第十届手征有效场论研讨会

Role of $a_0(980)$ in the decays of $D^0 \to K^+K^-\eta$ and $\pi^+\pi^-\eta$

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§1. Introduction

$a_0(980)$

$I^G(J^{PC}) = 1^-(0^{++})$

VALUE (MeV)
980±20 OUR ESTIMATE
$\eta\pi$ FINAL STATE ONLY

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
50 to 100 OUR ESTIMATE					

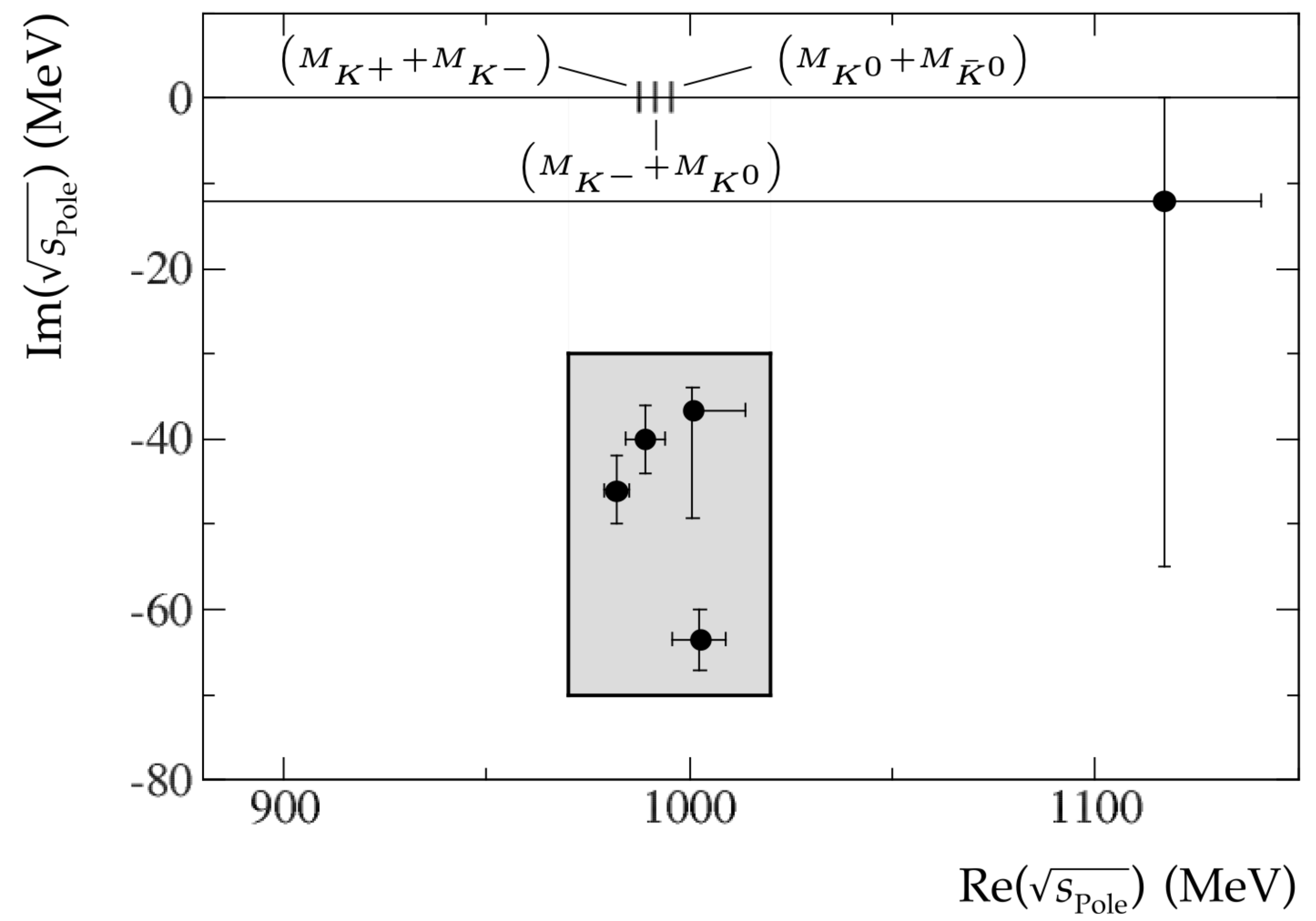
Width determination very model dependent. Peak width in $\eta\pi$ is about 60 MeV, but decay width can be much larger.

A. Astier, L. Montanet, M. Baubillier and J. Duboc, Phys. Lett. B 25, 294 (1967).
R. Ammar et al., Phys. Rev. Lett. 21, 1832 (1968).
C. Defoix, P. Rivet, J. Siaud, B. Conforto, M. Widgoff and F. Shively, Phys. Lett. B 28, 353 (1968).

PDG review

64. Scalar Mesons below 1 GeV

Revised August 2023 by T. Gutsche (Tübingen U.), C. Hanhart (FZ Jülich), R.E. Mitchell (Indiana U.) and S. Spanier (Tennessee U.).



$$\sqrt{s_{\mathrm{Pole}}^{a_0(980)}} = (970 - 1020) - i(30 - 70)\ \mathrm{MeV}$$

Conventional scalar $q\bar{q}$ states:

S. Godfrey and N. Isgur, Phys. Rev. D 32, 189-231 (1985).
D. Morgan and M. R. Pennington, Phys. Rev. D 48, 1185-1204 (1993).
N. A. Tornqvist and M. Roos, Phys. Rev. Lett. 76, 1575-1578 (1996).

