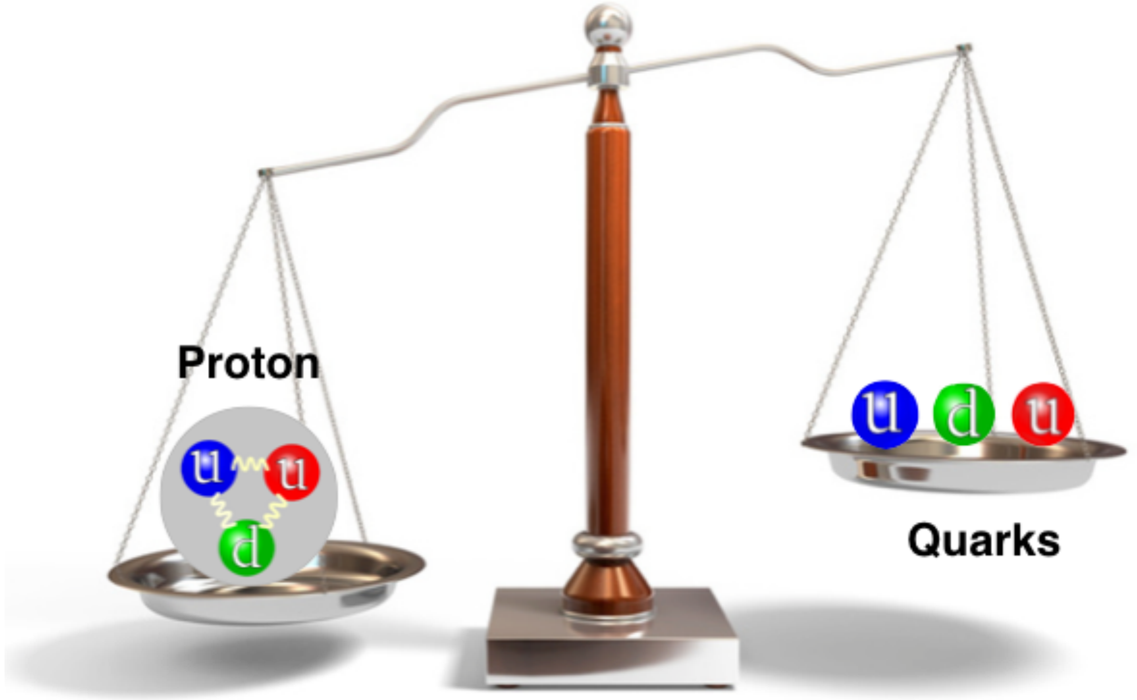
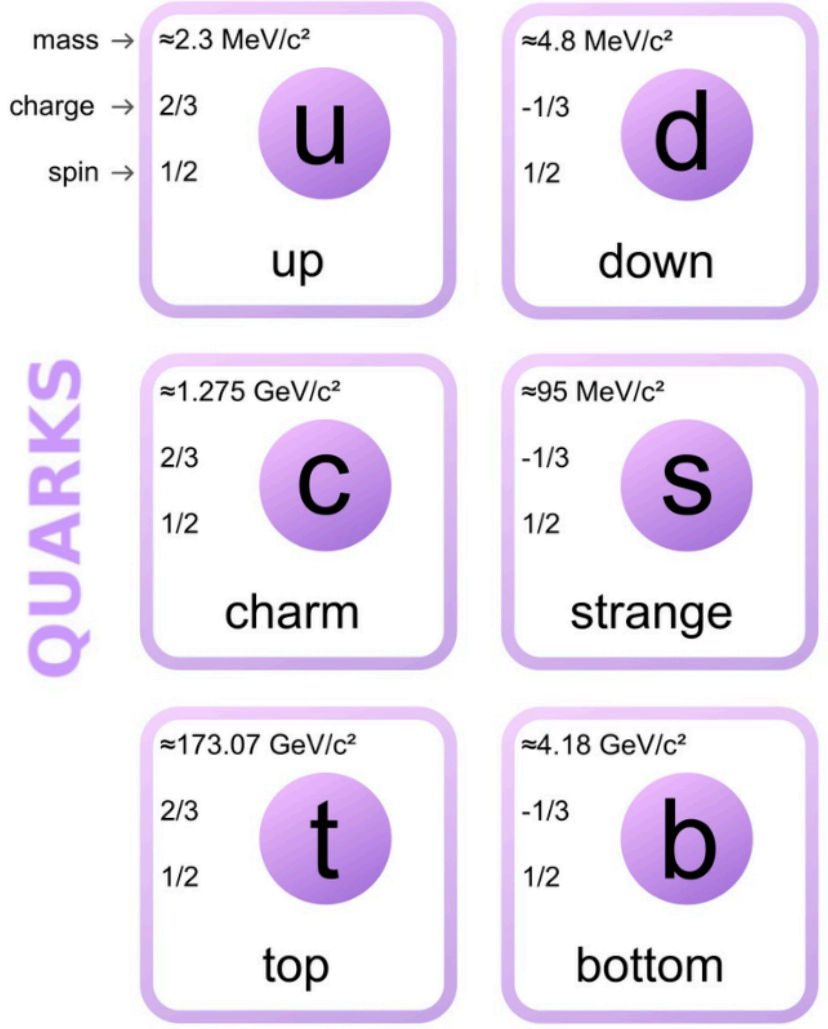
SU(3) flavor symmetry breaking in octet baryon from quark and gluon spin

Yi-Bo Yang
For CLQCD collaboration

Baryon mass and spin

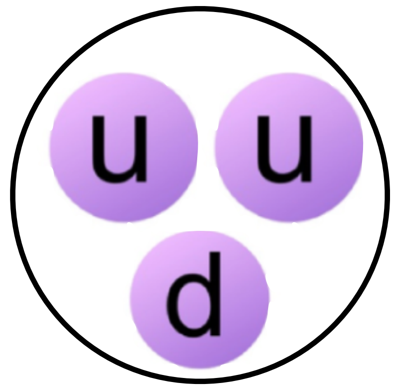
and their strong interaction origin

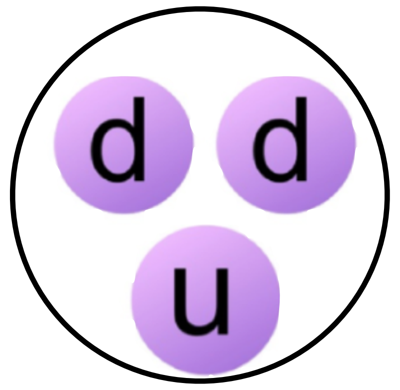




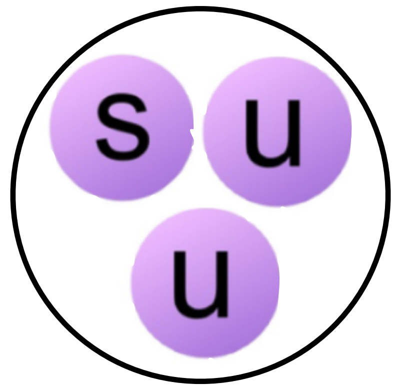
?

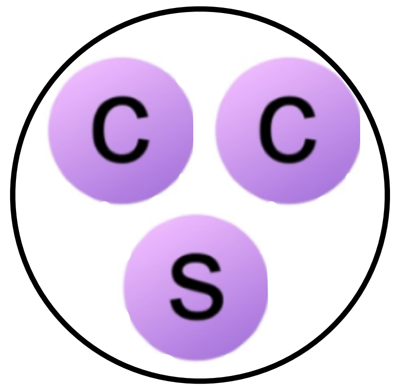
$m_p = 0.9383$ GeV $\quad m_n = 0.9396$ GeV

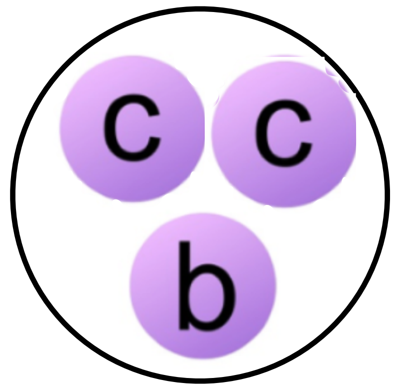




$m_{\Sigma^+} = 1.189$ GeV, $m_{\Xi^0} = 1.314$ GeV, $\quad m_{\Omega_{ccb}} = ?$

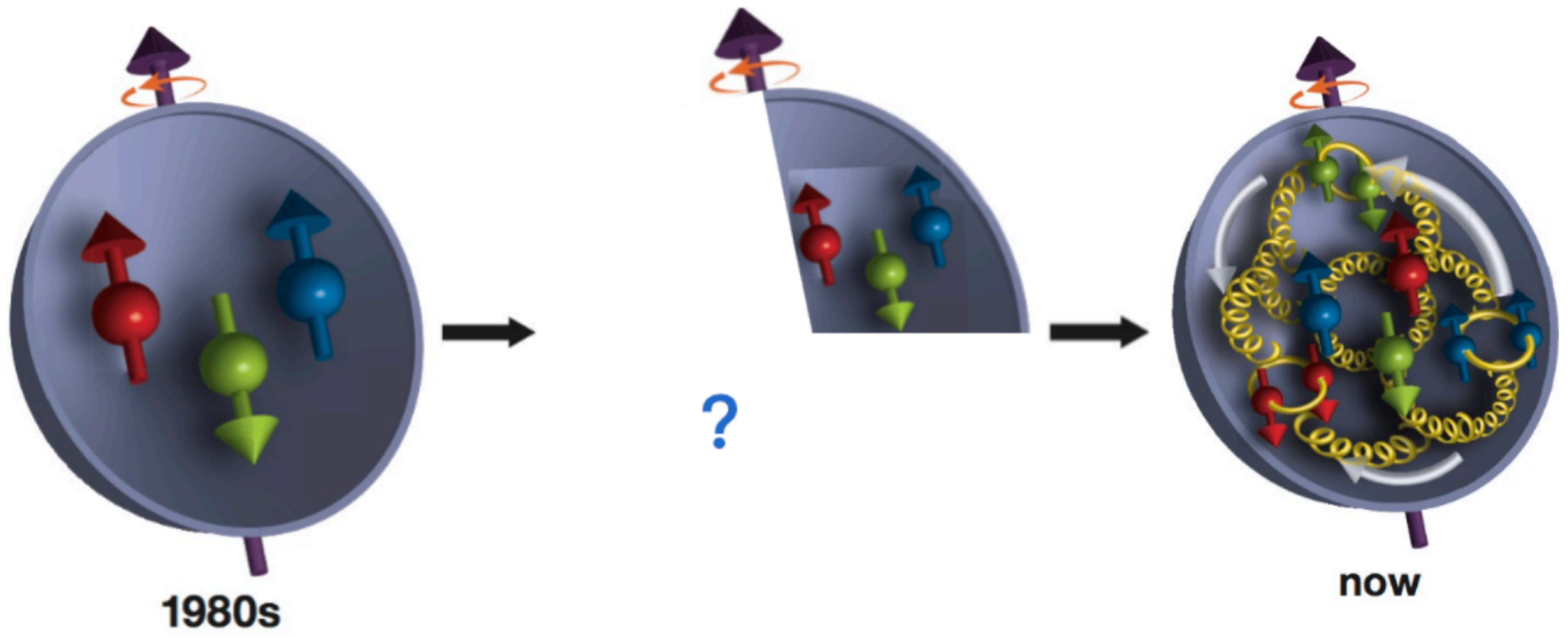






$$S^H = \frac{1}{2}\sum_q \Delta^H q + \Delta G^H + \sum_{i=q,g} L_i^H$$

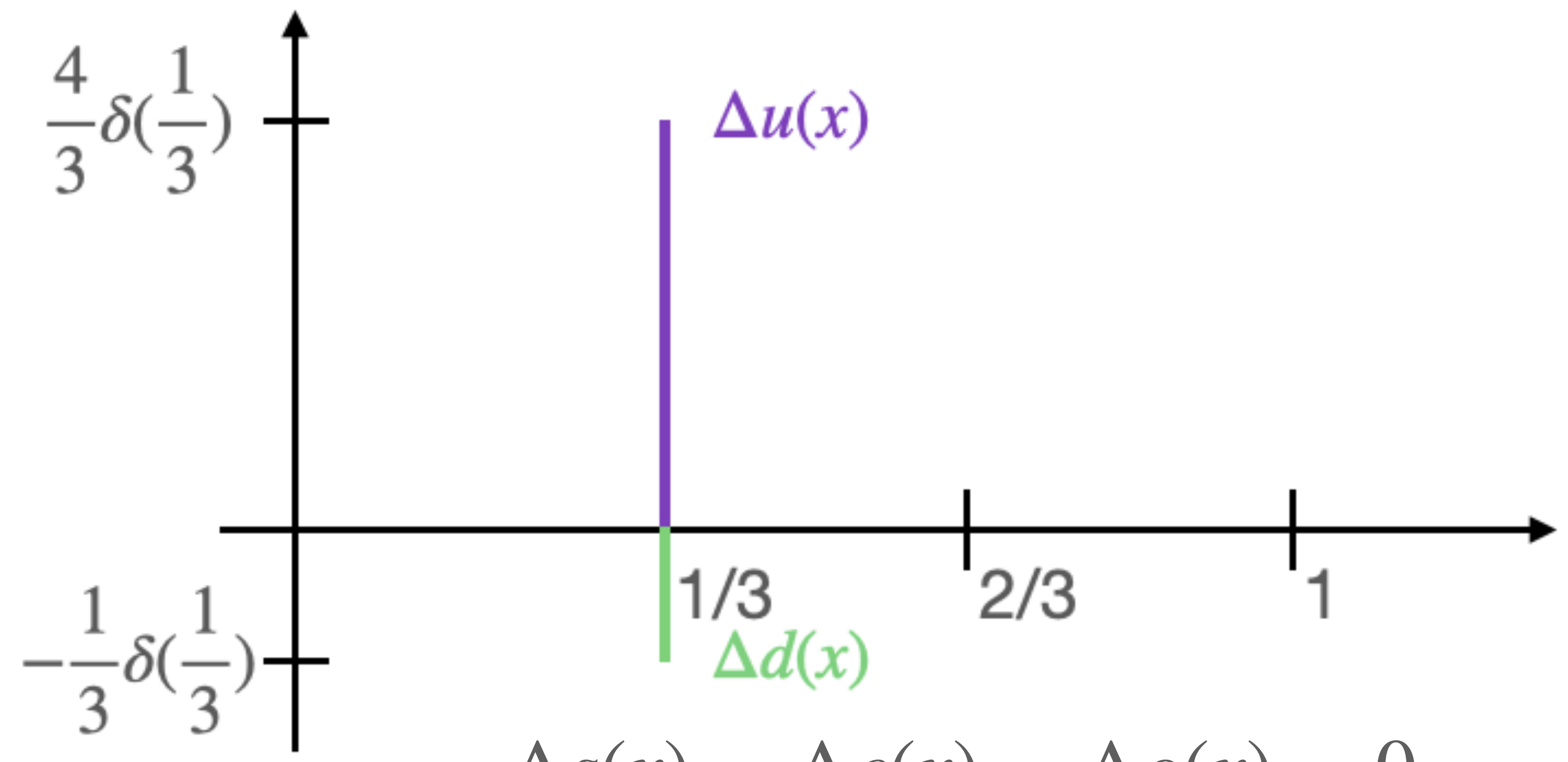
$\Sigma_q \Delta q_q^{\Lambda} = ?$ $\quad \Sigma_q \Delta q_q^{\Xi} = ?$ $\quad \Sigma_q \Delta q_q^{\Omega_{ccb}} = ?$



?

