第十届手征有效场论研讨会

The DD* scattering in Lattice QCD calculation

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Preparing……

南京师范大学

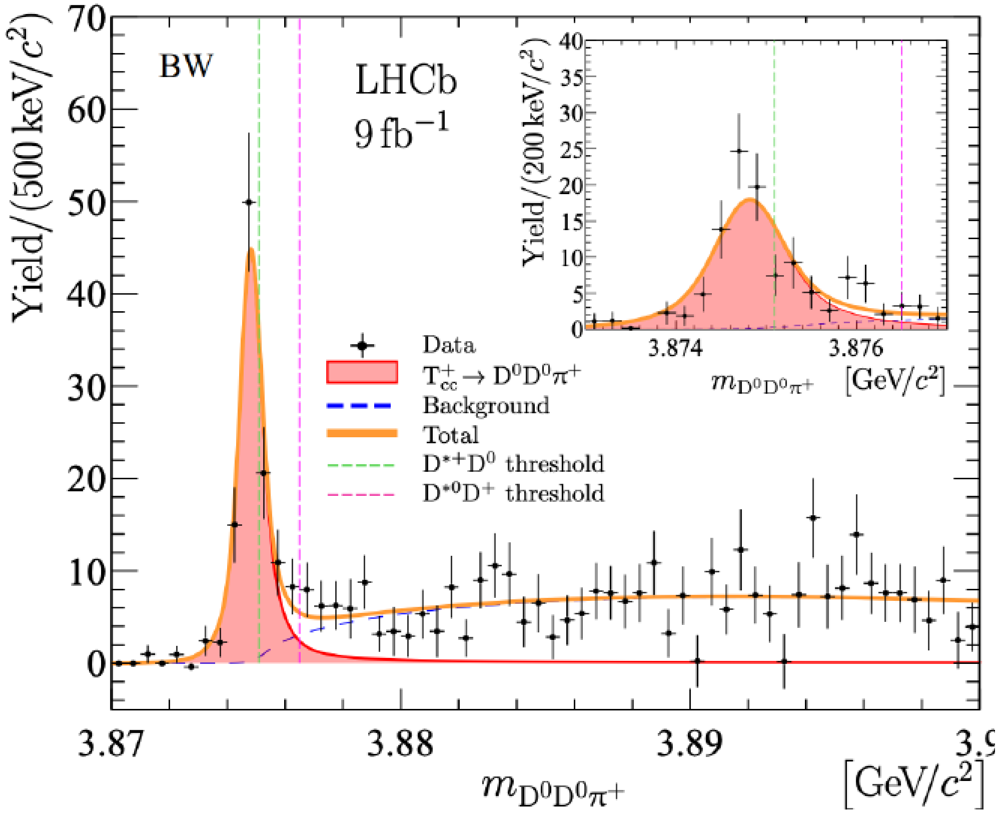
2025.10.18

中国科学院大学
University of Chinese Academy of Sciences

Outline

- Background
- The Lattice operator
- The long range force finite volume effect
- Summary and Outlook

Background



Breit-Wigner fitting

$$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0} = -273 \pm 61 \text{ keV}$$
$$\Gamma_{BW} = 410 \pm 165 \text{keV}$$

EPS-HEP conference, Ivan Polyakov's talk,29/07/2021; Nature Physics,22'

Nature Phys. 18 (2022) 7, 751-754

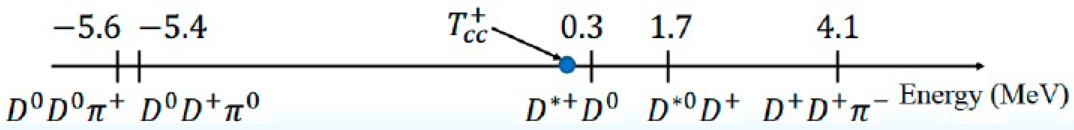
511 citations

Unitary fitting

$$\delta m_U = m_{T_{cc}} - m_{D^{*+}D^0} = -361 \pm 40 \text{ keV}$$
$$\Gamma_U = 47.8 \pm 1.9 \text{ keV}$$

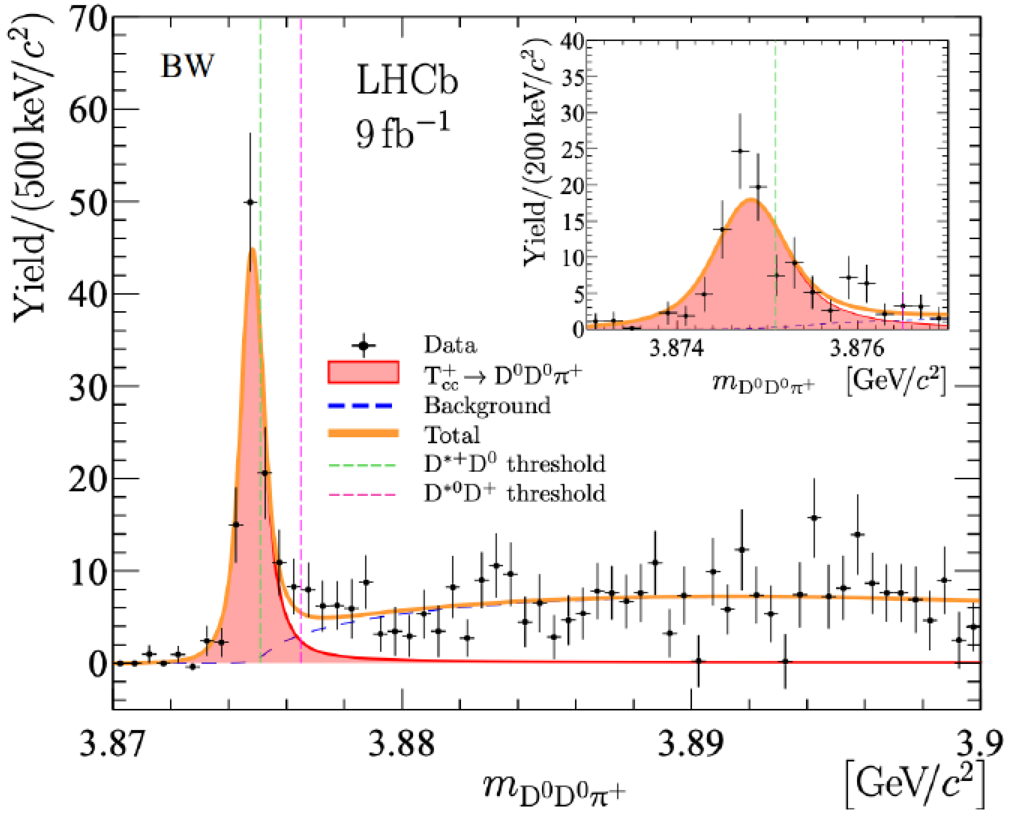
LHCb, Nature Commun. 13 (2022) 1, 3351

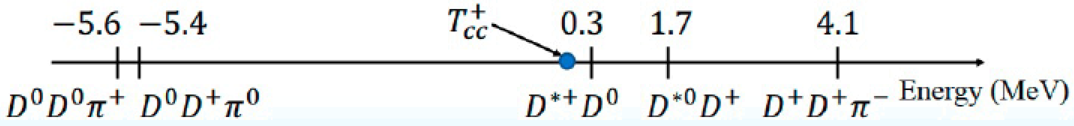
447 citations



L. Meng, et al. Phys. Rept. 1019 (2023) 1-149

Background





L. Meng, et al. Phys. Rept. 1019 (2023) 1-149

Prefer DD* molecular state

Before the observation

- E. Braaten, et.al., PRD103(2021)016001, Not bound
- J. Chen et.al., CPC45(2021)043102, Not bound
- Faustov et. al., universe, not bound
- M.Z.Liu et. al., PRD102(2020)091502, OBE, loosely bound
- Q.Lv et.al., PRD102(2020)034012
- C. Deng, et.al., EPJA56(2020),9, deeply bound -150keV
- P. Junnarkar et.al., PRD99(2019)034057
- W. Park et.al., NPA983(2019)1
- Z.G. Wang ACTA Physica Polonica B(2018) bound
- E.J.Eichten et.al., PRL119(2017)202002, not bound
- M. Karliner et.al., PRL2017, 7MeV above D^0D^{*+} threshold
- Lattice QCD simulation, PLB729(2014)85, $j^P = 1^+$, $I = 0$ attractiv
- G.Q. Feng, et.al., arXiv:1309.7813(2013), bound
- N.L., et al., PRD88(2013)114008, loosely bound
- ……

After the observation

SU(3) flavor partners

- M.Karliner, et.al., PRD105(2022)034020
- H.W.Ke, et.al., PRD105(2022)114019
- L.R.Dai, et.al., PRD105(2022)074017
- K. Chen, et.al., PRD105(2022)096004
- G. Yang, et.al., PRD104(2021)094035
- ……

