粲介子半轻衰变中的同位旋标量介子

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Overview

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II: 重介子四体半轻衰变的初步探索

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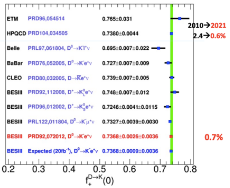
- ii: $D_s \to [\pi\pi]_{\rm S}\, e^+\nu_e$ 和 $f_0, [\pi\pi]_{\rm S}$ 结构初探

III: 总结和展望

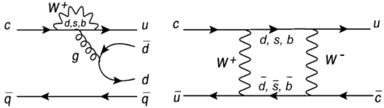
粲介子半轻衰变



- fundamental parameters, like the CKM matrix element $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]
  - the result measured via $D \to Kl\nu$ and $D_s \to \mu\nu_\mu$ consist with each other ($\sim 1.5\sigma$ derivation)

- new physical mechanism via the FCNC
  - anomalous measured in $B \to K^*\mu^+\mu^-$, 3.6σ derivation of $d\mathcal{B}/dq^2$ in $q^2 \in [1,6]$ GeV^2
  - a plausible effect in up-type FCNC process $c \to ull$ [Bharucha 2011.12856] SM $\mathcal{B}(D \to \pi l^+ l^-) \sim \mathcal{O}(10^{-9})$, current best-world limit $\mathcal{O}(10^{-8})$
  - LCSRs prediction $\mathcal{B}(D \to \pi\mu\mu) = 1.33^{+0.17}_{-0.24} \times 10^{-8}$, at the same order of LHCb limit 6.7×10^{-8} [A. Bansal, A. Khodjamirian and T. Mannel 2505.21369[hep-ph]],

  - first measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb 2412.09414] $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ
  - LD contribution from ϕ, $D_s \to \pi, \rho(\phi \to)e^+e^- \sim 10^{-5}$ [BESIII 2404.05973]



New physics hunter $D \to \pi\mu^+\mu^-$

- my talk in the " 超级陶粲装置研讨会" at LZU, July 8th, 2024

  - Experimentail potentials

Experiment	Measurement	Sensitivity
LHCb	Angular observables	$\sim 0.2\%$ with $50\,\mathrm{fb}^{-1}$, $\sim 0.08\%$ with $300\,\mathrm{fb}^{-1}$
LHCb	Branching ratio	$\sim 10^{-8}$ with $50\,\mathrm{fb}^{-1}$, $\sim 3\times10^{-9}$ with $300\,\mathrm{fb}^{-1}$
Belle-II	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar)

[talk at Towards the Ultimate Precision in Flavour Physics, Durham U.K. (2019)]

[BABAR Collaboration 1107.4465]

Run 4 $\sim$ 2030
Run 5 $\sim$ 2038

$N(D\bar{D}) \sim 10^9/\mathrm{ab}^{-1}$ angular observables $\sim 0.2\%$

  - BESIII Collaboration in the electron channel [BESIII Collaboration 1802.09752]
    $\mathcal{B}(D \to \pi^+\pi^-e^+e^-) < 0.7 \times 10^{-5}$ with $N(c\bar{c}) = 2 \times 10^7$ at 3.7 GeV

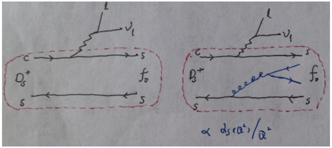
3.770	1	$D^0\bar{D}^0$	3.6	3.6×10^9	
		$D^+\bar{D}^-$	2.8	2.8×10^9	
		$D^0\bar{D}^0$		7.9×10^8	Single Tag
		$D^+\bar{D}^-$		5.5×10^8	Single Tag

STCF $N(D\bar{D}) \sim 8\times10^9$ Branching ratio $\sim 10^{-8}$

- STCF is still competitive in hunting the NP via $D \to \pi\mu^+\mu^-, \pi\pi\mu^+\mu^-$

粲介子半轻衰变 理解标量介子结构的干净平台

$$|f_0(980)\rangle, \quad |[\pi\pi]_S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$



- ⋆ in semileptonic $B_{(s)}$ decays
- ⋆ tetraquark contribution is suppressed doubly by strong coupling and power
- ⋆ FSI is weak too

- **how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?**

† the produced $q\bar{q}$ is nonrelativistic, likely proceeds through the creation of additional g or dynamical $q\bar{q}$

† hadron spectroscopy analysis provides evidence of the $q\bar{q}q\bar{q}$ states (compact tetraquarks or molecules)

† the cascade decay analyses of $D_s \to (f_0 \to \pi\pi)e\nu$ under $q\bar{q}$ ansatz consists with data, with $D_s \to f_0$ FFs and Flatté model of f_0 resonant

⋆ physical observables are usually written in a QCD convolution

$$\frac{d\sigma}{d\Omega} = \sum_t \int_0^1 dx_i\, \mathcal{H}^t(x_i, Q)\psi^t(x_i, \mu)$$

⋆ ψ^t is universal, however H^t is process dependent, hence different observables might highlike the contributions from different components

重介子四体半轻衰变的初步探索

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LCDAs 的共形展开

大动量转移过程用 LCDAs 来描述强子结构 [Braun, Korchemsky, Müller 2003]

- A convenient tool to study DAs is provided by conformal expansion
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD* *under conformal trans.* *VS* *rotation symmetry*
- *longitudinal* $\otimes$ *transversal* dofs *VS* *angular* $\otimes$ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group
  collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

‡ Three independent kinematic variables

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=0,\mathrm{even}}^{\infty} \sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=1,\mathrm{odd}}^{\infty} \sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass k^2** ?

