Meson–baryon interactions
within a renormalizable framework of ChEFT

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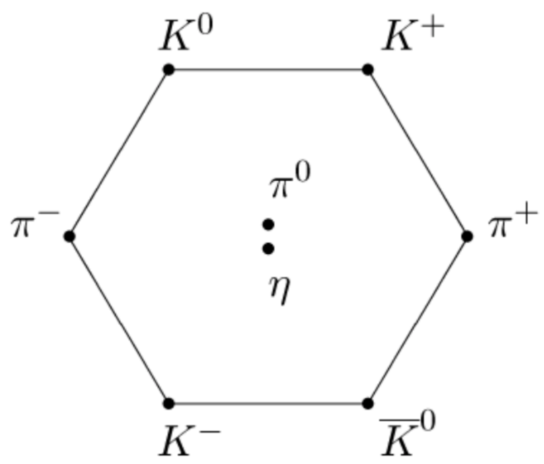
RUHR UNIVERSITÄT BOCHUM

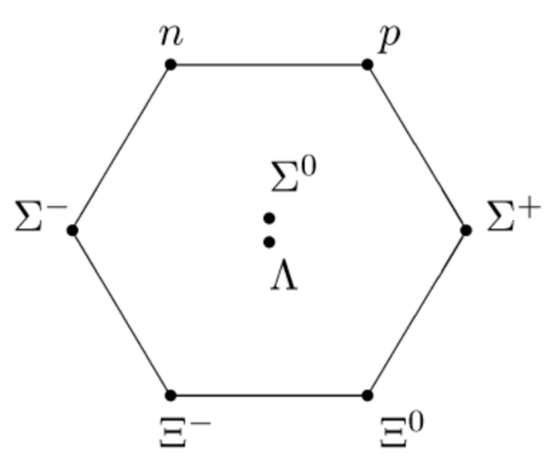
OUTLINE

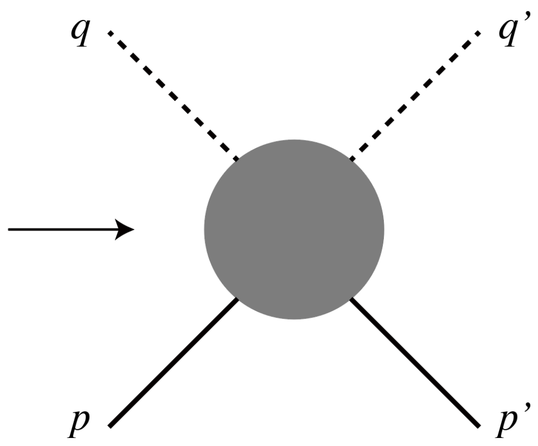
- Introduction
- Theoretical framework
- Results and discussion
- Summary

