Matching from quark to hadronic operators: external source vs spurion methods

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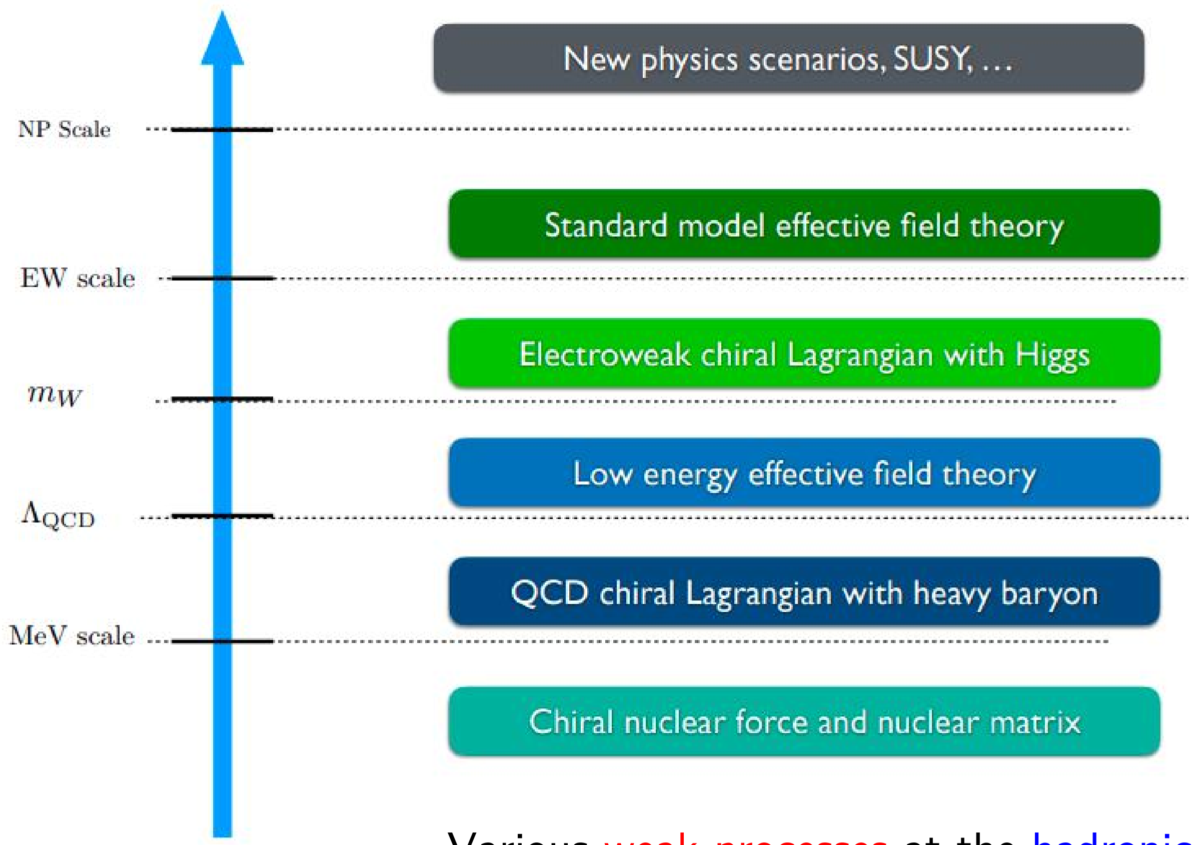
Effective Field Theory

Bottom-up approach to new physics:



credit: Jiang-Hao Yu

Various weak processes at the hadronic scale have been utilized to search for new physics at high energy scale

Effective Field Theory

Three tasks: operator, matching, RGE

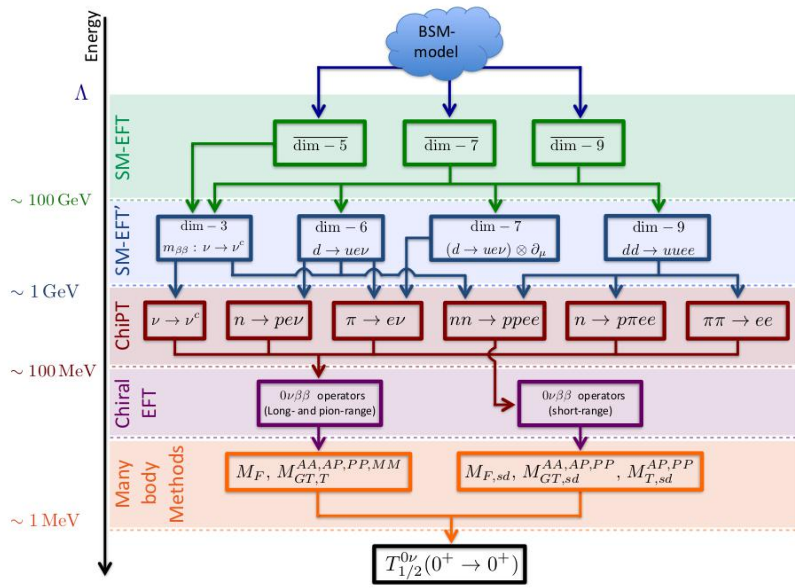
SMEFT
Scale: $\gtrsim$ 100 GeV
Operators: ✓
RGE: ✓