Quark and gluon spin

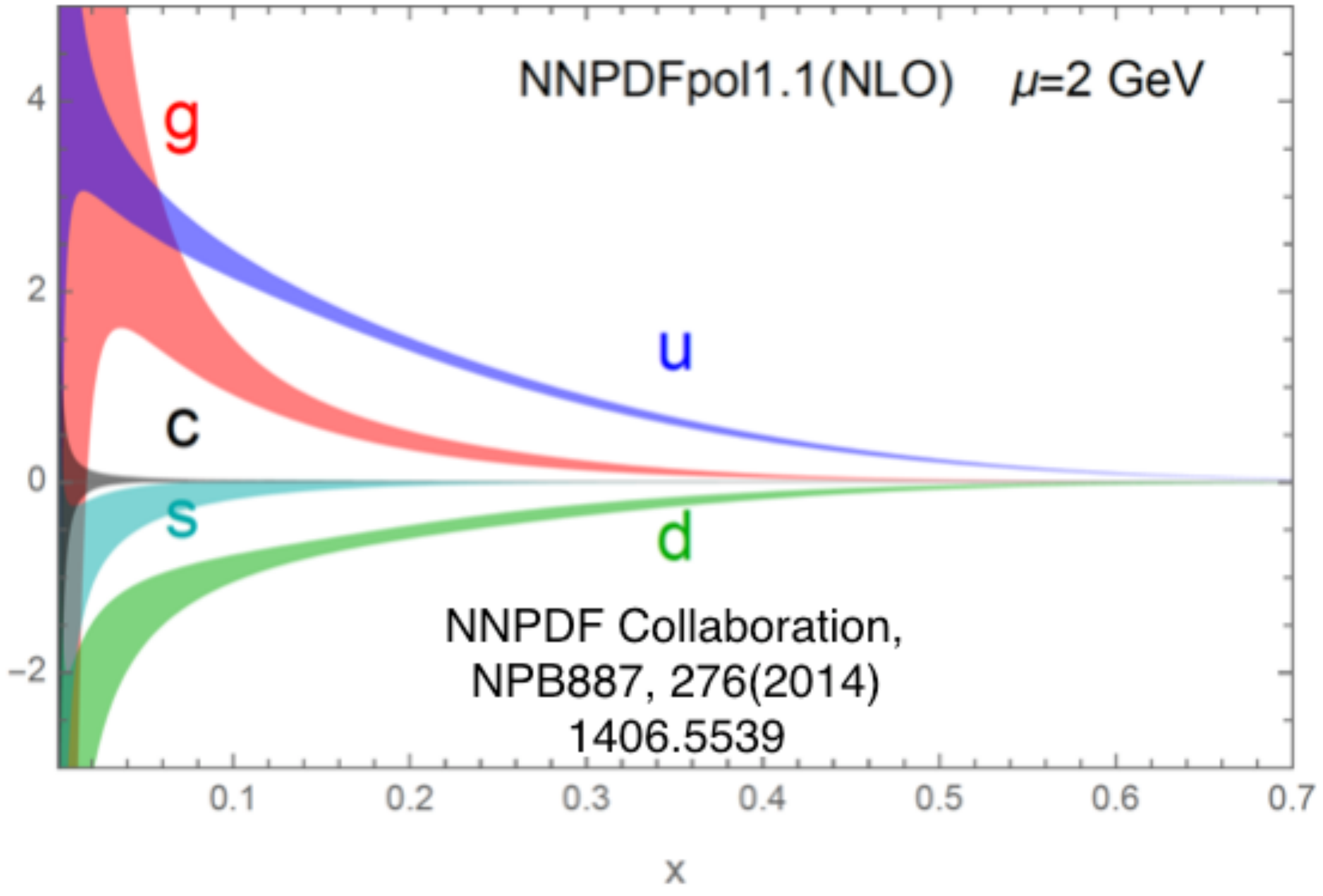
As functions of quark mass

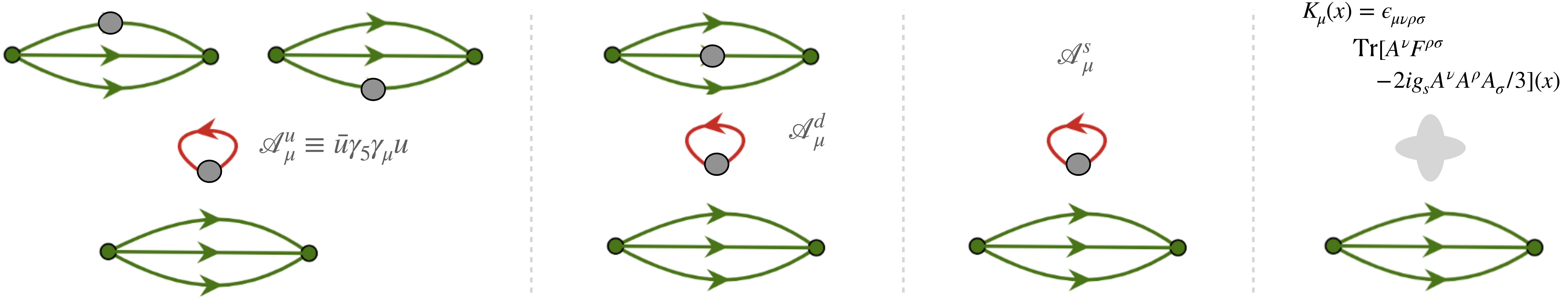


$\Delta s(x) = \Delta c(x) = \Delta g(x) = 0$

$m_q \to 0$

$\infty \leftarrow m_q$





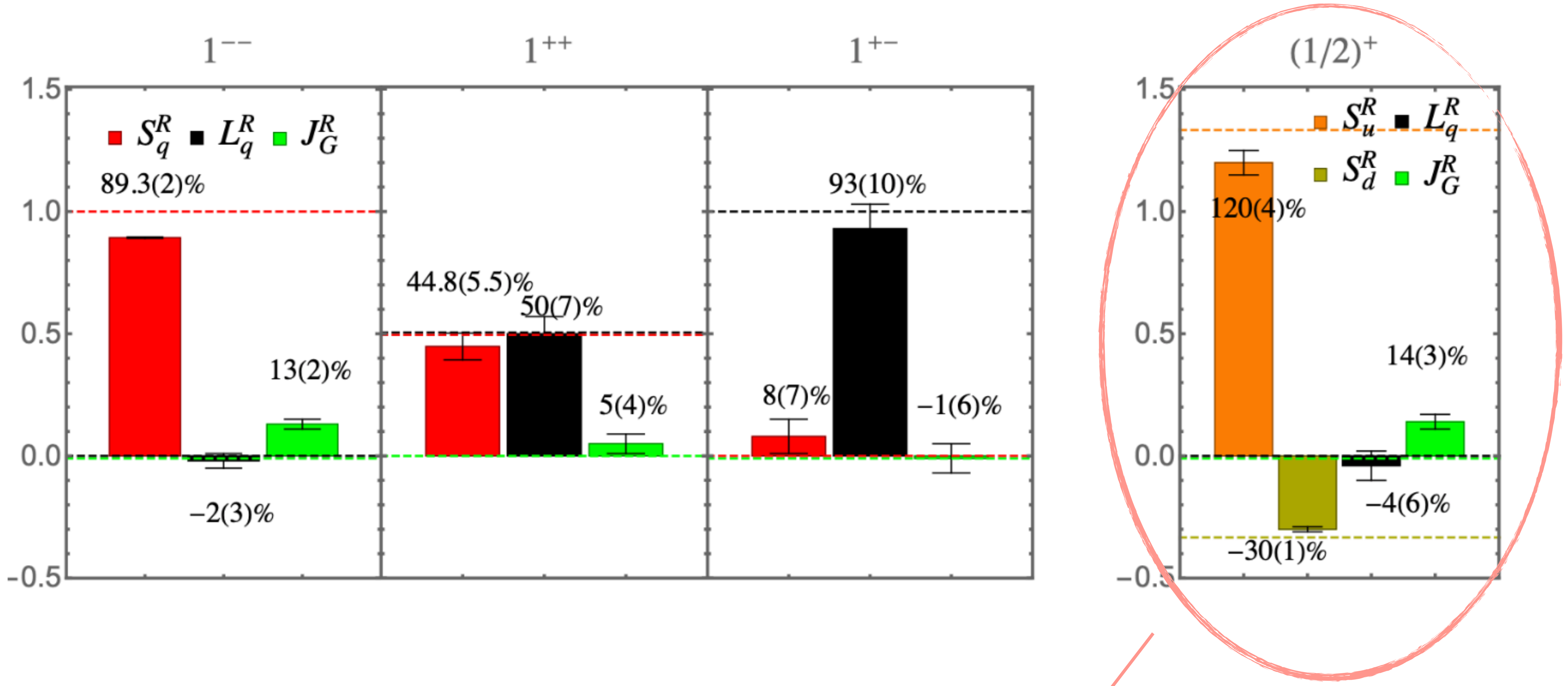
$\mathscr{A}^u_\mu \equiv \bar{u}\gamma_5\gamma_\mu u$

$\mathscr{A}^d_\mu$

$\mathscr{A}^s_\mu$

$K_\mu(x) = \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}[A^\nu F^{\rho\sigma} - 2ig_s A^\nu A^\rho A_\sigma/3](x)$

Hadron spin decomposition

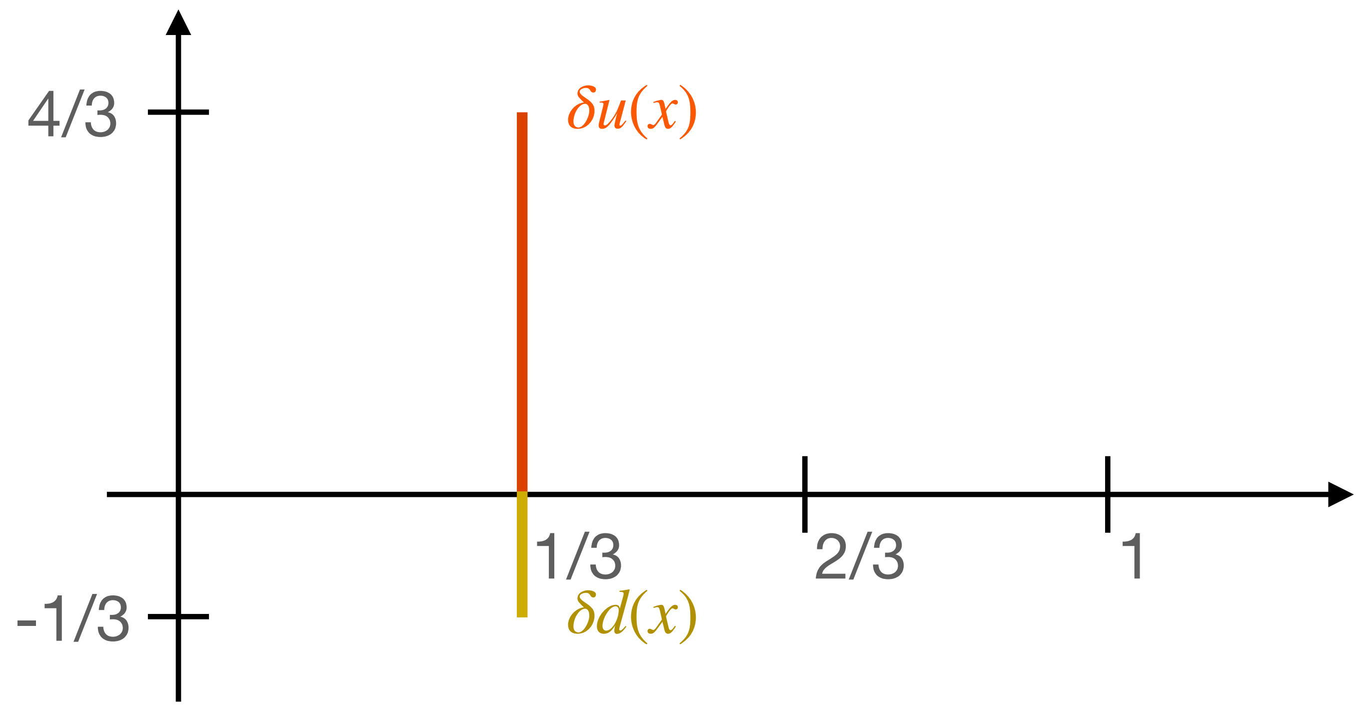


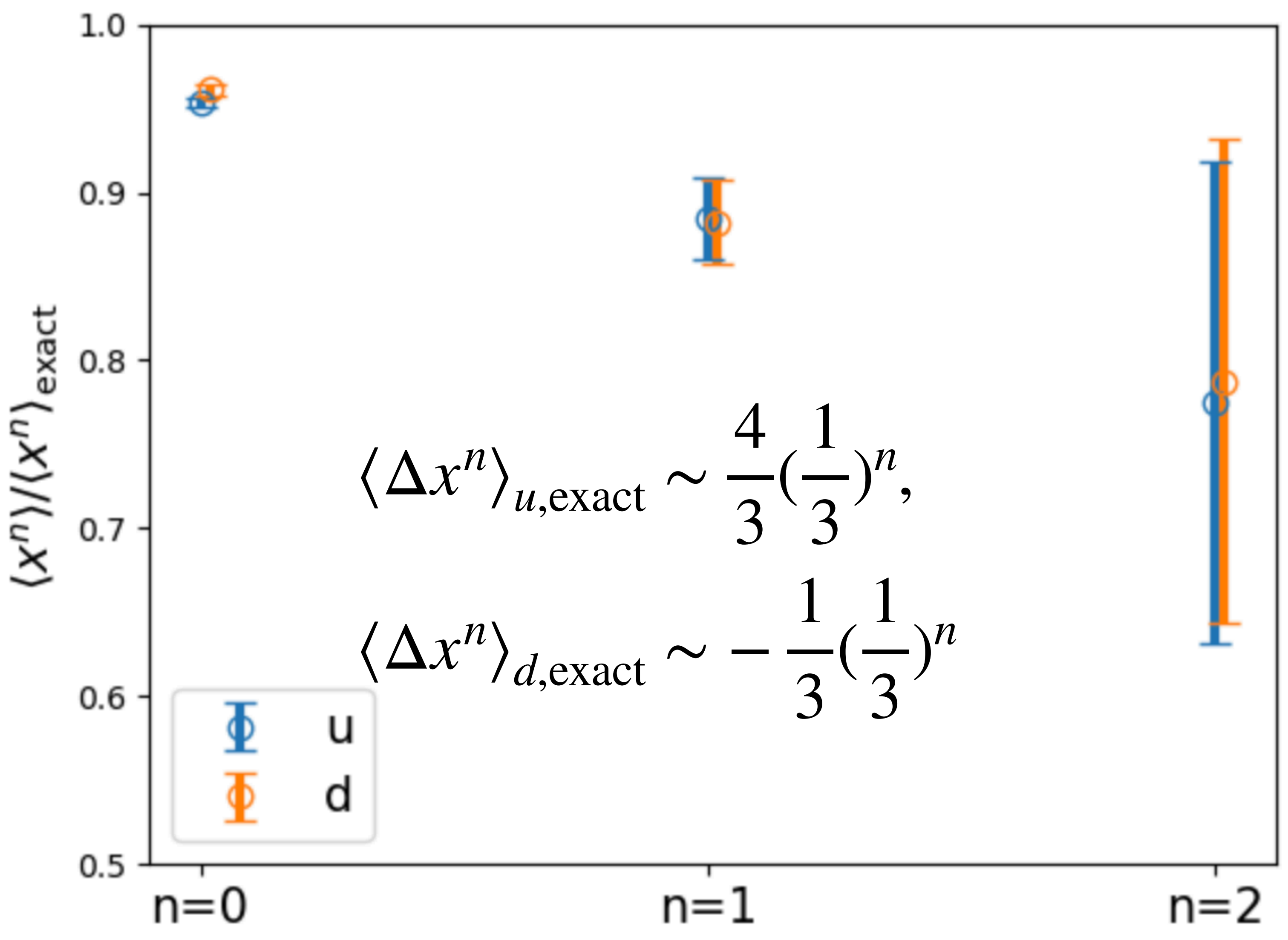
When the quark mass is as heavy as $m_q = m_c \sim 1.2$ GeV:

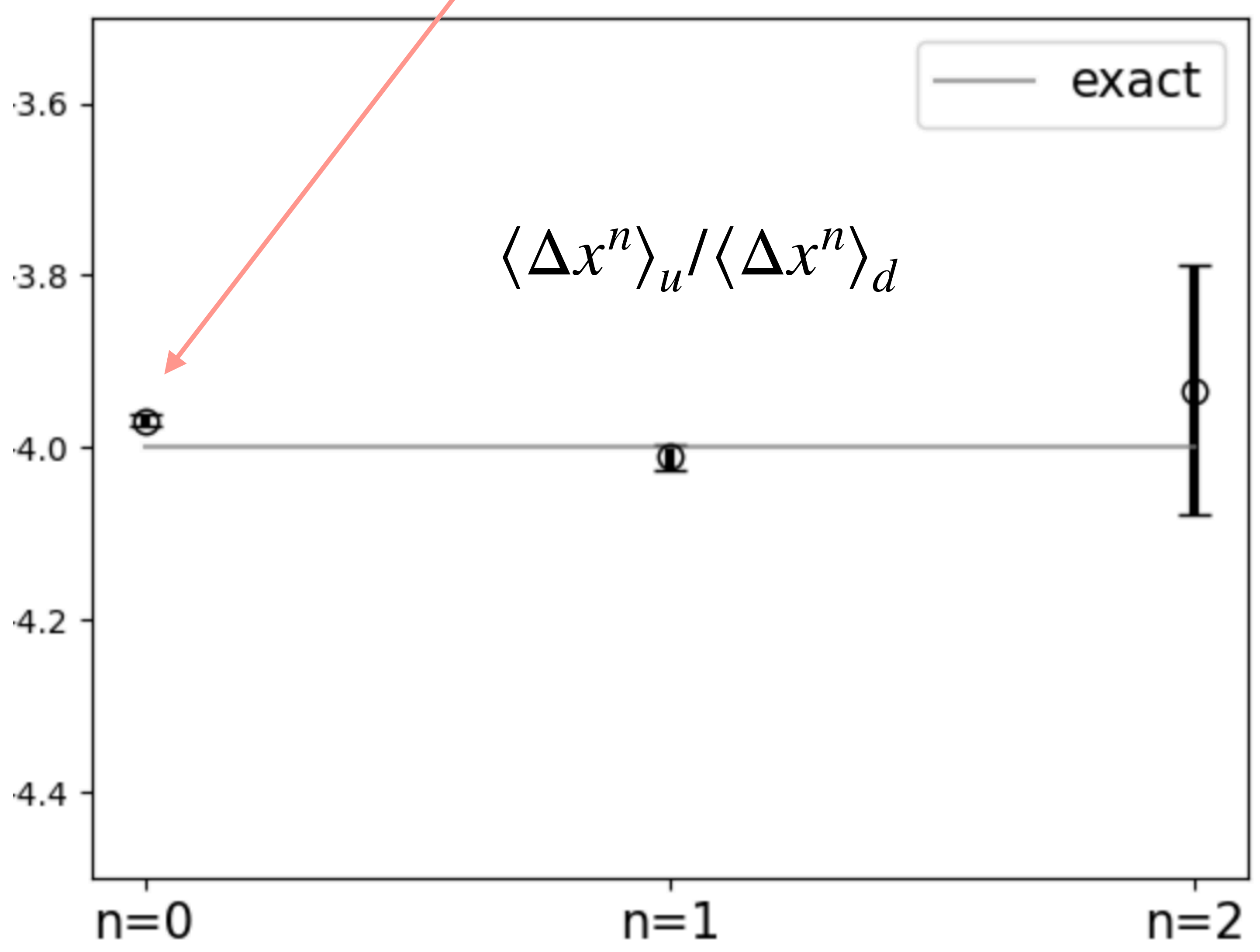
- Quark spin contribution agree with the quark model prediction at 90% level;

