Forward Particle Production in pA Collisions

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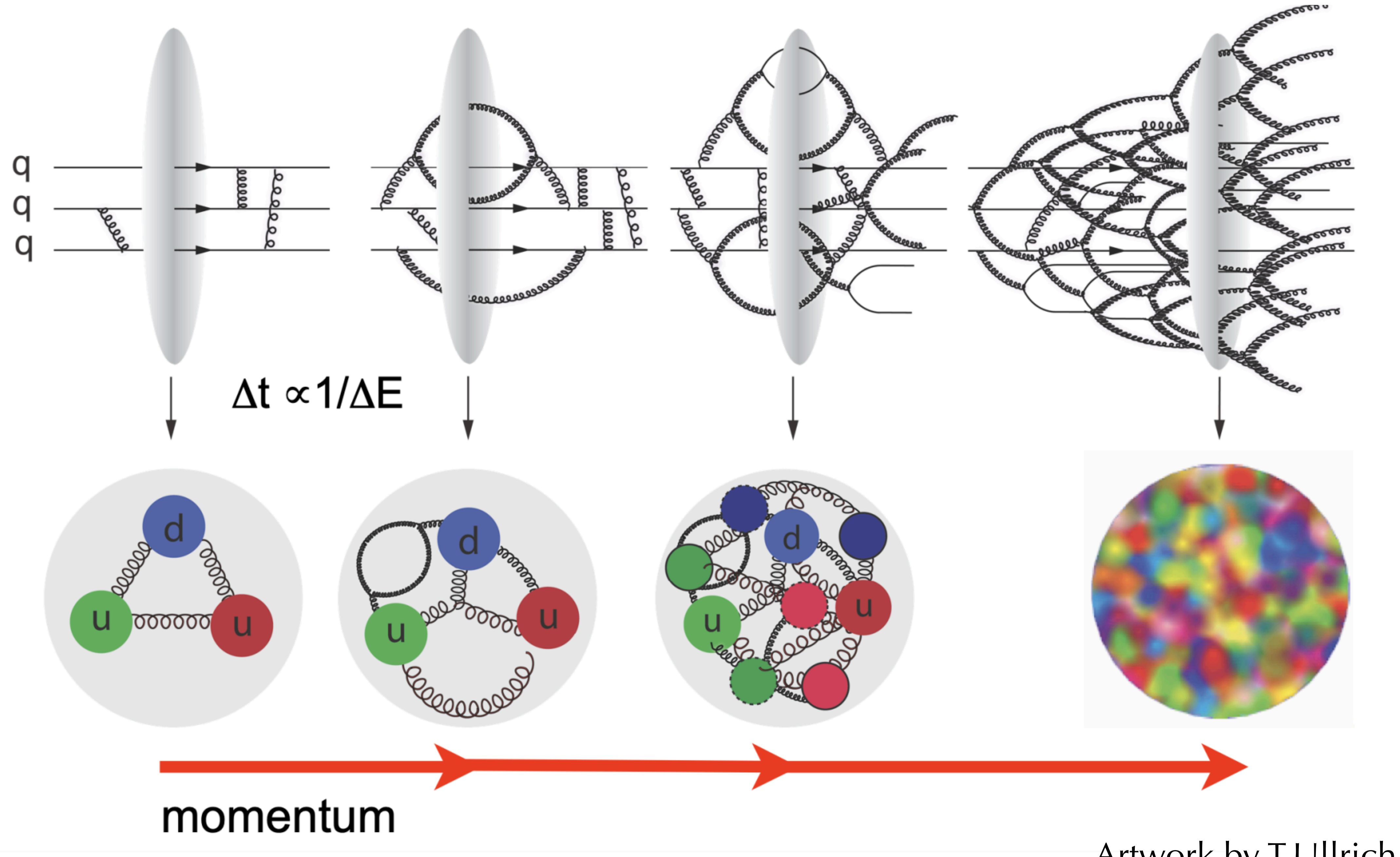
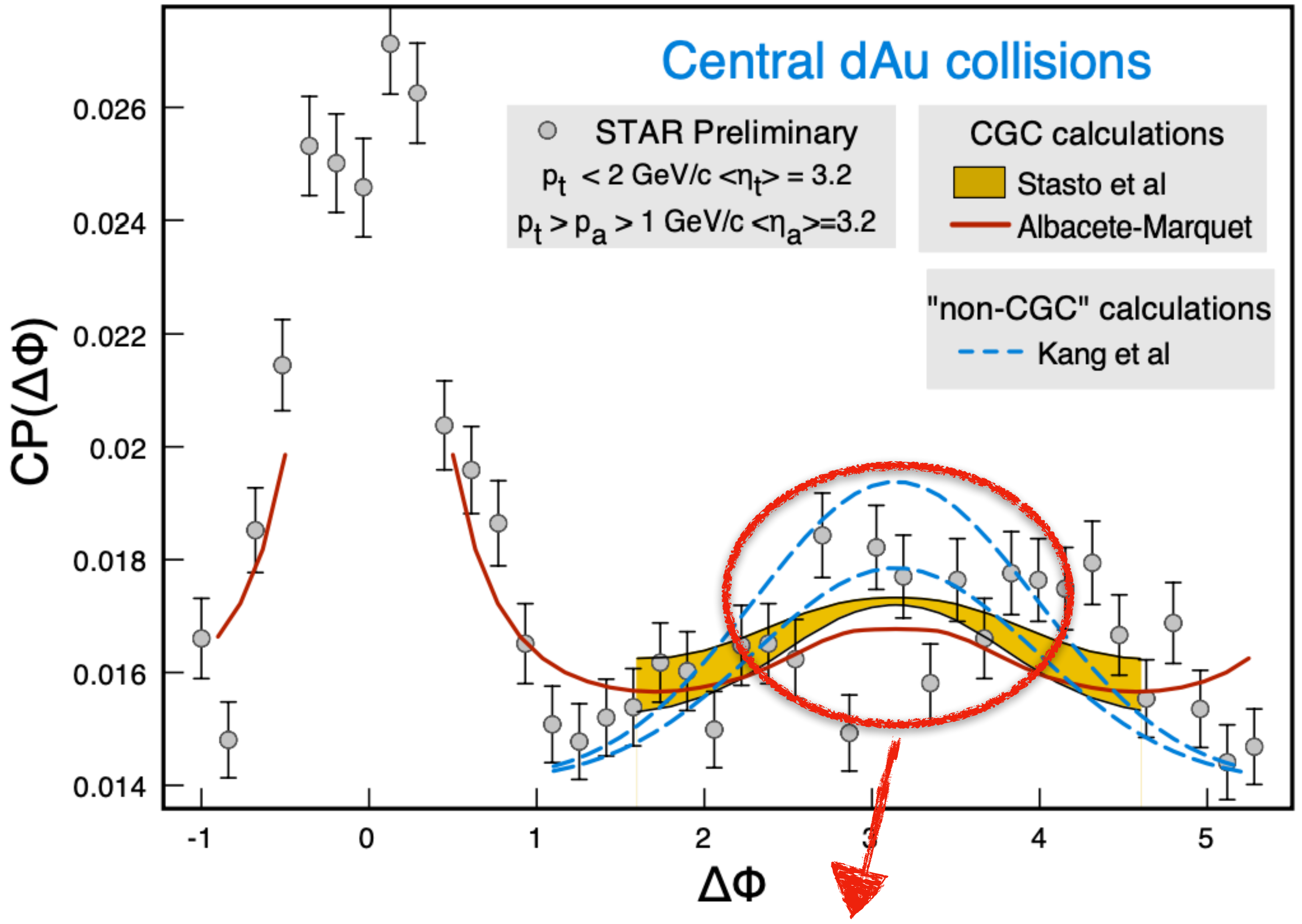
1910.10166, 2004.11990, 2204.03026

Precision Matters



Artwork by T.Ullrich



Compatible with both CGC and collinear high twist predictions

- Highly dense region: gluon splittings and recombinations
- Non-linear evolution, gluon saturation
- Pillar scientific goal of EIC

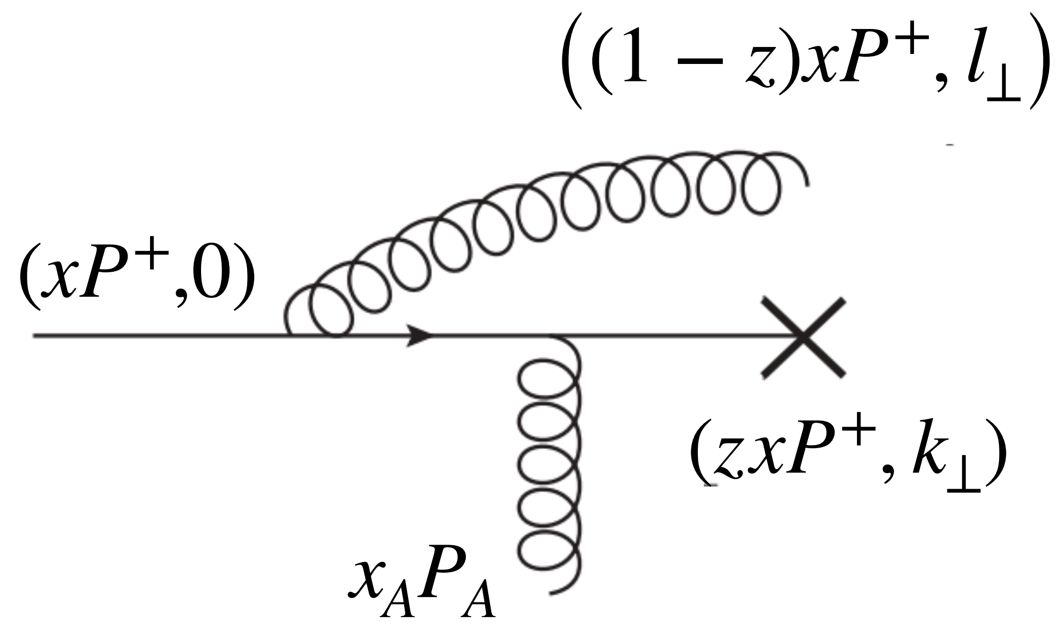
- Implications but not nailed down
- Needs different probes to differentiate
- **Needs precision: go beyond LO**

See Bowen's talk for recent progress within CGC EFT

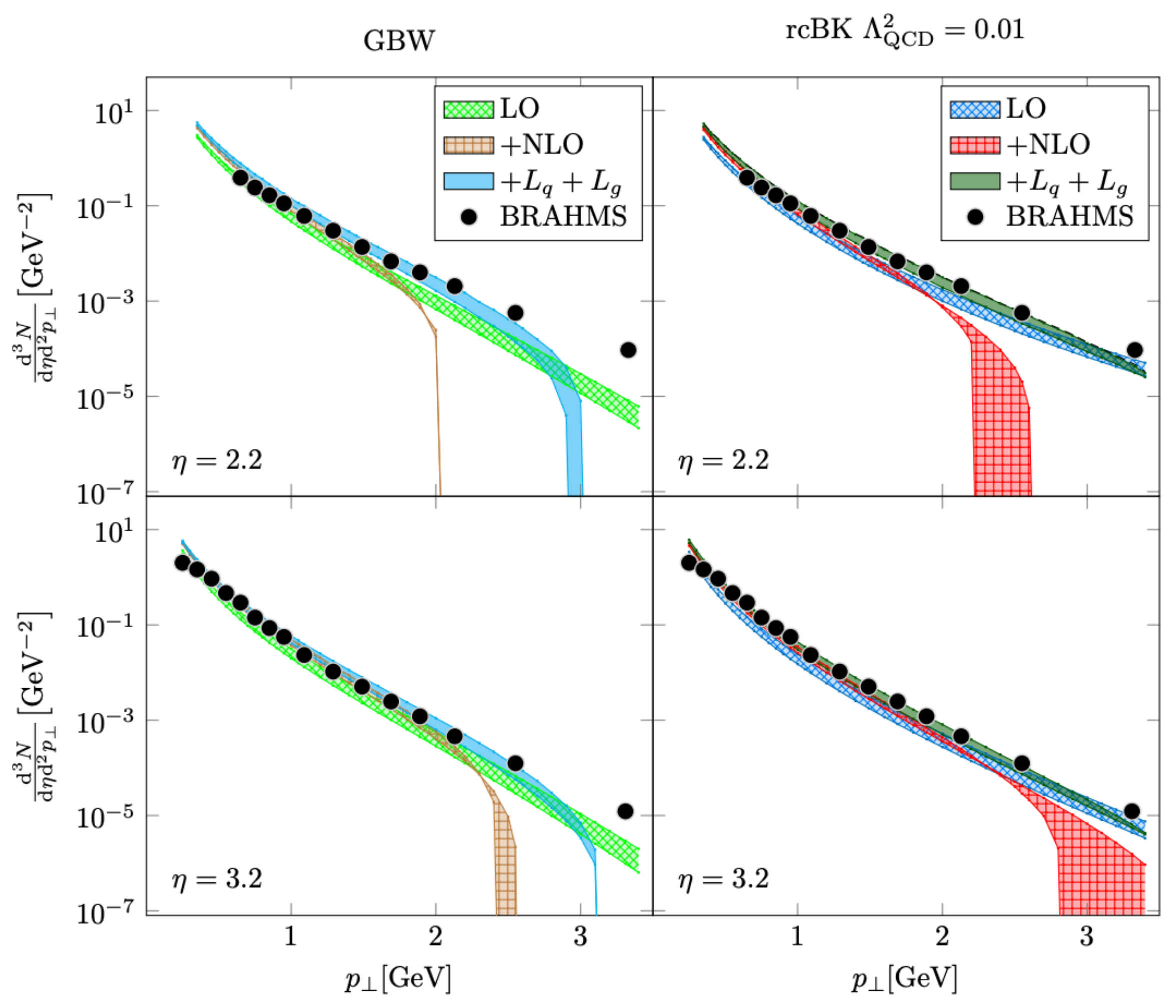
Issues at NLO in CGC

- Kinematic constraint, implemented by hand 1505.05183

$$z \leq 1 - \frac{l_\perp^2}{xs}, \quad \psi(u_\perp) \to \psi(u_\perp)\left[1 - J_0(u_\perp \Delta)\right]$$



- Negative cross section, calling for threshold resummation 1806.03522, 2112.06975



- …

See Bowen's talk for more details and his solution to these issues

Issues at NLO in CGC

- Kinematic constraint, implemented by hand

$$z \leq 1 - \frac{l_\perp^2}{xs}, \quad \psi(u_\perp) \to \psi(u_\perp)\left[1 - J_0(u_\perp \Delta)\right]$$

