\hat{\mathcal{H}}s}$,

$$G(x,x') = -\int_0^\infty ds \, e^{-m^2 s} \, \langle x | \, \hat{U}(s)(\gamma \cdot \hat{\Pi}(s) + m) | x' \rangle. \tag{4}$$

Similar to the transformation from the Schrödinger picture to the Heisenberg picture in quantum mechanics, the s-dependence of the momentum and coordinate operators are controlled by the Heisenberg-like equations,

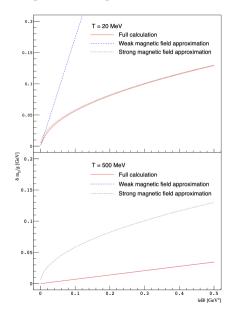
$$\partial_{s} \hat{\Pi}_{\mu}(s) = [\hat{\mathcal{H}}, \hat{\Pi}_{\mu}(s)] = 2iq F_{\mu\nu} \Pi^{\nu}(s),
\partial_{s} \hat{x}_{\mu}(s) = [\hat{\mathcal{H}}, \hat{x}_{\mu}(s)] = -2i\hat{\Pi}_{\mu}(s),$$
(5)

a more computational efficient Schwinger propagator formula

$$G(p) = -\int_{0}^{\infty} \frac{dv}{|qB|} \left\{ \left[m + (\gamma \cdot p)_{||} \right] \left[1 - i \operatorname{sgn}(q) \gamma_{1} \gamma_{2} \tanh v \right] - \frac{(\gamma \cdot p)_{\perp}}{\cosh^{2} v} \right\} \times e^{-\frac{v}{|qB|} \left(m^{2} - p_{||}^{2} + \frac{\tanh v}{v} p_{\perp}^{2} \right)}.$$

$$(6)$$

high temperatures require weak-field treatment of magnetism



Debye screening mass (leading order

$$m_D^2 = \Pi_{00}(k_0 = 0, \mathbf{k} \to \mathbf{0})$$
:

$$m_Q^2(T,B) = -\frac{g^2}{2}T\sum_{p_0}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\mathrm{Tr}\left[\gamma_0G(\boldsymbol{p})\gamma_0G(\boldsymbol{p})\right]\rho_B(\boldsymbol{p}_\perp^2),$$

with ρ_B being

$$\rho_B(\boldsymbol{p}_\perp^2) = 2|\boldsymbol{q}B| \sum_{l=0}^\infty \bigg(1 - \frac{\delta_{l,0}}{2}\bigg) \delta(\boldsymbol{p}_\perp^2 - 2l|\boldsymbol{q}B|),$$

- The magnetic field at RHIC and LHC energies: maybe strongest in nature;
- Should be treated as a weak field in comparison with the fireball's high temperature.

[1] G. Huang, J. Zhao and P. Zhuang, Phys.Rev.D 107, 114035 (2023)

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Rotation-induced confinemen

the $\hat{\mathcal{H}}$'s positive definite break in Schwinger propagator

Solve the equations and obtain the quark propagator in the external magnetic field,

$$G(p) = -\int_{0}^{\infty} \frac{dv}{|qB|} \left\{ [m + p_{\parallel}] \left[1 - i\operatorname{sgn}(q)\gamma_{1}\gamma_{2}\tanh v \right] - \frac{p_{\perp}}{\cosh^{2}v} \right\} \times e^{-\frac{v}{|qB|}} \left(m^{2} - p_{\parallel}^{2} + \frac{\tanh v}{v} p_{\perp}^{2} \right).$$

$$(7)$$

In thermal dense QCD matters, one needs to consider the quarks' chemical potential μ_f , and

$$\hat{\mathcal{H}}|p\rangle = \left[-(p_0 + \mu_f)^2 + p_z^2 + \epsilon_{lm_l m_s}^2\right]|p\rangle \equiv \mathcal{H}_p|p\rangle \tag{8}$$

with
$$p_0=-i\omega_n$$
, $\operatorname{Re}(\mathcal{H}_p)=\omega_n^2-\mu_f^2+p_z^2+\epsilon_{lm_lm_s}^2$ and $\operatorname{Im}(\mathcal{H}_p)=2\omega_n\mu_f$.

There exist the condition that $Re(\mathcal{H}_p) < 0$ if $\mu_f > \pi T$.

A naive substitution of p_0 with $p_0 + \mu_f$ in the Schwinger propagator is incorrect!

Key Method: avoid the integral divergence of s.