Meson–baryon scattering

Lowest-lying Meson-Baryon scattering

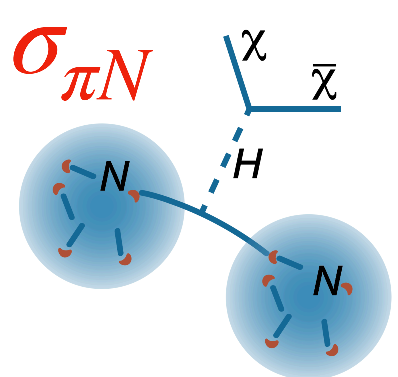




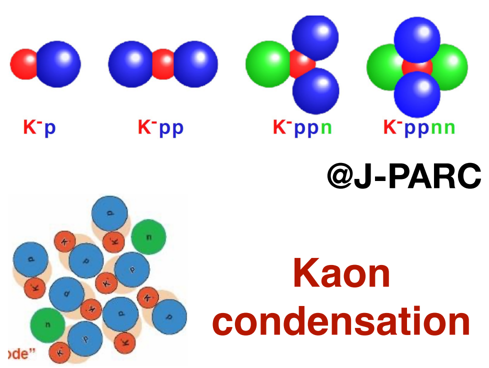


Interesting phenomena

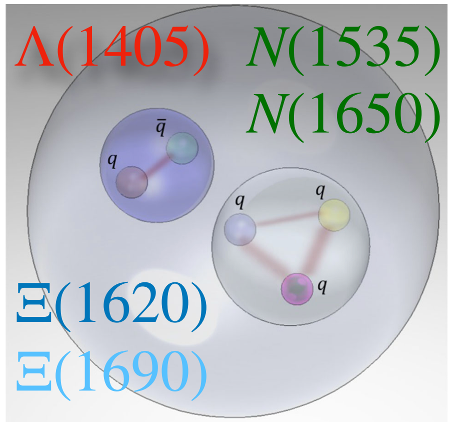
πN scattering



$\bar{K}N$ scattering



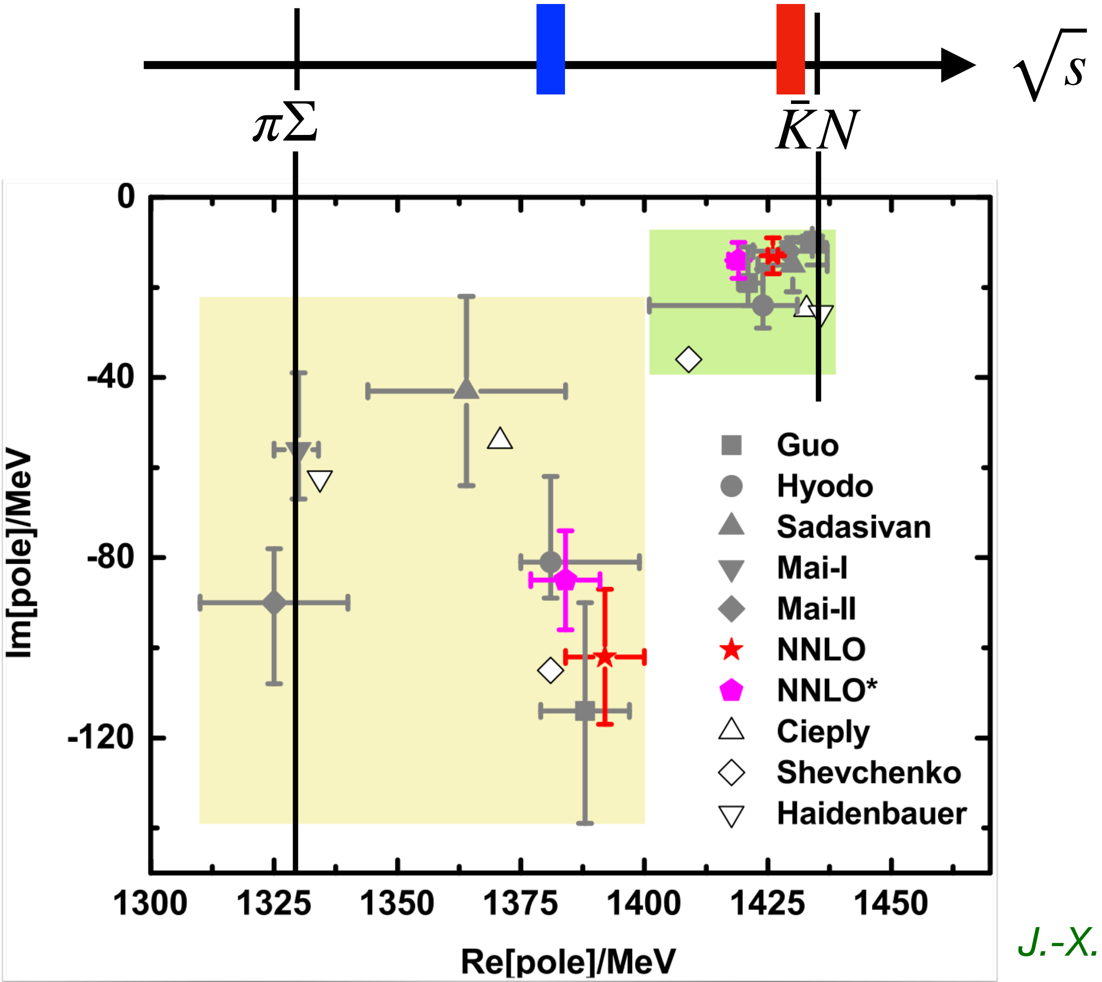
Resonances



- Deepen understanding of SU(3) dynamics in nonperturbative QCD

Structure of $\Lambda(1405)$ resonance

- Double-pole predicted by **chiral unitary approach**



$\Lambda(1405)\ 1/2^-$ $\quad I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