Other partners

- Z.Y. Yang, et.al., arXiv: 2206.06051
- S.Q. Luo, et.al., arXiv: 2206.04586
- S.Q.Luo, et.al., PRD105(2022)074033
- X.Z.Ling, et.al., EPJC81(2021)1090
- X.Z.Weng, et.al., PRD105(2022)034026
- F.L.Wang et.al., PRD104(2021)094030
- Y.W.Pan et.al., PRD105(2022)114048
- C.W.Shen et.al., PLB831(2022)137
- T.Guo et.al., PRD105(2022)014021
- Q.Qin et.al., PRD105(2022)L031902
- R.Chen et.al., PRD104(2021)114042
- ……

HQSS partners

- H.W.Ke, et.al., EPJC82(2022)144
- M.J.Zhao, et.al., PRD105(2022)096016
- C.R.Deng, et.al., PRD105(2022)054015
- ……

Dynamics

- S.Y. Chen, et.al., arXiv: 2206.06185
- Z.Y.Lin, et.al., arXiv: 2205.14628
- M.Albaladejo et.al., PLB829(2022)137052
- J.B. Cheng, et.al., arXiv: 2205.13354
- N.N.Achasov, et.al., PRD105(2022)096038
- J. He et.al., EPJC82(2022)387
- J.H.Liu et.al., PRD105(2022)076013
- L.Y.Dai et.al., PRD105(2022)L051507
- L. Meng et.al., PRD104(2021)051502
- X.Z.Ling et.al., PLB826(2022)136897
- M.Y. Yan et.al., PRD105(2022)014007
- A. Feijoo et.al., PRD104(2021)114015
- ……

Isospin property and other properties

- J.Shi, et.al., arXiv: 2205.05234
- ……

Guang-Juan Wang, Zhi Yang, Jia-Jun Wu, Makoto Oka, Shi-Lin Zhu *Sci.Bull.* 69 (2024) 3036-3041

Previous Lattice Calculations

P. Junnarkar, N. Mathur, and M. Padmanath, Phys. Rev. D 99, 034507 (2019), I=0 for $qq\bar{Q}\bar{Q}$ system with J=0,1, an energy level -23 ± 11 MeV relative to the DD threshold, m_π>260 MeV

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022), virtual bound state pole, with ERE and Lüscher, m_π ~280 MeV

S. Collins, A. Nefediev, M. Padmanath, and S. Prelovsek, Phys. Rev. D 109, 094509 (2024), virtual bound state pole, with ERE/LSE and Lüscher, m_π ~280 MeV, study the quark mass dependence

L. Meng, E. Ortiz-Pacheco, V. Baru, E. Epelbaum, M. Padmanath, and S. Prelovsek Phys.Rev.D 111 (2025) 3, 3, 2411.06266, the standard Lüscher method and the recently proposed effective-field-theory-based approach in the plane-wave basis for I=1 case

L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, Phys. Rev. D 109, L071506 (2024) effective-field-theory-based approach in the plane-wave basis for I=0 case

Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. In oue, N. Ishii, K. Murano, H. Nemura, and K. Sasaki, Phys. Lett. B 729, 85 (2014), The phase shifts in the isospin triplet (I = 1) channels indicate repulsive interactions, while those in the I = 0 channels suggest attraction, growing as m_π decreases. HAL QCD method

Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng (2023), *Phys.Rev.Lett.* 131 (2023) 16, 161901, virtual state, and the virtual state is shown to evolve into a loosely bound state as m_π decreases,, ERE expansion.

Hadron Spectrum Collaboration, Phys. Rev. D 111, 034511 (2025), 2405.15741, I=0 for DD*, D*D* coupled-channel, virtual bound state pole, with ERE and Lüscher, m_π ~ 391 MeV

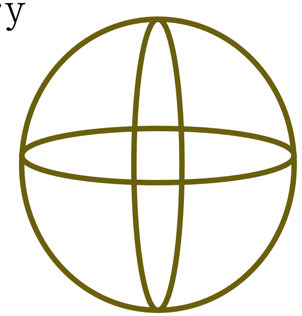
Hadron Spectrum Collaboration arXiv:2505.01363, I=1 for DD, DD*, D*D* coupled-channel

S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun, and R. Zhang, Phys. Lett. B 833, 137391 (2022), I = 0 attractive and I=1 replusive, m_π ~350 MeV, with ERE and Lüscher

P.-P. Shi, F.-K. Guo, C. Liu, L. Liu, P. Sun, J.-J. Wu, and H. Xing (2025), *DD* scattering in lattice QCD.

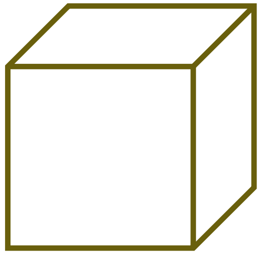
Coordinate space： Dumbbell Method

Symmetry



SO(3)

$L^P = 0^+, 1^-, 2^+, 3^-$, ……



O_h

$A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$

How to project the correlation function into a specific irreducible representation ?

Momentum space

Coordinate space

Momentum space： substantial storage and computational resources, for example $\pi\pi$：

$$(\pi\pi)^{[\vec{k}_1,\vec{k}_2]\dagger}_{\vec{P},\Lambda,\mu} = \sum_{\substack{\vec{k}_1\in\{\vec{k}_1\}^\star \\ \vec{k}_2\in\{\vec{k}_2\}^\star \\ \vec{k}_1+\vec{k}_2=\vec{P}}} \mathcal{C}(\vec{P},\Lambda,\mu;\ \vec{k}_1;\ \vec{k}_2)\ \pi^\dagger(\vec{k}_1)\ \pi^\dagger(\vec{k}_2)$$

$$\pi^\dagger(\vec{k},t) = \sum_{\vec{x}} e^{i\vec{k}\cdot\vec{x}} \left[\bar{\psi}\Box_\sigma \mathbf{\Gamma}_t^\dagger \Box_\sigma \psi\right](\vec{x},t)$$

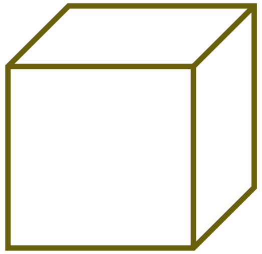
Advantage: It allows for precise control over the scattering energy levels for each lattice momentum.

Coordinate space： Dumbbell Method

How to project the correlation function into a specific irreducible representation ?

Momentum space

Coordinate space



Coordinate space: reduces storage and computational requirements.

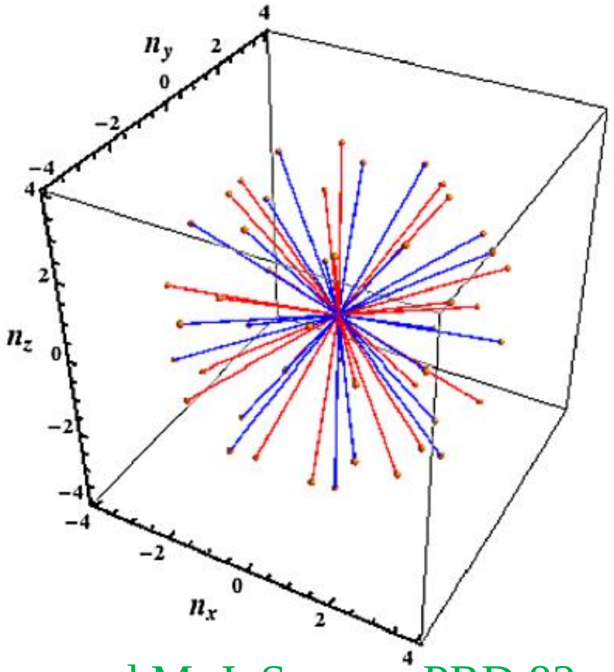
Disadvantage: only ground state for each irreducible representation.