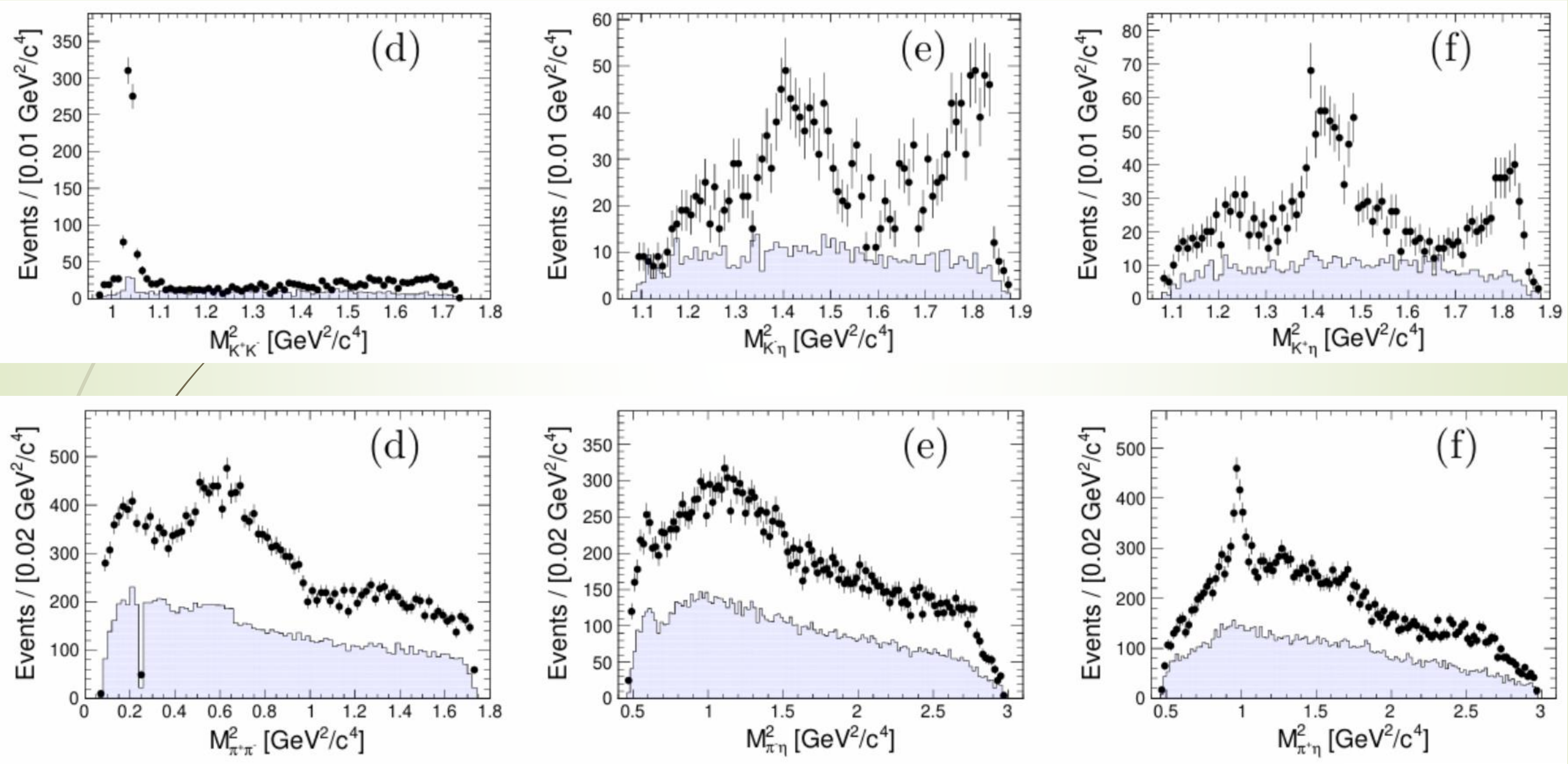
Tetraquarks, composed of four quarks $qq\bar{q}\bar{q}$ states:

R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).
N. N. Achasov, Nucl. Phys. A 728, 425-438 (2003).

Molecular state, composed of $K\bar{K}$:

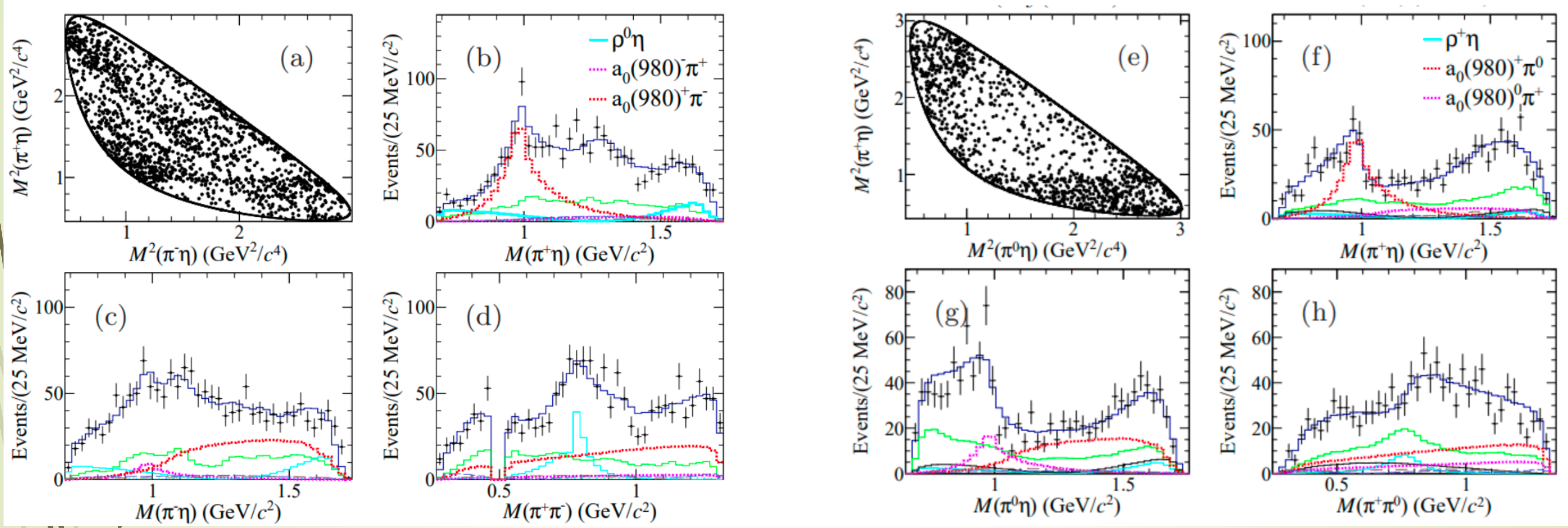
B. S. Zou and D. V. Bugg, Phys. Rev. D 50, 591-594 (1994).
G. Janssen, B. C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690-2700 (1995).
J. A. Oller and E. Oset, Nucl. Phys. A 620, 438-456 (1997) [erratum: Nucl. Phys. A 652, 407-409 (1999)].
M. P. Locher, V. E. Markushin and H. Q. Zheng, Eur. Phys. J. C 4, 317-326 (1998).
J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. 80, 3452-3455 (1998).

