Probing the nature of the anticharmedstrange pentaquark states

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Outline

- ➤ Backgroud and motivation
- ➤ Quark Model and Calculation Method

- Results of the Anticharmed-Strange Pentaquark Candidates
- > Summary

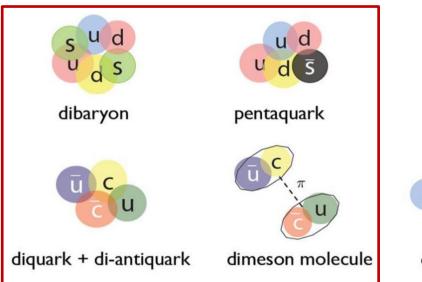
Background and motivation

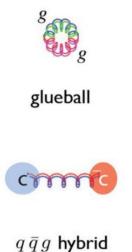
> Conventional hadron

- \checkmark Mesons($\overline{q}q$)
- \checkmark Baryons(qqq)

- **9**
- gg

- **Exotic hadron states**
 - ✓ Multiquarks(qqqq....)
 - √ Hybrids(qqg)
 - ✓ Glueballs(gg,ggg....)



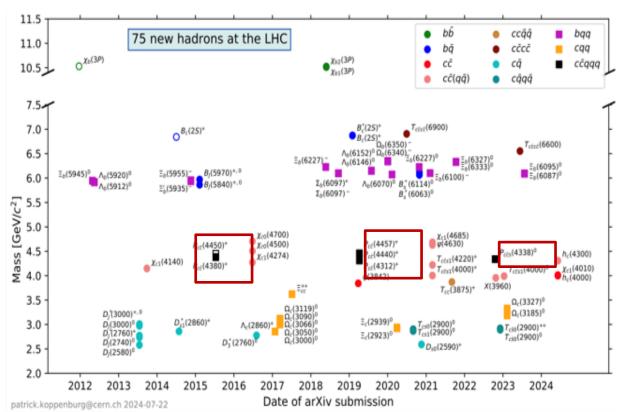


Searching for exotic hadron states remains an important ongoing research area in hadron physics!

Background and motivation

> Experimental breakthroughs

• X(3872) was first observed by Belle Collabortaion [Belle] PRL91,262001 (2003) ----the study of multiquark states has entered upon a new era

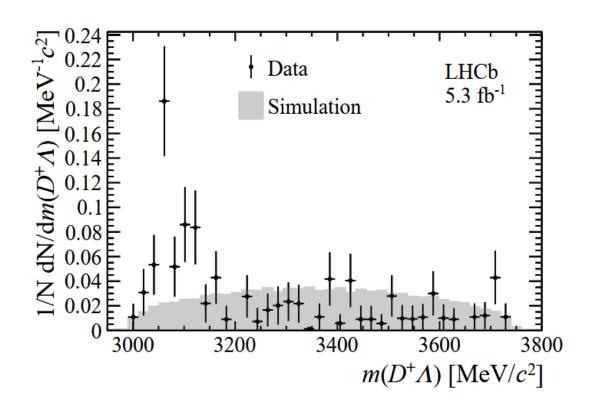


- Since 2015, a series of hidden-charm pentaquark states (P_c, P_{cs}) have been observed by the LHCb Collabortaion
- Theoretical interpretations:
 - ✓ molecular states: H. X. Huang et al, Phys. Rev. D 99 (2019) 1, 014010. H. X. Chen et al, Phys.Rev.D 100 (2019) 5, 051501. M. Z. Liu, et al, Phys. Rev. Lett. 122, 242001 (2019).
 - Compact states: Ahmed Ali et al. Phys.Lett.B 793 (2019) 365-371. Z.-G. Wang, Int. J. Mod. Phys. A 35, 2050003 (2020). A. Ali, I. Ahmed et al. J. High Energy Phys. 10 (2019) 256.
 - ✓ No-resonant: threshold effect(cusp, triangle singularity) F.-K. Guo et al, Phys. Rev. D 92, 071502 (2015). S. X. Nakamura, Phys. Rev. D 103, L111503 (2021).

