

Unveiling the structure of $T_{c\bar{s}}$ state in the $D_s\pi\pi$ final state

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Based on [arXiv:2510.01564](https://arxiv.org/abs/2510.01564)

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第十届手征有效场论研讨会, 南京, 2025/10/18

Hadron spectra and hadron structures : Exotic states



Volume 8, number 3

PHYSICS LETTERS

1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" ¹⁻³, we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone ⁴. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^- , s^- , u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" ⁶ q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest



Compact multiquark

8419/TH.412

21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig

CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

- 6) In general, we would expect that baryons are built not only from the product of three aces, AAA , but also from $\bar{A}AAAA$, $\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}AAA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA , that is, "deuces and treys".

Hadronic molecule



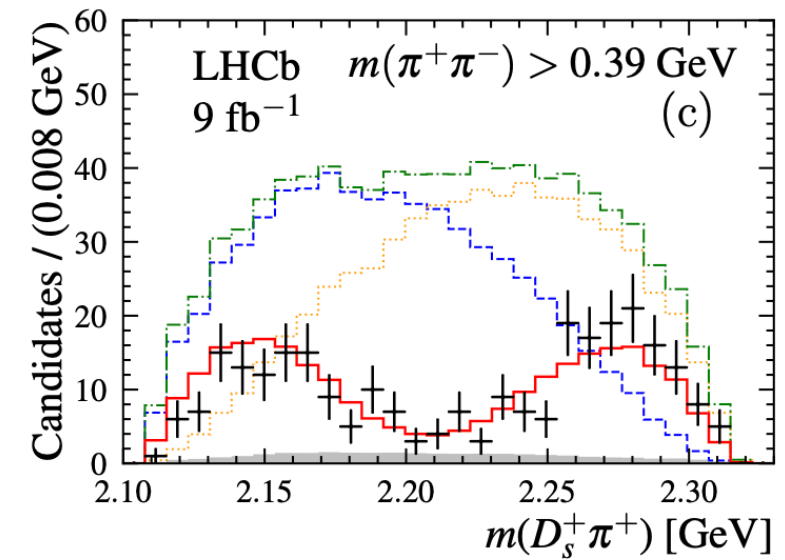
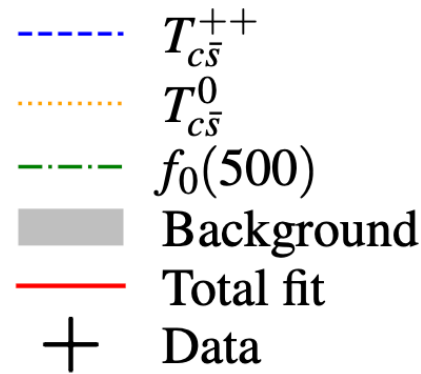
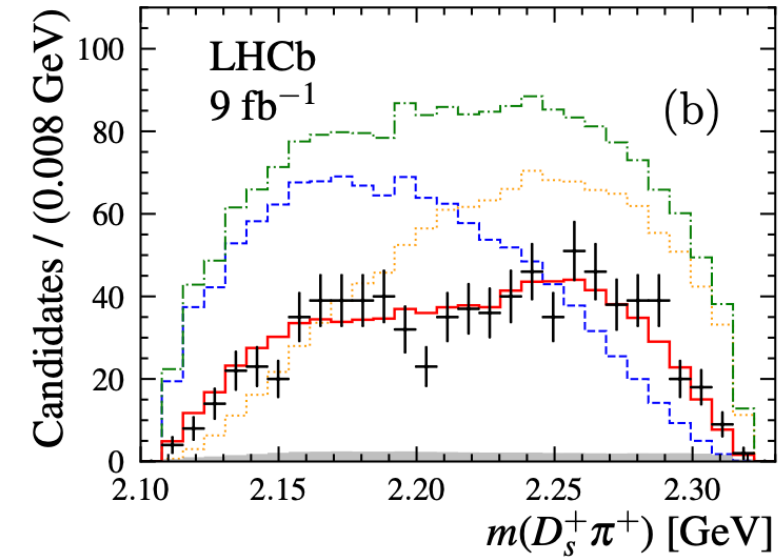
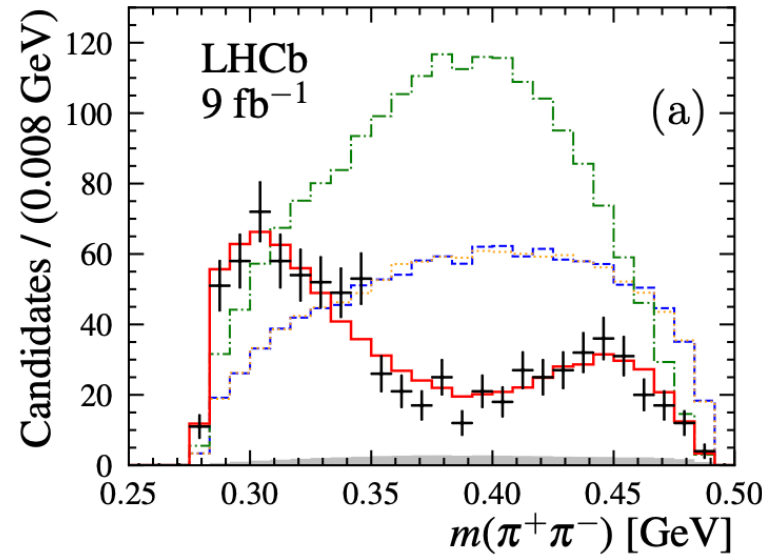
Exotic structure in $B \rightarrow D^{(*)} D_s^+ \pi^+ \pi^-$ decays

LHCb, arXiv:2411.03399

$$B \rightarrow D^{(*)} D_{s1}(2460)$$

$$\downarrow$$

$$D_s^+ \pi^+ \pi^-$$



Model	Resonance	Mass (MeV)	Width (MeV)
$f_0(500) + \text{RBW } T_{c\bar{s}}(0^+)$	$f_0(500)$	$464 \pm 23 \pm 14$	$214 \pm 28 \pm 8$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^0$	$2312 \pm 27 \pm 11$	$264 \pm 46 \pm 21$
$f_0(500) + \text{K-matrix } T_{c\bar{s}}(0^+)$	$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^0$	$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