F. He et al. [χQCD], 2410.08046

- For the nucleon-like heavy quark baryon state, one also expects $\langle \Delta x^n \rangle_u \sim \frac{4}{3}(\frac{1}{3})^n$, $\langle \Delta x^n \rangle_d \sim -\frac{1}{3}(\frac{1}{3})^n$;





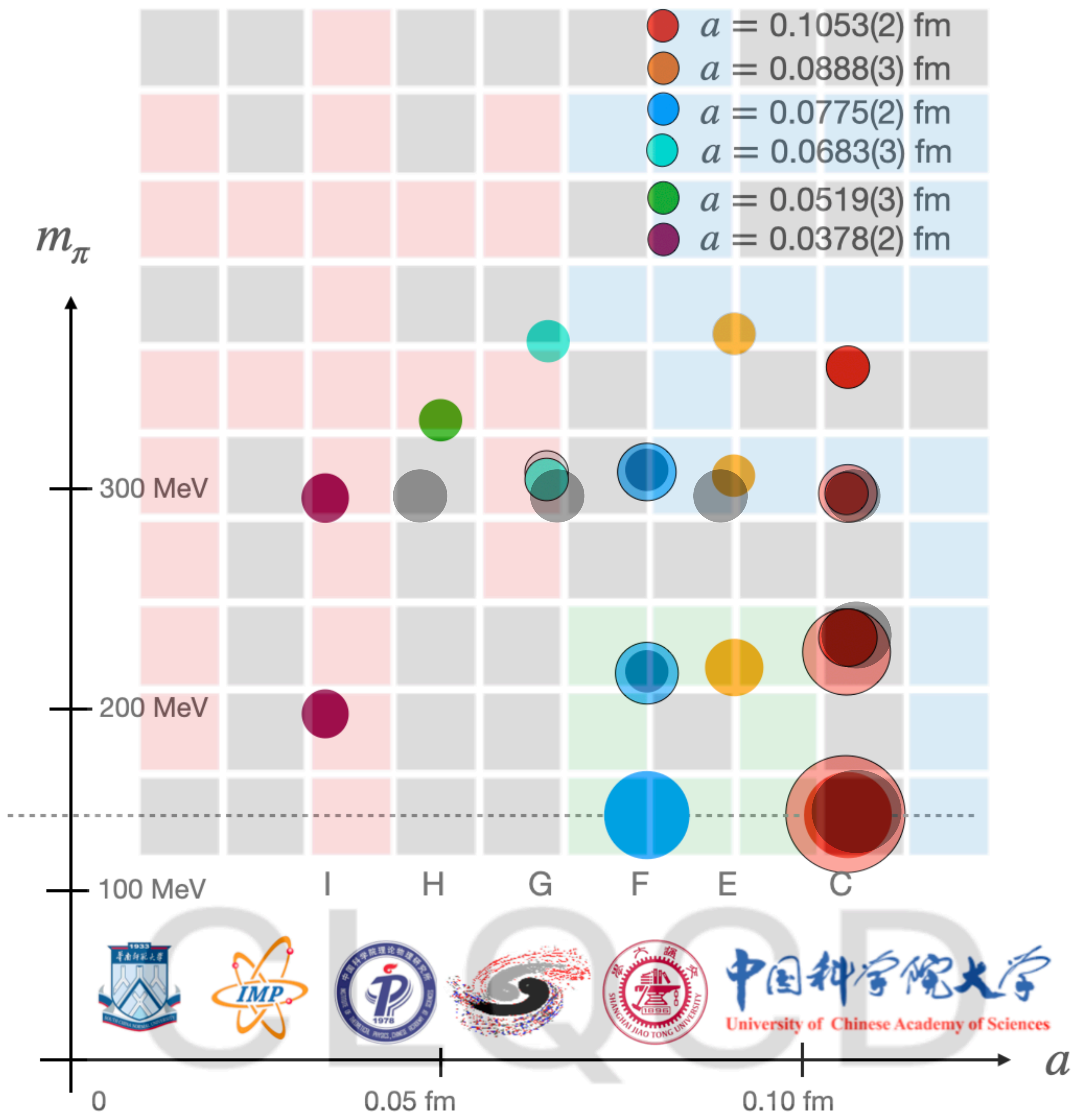


J.H. Wang et al. [CLQCD], In preparation

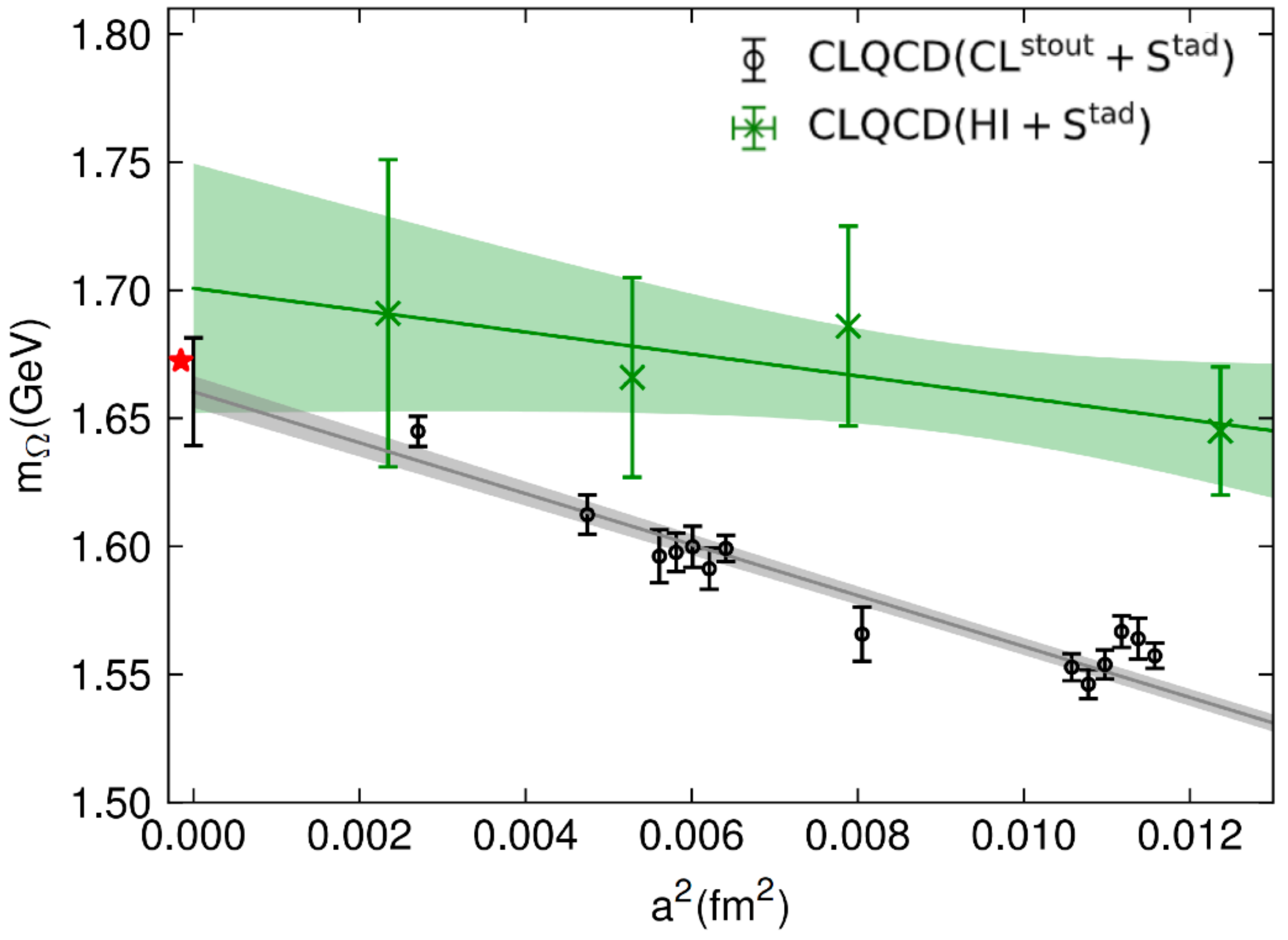
- Practical calculation also verifies this.

CLQCD ensembles

	Country/ Region	Smallest lattice spacing	No. of physical point ensembles	Largest spacial size	No. of fermion discretization
MILC	US	0.03 fm	5	5.8 fm	1
RBC	US	0.06 fm	3	5.5 fm	1
BMW	EN	0.05 fm	15	10 fm	2
CLS	EN	0.04 fm	2	5.5 fm	1
ETM	EN	0.05 fm	5	6.3 fm	1
PACS	JP	0.06 fm	3	10 fm	1
CLQCD	**CN**	**0.04 fm**	**3**	**6.7 fm**	**2**

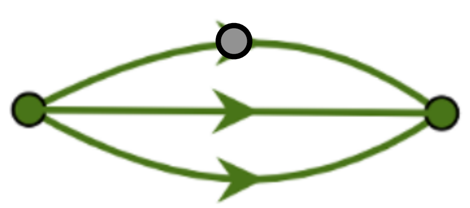


- The first ensemble set from China which can control most of the systematic uncertainties;
- Unique advantage on finite volume studies.

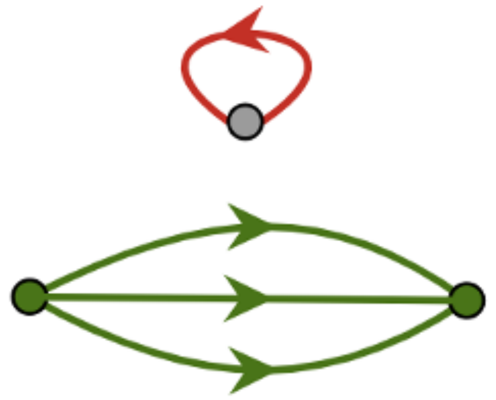


- New ensembles $(\mathrm{HI}+\mathrm{S}^{\mathrm{tad}})$ with 2+1+1 flavor HISQ fermion can provide proper estimate of the charm sea effects;
- Compared to the current 2+1 flavor Clover fermion ensembles $(\mathrm{CL}^{\mathrm{stout}}+\mathrm{S}^{\mathrm{tad}})$, the discretization errors are also suppressed in kinds of the cases.

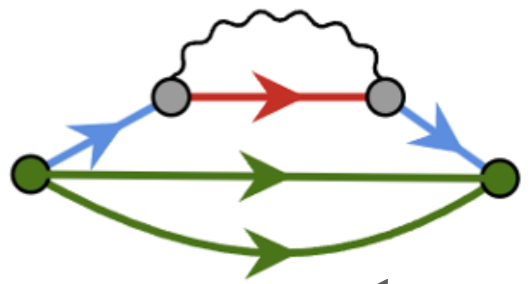
Blending method



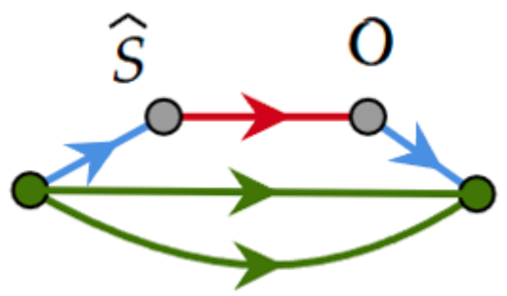
$\propto 1$



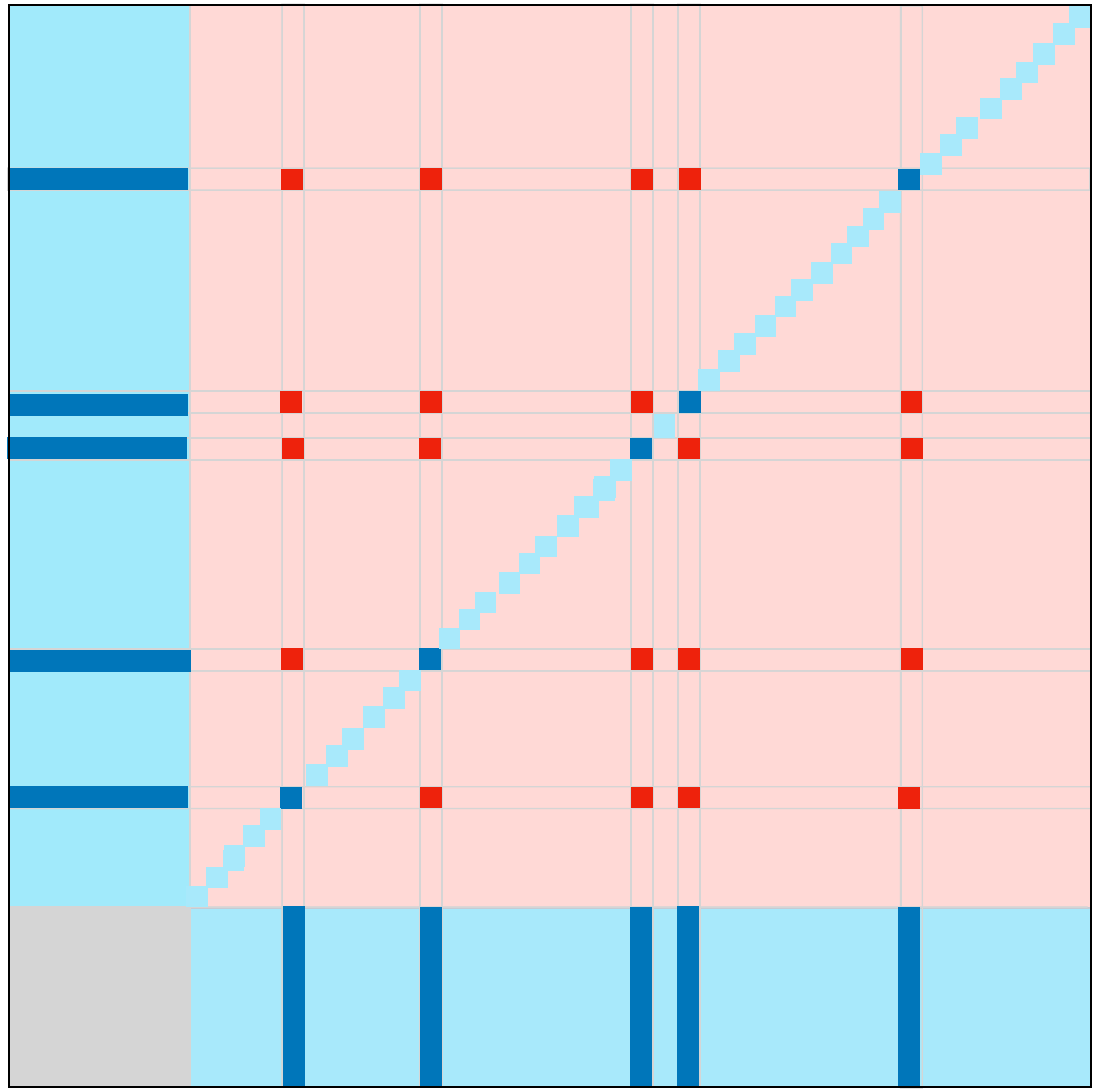
$\propto \alpha_s^2$



$\propto \alpha_{\rm QED} = \dfrac{1}{137}$



$\propto \dfrac{m_d - m_u}{\Lambda_{\rm QCD}} \sim 1\,\%$



- The "blending" method projects the all-to-all propagators into exact low momentum modes plus stochastic samples of high momentum modes;

- And allow us to calculate the hadron matrix element efficiently with kinds of high order diagrams (disconnected quark diagram, QED correction, iso-spin breaking effect et. al.).

