NANJING UNIVERSITY

Craig Roberts … http://inp.nju.edu.cn/

Grant no. 12135007

国家自然科学基金委员会
National Natural Science Foundation of China
NSFC

Hadron Structural Features Behind the Efficacy of ChPT

ChEFT

- Basic Features
  - Systematic, order-by-order expansion of hadron and nuclear observables in powers of the ratio of a low momentum to a high-energy scale
- Success depends on dynamical chiral symmetry breaking (DCSB) in QCD
  - Emergence of a large mass-scale against which some hadron masses and momenta may be considered small
- The large scale is supposed to be $\approx 32\,\pi\, f_\pi^2 \approx 1\,\mathrm{GeV}^2$ (see $\pi\pi$ scattering)
- How does that scale emerge in a theory described by a scale-free Action?
  - $f_\pi = 0$ in a scale-free theory
  - $m_p = 0$ in a scale-free theory
- This is the question that underlies all of physics:
  - How does the proton mass emerge from a massless, scale invariant theory?

The Pion

"There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

- The pion is the basic degree of freedom in ChEFT
- Many practitioners take it for granted
- However, the pion is NOT elementary, and its structure is NOT understood
  - Form factor?
  - Distribution amplitude?
  - Distribution functions … valence, sea, glue?
  - Fragmentation functions?
  - Generalised parton DFs?
  - Transverse momentum dependent DFs
  - How do pion DFs differ from those of the proton, which are also unknown on a large domain of light-front momentum fraction

Quantum Chromodynamics

$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\,\frac{\lambda^a}{2}\gamma \cdot A^a(x)\right]\psi(x)$$

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f^{abc} A^b_\mu(x) A^c_\nu(x)$$

- One-line Lagrangian – expressed in terms of massless gluon (A^a_μ) and Higgs-mass quark (ψ) partons
- Which are NOT the degrees-of-freedom measured in detectors

Gluon and Quark Confinement

Advantages of the Color Octet Gluon Picture,
H. Fritzsch, Murray Gell-Mann, H. Leutwyler, Phys. Lett. B 47 (1973) 365-368

- "The quarks come in three "colors," but all physical states and interactions are supposed to be singlets with respect to the SU(3) of color. Thus, we do not accept theories in which quarks are real, observable particles; nor do we allow any scheme in which the color non-singlet degrees of freedom can be excited."

Quantum Chromodynamics

$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\, \frac{\lambda^a}{2} \gamma \cdot A^a(x)\right] \psi(x)$$

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f^{abc} A^b_\mu(x) A^c_\nu(x)$$

Gluon and Quark Confinement

- One-line Lagrangian – expressed in terms of gluon and quark partons
- Which are NOT the degrees-of-freedom measured in detectors

Questions

- What are the (asymptotic) detectable degrees-of-freedom?
- How are they built from the Lagrangian degrees-of-freedom?
- Is QCD really the theory of strong interactions?
- Is QCD really a theory … or just another EFT?

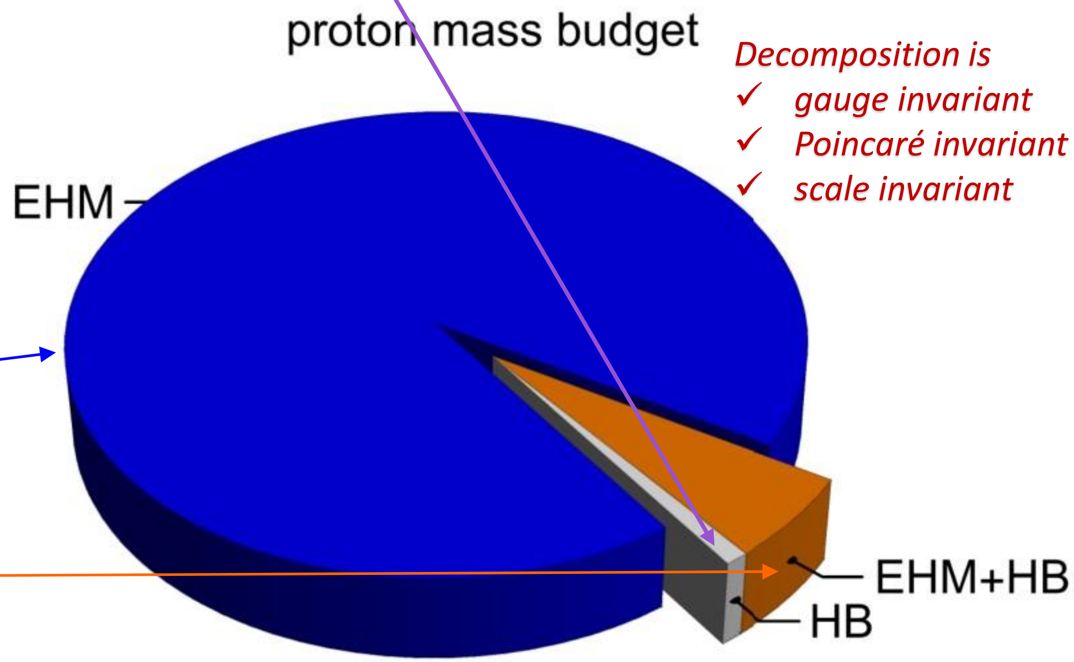
  ⇒ Implications far beyond Standard Model

Emergence of Hadron Mass

- Standard Model of Particle Physics has one obvious mass-generating mechanism

  = Higgs Boson … impacts are critical to evolution of Universe as we know it
- However, Higgs boson alone is responsible for just ~ 1% of the visible mass in the Universe
- Proton mass budget … only 9 MeV/939 MeV is directly from Higgs
- Evidently, Nature has another very effective mechanism for producing mass:

  Emergent Hadron Mass (EHM)
  - ✓ Alone, it produces 94% of the proton's mass
  - ✓ Remaining 5% is generated by constructive interference between EHM and Higgs-boson
- ***What is the origin of EHM?***



Decomposition is
- ✓ *gauge invariant*
- ✓ *Poincaré invariant*
- ✓ *scale invariant*

GENESIS

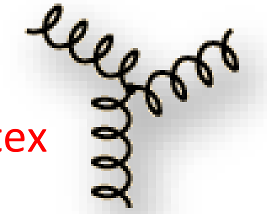
GLUON

In the Beginning…

Ancient History

Cornwall's estimate: $\mathrm{m_g} = 0.5(2)\mathrm{GeV} \approx \frac{1}{2} m_p$

3-gluon vertex

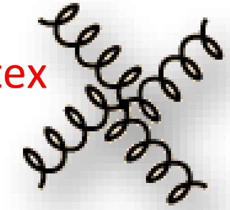


- ≈ 45 years ago

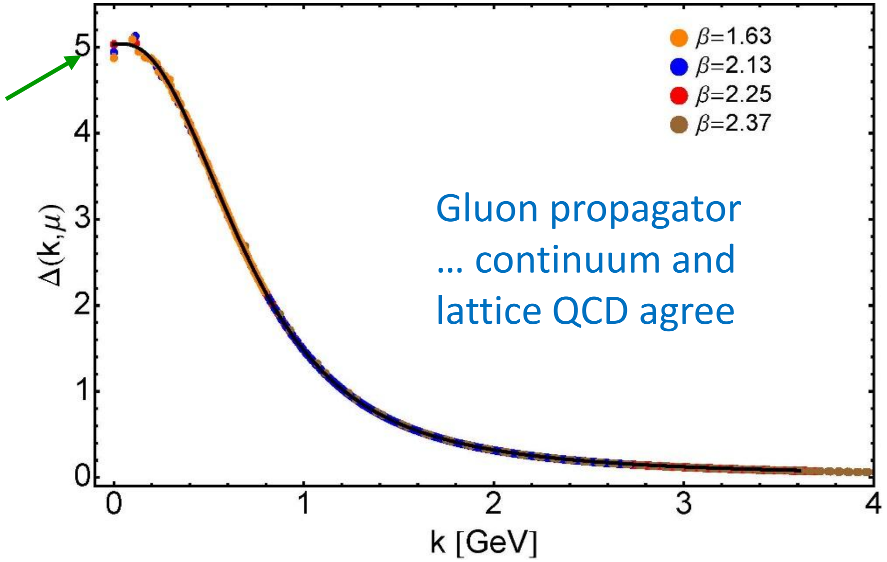
  Dynamical mass generation in continuum quantum chromodynamics,
  J.M. Cornwall, Phys. Rev. D **26** (1981) 1453 … $>$ 1100 citations

- Owing to strong self-interactions, gluon partons ⇒ gluon quasiparticles, described by a mass function that is large at infrared momenta