O_h $A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$

Key method:

1. Build operator by **rotations**
2. Generate the regular representation space
3. Project the operator for irrep
4. Calculate the correlation function
5. Obtain the energy levels

$\hat{R}$	Rotation Axis	Rotation Angle	(x_1,x_2,x_3)
$\hat{R}_1$	Any	0^o	(x_1,x_2,x_3)
$\hat{R}_2$	(1,1,1)	-120^o	(x_2,x_3,x_1)
…	…	…	…
$\hat{R}_{24}$	(0,0,1)	-180^o	$(-x_1,-x_2,x_3)$
$\hat{R}_{25}=\hat{\pi}\hat{R}_1$	-	-	$(-x_1,-x_2,-x_3)$
$\hat{R}_{26}=\hat{\pi}\hat{R}_2$	-	-	$(-x_2,-x_3,-x_1)$
…	…	…	…
$\hat{R}_{48}=\hat{\pi}\hat{R}_{24}$	-	-	$(x_1,x_2,-x_3)$



T. Luu and M. J. Savage, PRD 83 114508(2011)

Coordinate space： Dumbbell Method

Key method:
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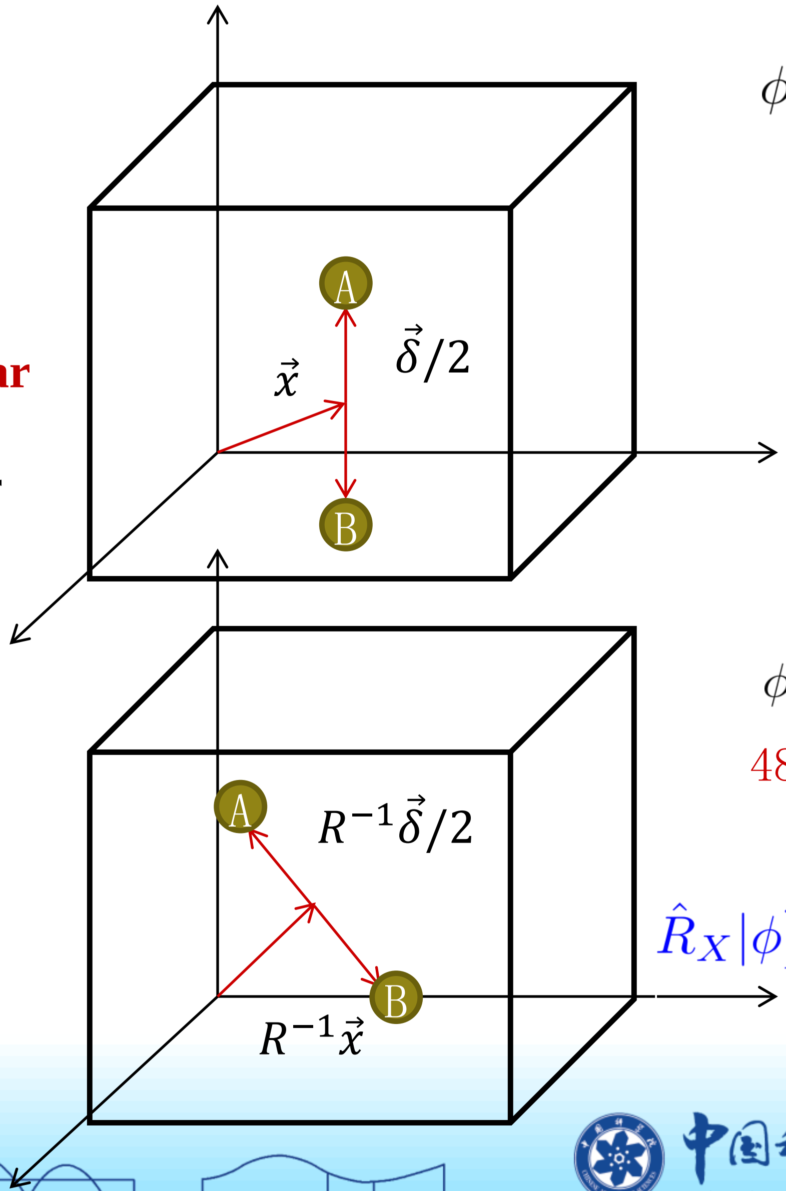

$$\phi(\vec{x},\vec{\delta},t)=\chi_A(\vec{x}+\vec{\delta}/2,t)\chi_B(\vec{x}-\vec{\delta}/2,t)$$

Rotation Elements: $\hat{R}=\{\hat{R}_1,\hat{R}_2,\ldots,\hat{R}_{48}\}$

$$\vec{\delta}=(\delta_x,\delta_y,\delta_z)$$

$$\delta_x\neq\delta_y\neq\delta_z$$

$$\vec{x}\pm\vec{\delta}/2=\vec{n}\in Z^3$$

$$\vec{x}=(\frac{1}{2},\frac{1}{2},\frac{1}{2})\ \text{and}\ \vec{\delta}=(1,3,5)$$

Simplifies continuum rotation symmetry

$$\phi_R(\vec{x},\vec{\delta},t)=\chi_A(\hat{R}^{-1}(\vec{x}+\vec{\delta}/2),t)\chi_B(\hat{R}^{-1}(\vec{x}-\vec{\delta}/2),t)$$

48 two-hadron operator ϕ_R.

$$\phi_R^\dagger(\vec{x},\vec{\delta},t)|\Omega\rangle=\phi_R^\dagger(\vec{x},\vec{\delta},t)\sim\phi_R^\dagger(\vec{x},\vec{\delta})$$

$$\hat{R}_X|\phi_R^\dagger(\vec{x},\vec{\delta})\rangle=|\phi_{R_XR}^\dagger(\vec{x},\vec{\delta})\rangle=\sum_{R'}|\phi_{R'}^\dagger(\vec{x},\vec{\delta})\rangle\left[\Gamma_{\mathrm{reg}}(R_X)\right]_{R'R}$$

Coordinate space： Dumbbell Method

Key method:
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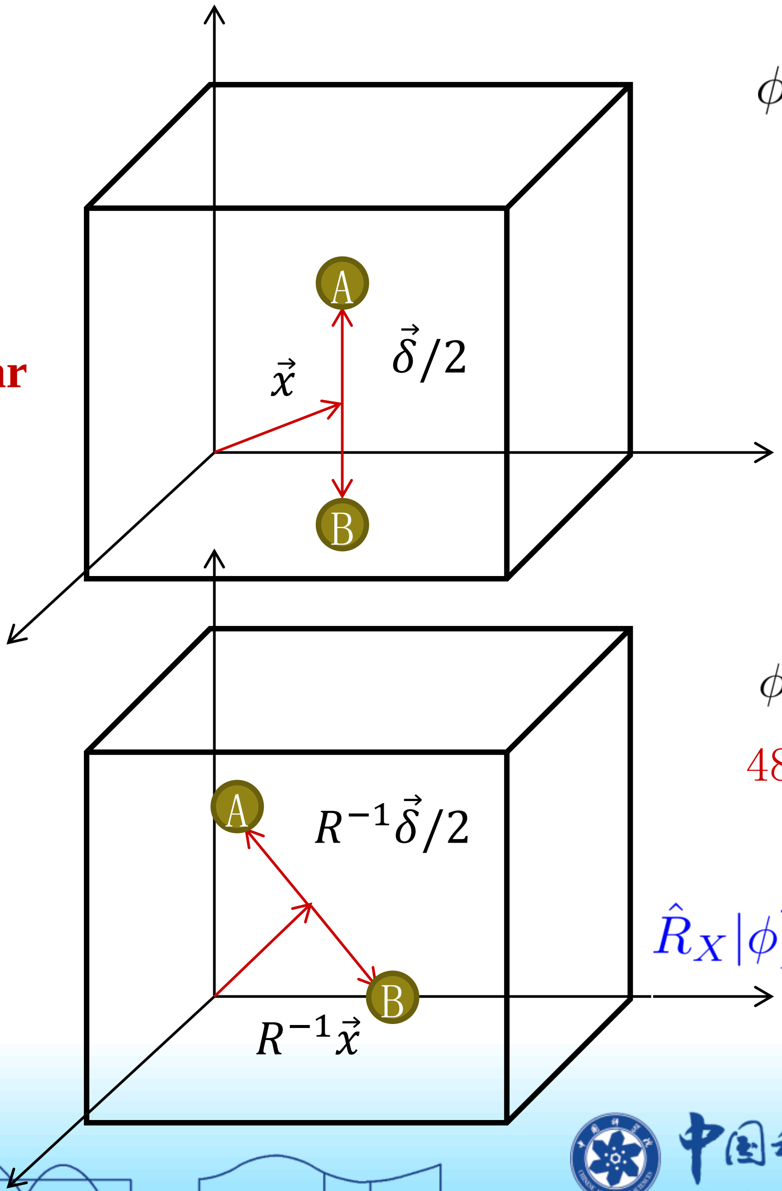

$$\phi(\vec{x},\vec{\delta},t) = \chi_A(\vec{x}+\vec{\delta}/2,t)\chi_B(\vec{x}-\vec{\delta}/2,t)$$

Rotation Elements: $\hat{R} = \{\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_{48}\}$

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Simplifies continuum rotation symmetry

$$\phi_R(\vec{x},\vec{\delta},t) = \chi_A(\hat{R}^{-1}(\vec{x}+\vec{\delta}/2),t)\chi_B(\hat{R}^{-1}(\vec{x}-\vec{\delta}/2),t)$$