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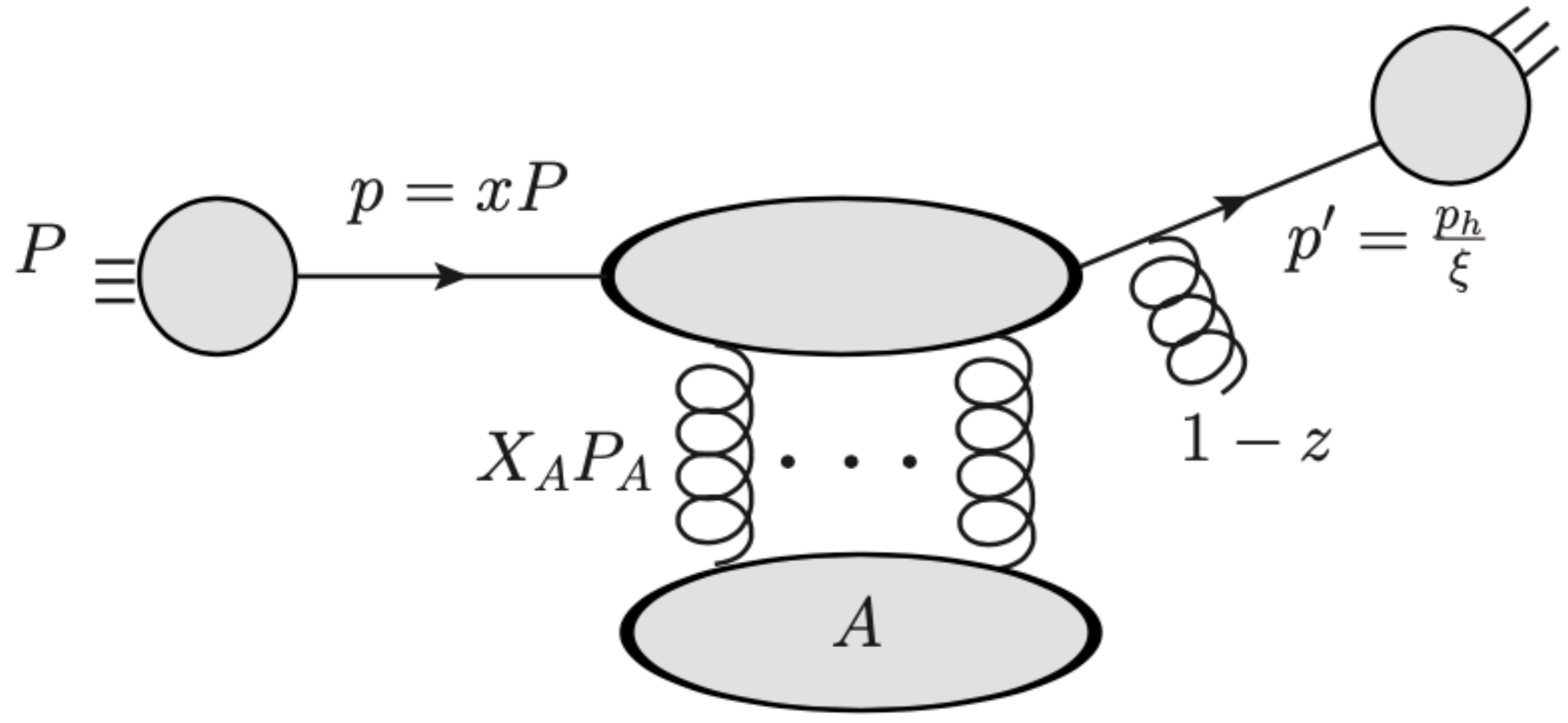
- …

This talk:

- **Try to understand these problems within "textbook" QCD**
- **Fitting into the philosophy of effective field theory**

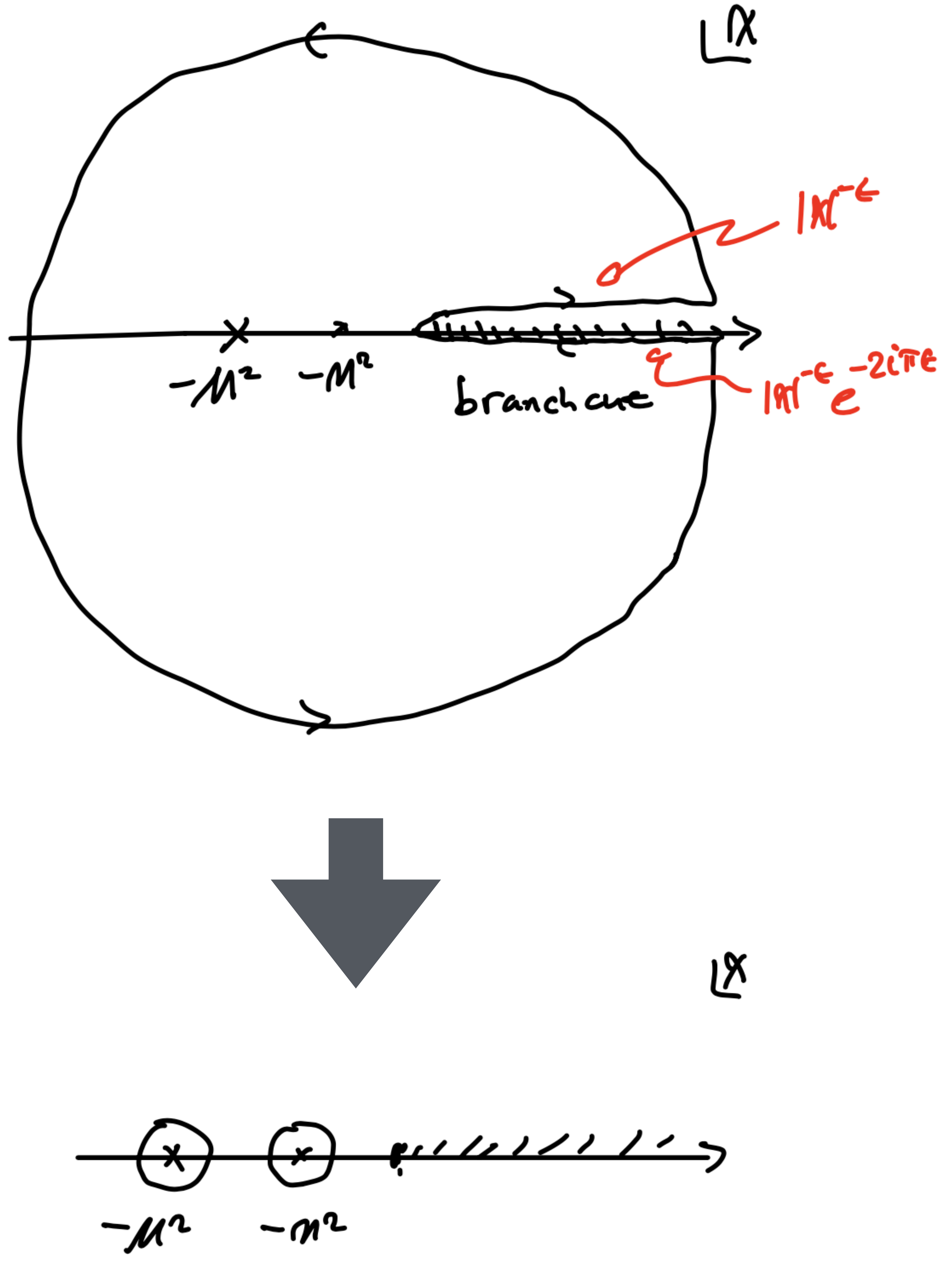


Idea of Effective Field Theories

Toy Model Full Theory:

$$I_{\text{full}} = \frac{\lambda^2}{2}\int_0^\infty dk \frac{k}{(k^2+m^2)(k^2+M^2)} \underset{x=k^2}{=} \frac{\lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}} \oint_C dx \frac{\lambda^2 (x/\mu^2)^{-\epsilon}}{(x+m^2)(x+M^2)}$$

$$= \frac{2\pi i \lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}} Res \frac{\lambda^2 (x/\mu^2)^{-\epsilon}}{(x+m^2)(x+M^2)} = \frac{\lambda^2 \ln\frac{m^2}{M^2}}{m^2-M^2} \quad \approx_{M\gg m} -\lambda^2 \frac{\ln\frac{m^2}{M^2}}{M^2} + \mathcal{O}\left(\frac{m^2}{M^2}\right)$$



- I_{full} is finite, $\sim_{|x|\to\infty} |x|^{-1} \to 0$, $\sim_{|x|\to 0} 0$, $x^{-\epsilon}$ is barely a mathematic trick
- Dominant contribution from $k^2 \sim -m^2$ and $k^2 \sim -M^2$
- Logarithmic enhancement when there exits hierarchy ($M \gg m$), $\lambda^2 \ln\frac{m^2}{M^2} \gtrsim 1$, which spoils the perturbative expansion.

Idea of Effective Field Theories

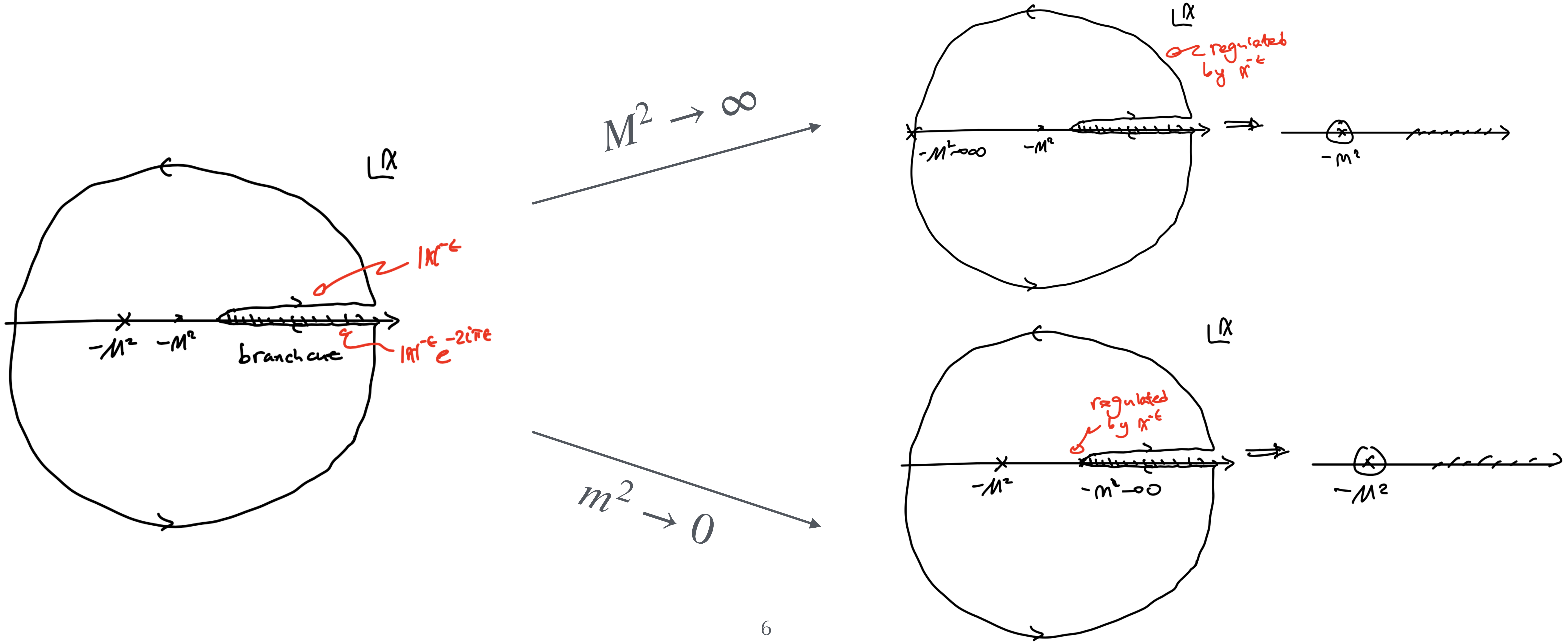
EFT: Focus on one single scale each time

Now, UV regulator

$$I_{k^2\sim -m^2} = \frac{\lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}}\oint_C dx \frac{\lambda^2 (x/\mu^2)^{-\epsilon}}{(x+m^2)(x+M^2)} \approx_{x\sim m^2 \ll M^2} \frac{\lambda^2 \lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}}\oint_C dx\frac{\boxed{(x/\mu^2)^{-\epsilon}}}{(x+m^2)(M^2)} = \frac{\lambda^2}{M^2\epsilon} - \lambda^2\frac{\ln(m/\mu)^2}{M^2}$$

Now, IR regulator

$$I_{k^2\sim -M^2}\frac{\lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}}\oint_C dx \frac{\lambda^2 (x/\mu^2)^{-\epsilon}}{(x+m^2)(x+M^2)} \approx_{x\sim M^2 \gg m^2} \frac{\lambda^2 \lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}}\oint_C dx\frac{\boxed{(x/\mu^2)^{-\epsilon}}}{x(x+M^2)} = -\frac{\lambda^2}{M^2\epsilon} + \lambda^2\frac{\ln(M/\mu)^2}{M^2}$$



Idea of Effective Field Theories

EFT: Focus on one single scale each time

Now, UV regulator

$$I_{k^2\sim -m^2} = \frac{\lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}}\oint_C dx \frac{\lambda^2 (x/\mu^2)^{-\epsilon}}{(x+m^2)(x+M^2)} \approx_{x\sim m^2 \ll M^2} \frac{\lambda^2 \lim_{\epsilon\to 0}}{1-e^{-2i\pi\epsilon}}\oint_C dx \frac{\boxed{(x/\mu^2)^{-\epsilon}}}{(x+m^2)(M^2)} = \frac{\lambda^2}{M^2\epsilon} - \lambda^2 \frac{\ln(m/\mu)^2}{M^2}$$