- Double peak in $\pi\pi$ distribution

Tang, Lin, Guo, Hanhart and Meissner, arXiv:2303.18225

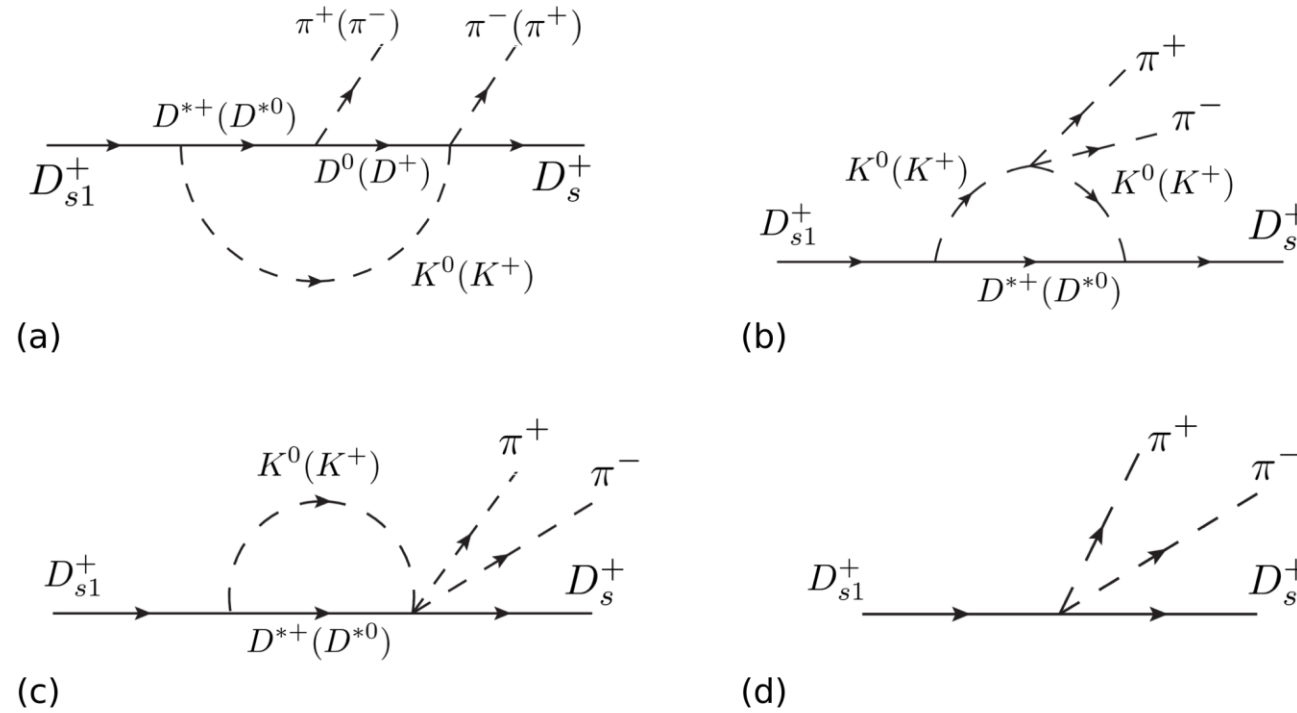
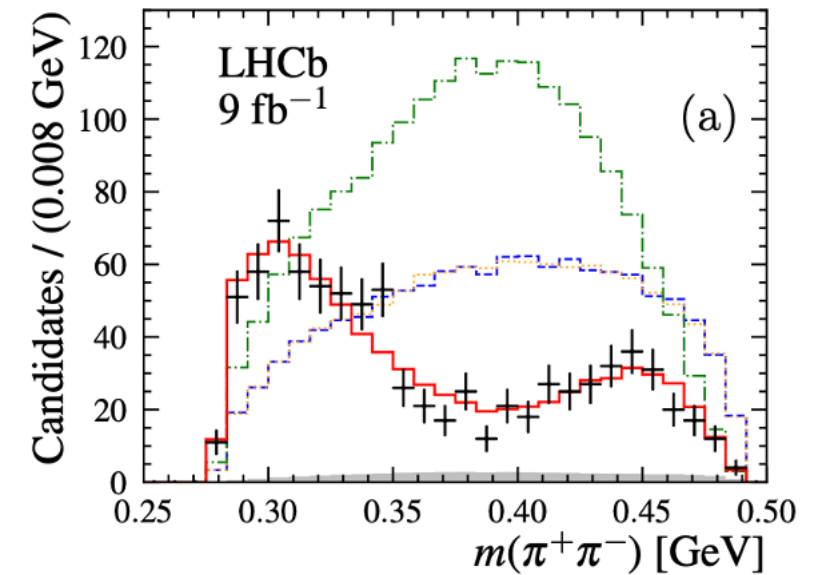
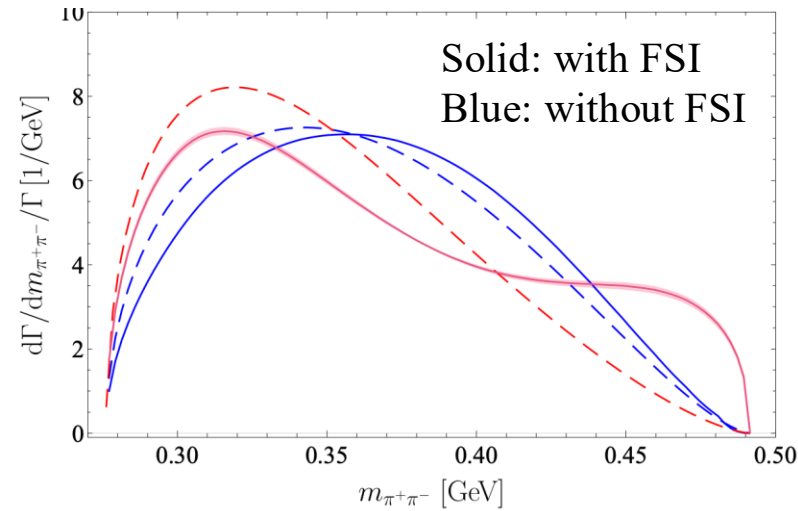
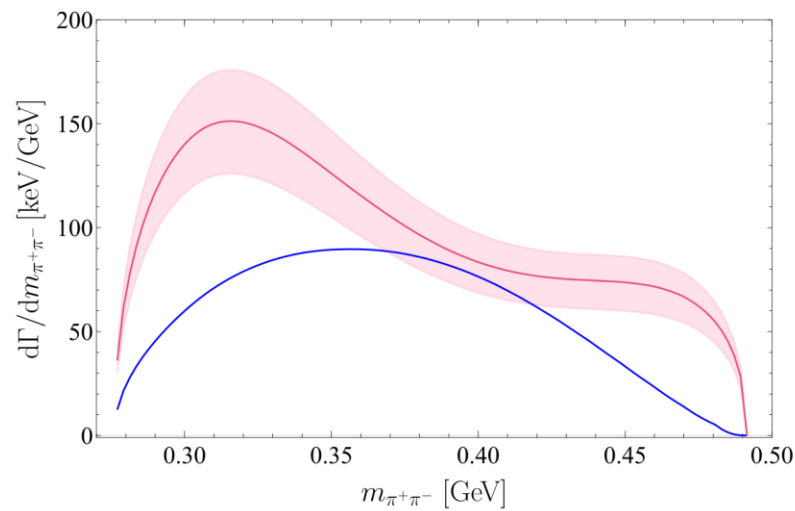


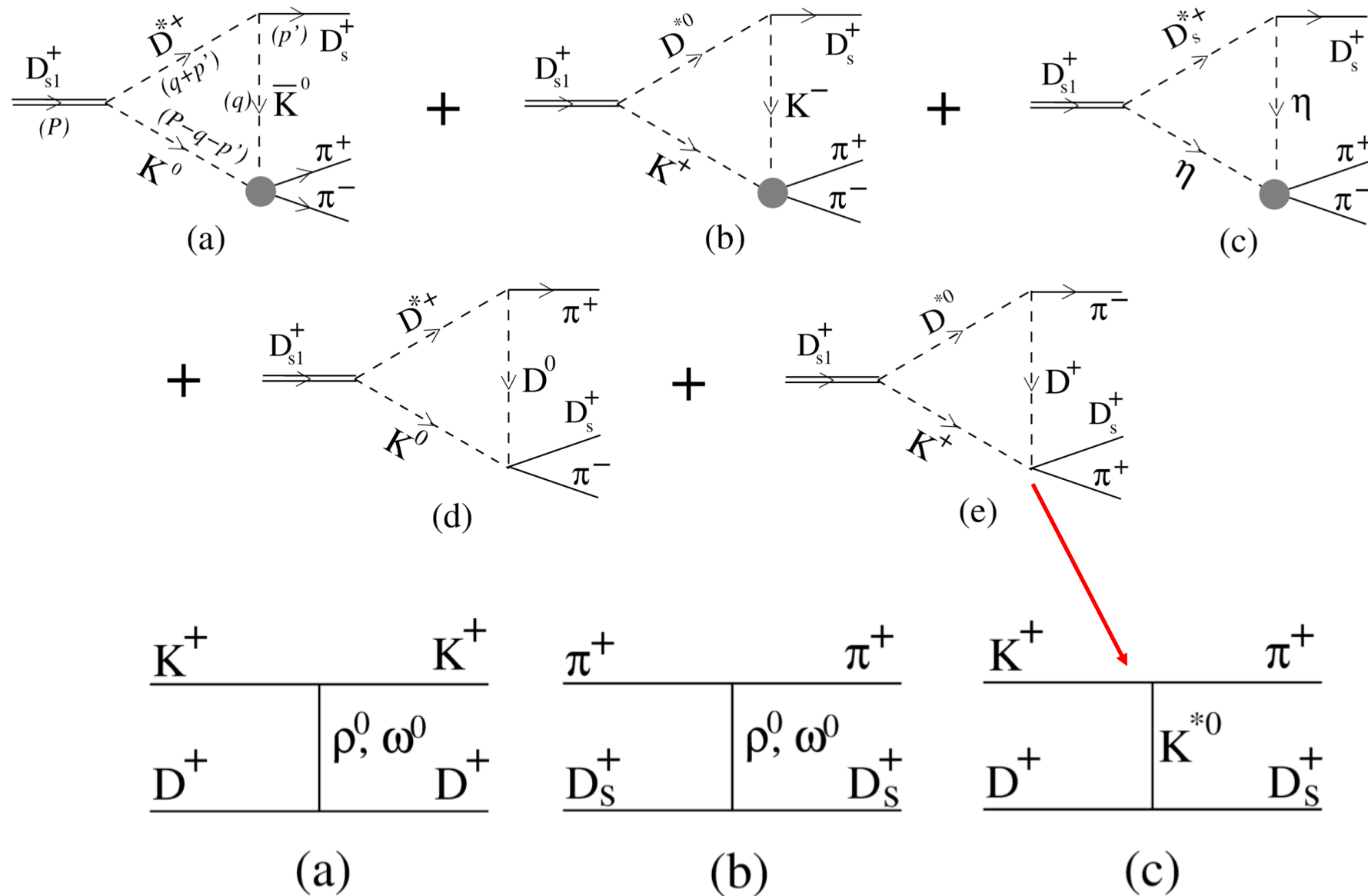
Figure 1. Diagrams for the decay $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$ with (a+b+c) and without (d) the $D^* K$ contribution.