Solution: **Do the analytic continuation after** *s***-integration!**

analytic continuation to introduce density effect

Key process: Analytic Continuation after s-integral

The increment of the square of Debye screening mass:

$$\delta m_D^2(T, B) = g^2 T \sum_n \int \frac{\mathrm{d}p_z}{(2\pi)^2} \frac{p_z^2 - \omega_n^2}{|qB|} \mathcal{K}\left(\frac{p_z^2 + \omega_n^2}{2|qB|}\right), \tag{9}$$

with $\omega_n = (2n+1)\pi T$. The $\mathcal{K}(x)$ can be represented by the following integral formula if Re(x) > 0 ($b = |qB|/(4\pi^2 T^2)$)

$$\mathcal{K}(x) = 4b \int_0^\infty d\xi \, e^{-2bx\xi^2} \xi [1 - b\xi^2 \coth(b\xi^2)]. \tag{10}$$

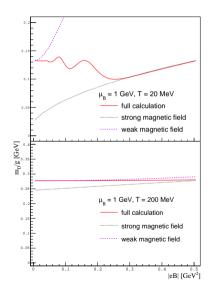
The analytic continuation $(\omega_n \to \omega_n + i\mu_q)$ can not keep the real part of $x = [p_z^2 + (\omega_n + i\mu_q)^2]/(2|qB|)$ being positive, use the following relation

$$\mathcal{K}(x) = \frac{1}{2x^2} + \frac{1}{x} - \frac{1}{2(x+N)^2} - \frac{1}{x+N} + \mathcal{K}(x+N) - \sum_{n=0}^{N-1} \frac{1}{(x+n)^2},\tag{11}$$

$$N = \left[\text{Floor} \left(\frac{\mu_q^2 - \pi^2 T^2}{2|qB|} \right) + 1 \right] \theta(\mu_q - \pi T), \tag{12}$$

then $(p_z^2 + \omega_n^2 - \mu_q^2)/(2|qB|) + N \ge (\pi^2 T^2 - \mu_q^2)/(2|qB|) + N > 0$, and $\mathcal{K}(x+N)$ can be represented by the ξ integral.

comparison with the weak and strong magnetic field approximations



The magnetic field dependence of m_D/g and the comparison with the approximations of weak and strong magnetic field are shown in the Figure at high baryon chemical potential $\mu_B=1$ GeV for low and high temperatures T=20 MeV and T=100 MeV.

Debye screening mass in thermal, dense, magnetized QCD matter

- $T \rightarrow 0 \colon m_D/g \text{ oscillates } \mu_q^2 = 2l|qB|$ $\mu\text{-}B\text{-}\text{oscillation}(\text{SI}) \colon B_l = \mu_q^2/(2lqc^2\hbar)$
- Landau energy level matches the Fermi surface of quarks:

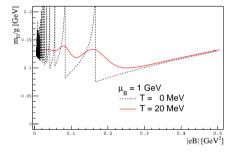
$$m_D/g \to +\infty$$
,

Color interaction between a pair of quarks is completely screened.

The low temperatures in compact star cores justify classifying the magnetic field as strong based on our prior criteria, which implies the expected emergence of the μ-B-oscillation.

Fermi-Landau Matching Triggers Debye Mass Divergence:

connection with Shubnikov-de Haas effect (SdH)



Shubnikov-de Haas effect (SdH) $\Rightarrow B_l = n_e h/(2le).$ n_e : the 2-d electron number density. electrical resistivity oscillation PRL 108, 216803 (2012).

Debye screening mass in thermal, dense, magnetized QCD matter

ightharpoonup T o 0, the screening mass oscillates $\mu_q^2 = 2l|qB|$

$$\mu\text{-}B\text{-}\text{oscillation(SI):}\ B_l=\mu_q^2/(2lqc^2\hbar)$$

- 2-d massless quark gas's number density, $n_q = \int_0^\infty dE \frac{E/(\pi \hbar^2 c^2)}{(E-\mu_0)/(k_B T) + 1},$
- states number for T=0, $g(k)=g_k(dV/dE)=E/(\pi\hbar^2c^2)$, degeneracy: $g_k=2/(2\pi)^2$, 2-d system: $V=\pi k^2$, massless quark: $E(k)=\hbar kc$.
- $n_q = \mu_q^2/(2\pi\hbar^2c^2)$

 μ -B-oscillation(SI): $B_l = n_q h/(2lq)$

screening mass⇔electrical resistivity

validation of analytic continuation via magnetic field and thermal limits

- ▶ Not only do the extreme conditions of strong and weak magnetic fields agree with our theory, but at low temperatures, our theory is also in agreement with the experimentally measured results (Shubnikov-de Haas effect).
- ▶ More importantly, the above results demonstrate the correctness of the analytic continuation procedure: specifically, performing the s-integration first, followed by the shift $p_0 \rightarrow p_0 + \mu_q$.