‡ Watson theorem of π-π scattering amplitudes

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\, \mathrm{Exp}\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

2πDAs

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^π, a_2^ρ [**SC** 2019, 2023]

- All the above discussions are at leading twist
- twist three 2πDAs are studied recently [S. Cheng, arXiv:2502.07333]

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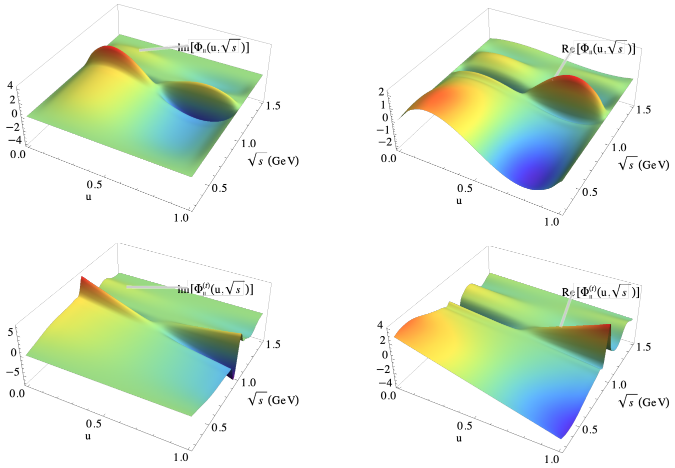
pion EMFF widely used in the three-body B decays studied from pQCD and QCDF are the asymptotic formula of 2πDAs

[J. Chai, **SC** and A.J Ma PRD 105 (2022) 033003]

normalized to unit as $\Gamma^{I=1}_{M_1M_2}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nl}(s,\mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s,\mu) \to a_n(\mu)\Gamma^{I=1}_{M_1M_2}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.

2πDAs



- Leading twist and twist-three 2πDAs for isospin scalar two-pion system
- asymmetry of the twist-three 2πDAs in the parton momentum fraction u

- symmetric twist-three LCDAs of f_0 obtained under the narrow-width approximation
- where QCD sum rules dictate that the asymmetric component vanishes

$D_s \to [f_0, \cdots \to] \, \pi\pi e^+ \nu_e$ 实验研究方案

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons
  - $D_s \to f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \to a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII '19]
  - $D_s \to f_0(\to \pi^0\pi^0, K_s K_s) e^+ \nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-) e^+ \nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $D_s \to f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \; D_s \to f_0 \text{ FF}$$

- improvement with the width effect by resonant models $D_s \to [f_0 \to] \, \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S \, l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))|^2}, \; \text{BESIII}$$

- calculate directly the signal channel $D_s \to [\pi\pi]_S \, e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S \, l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2 \, \beta_{\pi\pi}(k^2) \sqrt{\lambda_{D_s}} q^2}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, k^2)|^2, \; D_s \to \pi\pi \text{ FF}$$

$D_s \to f_0$ 形状因子和 $D_s \to (f_0 \to)\, \pi\pi e^+\nu$ 衰变

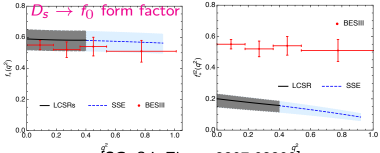
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\}\mathrm{GeV}^2$

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

- the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

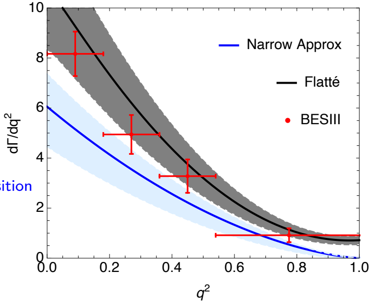
different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



[**SC**, S.L Zhang 2307.02309]

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0$ transition
  - the uncertainty estimation is conservative
  - without NLO correction
  - we need a model independent calculation
  - not only for the QCD understanding
  - but also for the future partial-wave measurement



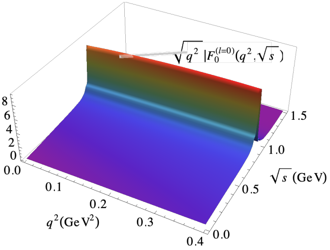
Differential decay width of $D_s^+ \to (f_0, [\pi\pi]_S)e^+\nu_e$

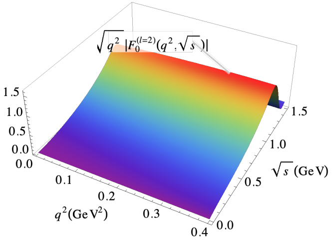
$D_s \to [\pi\pi]_S$ 形状因子和 $D_s \to [\pi\pi]_S\, e^+\nu$ 衰变

- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_S$ form factors ($\ell' = \mathrm{even}$ & $\ell' \leqslant n+1$)

$$\sqrt{q^2} F_0^{(\ell')}(q^2, k^2) = \frac{m_c(m_c + m_s)\sqrt{q^2}\sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1,\mathrm{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2, k^2, M^2, s_0)\, B_{n\ell,\parallel}^{l=0}(k^2)\, I_{\ell\ell'}$$

- Near threshold, B_{nl} can be determined from the low-energy effective theory of pions interacting with massive "constituent" quarks, based on the instanton model of the QCD vacuum
- For the resonant regions, Watson's theorem yields the k^2-dependence via Omnés solutions with the phase shifts mplemented through subtracted dispersion relations





- S- and D-wave FFs: $\sqrt{q^2}|F_0^{(l=0)}(\sqrt{s}, q^2)|$ and $\sqrt{q^2}|F_0^{(l=2)}(\sqrt{s}, q^2)|$

2πDAs

- Twist-2 and twist-3 contributions to the $D_s \to \pi\pi$ ($D_s \to f_0$) form factors at $q^2 = 0$

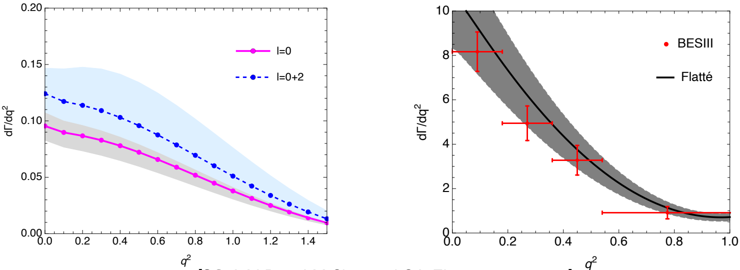
Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2}F_0^{(l=0)}(0) = \sqrt{q^2}F_t^{(l=0)}(0)$	$0.20^{+0.02}_{-0.02} - i0.24^{+0.02}_{-0.02}$	$-0.41^{+0.04}_{-0.05} + i0.51^{+0.02}_{-0.04}$	$-0.21^{+0.02}_{-0.01} + i0.27^{+0.03}_{-0.02}$
$\sqrt{q^2}F_0^{(l=2)}(0) = \sqrt{q^2}F_t^{(l=2)}(0)$	$0.27^{+0.03}_{-0.02} + i0.21^{+0.02}_{-0.01}$	$-0.55^{+0.02}_{-0.03} - i0.41^{+0.05}_{-0.04}$	$-0.28^{+0.02}_{-0.02} - i0.20^{+0.02}_{-0.01}$
$f_+(0) = f_0(0)$	$0.20^{+0.03}_{-0.05}$	$0.41^{+0.04}_{-0.06}$	$0.61^{+0.05}_{-0.07}$

- The widths (in unit of 10^{-4}) of $D_s \to [\pi\pi]_S\, e^+\nu$ obtained from $D_s \to [\pi\pi]$ and $D_s \to f_0$ transitions, under the $q\bar{q}$ component assumption.

$D_s \to [\pi\pi]_S\, e^+\nu_e$	$D_s \to [f_0 \to \pi\pi]\, e^+\nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

$D_s \to [\pi\pi]_S$ 形状因子和 $D_s \to [\pi\pi]_S\, e^+\nu$ 衰变

- The differential decay width on momentum transfers



[**SC**, L.Y Dai, J.M Shen and S.L Zhang, 2509.15659]

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- the FFs at the two-particle level are one-order lower than the required
- the conventional $q\bar{q}$ is not the dominate component in the charm decays
- we have to go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)
- much different in B decays leading twist dominated [**SC**, arXiv: 2502.07333]

总结和展望

- 2πDAs 提供了 $\pi\pi$ 质量谱的最一般描述

  † 准确厘清不同分波和同一个分波中不同共振态的贡献， † 有效分离、清晰描述强的非共振态背景的贡献，

  † 基于交换对称性来理解 SPD/GPD

- 2πDAs 是重介子四体半轻衰变 (H_{l4}) 研究的关键输入

  † 为 $|V_{ub}|$ 疑难和 FCNC 反常研究提供了新动能， † 为重味强子分布振幅的研究提供了并行检测

- 首次在三扭度水平研究了 2πDAs，实现了在 B_{l4} 过程抽取 $|V_{ub}|$
  [**SC**, 2502.07333]
- 理解重介子衰变中的同位旋标量介子 (对) 的部分子结构和多粒子图像
  [**SC**, L.Y Dai, J.M Shen and S.L Zhang 2509.15659]
- 在 $D \to \pi\pi l^+ l^-$ 衰变中深入理解 FCNC 过程可能的反常行为 wishlists
- 在 e^+e^- 湮灭 $\gamma^* \to \pi\pi\gamma, \gamma^*\gamma \to \pi\pi$ 过程中进一步理解 2πDAs

Thank you for your patience.