$$M_{\mathcal{O}}(x,y) = \int \mathrm{d}^3z\mathrm{d}^3w \hat{I}_{x,z}\mathcal{O}(z,w)\hat{I}_{w,y} = \sum_{i=1}^{N_e+N_{st}}\sum_{j=1}^{N_e+N_{st}} |\phi_i(x)\rangle \mathcal{O}_{ij}\langle\phi_j(y)| + \mathcal{O}(\frac{1}{N_{st}}),$$

$$\mathcal{O}_{ij} \equiv \Omega_{ij}^{(2)}\int \mathrm{d}^3z\mathrm{d}^3w \langle\phi_i(z)|M_{\mathcal{O}}(z,w)|\phi_j(w)\rangle$$

$$\Omega_{ij}^{(2)} = \begin{cases} 1 & \text{for } i,j \le N_e, \\ \Pi_{i=0}^{1}\dfrac{V-N_e-i}{N_{st}-i} & \text{for } i,j > N_e,\ i \neq j \\ \dfrac{V-N_e}{N_{st}} & \text{for the other cases,} \end{cases}$$

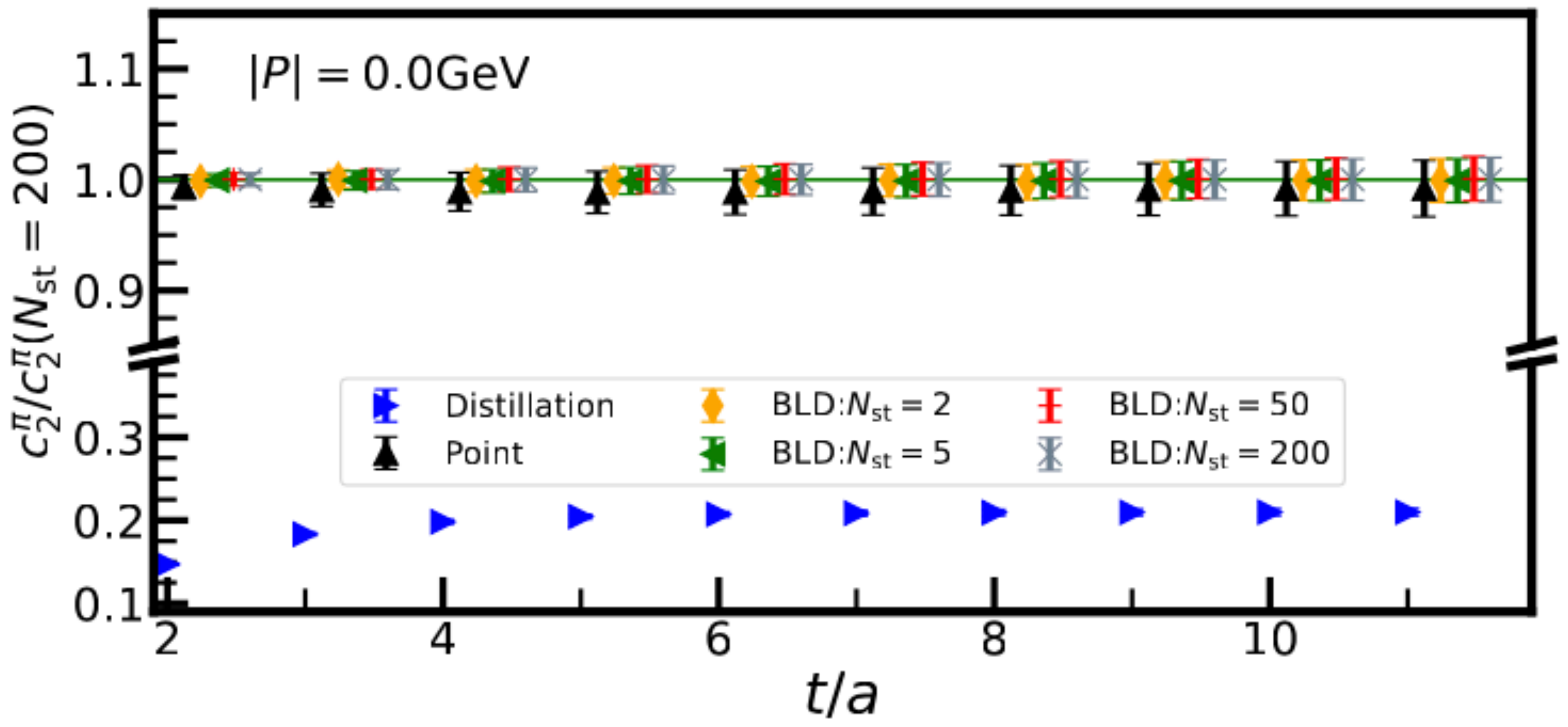
Oct.11th 16:40-17:00 分会1 胡志成

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

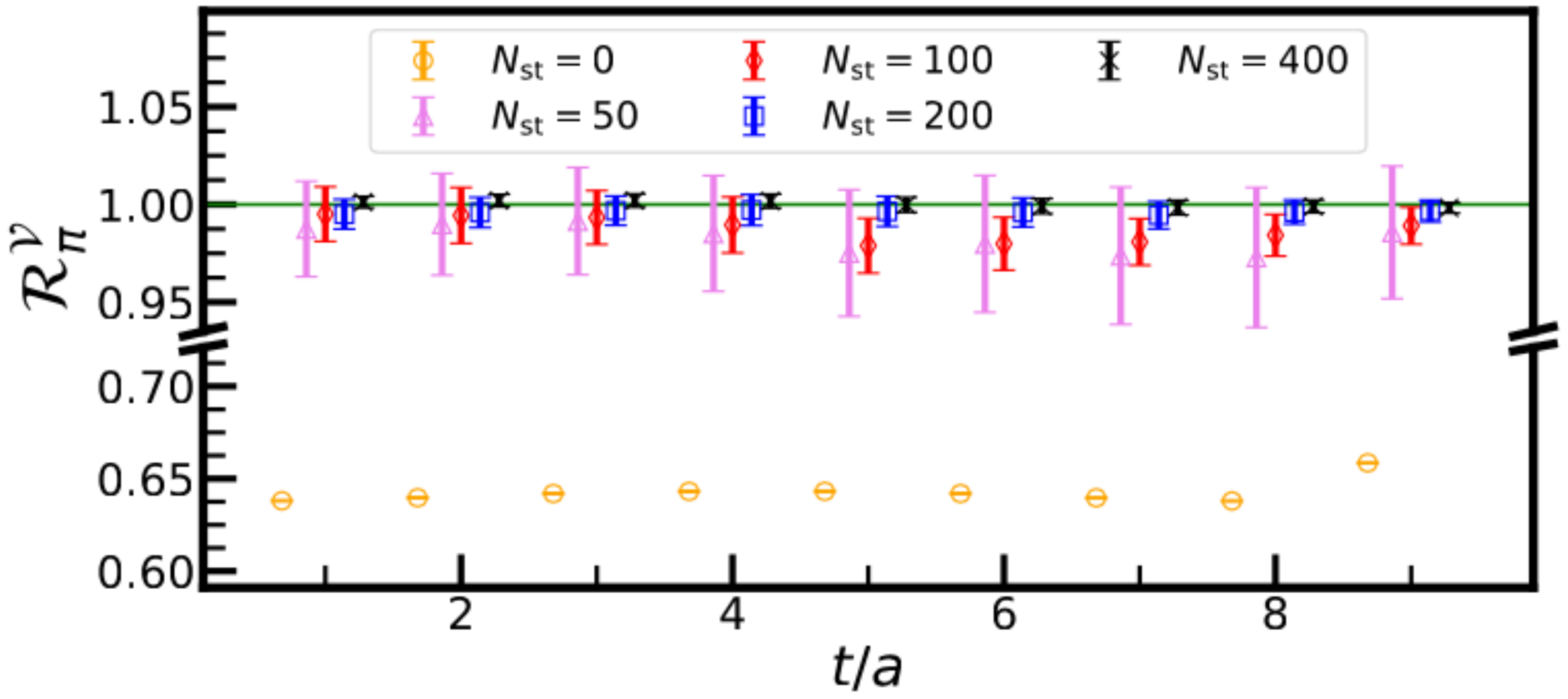
Blending method

Verification

$C_2^{\text{BLD}}(t)/C_2^{\text{pt}}(t) = 1$

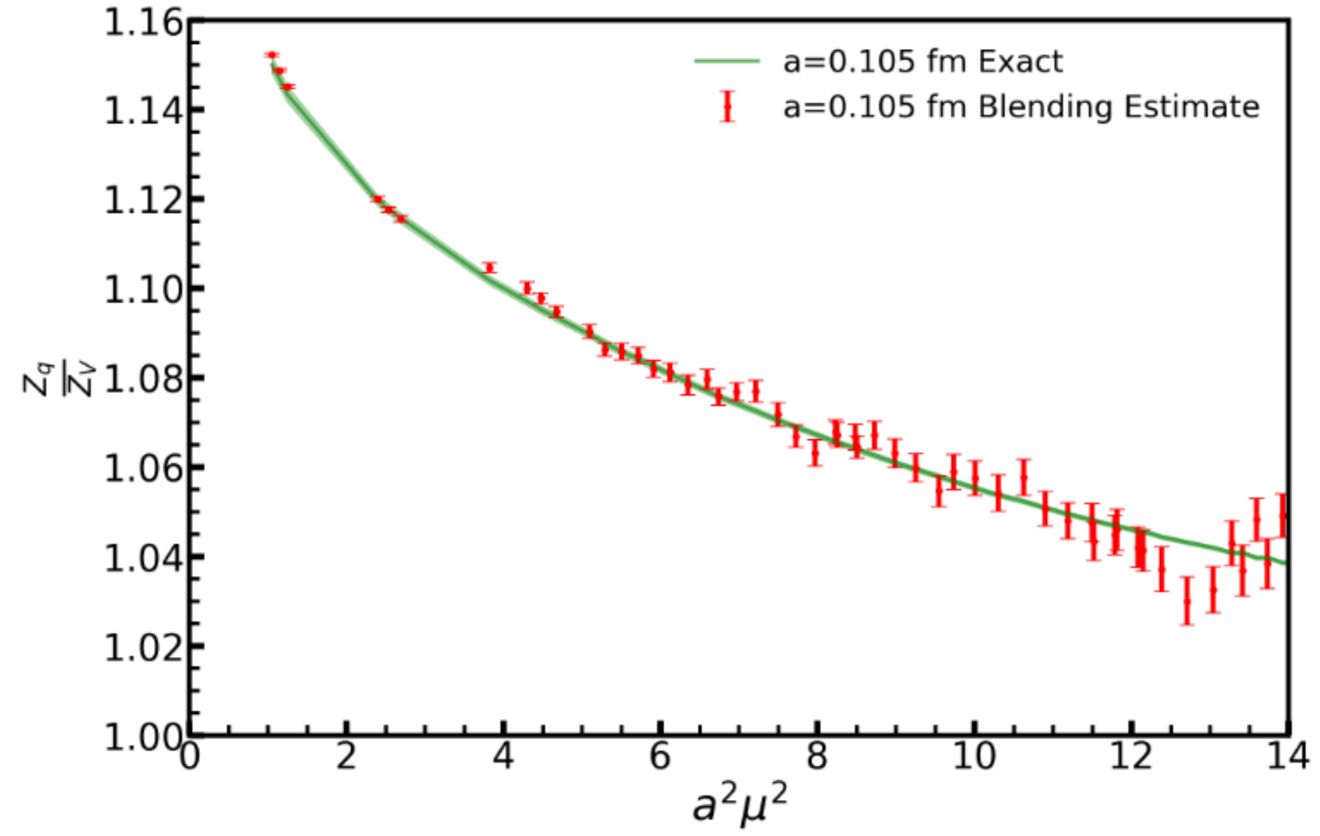


$\langle \mathcal{V}_4^{\text{conserved}} \rangle_\pi = 1$

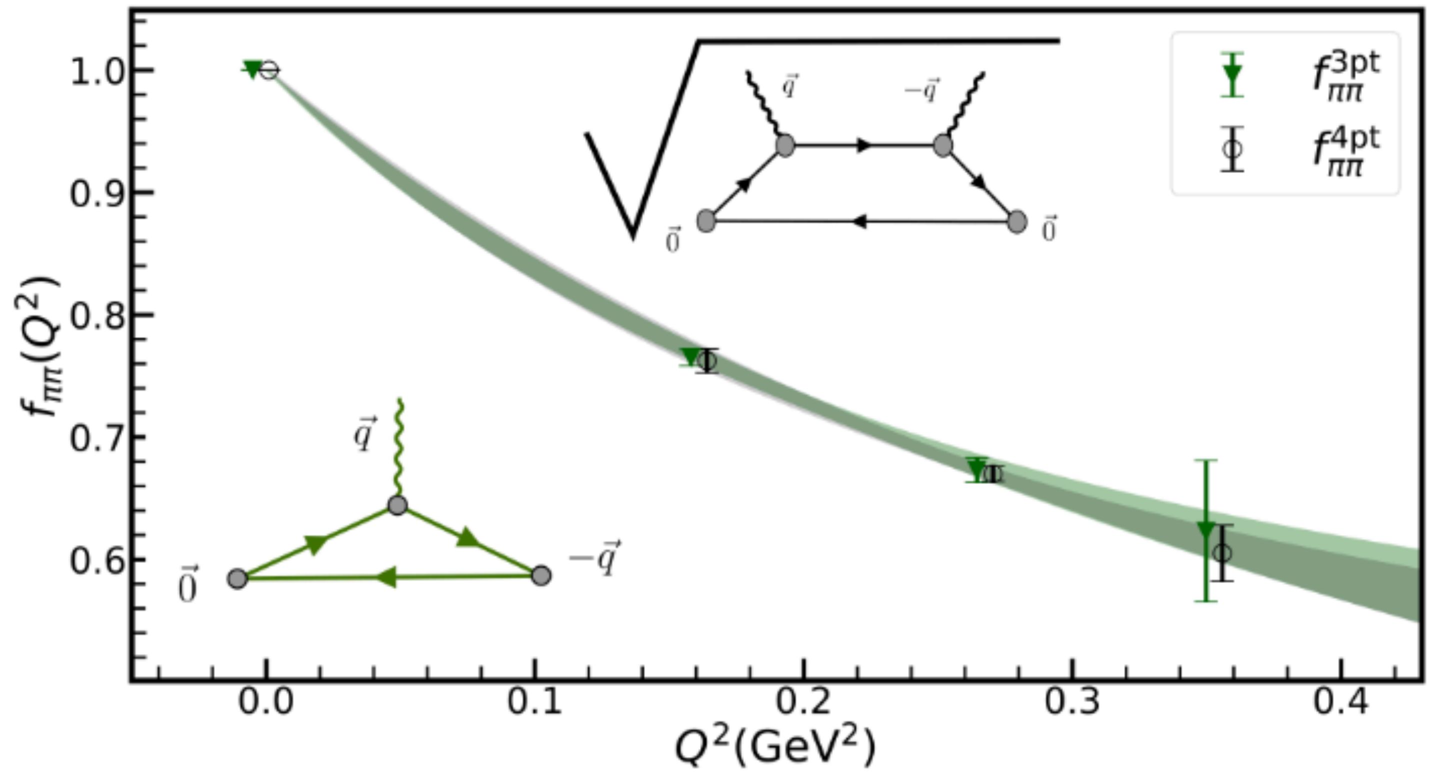


- All-to-all propagator can be approximated unbiasedly with $\mathcal{O}(1\%)$ inversions;
- The high momentum mode turns out to be important in the full correlation functions and can not be ignored.
- Unbiasedness has been verified using two-, three-, and four-point functions.