[2] G. Huang, J. Zhao and P. Zhuang, Phys.Rev.D 108, L091503 (2023)

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Rotation-induced confinement

represent rotation by gravity

cube model: addressing faster-than-light issue at boundaries

Rotation-induced confinement: motivation to study the density-rotation effect



Since rotation is similar to magnetic field in the sense of breaking the spherical symmetry, the phenomenon similar to the Shubnikov-de Haas effect is expected to happen in a rotational QCD system.

represent rotation by gravity





$$x^{\mu} = (t, x, y, z)$$

$$\overline{x}^{\mu} = (\overline{t}, \overline{x}, \overline{y}, \overline{z})$$

 $t = \bar{t}, \ z = \bar{z}, \ \boldsymbol{\rho} = \exp(-\mathrm{i}\omega\bar{t}\sigma_y)\bar{\boldsymbol{\rho}}.$

- ▶ In analogy with the weightlessness case in uniform circular motion space station. In a rotating frame (\overline{x}^{μ}) with the same angular velocity of the system, the QCD matters (astronauts) kinematically feel *weightless*, which corresponding to the *no gravity case*, or *flat space case*.
- Then the *Einstein's Strong Equivalence Principle* tells that this coordinate systems' transformation can equally be described by the curved gravity (x^{μ}), and the transformation between the curved gravity and its flat one should be <u>covariant</u>, due to the homogenous transformation of the **tensor** in General Relativity.

Proof of the gluon propagator to be tensor

The action and partition function in a curved space are defined as

$$S[G_a^{\mu}, J_{\mu}^a] = \int d^4x \sqrt{-\det(g_{\mu\nu}(x))} \left[\mathcal{L}\left(G_a^{\mu}(x), G_{a;\nu}^{\mu}(x)\right) + \frac{1}{\xi} \left(G_{a;\alpha}^{\alpha}(x)\right)^2 + J_{\mu}^a(x)G_a^{\mu}(x) \right], \quad (13)$$

$$Z[J_{\mu}^a] = \int DG_a^{\mu} \sqrt{-\det(g_{\mu\nu}(x))} e^{iS[G_a^{\mu}, J_{\mu}^a]}. \quad (14)$$

The gluon propagator can be obtained by the functional derivative with respect to the auxiliary field $J^a_{\mu}(x)$,

$$D_{ab}^{\mu\nu}(x,y) = -\left[\det(g_{\mu\nu}(x))\det(g_{\mu\nu}(y))\right]^{-1/2} \frac{1}{Z[J_a^a]} \frac{\delta^2 Z[J_\mu^a]}{\delta J_{a}^a(x)\delta J_{b}^b(y)}.$$
 (15)

Taking into account the transformation for the auxiliary field and the definition for the δ function in the curved space,

$$J_{\sigma}^{a}(\overline{x}) = \frac{\partial x^{\mu}}{\partial \overline{x}^{\sigma}} J_{\mu}^{a}(x),$$

$$\delta^{4}(x-y) = \frac{1}{|\det(\partial x^{\mu}/\partial \overline{x}^{\sigma})|} \delta^{4}(\overline{x}-\overline{y}),$$
(16)

there is

$$\frac{\delta}{\delta J_{\mu}^{a}(x)} \quad \Leftrightarrow \quad \frac{\partial x^{\mu}}{\partial \overline{x}^{\sigma}} \frac{1}{|\det(\partial x^{\mu}/\partial \overline{x}^{\sigma})|} \frac{\delta}{\delta J_{\sigma}^{a}(\overline{x})}. \tag{17}$$

Considering the relations $|\det(\partial x^{\mu}/\partial \overline{x}^{\sigma})| = [-\det(g_{\mu\nu}(x))]^{-1/2}$, $Z[J^a_{\mu}(x)] = Z[J^a_{\sigma}(\overline{x})]$, the gluon propagator in the curved space can be represented by the one in the flat space,

$$D_{ab}^{\mu\nu}(x,y) = -\frac{\partial x^{\mu}}{\partial \overline{x}^{\sigma}} \frac{\partial y^{\nu}}{\partial \overline{y}^{\rho}} \frac{1}{Z[J_{\underline{q}}]} \frac{\delta^{2}Z[J_{\underline{q}}^{\mu}]}{\delta J_{\underline{q}}(\overline{x})\delta J_{\underline{p}}^{b}(\overline{y})} = \frac{\partial x^{\mu}}{\partial \overline{x}^{\sigma}} \frac{\partial y^{\nu}}{\partial \overline{y}^{\rho}} \overline{D}_{ab}^{\sigma\rho}(\overline{x} - \overline{y}). \tag{18}$$