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$$\hat{R}_X|\phi_R^\dagger(\vec{x},\vec{\delta})\rangle = |\phi_{R_X R}^\dagger(\vec{x},\vec{\delta})\rangle = \sum_{R'} |\phi_{R'}^\dagger(\vec{x},\vec{\delta})\rangle \left[\bar{\Gamma}_{\mathrm{reg}}(R_X)\right]_{R'R}$$

$$S^{-1}\bar{\Gamma}_{reg}(R_X)\,S \;=\; A_1^{+} \oplus A_2^{+} \oplus 2E^{+} \oplus 3T_1^{+} \oplus 3T_2^{+} \oplus A_1^{-} \oplus A_2^{-} \oplus 2E^{-} \oplus 3T_1^{-} \oplus 3T_2^{-}$$

Coordinate space： Dumbbell Method

$$\hat{R}_X|\phi^\dagger_R(\vec{x},\vec{\delta})\rangle = |\phi^\dagger_{R_X R}(\vec{x},\vec{\delta})\rangle = \sum_{R'} |\phi^\dagger_{R'}(\vec{x},\vec{\delta})\rangle \left[\bar{\Gamma}_{\text{reg}}(R_X)\right]_{R'R}$$

$$\left|\Phi^\dagger_{i,\Gamma,n}\left(\vec{\delta}\right)\right\rangle$$

Key method:

1. Build operator by rotations
2. Generate the regular representation space
3. **Project** the operator for **irrep**
4. Calculate the correlation function
5. Obtain the energy levels

$$S^{-1}\bar{\Gamma}_{reg}\, S \;=\; A_1{}^+ \oplus A_2{}^+ \oplus 2E^+ \oplus 3T_1{}^+ \oplus 3T_2{}^+ \oplus A_1{}^- \oplus A_2{}^- \oplus 2E^- \oplus 3T_1{}^- \oplus 3T_2{}^-$$

$$48 = 1+1+2\times 2+3\times 3+3\times 3 +1+1+2\times 2+3\times 3+3\times 3$$

e. g.

i： 1, 2 　　i： 1, 2, 3

Γ： E 　　Γ： T_2

n： 1, 2 　　n： 1, 2, 3

This block-diagonal matrices will lead to a new set of basis states:

$$\hat{R}_X|\Phi^\dagger_{i,\Gamma,n}(\vec{x},\vec{\delta})\rangle = \sum_{i',\Gamma',n'} |\Phi^\dagger_{i',\Gamma',n'}(\vec{x},\vec{\delta})\rangle \left[S^{-1}\bar{\Gamma}_{\text{reg}}(R_X)S\right]_{i'\Gamma'n',i\Gamma n}$$

$$\left|\phi_R\left(\vec{\delta}\right)\right\rangle \longrightarrow \left|\Phi_{i,\Gamma,n}\left(\vec{\delta}\right)\right\rangle$$

$$|\Phi^\dagger_{i,\Gamma,n}(\vec{x},\vec{\delta})\rangle = \sum_R |\phi^\dagger_R(\vec{x},\vec{\delta})\rangle \left[S\right]_{R,i\Gamma n}$$

Coordinate space： Dumbbell Method

$$\sum_{\vec{y}-\vec{z}\in Z^3} e^{i\vec{p}.(\vec{y}-\vec{x})}\langle\Omega|\Phi_{i,\Gamma,n}(\vec{y},\,\vec{\delta},\,t)\Phi^{\dagger}_{i',\Gamma',n'}(\vec{x},\,\vec{\delta},\,0)|\Omega\rangle$$

$$= \delta_{i,i'}\delta_{\Gamma,\Gamma'}\delta_{n,n'}\sum_{\vec{y}-\vec{x}\in Z^3} e^{i\vec{p}.(\vec{y}-\vec{x})}\frac{h}{l_\Gamma}\sum_{R}\chi_\Gamma(R)\langle\Omega|\phi_R(\vec{y},\,\vec{\delta},\,t)\phi^{\dagger}(\vec{x},\,\vec{\delta},\,0)|\Omega\rangle$$

Key method:
1. Build operator by rotations
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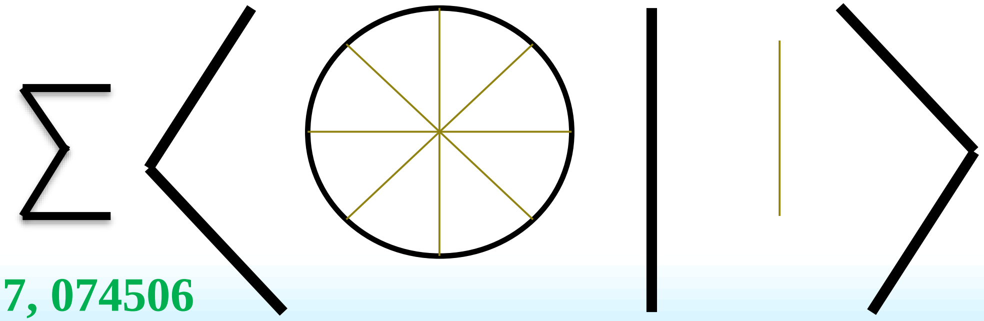
Diagonal and Just rely on Γ

Character Number of Γ

"Shell" Sink

Two source locations as $\vec{x}+\frac{\vec{\delta}}{2}$ and $\vec{x}-\frac{\vec{\delta}}{2}$

h and l_Γ are the orders of Oh group and irreducible representation Γ .



CSSM-QCDSF Collaboration Jia-Jun Wu et al.*PRD* 105 (2022) 7, 074506

Coordinate space： Dumbbell Method

$$\phi(x) \equiv \sum_a \bar{u}^a(x)\gamma_5 c^a(x)\,, \qquad D^0$$

$$\phi_\mu(x) \equiv \sum_a \bar{d}^a(x)\gamma_\mu c^a(x)\,,\ \ \mu = x, y, z\,, \qquad D^{*+}$$

$$\Phi_{\mu\hat{R}}(x,\delta) \equiv \hat{P}_{\hat{R}}\phi(x+\delta/2)\phi_\mu(x-\delta/2)\hat{P}_{\hat{R}^{-1}}$$

$$= \phi(\hat{R}^{-1}(x+\delta/2))\sum_\nu \bar{D}_{\mu\nu}(R_{\rm in})\phi_\nu(\hat{R}^{-1}(x-\delta/2))\,.$$

$$|\Phi_{\mu\hat{R}}(x,\delta)\rangle \xrightarrow{S} |\Phi_{\mu,i'\Gamma'n'}(x,\delta)\rangle \xrightarrow{S'} |\Phi_{i\Gamma n}(x,\delta)\rangle\,.$$

Reduce Regular Representation
Orbit angular momentum L

“irreps” x T1 => irrep
J=L+S

DD*

Γ	$T_1^+ \otimes \Gamma'(\ell)$
A_1^+	$T_1^+ \otimes T_1^+(4)$
A_2^+	$T_1^+ \otimes T_2^+(2)$
E^+	$T_1^+ \otimes T_1^+(2)$, $T_1^+ \otimes T_2^+(2)$
T_1^+	$T_1^+ \otimes A_1^+(0)$, $T_1^+ \otimes E^+(2)$, $T_1^+ \otimes T_1^+(4)$, $T_1^+ \otimes T_2^+(2)$
T_2^+	$T_1^+ \otimes A_2^+(6)$, $T_1^+ \otimes E^+(2)$, $T_1^+ \otimes T_1^+(4)$, $T_1^+ \otimes T_2^+(2)$
A_1^-	$T_1^+ \otimes T_1^-(1)$
A_2^-	$T_1^+ \otimes T_2^-(3)$
E^-	$T_1^+ \otimes T_1^-(1)$, $T_1^+ \otimes T_2^-(3)$
T_1^-	$T_1^+ \otimes A_1^-(9)$, $T_1^+ \otimes E^-(5)$, $T_1^+ \otimes T_1^-(1)$, $T_1^+ \otimes T_2^-(3)$
T_2^-	$T_1^+ \otimes A_2^-(9)$, $T_1^+ \otimes E^-(5)$, $T_1^+ \otimes T_1^-(1)$, $T_1^+ \otimes T_2^-(3)$

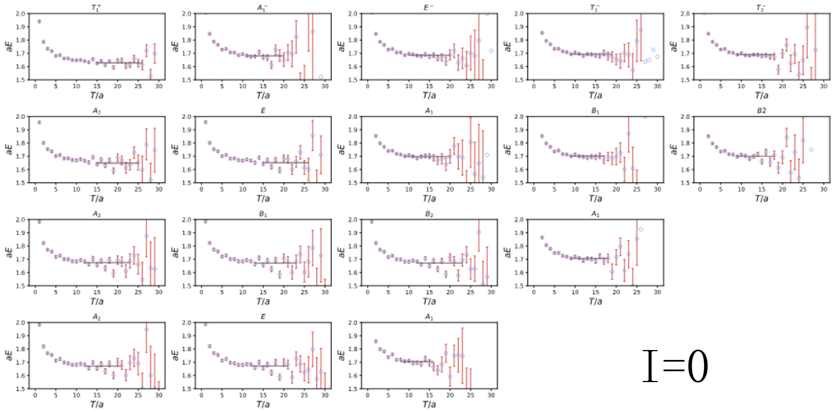
DD* CM system， P=0, belong to O_h group

DD* Moving system，P=(001), belong to the subgroup of O_h， and T_1^+ will reduce to $A_2 \oplus E$。