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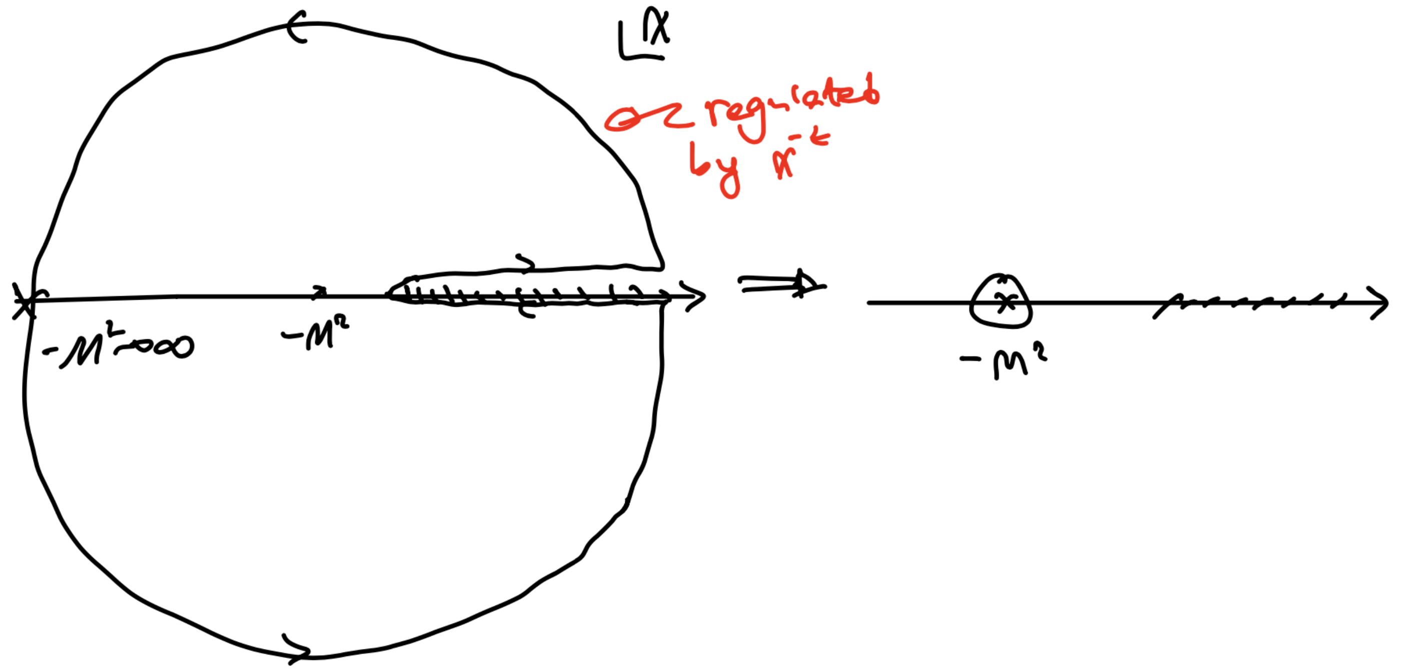
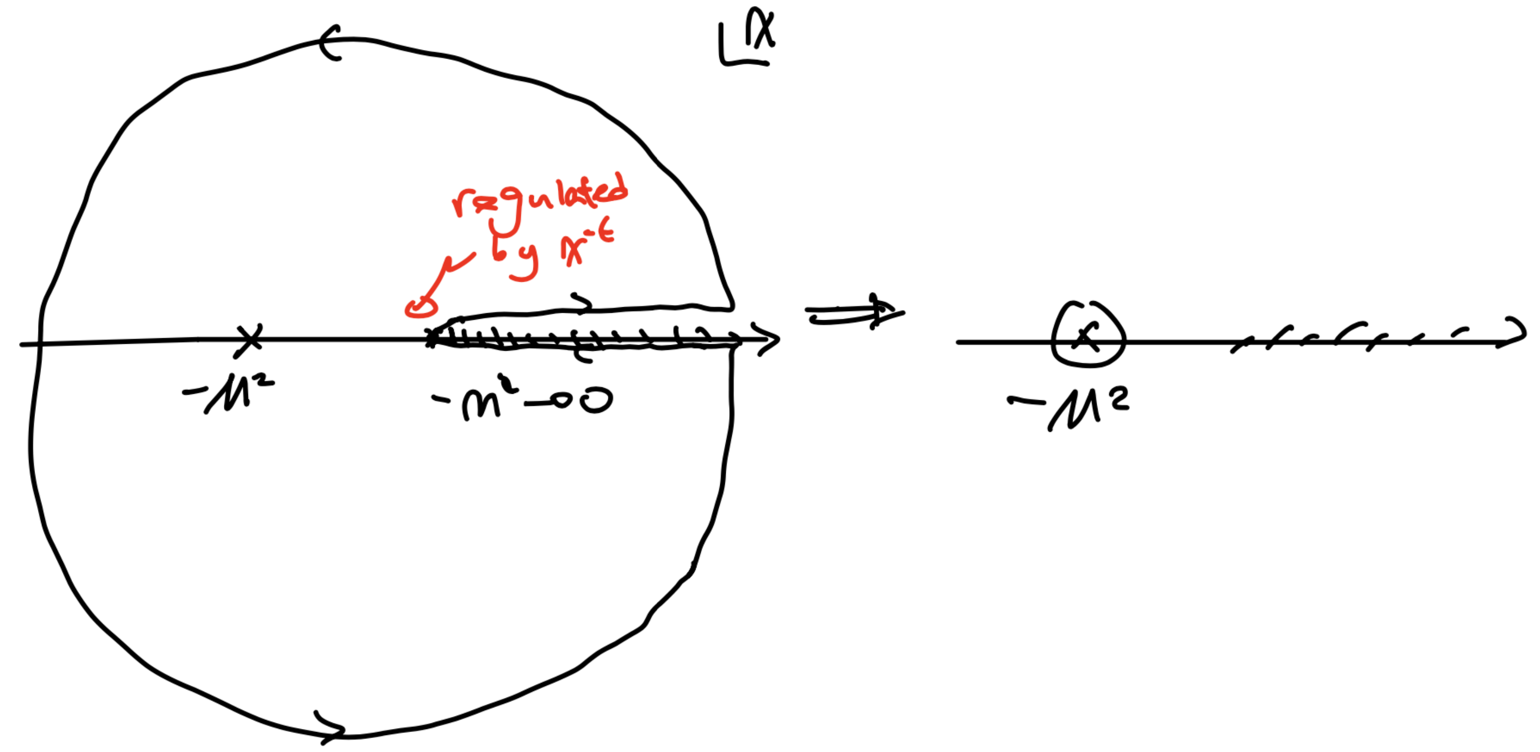
- Power expansion at the integrand level (different modes), seemingly same integration boundary but **different domain.**
- Neither integral (effective theory) is finite, UV for $I_{k^2\sim -m^2}$ and IR for $I_{k^2\sim -M^2}$. Both need a regulator, which leads to μ dependence. (Allow for RGE to resum logs)
- $I_{k^2\sim -m^2} + I_{k^2\sim -M^2} = I_{\text{full}}$, poles cancel, independent of μ



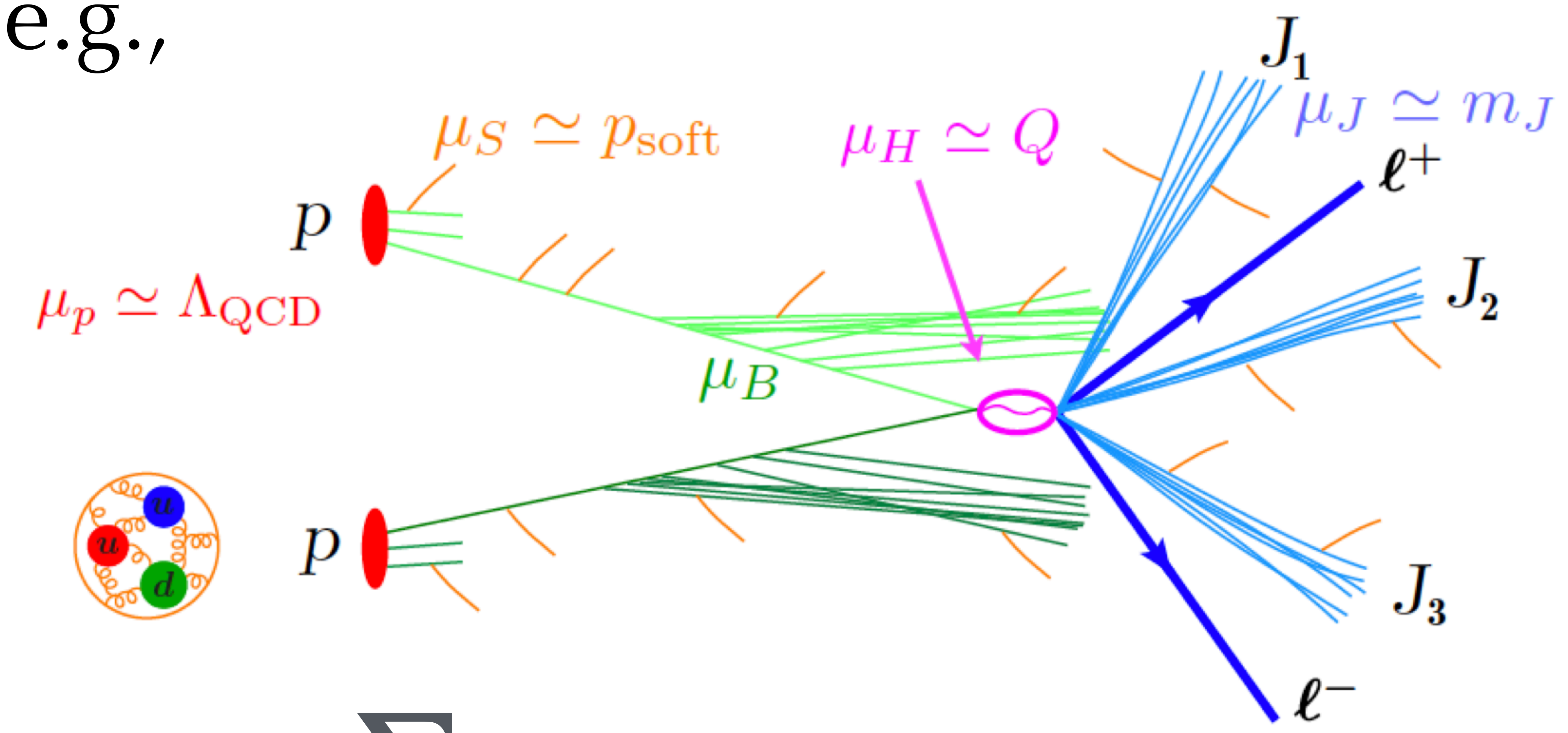


Idea of Effective Field Theories

EFT: Focus on one single scale each time

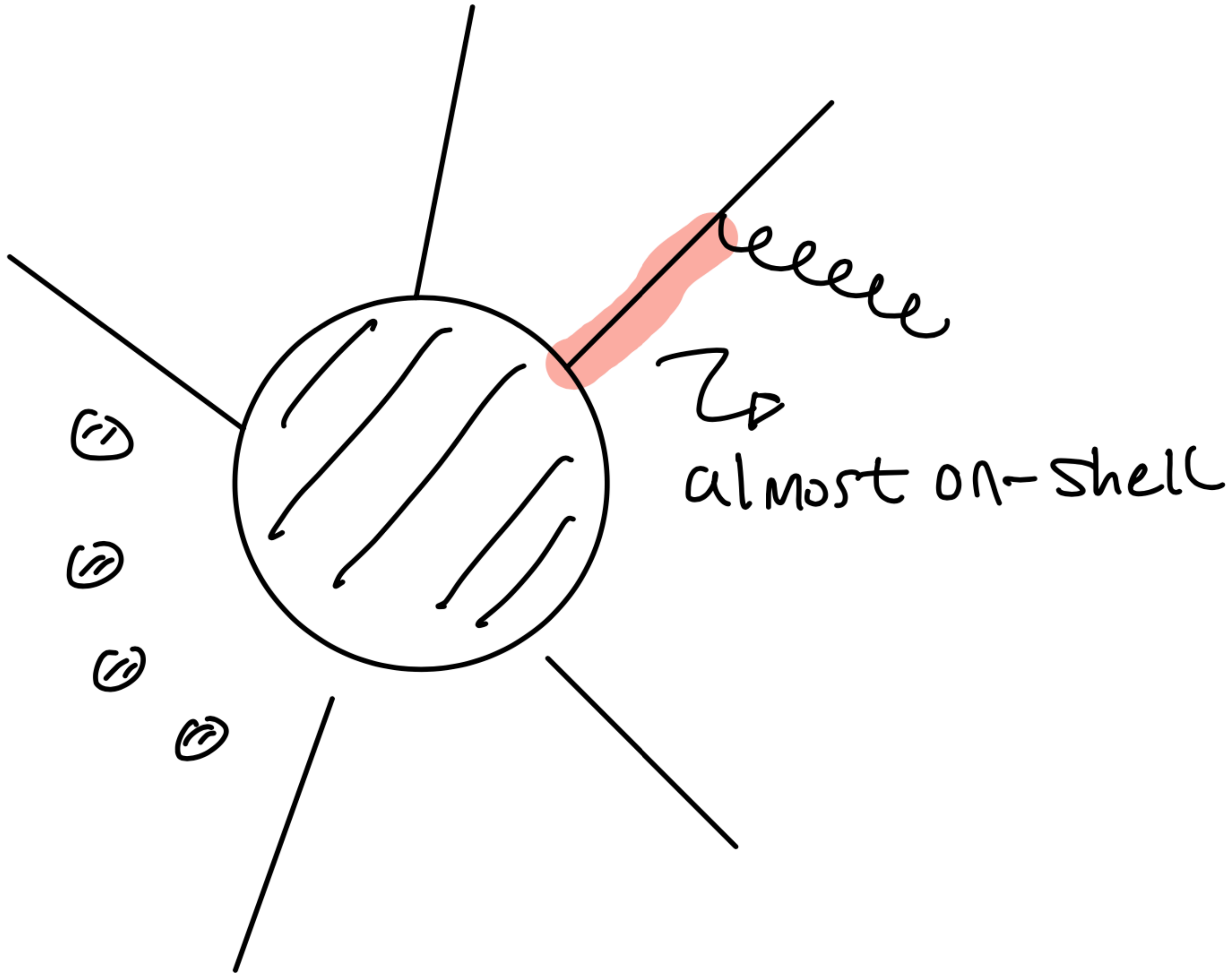
Mode expansion based on v (set by the observable/constraint)

e.g.,



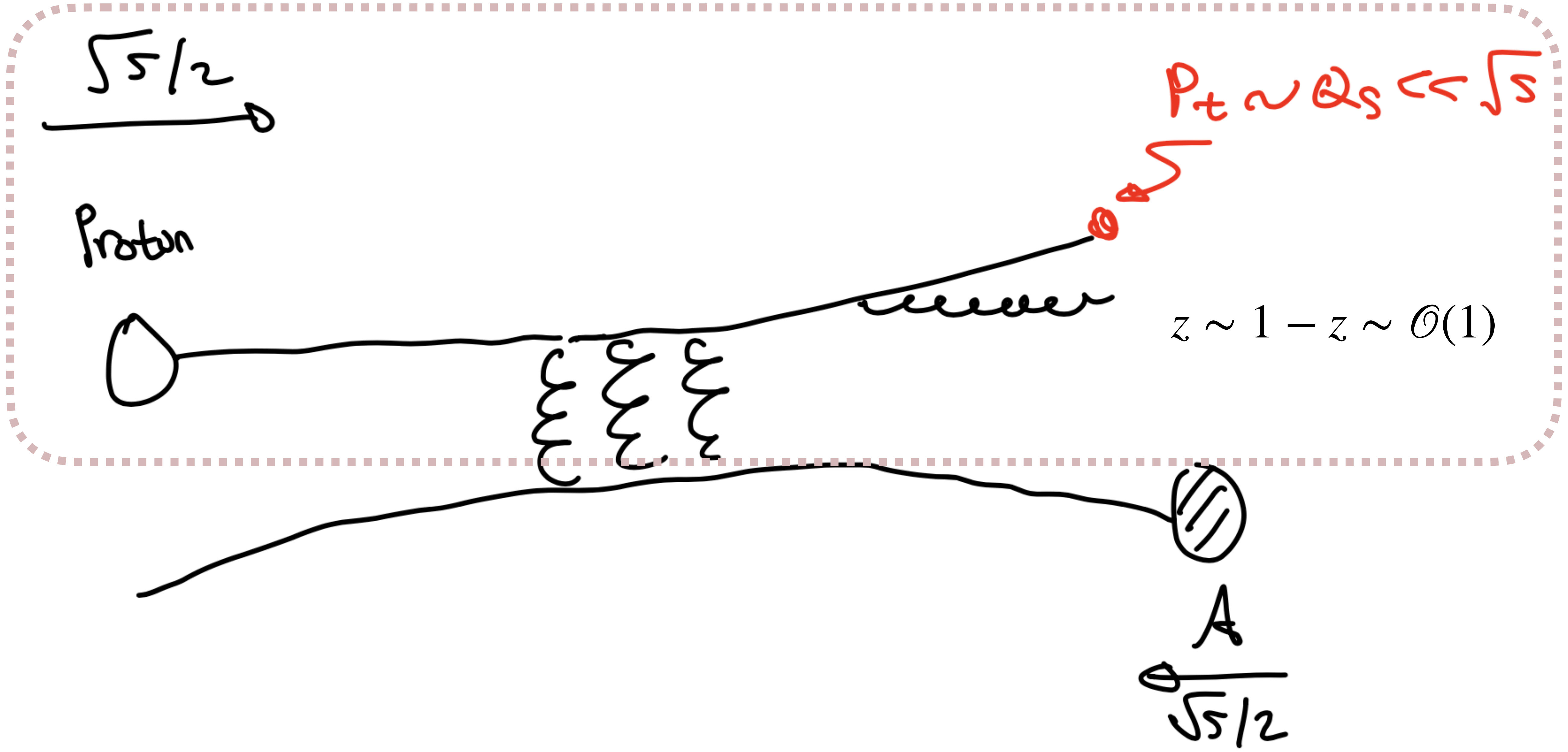
$$\phi = \sum_{i=H,S,J} \phi_i$$

$$\sigma(v) \to_{v\to 0} H \times S \times J_i \ldots$$



- Looking for configurations that makes internal line almost on-shel
- v will enforce the configurations