Background and motivation

• LHCb Collaboration utilized proton-proton collision data for the first time to study the $\Lambda_b^0 \to \Lambda D^+ D^-$ at nearly 13 TeV energy levels.

[LHCb] *JHEP* 07 (2024) 140



- Form the invariant mass distributions of ΛD^+ , a rich presence of intermediate resonances in the decay.
- These hints imply the possible existence of open-charm pentaquark states with quark content $\bar{c}sqqq$.

Our goal: To predict the stable states and decay properties of the csqqq

- > Quark delocalization color screening model (QDCSM)
 - QDCSM was developed by Nanjing-Los Alamos collaboration in 1990s aimed to multi-quark study. (PRL 69, 2901, 1992)
 - Two new ingredients

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quark delocalization (orbital excitation) color screening (color structure)
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• Apply to the study of baryon-baryon interaction and dibaryons deuteron, d^* , NN, N Λ , N Ω ,...

Apply to the study of baryon-meson interaction and pentaquarks

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NK, N\pi, P_c,...
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$$H = \sum_{i=1}^{5} \left(m_i + \frac{p_i^2}{2m_i} \right) - T_c + \sum_{i < j} \left[V^G(r_{ij}) + V^{\chi}(r_{ij}) + V^C(r_{ij}) \right],$$

$$V^G(r_{ij}) = \frac{1}{4} \alpha_s \lambda_i \cdot \lambda_j \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_i m_j} \right) \delta(r_{ij}) - \frac{3}{4m_i m_j r_{ij}^3} S_{ij} \right],$$

$$V^{\chi}(r_{ij}) = \frac{1}{3} \alpha_{ch} \frac{\Lambda^2}{\Lambda^2 - m_{\chi}^2} m_{\chi} \left\{ \left[Y(m_{\chi} r_{ij}) - \frac{\Lambda^3}{m_{\chi}^3} Y(\Lambda r_{ij}) \right] \sigma_i \cdot \sigma_j + \left[H(m_{\chi} r_{ij}) - \frac{\Lambda^3}{m_{\chi}^3} H(\Lambda r_{ij}) \right] S_{ij} \right\} \mathbf{F}_i \cdot \mathbf{F}_j, \qquad \chi = \pi, K, \eta,$$

$$V^C(r_{ij}) = -a_c \lambda_i \cdot \lambda_j [f(r_{ij}) + V_0],$$

$$f(r_{ij}) = \begin{cases} r_{ij}^2 & \text{if } i, \text{joccur in the same baryon orbit,} \\ \frac{1 - e^{-m_i r_{ij}^2}}{\mu_{ij}} & \text{if } i, \text{joccur in different baryon orbits,} \end{cases}$$

$$S_{ij} = \frac{(\sigma_i \cdot \mathbf{r}_{ij}) (\sigma_j \cdot \mathbf{r}_{ij})}{r_{ii}^2} - \frac{1}{3} \sigma_i \cdot \sigma_j, \qquad (1)$$

> Resonating group method (RGM)

J. A. Wheeler, Phys. Rev. 52, 1107-1122 (1937)

In RGM, the multiquark wave function is approximated by the cluster wave function

$$\Psi = \mathcal{A}\left[\left[\phi_{B_1}(\vec{\rho}_1, \vec{\lambda}_1) \phi_{B_2}(\vec{\rho}_2, \vec{\lambda}_2) \right]^{[\sigma]IS} \chi(R) Z(\vec{R}_C) \right],$$

