One-Loop Renormalization of GraviChPT with Heat-Kernel Technique

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What is GraviChPT?

Why Important?

How to Renormalize?

Preliminary Results

ChPT in Curved Spacetime[1]

- Ordinary terms present in flat spacetime is generalized by replacing $\eta_{\mu\nu}$ with $g_{\mu\nu}(x)$
- Incorporate also interactions between pion, nucleon and gravitational field

$$\mathcal{L}_{\pi N}^{(2)} = \frac{c_8}{8} R\bar{\Psi}\Psi + \frac{ic_9}{m} R^{\mu\nu}\left[\bar{\Psi}\gamma_\mu\left(\overrightarrow{\nabla}_\nu - \overleftarrow{\nabla}_\nu\right)\Psi\right]$$

$$\begin{aligned}\mathcal{L}_{\pi N}^{(3)} &= \tilde{d}_{g1} R\bar{\Psi}\gamma^\mu\gamma_5 u_\mu\Psi + \tilde{d}_{g2} R^{\mu\nu}\bar{\Psi}\gamma_\mu\gamma_5 u_\nu\Psi \\ &+ \tilde{d}_{g3} R^{\alpha\beta\mu\nu}\left[\bar{\Psi}\sigma_{\alpha\beta}\gamma_5 u_\mu\left(\overrightarrow{\nabla}_\nu + \overleftarrow{\nabla}_\nu\right)\Psi\right] \\ &+ \tilde{d}_{g4} \nabla_\nu R^{\alpha\beta\mu\nu}\left[\bar{\Psi}\sigma_{\alpha\beta}\gamma_5\left(\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu\right)\Psi\right]\end{aligned}$$

[1] Alharazin, H., Djukanovic, D., Gegelia, J. & Polyakov, M. V. Chiral theory of nucleons and pions in the presence of an external gravitational field. *Phys. Rev. D* **102**, 076023 (2020).

$$T_{\mu\nu} = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}} e^{a}_{\nu} + \mu \leftrightarrow \nu\right]$$ Definition in Curved Spacetime

$$T_{\mu\nu} \to \frac{c_8}{4}\left(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right)\bar{\Psi}\Psi + \cdots$$ Limit in Flat Spacetime

$$\langle N|T_{\mu\nu}|N\rangle = \bar{u}\left[A(t)\frac{P_\mu P_\nu}{m_N} + iJ(t)\frac{P_\mu \sigma_{\nu\alpha}\Delta^\alpha + P_\nu \sigma_{\mu\alpha}\Delta^\alpha}{2m_N} + D(t)\frac{\Delta_\mu\Delta_\nu - \eta_{\mu\nu}\Delta^2}{4m_N}\right]u$$

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$$A_{\text{tree}}(t) = 1 - \frac{2\boldsymbol{c_9}}{m_N}t + \cdots \quad , \quad J_{\text{tree}}(t) = \frac{1}{2} - \frac{\boldsymbol{c_9}}{m_N}t \quad , \quad D_{\text{tree}}(t) = \boldsymbol{c_8} m_N + \cdots$$

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Feynman-Diagram Technique | Cumbersome

$$\mathrm{div}[W_{L=1}] = \mathrm{div}\left[\sum_{\text{One-Loop 1PI}} \textbf{Feynman Diagram}\right]$$

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Heat-Kernel Technique | Algebraic & Systematic

$$\mathrm{div}[W_{L=1}] \propto \frac{1}{\epsilon}\int d^d x\sqrt{g}\,\mathrm{str}\left[\boldsymbol{A}^{(2)}(\boldsymbol{x},\boldsymbol{x})\right]$$

$$W_{L=1} = \frac{i}{2}\mathrm{Str}\{\ln[\boldsymbol{M}]\}$$

$$(\partial_\tau + \boldsymbol{M})K(x,y|\tau) = 0,\ K(x,y|\tau) \propto \sum_n \tau^n \boldsymbol{A}^{(n)}(\boldsymbol{x},\boldsymbol{y})$$

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Momentum-Space Iterative Solution for Heat Problem

- A symbolic program in Python has been implemented and is to be published.
- Details can be found in the poster.

Conter-Terms

$$c_8 = c_8^R - \frac{15 g_A^2 m}{2}\Lambda \quad c_9 = c_9^R + \frac{25 g_A^2 m}{864 F^2}\Lambda$$

$$\tilde{d}_{g1} = \tilde{d}_{g1}^R + \left(\frac{10 g_A^3}{23 F^2} - \frac{g_A}{3F^2}\right)\Lambda \quad \tilde{d}_{g2} = \tilde{d}_{g2}^R + \frac{g_A^3}{72 F^2}\Lambda \quad \tilde{d}_{g3} = \tilde{d}_{g3}^R \quad \tilde{d}_{g4} = \tilde{d}_{g4}^R$$

$$\Lambda = \frac{1}{32\pi^2}\left(\frac{2}{d-4} - 1 + \gamma - \ln(4\pi)\right)$$

Thank You for Your Attention