4-gluon vertex



$\frac{1}{m_g^2}$



Truly mass from nothing
An interacting theory, written in terms of massless gluon fields, produces dressed gluon fields that are characterised by a mass function that is large at infrared momenta

✓ *QCD fact*

✓ *Continuum theory and lattice simulations agree*

✓ *Empirical verification?*

D. Binosi, *Emergent Hadron Mass in Strong Dynamics*, Few Body Syst. **63** (2022) 42.

M. N. Ferreira, J. Papavassiliou, *Gauge Sector Dynamics in QCD*, Particles **6** (1) (2023) 312–363.

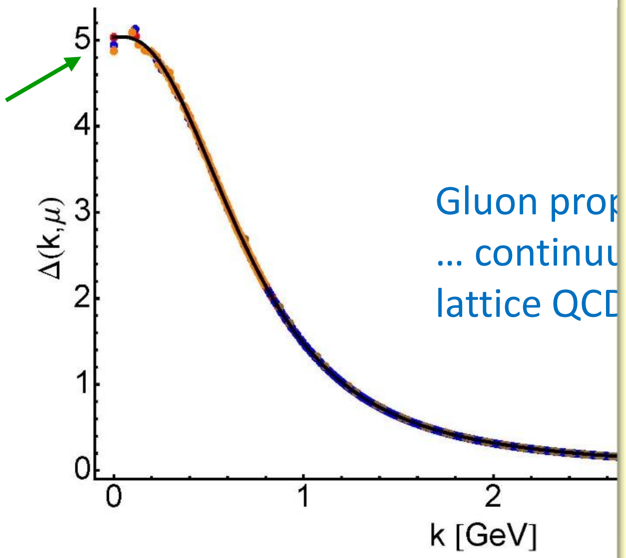
Ancient History

Today's best estimate: $m_g^{\mathrm{RGI}} = 0.43(1)\mathrm{GeV}$

Gauge Invariance and Mass,
J. S. Schwinger
Phys. Rev. 125 (1962) 397-398

It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong.

$\dfrac{1}{m_g^2}$



Gluon prop
… continuu
lattice QCD

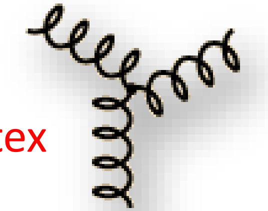
EHM means Gluons are massive via Schwinger Mechanism

ics,

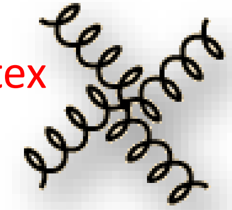
siparticles,
menta

n

3-gluon vertex



4-gluon vertex

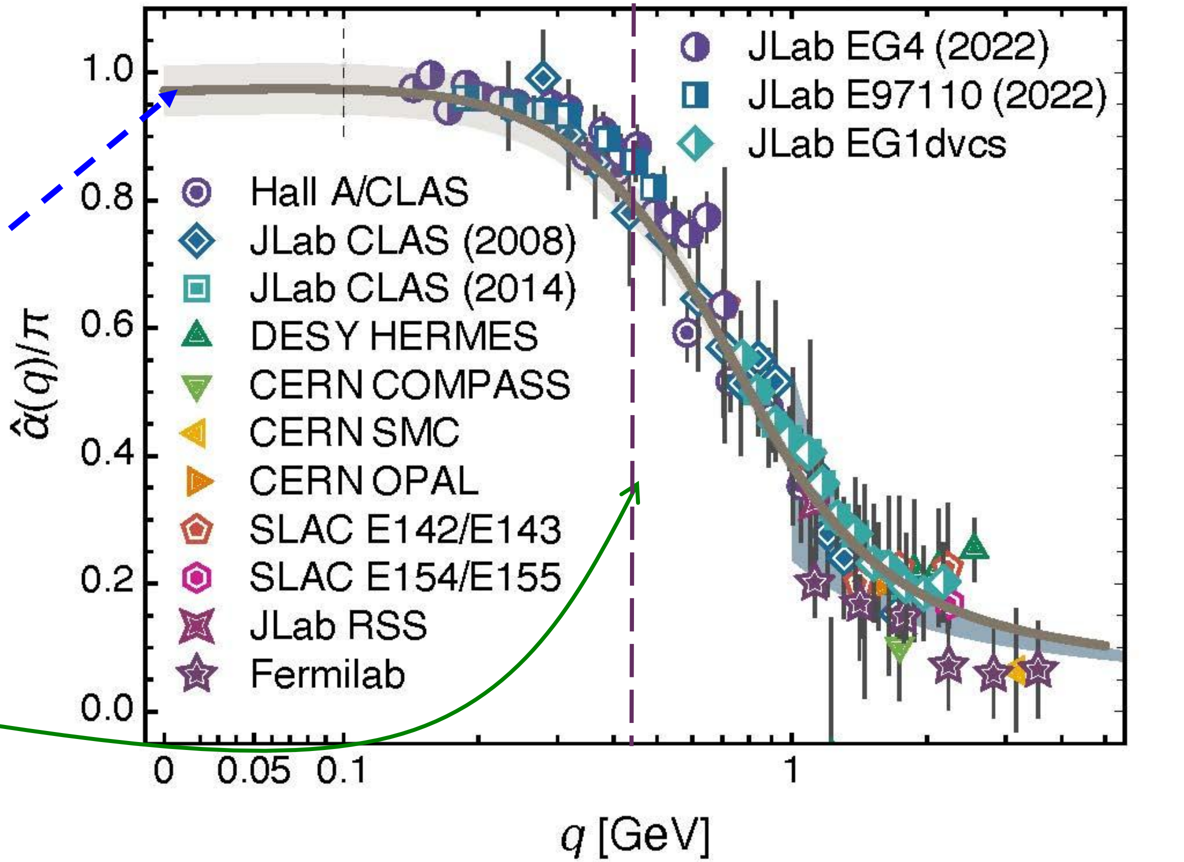


✓ *QCD fact*
✓ *Continuum theory and lattice simulations agree*
✓ *Empirical verification?*

Effective charge from lattice QCD, *Zhu-Fang Cui, Jin-Li Zhang et al.*, *NJU-INP 014/19, arXiv:1912.08232 [hep-ph], Chin. Phys. C 44 (2020) 083102/1-10*

Process independent effective charge = running coupling

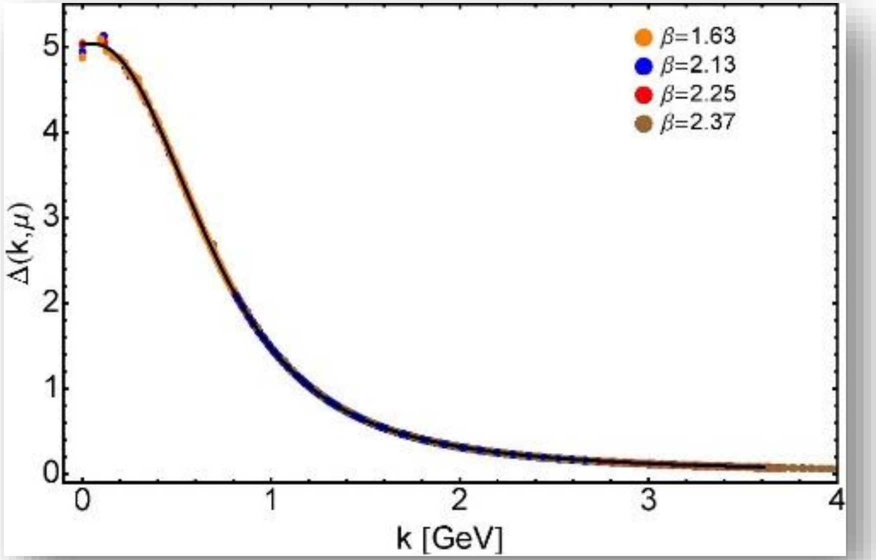
- Modern theory – exploiting pinch technique and background field method – enables unique QCD analogue of
  "Gell-Mann – Low"
  running charge to be
  rigorously defined and calculated
- Analysis of QCD's gauge sector
  yields a *parameter-free prediction*
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$
- No Landau Pole
- Below $k \sim \hat{m}_0$, interactions become scale independent, just as they were in the Lagrangian; so, QCD becomes practically conformal again