Red: with (a) (b) (c) (d)
Blue: only with (d)

- Molecular $D_{s1}(2460)[D^*K, D_s^*\eta]$ state decay

Roca, Dias and Oset, arXiv:2502.18401

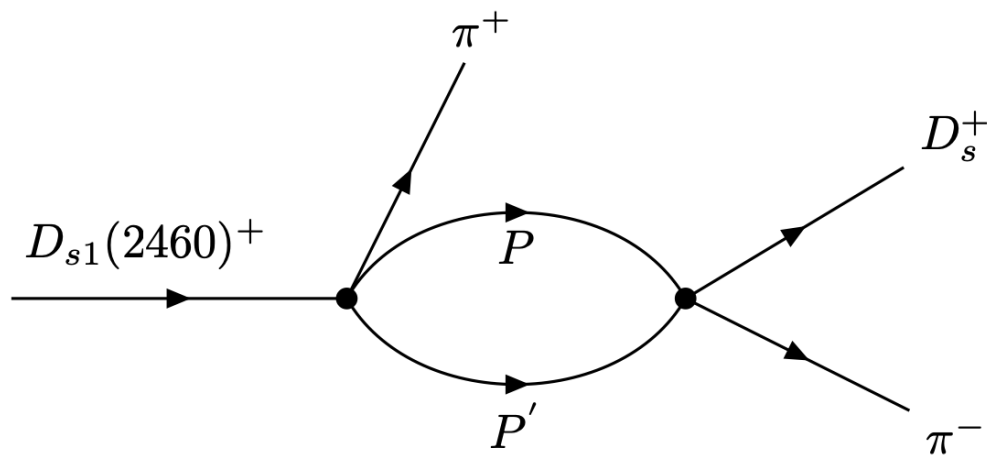
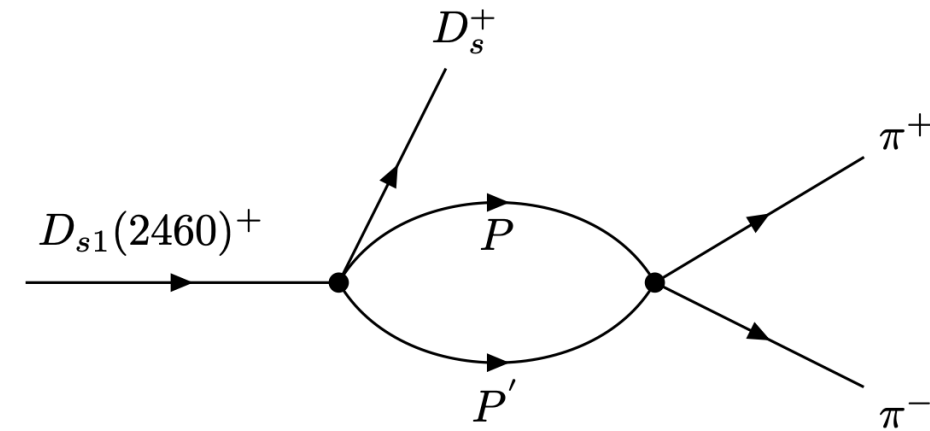
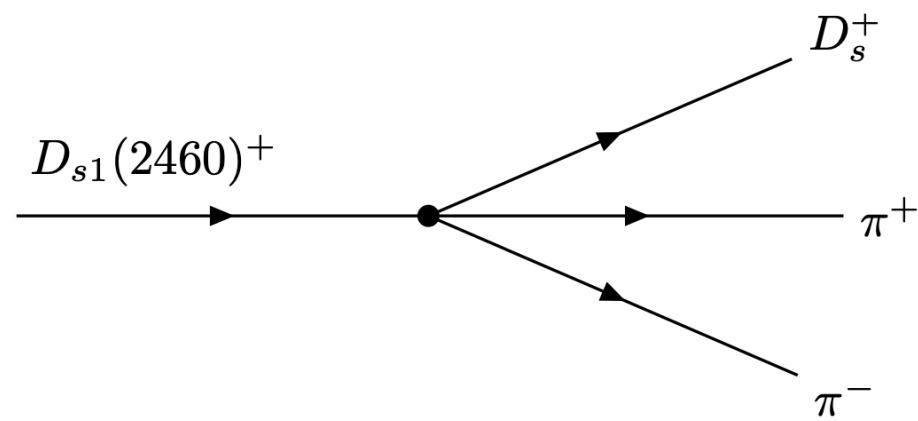


$$V_{K^+ D^+, K^+ D^+} = 0; \quad V_{\pi^+ D_s^+, \pi^+ D_s^+} = 0;$$

$$V_{K^+ D^+, \pi^+ D_s^+} = \frac{g^2}{m_{K^*}^2} (p_1 + p_3)_\mu (p_2 + p_4)^\mu.$$

- $D_s\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009



$$V_{D_s\pi \rightarrow D_s\pi}^{I=1} = 0,$$

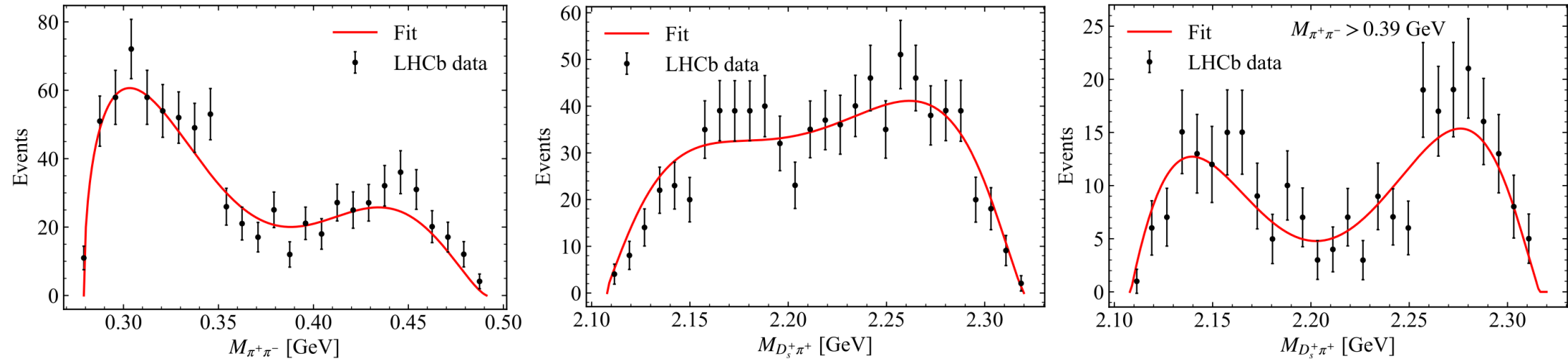
$$V_{D_s\pi \rightarrow DK}^{I=1} = -\frac{1}{8f^2} \left(3s - (M_1^2 + M_2^2 + m_1^2 + m_2^2) - \frac{\Delta_1 \Delta_2}{s} \right),$$

$$V_{DK \rightarrow DK}^{I=1} = 0,$$

Theoretical interpretation

- $D_S\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009



- A pole at $\sqrt{s_p} = (2.394 - 0.057i)\text{GeV}$ on the second Riemann sheet; a pole at $\sqrt{s_p} = (2.246 - 0.093i)\text{GeV}$ on the third Riemann sheet.