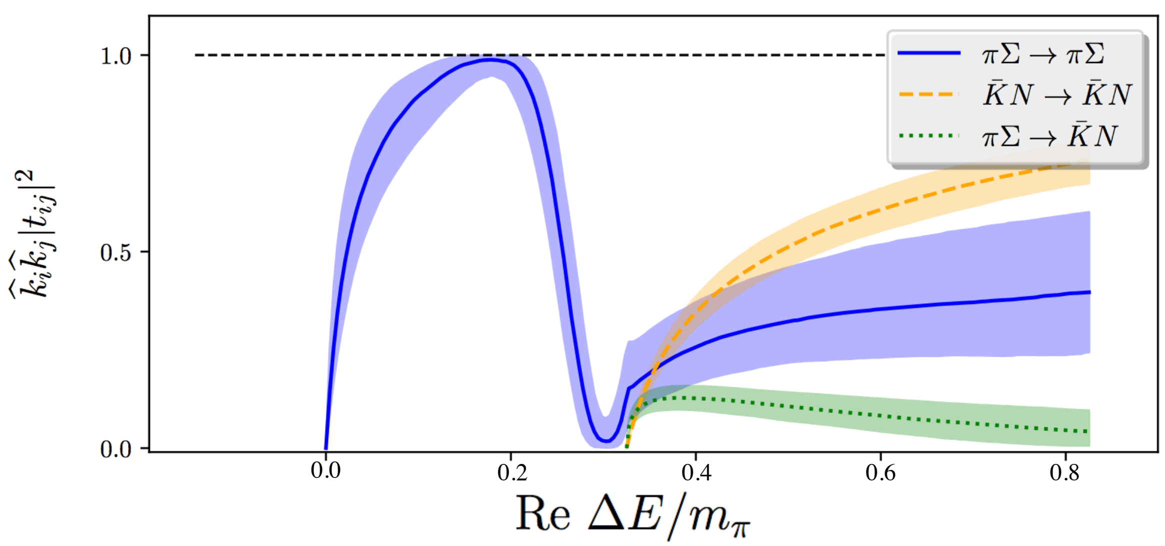
$\Lambda(1380)\ 1/2^-$ $\quad J^P = \frac{1}{2}^-$ Status: ∗∗

✓ Pole 1: Λ(1405) is around 1420 MeV

✓ Pole 2: Λ(1380) needs further studies to fix its position

J.-X. Lu, et al., PRL130(2023)071902

- Double-pole structure verified by LQCD



$m_\pi \approx 200$ MeV, $m_K \approx 487$ MeV

Lower Pole : $E_1 = 1392(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}$ MeV

Higher Pole : $E_2 = 1455(13)_{\rm stat}(2)_{\rm model}(17)_{\rm a} - i11.5(4.4)_{\rm stat}(4.0)_{\rm model}(0.1)_{\rm a}$ MeV

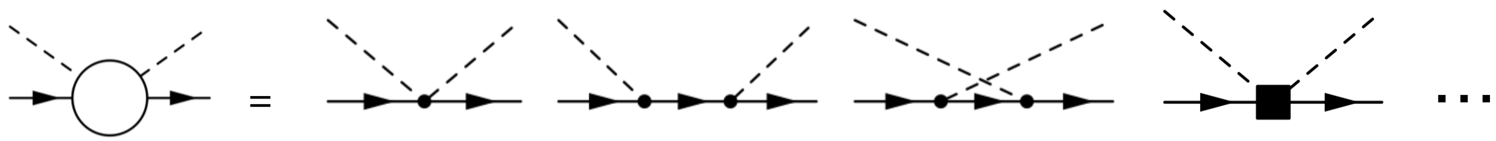
Baryon Scattering Coll., PRL132(2024)051901