$\downarrow$ Matching: ✓

LEFT
Scale: $\sim$ 1-100 GeV
Operators: ✓
RGE: ✓

$\downarrow$ Matching: Y/N

ChPT
Scale: $\lesssim$ 1 GeV
Operators: Y/N
RGE: -



Y for SM interactions
N for BSM interactions

How to systematically match LEFT to ChPT?

Outline

- Basics of external source method
- Conventional spurion method
- Systematic spurion method

External source method

Quark-level interactions:

$$\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \mathcal{L}_{\rm ext}$$

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465–516

$$\mathcal{L}_{\rm ext} = \bar{q}_L \gamma^\mu \ell_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R - [\bar{q}_L (s - ip) q_R + \text{ h.c. }]$$

under chiral symmetry $SU(2)_L \times SU(2)_R$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_L \to L q_L, \quad q_R \to R q_R$$

guiding rules of constructing chiral Lagrangian:

① **Real (Hermiticity)** ② **Flavor neutral (Trace)** ③ **Scalar (Parity even)**

④ **Chiral transformation** ⑤ **Proper Lorentz transformations**

⑥ **Charge conjugation C** ⑦ **Parity P** ⑧ **Time reversal T**

credit: De-Liang Yao

External source method

Matching from quark to hadronic operators:

under chiral symmetry:

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465–516

$$U(x) = \exp\left(i\frac{\vec{\phi}\cdot\vec{\tau}}{F_0}\right) \qquad U \to RUL^\dagger$$

covariant derivate:

$$D_\mu U = \partial_\mu U - ir_\mu U + iU\ell_\mu$$

local chiral symmetry

ingredients:

$$\begin{aligned} \chi &= 2B(s+ip) \\ F_L^{\mu\nu} &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i\left[\ell^\mu, \ell^\nu\right] \\ F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i\left[r^\mu, r^\nu\right] \end{aligned}$$

Building blocks:

$$(U, U^\dagger, \chi, \chi^\dagger, F_L^{\mu\nu}, F_R^{\mu\nu}, D_\mu)$$

U-parameterization

External source method

Matching from quark to hadronic operators:

p^2 order:

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465–516

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] + \frac{F_0^2}{4}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)$$

p^4 order:

$$\begin{aligned}
\mathcal{L}_4 =& \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger\right]\mathrm{Tr}\left[D^\mu U (D^\nu U)^\dagger\right] \\
&+ \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U (D^\mu \chi)^\dagger + D_\mu \chi (D^\mu U)^\dagger\right] \\
&+ l_5\left[\mathrm{Tr}\left(F_{R\mu\nu} U F_L^{\mu\nu} U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(F_{L\mu\nu}F_L^{\mu\nu} + F_{R\mu\nu}F_R^{\mu\nu}\right)\right] \\
&+ i\frac{l_6}{2}\mathrm{Tr}\left[F_{R\mu\nu} D^\mu U (D^\nu U)^\dagger + F_{L\mu\nu}(D^\mu U)^\dagger D^\nu U\right] \\
&- \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1 + h_3}{16}\mathrm{Tr}\left(U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger\right) \\
&- \frac{h_1 - h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} \\
&- 2h_2\,\mathrm{Tr}\left(F_{L\mu\nu}F_L^{\mu\nu} + F_{R\mu\nu}F_R^{\mu\nu}\right)
\end{aligned}$$

LR basis

External source method

Matching from quark to hadronic operators:

$$u(x) = \sqrt{U} = \exp\left(i\frac{\vec{\phi}\cdot\vec{\tau}}{2F_0}\right)$$

Ecker, Gasser, Pich, de Rafael, Nucl.Phys.B 321 (1989) 311

under chiral symmetry $SU(2)_L \times SU(2)_R$

$$u \to RuK^\dagger = KuL^\dagger, \quad u^\dagger \to Lu^\dagger K^\dagger = Ku^\dagger R^\dagger$$

ingredients:

$$\begin{aligned}
\chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \\
u_\mu &= i\left[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right] = iu^\dagger D_\mu U u^\dagger \\
f_\pm^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u,
\end{aligned}$$

$$X \to KXK^\dagger, \quad K \in SU(2)_V \qquad X = \chi_\pm, u^\mu, f_\pm^{\mu\nu}$$