Modes in pA forward scattering

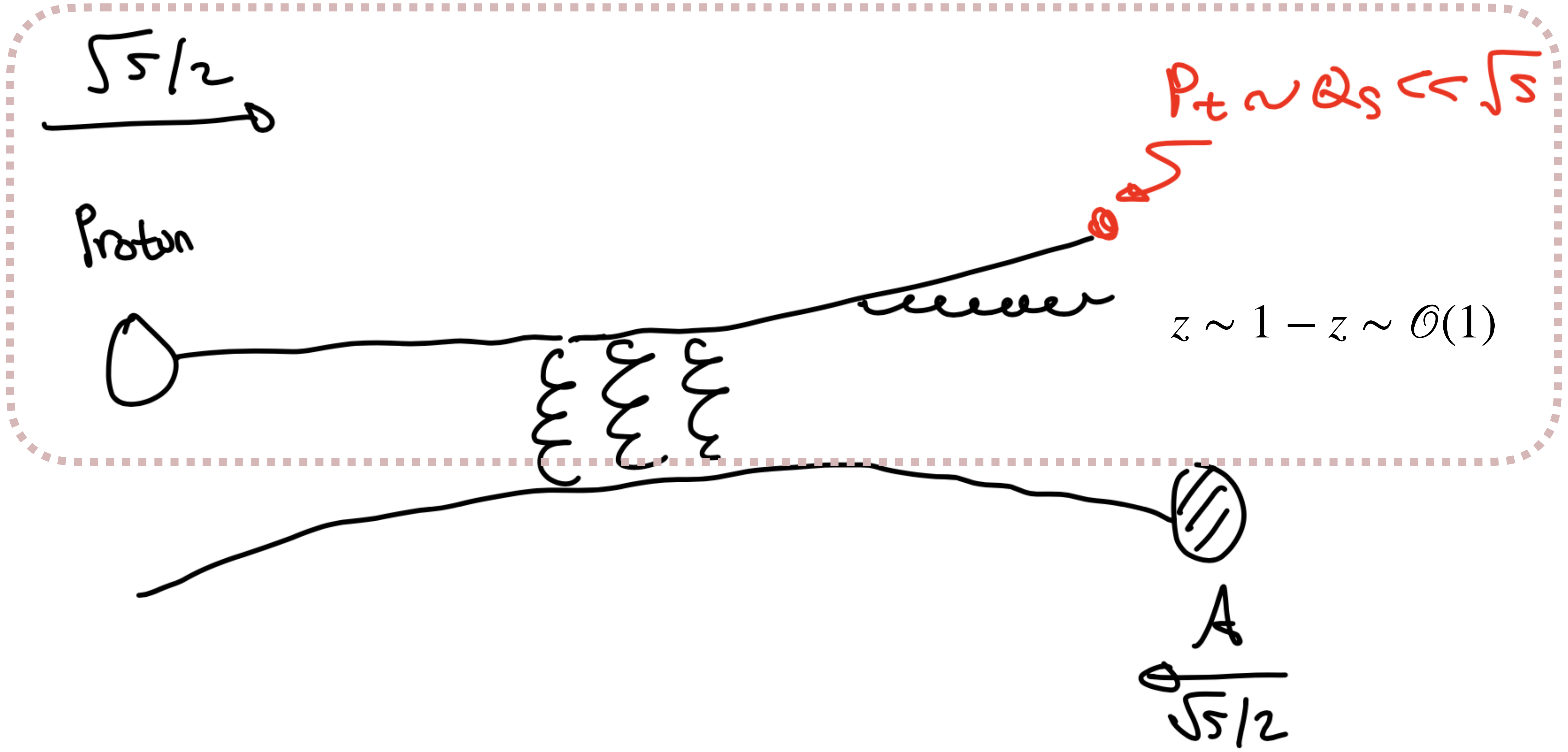


$z \sim 1 - z \sim \mathcal{O}(1)$

$$v \sim p_t/\sqrt{s} \sim Q_s/\sqrt{s} \ll 1$$

$$p^+ \equiv p^0 + p^3\,, \qquad p^- \equiv p^0 - p^3$$

Modes in pA forward scattering



$$v \sim p_t/\sqrt{s} \sim Q_s/\sqrt{s} \ll 1$$

$+ \; t \; -$

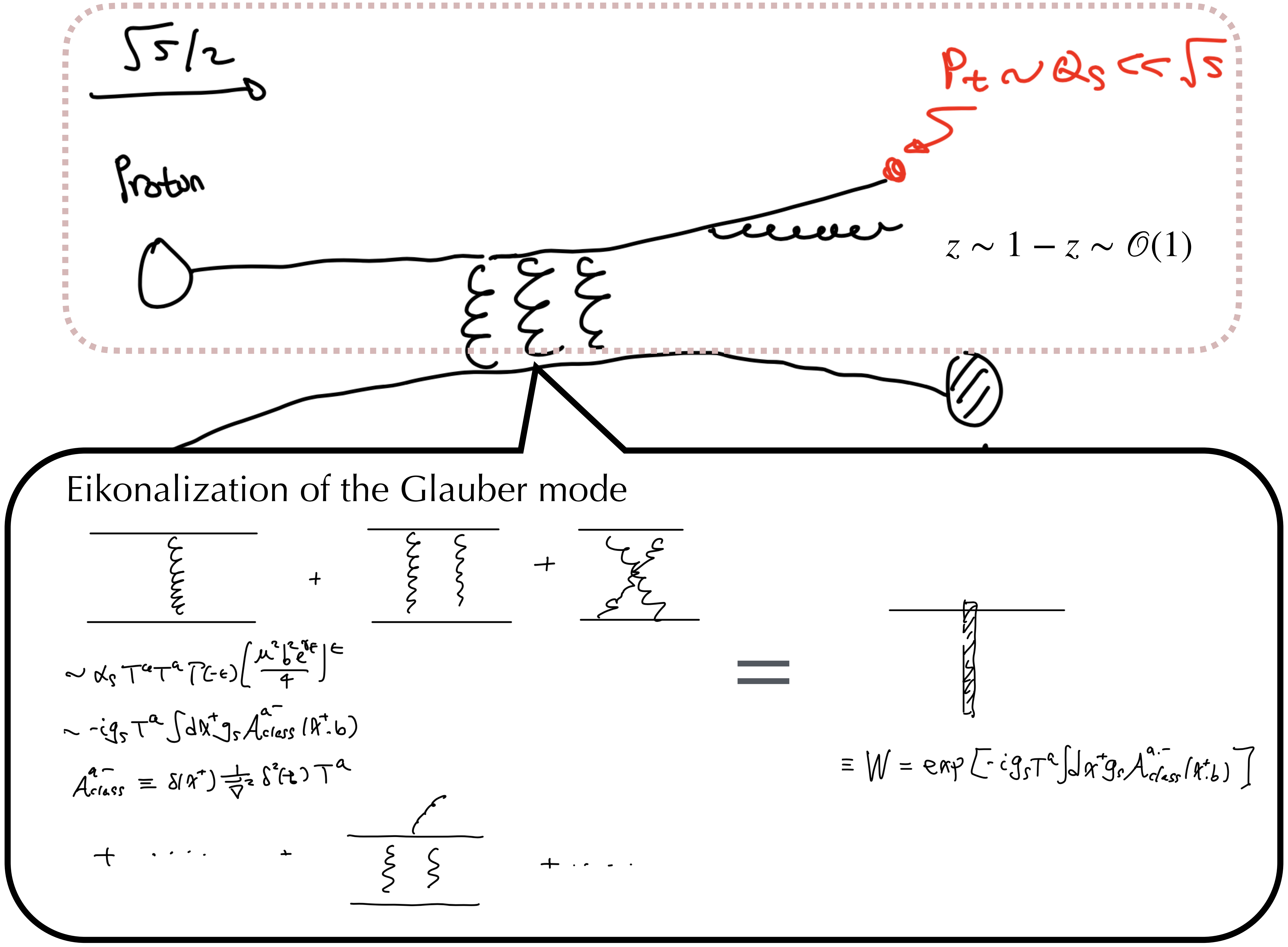
Collinear: $P_c = \frac{\sqrt{s}}{2}(1, v, v^2)$

$\bar{\text{C}}$ollinear: $P_{\bar{c}} = \frac{\sqrt{s}}{2}(v^2, v, 1)$

Glauber: $q \sim \Delta P_c \sim \Delta P_{\bar{c}} \sim \frac{\sqrt{s}}{2}(v^2, v, v^2)$

$$p^+ \equiv p^0 + p^3, \qquad p^- \equiv p^0 - p^3$$

Modes in pA forward scattering



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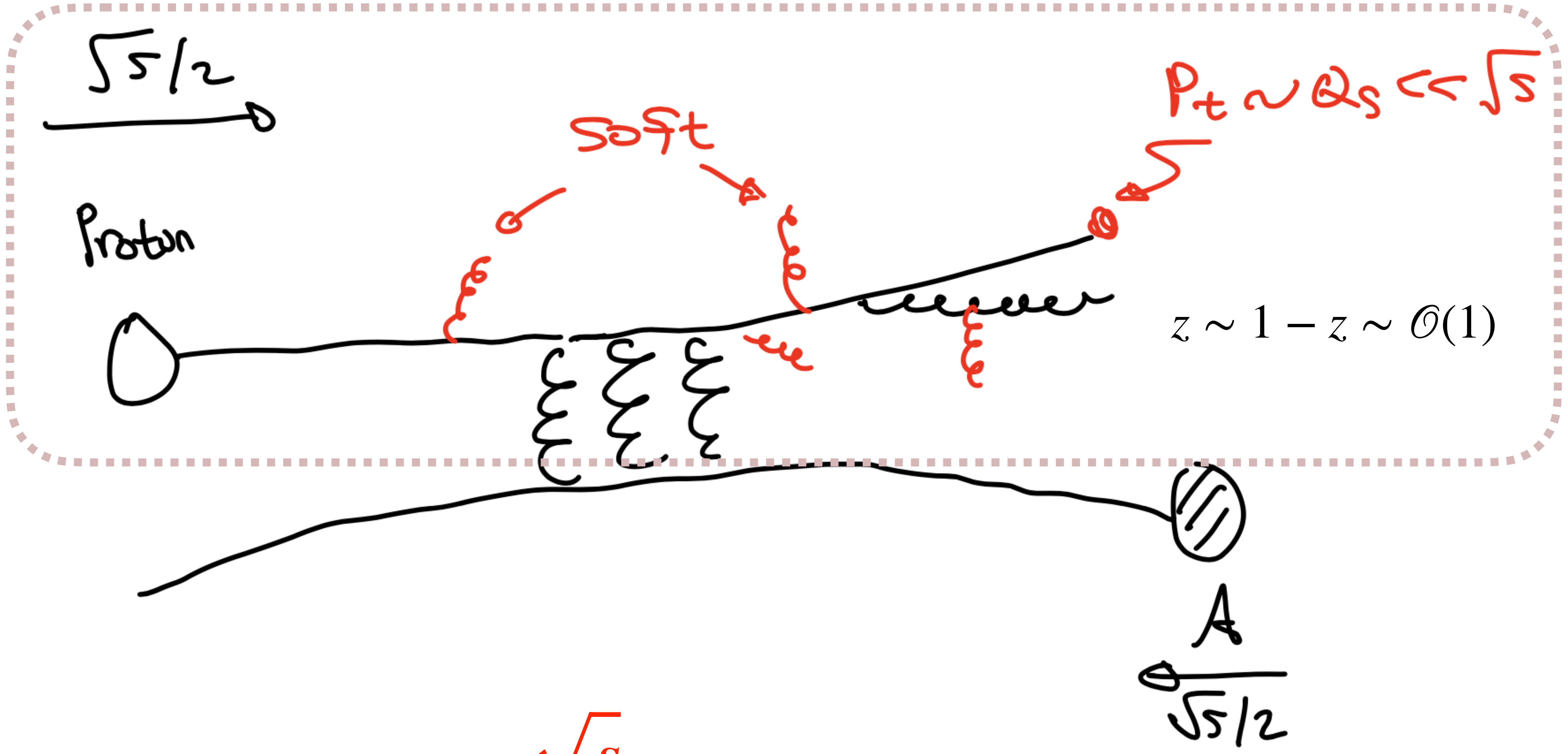
Glauber: $q \sim \Delta P_c \sim \Delta P_{\bar{c}} \sim \frac{\sqrt{s}}{2}(v^2, v, v^2)$

Reproduce CGC (modes) and the CGC "Wilson line" (shock wave)

Modes in pA forward scattering



$$v \sim p_t/\sqrt{s} \sim Q_s/\sqrt{s} \ll 1$$

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Glauber: $q \sim \Delta P_c \sim \Delta P_{\bar{c}} \sim \frac{\sqrt{s}}{2}(v^2, v, v^2)$

Soft: $P_s \sim \frac{\sqrt{s}}{2}(v, v, v)$

- Absent in the CGC EFT
- Crucial for kinematic constraint, producing correctly the poles …
- Threshold resummation