Variational principle

$$\langle \delta \Psi'' | H - E | \Psi' \rangle = 0.$$

Eiple
$$\Psi'' = \int \mathcal{A} \left[\phi_1(\rho_1, \lambda_1) \phi_2(\rho_2, \lambda_2) \delta(R - R'') \right] \chi(R'') dR'',$$

$$\Psi' = \int \mathcal{A} \left[\phi_1(\rho_1, \lambda_1) \phi_2(\rho_2, \lambda_2) \delta(R - R'') \right] \chi(R'') dR'',$$

$$\Psi' = \int \mathcal{A} \left[\phi_1(\rho_1, \lambda_1) \phi_2(\rho_2, \lambda_2) \delta(R - R'') \right] \chi(R'') dR'',$$

$$\delta \Psi'' = \int \mathcal{A} \left[\phi_1(\rho_1, \lambda_1) \phi_2(\rho_2, \lambda_2) \delta(R - R'') \right] \delta \chi(R'') dR''.$$

The relative motion wave function satisfies the following RGM equation

$$\int H(R'',R')\chi(R')dR' = E \int N(R'',R')\chi(R')dR',$$

$$H(R'',R') = \langle [\phi_1 \phi_2 \delta(R - R'')] | H | \mathcal{A} [\phi_1 \phi_2 \delta(R - R')] \rangle,$$

$$N(R'',R') = \langle [\phi_1 \phi_2 \delta(R - R'')] | \mathcal{A} [\phi_1 \phi_2 \delta(R - R')] \rangle.$$

$$H(R'',R') = H^D(R'',R') \delta(R - R'') + H^{EX}(R'',R'),$$

$$N(R'',R') = N^D(R'',R') \delta(R - R'') + N^{EX}(R'',R').$$

> Kohn-Hulthen-Kato (KHK) variational method

 $u_t(R) = \sum_{i=0}^n c_i u_i(R)$ M. Kamimura, Prog. Theor. Phys. Suppl. 62(1977) 236 $u_i(R) = \begin{cases} \alpha_i u_i^{(in)}(R), & R < R_c, \\ (h_L^{(-)}(k,R) + s_i h_L^{(+)}(k,R))R, & R > R_c \end{cases}$

$$\frac{u_i^{(in)}(R)}{R} = \sqrt{4\pi} \left(\frac{3}{2\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{3}{4b^2}(R^2 + r_i^2)} i^L j_L(-i\frac{3}{2b^2}Rr_i)$$

$$\sum_{i=0}^{n} c_i = 1, \qquad \sum_{i=0}^{n} c_i s_i = S_t.$$

$$c_0 = 1 - \sum_{i=1}^{n} c_i$$
 $u_t(R) = u_0(R) + \sum_{i=1}^{n} c_i(u_i(R) - u_0(R))$

$$\langle \delta \Psi' | H - E | \Psi \rangle = 0$$

$$\sum_{i=1}^{n} \mathcal{L}_{ij} c_j = \mathcal{M}_i, (i = 1 \sim n)$$

$$\mathcal{L}_{ij} = \mathcal{K}_{ij} - \mathcal{K}_{i0} - \mathcal{K}_{0j} + \mathcal{K}_{00},$$
$$\mathcal{M}_i = \mathcal{K}_{00} - \mathcal{K}_{i0}$$

$$\mathcal{K}_{ij} = \langle \phi_A(\xi_A)\phi_B(\xi_B)u_i(R)/R \cdot Y_{LM}(\hat{R})|H - E|\mathcal{A}[\phi_A(\xi_A)\phi_B(\xi_B)u_j(R)/R \cdot Y_{LM}(\hat{R})]\rangle$$

$$\sum_{i=0}^{n} c_{i} s_{i} = S_{t}
C_{i} \qquad S_{L} = |S_{L}| e^{2i\delta_{L}}
S_{L} \qquad \delta_{L}$$

$$\sigma_L = \frac{4\pi}{k^2} \cdot (2l+1) \cdot \sin^2 \delta_L$$

1

• Here, we only consider the S-wave channels for the anticharmed-strange pentaquarks

TABLE I. The relevant channels for all possible states with different J^P quantum numbers.