Chiral Unitary approach

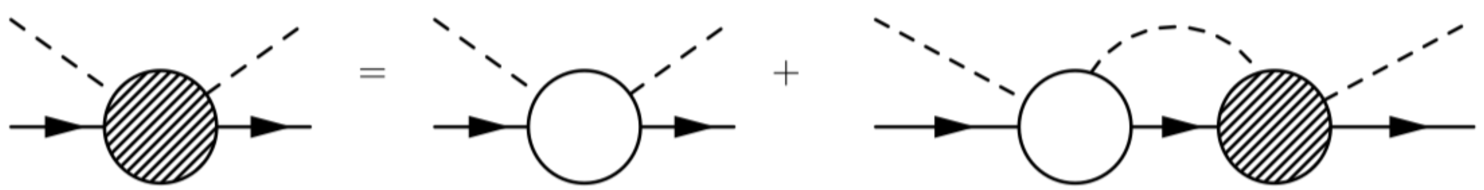
- **Chiral symmetry** of low-energy QCD + **Unitary Relation**

  *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP*120 (2021)103868 …

- Interaction kernel V: calculate in ChPT order by order
  - Leading, next-to-leading order, …



- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^4k}{(2\pi)^4} V(p',k)\, G(k)\, T(k,p)$$

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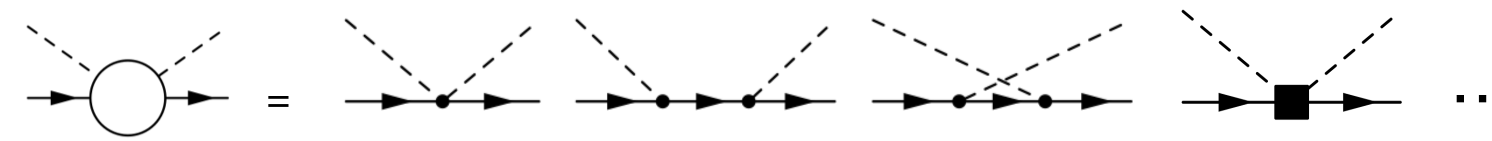

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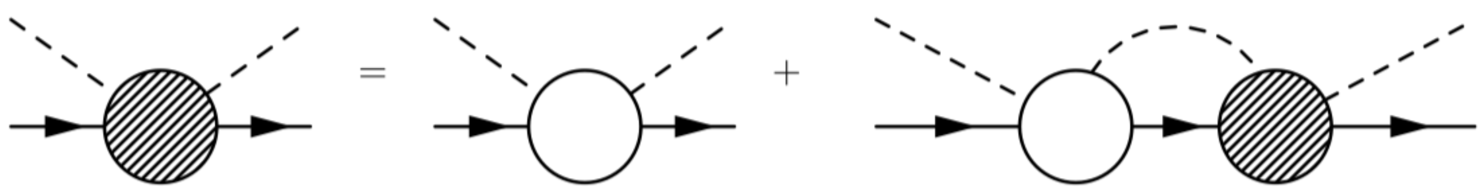

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- On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k)\right) T(p',p)$

Neglecting off-shell effect

→ cause troubles in the study of three-body interaction?

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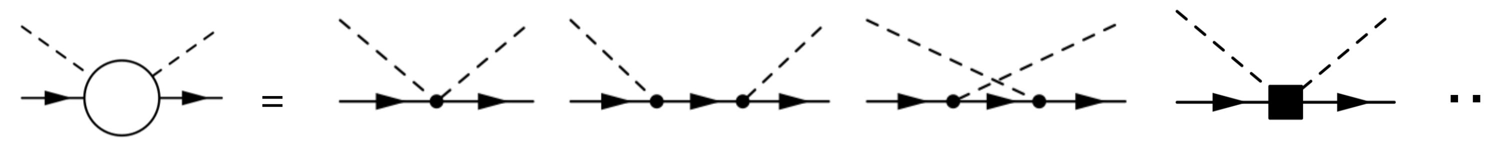

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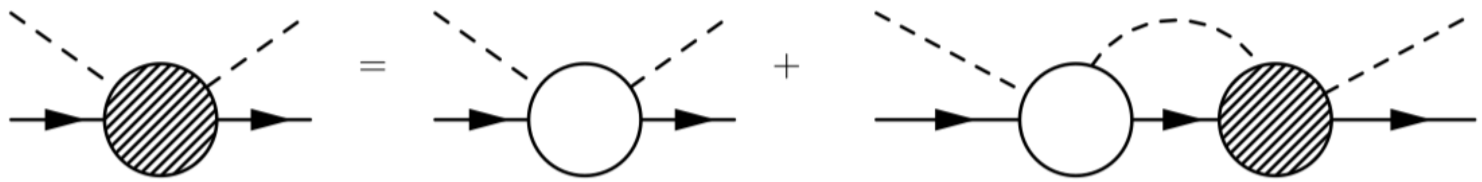

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  Neglecting off-shell effect

  → cause troubles in the study of three-body interaction?