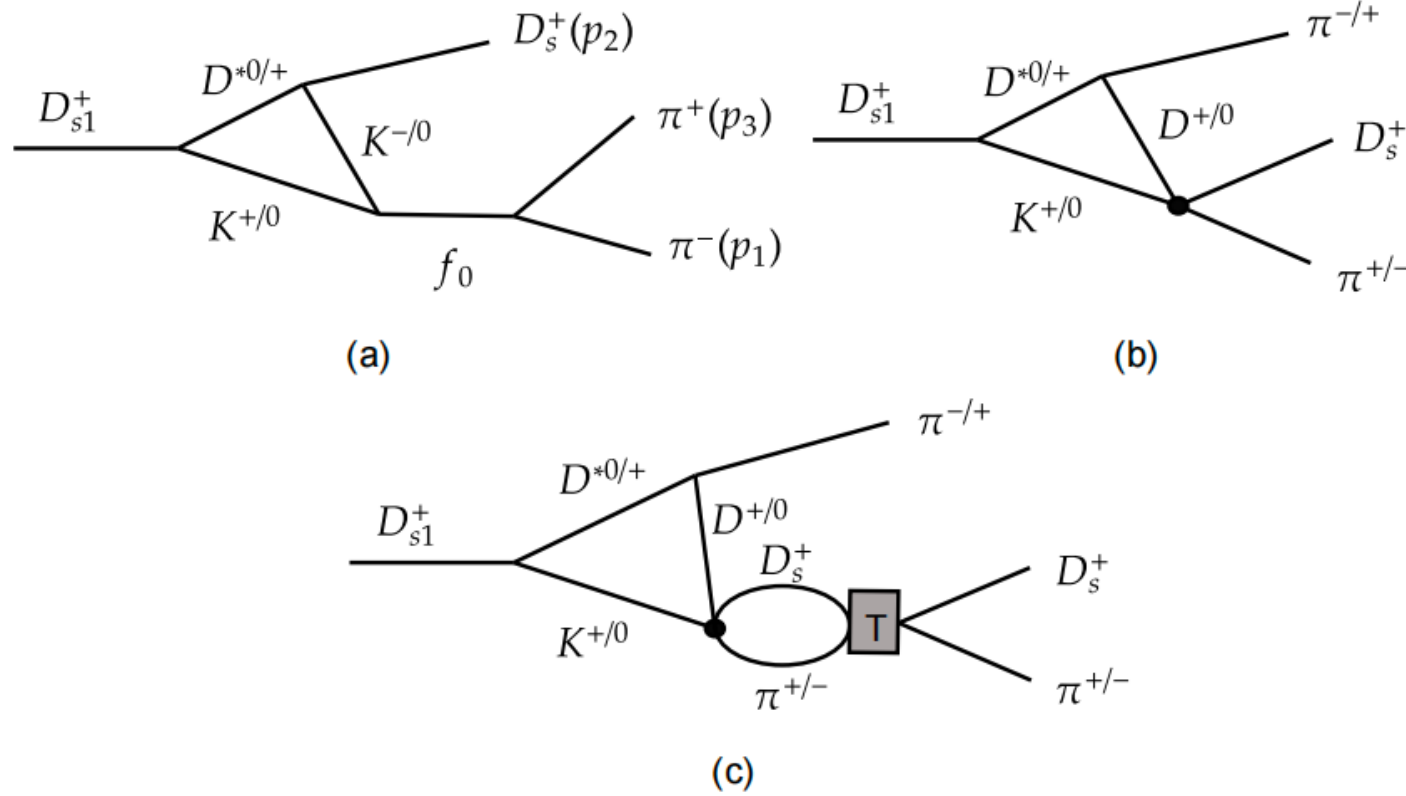
(S, I)	$h'_5 = +1$			$h'_5 = -1$		
	Re	Im	RS	Re	Im	RS
$(0, \frac{1}{2})$	2107	± 123	II	2107	± 105	II
	2452	± 17	III	2519	± 69	III
$(0, \frac{1}{2}) (V_{ii} = 0)$	2466	± 24	III	2388	± 49	III
$(1, 0)$	2318	0	I	2318	0	I
$(1, 1)$	2309	± 111	III	2283	± 196	III

(S, I)	RS	$\sqrt{s_{pole}}$ [MeV]
$(1, 1)$	II	$2466^{+32}_{-27} - i 271^{+4}_{-5}$
	III	$2225^{+12}_{-9} - i 178^{+19}_{-17}$

Guo, Hanhart and Meissner,
Eur. Phys. J. A40, 171-179(2009)

Guo, Meissner and Yao,
Phys.Rev.D 92 (2015) 9, 094008

- The feynman diagrams



- The amplitude

$$i\mathcal{M}_a = \frac{r_1}{m_{13}^2 - m_{f_0} + im_{f_0}\Gamma_{f_0}} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2 - m_D^2][(q + p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]},$$

$$i\mathcal{M}_b = r_2 \int \sqrt{\pi} \sin\theta d\theta \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2 - m_D^2][(q + p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),$$

$$i\mathcal{M}_c = r_2 \int \frac{\sin\theta d\theta k^2 dk}{2\pi^{3/2}} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi \rightarrow D_s\pi}(k, p_{\text{on}}, m_{23})}{[q^2 - m_D^2][(q + p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),$$

$e^{i\phi}$

- For the T-matrix,

$$T(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) + \int d\vec{q} \frac{\mathcal{V}(\vec{k}_{D^*}, \vec{q}; E) T(\vec{q}, \vec{k}'_{D^*}; E)}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_{D^*}^2 + q^2} + i\epsilon}$$

$$\mathcal{V}_{DK \rightarrow D_S \pi}^{K^*} = \frac{g_{K^*}}{m_{K^*}^2} (p_1 + p_3)_\mu (p_2 + p_4)^\mu$$

K^+	π^+
D^+	$K^{*0} D_S^+$

Similar for $V_{DK \rightarrow D_S \pi}^{D^*}$. The resulting effective potential reads

$$\mathcal{V}_{\alpha\beta} = \left(V_{\alpha\beta}^{K^*} + V_{\alpha\beta}^{D^*} \right) \left(\frac{\Lambda_\alpha^2}{\Lambda_\alpha^2 + p_\alpha^2} \right)^2 \left(\frac{\Lambda_\beta^2}{\Lambda_\beta^2 + p_\beta'^2} \right)^2$$

$\alpha, \beta = 1, 2$ for $D_S \pi$ and DK channel, respectively. The cutoff for these two channels can be different.

UV divergence in the triangle loop

- S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^\mu \epsilon_{j,\mu}^\dagger, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^\mu \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