$D^0 \to K^+K^-\eta, \quad \pi^+\pi^-\eta$



L. K. Li et al. [Belle], JHEP 09, 075 (2021)

$D^0 \to \pi^+\pi^-\eta, \quad D^+ \to \pi^+\pi^0\eta$



M. Ablikim et al. [BESIII], Phys. Rev. D 110, L111102 (2024)

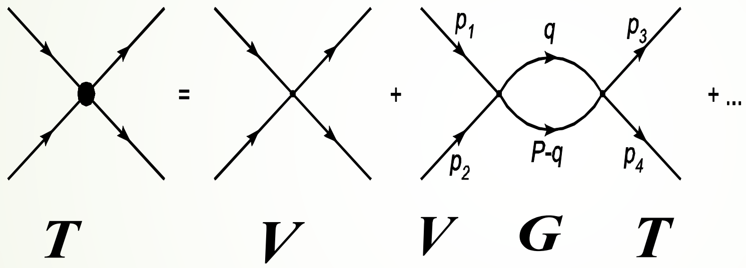
What can we learn from the invariant mass distributions of the experimental measurement?

§2. Formalism

(1) Coupled channel interaction from the chiral unitary approach

- **Chiral Unitary Approach**: solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



D. L. Yao, L. Y. Dai, H. Q. Zheng and Z. Y. Zhou, Rept. Prog. Phys. 84, 076201 (2021)

where V matrix (potentials) can be evaluated from the interaction Lagrangians.

$I = 0$ sector	$\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\bar{K}^0$, $\eta\eta$
$I = 1$ sector	K^+K^-, $K^0\bar{K}^0$, $\pi^0\eta$
$I = 1/2$ sector	$K^+\pi^-$, $K^0\pi^0$, $K^0\eta$

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99
J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2-m_{l1}^2+i\varepsilon}\ \frac{1}{q^2-m_{l2}^2+i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

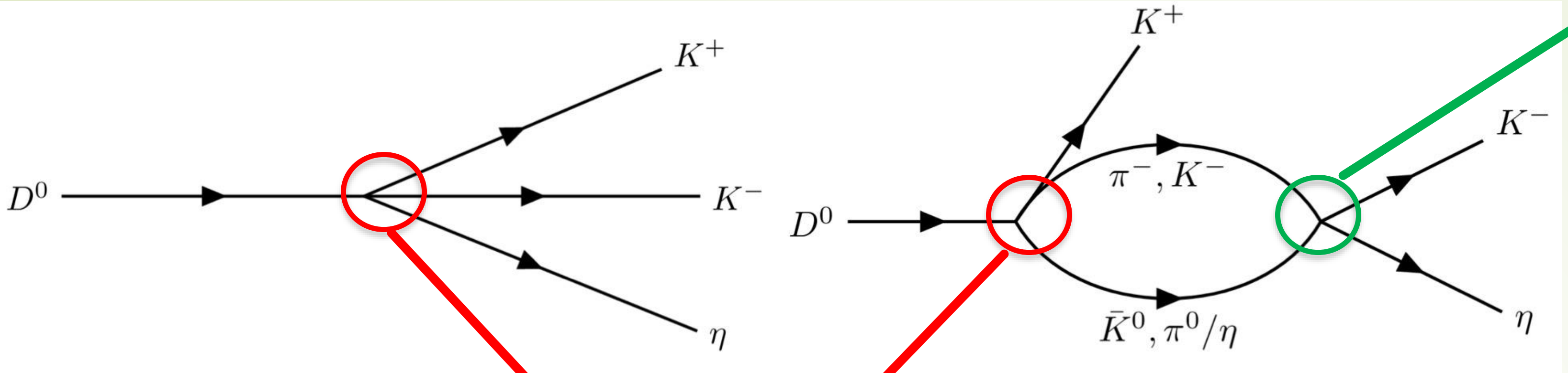
$$\mathrm{Im}\ T_{ij} = T_{in}\,\sigma_{nn}\,T^*_{nj}$$

$$\sigma_{nn} \equiv \mathrm{Im}\ G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s-(m_1+m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G^{II}_{ll}(s) = G^{I}_{ll}(s) + i\,\frac{q_{cm}}{4\pi\sqrt{s}}$$

(2) Final state interaction



(a) Tree-level production.