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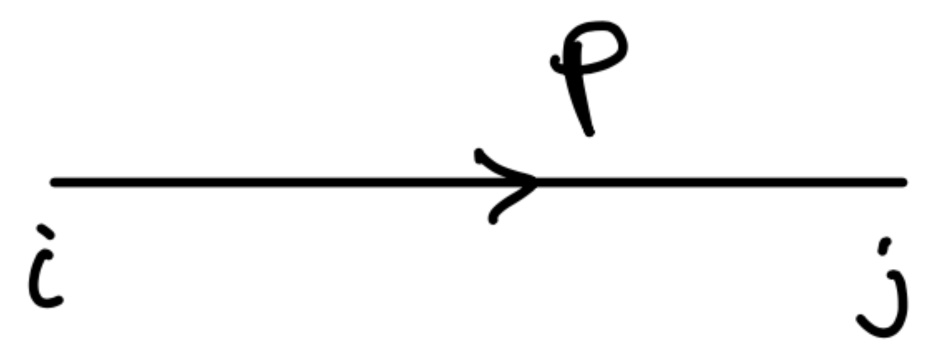
Interaction in pA forward scattering

Start with the QCD Lagrangian and expand it in terms of v: $\phi = \sum_{i=modes} \phi_i$

- Collinear propagators:



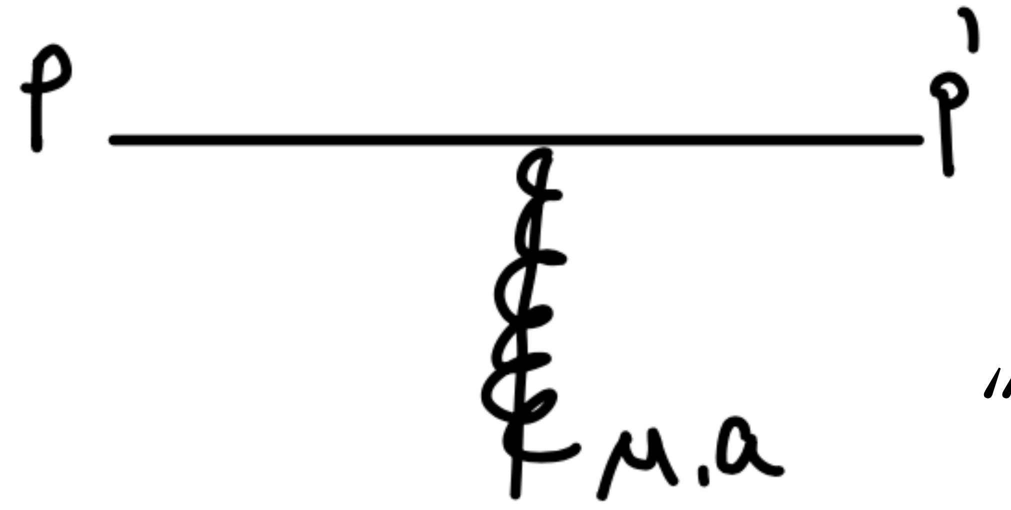
$$= \delta_{ij} \frac{i p^+}{p^2 + i0^+} \frac{\not{n}}{2}$$

$$\not{p} = p^+ \frac{\not{n}}{2} + p^- \frac{\not{\bar{n}}}{2} + \not{p}_t$$

Keep the leading contribution in v

- Collinear-collinear interaction:

Similarly we have gluon Feyn rules



$$= i g_s T^a \left(n^\mu + \frac{\gamma_t^\mu \not{p}_t}{p^+} + \frac{\not{p}_t' \gamma_t^\mu}{p'^+} \right)$$

"Bad components in terms of the good components"

$n = (1,0,0,1)$, $\bar{n} = (1,0,0,-1)$ $\bar{n} \cdot A = 0$

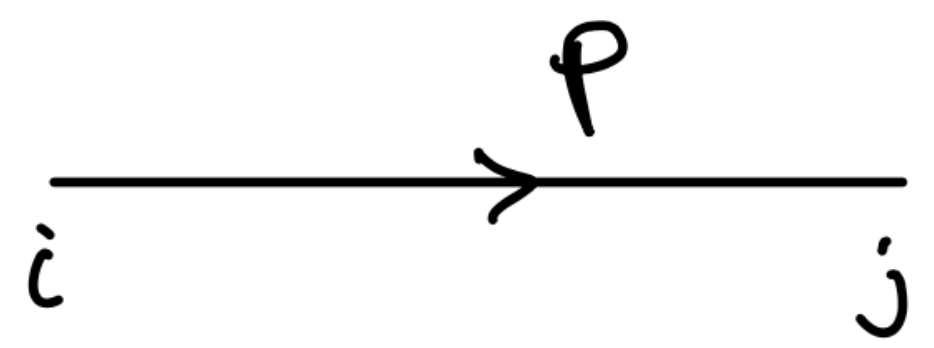
Interaction in pA forward scattering

Start with the QCD Lagrangian and expand it in terms of v: $\phi = \sum_{i=modes} \phi_i$

$$\langle i_c\, j_b \cdots | T_i^a | i_{c'}\, j_{b'} \cdots \rangle = T^a_{cc'}\, \delta_{bb'} \cdots$$

ith particle

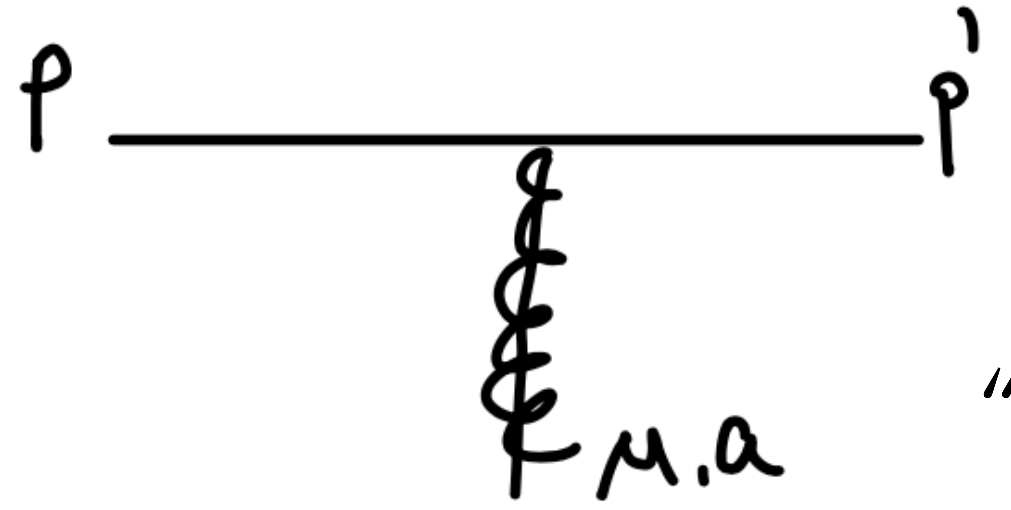
- Collinear propagators:



$$= \delta_{ij} \frac{i p^+}{p^2 + i0^+} \frac{\not{\bar{n}}}{2}$$

$$T^a_{cc'} = \begin{cases} t^a_{cc'} & \text{quark} \\ -t^a_{c'c} & \text{anti-quark} \\ i f_{cac'} & \text{gluon} \end{cases}$$

- Collinear-collinear interaction:



$$= i g_s T^a \left(n^\mu + \frac{\gamma_\perp^\mu \not{p}_\perp}{p^+} + \frac{\not{p}'_\perp \gamma_\perp^\mu}{p'^+} \right)$$

Rotation in the color space when emitting a gluon

"Bad components in terms of the good components"

$n = (1,0,0,1)$, $\quad \bar{n} = (1,0,0,-1)$ $\quad \bar{n} \cdot A = 0$

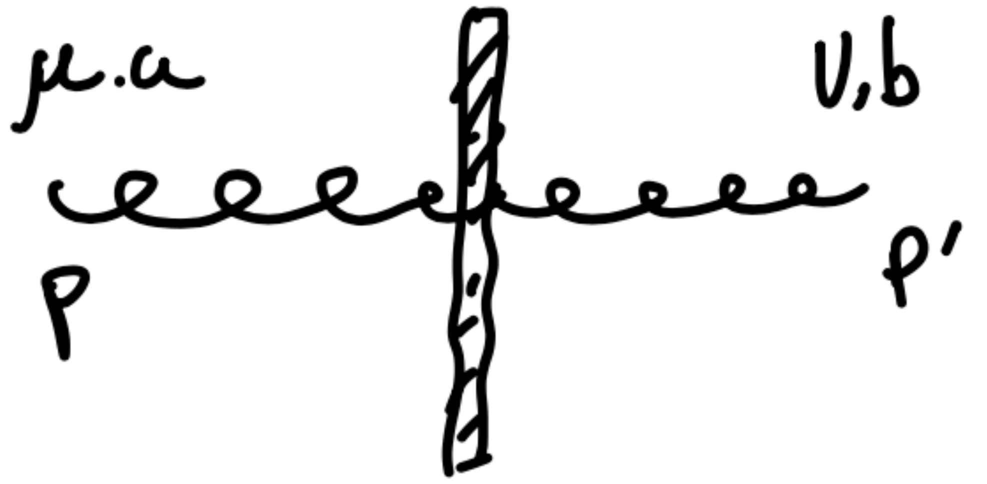
Interaction in pA forward scattering

Start with the QCD Lagrangian and expand it in terms of v: $\phi = \sum_{i=modes} \phi_i$

- Interaction with soft (eikonal):

$$= -g_s T^a \frac{n^\mu}{k^-}$$

- Interaction with the Glauber (or shockwave interaction):



$$= -g_{\mu\nu}(4\pi)\delta(p'^+ - p^+)\, p^+ \int d\vec{b}_t\, e^{i(\vec{p}'_t - \vec{p}_t)\cdot \vec{b}_t}\, W_{ba}(\vec{b}_t)$$

Same for collinear and soft, due to small Glauber "+" component, $\sim \mathcal{O}(v^2)$

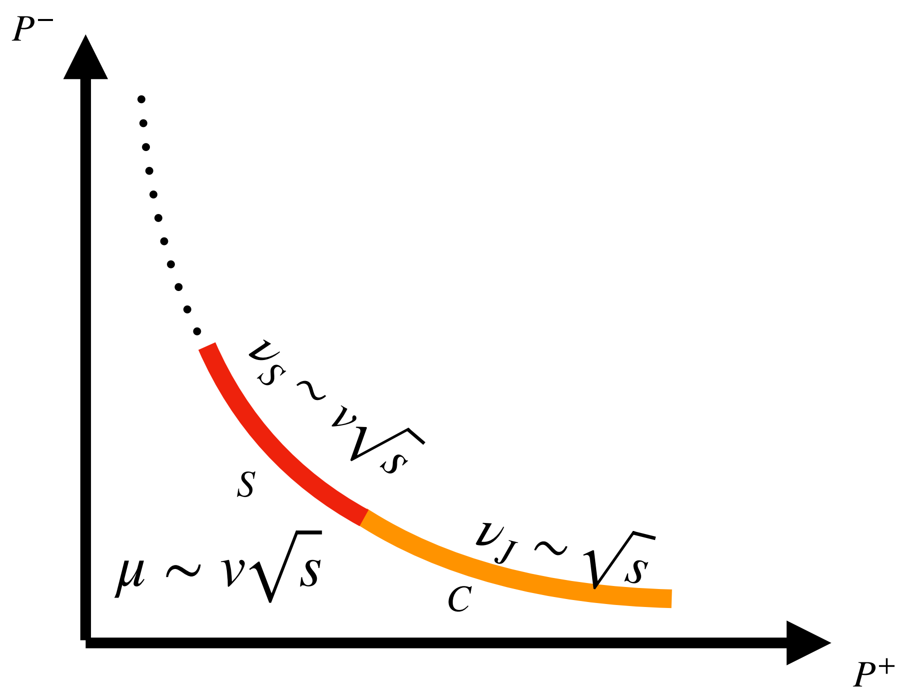
$n = (1,0,0,1)$, $\bar{n} = (1,0,0,-1)$ $\bar{n}\cdot A = 0$

Interaction in pA forward scattering

Start with the QCD Lagrangian and expand it in terms of v: $\phi = \sum_{i=modes} \phi_i$

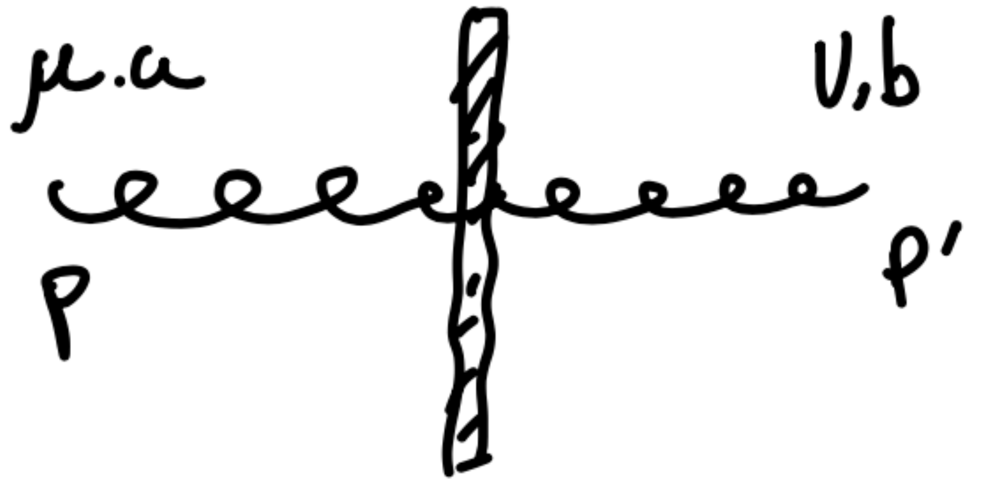
- Interaction with soft (eikonal):

$$= -g_s T^a \frac{n^\mu}{k^-} \longrightarrow -g_s T^a \frac{n^\mu}{k^-}\left(\frac{\nu}{k_t}\right)^{\eta/2} e^{-\eta/2\,|\eta_k|}$$



Rapidity regulator (similar to TMD),
ν: rapidity scale

- Interaction with the Glauber (or shockwave interaction):



$$= -g_{\mu\nu}(4\pi)\delta(p'^+ - p^+)\,p^+\int d\vec{b}_t\, e^{i(\vec{p}\,'_t - \vec{p}_t)\cdot \vec{b}_t}\, W_{ba}(\vec{b}_t)$$

Same for collinear and soft, due to small Glauber "+" component, $\sim \mathcal{O}(v^2)$