- The amplitude

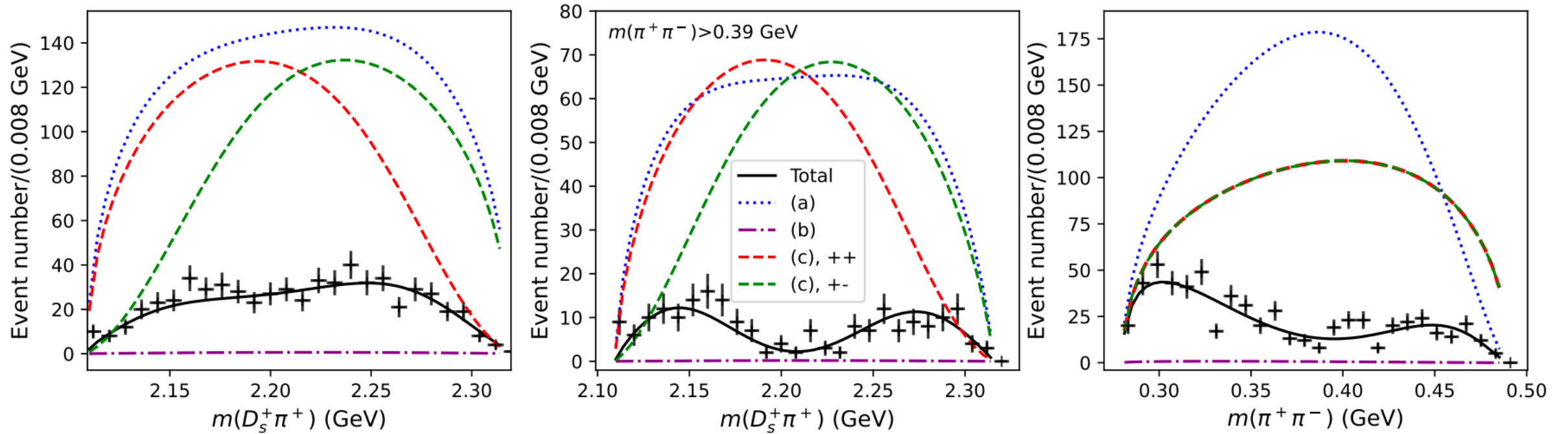
$$\begin{aligned} i\mathcal{M}_a &= \frac{r_1}{m_{13}^2 - m_{f_0} + im_{f_0}\Gamma_{f_0}} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2 - m_D^2][(q + p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\ i\mathcal{M}_b &= r_2 \int \sqrt{\pi} \sin\theta d\theta \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2 - m_D^2][(q + p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \\ i\mathcal{M}_c &= r_2 \int \frac{\sin\theta d\theta k^2 dk}{2\pi^{3/2}} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi \rightarrow D_s\pi}(k, p_{\text{on}}, m_{23})}{[q^2 - m_D^2][(q + p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \end{aligned}$$

$$\begin{aligned} L_\mu &= P_{\mu\nu}(p_2 + q, m_{D^*})(p_2 - q)^\nu \\ N_\mu &= P_{\mu\nu}(p_1 + q, m_{D^*})(p_1 - q)^\nu (p_2 - q + 2p_3)^\alpha \\ &\quad \times P_{\alpha\beta}(p_2 - q, m_{K^*})(p_2 + q)^\beta \end{aligned} \quad P_{\mu\nu}(p, m) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

- Regularize the loop integral using dimensional regularization, and use the $\overline{\text{MS}}$ renormalization scheme.

Fit to the experimental lineshapes

- $\chi^2/dof = 1.5$



- The parameters of the model determined from the fit :

Parameter	
Λ_1 [GeV]	$2.18^{+0.24}_{-0.04}$
Λ_2 [GeV]	0.5 (fixed)
g_{K^*}	$55.3^{+0.8}_{-2.5}$
ϕ [Rad]	$3.78^{+0.38}_{-0.26}$
r_1	215^{+51}_{-86}
r_2	$-9.0^{+4.3}_{-0.8}$
m_{f_0} [MeV]	519^{+31}_{-89}
Γ_{f_0} [MeV]	242^{+90}_{-88}
$\chi^2/d.o.f.$	1.43
E_p [MeV]	$2288.4^{+11.7}_{-13.4} - 89.6^{+6.5}_{-6.4}i$

Experimental position	
Mass (MeV)	Width (MeV)
$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

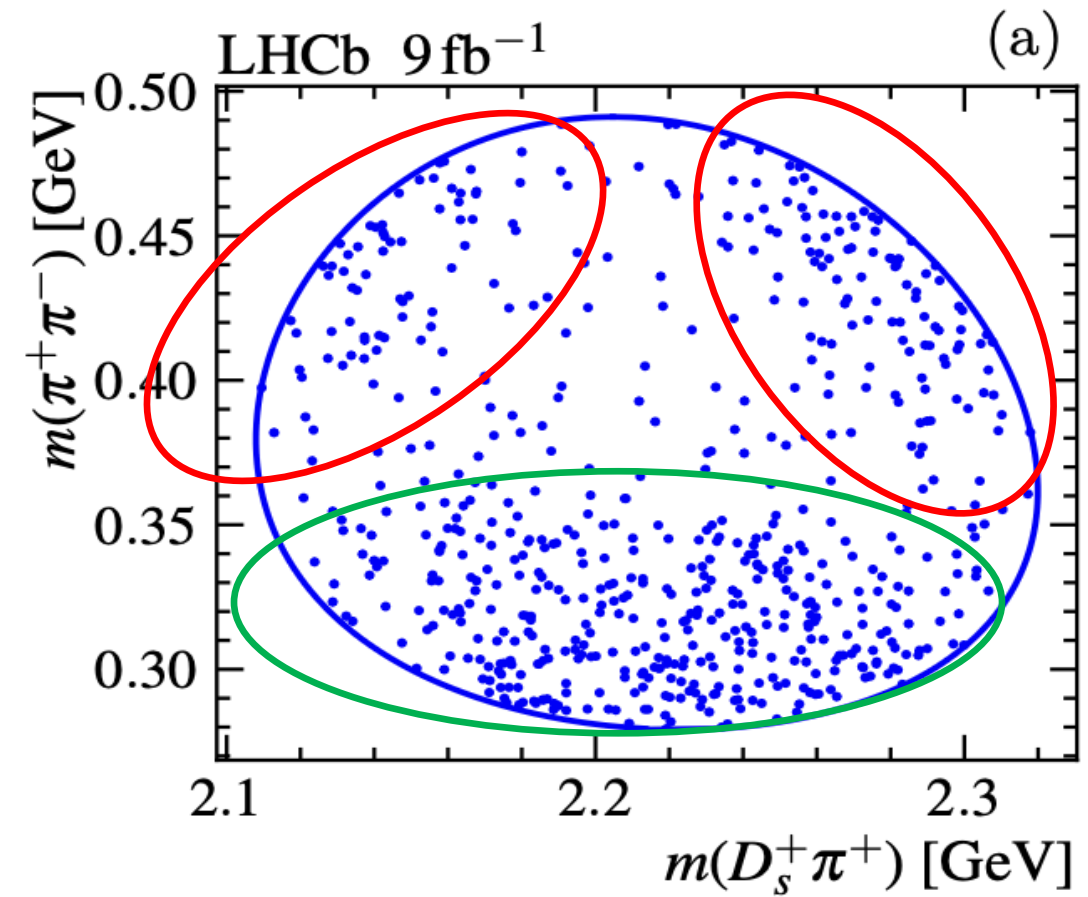
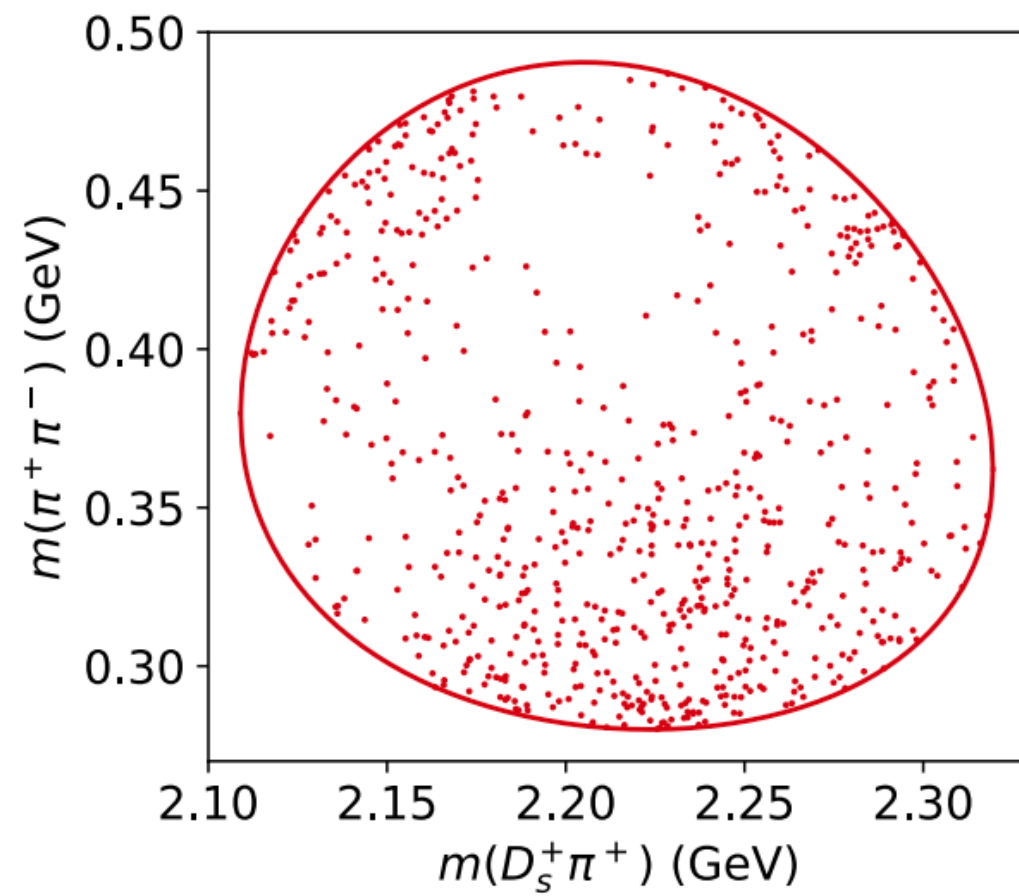
LHCb, arXiv:2411.03399