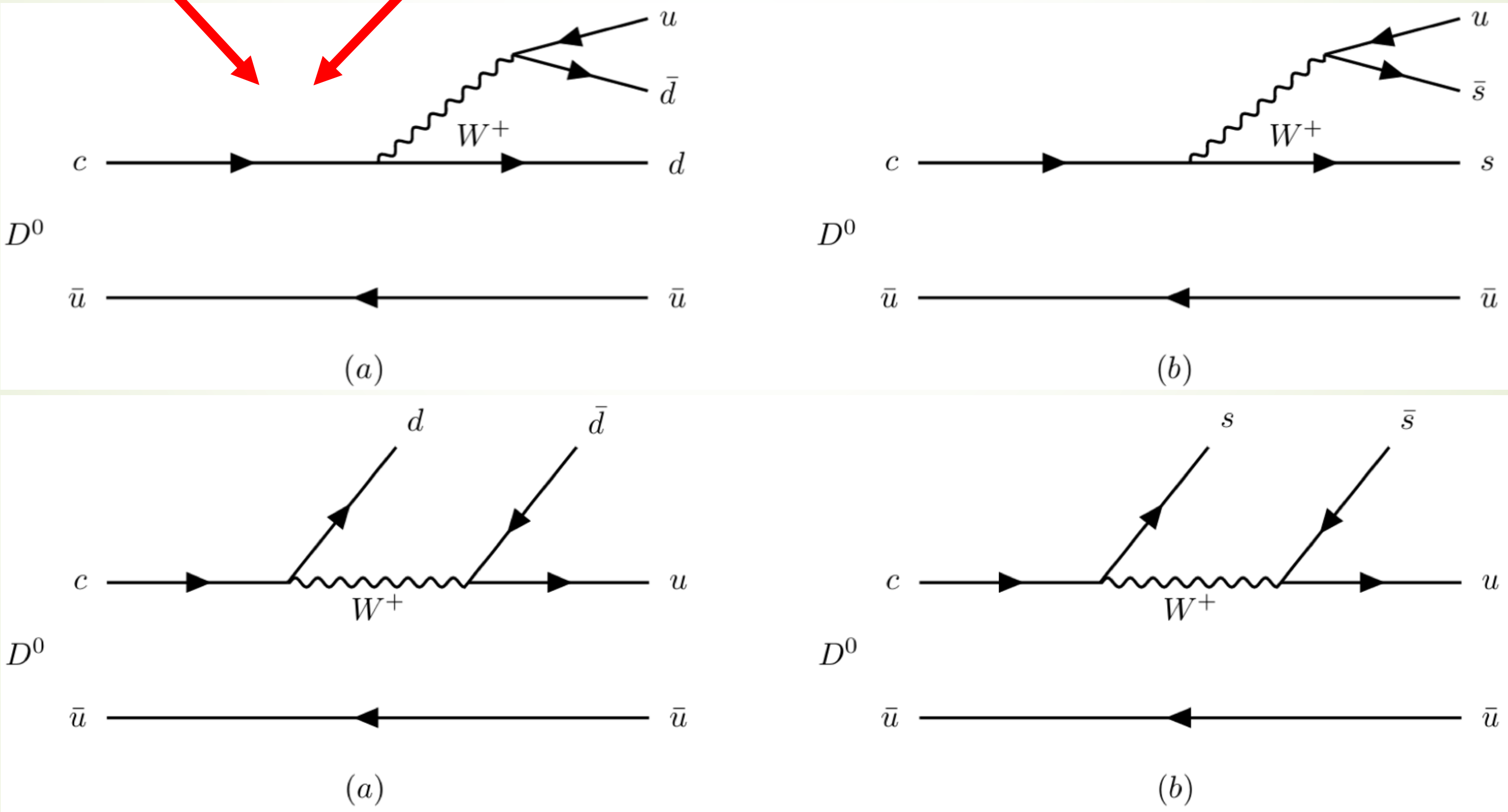
(b) Rescattering of $\pi^-\bar{K}^0$, $K^-\pi^0$ and $K^-\eta$.

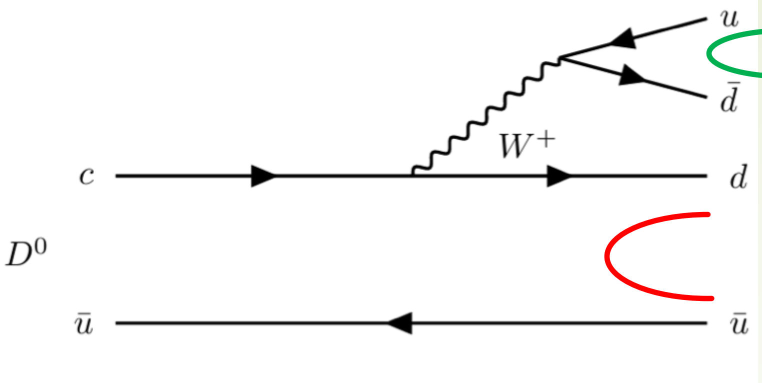
$$T = [1 - VG]^{-1}V$$

S-wave

The final state interaction at the hadron level

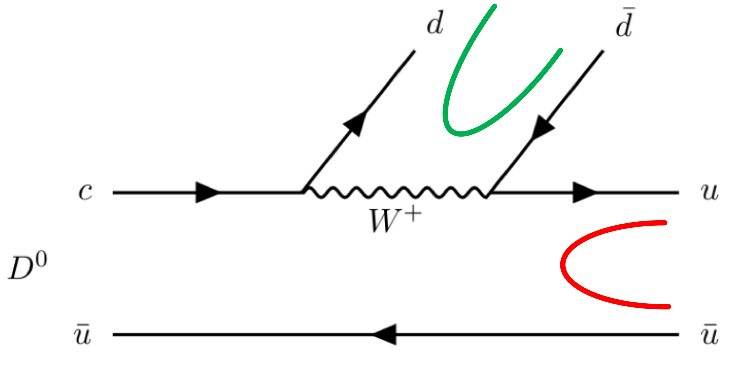
The weak decay process at the quark level





(a)

$$
\begin{aligned}
H^{(1a)} &= V_P V_{cd} V_{ud} \{ (u\bar d \to \pi^+) \left[d\bar u \to d\bar u \left(u\bar u + d\bar d + s\bar s \right) \right] \\
&\quad + (d\bar u \to \pi^-) \left[u\bar d \to u\bar d \left(u\bar u + d\bar d + s\bar s \right) \right] \} \\
&= V_P V_{cd} V_{ud} \{ (u\bar d \to \pi^+) \left[M_{21} \to (M \cdot M)_{21} \right] \\
&\quad + (d\bar u \to \pi^-) \left[(M \cdot M)_{12} \right] \},
\end{aligned}
$$



(a)

$$
\begin{aligned}
H^{(2a)} &= \beta V_P V_{cd} V_{ud} \Bigg[\left(d\bar d \to -\frac{1}{\sqrt{2}} \pi^0 \right) \left[u\bar u \to u\bar u \left(u\bar u + d\bar d + s\bar s \right) \right] \\
&\quad + \left(d\bar d \to \frac{1}{\sqrt{6}} \eta \right) \left[u\bar u \to u\bar u \left(u\bar u + d\bar d + s\bar s \right) \right] \\
&\quad + \left(u\bar u \to \frac{1}{\sqrt{2}} \pi^0 \right) \left[d\bar d \to d\bar d \left(u\bar u + d\bar d + s\bar s \right) \right] \\
&\quad + \left(u\bar u \to \frac{1}{\sqrt{6}} \eta \right) \left[d\bar d \to d\bar d \left(u\bar u + d\bar d + s\bar s \right) \right] \Bigg] \\
&= \beta V_P V_{cd} V_{ud} \Bigg[\left(d\bar d \to -\frac{1}{\sqrt{2}} \pi^0 \right) \left[M_{11} \to (M \cdot M)_{11} \right] \\
&\quad + \left(d\bar d \to \frac{1}{\sqrt{6}} \eta \right) \left[M_{11} \to (M \cdot M)_{11} \right] \\
&\quad + \left(u\bar u \to \frac{1}{\sqrt{2}} \pi^0 \right) \left[M_{22} \to (M \cdot M)_{22} \right] \\
&\quad + \left(u\bar u \to \frac{1}{\sqrt{6}} \eta \right) \left[M_{22} \to (M \cdot M)_{22} \right] \Bigg],
\end{aligned}
$$

$$M=\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}=\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$(M\cdot M)_{12}=\frac{2}{\sqrt{6}}\pi^+\eta+K^+\bar{K}^0$$

$$(M\cdot M)_{21}=\frac{2}{\sqrt{6}}\pi^-\eta+K^0K^-$$

$$(M\cdot M)_{11}=\frac{1}{2}\pi^0\pi^0+\frac{1}{6}\eta\eta+\frac{1}{\sqrt{3}}\pi^0\eta+\pi^+\pi^-+K^+K^-$$

$$(M\cdot M)_{22}=\pi^+\pi^-+\frac{1}{2}\pi^0\pi^0+\frac{1}{6}\eta\eta-\frac{1}{\sqrt{3}}\pi^0\eta+K^0\bar{K}^0$$