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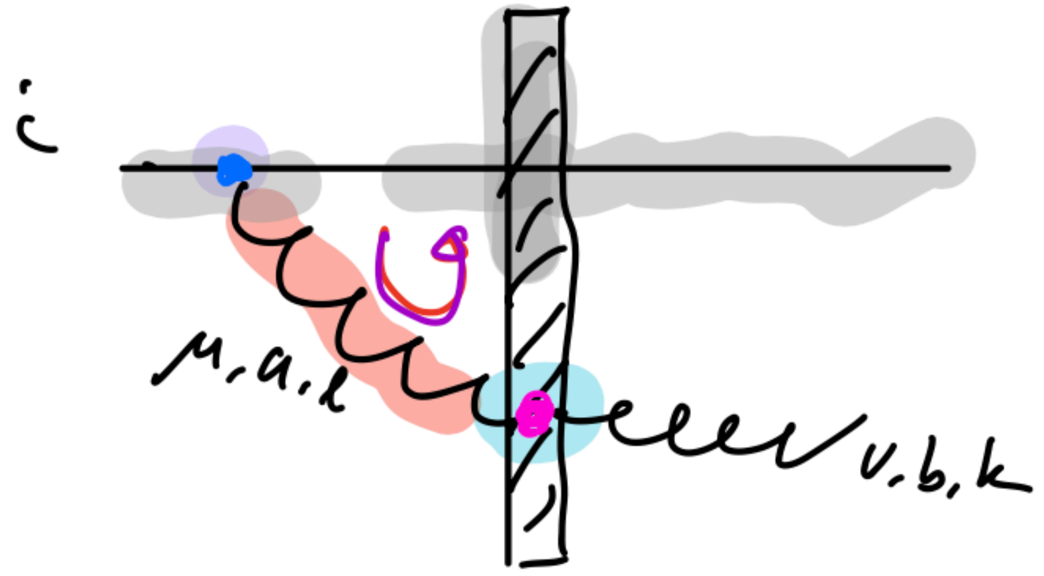
Interaction in pA forward scattering

e.g., Soft ISR:



Typical color structure in the small-x physics

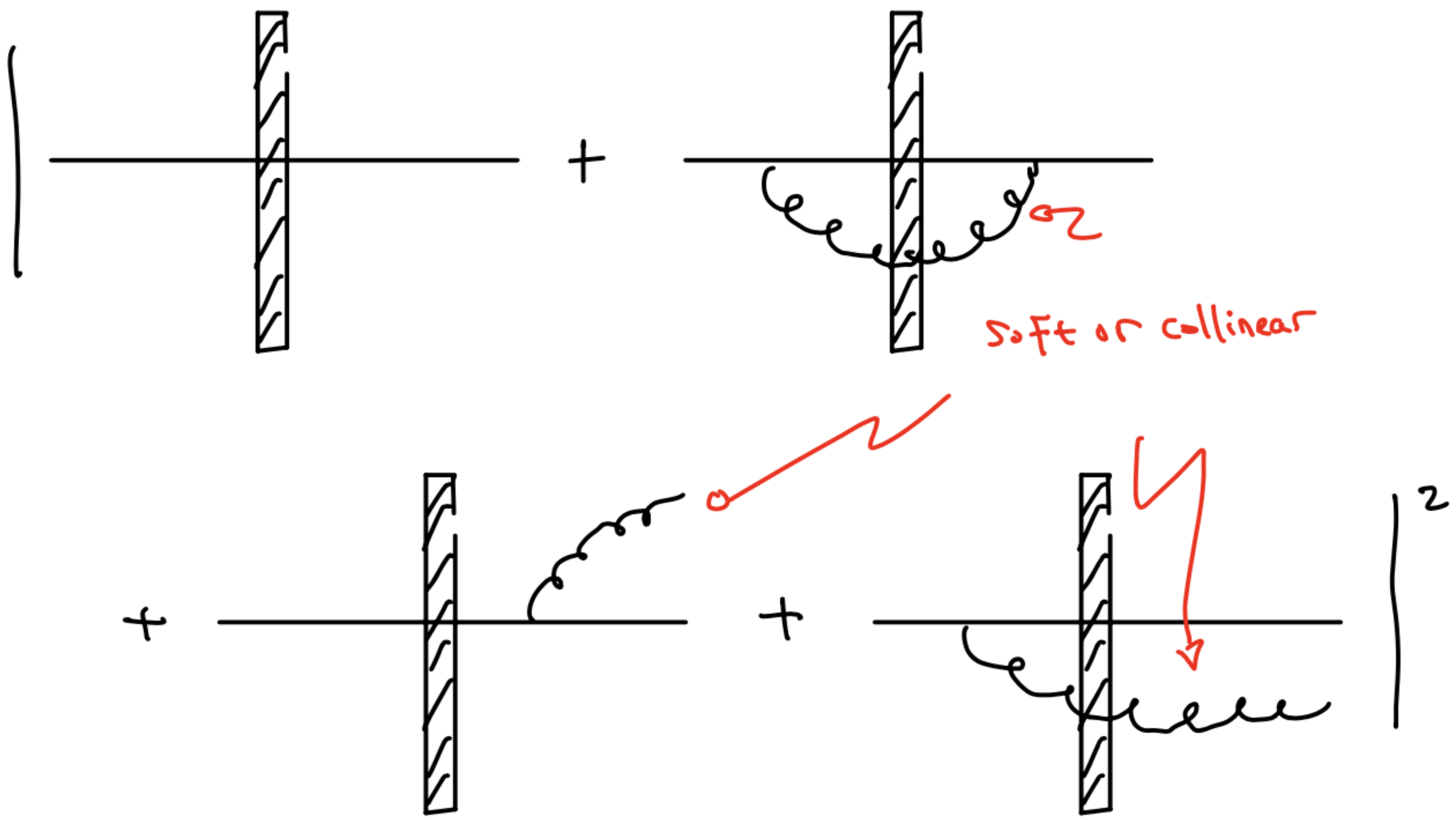
$$= -g_s \int \frac{d^d l}{(2\pi)^d} \frac{n^\mu}{n\cdot l} \left(\frac{\nu}{l_t}\right)^{\eta/2} e^{-\frac{\eta}{2}|y_l|} \frac{i\, d_{\mu\nu}}{l^2 + i0^+} (-\epsilon^\nu) (4\pi)\, \delta(k^+ - l^+)\, k^+$$

$$\int d\vec{b}_t\, e^{i(\vec{k}_t - \vec{l}_t)\cdot \vec{b}_t} \boxed{W_{ba}(\vec{b}_t)\, T_i^a}\, |W(\vec{r}_t)\rangle$$

$$= \frac{i g_s}{(4\pi)^{1+\epsilon}} \mu^{2\epsilon} \nu^{\eta/2} e^{-\frac{\eta}{2}|y_k|} \frac{\Gamma[1-\epsilon-\eta/4]}{\Gamma[1+\eta/4]} \int d\vec{b}_t\, b_t^\nu \left[\frac{b_t^2}{4}\right]^{-1+\epsilon+\eta/4} e^{-i\, k_t \cdot b_t}$$

$$\times\, W_{ba}(b_t)\, T_i^a\, |W(\vec{r}_t)\rangle$$

pA forward scattering: generic case



- $z \sim 1 - z \gg v$ … (1)
- Both collinear and soft radiations are allowed
- Same topology for both collinear and soft radiation, but different Feyn. Rules
- Different phase space constraint. e.g. Soft $\propto \delta(1 - z - E_g) \approx \delta(1 - z)$, since (1)

pA forward scattering: generic case

- Reproduce the known NLO calculation

- Pole terms:

Collinear:

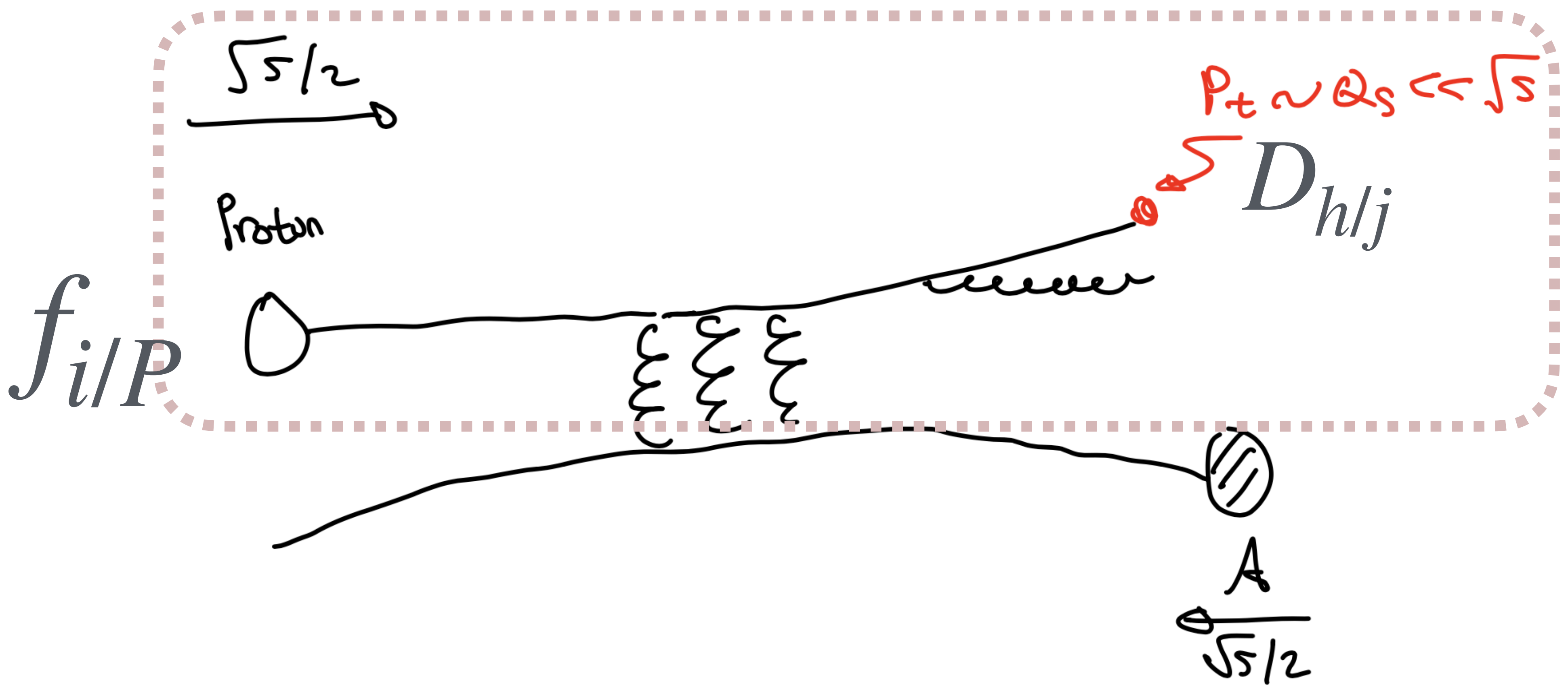
ISR FSR

$$-\frac{\alpha_s}{2\pi\epsilon}P_{ij}(z)\left[1+\frac{1}{z^2}e^{-i\frac{1-z}{z}\vec{p}_t\cdot\vec{r}_t}\right]\langle W_{ij}|\delta_{jj'}|W_{j'i}\rangle$$

$$-\frac{\alpha_s}{\pi\eta}\left[1+\eta\log\frac{\nu}{p_t}\right]\frac{1}{\pi}\left[\frac{\vec{r}_t^{\,2}}{\vec{r}_t^{\,\prime 2}\,\vec{r}_t^{\,\prime\prime 2}}\right]_+\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i}\rangle$$

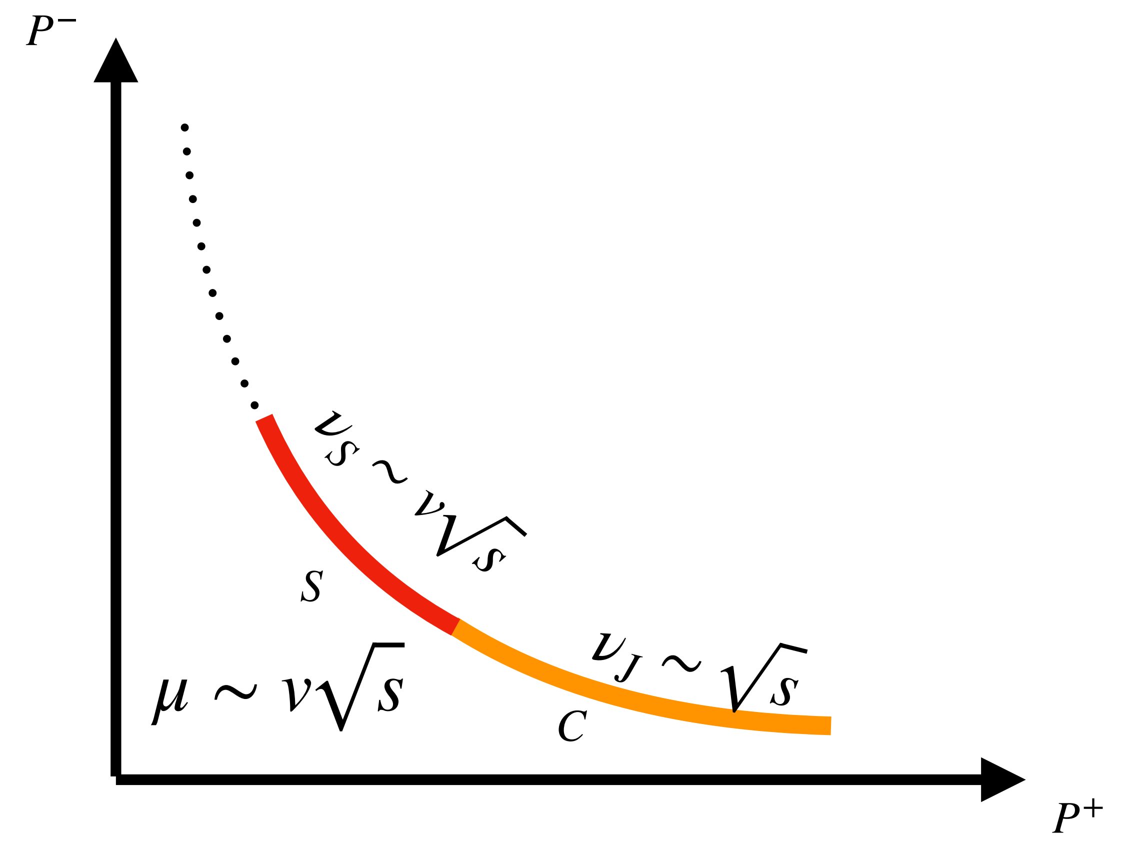
Soft:

$$\frac{2\alpha_s}{\pi\eta}\left[1+\eta\log\frac{\nu}{p_t}\right]\frac{1}{\pi}\left[\frac{\vec{r}_t^{\,2}}{\vec{r}_t^{\,\prime 2}\,\vec{r}_t^{\,\prime\prime 2}}\right]_+\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i}\rangle$$



- ϵ^{-1} poles in the collinear sector, proportional to the splitting function, to be absorbed by the PDF and FF, leads to DGLAP evolution

pA forward scattering: generic case



- Reproduce the known NLO calculation
- Pole terms:

Collinear:

ISR FSR

$$-\frac{\alpha_s}{2\pi\epsilon} P_{ij}(z)\left[1+\frac{1}{z^2}e^{-i\frac{1-z}{z}\vec{p}_t\cdot\vec{r}_t}\right]\langle W_{ij}|\delta_{jj'}|W_{j'i}\rangle$$

$$-\frac{\alpha_s}{\pi\eta}\left[1+\eta\log\frac{\nu}{p^+}\right]\frac{1}{\pi}\left[\frac{\vec{r}_t^{\,2}}{\vec{r}_t^{\,\prime 2}\,\vec{r}_t^{\,\prime\prime 2}}\right]_+\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i}\rangle$$

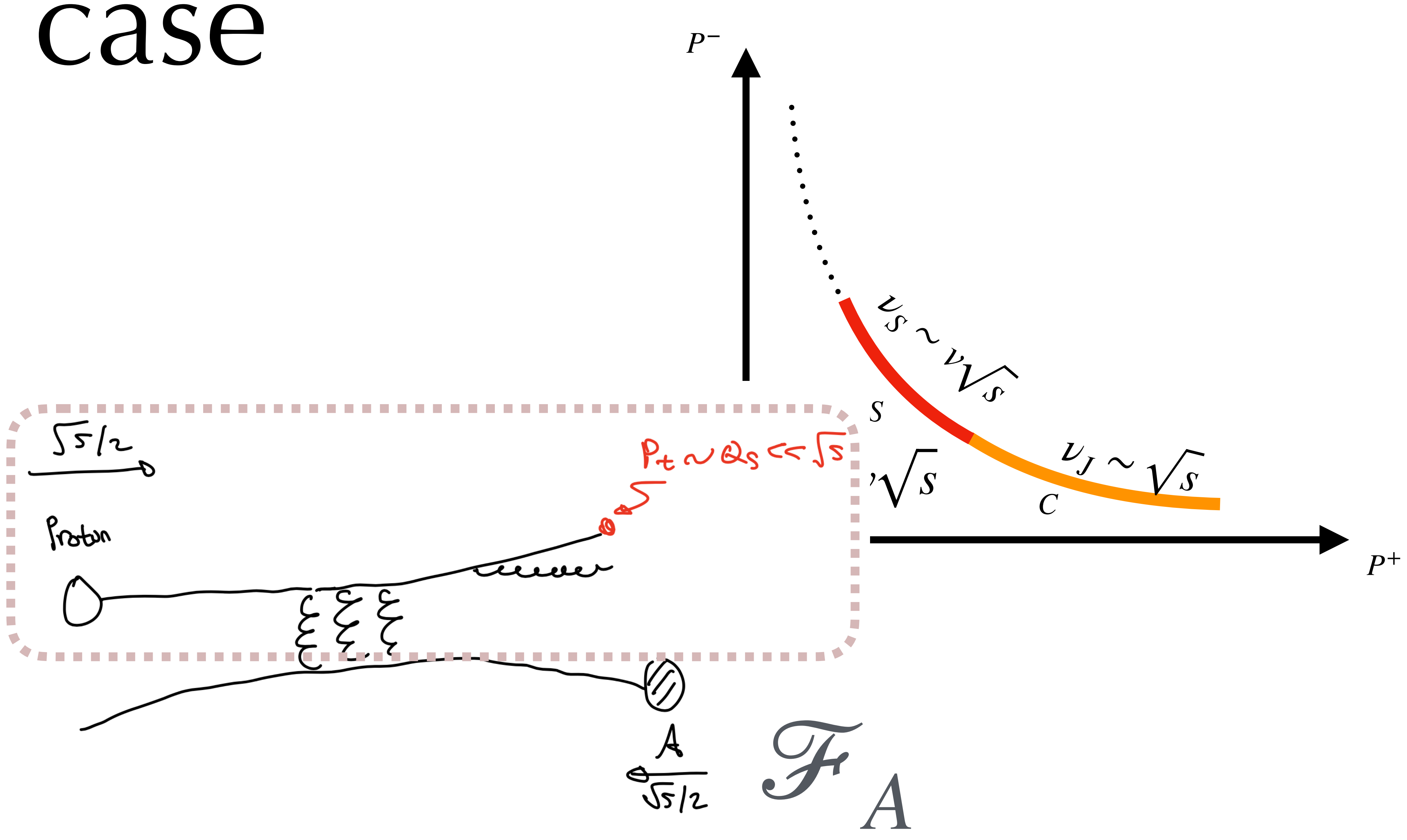
- ϵ^{-1} poles in the collinear sector, proportional to the splitting function, to be absorbed by the PDF and FF, leads to DGLAP evolution

Soft:

$$\frac{2\alpha_s}{\pi\eta}\left[1+\eta\log\frac{\nu}{p_t}\right]\frac{1}{\pi}\left[\frac{\vec{r}_t^{\,2}}{\vec{r}_t^{\,\prime 2}\,\vec{r}_t^{\,\prime\prime 2}}\right]_+\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i}\rangle$$

- Additional rapidity η^{-1} poles in both sectors, characterizing the rapidity divergence as $P^- \to 0$ in coll. or $P^{\pm} \to 0$ in soft
- Logs indicate the natural scale for each sector

pA forward scattering: generic case



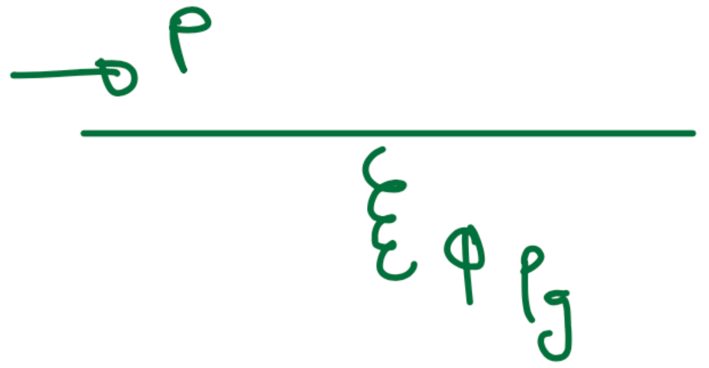
- Reproduce the known NLO calculation

- Pole terms:

Collinear + Soft:

$$\frac{\alpha_s}{\pi\eta}\left[1+\eta\log\frac{\nu}{p_t^2/p^+}\right]\frac{1}{\pi}\left[\frac{\vec{r}_t^{\,2}}{\vec{r}_t'^{\,2}\,\vec{r}_t''^{\,2}}\right]_+\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i'}\rangle$$

& $p_g^- \sim x_A \cdot P_A$ small-x scale!



$$(p+p_g)^2 = p^+p_g^- - \vec{p}_{gt}^{\,2} = 0$$

$$\Rightarrow p_g^- = \frac{\vec{p}_{gt}^{\,2}}{p^+}$$

- η^{-1}-pole to be absorbed by the nucleus structure function $\mathscr{F}_A$
- Log indicates correctly the small-x scale
- η^{-1}-pole produces the rapidity RG equation for the nucleus structure function = BK!

$$\nu\frac{d}{d\nu}\langle W_{ij}|\delta_{ii'}|W_{j'i'}\rangle$$

$$= \frac{\alpha_s}{\pi}\int\frac{d^2\vec{x}_t}{\pi}\left[\frac{\vec{r}_t^{\,2}}{\vec{r}_t'^{\,2}\,\vec{r}_t''^{\,2}}\right]_+\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i'}\rangle$$

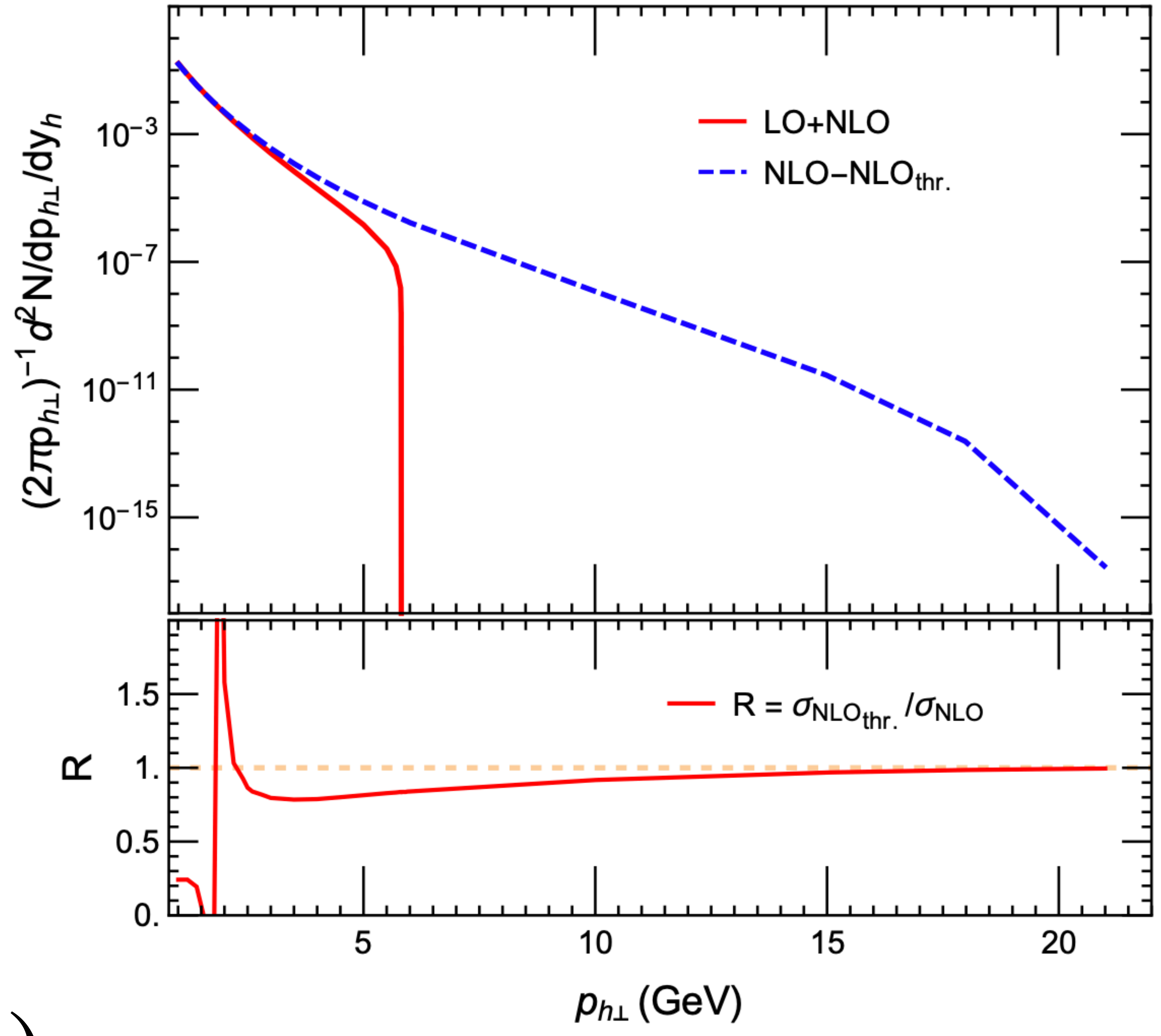
pA forward scattering: generic case

- Finite terms:

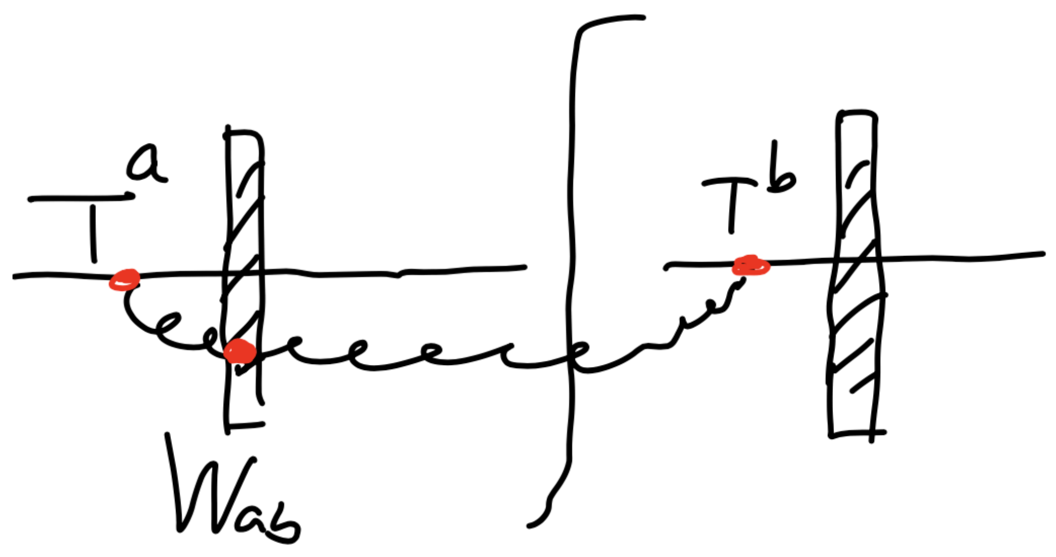
Collinear + Soft:

$$-\frac{\alpha_s}{2\pi} P(z) \ln\frac{\vec{r}_t^{\,2}\mu^2}{c_0^2}\left(1+\frac{1}{z^2} e^{-i\frac{1-z}{z}\vec{p}_t\cdot\vec{r}_t}\right)\langle W_{ij}|\delta_{jj'}|W_{j'i}\rangle$$