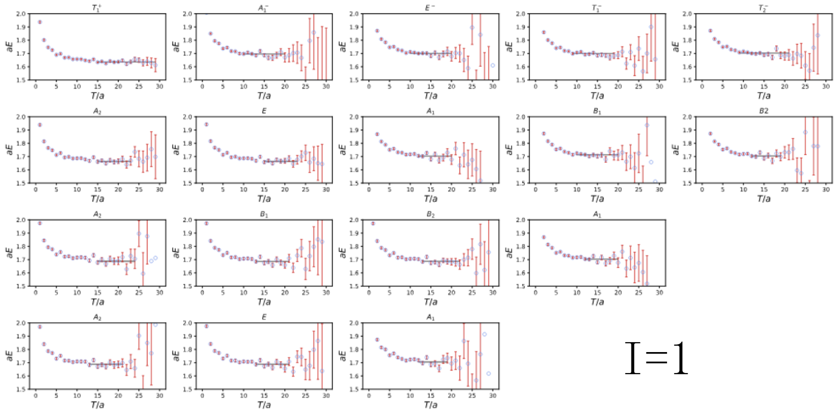
Γ	$(A_2 \oplus E) \otimes \Gamma'(\ell)$
A_1	$A_2 \otimes A_2(4)$, $E \otimes E(1)$
A_2	$A_2 \otimes A_1(0)$, $E \otimes E(1)$
B_1	$A_2 \otimes B_2(2)$, $E \otimes E(1)$
B_2	$A_2 \otimes B_1(2)$, $E \otimes E(1)$
E	$E \otimes A_1(0)$, $A_2 \otimes E(1)$

Lattice spectrum

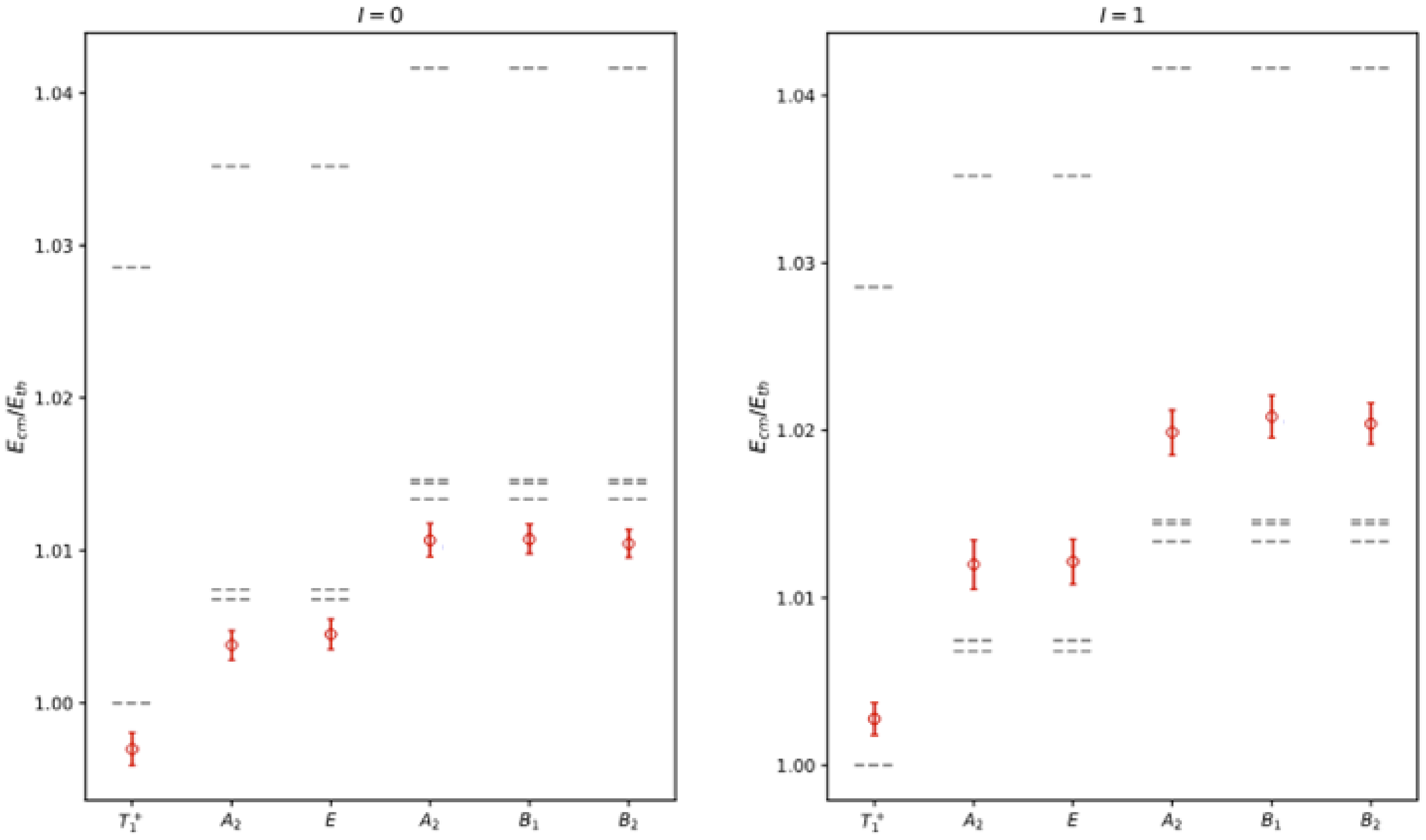
DD*



I=0



I=1



$|P|^2 = 0$ 1 1 2 2 2

$32^3 \times 48$ volume, lattice spacing $a = 0.074$ fm, $m\pi \simeq 359.4$MeV, $m_D \simeq 2121.2$MeV, and $m_{D*} \simeq 2225.4$MeV, from the QCDSF Collaboration **G. S. Bali et al., Nucl. Phys. B 866, 1 (2013)**

Lattice spectrum analysis

DD*

Lüscher formular

$$\det\left(M^{\Gamma,\boldsymbol{p}}_{Jln,J'l'n'}(E) - \delta_{JJ'}\delta_{ll'}\delta_{nn'}\cot\delta_{Jl}\right) = 0$$

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

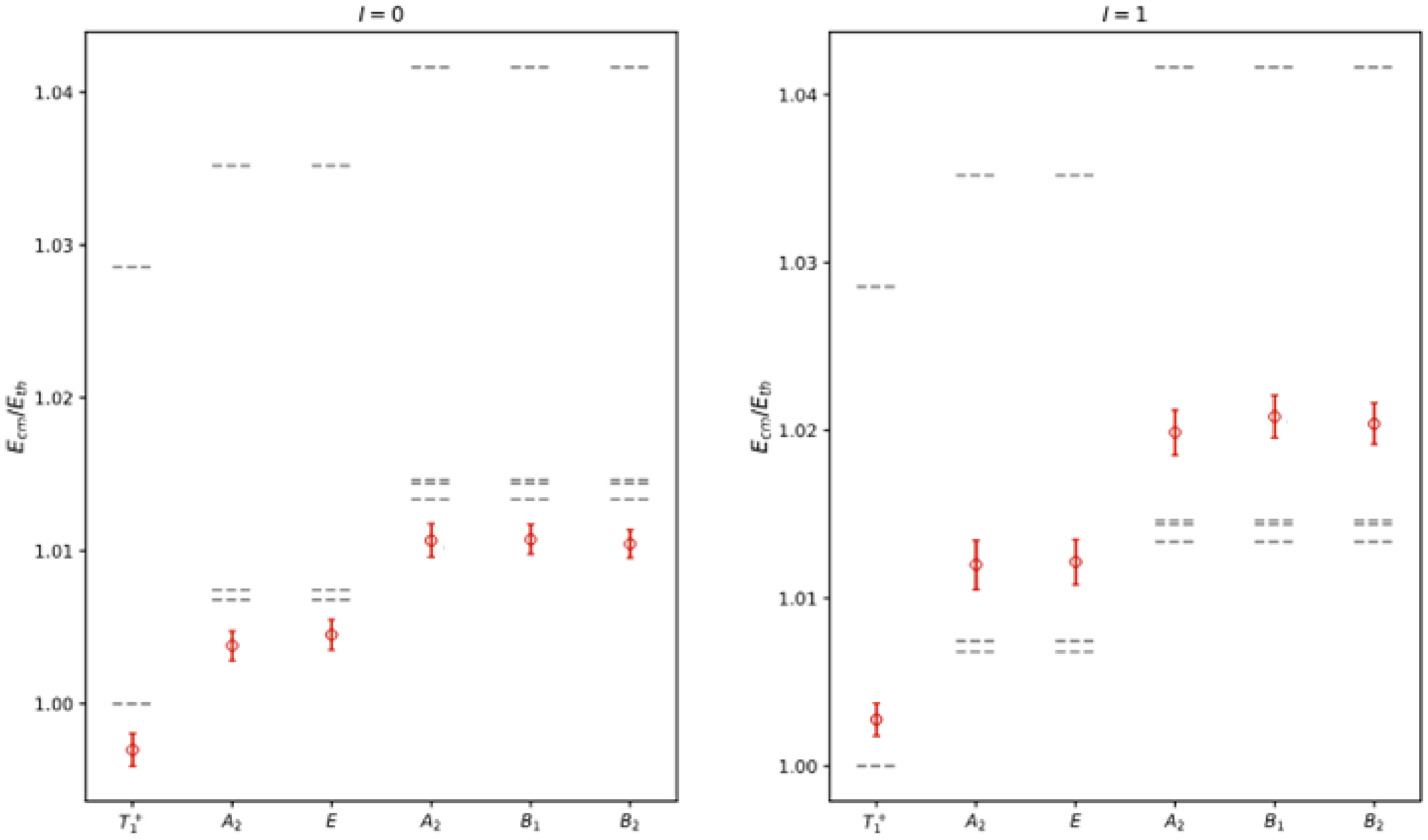
Constrain to the short range interaction,
Long range interaction will bring two problems