calculating the curved-space gluon self-energy

The coordinate transformation for a constant rotation ω around z-axis is

$$t = \bar{t}, \tag{19}$$

$$x = \overline{x}\cos(\omega \overline{t}) - \overline{y}\sin(\omega \overline{t}), \tag{20}$$

$$y = \overline{x}\sin(\omega \overline{t}) + \overline{y}\cos(\omega \overline{t}), \tag{21}$$

$$z = \overline{z}, \tag{22}$$

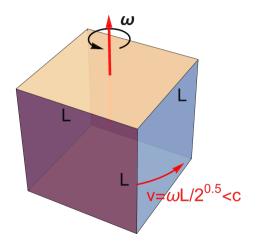
then for the inverse gluon propagator D^{-1} , the covariant transformation should be

$$[D^{-1}]_{\mu\nu}^{ab}(x,y) = \frac{\partial \overline{x}^{\sigma}}{\partial x^{\mu}} \frac{\partial \overline{y}^{\rho}}{\partial y^{\nu}} [\overline{D}^{-1}]_{\sigma\rho}^{ab} (\overline{x} - \overline{y}). \tag{23}$$

Propagator transformation between different coordinate frames are expected to be simple!

This transformation keeps valid for both D^{-1} and D_0^{-1} , then Π , for $\Pi_{\mu\nu}^{ab} = [D^{-1}]_{\mu\nu}^{ab} - [D_0^{-1}]_{\mu\nu}^{ab}$ in both coordinate frames.

cube model: addressing faster-than-light issue at boundaries



- The collisional fireballs are not necessarily being rotational symmetric, in shape.
- To quantitatively involve the constraint from the causality in our treatment, we consider a cube of QCD matter with side length L. In this case, the angular velocity ω should be smaller than $\sqrt{2}/L$.
- ► The finite volume changes the continuous momenta to discrete ones

$$\begin{split} & \boldsymbol{p} = (2\pi/L)\boldsymbol{n}, \overline{\boldsymbol{p}} = (2\pi/L)\overline{\boldsymbol{n}}, \\ & \boldsymbol{n} = (n_1, n_2, n_3), \overline{\boldsymbol{n}} = (\overline{n}_1, \overline{n}_2, \overline{n}_3), \\ & n_i, \overline{n}_i \in \mathbb{Z}, (i = 1, 2, 3). \end{split}$$

The cube model makes the calculation much easier.

Momentum space representation of the covariant transformation

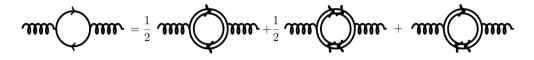
Covariant transformation: coordinate \rightarrow momentum space, the gluon self-energy in curved space can be expressed as $\Pi^{ab}_{\mu\nu}(p,p') = \int \frac{d^4 \overline{p}}{(2\pi)^4} T^{\sigma\rho}_{\mu\nu}(\overline{p}|p,p') \overline{\Pi}^{ab}_{\sigma\rho}(\overline{p}).$

► The second-order and first-order translation tensors

$$\begin{split} T^{\sigma\rho}_{\mu\nu}(\overline{p}|p,p') &= h^{\sigma}_{\;\mu}(\overline{p}|p) h^{\rho}_{\;\nu}(-\overline{p}|p'), \\ h^{\sigma}_{\;\mu}(\overline{p}|p) &= \int d^4x \sqrt{-\det(g_{\alpha\beta}(x))} \frac{\partial \overline{x}^{\sigma}}{\partial x^{\mu}} e^{i(p\cdot x - \overline{p}\cdot \overline{x})}, \\ g_{\alpha\beta}(x) &= \begin{pmatrix} 1 - (x^2 + y^2)\omega^2 & -y\omega & x\omega & 0 \\ -y\omega & -1 & 0 & 0 \\ x\omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \end{split}$$

Zero-temperature gluon self-energy at one-loop in flat spacetime

Use the energy projector method to obtain the gluon self-energy of quark loops, the double lines indicate quark modes with positive (one arrow) and negative (double arrow) energies.