External source method

Matching from quark to hadronic operators:

covariant derivate:

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X] \qquad\qquad \Gamma_\mu \equiv \frac{1}{2}\left[u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right]$$

Building blocks:

$$\left(u^\mu, \chi_\pm, f_\pm^{\mu\nu}, \nabla^\mu\right)$$

u-parameterization

p^2 order:

$$\mathcal{L}_2' = \frac{F_0^2}{4}\langle u^\mu u_\mu + \chi_+\rangle$$

p^4 order:

$$\begin{aligned}
\mathcal{L}_4' = {} & L_1 \langle u_\mu u^\mu\rangle \langle u_\nu u^\nu\rangle + L_2 \langle u_\mu u_\nu\rangle \langle u^\mu u^\nu\rangle + L_3 \langle \chi_+\rangle \langle u^\mu u_\mu\rangle \\
& + L_4 \left\langle f_+^{\mu\nu} u_\mu u_\nu\right\rangle + L_5 \left\langle f_+^{\mu\nu} f_{+\mu\nu}\right\rangle + L_6 \left\langle f_+^{\mu\nu} f_{+\mu\nu}\right\rangle \\
& + L_7 \langle \chi_+ \chi_+\rangle + L_8 \langle \chi_- \chi_-\rangle + L_9 \langle \chi_+\rangle \langle \chi_+\rangle + L_{10} \langle \chi_-\rangle \langle \chi_-\rangle
\end{aligned}$$

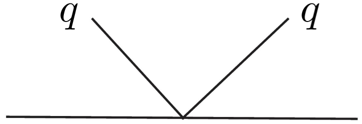
u basis

New physics scenarios

External source method provides convenient frameworks

- Tensor current

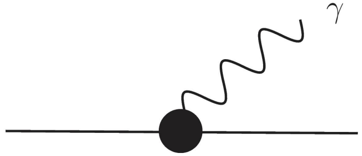
$$\mathcal{L}_{\rm ext} \supset \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu}q = \bar{q}_L\sigma_{\mu\nu}t^{\mu\nu\dagger}q_R + \bar{q}_R\sigma_{\mu\nu}t^{\mu\nu}q_L$$



$$t_+^{\mu\nu} = u^\dagger t^{\mu\nu}u^\dagger + ut^{\mu\nu\dagger}u$$

Cata, Mateu, JHEP 09 (2007) 078

chiral Lagrangian:



$$\mathcal{L}_4 \supset \Lambda_1\,\mathrm{Tr}\left(t_+^{\mu\nu}f_{+\mu\nu}\right)$$

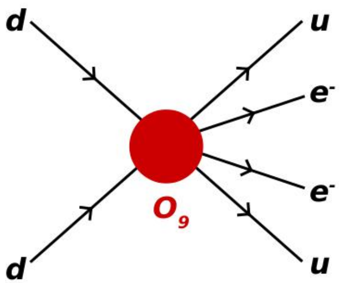
applications: $\mu \to e\gamma$, DM-nucleon, lepton EDMs

Dekens, Jenkins, Manohar, Stoffer, 1810.05675 (JHEP)
J.-H. Liang, Y. Liao, X.-D. Ma, H.-L. Wang, 2401.05005 (CPC)
Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer, 2102.08954 (JHEP)

New physics scenarios

External source method does not apply to more quark bilinears

- $0\nu\beta\beta$ decay



$\bar{u}\Gamma_1 d\ \bar{u}\Gamma_2 d\ \bar{e}\Gamma_3 e^c$

$$
\begin{aligned}
O_1^{(9)} &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha\ \bar{q}_L^\beta \gamma^\mu \tau^+ q_L^\beta\ , & O_1^{(9)\prime} &= \bar{q}_R^\alpha \gamma_\mu \tau^+ q_R^\alpha\ \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta\ ,\\
O_2^{(9)} &= \bar{q}_R^\alpha \tau^+ q_L^\alpha\ \bar{q}_R^\beta \tau^+ q_L^\beta\ , & O_2^{(9)\prime} &= \bar{q}_L^\alpha \tau^+ q_R^\alpha\ \bar{q}_L^\beta \tau^+ q_R^\beta\ ,\\
O_3^{(9)} &= \bar{q}_R^\alpha \tau^+ q_L^\beta\ \bar{q}_R^\beta \tau^+ q_L^\alpha\ , & O_3^{(9)\prime} &= \bar{q}_L^\alpha \tau^+ q_R^\beta\ \bar{q}_L^\beta \tau^+ q_R^\alpha\ ,\\
O_4^{(9)} &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha\ \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta\ , & &\\
O_5^{(9)} &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\beta\ \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\alpha\ , & &\\
O_6^{\mu(9)} &= \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_L \tau^+ q_R\right)\ , & O_6^{\mu(9)\prime} &= \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_R \tau^+ q_L\right)\ ,\\
O_7^{\mu(9)} &= \left(\bar{q}_L T^A \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_L T^A \tau^+ q_R\right)\ , & O_7^{\mu(9)\prime} &= \left(\bar{q}_R T^A \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_R T^A \tau^+ q_L\right)\ ,\\
O_8^{\mu(9)} &= \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_R \tau^+ q_L\right)\ , & O_8^{\mu(9)\prime} &= \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_L \tau^+ q_R\right)\ ,\\
O_9^{\mu(9)} &= \left(\bar{q}_L T^A \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_R T^A \tau^+ q_L\right)\ , & O_9^{\mu(9)\prime} &= \left(\bar{q}_R T^A \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_L T^A \tau^+ q_R\right)\ ,
\end{aligned}
$$