- The pole position is in second Riemann sheet.

Dalitz plot



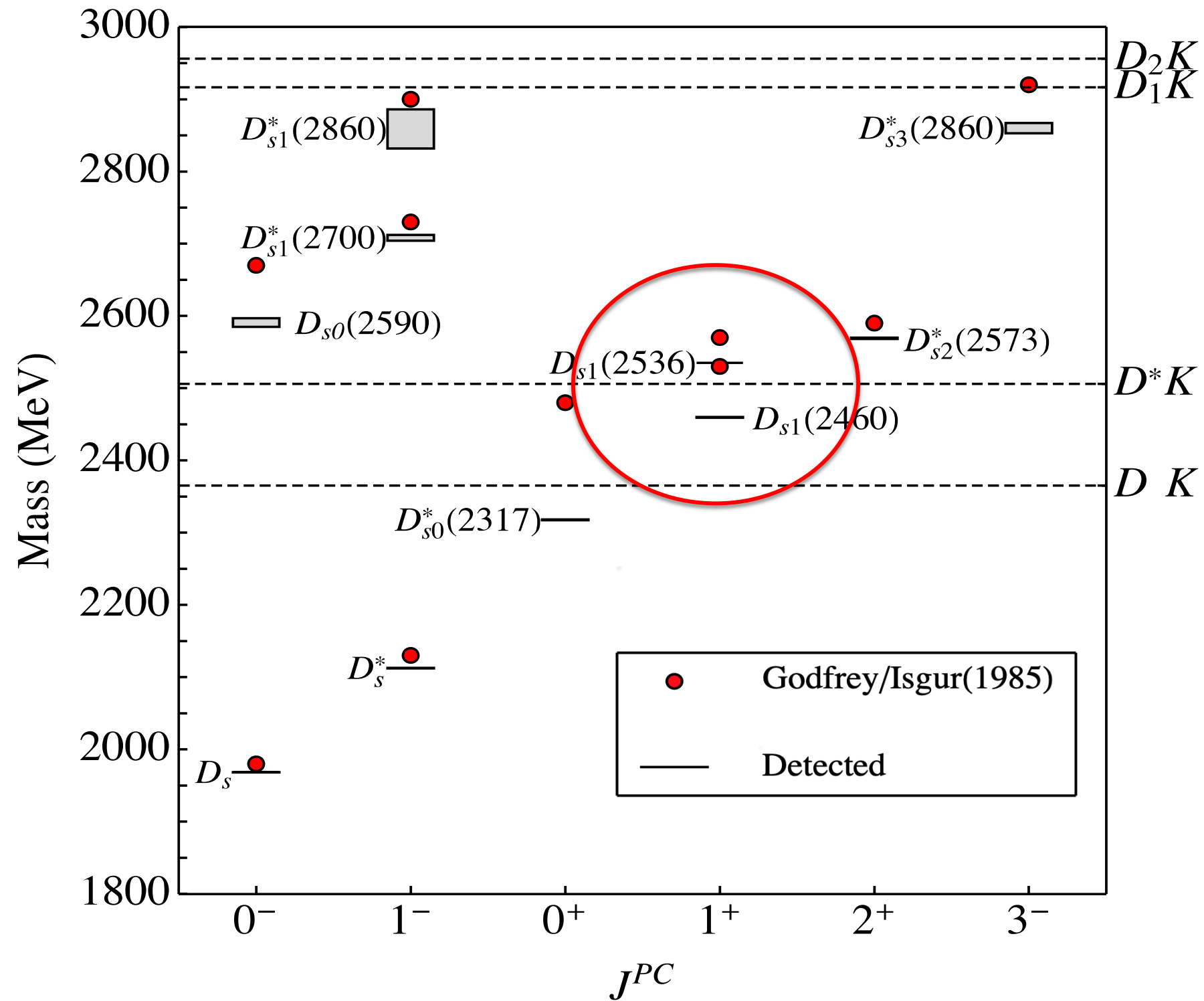
LHCb, arXiv:2411.03399



Is possible to investigate the structure of D_{s1} states?

- $D_{s1}(2460) \rightarrow D_s^+ \pi^+ \pi^-$
- $D_{s1}(2536) \rightarrow D_s^+ \pi^+ \pi^-$

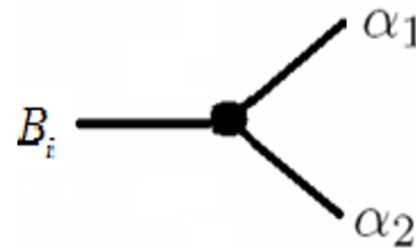
D_{s1} exotic structure



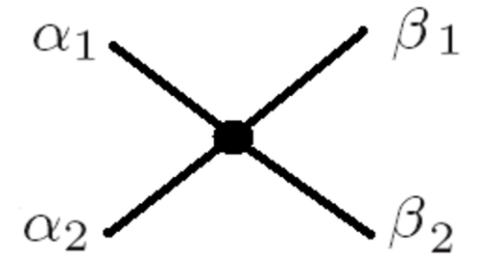
- Quark model bare state + hadronic molecule

Yang, Wang, Wu, Makoto, Zhu, Phys.Rev.Lett. 128,112001(2022)

bare state core $\rightarrow$ channel:



channel $\rightarrow$ channel:



$$g = \sum_{\alpha, B} \int d^3 \vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

$$v = \sum_{\alpha, \beta} \int d^3 \vec{k} d^3 \vec{k}' |\alpha(\vec{k})\rangle V_{\alpha, \beta}^L(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

P. G. Ortega, et al,
Phys. Rev. D 94, 074037 (2016)

truncate the hard vertices given
by usual QPC

Effective Lagrangian: (exchanging mesons, e.g. ρ/ω)

Form factor: $\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2} \right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2} \right)^2$

Component and pole mass of D_s states



Yang, Wang, Wu, Makoto, Zhu Phys.Rev.Lett. 128,112001(2022)

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

$D_{s1}(2460)$

- Both the bare $c\bar{s}$ core and molecular components are significant and essential.

$D_{s1}(2536)$

- Mainly pure $c\bar{s}$.