	S =	$=\frac{1}{2}$	S =	$=\frac{3}{2}$	$S = \frac{5}{2}$		
$I = \frac{1}{2}$	$\Lambdaar{D} \ \Sigmaar{D} \ \Sigma^*ar{D}^* \ ND_s^{*-}$	$\Lambda ar{D}^* \ \Sigma ar{D}^* \ ND_s^-$	$\Lambda ar{D}^* \ \Sigma^* ar{D} \ ND_s^{*-}$	$\Sigma ar{D}^* \ \Sigma^* ar{D}^*$	$\Sigma^*ar{D}^*$		
$I=\frac{3}{2}$	$\Sigmaar{D} \ \Sigma^*ar{D}^*$	$\Sigma ar{D}^* \ \Delta D_s^{*-}$	$\Sigma ar{D}^* \ \Sigma^* ar{D}^* \ \Delta D_s^{*-}$	$\Sigma^*ar{D}$ ΔD_s^-	$\Sigma^*ar{D}^*$	ΔD_s^{*-}	

1

> The effective potential analysis

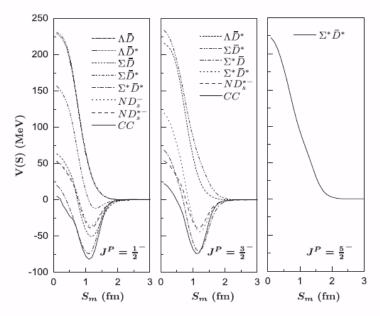


FIG. 1. The effective potentials defined in Eq. (12) for different channels of the anticharmed-strange pentaquark systems with I = 1/2 in QDCSM. CC stands for the effective potentials after considering the all-channel coupling.

• Strongest attraction:

For $J^P = 1/2^-$, the $\Sigma^* \overline{D}^*$ and $\Sigma \overline{D}$ channels are the most attractive. For $J^P = 3/2^-$, the $\Sigma^* \overline{D}$ channel shows significant attraction.

• Replusive channels: channels like $\Lambda \overline{D}$ and $\Lambda \overline{D}^*$ are repulsive.

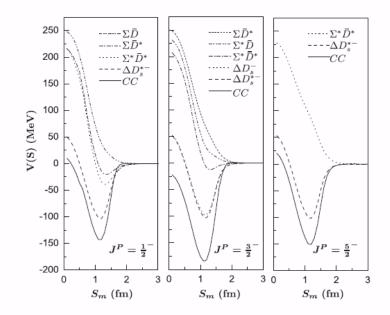


FIG. 2. The effective potentials defined in Eq. (12) for different channels of the anticharmed-strange pentaquark systems with I = 3/2 in QDCSM. CC stands for the effective potentials after considering the all-channel coupling.

- Strongest attraction: channels involving the Δ baryon, such as ΔD_s^* and ΔD_s^- , consistently show the strongest attraction.
- Replusive channels: channels like $\Sigma^* \overline{D}^*$ and $\Sigma \overline{D}$ are repulsive.

Crucial role of channel coupling: The resulting attraction after coupling is the strongest, strongly suggesting the presence of bound states

Bound state predictions

TABLE II. The binding energies and the masses of every single channel and those of channel coupling for the pentaquarks with I = 1/2. The values are provided in units of MeV.

$\overline{I(J^P)}$	Channel	E_{sc}	$E_{th}^{ m Model}$	E_B	E_{th}^{Exp}	E'_{sc}	E_{cc}/E_B	E'_{cc}	
$\frac{1}{2}(\frac{1}{2})$	$\Lambda ar{D}$	2992	2988	4	3122	3126	2873/-21	2886	Rms: 1.2 fm
2 \ 2 /	$\Lambda ar{D}^*$	3027	3023	4	3227	3231			ND_{s}^{-} : 96%
	$\Sigma ar{D}$	3101	3102	-1	3058	3057			
	$\Sigma ar{D}^*$	3140	3137	3	3196	3199			
	$\Sigma^*ar{D}^*$	3251	3259	-8	3392	3384			
	ND_s^-	2896	2894	2	2907	2909			
	ND_s^{*-}	2910	2908	2	3051	3053			
$\frac{1}{2}(\frac{3}{2})$	$\Lambda ar{D}^*$	3027	3023	4	3122	3126	2896/-12	3039	Rms: 1.5 fm
2 \2 /	$\Sigma ar{D}^*$	3141	3137	4	3196	3200			$ND_{S}^{*-}:97\%$
	$\Sigma^*ar{D}$	3218	3224	-6	3254	3248			
	$\Sigma^*ar{D}^*$	3261	3259	2	3392	3394			
	ND_s^{*-}	2910	2908	2	3051	3053			
$\frac{1}{2}\left(\frac{5}{2}^{-}\right)$	$\Sigma^*ar{D}^*$	3263	3259	4	3392	3396	3263/4	3396	