- Finite cutoff or subtraction constant
  to renormalize the loop integral

$$\downarrow$$

$$G^R(E,\Lambda) \text{ or } G^R(E,\alpha_i)$$

Cutoff / Model dependence

In this work

- Facing the rapid progress of precision experiments
  - Avoid the above approximations or obstacles
  - → **Model-independent formalism**

ALICE, AMADEUS, J-PARC, STAR…

- We tentatively proposed **a renormalizable framework** of ChEFT for meson-baryon scattering by using **time-ordered perturbation theory** with the covariant chiral Lagrangians
  - Apply to the SU(2) sector: pion-nucleon scattering

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406
  - Extend to the SU(3) sector: $\bar{K}N$ scattering and $\Lambda(1405)$ state

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582
  - Investigate the light-quark mass dependence of $\Lambda(1405)$

    XLR, Phys. Lett. B 855 (2024) 138802
  - Next-to-leading order studies: e.g. $\pi N, KN$

    XLR, et al., In preparation

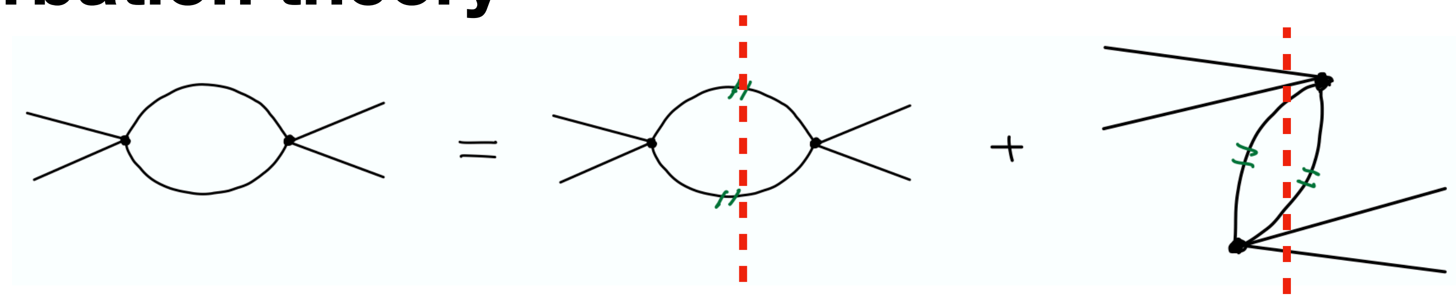
Theoretical framework

Time-ordered perturbation theory

- Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

  - Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
    - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
  - **TOPT or old-fashioned perturbation theory**



- Advantages
  - Explicitly show the unitarity
  - Easily to tell the contributions of a particular diagram

- Obtain the rules for time-ordered diagrams
  - Perform Feynman integrations over the zeroth components of the loop momenta
  - Decompose Feynman diagram into sums of time-ordered diagrams
  - Match to the rules of time-ordered diagrams

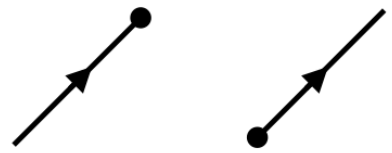
Diagrammatic rules in TOPT

- **External lines** — *XLR, PoS(CD2021)007*

Spin 0 boson (in, out)

1

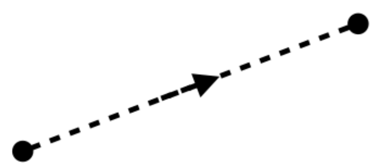
Spin 1/2 fermion (in, out)