❖ S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^\mu \epsilon_{j,\mu}^\dagger, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^\mu \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

The coupling of D_{s1} to D^*K can be obtained from the residue of the T-matrix:

$$\begin{aligned} g_i^2 &= \frac{r}{2\pi} \int_0^{2\pi} T_{ii}(z(\theta)) e^{i\theta} d\theta \\ &= \lim_{s \rightarrow z_p} (s - z_p) T_{ii}(s) \end{aligned}$$

- $D_{s1}(2460)$: $g_S \gg g_D$
- $D_{s1}(2536)$: $\frac{g_S}{g_D} = 0.08 e^{2.67i}$

❖ The $D_{s1}(2536)$ coupling can also be obtained using the experimental measurement on the S- and D-wave decay:

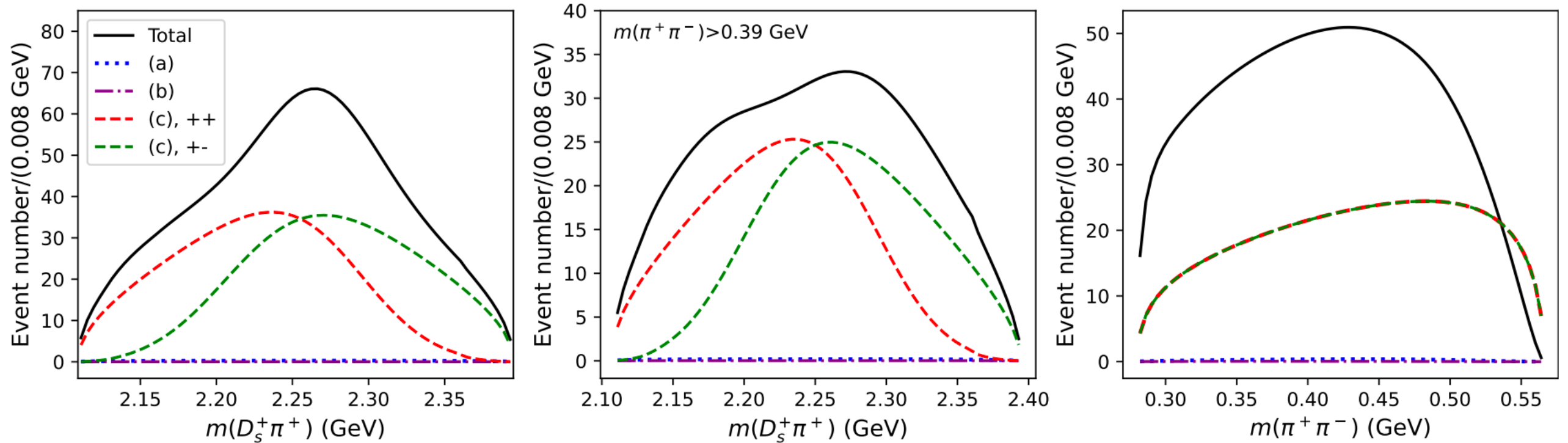
$$\frac{g_S}{g_D} = 0.1 e^{-0.7i}$$

LHCb, JHEP 10 (2023) 106

Estimate the lineshape for $D_{s1}(2536)$ decay

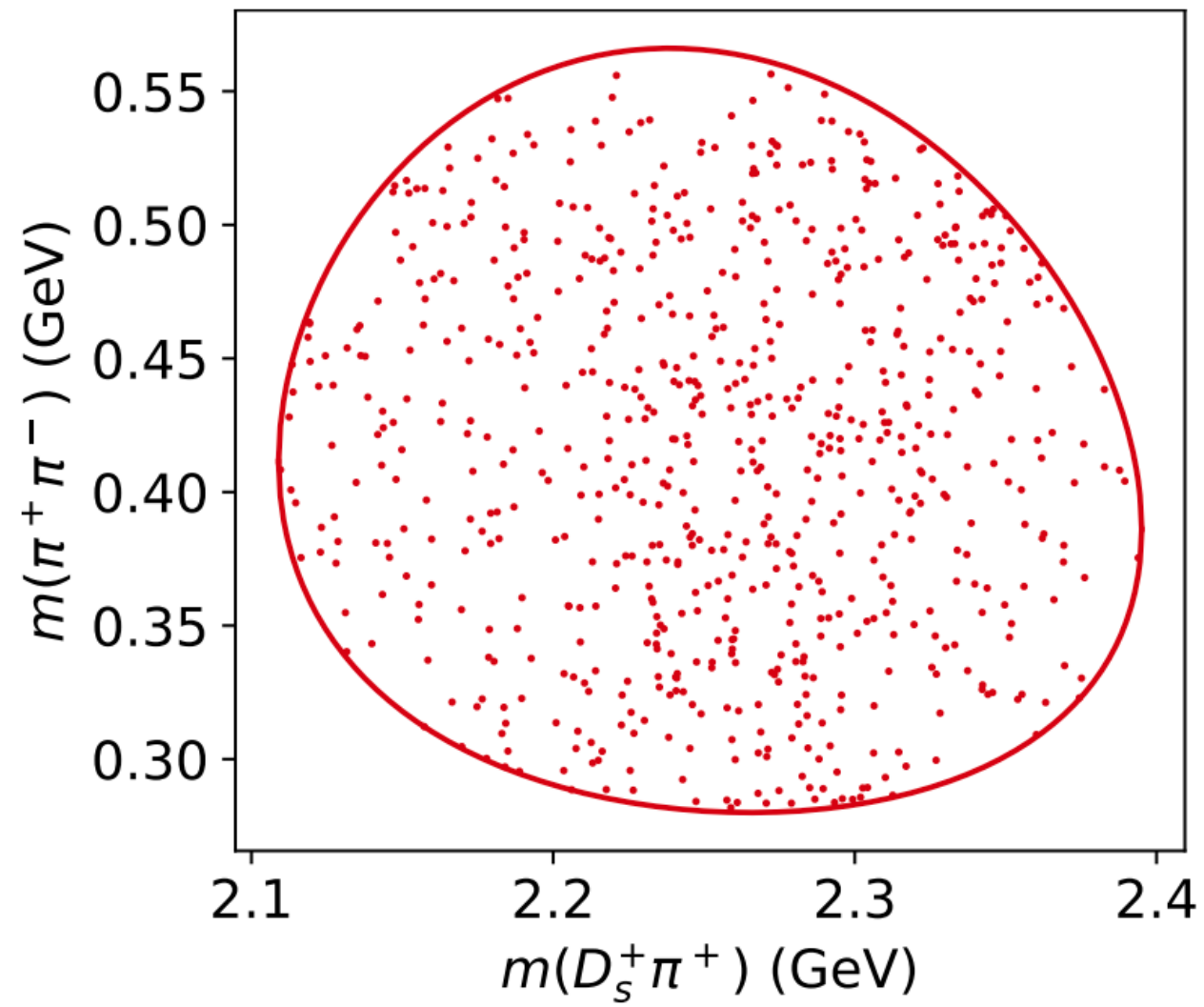


- No two-peak structure at $m(\pi^+\pi^-) > 0.39$ GeV



- If a stricter cut is applied, for instance, $m(\pi^+\pi^-) > 0.45$ GeV, the two-peak structure will appear.

Dalitz plot for $D_{s1}(2536)$ decay



- The $T_{c\bar{s}}$ lineshape can be described through $D_s\pi - DK$ coupled-channel interactions, revealing how off-diagonal potential terms generate the observed resonance pole;
- Predictive lineshape for $D_{s1}(2536)$ decay, no two-peak structure.