1. ERE vs Left hand cut

Du, Filin, Baru, Dong, Epel-baum, Guo, Hanhart, Nefediev, Nieves,and Wang, PRL 131, 131903 (2023)

2. The correction of Lüscher formulas become large by high partial wave contribution and left hand cut

Raposo and Hansen, JHEP 08, 075 (2024)
Bubna, Hammer, Müller, Pang, Rusetsky, and Wu, JHEP 05, 168 (2024)
Meng, Baru, Epelbaum, Filin, and Gasparyan, PRD 109, L071506 (2024)
Yu, Wang, Wu, and Yang, JHEP 04 108(2025)



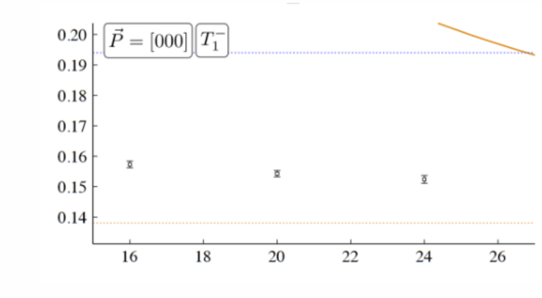
$|P|^2 =$ 0 1 1 2 2 2

In HEFT, we can include π exchange naturally.

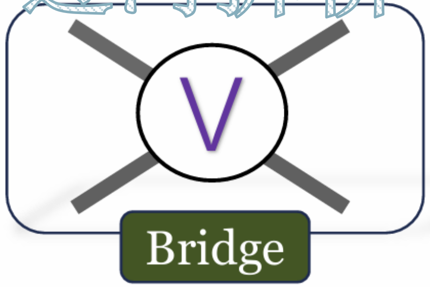
The quantization condition in the finite volume

Standard Lüscher Formula

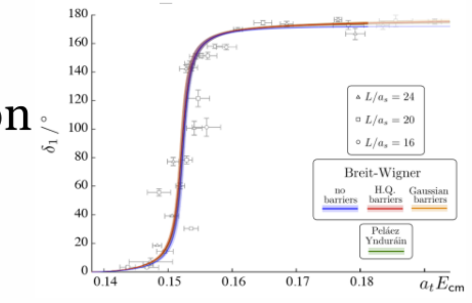
过河拆桥



solve Eigenvalues



solve
scattering equation



$$T = V + \int VGT$$

$$T^{FV} = V^{FV} + \sum V^{FV} G T^{FV}$$

Lattice spectra are the poles of T^{FV}

$$T^{FV} = T + TMT^{FV}$$

$$M \sim \frac{\sum V^{FV} G - \int VG}{V_{on}}$$

$$\left(T^{FV}\right)^{-1} = T^{-1} - M^{-1}$$

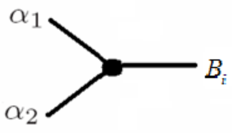
V has no pole or pole is far away --- **Usual Lüscher's**

V has pole close to threshold --- **Left hand cut/ Long range interaction**

V has a pole above threshold --- **Three-body**

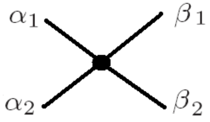
HEFT Introduction

$$H = H_0 + H_I \qquad H_I = \hat{g} + \hat{v}$$



$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[\left|\alpha(k_\alpha)\right\rangle g^{+}_{i,\alpha} \left\langle B_i \right| + \left| B_i \right\rangle g_{i,\alpha} \left\langle \alpha(k_\alpha) \right| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} \left|\alpha(k_\alpha)\right\rangle v_{\alpha,\beta} \left\langle \beta(k_\beta)\right|$$



Continuous → Discrete

$\int d\vec{k}$ and	$\left|\alpha(\vec{k}_\alpha)\right\rangle$ and	$\left\langle \beta(\vec{k}_\beta) \middle| \alpha(\vec{k}_\alpha)\right\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓	↓
$\sum_i \left(2\pi/L\right)^3$ and	$\left(2\pi/L\right)^{-3/2} \left|\vec{k}_i, -\vec{k}_i\right\rangle_\alpha$ and	${}_\beta\left\langle \vec{k}_j, -\vec{k}_j \middle| \vec{k}_i, -\vec{k}_i \right\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0 + H_I)\,|\Psi\rangle = E\,|\Psi\rangle$$

Eigen-Value ⟷ Lattice Spectrum

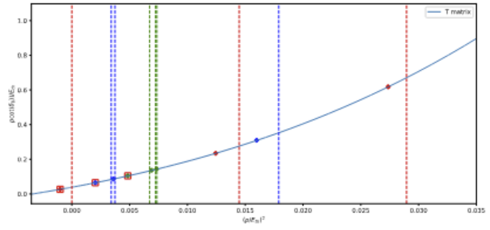
Resonance

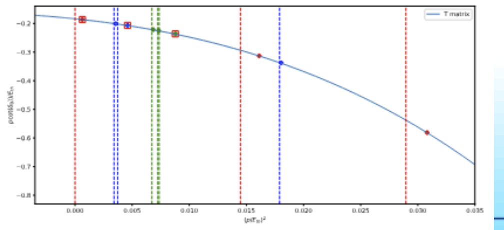
(Mass, width, pole position, coupling)

HEFT

T matrix
(phase shift, Inelascity)

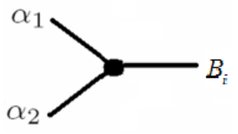
Lattice spectrum





HEFT Introduction

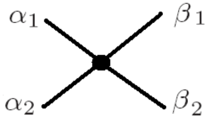
$$H = H_0 + H_I$$



$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[\left| \alpha(k_\alpha) \right\rangle g^{+}_{i,\alpha} \left\langle B_i \right| + \left| B_i \right\rangle g_{i,\alpha} \left\langle \alpha(k_\alpha) \right| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} \left| \alpha(k_\alpha) \right\rangle v_{\alpha,\beta} \left\langle \beta(k_\beta) \right|$$



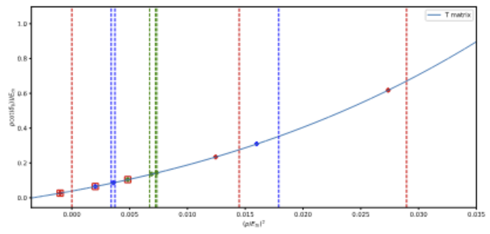
Continuous	$\int d\vec{k}$ and	$\left|\alpha(\vec{k}_\alpha)\right\rangle$	and	$\left\langle \beta(\vec{k}_\beta) \middle| \alpha(\vec{k}_\alpha) \right\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓	↓		↓
Discrete	$\sum_{i} \left({}^{2\pi}\!/_{L} \right)^3$ and	$\left({}^{2\pi}\!/_{L} \right)^{-3/2} \left| \vec{k}_i, -\vec{k}_i \right\rangle_\alpha$	and	${}_\beta\left\langle \vec{k}_j, -\vec{k}_j \middle| \vec{k}_i, -\vec{k}_i \right\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