$Z_q^{\text{BLD}}(a^2\mu^2)/Z_q^{\text{vol}}(a^2\mu^2) = 1$



$f_{\pi\pi}^{\text{3pt,BLD}}(Q^2) = f_{\pi\pi}^{\text{4pt,BLD}}(Q^2)$

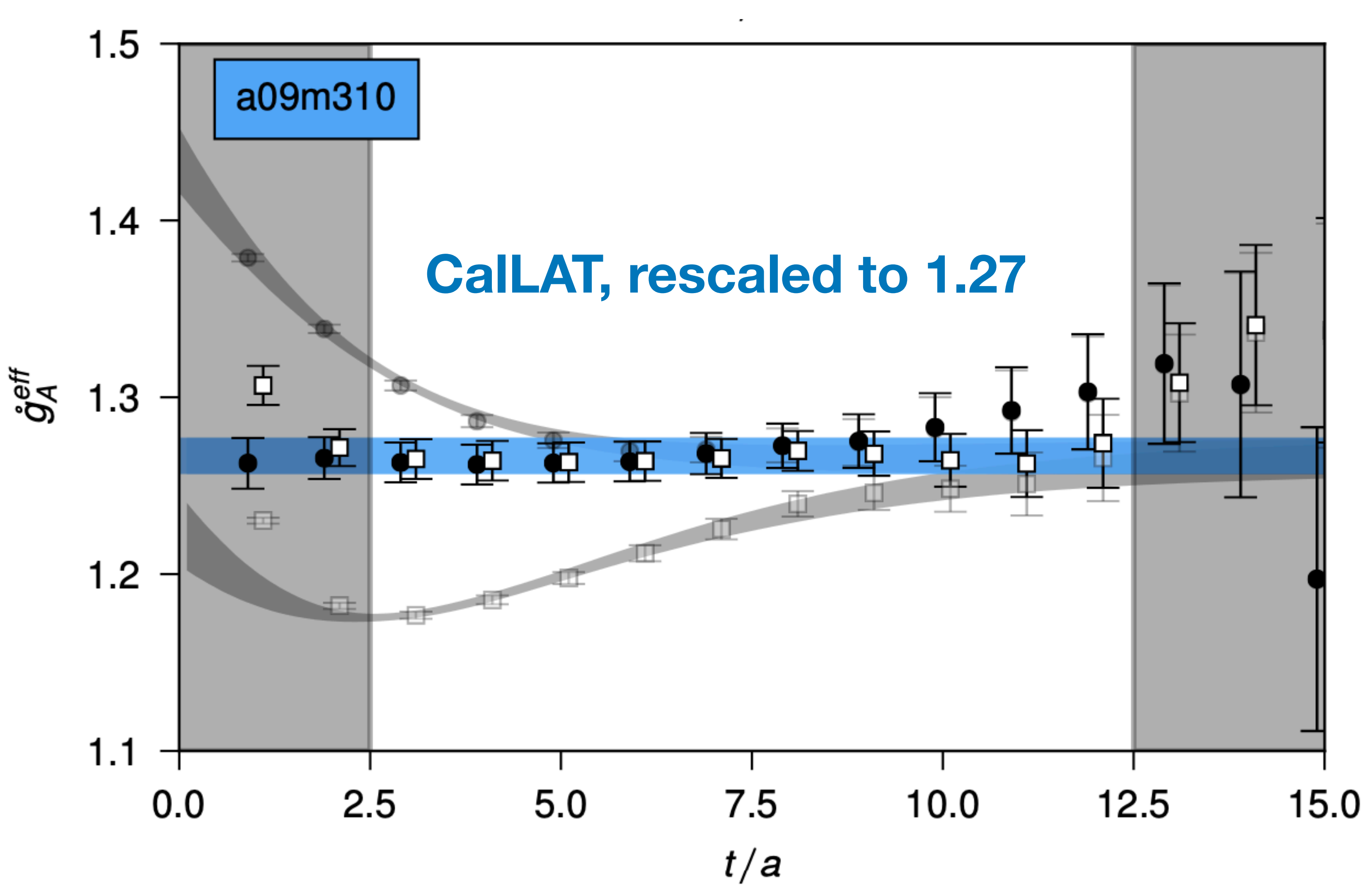


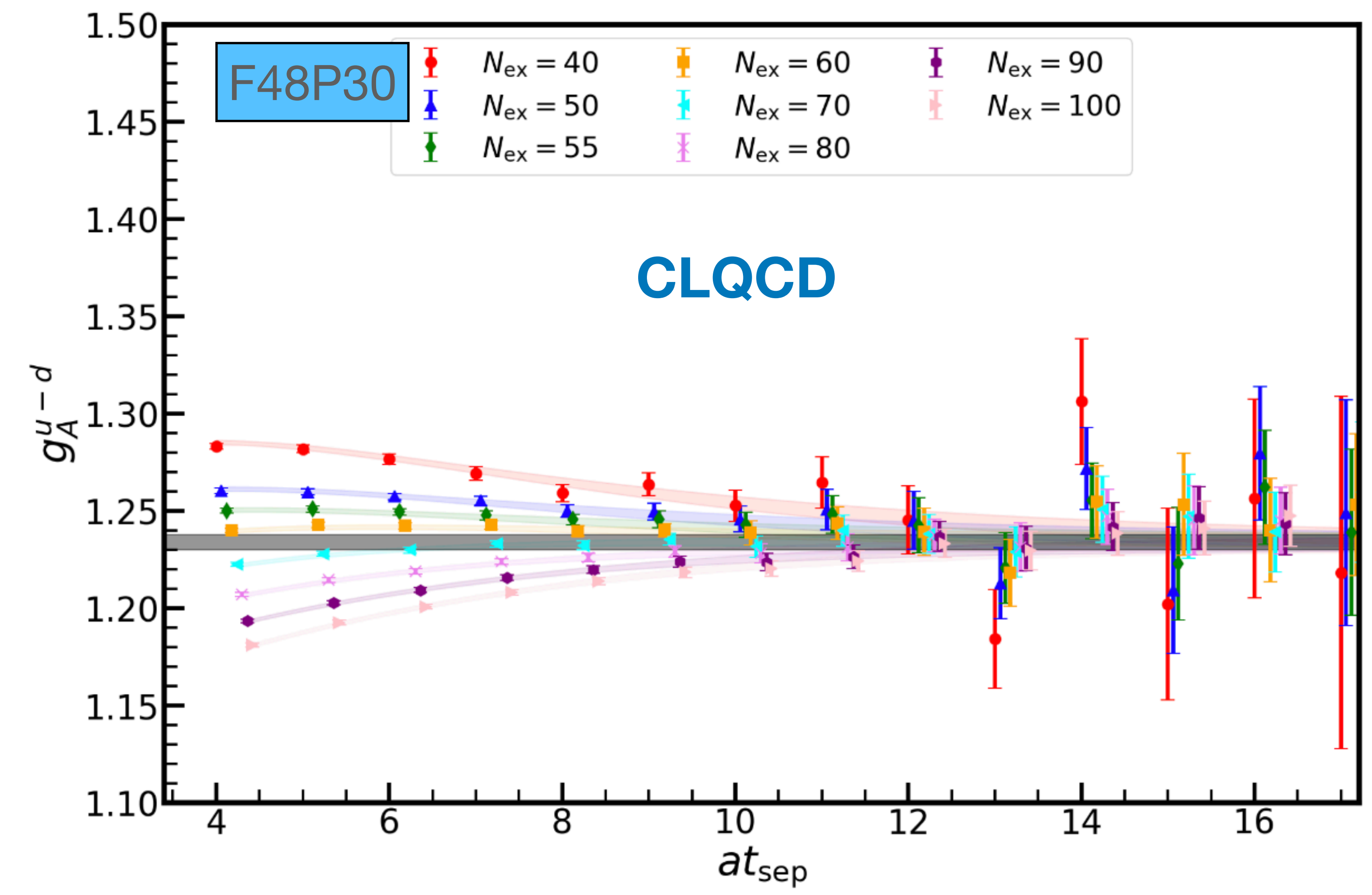
Oct.11th 16:40-17:00 分会1 胡志成

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

Blending method

Cost comparison





	Ensemble	L	T	a(fm)	m_π	n_{cfg}	g_A^{u-d}	Propagators	Propagators for 0.35% error
CLQCD	F48P30	48	96	0.078	300	**40**	**1.234(04)**	**0.19M**	**0.19M**
CalLAT	a09m310	32	96	0.090	310	784	1.235(11)	0.06M	0.37M
RQCD	N450	48	128	0.076	287	1132	1.238(24)	0.02M	0.44M

C.C.Chang et. al., [CalLAT], Nature 558(2018)91
G.S.Bali et. al., [RQCD], PRD108(2023)034512

- Blending method can reach the same precision with **50%** cost of either FH-inspired (CalLAT) or stochastic (RQCD) method, and also

1. Provide the information from arbitrary source-current-sink separations;
2. Reuse data in any other calculation which needs light quark propagators, including disconnected insertions.

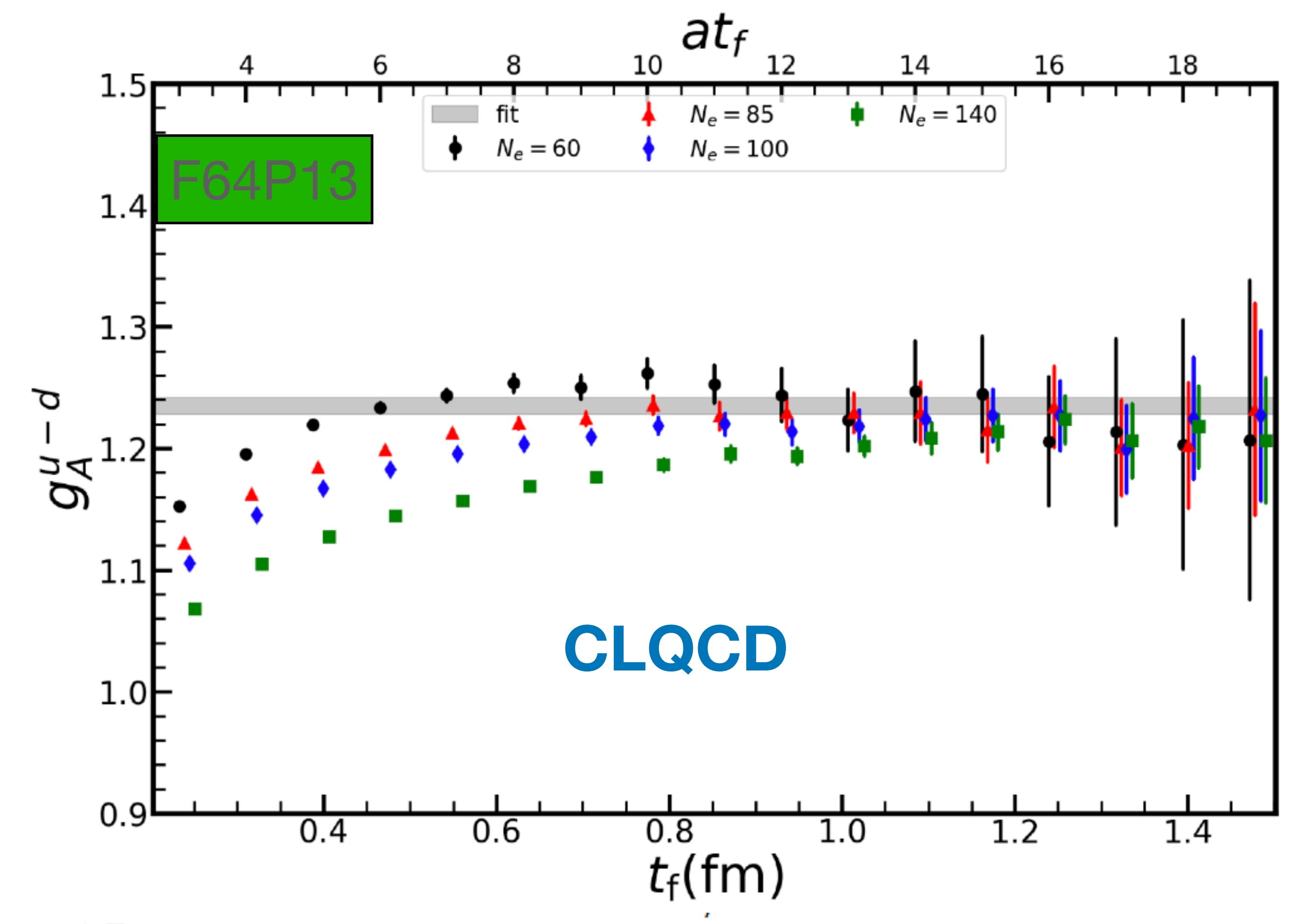
	Ensemble	L	T	a(fm)	m_π	n_{cfg}	$g_A^{u,DI}$	Propagators	Propagators for 17% error
CLQCD	F48P30	48	96	0.078	300	**40**	**-0.052(9)**	$0^\dagger$	$0^\dagger$
PNDME	a09m310	32	96	0.090	310	1081	-0.053(6)	4.0M	1.8M

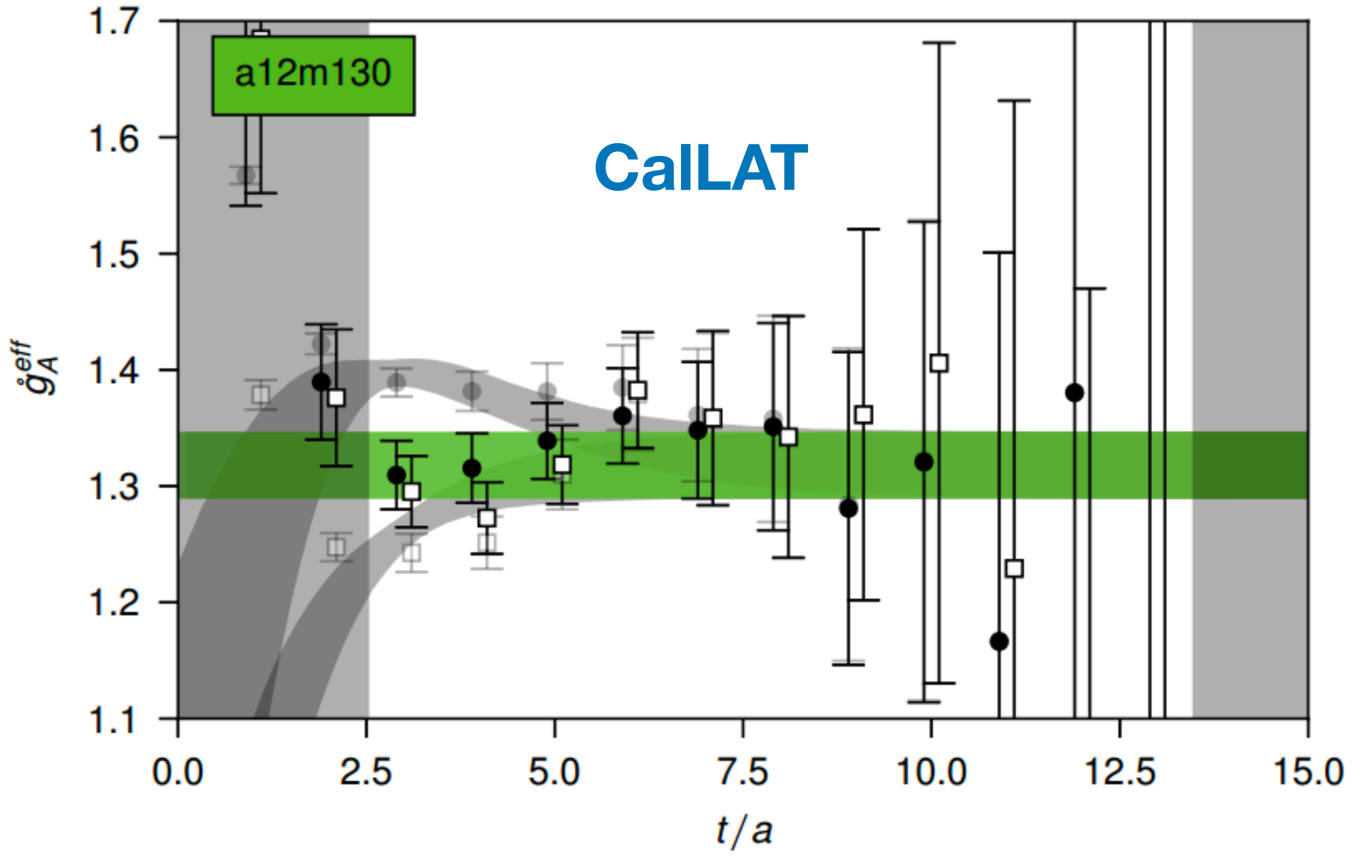
† Shared with that for g_A^{u-d}

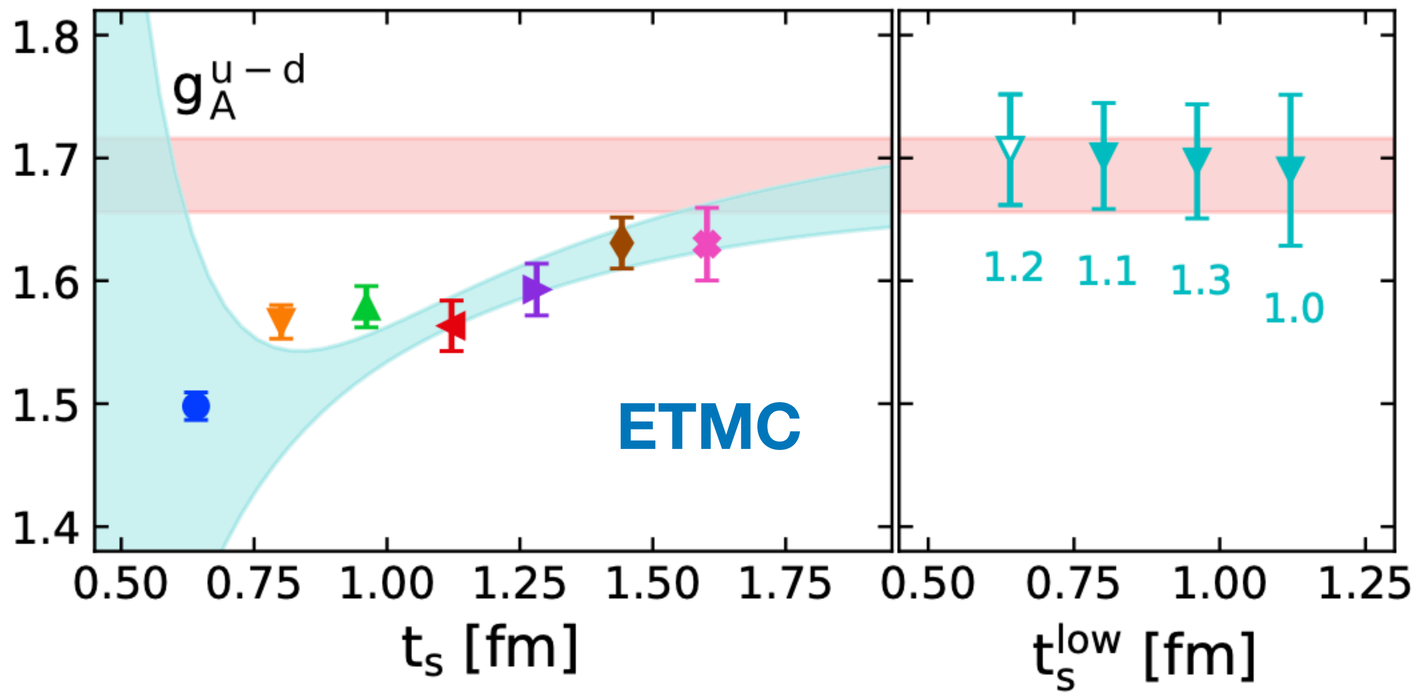
Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