Bound state predictions

TABLE III. The binding energies and the masses of every single channel and those of channel coupling for the pentaquarks with I = 3/2. The values are provided in units of MeV.

$I(J^P)$	Channel	E_{sc}	$E_{th}^{ m Model}$	E_B	$E_{th}^{ m Exp}$	E'_{sc}	E_{cc}/E_B	E_{cc}^{\prime}	
$\frac{3}{2}(\frac{1}{2})$	$\Sigma ar{D}$	3106	3102	4	3058	3062	3103/1	3059	
2 \ 2 /	$\Sigma ar{D}^*$	3140	3137	3	3196	3199			
	$\Sigma^*ar{D}^*$	3261	3259	2	3392	3394			
	ΔD_s^{*-}	3167	3201	-34	3344	3310			
$\frac{3}{2}(\frac{3}{2})$	$\Sigma ar{D}^*$	3141	3137	4	3196	3200	3094/-43	3153	Rms: 2.1 fm \overline{D}_{*} , 070/
2 \ 2 \ /	$\Sigma^*ar{D}$	3228	3224	4	3254	3248			$\Sigma \overline{D}^*$: 97%
	$\Sigma^*ar{D}^*$	3263	3259	4	3392	3394			
	ΔD_s^-	3162	3187	-25	3200	3175			
	ΔD_s^{*-}	3155	3201	-45	3344	3299			
$\frac{3}{2}(\frac{5}{2}^{-})$	$\Sigma^*ar{D}^*$	3263	3259	4	3392	3396	3202/1	3345	
2 \ 2 /	ΔD_s^{*-}	3206	3201	5	3344	3349			

Channel coupling is crucial for the formation of bound states

Bound state confirmation

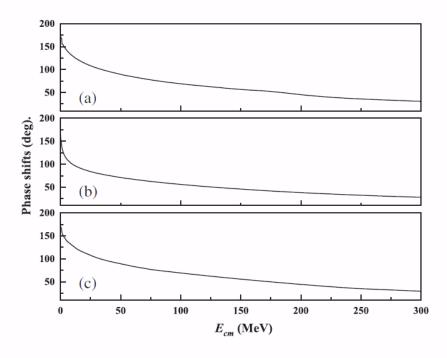


FIG. 3. The phase shifts of the ND_s^- with $I(J^P)=\frac{1}{2}(\frac{1}{2}^-)$, ND_s^{*-} with $I(J^P)=\frac{1}{2}(\frac{3}{2}^-)$, and $\Sigma\bar{D}^*$ with $I(J^P)=\frac{3}{2}(\frac{3}{2}^-)$ in the coupled-channel estimations. (a) Stands for ND_s^- with $I(J^P)=\frac{1}{2}(\frac{1}{2}^-)$, (b) represents ND_s^{*-} with $I(J^P)=\frac{1}{2}(\frac{3}{2}^-)$, (c) stands for $\Sigma\bar{D}^*$ with $I(J^P)=\frac{3}{2}(\frac{3}{2}^-)$.

TABLE IV. The the scattering length a_0 , the effective range r_0 and the binding energy E'_B determined by the variation method.