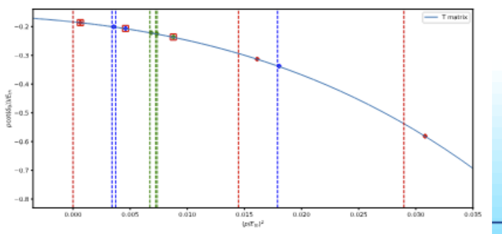
$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g^V_1(k_0) & g^V_2(k_0) & \cdots & g^V_{n_c}(k_0) & g^V_1(k_1) & \cdots \\ g^V_1(k_0) & v^V_{1,1}(k_0,k_0) & v^V_{1,2}(k_0,k_0) & \cdots & v^V_{1,n_c}(k_0,k_0) & v^V_{1,1}(k_0,k_1) & \cdots \\ g^V_2(k_0) & v^V_{2,1}(k_0,k_0) & v^V_{2,2}(k_0,k_0) & \cdots & v^V_{2,n_c}(k_0,k_0) & v^V_{2,1}(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g^V_{n_c}(k_0) & v^V_{n_c,1}(k_0,k_0) & v^V_{n_c,2}(k_0,k_0) & \cdots & v^V_{n_c,n_c}(k_0,k_0) & v^V_{n_c,1}(k_0,k_1) & \cdots \\ g^V_1(k_1) & v^V_{1,1}(k_1,k_0) & v^V_{1,2}(k_1,k_0) & \cdots & v^V_{1,n_c}(k_1,k_0) & v^V_{1,1}(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0 + H_I)\,|\Psi> = E\,|\Psi>$$

Eigen-Value ↔ Lattice Spectrum





Simplest way, not project to the irrep, no moving system, ……

More detail information:

Kang Yu, Guang-Juan Wang, Jia-Jun Wu, Zhi Yang, JHEP04(2025)108

Yan Li, Jia-Jun Wu, T.-S.H. Lee, R.D. Young, *JHEP*08(2024)178

Lattice spectrum analysis

DD*

$$H = H_0 + H_I$$

$$H_I = v = V_{Short} + V_{Long}$$

$$V(\boldsymbol{p},\boldsymbol{k}) = V^I_{\text{Short}}(\boldsymbol{p},\boldsymbol{k}) \equiv c^I_S \left(\frac{\Lambda^2}{\boldsymbol{k}^2+\Lambda^2}\right)^2 \left(\frac{\Lambda^2}{\boldsymbol{p}^2+\Lambda^2}\right)^2$$

$$V_{\text{Long}}(|\boldsymbol{p}|,|\boldsymbol{k}|) = \frac{g_c}{8|\boldsymbol{p}||\boldsymbol{k}|f_\pi^2(2\pi)^2}\ln\left[\frac{(|\boldsymbol{p}|+|\boldsymbol{k}|)^2+m^2_{\text{eff}}}{(|\boldsymbol{p}|-|\boldsymbol{k}|)^2+m^2_{\text{eff}}}\right]$$

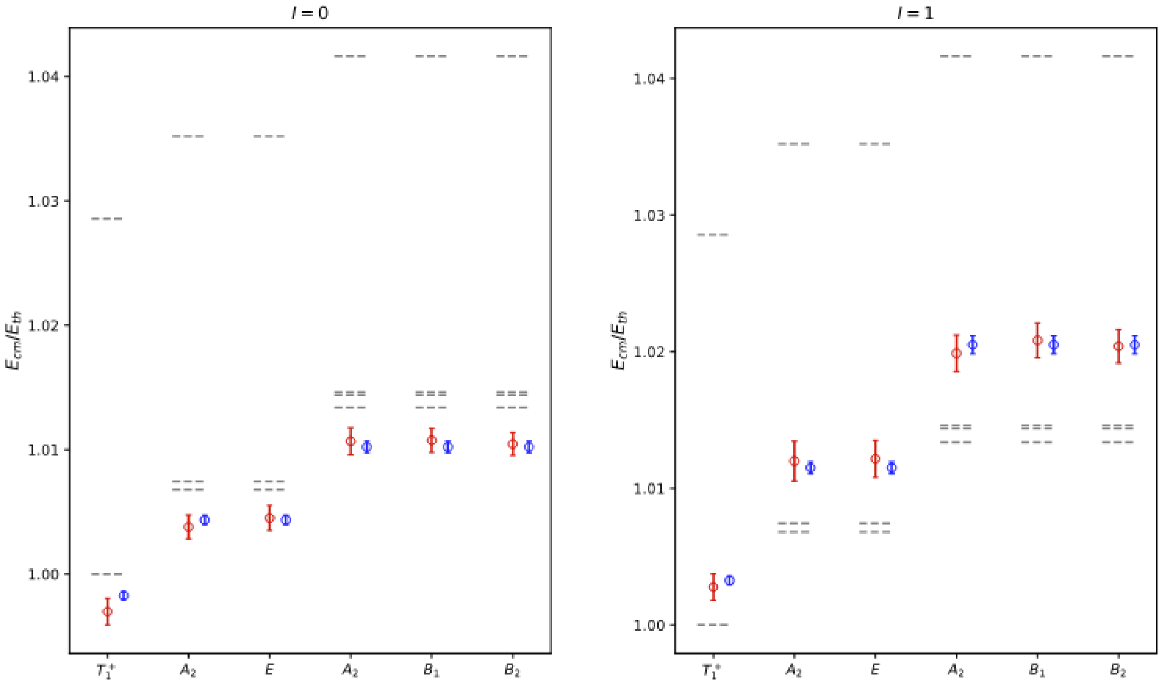
f_π = 112.3 MeV when $m_\pi \simeq$ 359.4 MeV

D. Becirevic and F. Sanfilippo, Phys. Lett. B 721, 94(2013)

g_c = 0.550 when a = 0.074 fm and $m_\pi \simeq$359.4 MeV

Meng, Baru, Epelbaum, Filin, and Gasparyan, PRD 109, L071506 (2024)

Only one free parameter



$|P|^2 =$ 0 1 1 2 2 2

Table VI. Fitting results of coupling constant c_S and the pole position.

isopsin	potential	$c^I_S[10^{-5}\text{MeV}^2]$	χ^2/dof	$E^{\text{Pole}} - E_{th}$[MeV]
0	without OPE	-1.05(0.12)	2.75/5	-12.2(4.1)
0	with OPE	-1.55(0.15)	2.35/5	-10.2(2.4)±5.1(3.2)i
1	without OPE	3.37(0.61)	6.15/5	41.9(2.7)± 2.3(0.2)i
1	with OPE	4.42(0.90)	5.55/5	42.5(4.1)± 2.7(0.3)i

Lattice spectrum analysis

DD*

$$V(\boldsymbol{p},\boldsymbol{k}) = V^{I}_{\text{Short}}(\boldsymbol{p},\boldsymbol{k}) \equiv c^{I}_{S}\left(\frac{\Lambda^2}{\boldsymbol{k}^2+\Lambda^2}\right)^2\left(\frac{\Lambda^2}{\boldsymbol{p}^2+\Lambda^2}\right)^2$$