Blending method

Cost comparison at physical pion mass







	Ensemble	L	T	a(fm)	m_π	$n_{\rm cfg}$	g_A^{u-d}	Propagators	Propagators for 1% error
CLQCD	F64P13	64	128	0.078	134	40	1.24(01)	0.34M	0.11M
CalLAT	a12m130	48	64	0.121	131	1000	1.29(03)	0.03M	0.15M
ETMC	cB211.072.64	64	128	0.080	139	750	1.29(02)	1.71M	5.5M
RQCD	D452	64	128	0.076	156	1000	1.19(25)	0.01M	5.2M
PNDME	a09m130	64	96	0.090	138	1290	1.32(03)	1.69M	11.2M

Z.C.Hu, et. al.,[CLQCD] in preparation

C.C.Chang et. al., [CalLAT], Nature 558(2018)91

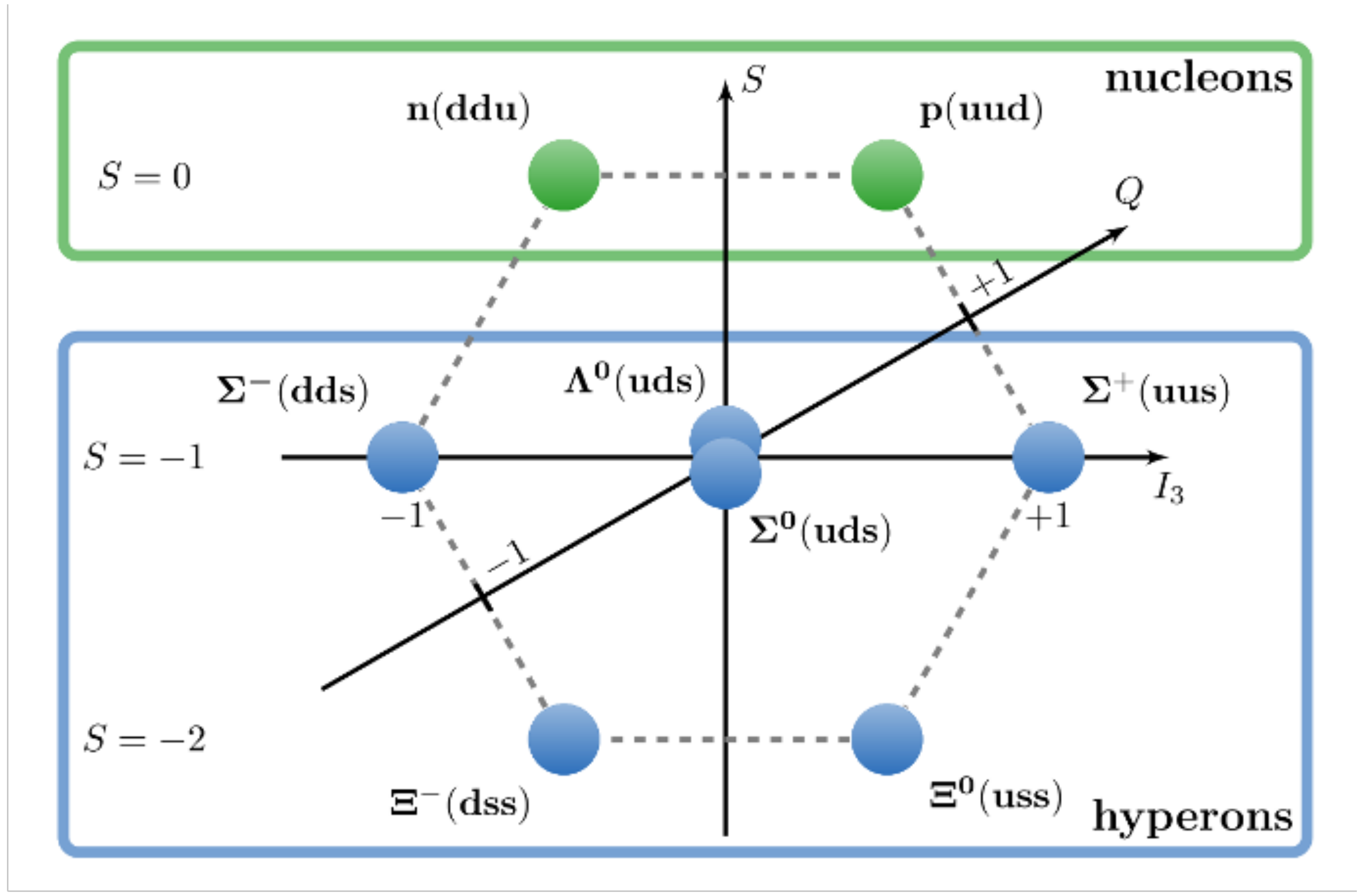
C.Alexandrou et. al., [ETMC], PRD102(2020)054517

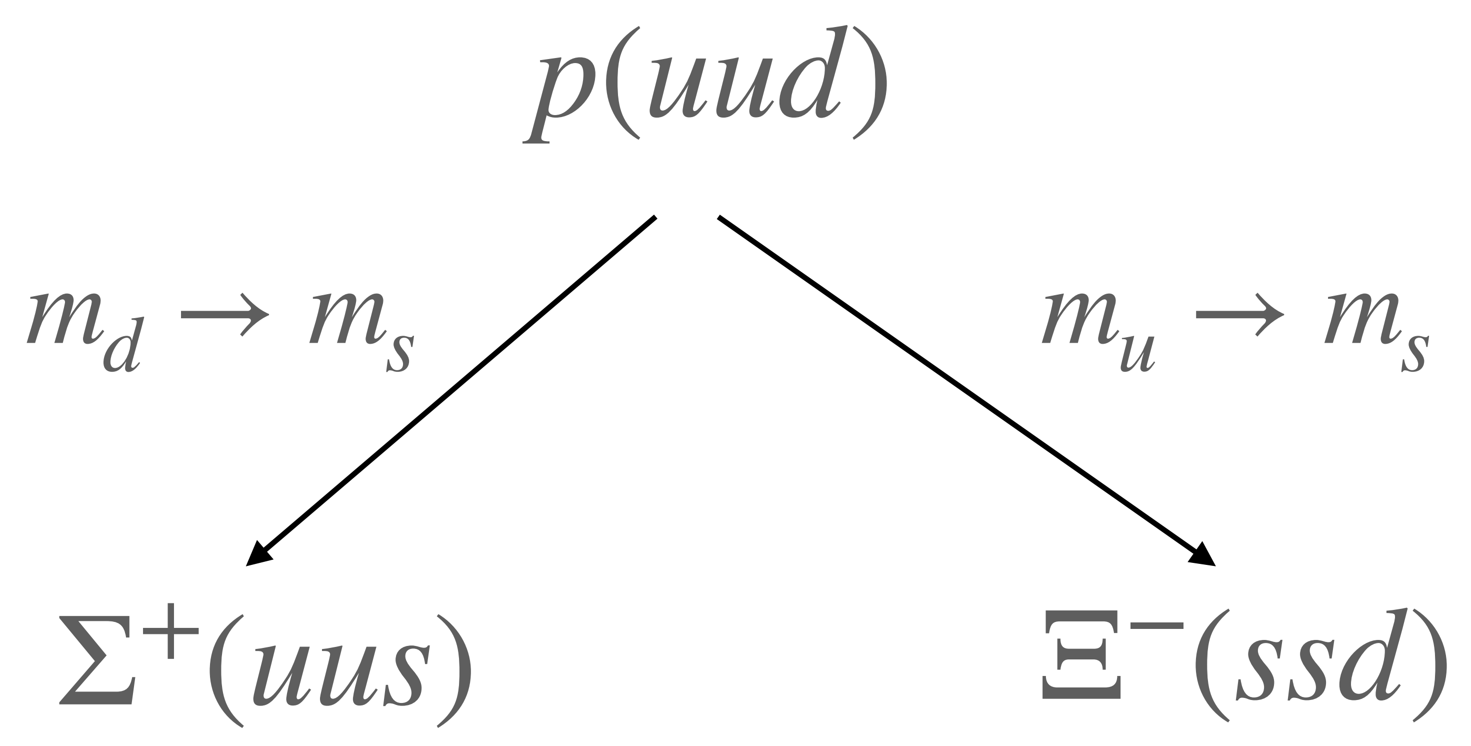
G.S.Bali et. al., [RQCD], PRD108(2023)034512

Y.C.Jang et. al., [PNDME], PRD109(2023)014503

- Advantage becomes much more significant at the physical pion mass, except the CalLAT results which is only available at much larger lattice spacing;
- And also provide much more information on different source-current-sink separations and nucleon interpolation fields, which allow us to have much better control on the excited state contaminations.

Quark spin





$$m_u^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 2.14(8)\ \mathrm{MeV}$$
$$m_d^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 4.70(5)\ \mathrm{MeV}$$
$$m_s^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 93.5(6)\ \mathrm{MeV}$$

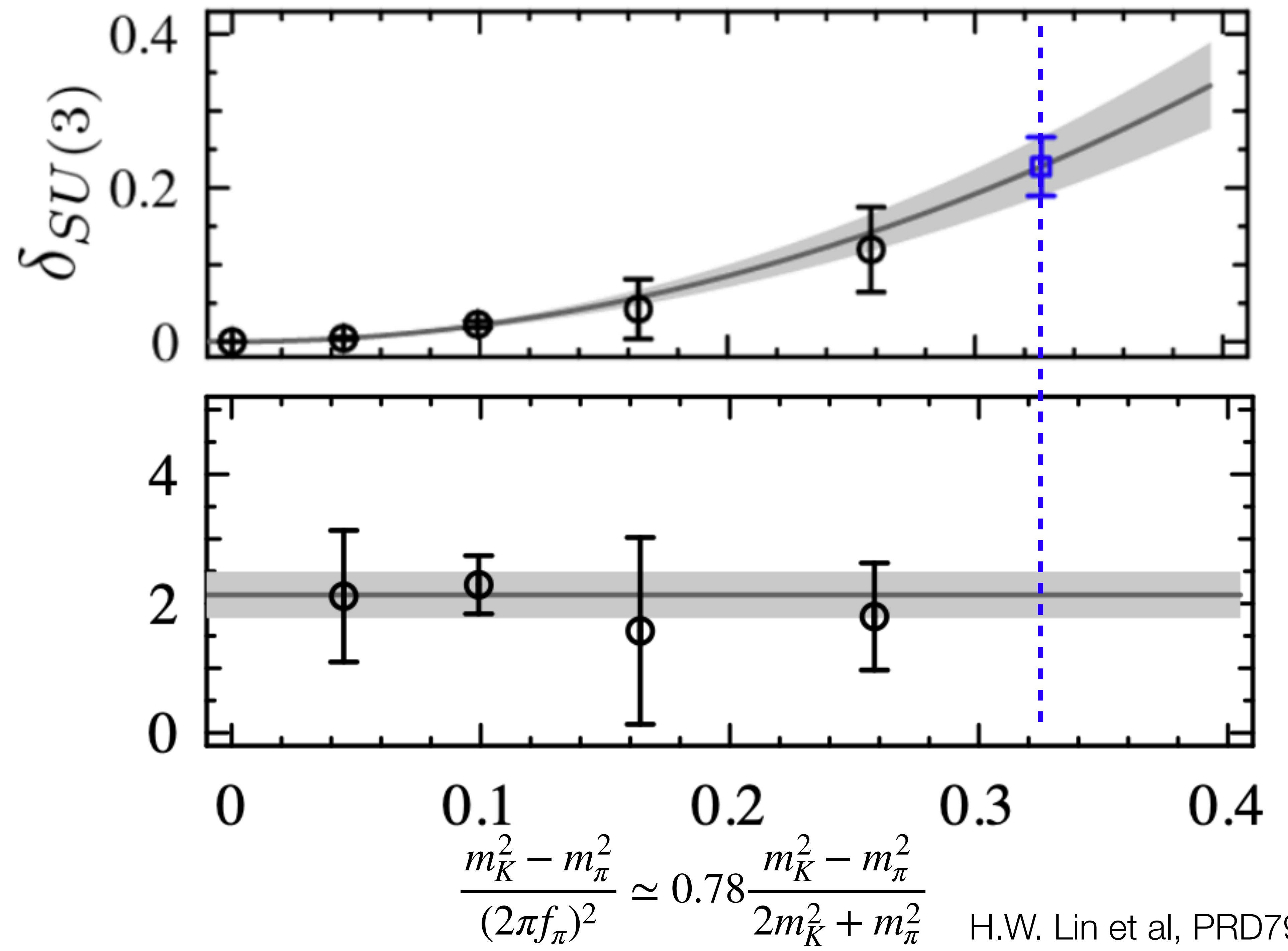
Flavor lattice average group, 2411.04268

$$\delta_{\mathrm{SU(3)}} = g^p_{A^{u-d}} - g^{\Sigma^+}_{A^{u-d}} + g^{\Xi^+}_{A^{u-d}}$$
$$\sim g^p_{A^{u-d}} - g^p_{A^u_{\mathrm{CI}}} + g^p_{A^d_{\mathrm{CI}}} = 0$$

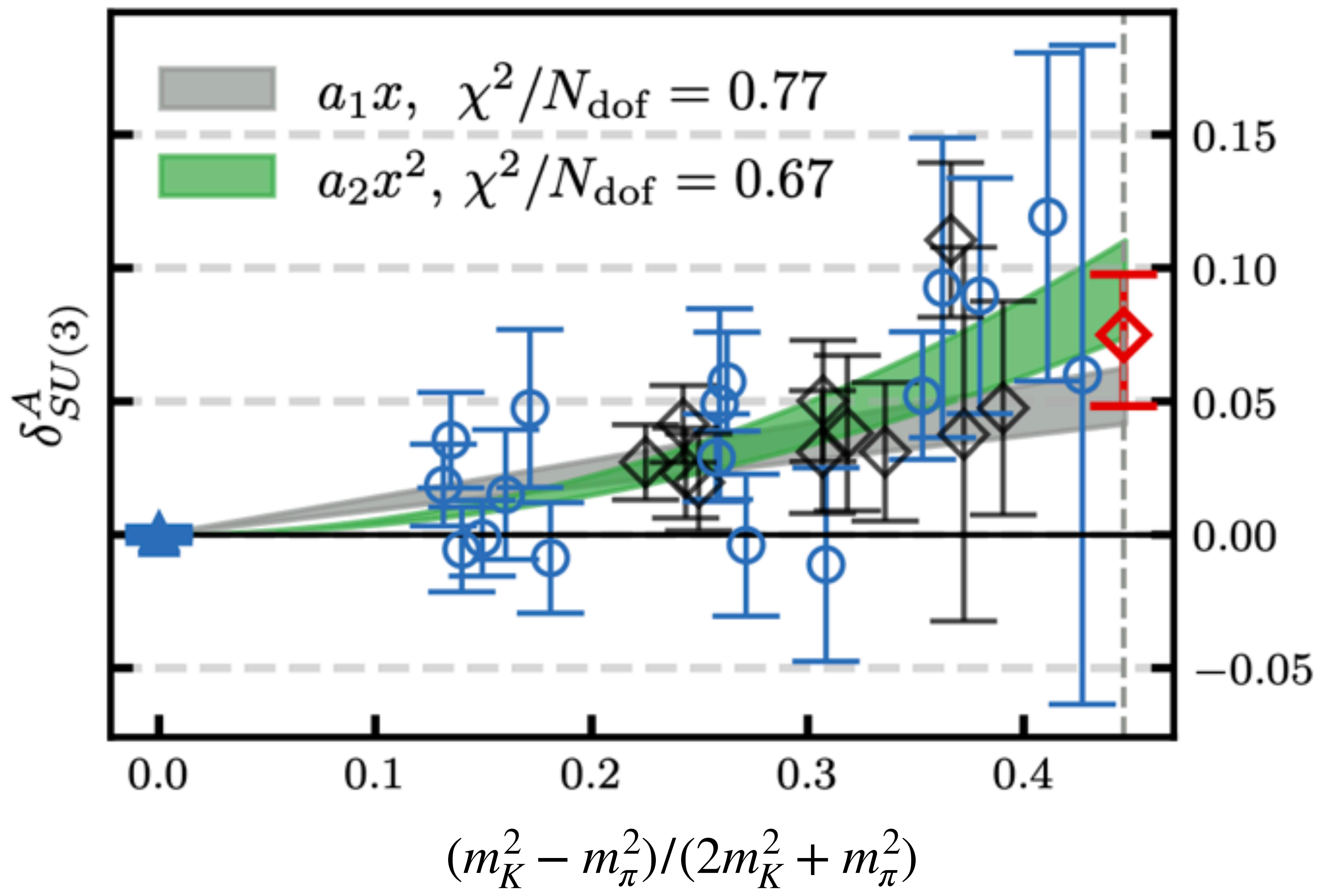
- One can calculate the quark spin of SU(3) octet baryon using the blended light and strange quark propagators.
- The quantity has the best signal is $\delta_{\mathrm{SU(3)}}$ with the connected insertion part of the u and d quarks only.