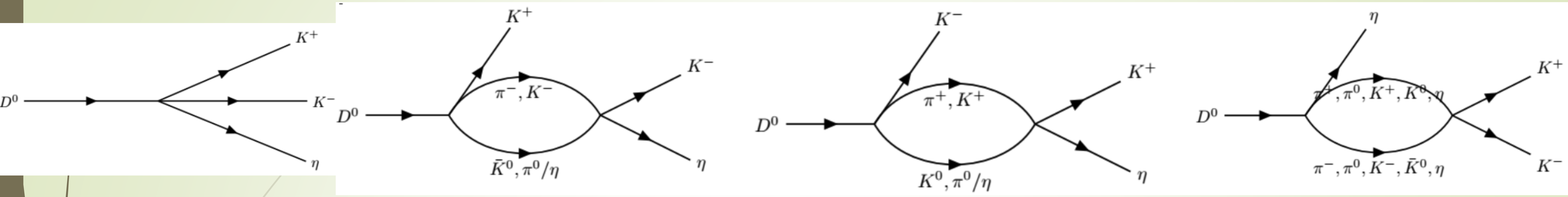
$$H^{(1a)}=V_PV_{cd}V_{ud}\left(\frac{4}{\sqrt{6}}\underline{\pi^+\pi^-\eta}+\pi^+K^0K^-+\pi^-K^+\bar{K}^0\right)$$

$$H^{(2a)}=\beta V_PV_{cd}V_{ud}\left(\frac{1}{3\sqrt{6}}\eta\eta\eta-\frac{1}{\sqrt{2}}\pi^0K^+K^-+\frac{2}{\sqrt{6}}\underline{\pi^+\pi^-\eta}+\frac{1}{\sqrt{6}}\underline{K^+K^-\eta}\right.$$
$$\left.-\frac{1}{\sqrt{6}}\pi^0\pi^0\eta+\frac{1}{\sqrt{2}}\pi^0K^0\bar{K}^0+\frac{1}{\sqrt{6}}K^0\bar{K}^0\eta\right),$$

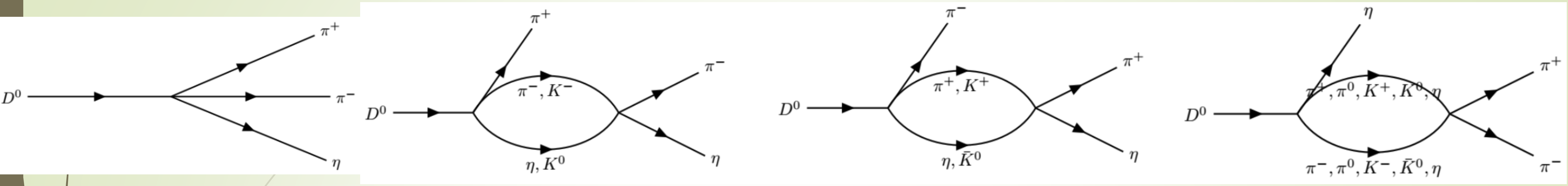
$D^0 \to \pi^+\pi^-\eta$ and $K^+K^-\eta$, having

$$H = H^{(1a)} + H^{(1b)} + H^{(2a)} + H^{(2b)}$$

$$= \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)\boxed{\pi^+\pi^-\eta} + (C_1 - C_2)\pi^+K^0K^- + (C_1 - C_2)\pi^-K^+\bar{K}^0$$
$$- \frac{1}{\sqrt{2}}(2C_2 + \beta C_1 + \beta C_2)K^+K^-\pi^0 + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)\boxed{K^+K^-\eta}$$
$$+ \frac{1}{3\sqrt{6}}\beta(C_1 - C_2)\eta\eta\eta + \frac{1}{\sqrt{6}}\beta(C_2 - C_1)\pi^0\pi^0\eta + \frac{1}{\sqrt{2}}\beta(C_1 - C_2)\pi^0K^0\bar{K}^0$$
$$+ \frac{1}{\sqrt{6}}\beta(C_1 - C_2)K^0\bar{K}^0\eta,$$