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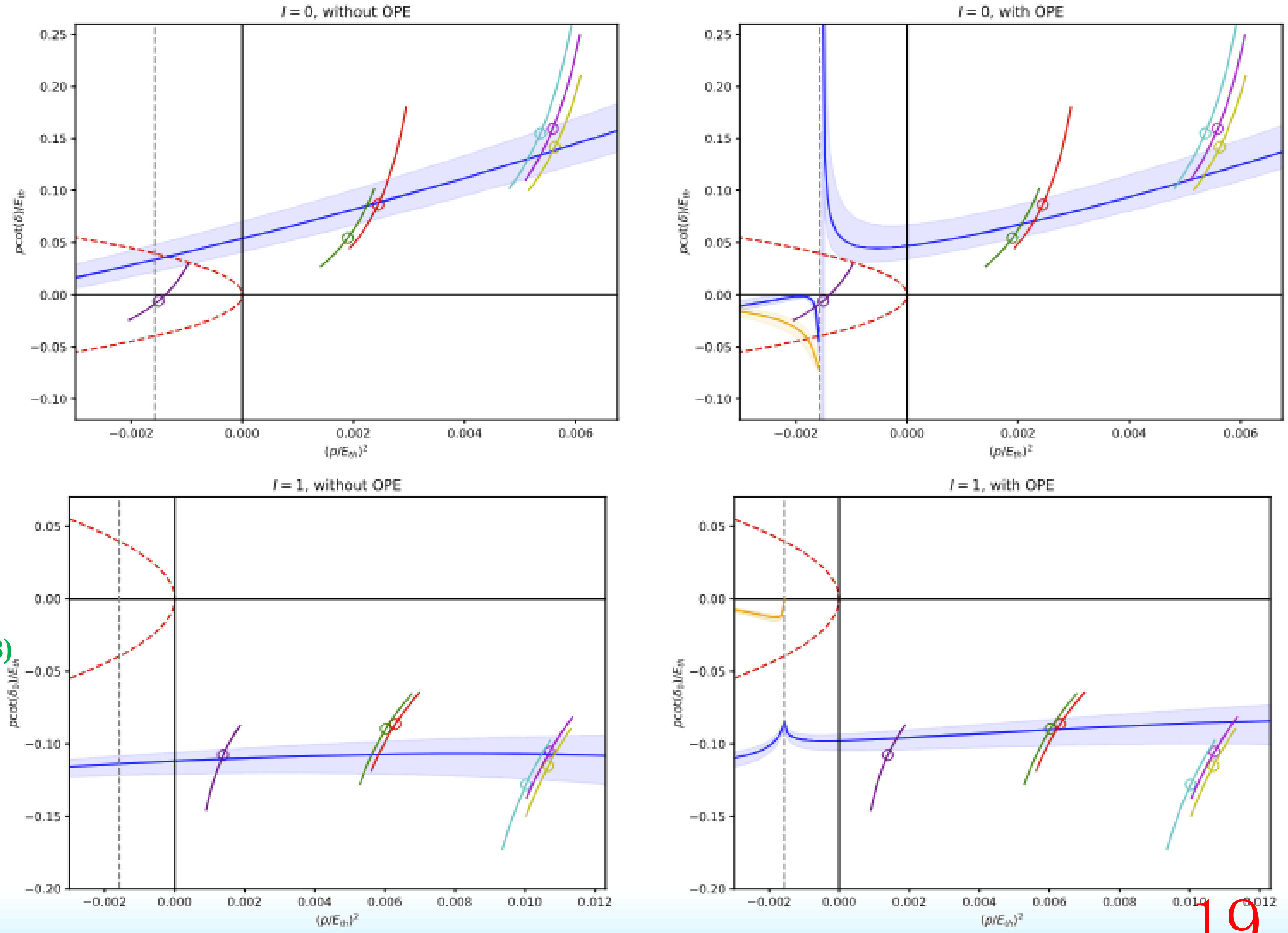
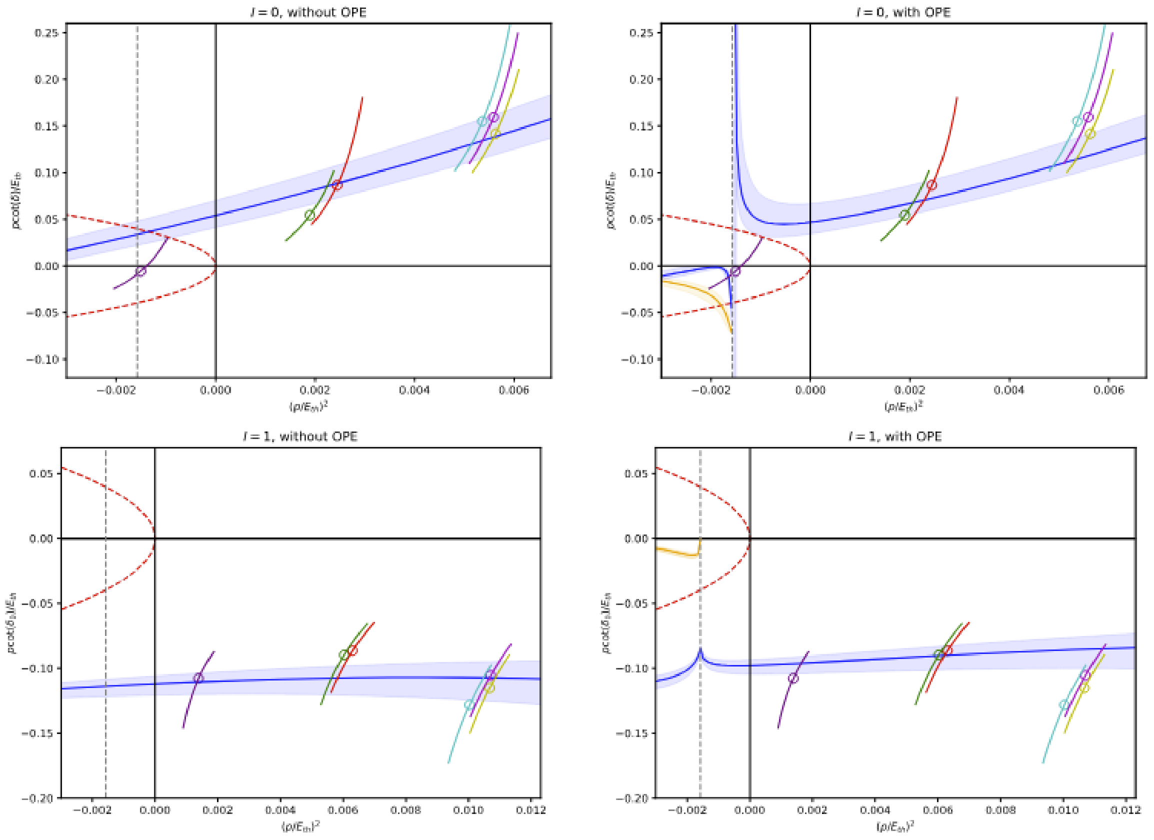
1. I=0 complex pole below the threshold --- crazy pole **YF Wang, DL Yao, HQ Zheng, EPJC 78, 543 (2018)**
2. I=1 similar to short range interaction case
3. The attraction and repulsion of the short-range potential determine the behavior near the left-hand cut generated by the long-range potentials



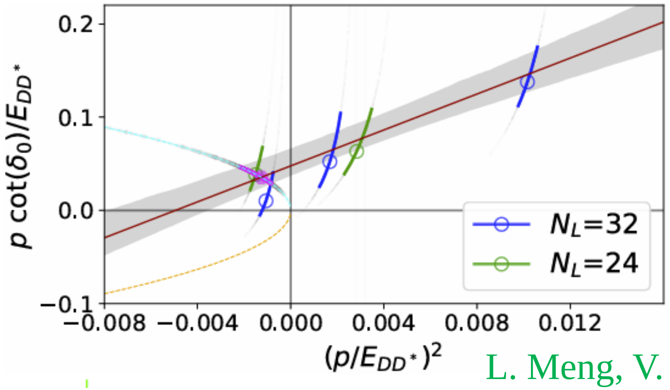
University of Chinese Academy of Sciences

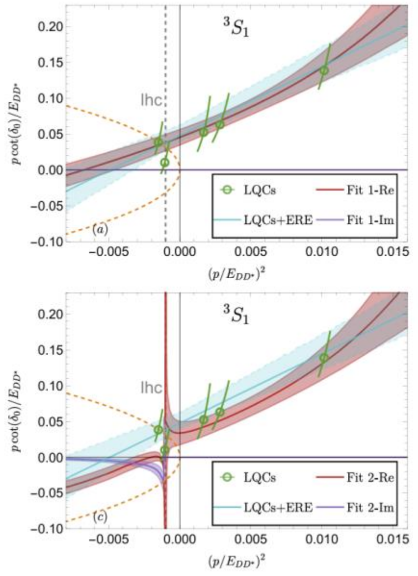
Lattice spectrum analysis

DD*

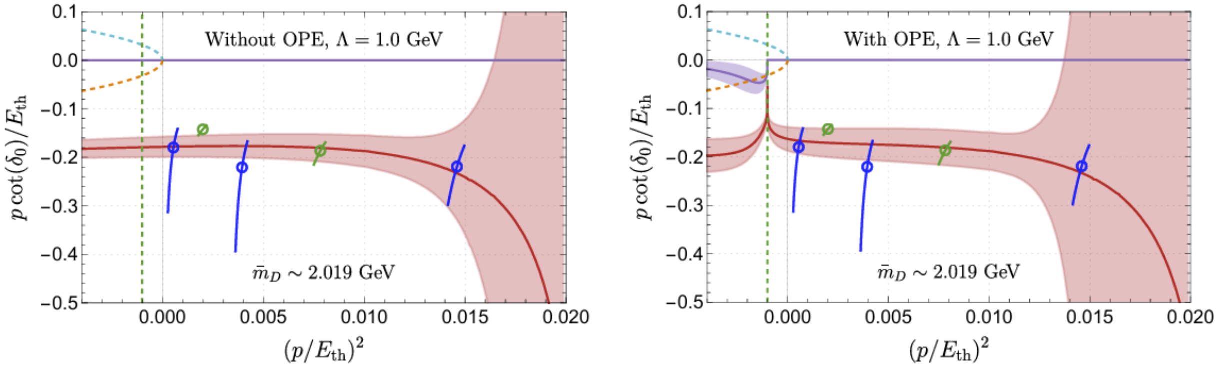


M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022),





L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, Phys. Rev. D 109, L071506 (2024)



L. Meng, E. Ortiz-Pacheco, V. Baru, E. Epelbaum, M. Padmanath, and S. Prelovsek Phys.Rev.D 111 (2025) 3, 3, 2411.06266,

Summary

- Introduce an new method to organize operator in lattice
- Long range force problem
- The pole of T_{cc} from lattice QCD calculation

Table VI. Fitting results of coupling constant c_S and the pole position.

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Thanks for attention