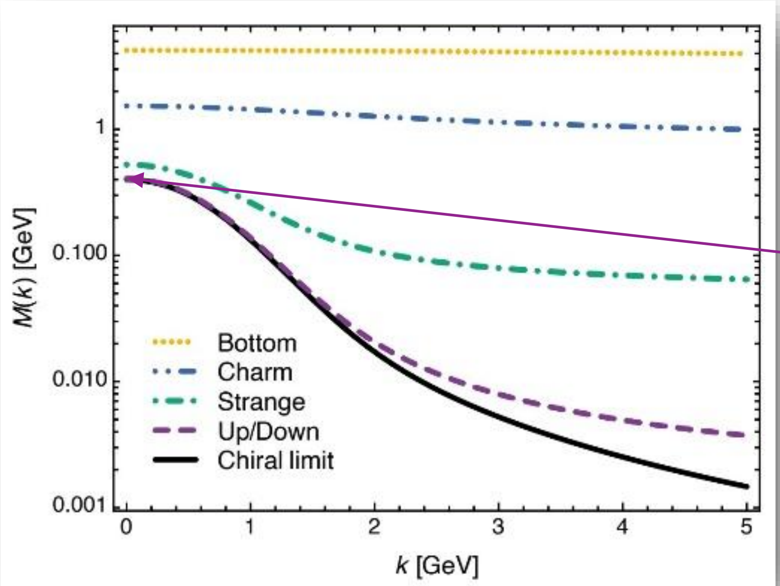
- ✓ *Process independent strong running coupling* Daniele Binosi *et al.*, arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
- ✓ *Experimental determination of the QCD effective charge $\alpha_{g1}(Q)$.* A. Deur; V. Burkert; J.-P. Chen; W. Korsch, Particles 5 (2022) 171
- ✓ *QCD Running Couplings and Effective Charges,* Alexandre Deur, Stanley J. Brodsky and Craig Roberts, e-Print: 2303.00723 [hep-ph], Prog. Part. Nucl. Phys. **134** (2024) 104081

EHM Basics

- Absent Higgs boson couplings,
  QCD Lagrangian is scale invariant … yet …
  - ✓ Massless gluons become massive
  - ✓ A momentum-dependent, IR saturating charge is produced
  - ✓ Massless quarks become massive

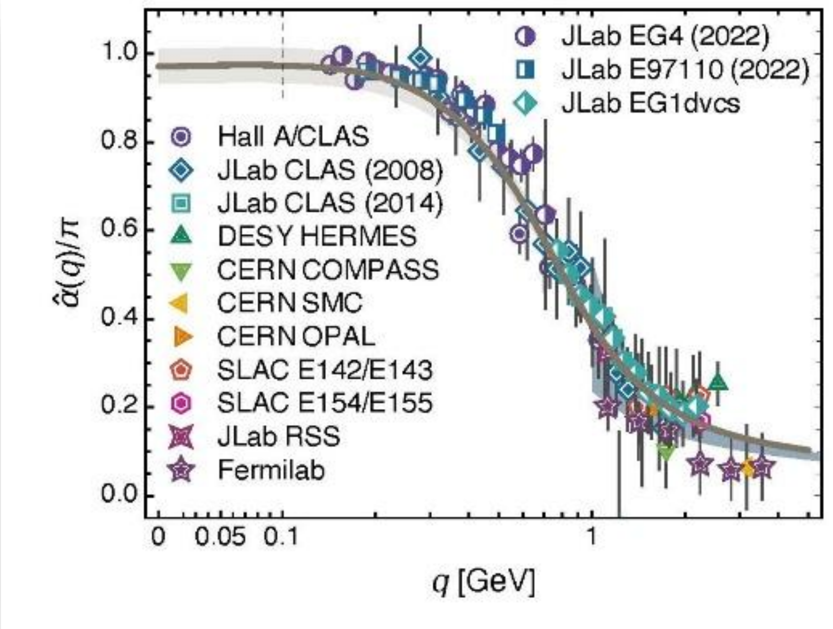


THREE PILLARS OF EHM



✓ $3 \times M(0)$ sets the scale of the proton mass.

✓ Meson-loops provide 20% quantum corrections

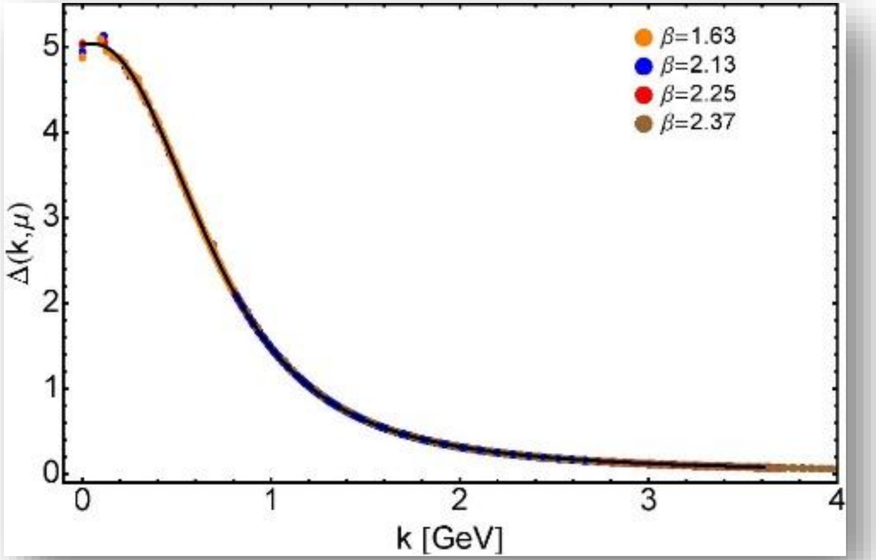


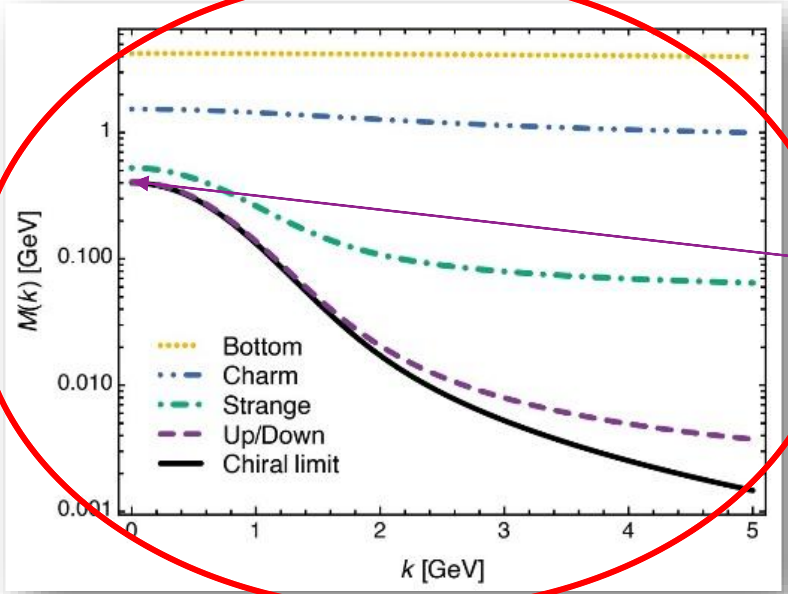
EHM Basics

This one is the key to ChEFT
Massless quarks become massive

$f_\pi \sim M_0^{\overline{m}}(0)$

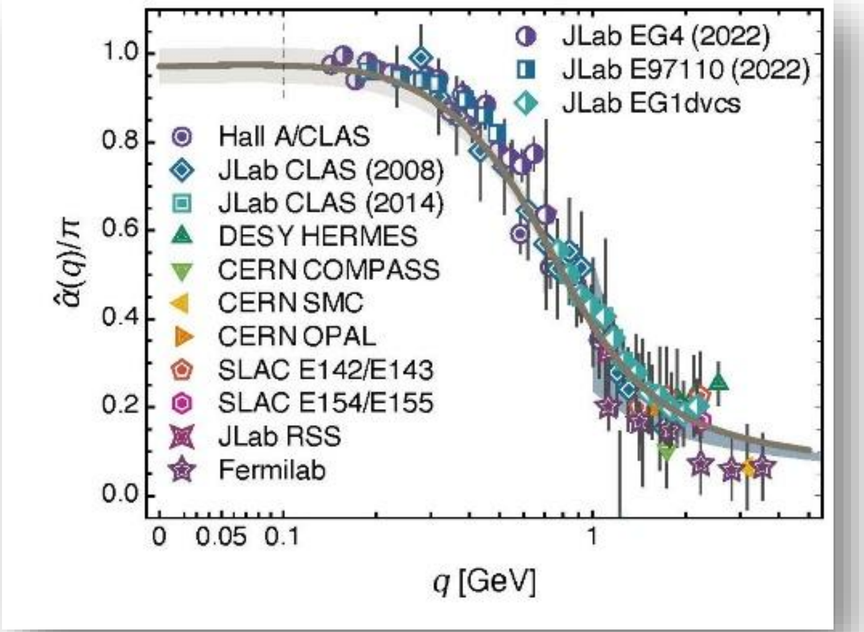
- Absent Higgs boson couplings,
  QCD Lagrangian is scale invariant … yet …
  - ✓ Massless gluons become massive
  - ✓ A momentum-dependent, IR saturating charge is produced
  - ✓ Massless quarks become massive