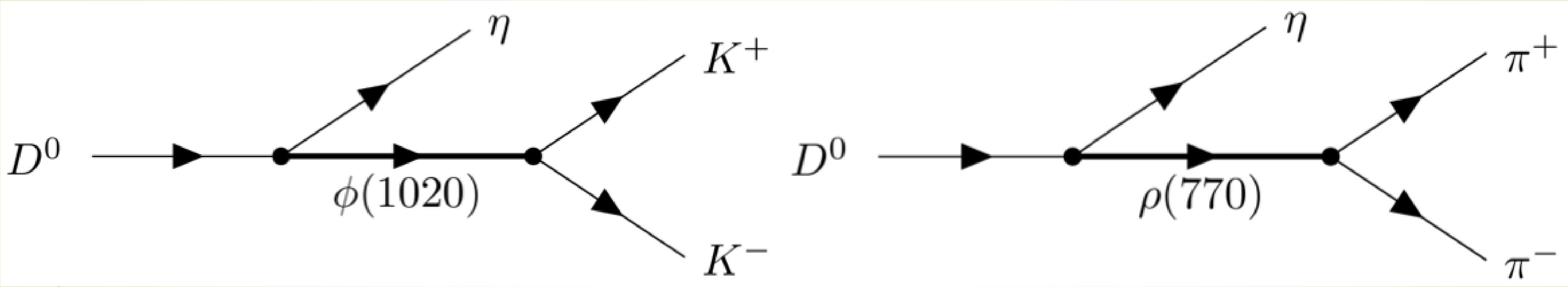


$$
\begin{aligned}
t_{D^0\to K^+K^-\eta}(s_{12}, s_{23}) = {} & \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)G_{\pi^+\pi^-}(s_{12})\,T_{\pi^+\pi^-\to K^+K^-}(s_{12}) \\
& + \underline{\frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)} + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)\,G_{K^+K^-}(s_{12})\,T_{K^+K^-\to K^+K^-}(s_{12}) \\
& + \frac{1}{3\sqrt{6}}\beta(C_1 - C_2)G_{\eta\eta}(s_{12})\,T_{\eta\eta\to K^+K^-}(s_{12}) - \frac{1}{\sqrt{6}}\beta(C_1 - C_2)G_{\pi^0\pi^0}(s_{12})\,T_{\pi^0\pi^0\to K^+K^-}(s_{12}) \\
& + \frac{1}{\sqrt{6}}\beta(C_1 - C_2)G_{K^0\bar{K}^0}(s_{12})\,T_{K^0\bar{K}^0\to K^+K^-}(s_{12}) + (C_1 - C_2)G_{\pi^-\bar{K}^0}(s_{23})\,T_{\pi^-\bar{K}^0\to K^-\eta}(s_{23}) \\
& - \frac{1}{\sqrt{2}}(2C_2 + \beta C_1 + \beta C_2)\,G_{K^-\pi^0}(s_{23})\,T_{K^-\pi^0\to K^-\eta}(s_{23}) + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)\,G_{K^-\eta}(s_{23})\,T_{K^-\eta\to K^-\eta}(s_{23}) \\
& + (C_1 - C_2)G_{\pi^+K^0}(s_{13})\,T_{\pi^+K^0\to K^+\eta}(s_{13}) - \frac{1}{\sqrt{2}}(2C_2 + \beta C_1 + \beta C_2)\,G_{K^+\pi^0}(s_{13})\,T_{K^+\pi^0\to K^+\eta}(s_{13}) \\
& + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)\,G_{K^+\eta}(s_{13})\,T_{K^+\eta\to K^+\eta}(s_{13}),
\end{aligned}
$$



$$
\begin{aligned}
t_{D^0\to\pi^+\pi^-\eta}(s_{12}, s_{23}) = {} & \underline{\frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)} + \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2) G_{\pi^+\pi^-}(s_{12})\, T_{\pi^+\pi^-\to\pi^+\pi^-}(s_{12}) \\
& + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)\, G_{K^+K^-}(s_{12})\, T_{K^+K^-\to\pi^+\pi^-}(s_{12}) + \frac{1}{3\sqrt{6}}\beta(C_1 - C_2) G_{\eta\eta}(s_{12})\, T_{\eta\eta\to\pi^+\pi^-}(s_{12}) \\
& - \frac{1}{\sqrt{6}}\beta(C_1 - C_2) G_{\pi^0\pi^0}(s_{12})\, T_{\pi^0\pi^0\to\pi^+\pi^-}(s_{12}) + \frac{1}{\sqrt{6}}\beta(C_1 - C_2) G_{K^0\bar{K}^0}(s_{12})\, T_{K^0\bar{K}^0\to\pi^+\pi^-}(s_{12}) \\
& + \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2) G_{\pi^-\eta}(s_{23})\, T_{\pi^-\eta\to\pi^-\eta}(s_{23}) + \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2) G_{\pi^+\eta}(s_{13})\, T_{\pi^+\eta\to\pi^+\eta}(s_{13}) \\
& + (C_1 - C_2) G_{K^0K^-}(s_{23})\, T_{K^0K^-\to\pi^-\eta}(s_{23}) + (C_1 - C_2) G_{K^+\bar{K}^0}(s_{13})\, T_{K^+\bar{K}^0\to\pi^+\eta}(s_{13}),
\end{aligned}
$$

(3) Contributions from P-wave



$$M_\phi\left(s_{12}, s_{23}\right)=\frac{D_\phi e^{i\alpha_\phi}}{s_{12}-m_\phi^2+i m_\phi \Gamma_\phi}\left(s_{23}-s_{13}\right)$$

$$M_\rho\left(s_{12}, s_{23}\right)=\frac{D_\rho e^{i\alpha_\rho}}{s_{12}-m_\rho^2+i m_\rho \Gamma_\rho}\left(s_{23}-s_{13}\right)$$

$$\frac{d^2\Gamma}{ds_{12}ds_{23}}=\frac{1}{(2\pi)^3}\frac{1}{32 m_{D^0}^3}\left|t_{D^0\to\pi^+\pi^-\eta}\left(s_{12}, s_{23}\right)+M_\rho\right|^2$$

$$\frac{d^2\Gamma}{ds_{12}ds_{23}}=\frac{1}{(2\pi)^3}\frac{1}{32 m_{D^0}^3}\left|t_{D^0\to K^+K^-\eta}\left(s_{12}, s_{23}\right)+M_\phi\right|^2$$

$$G(s)T(s)=G(s_{cut})T(s_{cut})e^{-\alpha(\sqrt{s}-\sqrt{s_{cut}})}, \quad \text{for } \sqrt{s}>\sqrt{s_{cut}}, \quad \sqrt{s_{cut}}=1.1 \text{ GeV}$$