Quark spin

SU(3) breaking effects

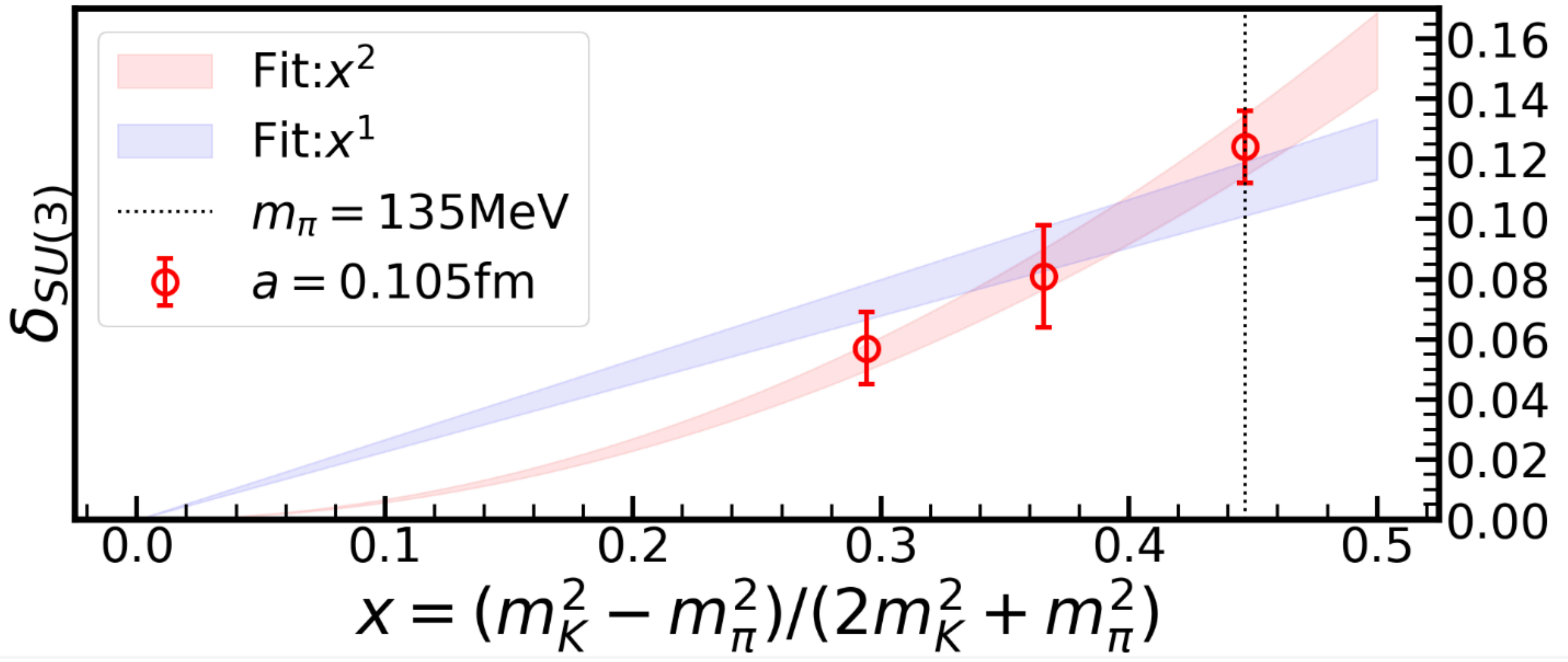


H.W. Lin et al, PRD79(2009)034507



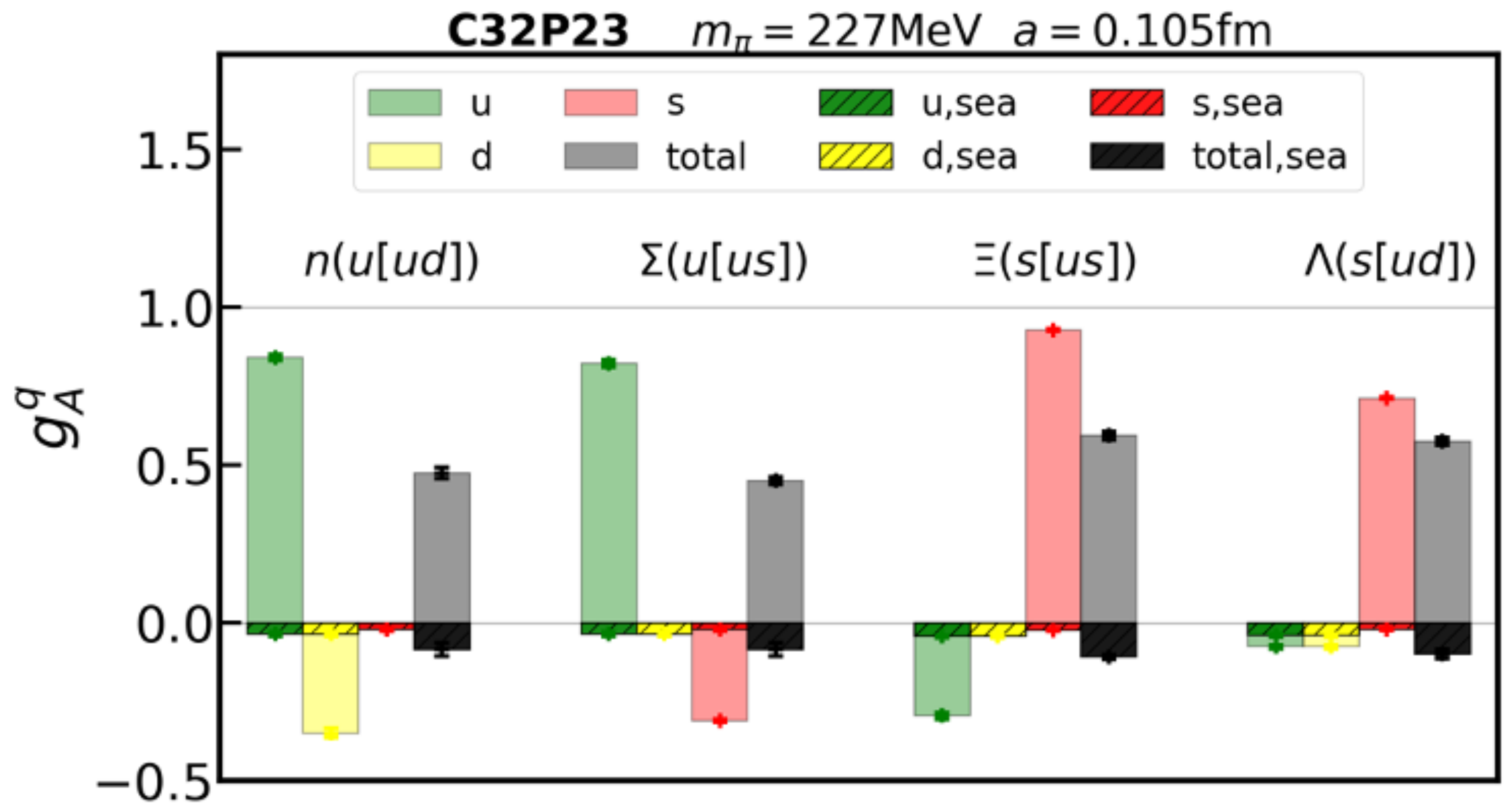
G.S.Bali et. al., [RQCD], PRD108(2023)034512

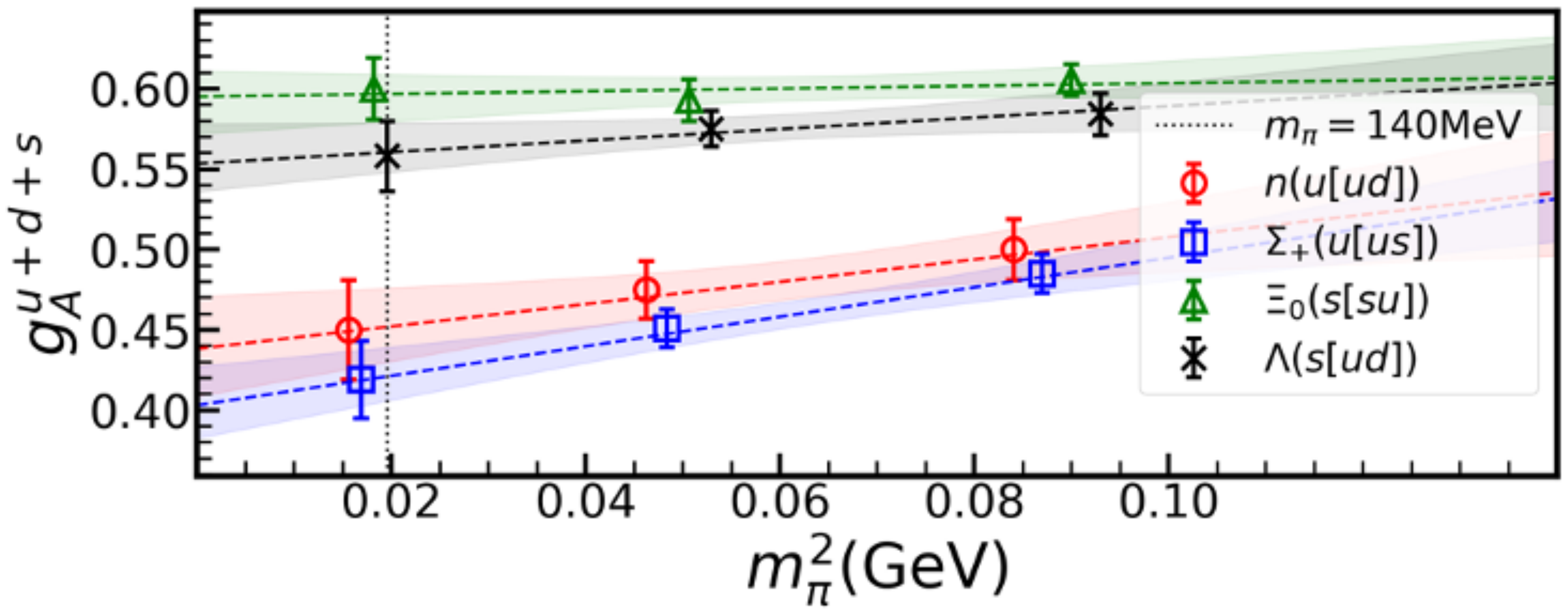
- The previous predictions on $\delta_{\rm SU(3)}$ has huge uncertainty around the physical point and rely on the functional form of $x \propto m_s - m_l$ ($\delta_{\rm SU(3)}(x) \propto x$ or x^2);
- Our prediction directly at the physical point can be even more precise than the existed ones based on extrapolations.



Z.C.Hu, et. al., [CLQCD], in preparation

Quark spin



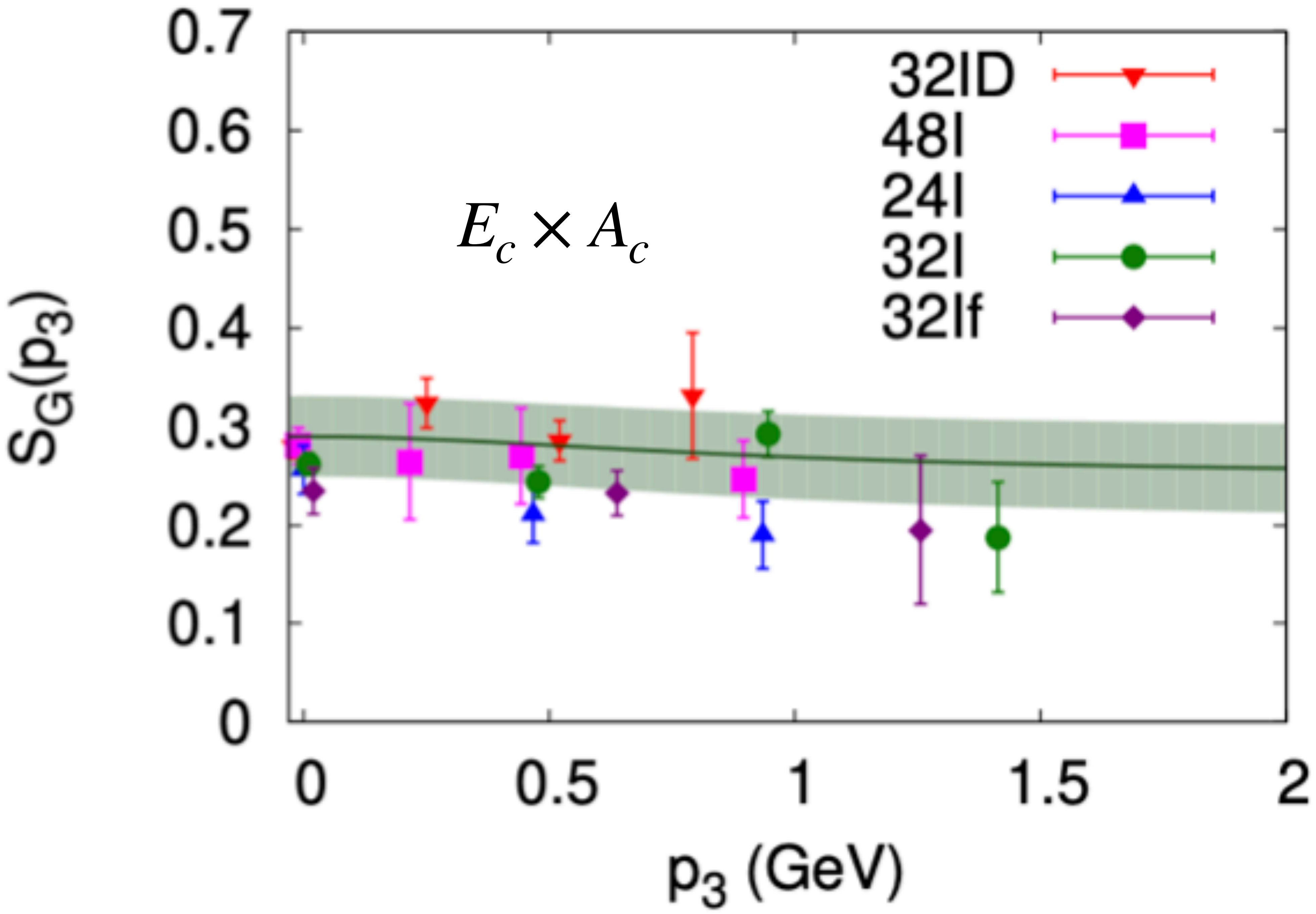


- Further more, one can calculate the contribution by reusing the same blended propagators.
- The results suggest that SU(3) breaking effect is indeed small, and eventually the quark spin contribution to any of the octet baryons are around 50%.
- Repeating the calculation on the other ensemble can significantly suppress the statistical uncertainty after the continuum and chiral extrapolations.