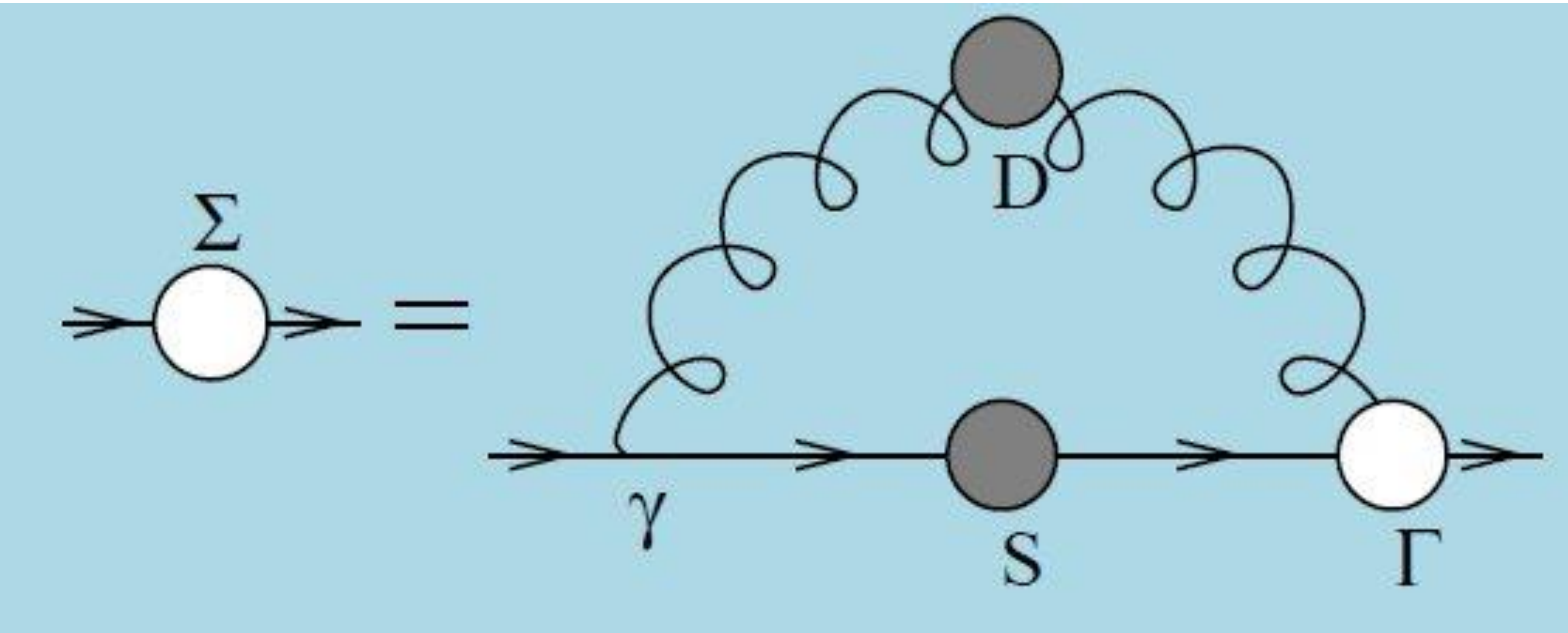


THREE PILLARS OF EHM

- ✓ $3 \times M(0)$ sets the scale of the proton mass.
- ✓ Meson-loops provide 20% quantum corrections



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Quark Gap Equation

Axialvector Ward Green Takahashi Identity

- Expression of (partial) conservation of axialvector (chirality) current
- Identity between

$$P_\mu \Gamma_{5\mu}^{fg}(k;P) + i\,[m_f(\zeta) + m_g(\zeta)]\,\Gamma_5^{fg}(k;P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-)$$

axial vector vertex pseudoscalar vertex dressed quark propagator

- Chiral limit

$$P_\mu \Gamma_{5\mu}^{fg}(k;P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-)$$

- Quantum numbers ⇒ pion appears as pole in axial vector vertex

$$P_\mu \Gamma_{5\mu}^{ud}(k;0) = -f_\pi \Gamma_\pi(k;0)$$

- This says that leading amplitude in pion "wave function" is completely determined by the dressed-quark propagator

Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Pion structure (almost) completely determined by dressed quark mass

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude
  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator

$$S(p)=\frac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$$

$$M(p^2)=\frac{B(p^2)}{A(p^2)}$$

- Symmetry demands, via Axial-vector Ward-Takahashi identity:

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

Miracle: **two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

Pion exists if, and only if, mass emerges dynamically

QCD Fact — Pion (Nambu-Goldstone modes) and mass

- Higgs boson couplings → 0
- Pion exists and is massless
- Pion Bethe-Salpeter amplitude

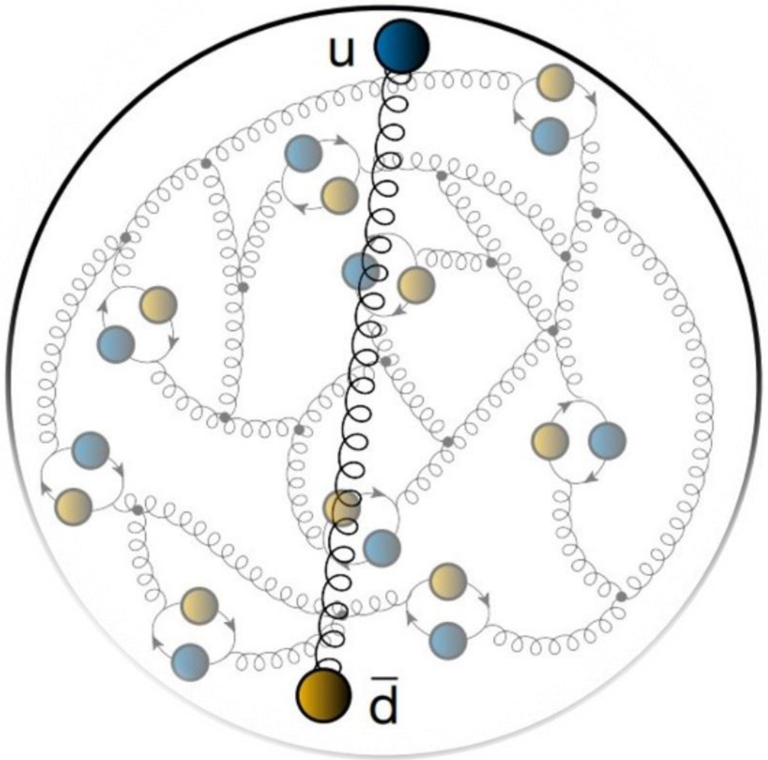
EHM demands equivalence between
one-body mass and two-body correlation strength
in Nature's most fundamental Nambu-Goldstone bosons

$$f_\pi E_\pi(p^2) = B(p^2)$$

Pion wave function

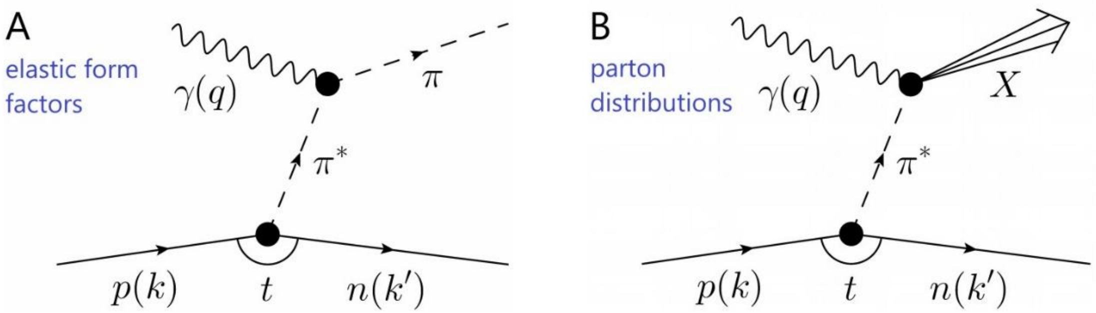
quark mass function

- **Thus, enigmatically, properties of the nearly massless pion are the cleanest expression of EHM in the Standard Model !**