How to match them to chiral Lagrangian?

- spurion method

Outline

- Basics of external source method
- Conventional spurion method
- Systematic spurion method

Conventional spurion method

Spurion field:

- External sources can be regarded as spurions

$$(v_\mu - a_\mu)' = L(v_\mu - a_\mu + i\partial_\mu)L^\dagger$$
$$(v_\mu + a_\mu)' = R(v_\mu + a_\mu + i\partial_\mu)R^\dagger$$
$$(s - ip)' = L(s - ip)R^\dagger$$

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142

- Non-leptonic weak decays

Manohar, Georgi, Nucl.Phys.B 234 (1984) 189-212; Donoghue, Grinstein, Rey, Wise Phys.Rev.D 33 (1986) 1495

$$T^{bd}_{ac}\,(\bar{q}^a_L\gamma^\mu q_{Lb})(\bar{q}^c_L\gamma_\mu q_{Ld})$$

$$T^{ab}_{cd} \to T^{\alpha\beta}_{\rho\sigma} L^\rho_c L^\sigma_d L^{\dagger a}_\alpha L^{\dagger b}_\beta$$

a, b, c, d are $SU(3)$ or $SU(2)$ flavor indices

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix} \qquad P_{L/R} \equiv (1 \mp \gamma^5)/2$$

Conventional spurion method

Map each quark field to u field:

$$\begin{cases} q_L \to L q_L, \quad q_R \to R q_R \\ u \to R u K^\dagger = K u L^\dagger, \quad u^\dagger \to L u^\dagger K^\dagger = K u^\dagger R^\dagger \end{cases}$$

chiral basis

LO: $q_L \to u^\dagger, \quad q_R \to u, \quad \bar{q}_L \to u, \quad \bar{q}_R \to u^\dagger$

NLO: $q_L \to (D_\mu u)^\dagger, \quad q_R \to \left(D_\mu u^\dagger\right)^\dagger, \quad \bar{q}_L \to D_\mu u, \quad \bar{q}_R \to D_\mu u^\dagger$

$$D_\mu = \partial_\mu - i\mathcal{V}_\mu, \quad \mathcal{V}_\mu = \frac{i}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)$$

global chiral symmetry

M. L. Graesser, 1606.04549 (JHEP)
Y. Liao, X.-D. Ma, H.-L. Wang, 1909.06272 (JHEP)

Conventional spurion method

Construct the chiral Lagrangian:

- Map each quark field to u field in the chiral basis
- Spurions remain invariant (different contractions)
- Check the CP transformation properties
- Eliminate the redundancies using EOM, IBP, and identities

$$(D_\mu u)u^\dagger = -u(D_\mu u)^\dagger, \qquad u^\dagger (D_\mu u) = -(D_\mu u)^\dagger u$$
$$\left(uD_\mu u^\dagger\right)^\dagger = -uD_\mu u^\dagger, \qquad \left(u^\dagger D_\mu u\right)^\dagger = -u^\dagger D_\mu u$$

- Convert from the u-parameterization to U-parameterization

$$uD_\mu u^\dagger = U\partial_\mu U^\dagger/2, \qquad u^\dagger D_\mu u = U^\dagger \partial_\mu U/2$$
$$u^\dagger D_\mu u^\dagger = \partial_\mu U^\dagger/2, \qquad uD_\mu u = \partial_\mu U/2$$

Conventional spurion method

Single quark bilinear:

- Scalar/pseudo-scalar interactions

$$\mathcal{L}^q_{S,P} = \bar{q}_L \lambda^\dagger q_R + \bar{q}_R \lambda q_L$$

Matching at p^2 order:

$$\begin{aligned}\mathcal{L}^{(0)}_{S,P} &= \mathbf{Tr}\left(u\lambda^\dagger u + u^\dagger \lambda u^\dagger\right) g_S^{(0)} \\ &= \mathbf{Tr}\left(\lambda^\dagger U + \lambda U^\dagger\right) g_S^{(0)}\end{aligned}$$