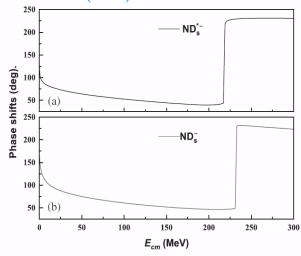
$\overline{I(J^P)}$	Channel	a_0 (fm)	r_0 (fm)	E_B' (MeV)
$\frac{1}{2}(\frac{1}{2})$	ND_s^-	2.06	0.98	-23.8
$\frac{1}{2}(\frac{3}{2})$	ND_s^{*-}	2.00	1.00	-11.7
$\frac{3}{2}(\frac{3}{2}^{-})$	$\Sigma ar{D}^*$	2.56	1.45	-40.7

- All scattering lengths (a_0) are positive, robustly confirming the presence of bound states
- The binding energies (E'_B) are in agreement with the binding energies obtained from the dynamical calculation

Both the qualitative behavior of the phase shifts and the quantitative analysis of the scattering parameters provide **strong and self-consistent confirmation** for the existence of the three predicted bound states.

> Resonance state and decay width

$$I(J^P) = \frac{1}{2} \left(\frac{1}{2} \right)$$



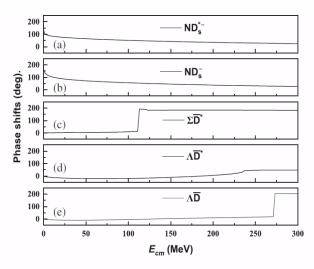


FIG. 4. The phase shifts of the open channels with two-channel coupling for $I(J^P)=1/2(1/2^-)$ in QDCSM. On the left, (a) corresponds to two channels coupling with $\Sigma\bar{D}$ and ND_s^{*-} , (b) denotes two-channel coupling with $\Sigma\bar{D}$ and ND_s^{-} . On the right, (a) stands for two-channel coupling with $\Sigma^*\bar{D}^*$ and ND_s^{-} , (c) represents two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (d) stands for two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$, (e) denotes two-channel coupling with $\Sigma^*\bar{D}^*$ and $\Sigma\bar{D}^*$ and

- $\Sigma \overline{D}$ resonance state was found in the phase shifts of open channel ND_s^- and ND_s^{*-}
- $\Sigma^* \overline{D}^*$ resonance state was seen in the phase shifts of open channel $\Sigma \overline{D}^*$ and $\Lambda \overline{D}$

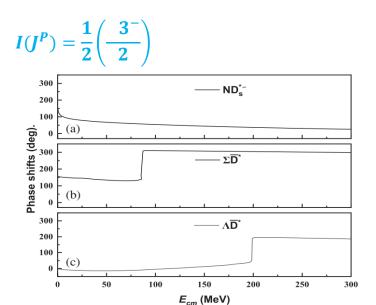


FIG. 6. The phase shifts of the open channels with two channels coupling with a closed channel $(\Sigma^*\bar{D})$ and one open channel $(ND_s^{*-},\ \Sigma\bar{D}^*,\ \Lambda\bar{D}^*)$ for $I(J^P)=1/2(3/2^-)$ in QDCSM. (a) represents two-channel coupling with $\Sigma^*\bar{D}$ and ND_s^{*-} , (b) shows two-channel coupling with $\Sigma^*\bar{D}$ and $\Sigma\bar{D}^*$. (c) denotes two-channel coupling with $\Sigma^*\bar{D}$ and $\Lambda\bar{D}^*$.

• $\Sigma^* \overline{D}$ resonance state was obtained in the phase shifts of open channel $\Sigma \overline{D}^*$ and $\Lambda \overline{D}^*$