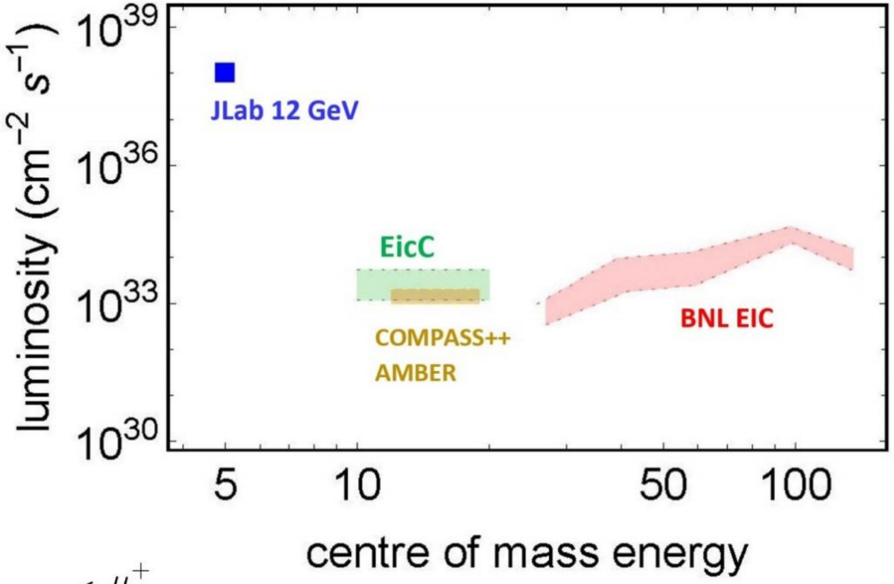


One aspect of pseudoscalar meson physics

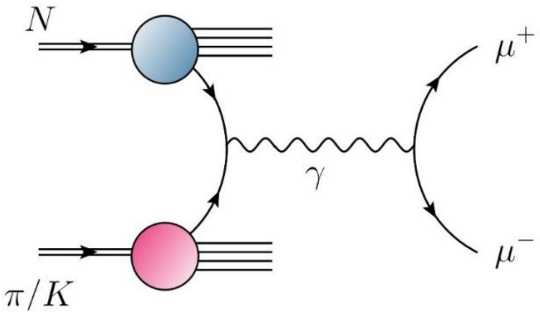
Era of Meson "Targets"



- JLab & EIC & EicC
  - High luminosity electron (+ ion) beams
  - Access to meson targets via the Sullivan Process,
    i.e., a baryon's "meson cloud"



- AMBER @ CERN SPS
  - High-intensity beams of pions
    ($\gtrsim 10^7$ pions/sec in Phase-1 = approved)
    and kaons (5×10^6 kaons/sec Phase-2 = proposal being prepared)
  - Drell-Yan, J/ψ production, prompt photon production
    … from proton and nuclear targets



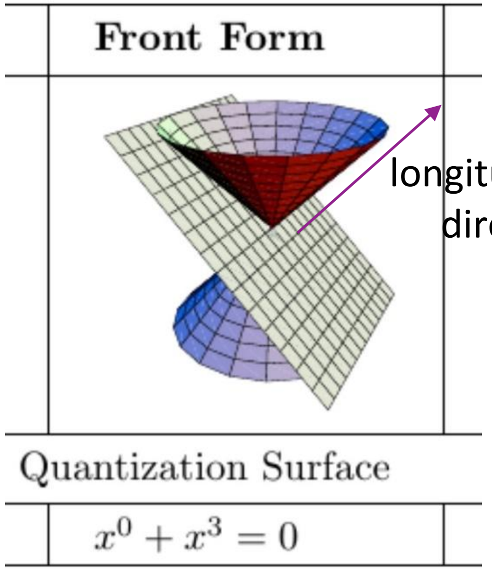
$$\Psi(x,t)$$

π & K DAs & Form Factors

Wave Functions of Nambu Goldstone Bosons

DAs are 1D projection of hadron's light-front wave function, obtained by integration $\sim \int d^2k_\perp \Psi(x,k_\perp)$

- Physics Goals:
  - Pion and kaon distribution amplitudes (DAs – $\varphi_{\pi,K}$)
  - Nearest thing in quantum field theory to Schrödinger wave function
  - Consequently, fundamental to understanding π and K structure.
- Scientific Context:
  - For 40 years, the x-dependence of the pion's dominant distribution amplitude (DA) has been controversial.
  - Modern theory ⇒ EHM expressed in x-dependence of $\varphi_{\pi,K}(x)$
  - $\varphi_\pi(x)$ is direct measure of dressed-quark running mass in chiral limit.
  - Kaon DA = asymmetric around midpoint of its domain of support (0<x<1)
    - Degree of asymmetry is signature of constructive interference between EHM and HB mass-generating mechanisms



Insights into the Emergence of Mass from Studies of Pion and Kaon Structure, Craig D. Roberts, David G. Richards, Tanja Horn and Lei Chang, NJU-INP 034/21, arXiv: 2102.01765 [hep-ph], Prog. Part. Nucl. Phys. **120** (2021) 103883/1-65

Pion's valence-quark Distribution Amplitude

- Methods were developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

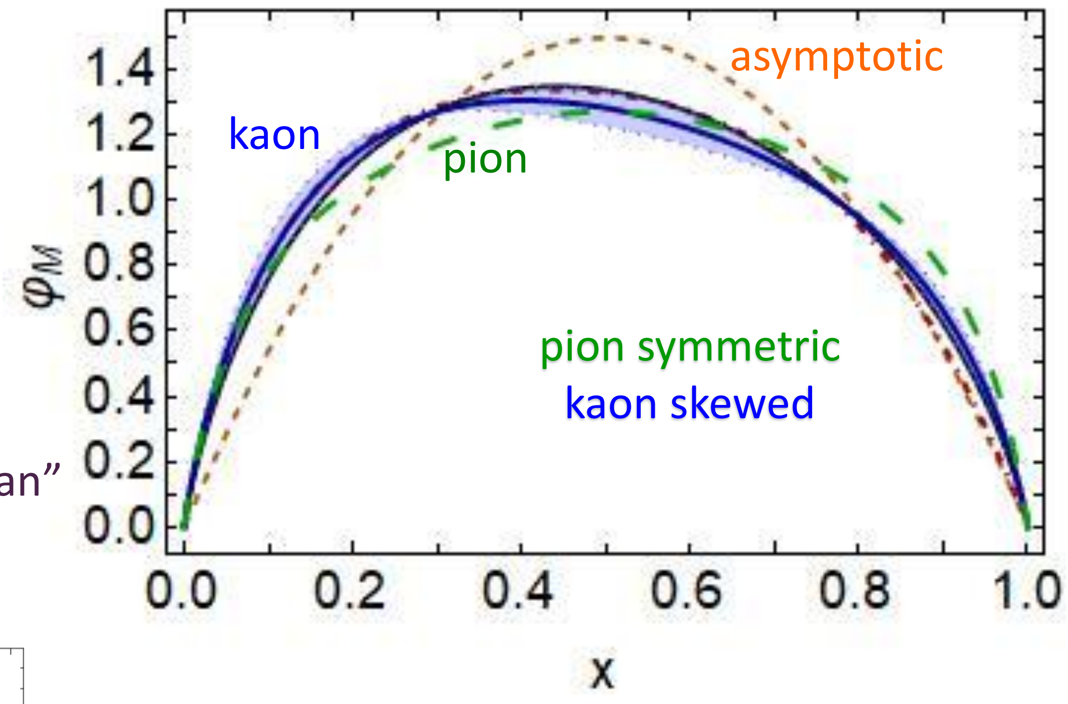
$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.