Compare with the result in the external source method

$$\lambda = -(s + ip) \qquad\qquad g_S^{(0)} = -\frac{F_0^2}{2} B$$

Similar for other interactions

Conventional spurion method

Two quark bilinears:

- $0\nu\beta\beta$ decay

$$O_1^{(9)} = \left(\bar{q}_L \gamma^\mu \tau^+ q_L\right)\left(\bar{q}_L \gamma_\mu \tau^+ q_L\right)$$

$$= T_{ac}^{bd}\left(\bar{q}_L^a \gamma^\mu q_{Lb}\right)\left(\bar{q}_L^c \gamma_\mu q_{Ld}\right) \qquad T_{ac}^{bd} \equiv \left(\tau^+\right)_a^b \left(\tau^+\right)_d^c$$

Matching at p^0 order:

$$\to T_{ac}^{bd} u_b^{\dagger i} u_d^{\dagger j} u_k^a u_l^c$$ (mapping from quark to hadronic fields)

leads to

$$\begin{cases} \mathrm{Tr}\left(u\tau^+ u^\dagger u \tau^+ u^\dagger\right) = 0 \\ \mathrm{Tr}\left(u\tau^+ u^\dagger\right)\mathrm{Tr}\left(u\tau^+ u^\dagger\right) = 0 \end{cases}$$

M. L. Graesser, 1606.04549 (JHEP)

Conventional spurion method

Two quark bilinears:

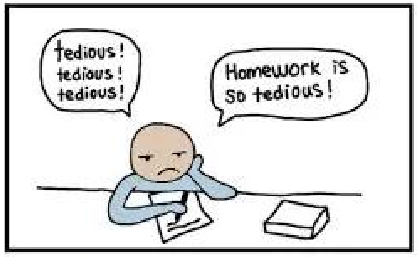
- $0\nu\beta\beta$ decay

$$O_1^{(9)} = (\bar{q}_L\gamma^\mu\tau^+ q_L)\,(\bar{q}_L\gamma_\mu\tau^+ q_L)$$

$$= T^{bd}_{ac}\,(\bar{q}^a_L\gamma^\mu q_{Lb})\,(\bar{q}^c_L\gamma_\mu q_{Ld}) \qquad\qquad T^{bd}_{ac} \equiv (\tau^+)^{\,b}_{\,a}(\tau^+)^{\,c}_{\,d}$$

Matching at p^2 order:

$$\begin{aligned}
&\mathrm{Tr}\left(D^\mu u\tau^+ D_\mu u^\dagger u\tau^+ u^\dagger\right)\\
&\mathrm{Tr}\left(D^\mu u\tau^+ u^\dagger D_\mu u\tau^+ u^\dagger\right)\\
&\mathrm{Tr}\left(u\tau^+ D^\mu u^\dagger u\tau^+ D_\mu u^\dagger\right)\\
&\mathrm{Tr}\left(D^\mu u\tau^+ D_\mu u^\dagger\right)\mathrm{Tr}\left(u\tau^+ u^\dagger\right)\\
&\mathrm{Tr}\left(D^\mu u\tau^+ u^\dagger\right)\mathrm{Tr}\left(D_\mu u\tau^+ u^\dagger\right)\\
&\mathrm{Tr}\left(u\tau^+ D^\mu u^\dagger\right)\mathrm{Tr}\left(u\tau^+ D_\mu u^\dagger\right)
\end{aligned}$$



Tedious (adj.) boring

M. L. Graesser, 1606.04549 (JHEP)

Conventional spurion method

Two quark bilinears:

- $0\nu\beta\beta$ decay

Introduce the building block:

$$X_{\mathrm{L}}^{+} = u\tau^{+}u^{\dagger} \qquad X_{\mathrm{L}}^{+} \rightarrow KuL^{\dagger}\tau^{+}Lu^{\dagger}K^{\dagger} \qquad \text{(chiral trans.)}$$

Matching:

$$\mathbf{Tr}\left(X_{\mathrm{L}}^{+}X_{\mathrm{L}}^{+}\right) = 0, \quad \mathbf{Tr}\left(X_{\mathrm{L}}^{+}\right)\mathbf{Tr}\left(X_{\mathrm{L}}^{+}\right) = 0 \qquad p^0 \text{ order}$$

$$\mathbf{Tr}\left(D^{\mu}X_{\mathrm{L}}^{+}D_{\mu}X_{\mathrm{L}}^{+}\right), \quad \mathbf{Tr}\left(X_{\mathrm{L}}^{+}D^{\mu}D_{\mu}X_{\mathrm{L}}^{+}\right) \qquad p^2 \text{ order}$$