> Resonance state and decay width

$$I(J^P) = \frac{1}{2} \left(\frac{1}{2} \right)$$

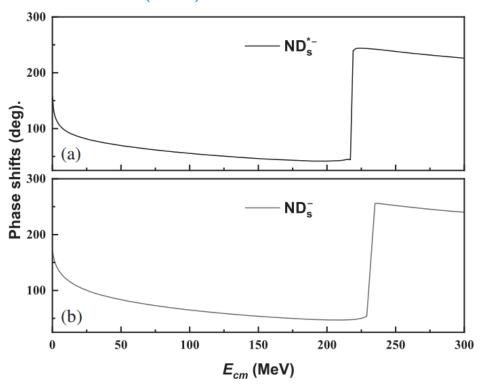


FIG. 5. The phase shifts of the open channels with three channels coupling with two closed channels ($\Sigma \bar{D}$ and $\Sigma^* \bar{D}^*$) and one open channel (ND_s^- or ND_s^{*-}) for $I(J^P)=1/2(1/2^-)$ in QDCSM. (a) stands for three-channel coupling with $\Sigma \bar{D}$, $\Sigma^* \bar{D}^*$, and ND_s^{*-} , (b) shows three-channel coupling with $\Sigma \bar{D}$, $\Sigma^* \bar{D}^*$, and ND_s^- .

• Only one resonance state $\Sigma \overline{D}$ appears in the scattering phase shifts of ND_s^- and ND_s^{*-}

> Resonance state and decay width

$$I=\frac{3}{2}$$

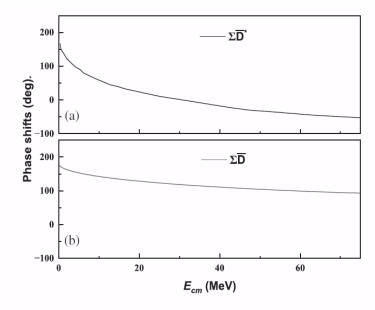


FIG. 7. The phase shifts of the open channels with two channels coupling with a closed channel (ΔD_s^{*-}) and one open channel $(\Sigma \bar{D}, \Sigma \bar{D}^*)$ for $I(J^P) = 3/2(1/2^-)$ in QDCSM. (a) indicates two-channel coupling with ΔD_s^{*-} and $\Sigma \bar{D}$, (b) presents two-channel coupling with ΔD_s^{*-} and $\Sigma \bar{D}^*$.

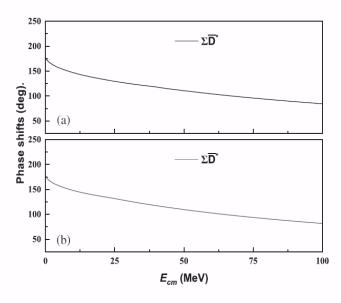


FIG. 8. The phase shifts of the open channels with two channels coupling with a closed channel $(\Delta D_s^-, \Delta D_s^{*-})$ and one open channel $(\Sigma \bar{D}^*)$ for $I(J^P) = 3/2(3/2^-)$ in QDCSM. (a) Represents two channels coupling with ΔD_s^{*-} and $\Sigma \bar{D}^*$, (b) denotes two channels coupling with ΔD_s^- and $\Sigma \bar{D}^*$.

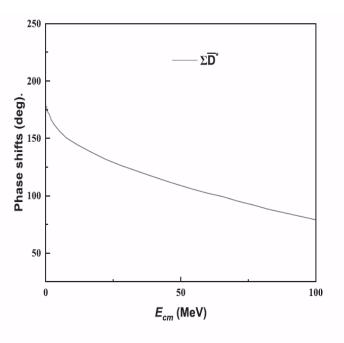


FIG. 9. The phase shifts of the open channels with three channels coupling with two closed channels $(\Delta D_s^-, \Delta D_s^{*-})$ and one open channel $(\Sigma \bar{D}^*)$ for $I(J^P) = 3/2(3/2^-)$ in QDCSM.

• No resonance states can be observed in the corresponding scattering phase shifts

> Resonance state and decay width

TABLE V. The masses and decay widths (in the unit of MeV) of resonance states with the difference scattering process. M_R^{th} stands for the sum of the corresponding theoretical threshold of the open channel and the incident energy, M_R represents the modified resonance mass. Γ_i is the partial decay width of the resonance state decaying to the *i*th open channel. Γ_{Total} is the total decay width of the resonance state.