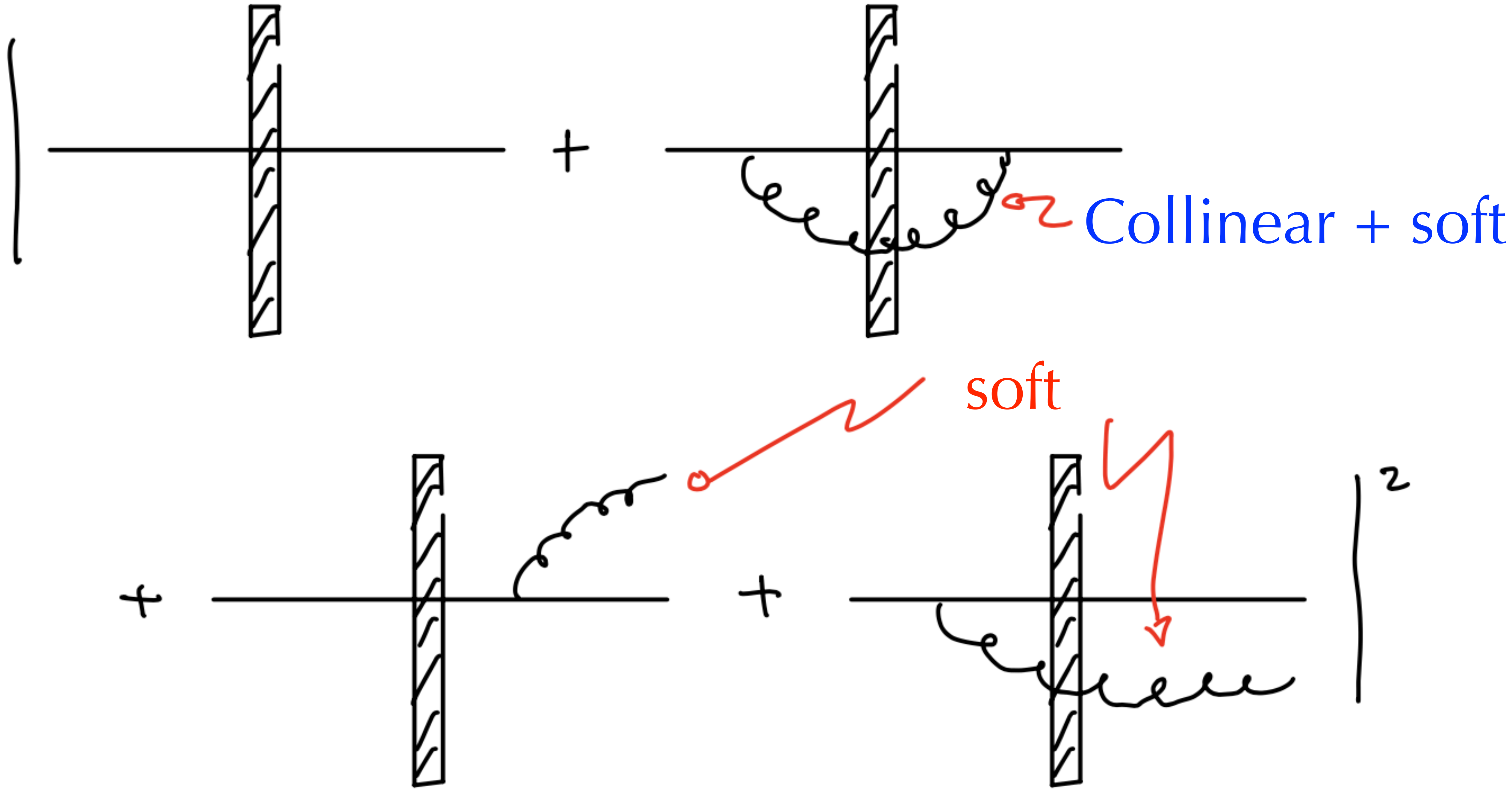
$$+\frac{\alpha_s}{\pi}\left\{\frac{1}{z}\tilde{P}(z) e^{-i\frac{1-z}{z}\vec{p}_t\cdot\vec{r}_t'}\frac{\vec{r}_t{}'\cdot\vec{r}_t{}''}{\vec{r}_t{}'^2\,\vec{r}_t{}''^2}+\cdots\right\}\langle W_{ij}|\mathbf{T}^a W_{ab}\mathbf{T}^b|W_{j'i}\rangle$$



- $z \to 1$, huge threshold logs, $P(z) \sim \tilde{P}(z) \sim 2/(1-z)_+$
- Threshold PDF and FF can be eliminated by $\mu \sim c_0/|r_t|$
- $\tilde{P}$ term: non-LO kinematic factor + color flow (BK real) due to interference



pA forward scattering: threshold case



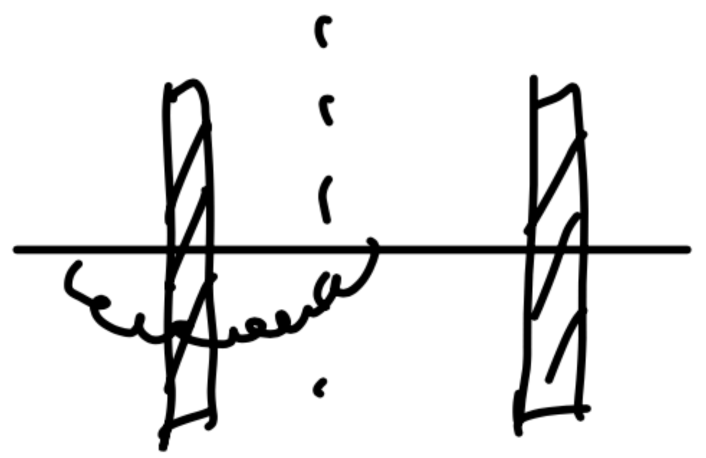
- $z \sim 1 \gg 1 - z \sim \nu \ldots$ (2)
- Only soft real emission are allowed
- Phase space constraint. Soft $\propto \delta(1 - z - E_g) \approx \cancel{\delta(1 - z)}$, can NOT be further expanded, since (2)

η^{-1}-pole $\Longrightarrow$ $U_J U_s =$

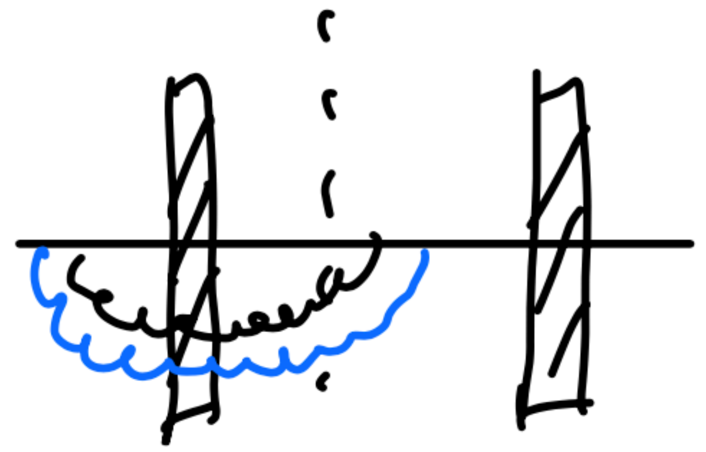
$\int_0^1 \frac{z^{N-1}}{(1-z)_+} dz \to -\ln(\bar{N})$ $\exp\left[\frac{\alpha_s}{\pi}\int\frac{d^2 x_t}{\pi}\left(-2\ln\bar{N}\,\frac{\vec{r}_t'\cdot\vec{r}_t''}{r_t'^2\, r_t''^2} + \ln\frac{\nu}{x_A P_A} I_{BK}\right) T^a W_{ab} T^b\right]$

Large threshold logs automatically resummed by rapidity RGE

pA forward scattering: threshold case



$$\sim T^a W_{ab} T^b$$



$$\sim \frac{1}{2!} T^a W_{ab} T^b \, T^{a'} W_{a'b'} T^{b'}$$

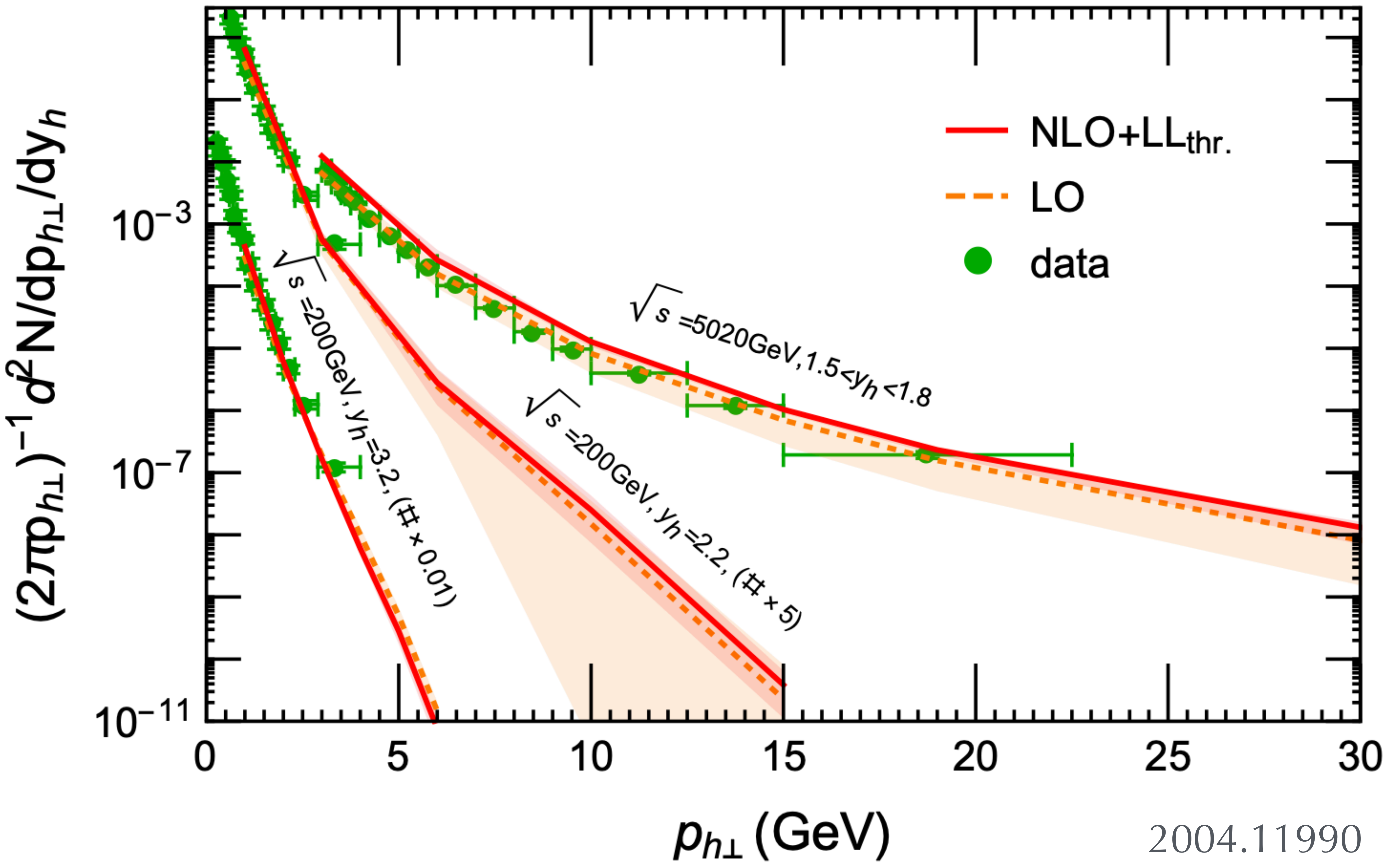
$$= \frac{1}{2!} \left(T^a W_{ab} T^b \right)^2$$

$$\vdots$$

$$= \quad U_J U_S = \exp\left[\frac{\alpha_s}{\pi} \int \frac{d\vec{x}_t}{\pi} \left(-2 \ln N \frac{\vec{r}_t' \cdot \vec{r}_t''}{r_t'^2 \, r_t''^2} + \ln \frac{\nu}{n x_A P_A} I_{BK} \right) T^a W_{ab} T^b \right]$$

Can also reproduce the RGE result by summing all order emissions in the strong order limit (LL)

pA forward scattering

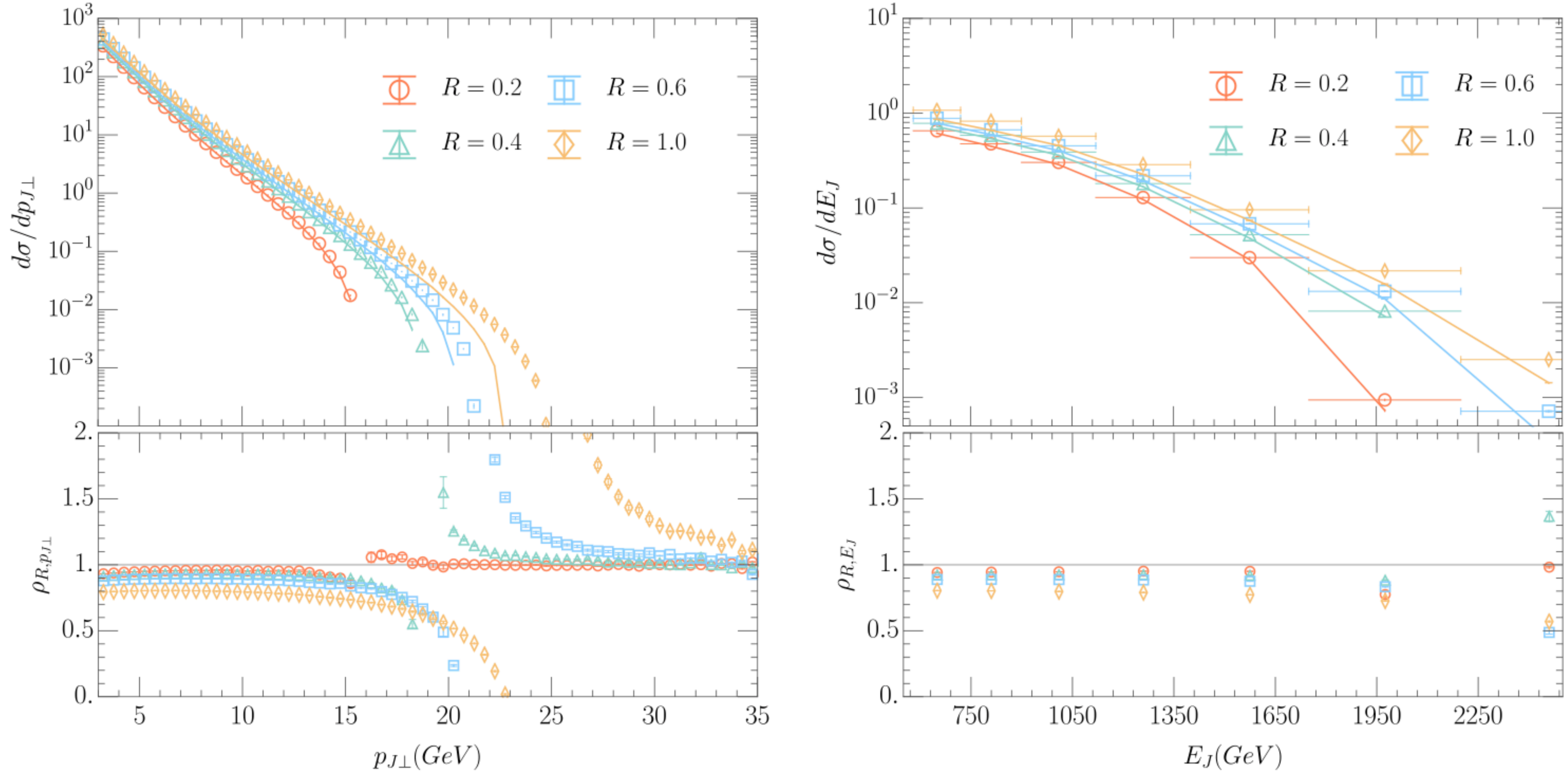


2004.11990

Good agreement with current data

pA forward scattering

$$\sqrt{s} = 5.02\,\mathrm{TeV}\,, \quad 5.2 < \eta_J < 6.6\,, \quad \text{anti-}k_T$$



Full jet algorithm (numerical monte carlo) v.s. small-R approximation (analytic)

2204.03026

Thanks!