Prezeau, Ramsey-Musolf, Vogel, PRD 68 (2003) 034016

In both ways, after some tedious work:

$$O_1^{(9)} \rightarrow \mathbf{Tr}\left(U^{\dagger}\partial^{\mu}U\tau^{+}U^{\dagger}\partial_{\mu}U\tau^{+}\right)$$

V. Cirigliano, et al., 1806.02780 (JHEP)

Outline

- Basics of external source method
- Conventional spurion method
- Systematic spurion method

q basis

Single quark bilinear:

- Scalar/pseudo-scalar interactions

$$\mathcal{L}^q_{S,P} = \bar{q}\lambda_S q + i\bar{q}\gamma^5\lambda_P q$$

$$\bar{q}\lambda_S q = \bar{q}_L\lambda_S q_R + \bar{q}_R\lambda_S^\dagger q_L \qquad \lambda_{S/P} \to L\lambda_{S/P}R^\dagger$$

$$\bar{q}\gamma^5\lambda_P q = \bar{q}_L\lambda_P q_R + \bar{q}_R\lambda_P^\dagger q_L \qquad \lambda_{S/P}^\dagger \to R\lambda_{S/P}^\dagger L^\dagger$$

Matching in the q/u bases:

$$\bar{q}\lambda_S q \to \mathrm{Tr}\left[u\lambda_S u + u^\dagger\lambda_S^\dagger u^\dagger\right]$$

$$\bar{q}\gamma^5 i\lambda_P q \to \mathrm{Tr}\left[ui\lambda_P u + u^\dagger(i\lambda_P)^\dagger u^\dagger\right]$$

This is similar to:

$$\mathscr{L}_{\text{s.b.}} = \frac{F_0^2 B}{2}\mathrm{Tr}(m_q U^\dagger + U m_q^\dagger)$$

We can get the same result by indentifying

$$\lambda_S = \left(\lambda^\dagger + \lambda\right)/2, \quad i\lambda_P = \left(\lambda^\dagger - \lambda\right)/2$$

q basis

Single quark bilinear:

- Similar for vector/axial-vector and tensor interactions

$$\mathcal{L}^q_{V,A} = \bar{q}\gamma_\mu\lambda^\mu_V q + \bar{q}\gamma_\mu\gamma^5\lambda^\mu_A q$$

$$\mathcal{L}^q_T = \bar{q}\sigma_{\mu\nu}\lambda^{\mu\nu}_T q + i\bar{q}\sigma_{\mu\nu}\gamma^5\lambda^{\mu\nu}_\varepsilon q$$

LEFT	ChPT
chiral basis	LR basis
q basis	u basis

- Four types of bilinears:

$$(\bar{q}_L\Gamma\Sigma_L q_L), \quad (\bar{q}_R\Gamma\Sigma_R q_R), \quad (\bar{q}_R\Gamma\Sigma q_L), \quad (\bar{q}_L\Gamma\Sigma^\dagger q_R)$$

even for derivative interaction

$$\mathcal{L}^q_{D,D5} = \bar{q}\lambda_D \overleftrightarrow{\partial}_\mu q + \bar{q}\gamma^5\lambda_{D5}\overleftrightarrow{\partial}_\mu q$$

Systematic spurion method

- Building blocks:

$$\Sigma_\pm = u\Sigma^\dagger u \pm u^\dagger \Sigma u^\dagger$$
$$Q_\pm = u^\dagger \Sigma_R u \pm u \Sigma_L u^\dagger$$

- Young tensor technique has been used to construct complete and independent basis of chiral Lagrangian for QCD at p^8 order in the u basis using the building blocks $(u^\mu, \chi_\pm, f_\pm^{\mu\nu}, \nabla^\mu)$

X.-H. Li, H. Sun, F.-J. Tang, J.-H. Yu, 2404.14152 (JHEP)

- Marriage spurion with Young tensor technique:

interactions in CP eigenstates

Y.T. $\longrightarrow$ matching

$$P: X \xrightarrow{P} \eta_X X,$$
$$C: \quad X \xrightarrow{C} (-1)^X X^T$$

$$X = \Sigma_\pm, Q_\pm, \hat{u}^\mu$$

q basis

u basis

Systematic spurion method

Single quark bilinear:

LEFT operator	χPT operators	
	$\mathcal{O}(p^2)$	$\mathcal{O}(p^4)$
$\mathcal{O}_5^{(6)} = (\bar{e}\gamma^\mu e)[\bar{q}\gamma_\mu(\Sigma_R P_R + \Sigma_L P_L)q]$	$(\bar{e}\gamma^\mu e)\langle Q_- \hat{u}_\mu\rangle$	$(\bar{e}\gamma^\mu e)\langle Q_- \hat{u}_\mu\rangle\langle \hat{u}^\nu \hat{u}_\nu\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_- \hat{u}_\nu\rangle\langle \hat{u}^\mu \hat{u}^\nu\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_- \hat{u}_\mu\rangle\langle \hat{\chi}_+\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_+ [\hat{u}_\mu, \hat{\chi}_-]\rangle$
$\mathcal{O}_7^{(6)} = (\bar{e}\gamma^\mu e)[\bar{q}\gamma_\mu(\Sigma_R P_R - \Sigma_L P_L)q]$	$(\bar{e}\gamma^\mu e)\langle Q_+ \hat{u}_\mu\rangle$	$(\bar{e}\gamma^\mu e)\langle Q_+ \hat{u}_\mu\rangle\langle \hat{u}^\nu \hat{u}_\nu\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_+ \hat{u}_\nu\rangle\langle \hat{u}^\mu \hat{u}^\nu\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_+ \hat{u}_\mu\rangle\langle \hat{\chi}_+\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_- [\hat{u}_\mu, \hat{\chi}_-]\rangle$
$\mathcal{O}_6^{(6)} = (\bar{e}\gamma^\mu\gamma^5 e)[\bar{q}\gamma_\mu(\Sigma_R P_R + \Sigma_L P_L)q]$	$(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_- \hat{u}_\mu\rangle$	$(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_- \hat{u}_\mu\rangle\langle \hat{u}^\nu \hat{u}_\nu\rangle$ $(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_- \hat{u}_\nu\rangle\langle \hat{u}^\mu \hat{u}^\nu\rangle$ $(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_- \hat{u}_\mu\rangle\langle \hat{\chi}_+\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_+ [\hat{u}_\mu, \hat{\chi}_-]\rangle$
$\mathcal{O}_8^{(6)} = (\bar{e}\gamma^\mu\gamma^5 e)[\bar{q}\gamma_\mu(\Sigma_R P_R - \Sigma_L P_L)q]$	$(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_+ \hat{u}_\mu\rangle$	$(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_+ \hat{u}_\mu\rangle\langle \hat{u}^\nu \hat{u}_\nu\rangle$ $(\bar{e}\gamma^\mu\gamma e)\langle Q_+ \hat{u}_\nu\rangle\langle \hat{u}^\mu \hat{u}^\nu\rangle$ $(\bar{e}\gamma^\mu\gamma^5 e)\langle Q_+ \hat{u}_\mu\rangle\langle \hat{\chi}_+\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_- [\hat{u}_\mu, \hat{\chi}_-]\rangle$

Systematic spurion method

Two quark bilinears:

- $0\nu\beta\beta$ decay

$$
\begin{aligned}
O_1^{(9)} &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \, \bar{q}_L^\beta \gamma^\mu \tau^+ q_L^\beta \,, & O_1^{(9)\prime} &= \bar{q}_R^\alpha \gamma_\mu \tau^+ q_R^\alpha \, \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta \,, \\
O_2^{(9)} &= \bar{q}_R^\alpha \tau^+ q_L^\alpha \, \bar{q}_R^\beta \tau^+ q_L^\beta \,, & O_2^{(9)\prime} &= \bar{q}_L^\alpha \tau^+ q_R^\alpha \, \bar{q}_L^\beta \tau^+ q_R^\beta \,, \\
O_3^{(9)} &= \bar{q}_R^\alpha \tau^+ q_L^\beta \, \bar{q}_R^\beta \tau^+ q_L^\alpha \,, & O_3^{(9)\prime} &= \bar{q}_L^\alpha \tau^+ q_R^\beta \, \bar{q}_L^\beta \tau^+ q_R^\alpha \,, \\
O_4^{(9)} &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \, \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta \,, & & \\
O_5^{(9)} &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\beta \, \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\alpha \,, & & \\
O_6^{\mu(9)} &= \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right) \left(\bar{q}_L \tau^+ q_R\right) \,, & O_6^{\mu(9)\prime} &= \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right) \left(\bar{q}_R \tau^+ q_L\right) \,, \\
O_7^{\mu(9)} &= \left(\bar{q}_L T^A \tau^+ \gamma^\mu q_L\right) \left(\bar{q}_L T^A \tau^+ q_R\right) \,, & O_7^{\mu(9)\prime} &= \left(\bar{q}_R T^A \tau^+ \gamma^\mu q_R\right) \left(\bar{q}_R T^A \tau^+ q_L\right) \,, \\
O_8^{\mu(9)} &= \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right) \left(\bar{q}_R \tau^+ q_L\right) \,, & O_8^{\mu(9)\prime} &= \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right) \left(\bar{q}_L \tau^+ q_R\right) \,, \\
O_9^{\mu(9)} &= \left(\bar{q}_L T^A \tau^+ \gamma^\mu q_L\right) \left(\bar{q}_R T^A \tau^+ q_L\right) \,, & O_9^{\mu(9)\prime} &= \left(\bar{q}_R T^A \tau^+ \gamma^\mu q_R\right) \left(\bar{q}_L T^A \tau^+ q_R\right) \,,
\end{aligned}
$$

basis trans.