Backup slides : Fit the lattice data of $D_s(2317,2460,2536)$



$$(H_0 + H_I)|\Psi\rangle = E|\Psi\rangle$$

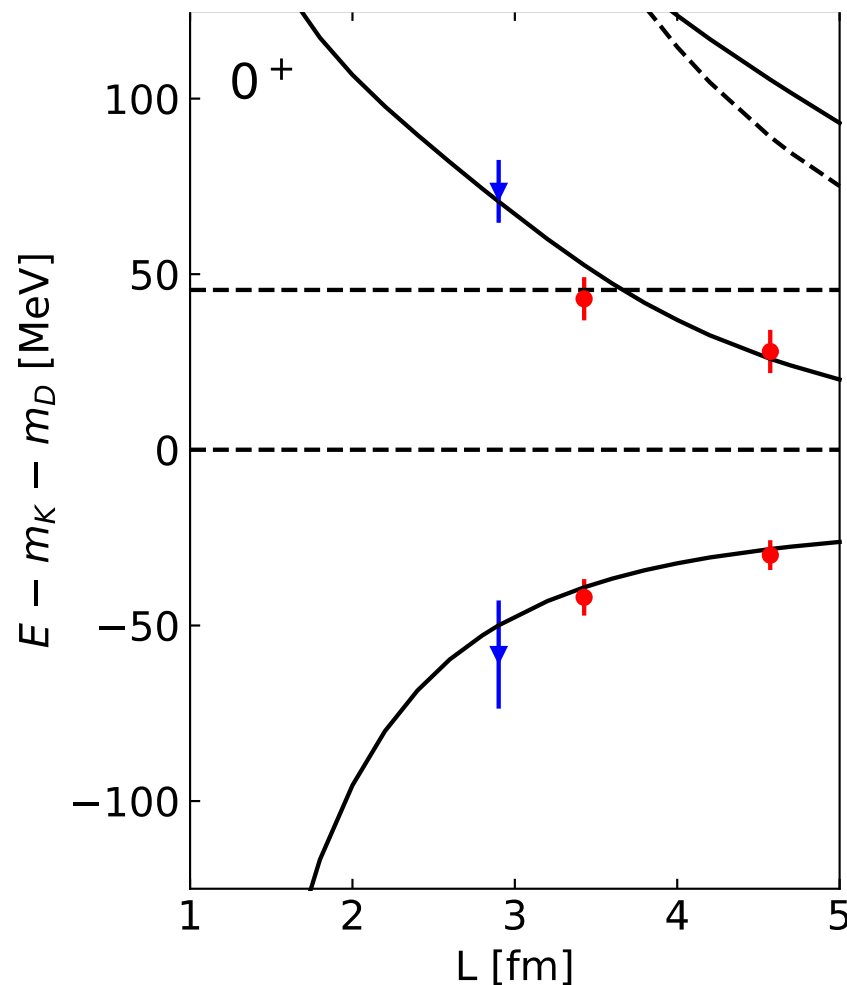
Eigenvalues



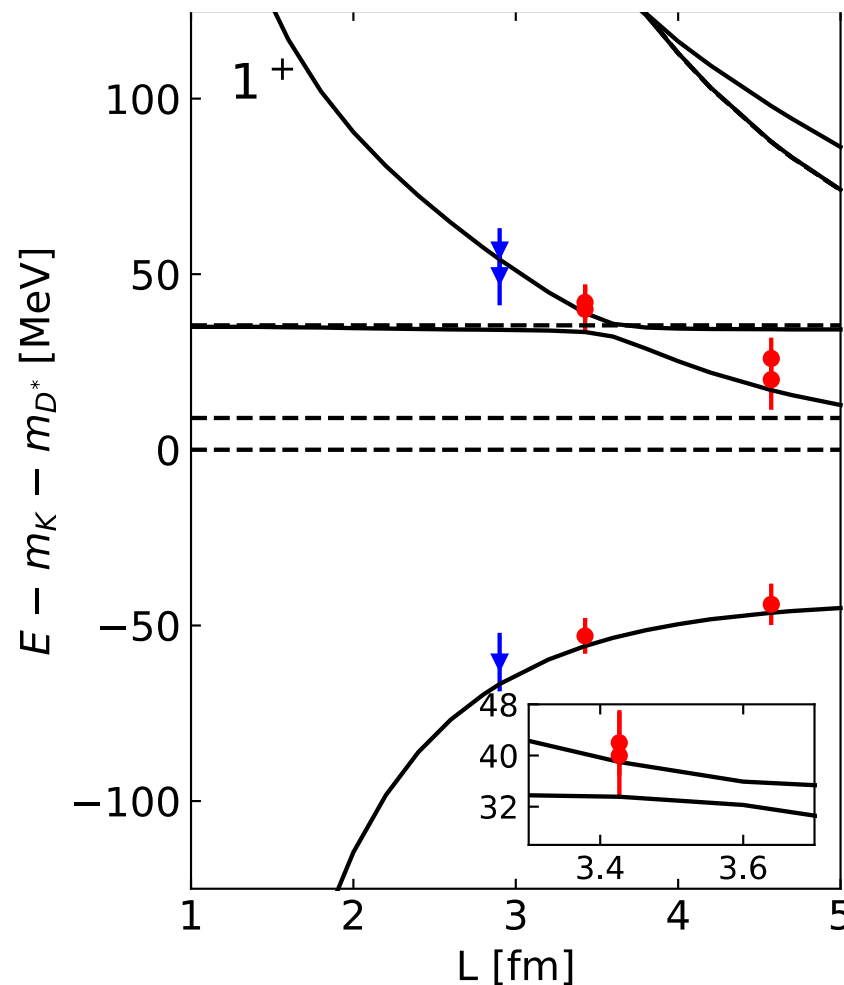
Lattice levels

Lattice data from: C. B. Lang et al., [Phys. Rev. D 90, 034510 \(2014\)](#);
G. S. Bali et al., [Phys. Rev. D 96, 074501 \(2017\)](#)

Fit

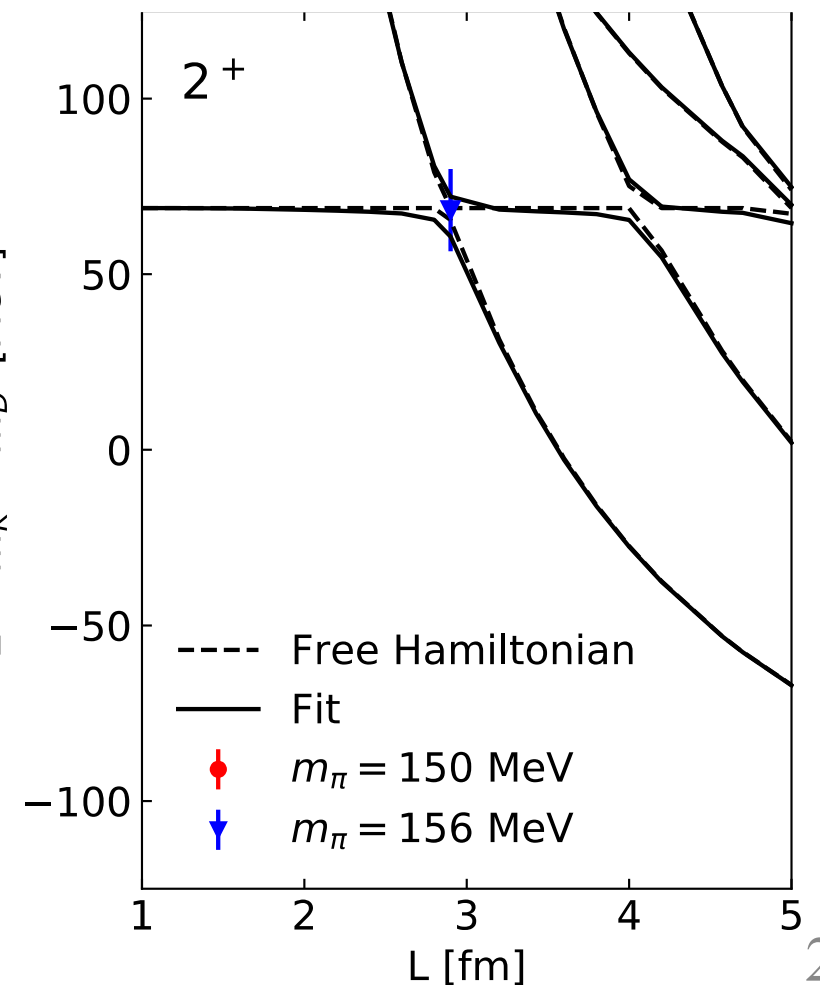


$E - m_K - m_{D^*}$ [MeV]



$E - m_K - m_{D^*}$ [MeV]

Postpredict



- Component

$$(H_0 + H_I)|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi_E\rangle = C_0|B\rangle + \sum_{\vec{k}_n = \frac{2\pi}{L}\vec{n}} C_E(\vec{k}_n)|\alpha(\vec{k}_n)\rangle$$

Eigenvector  Component

- Pole mass

In the infinite volume, the scattering T-matrix reads

$$T_{\alpha,\beta}(k,k';E) = \mathcal{V}_{\alpha,\beta}(k,k';E) + \sum_{\alpha'} \int q^2 dq \frac{\mathcal{V}_{\alpha,\alpha'}(k,q;E) T_{\alpha',\beta}(q,k';E)}{E - E_{\alpha'}(q) + i\epsilon}$$

where the effective potential reads

$$\mathcal{V}_{\alpha,\beta}(k,k';E) = \sum_B \frac{g_{\alpha B}(k) g_{\beta B}^*(k')}{E - m_B} + V_{\alpha,\beta}^L(k,k').$$

T-matrix  Pole mass