		Three-	-channe	l coupli	ing					Two-ch	nannel co	oupling			
		I	$(J^P) = $	$\frac{1}{2}(\frac{1}{2}^{-})$					$I(J^P)$ =	$=\frac{1}{2}\left(\frac{1}{2}^{-}\right)$			I(J	$P = \frac{1}{2} \left(\frac{3}{2}\right)$	_)
		$\Sigma ar{D}$			$\Sigma^*ar{D}^*$		$\Sigma ar{D}$				$\Sigma^*ar{D}^*$			$\Sigma^*ar{D}$	
Open channels	M_R^{th}	M_R	Γ_i	M_R^{th}	M_R	Γ_i	M_R^{th}	M_R	Γ_i	M_R^{th}	M_R	Γ_i	M_R^{th}	M_R	Γ_i
$\overline{ND_s^-}$	3097	3053	5.5				3098	3054	5.6						
ND_s^{*-}	3098	3054	7.5				3099	3055	7.8						
$\Lambda ar{D}^{\circ}$										3258	3390	1.1			
$\Lambda ar{D}^*$													3220	3250	1.3
$\Sigma ar{D}^*$										3257	3389	9.5	3222	3252	3.1
$\Gamma_{ ext{Total}}$			13.0						13.4			10.4			4.4

Three resonance states:

•
$$I(J^P) = \frac{1}{2} \left(\frac{1^-}{2} \right) \Sigma \overline{D}$$
 • $I(J^P) = \frac{1}{2} \left(\frac{1^-}{2} \right) \Sigma^* \overline{D}^*$ • $I(J^P) = \frac{1}{2} \left(\frac{3^-}{2} \right) \Sigma^* \overline{D}$

Experimental potential

TABLE VI. The possible decay channels of Bound states and resonance states.

$\overline{I(J^P)}$	Bound states	Possible decay channel
$\frac{\frac{1}{2}(\frac{3}{2}^{-})}{\frac{3}{2}(\frac{3}{2}^{-})}$	$ND_s^{*-} \ \Sigma ar{D}^*$	ND_s^- (<i>D</i> -wave) $\Sigma \bar{D}$ (<i>D</i> -wave)
$I(J^P)$ $\frac{1}{2}(\frac{1}{2}^-)$ $\frac{1}{2}(\frac{3}{2}^-)$	Resonance states $\Sigma \bar{D}$ $\Sigma^* \bar{D}^*$ $\Sigma^* \bar{D}$	Possible decay channel ND_s^-, ND_s^{*-} $\Lambda \bar{D}, \Sigma \bar{D}^*$ $\Lambda \bar{D}^*, \Sigma \bar{D}^*, \Sigma \bar{D}$ (D -wave)

Summary

- Prediction of three bound states: our dynamical calculations predict the existence of three bound states:
 - $I(J^P) = \frac{1}{2} \left(\frac{1}{2} \right)$ with a predicted mass of 2886 MeV.
 - $I(J^P) = \frac{1}{2} \left(\frac{3}{2} \right)$ with a predicted mass of 3039 MeV.
 - $I(J^P) = \frac{3}{2} \left(\frac{3}{2} \right)$ with a predicted mass of 3153 MeV.
- Prediction of three resonance states: by analyzing scattering phase shifts, three resonance states are identified:
 - $\Sigma \overline{D}$ resonance $I(J^P) = \frac{1}{2} \left(\frac{1}{2} \right)$ with a mass of 3053-3055 MeV and a width of 13-13.4 MeV
 - $\Sigma^* \overline{D}^*$ resonance $I(J^P) = \frac{1}{2} \left(\frac{1}{2} \right)$ with a mass of 3389-3390 MeV and a width of 10.4 MeV
 - $\Sigma^* \overline{D}$ resonance $I(J^P) = \frac{1}{2} \left(\frac{3}{2} \right)$ with a mass of 3250-3252 MeV and a width of 4.4 MeV

Thank you!