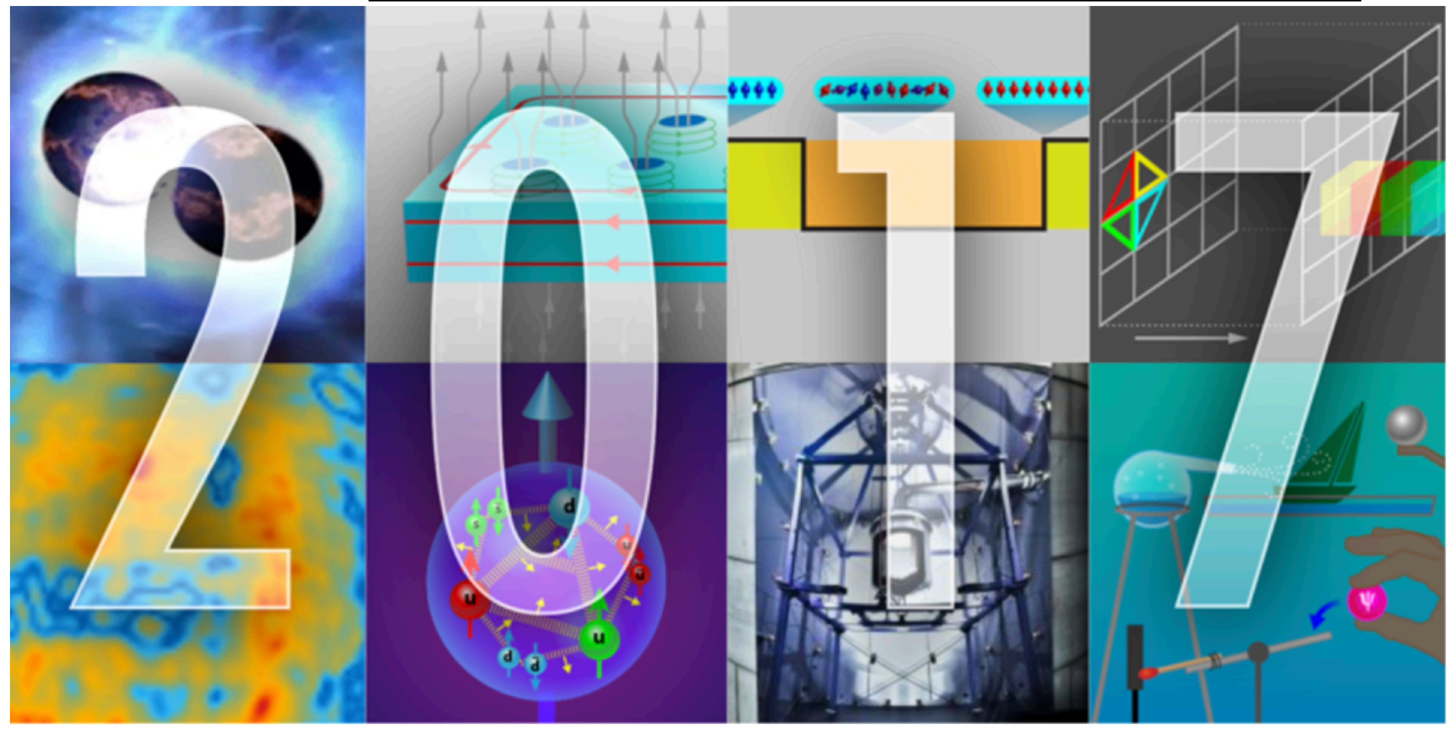
Z.C.Hu, J.H. Wang et. al., CLQCD, in preparation

Gluon spin

New lattice result



YBY et al. [χQCD], PRL118(2017)102001
one of eight APS highlight 2017

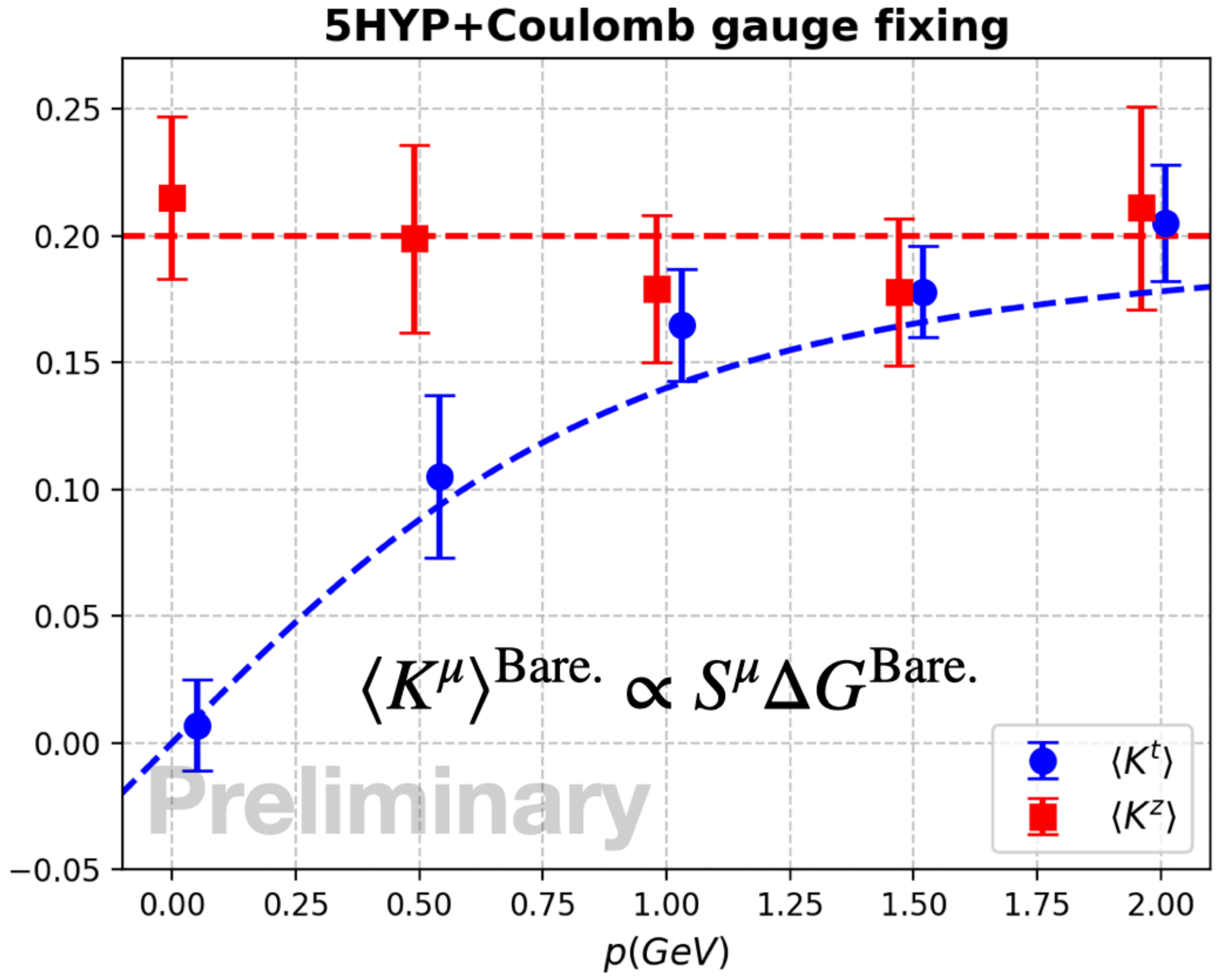


Zhuo-Yi Pang et al. JHEP07(2024)222

$$K^\mu(x) = \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[A_\nu F_{\rho\sigma} - 2ig_s A_\nu A_\rho A_\sigma/3](x)$$

Matching-free
Non-perturbative renormalizable

Distillation + momentum smearing



D.J. Zhao, et. al. [LPC], In preparation

- Experimental analysis and preliminary lattice calculation suggests the missing baryon spin would come from the gluon spin.
- Our recent work finds that the nucleon matrix element of the topological current K_μ can have similar kinetic feature as the axial current for quark spin, $\langle A_\mu^{u,d,s}\rangle_N \propto S_\mu g_A^{u,d,s}$.

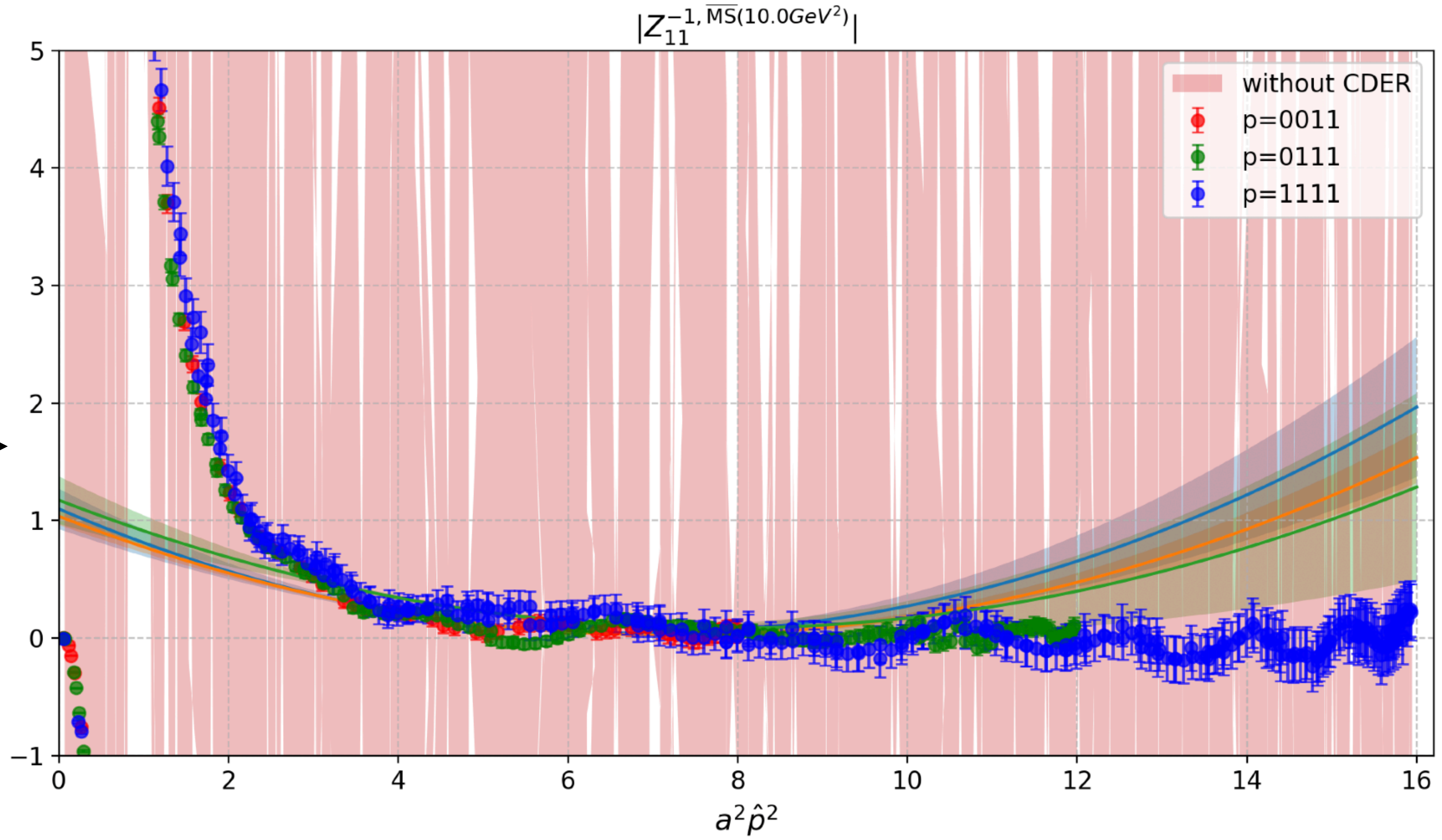
Gluon spin

Non-perturbative renormalization

$$S_G^{\overline{\rm MS}} = \left\{1 - \frac{g^2}{16\pi^2}\left[N_f\left(\frac{2}{3}\log(\mu^2a^2) + 1.27\right) - C_A\left(\frac{4}{3}\log(\mu^2a^2) + f_{gg}(g^2)\right)\right]\right\} S_G^L + \frac{g^2C_F}{16\pi^2}\left(\frac{5}{3}\log(\mu^2a^2) + 7.31\right)\Delta\Sigma^L + O(g^4)\ ,$$

$$f_{gg}(g^2 \in [2.5,3.4]) \in [1.7 - 2.4]$$

$$+\sum_{i=2}^{\infty}\alpha_s^i$$



YBY et al. [χQCD], PRL118(2017)102001
one of eight APS highlight 2017

YBY et al. [χQCD], PRD98(2018)074506

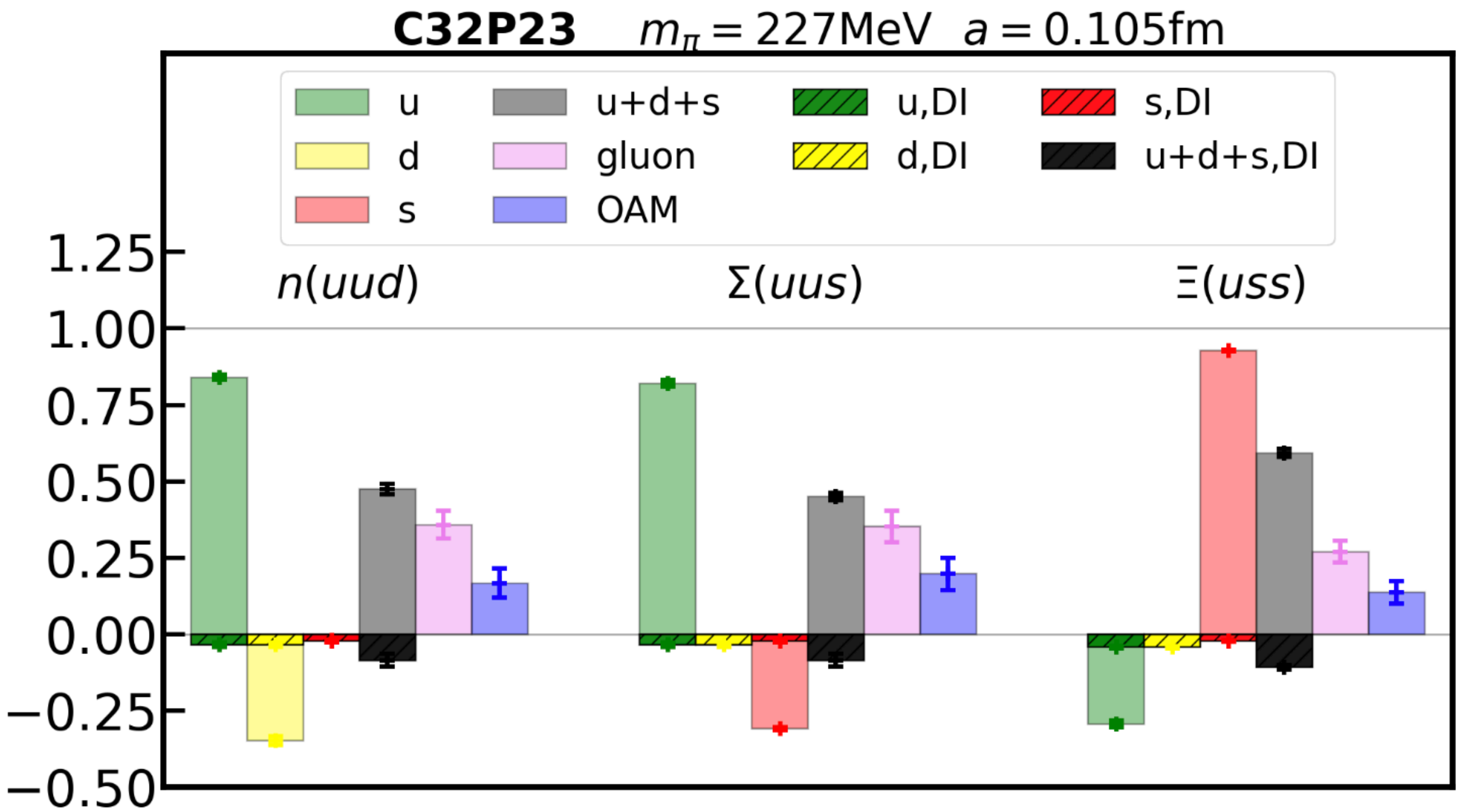
D.J. Zhao, et. al. [LPC], In preparation

- Previous study used the lattice perturbative theory to renormalize the gluon spin operator, and suffered from non-negligible higher order corrections.
- Using the cluster decomposition error reduction (CDER) method which can suppress the statistical uncertainty by $\mathcal{O}(100)$, one can renormalize the gluon spin operator non-perturbative **for the first time.**
- The final result including the mixing with the quark spin is in progress.

Summary

- CLQCD ensembles can now provide high precision hadron matrix elements through the blending method;

	Ensemble	L	T	a(fm)	m_π	n_{cfg}	g_A^{u-d}	Propagators	Propagators for 1% error
CLQCD	F64P13	64	128	0.078	134	**40**	**1.24(01)**	**0.34M**	**0.11M**
CalLAT	a12m130	48	64	0.121	131	1000	1.29(03)	0.03M	0.15M
ETMC	cB211.072.64	64	128	0.080	139	750	1.29(02)	1.71M	5.5M
RQCD	D452	64	128	0.076	156	1000	1.19(25)	0.01M	5.2M
PNDME	a09m130	64	96	0.090	138	1290	1.32(03)	1.69M	11.2M



- Update results on CLQCD ensembles suggest both quark and gluon contribute roughly half of the octet baryon spin;
- Control on the systematic uncertainties are in progress.