$$O_{XY} = \left(\bar{q} X \tau^+ q\right) \left(\bar{q} Y \tau^+ q\right), \quad X, Y = V, A, S, P$$

$$V = \gamma^\mu, A = \gamma^\mu \gamma^5, S = 1, P = \gamma^5$$

Systematic spurion method

Two quark bilinears:

- $0\nu\beta\beta$ decay

$$O_1^{(9)} = O_{VV} + O_{AA} - 2\,O_{VA}$$

CP property	four-quark operator	p^0	p^2
$C+P+$	O_{VV}	$\langle Q_- Q_- \rangle$	$\langle Q_- \hat{u}^\mu Q_- \hat{u}_\mu \rangle$
$C+P+$	O_{AA}	$\langle Q_+ Q_+ \rangle$	$\langle Q_+ \hat{u}^\mu Q_+ \hat{u}_\mu \rangle$
$C-P-$	O_{VA}	$\langle Q_- Q_+ \rangle$	$\langle Q_- \hat{u}^\mu Q_+ \hat{u}_\mu \rangle$

$$\hat{u}_\mu = i\left(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right)$$

- No need to introduce external sources and new spurions
- A simple algebraic calculation yields

p^0 order: $\quad O_1^{(9)} \to 0$

p^2 order: $\quad O_1^{(9)} \to -4\left\langle U^\dagger \partial^\mu U \tau^+ U^\dagger \partial_\mu U \tau^+ \right\rangle$

Summary

- We propose a systematic spurion method for the matching of LEFT to ChPT, which is particularly useful for LEFT at higher dimensions and for ChPT at higher orders of p.
- Key achievements:
  - identify a minimal set of building blocks and establish a one-to-one correspondence between LEFT and chiral operators.
  - avoids the irreducible representation decomposition, and the introduction of external sources and new spurions
  - applies to single quark bilinear, and two quark bilinears
- Outlook:
  - three quark bilinears (neutron-antineutron oscillation)
  - odd number of quark fields (baryon number violation)

Thank you

Upcoming νSTEP 2025

Neutrino Scattering: Theory, Experiment, Phenomenology (νSTEP 2025)

24-27 October 2025
Asia/Shanghai timezone

- Overview
- Timetable
- Contribution List
- Registration
- Participant List

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中国科学院理论物理研究所和中山大学将共同主办第二届中微子散射：理论、实验、唯象研讨会(νSTEP 2025)，会议将于2025年10月24-27日在北京举行，诚邀您注册参会！

随着中微子物理精确测量时代的到来，JUNO实验将在近期取数及中微子相干散射实验的开展，需要进一步降低中微子与物质相互作用的理论不确定性，在这些实验中寻找新物理信号。中微子散射是研究中微子相互作用的重要过程，涉及从原子核物理、强子物理到电弱能标物理的各个层次的理论描述，并与天文学和宇宙学密切相关。

本次会议将聚焦中微子相互作用的理论和实验最新进展以及相关核物理、强子物理、天文学和宇宙学的研究，以促进更多的交流与合作。会议主题包括：中微子理论和实验进展，中微子-原子核散射，中微子-强子散射，强子物理和核物理的弱流相互作用以及相关的中微子散射过程，对撞机与高能中微子探测中的深度非弹性散射过程，无中微子双贝塔衰变，中微子相关新物理，中微子天文学和宇宙学等。

This conference will focus on the latest theoretical and experimental developments in neutrino interactions, as well as related research in nuclear physics, hadronic physics, astronomy, and cosmology, with the aim of fostering greater exchange and collaboration. The topics include: advances in neutrino theory and experiments, neutrino–nucleus scattering, neutrino–hadron scattering, weak interactions in hadronic and nuclear physics and the associated neutrino scattering processes, deep inelastic scattering in colliders and high-energy neutrino detection, neutrinoless double beta decay, new physics related to neutrinos, and neutrino astronomy and cosmology.

如果您有意愿就相关课题做报告，请在注册时提供报告标题。

会议组委会：于江浩、唐健、葛韶峰、李玉峰、Sasha Tomalak、宋宁强、李刚

https://indico.itp.ac.cn/event/358/