The gluon self-energy can be represented as the summation of transverse and longitudinal parts $\overline{\Pi}_{\mu\nu}^{ab}(\overline{p}) = \delta^{ab}[P_{t\nu}^{T}(\overline{p})\overline{\Pi}_{T}(\overline{p}) + P_{t\nu}^{L}(\overline{p})\overline{\Pi}_{L}(\overline{p})]$, with

$$\begin{split} & \overline{\Pi}_T(\overline{p}) = -\frac{g^2 \mu_q^2}{12\pi^2} \left(\frac{2\overline{p}_0^2}{|\overline{p}|^2} + 1\right) + \frac{g^2}{384\pi^2 |\overline{p}|^3} \sum_{n,s=\pm} F_T(n\overline{p}_0, |\overline{p}|, s\mu_q), \\ & \overline{\Pi}_L(\overline{p}) = \frac{g^2 \mu_q^2}{2\pi^2} \left(\frac{\overline{p}_0^2}{|\overline{p}|^2} - 1\right) - \frac{g^2}{192\pi^2 |\overline{p}|^3} \sum_{n,s=\pm} F_L(n\overline{p}_0, |\overline{p}|, s\mu_q), \end{split}$$

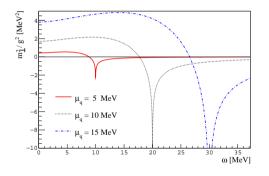
$$F_T(n\overline{p}_0,|\overline{p}|,s\mu_q) = \left(\overline{p}_0^2 - |\overline{p}|^2\right) \left(\overline{p}_0^2 + n\overline{p}_0|\overline{p}| + 4|\overline{p}|^2 + 4ns\mu_q\overline{p}_0 + 2s\mu_q|\overline{p}| + 4\mu_q^2\right) B_n^s \ln\left(\frac{\left(B_n^s\right)^2}{\left(n\overline{p}_n - |\overline{p}|\right)^2}\right),$$

$$F_L(n\overline{p}_0, |\overline{p}|, s\mu_q) = \left(\overline{p}_0^2 - |\overline{p}|^2\right) \left(n\overline{p}_0 + 2|\overline{p}| + 2s\mu_q\right) \left(B_n^s\right)^2 \ln\left(\frac{\left(B_n^s\right)^2}{\left(n\overline{p}_0 - |\overline{p}|\right)^2}\right).$$
 where B_n^s is defined as $B_n^s = n\overline{p}_0 - |\overline{p}| + 2s\mu_q$.

The result covers both the low and high density conditions, and when p_0 , $|p| \ll \mu_q$, the result goes back to the HDL.

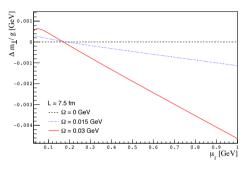
[3] G. Huang and P. Zhuang, Phys.Rev.D 104, 074001 (2021)

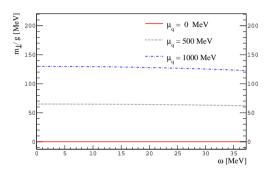
Dynamic mass: $m_{\rm dy}^2 = \Pi_{00}(k_0 \to 0, k = 0)$



- The de Haas-van Alphen effect (dHvA) happens every time the orbital states pass through the Fermi contour, $\mu_q = (n+1/2)\tilde{\omega}$, with $\tilde{\omega} = eH/mc$.
- Since there can be many Landau levels (n = 0, 1, 2, ···), many divergences will emerge with increasing magnetic field.
- For the rotational matter, however, there is only one positive energy level $\omega/2$, there is maximum one divergence for the gluon mass at $\omega=2\mu_q$.

High Density Reverses the Effect of Rotation on Dynamic Mass





The density will change the enhancement and attenuation effect of rotation on dynamic mass Focus on the area of $\mu_q > \omega/2$, the high density actually reverses the effect of rotation on dynamic mass:

$$T=0, \mu_q>\omega/2,$$
 but μ_q is small: $\omega\uparrow, m_\perp/g\uparrow$

$$T=0, \mu_q>\omega/2$$
, but μ_q is large: $\omega\uparrow$, $m_{\perp}/g\downarrow$

Summary

T-B competition:

▶ The magnetic field at RHIC and LHC energies: maybe strongest in nature, should be treated as a weak field in comparison with the fireball's high temperature.

μ_q -B oscillations:

▶ There may exist Shubnikov-de Haas effect (SdH) in the center of compact stars.

μ_{α} - ω oscillation:

- ▶ The μ_q - ω oscillation at low density condition.
- ▶ The enhancement and attenuation effect of rotation on dynamic mass will be changed by the density.

Thanks for listening!