$x^\alpha\ (1-x)^\alpha$, with $\alpha=0.5$

Meson leading-twist DAs

- Continuum results exist & lQCD results arriving
  - Common feature = broadening/dilation
  - Origin = EHM
- NO differences between π & K if EHM is all there is
  - Differences arise from Higgs-modulation of EHM mechanism
  - "Contrasting π & K properties reveals Higgs wave on EHM ocean"



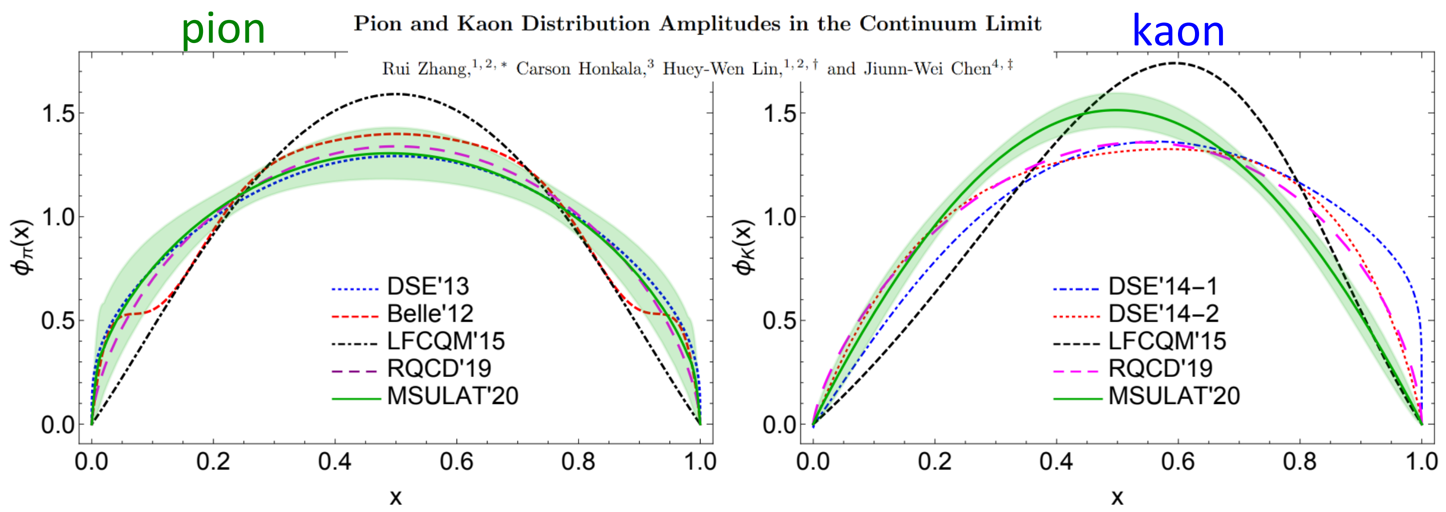


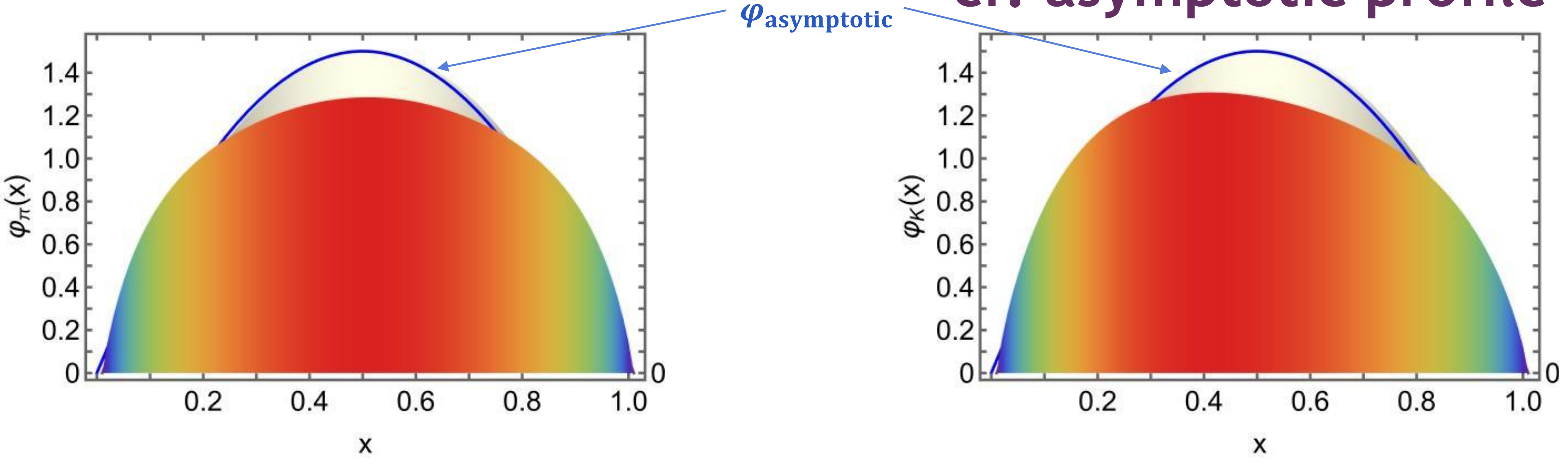
FIG. 10. Fit of the $P_z = 4\frac{2\pi}{L}$ pion (left) and kaon (right) data to the analytical form in Bjorken-x space, compared with previous calculations (with only central values shown). Although we do not impose the symmetric condition $m = n$, both results for the pion and kaon are symmetric around $x = 1/2$ within error.

- Kaon DA vs pion DA
  - almost as broad
  - peak shifted to x=0.40(5)
  - $\langle \xi^2 \rangle = 0.24(1)$, $\langle \xi \rangle = 0.035(5)$
- ERBL evolution logarithmic
- Broadening & skewing persist to *very* large resolving scales – beyond LHC

Exposing strangeness: projections for kaon electromagnetic form factors, Fei Gao *et al.*, arXiv:1703.04875 [nucl-th], Phys. Rev. D **96** (2017) 034024/1-8

Constraining the pion DA using Drell-Yan reactions on a proton, H.-Y. Xing (邢惠瑜) *et al.*, NJU-INP 077/23, e-Print: 2308.13695 [hep-ph]

π & K DAs cf. asymptotic profile



- EHM generates broadening in both π & K
- EHM + Higgs-boson interference responsible for skewing in kaon
  - HB-only ⇒ peak shifted to $\frac{m_u^{\text{HB}}}{m_s^{\text{HB}}} \times \frac{1}{2} \approx 0.02$ … *wrong*
  - Instead, EHM*HB for u and s quarks … $\frac{\text{EHM } m_u \to M_u}{\text{EHM } m_s \to M_s} \times \frac{1}{2} \approx 0.4$

These features have widespread impact in studies of (hard) exclusive processes.

Broadening can be verified, e.g., in measurements of π and K electric charge distributions (elastic form factor measurements at large Q^2) & in $\pi + p$ Drell-Yan measurements

Hard Exclusive Processes e.g, meson form factors

Normalisation of large Q^2 behaviour of meson form factors is largely determined by EHM, expressed in

- ✓ f_M = order parameter for DCSB (EHM)
- ✓ $\varphi_M(x)$ = meson wave function

Parton distribution functions (PDFs) and parton distribution amplitudes (PDAs) are connected through the hadron's light-front wave function

$$\varphi(x) \sim \int d^2k_\perp \psi(x, k_\perp^2)\,,$$

$$q(x) \sim \int d^2k_\perp |\psi(x, k_\perp^2)|^2$$

Hadron's LFWF is the key.

- ✓ EHM is expressed in every hadron LFWF
- ✓ LFWF correlates all observables

The impetus for measuring $F_M(Q^2)$ at large momentum transfers is a need to understand and validate a strict prediction of QCD [32–34], *viz.* $\exists Q_0 \gg \Lambda_{\rm QCD}$ such that

$$Q^2 F_M(Q^2) \stackrel{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_M^2 w_M^2(Q^2)\,, \qquad (1)$$

where α_s is the one-loop strong running coupling, $\Lambda_{\rm QCD} \approx 0.3\,{\rm GeV}$, $f_\pi = 0.092\,{\rm GeV}$, $f_K = 0.110\,{\rm GeV}$ [1]; and $w_M^2 = e_{\bar q} w_{\bar q}^2(Q^2) + e_u w_u^2(Q^2)$,

$$w_f = \tfrac{1}{3}\int_0^1 dx\, g_f(x)\, \varphi_M(x;Q^2)\,, \qquad (2)$$

$$g_u = \frac{1}{x},\; g_{\bar q} = \frac{1}{1-x}$$

- ➢ High-energy nuclear and particle physics have long focused on finding evidence for power-law scaling in experimental data.
- ➢ Certainly, an important first step; BUT it should be remembered that QCD is not found in scaling laws.
- ➢ Instead, since quantum field theory requires deviations from strict scaling, then QCD is found in the existence and character of scaling violations.