- **free parameters**

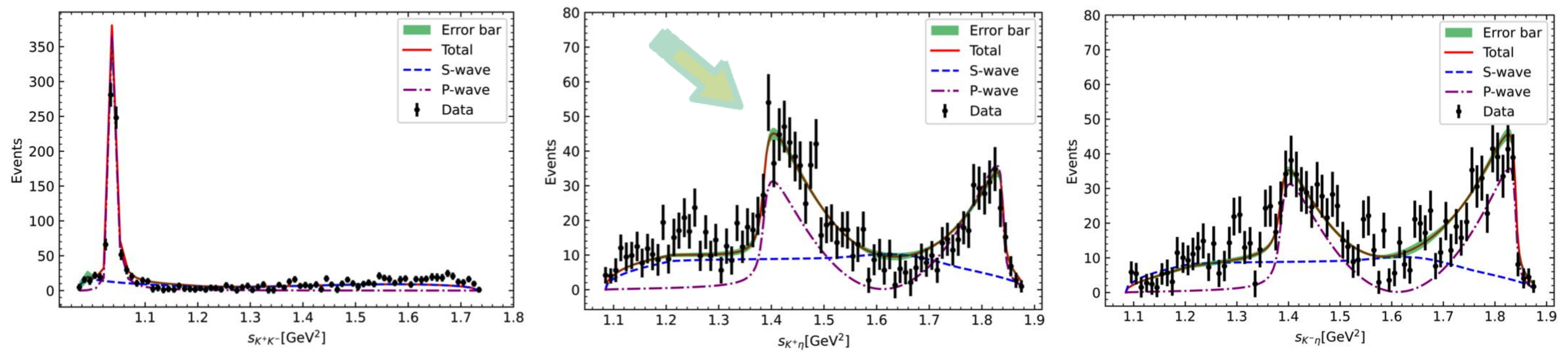
S-wave: C_1, C_2, β, α

P-wave: $D_\rho, \alpha_\rho, D_\phi, \alpha_\phi$

§3. Results

(1) $D0 \to K^+K^-\eta$ results (Belle)

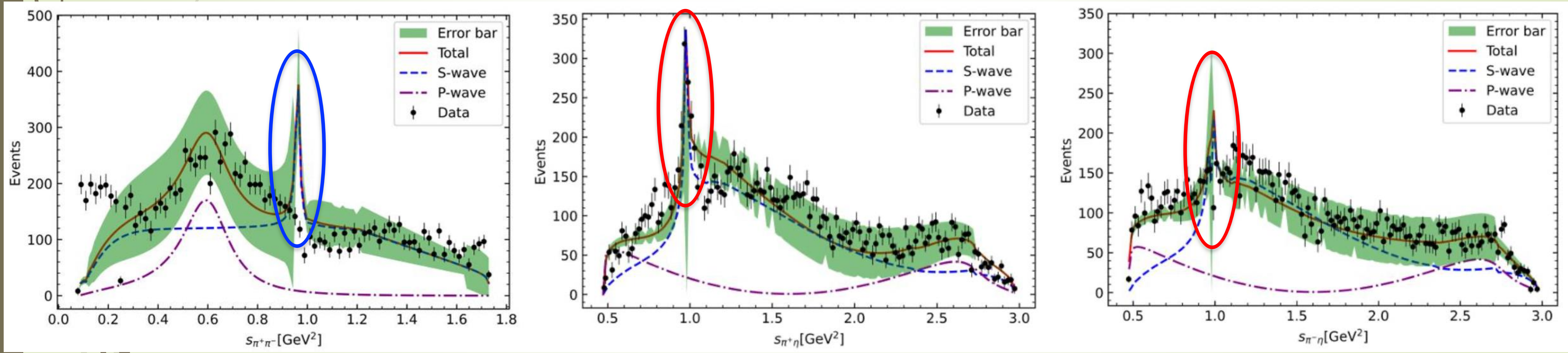
Parameters	C_1	C_2	β	α	$D_{\phi(1020)}$	$\alpha_{\phi(1020)}$	$\chi^2/dof.$
Fit	-3448.24±221.07	99.07±91.70	0.77±0.04	6.28±0.05	123.75±2.02	2.89±0.12	1.48



L. K. Li et al. [Belle], JHEP 09, 075 (2021)

(2) D0 → $\pi^+\pi^-\eta$ results (Belle)

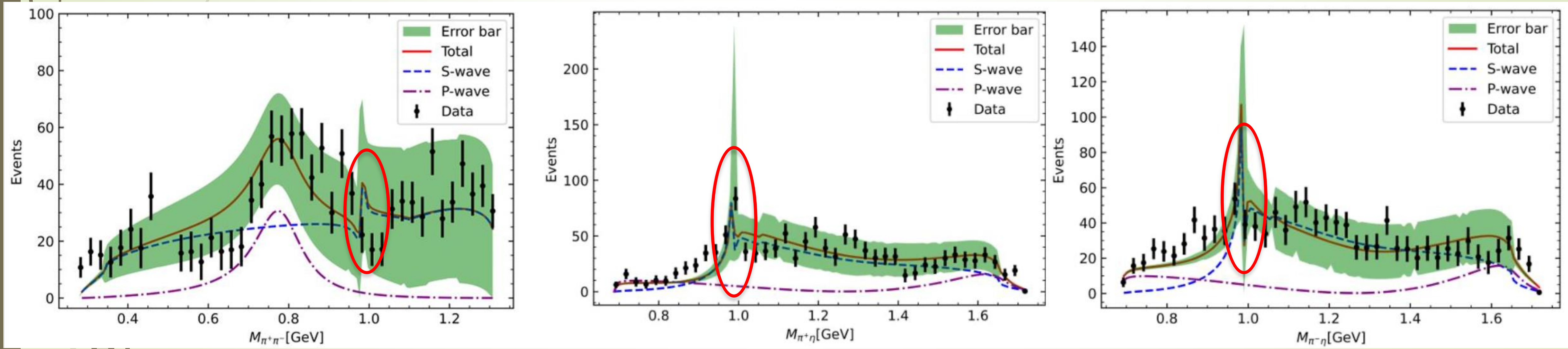
Parameters	C_1	C_2	β	α	$D_{\rho(770)}$	$\alpha_{\rho(770)}$	$\chi^2/dof.$
Fit	419.79± 23.61	2196.24±57.25	1.00±0.006	0.38±0.10	-170.97±2.90	2.20±0.03	5.23