Wave Functions of Nambu Goldstone Bosons

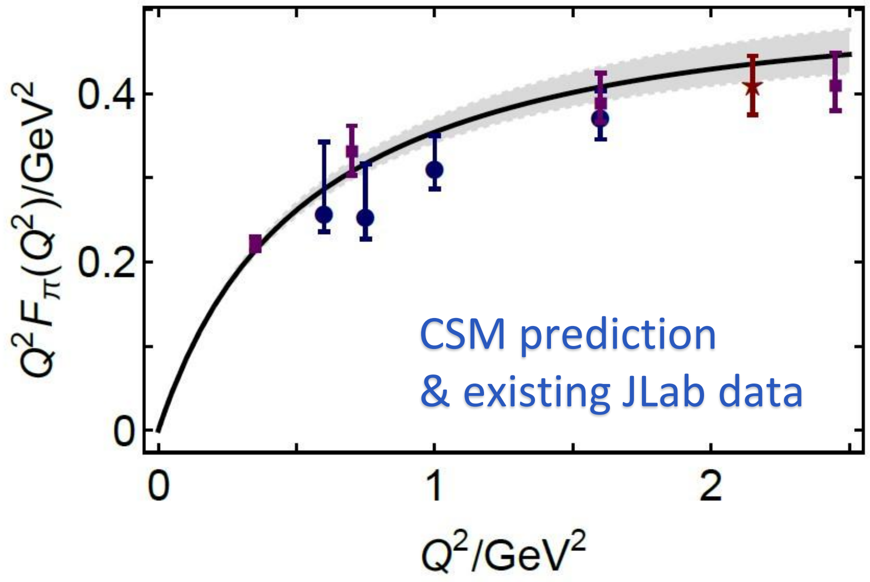
Revealing the structure of light pseudoscalar mesons at the Electron-Ion Collider, J. Arrington *et al.*, NJU-INP 036/21, arXiv: 2102.11788 [nucl-ex], J. Phys. G **48** (2021) 7, 075106

- Scientific Context:
  - Today, continuum and lattice are delivering consistent predictions for $\varphi_{\pi,K}(x)$
  - Can signature features of interference between EHM and HB, which are manifest in low-order Mellin moments of the DAs, be probed experimentally?
- Measurement

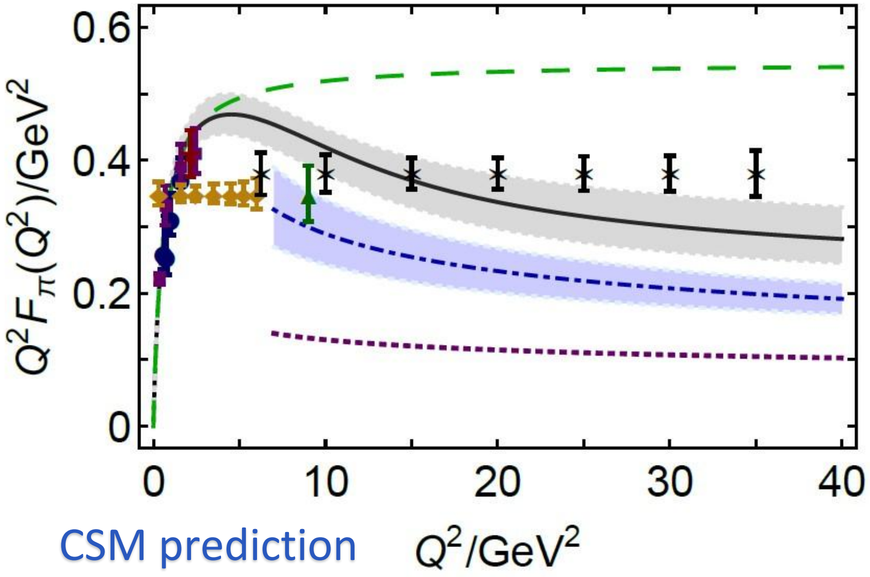
Elastic electromagnetic form factors

Breakaway from scaling at $Q^2 \approx 6\ GeV^2$
Thereafter, Q^2 dependence
is largely described by hard-scattering formula with realistic DA
Reveal QCD scaling violations for 1st time in hard elastic scattering

A



CSM prediction
& existing JLab data



CSM prediction
& JLab + EIC projections

Grant no. 12135007
国家自然科学基金委员会
National Natural Science Foundation of China

Emergent Hadron Mass

- QCD is unique amongst known fundamental theories of natural phenomena
  - The degrees-of-freedom used to express the scale-free Lagrangian are not directly observable
  - Massless gauge bosons become massive, with no "human" interference
  - Gluon mass ensures a stable, infrared completion of the theory through the appearance of a running coupling that saturates at infrared momenta, being everywhere finite
  - Massless fermions become massive, producing
    - Massive baryons and simultaneously Massless mesons
- These emergent features of QCD are expressed in every strong interaction observable
- They can also be revealed via

  EHM interference with Nature's other known source of mass = Higgs
- We are capable of building facilities that can validate these concepts, proving QCD to be

  the 1st well-defined four-dimensional quantum field theory ever contemplated
- *This may open doors that lead far beyond the Standard Model*

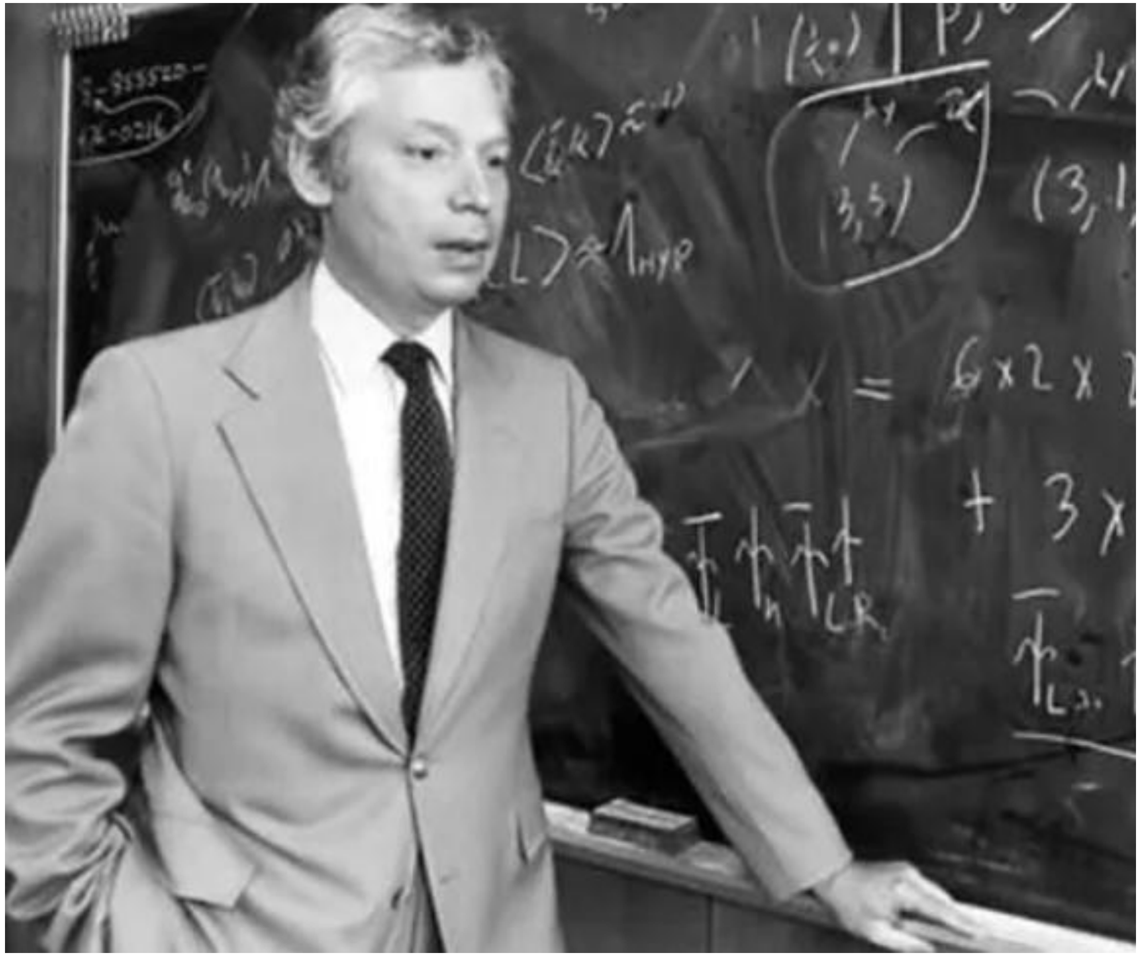
Can't build a Theory of Everything before status of QCD is decided

$$\mathcal{L}_{Nature} = ?$$

There are theories of many things,
But is there a theory of everything?

Thankyou

Nature of Physics



Steven Weinberg

Born: May 3, 1933, New York, NY

Died: July 23, 2021, Austin, TX

- ✓ "The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce and gives it some of the grace of tragedy."
- ✓ "Nothing in the last five hundred years has had so great an effect on the human spirit as the discoveries of modern science."
- ✓ "I know of no one who has participated actively in the advance of physics in the postwar period whose research has been significantly helped by the work of philosophers."
- ✓ "Science is the path to truth"


WHERE TO
BEGIN

Nucleon σ Term

- Expectation value of $H_{\rm QCD}$ mass term – $\int d^4x\, m(\zeta)\bar{q}(x;\zeta)q(x;\zeta)$ – in the proton eigenstate
  = total contribution to proton mass generated by Higgs-seeded interaction term
  Final (measured) value = result from constructive interference between
  Higgs quark mass (explicit chiral symmetry breaking)
  and emergent mass (dynamical chiral symmetry breaking) magnification factor
- Mathematical definition … forward matrix element, zero momentum insertion:
$$\left\langle p(k)\middle|\int d^4x\, m(\zeta)\bar{q}(x;\zeta)q(x;\zeta)\middle|p(k)\right\rangle = \left\langle p(k)\middle|\int d^4x\, \hat{m}[\widehat{\bar{q}(x)q(x)}]\middle|p(k)\right\rangle =: \sigma_p$$
  Normalisation: $\langle p(k)|p(k)\rangle = 2m_p$
- Equally (Feynman-Hellmann theorem): $\sigma_p = m(\zeta)\dfrac{dm_p}{dm(\zeta)} = \hat{m}\dfrac{dm_p}{d\hat{m}}$
- $\dfrac{dm_p}{d\hat{m}} = \left\langle p(k)\middle|\int d^4x\, [\widehat{\bar{q}(x)q(x)}]\middle|p(k)\right\rangle$ = in-proton expectation value of the chiral condensate operator, a nonzero value for which is a corollary of EHM
- Higgs-only contribution $= \hat{m}_d\left(1+\dfrac{2\hat{m}_u}{\hat{m}_d}\right) = \hat{m}_d(1.924) \approx 6.8\times 1.924 = 13\ \text{MeV} = 0.014\, m_p$

Nucleon σ Term & Nucleon Mass Budget

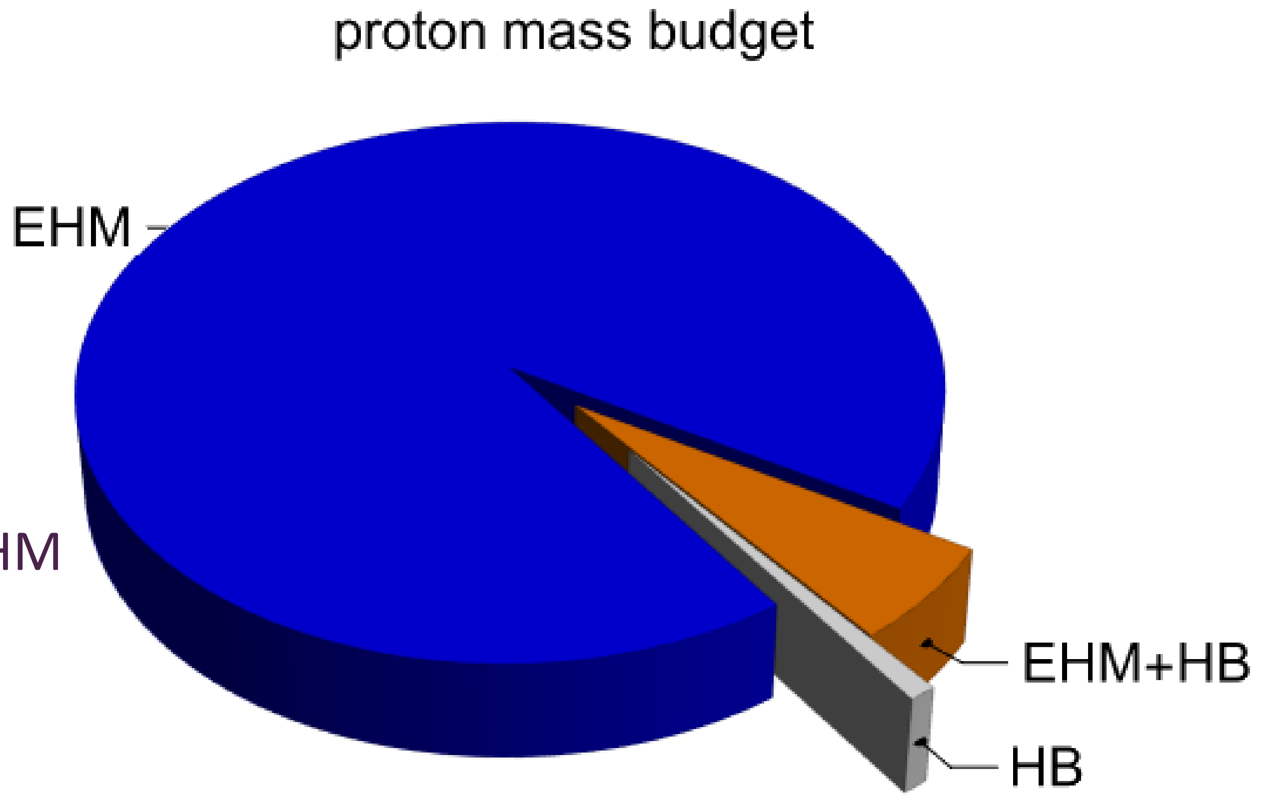
- Inferred value … $\sigma_p \approx 56$ MeV
- Higgs-only (explicit chiral symmetry breaking) contribution

$$\hat{m}_d\left(1+\frac{2\hat{m}_u}{\hat{m}_d}\right) = 13\text{ MeV} = 0.014\, m_p$$

- Constructive interference between Higgs and EHM

$$= \sigma_p - \hat{m}_d\left(1+\frac{2\hat{m}_u}{\hat{m}_d}\right) = 43\text{ MeV} = 0.046\, m_p$$

- ✓ Remaining 94% of m_p is pure EHM component
  = EHM-only
- ✓ Higgs (ECSB) alone is responsible for just 1.4% m_p

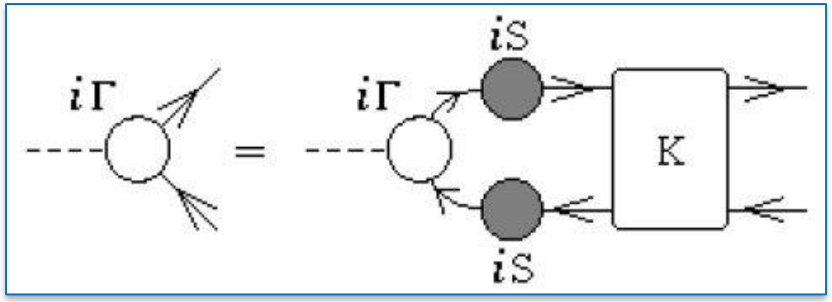


Renormalisation Group Invariant Running Quark Mass

- Chromodynamics is a Poincaré-invariant quantum non-Abelian gauge field theory
- It is thus common to use a Poincaré-invariant quantisation method ⇒ QCD
- Continuum and lattice Schwinger function methods provide such schemes
- In a manifestly Poincaré-invariant definition of QCD, the propagator of any quark takes the form $\tilde{S}(k) = \tilde{Z}(k^2)/[i\gamma \cdot k + \widetilde{M}(k^2)]$
- When working with the renormalised theory, with renormalisation scale ζ, this becomes $S(k;\zeta) = Z(k^2;\zeta)/[i\gamma \cdot k + M(k^2)]$
- $Z(k^2;\zeta)$ is the dressed quark wave function renormalisation function. It depends on ζ.
- $M(k^2)$ is the dressed quark mass function.
- It now follows from multiplicative renormalizability of QCD, *i.e.*, the bare and renormalised quark propagators are related by a single, overall multiplicative factor, that
  - $M(k^2)$ is does NOT depend on ζ.
  - Instead, $M(k^2)$ is a RGI quantity.

On any domain within which perturbation theory is valid,

$$M(k^2) = m(k^2) = \hat{m}\,[\alpha_{QCD}(k^2)/\pi\gamma_m]^{\gamma_m}$$



Pseudoscalar Mesons as Bound-States

- Bound-State amplitude is determined from the homogeneous Bethe-Salpeter equation (BSE):

$$[\Gamma_\pi^j(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4} [\chi_\pi^j(q;P)]_{sr} K_{tu}^{rs}(q,k;P)$$

  - $\chi(q;P) = S(q)\,\Gamma(q;P)\,S(q+P)$, S = dressed-quark propagator
  - r,s,t,u represent colour, isospin, spinor indices
  - *K* = quark-antiquark scattering amplitude
- A generalisation to quantum field theory of the Lippmann-Schwinger equation = one of the most-used equations in quantum mechanics … to describe scattering and bound states

Pseudoscalar Mesons as Bound-States

- Poincaré covariance entails that the bound-state (Bethe-Salpeter) amplitude for an isovector pseudoscalar meson (valence-quark and valence-antiquark) takes the form

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

  - $\{\tau^{i=1,2,3}\}$ = Pauli matrices
  - P = total momentum of system
  - k = relative momentum between valence degrees-of-freedom