L. K. Li et al. [Belle], JHEP 09, 075 (2021)

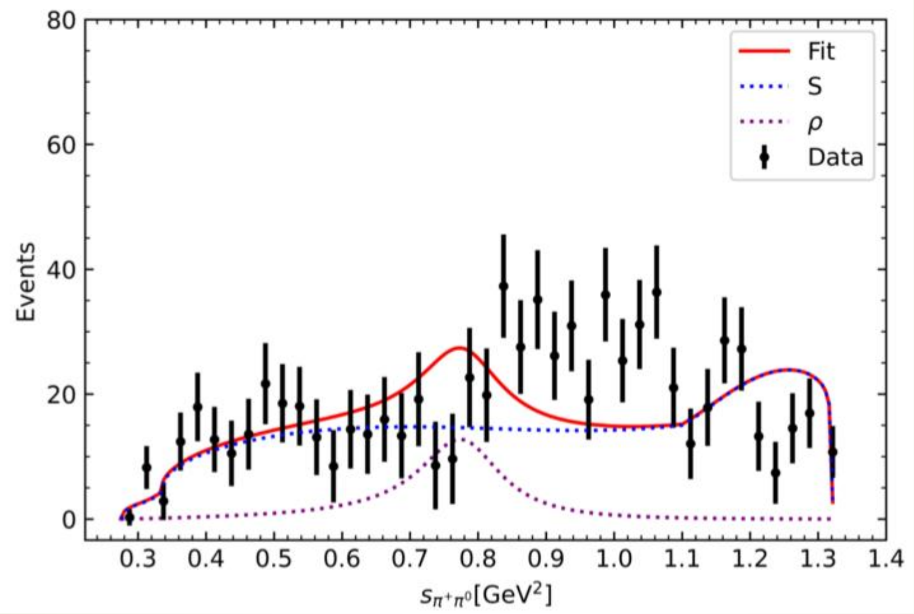
(3) $D0 \to \pi^+\pi^- \eta$ results (BESIII)

Parameters	C_1	C_2	β	α	$D_{\rho(770)}$	$\alpha_{\rho(770)}$	$\chi^2/dof.$
Fit	282.74±15.25	-516.33±26.32	1.00±0.03	2.47±0.44	58.41±2.98	4.36±0.13	2.52



M. Ablikim et al. [BESIII], Phys. Rev. D 110, L111102 (2024)

(4) $D+ \to \pi+\pi0\ \eta$ results (BESIII)



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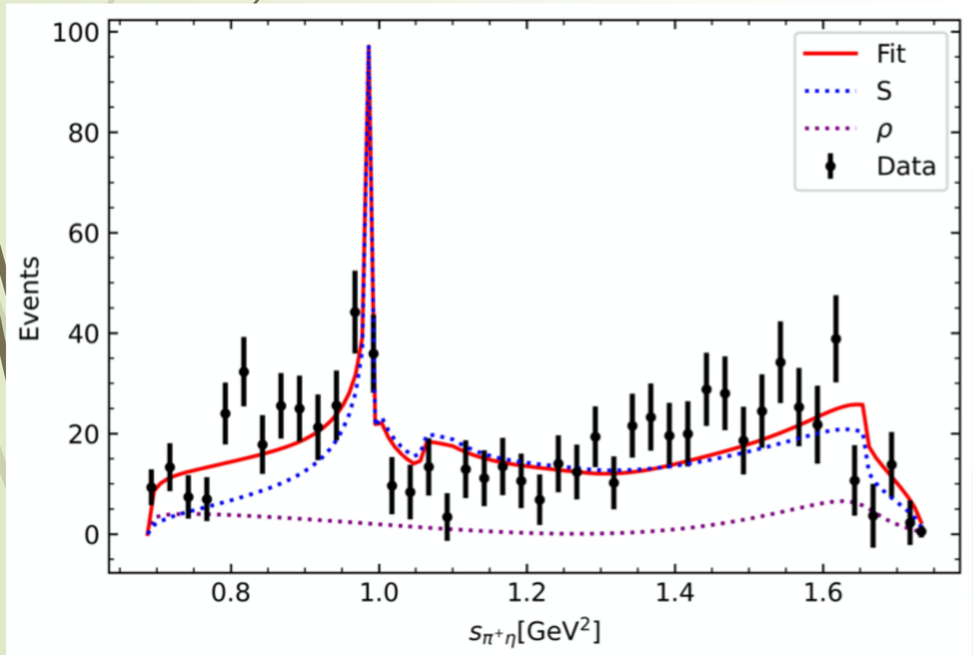

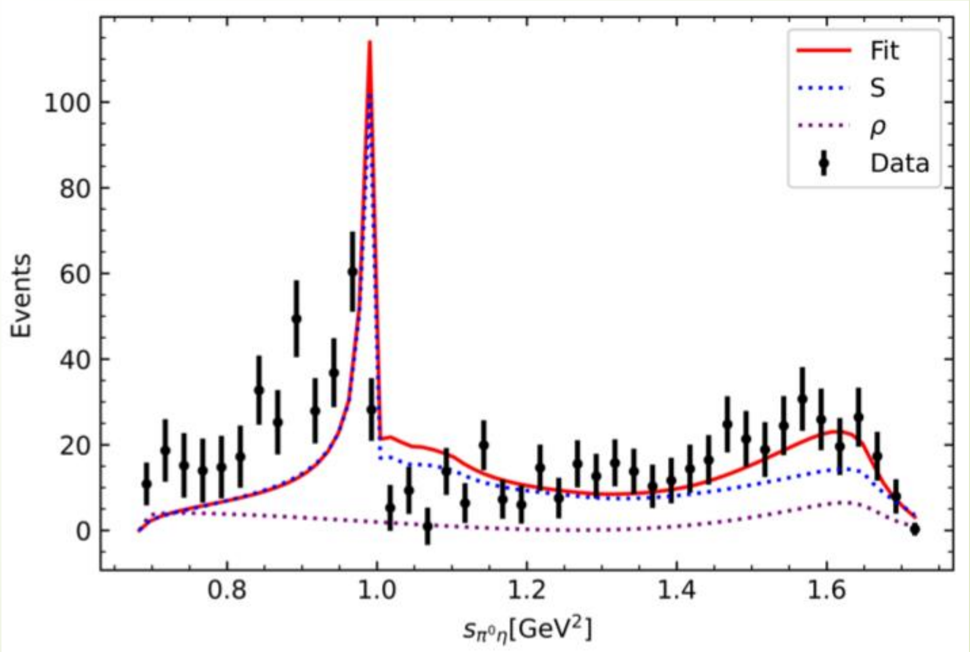

§4. Summary

- We use the chiral unitary approach to dynamically generate the state $a_0(980)$
- Taking the data from Belle, we fit the invariant mass distributions of the decays $\boldsymbol{D^0 \to K^+K^-\eta}$, $\boldsymbol{\pi^+\pi^-\eta}$
- Taking the data from BESIII, we fit the invariant mass distributions of the decays $\boldsymbol{D^0 \to \pi^+\pi^-\eta}$, $\boldsymbol{D^+ \to \pi^+\pi^0\eta}$
- Indicating the molecular nature of the $a_0(980)$

Thanks for your attention!

感谢大家的聆听！