$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

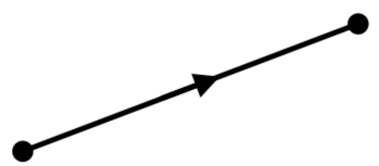
- **Internal lines**

Spin 0 (anti-)boson



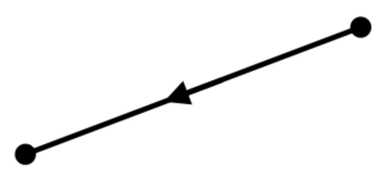
$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

Spin 1/2 fermion



$$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

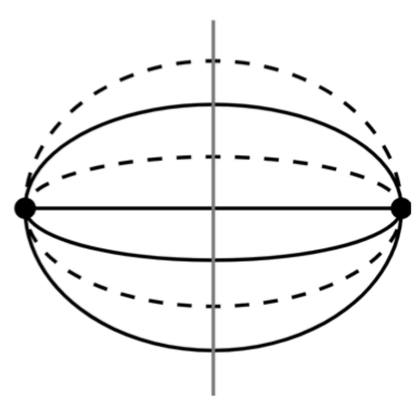
anti-fermion



$$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

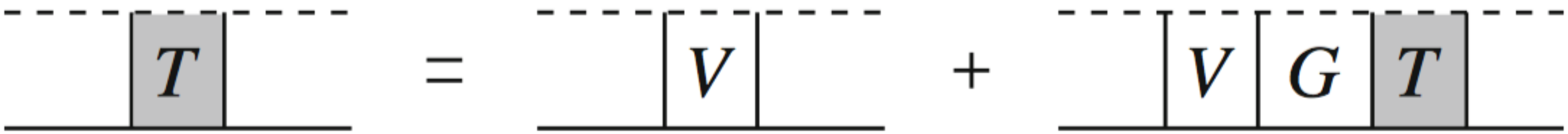
- Interaction vertices: **the standard Feynman rules**
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Meson–baryon scattering in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the one-meson and one-baryon **irreducible diagrams**
  - **Power counting:** Q/Λ_χ systematic ordering of all graphs
- Scattering equation



  - Coupled-channel integral equation for T-matrix

$$
\begin{aligned}
T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) &= V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) \\
&\quad + \sum_{MB}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)
\end{aligned}
$$

  - **Meson-baryon Green function in TOPT**

$$
G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}
$$

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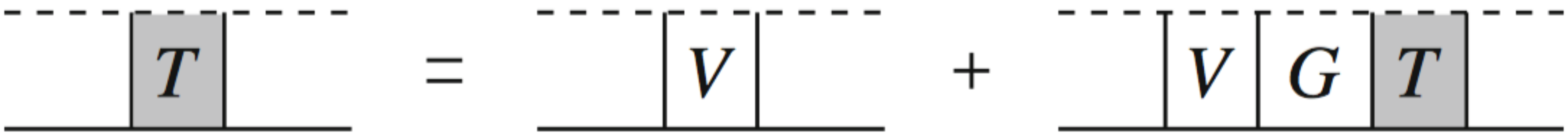

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&+ \sum_{MB}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)
\end{aligned}
$$

  - **Meson-baryon Green function in TOPT**

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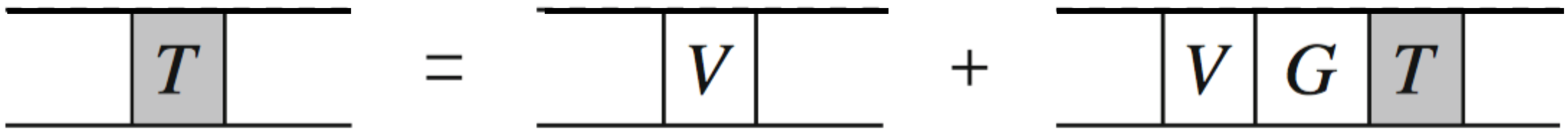
G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}
$$

Potential and scattering equation are obtained on an equal footing!

Baryon-baryon scattering in TOPT

Baryon-baryon scattering equation



- Potential V: sum up the two-baryon irreducible time-ordered diagrams
- **Two-baryon Green function**

$$G_{ij}^{BB}(E) = \frac{m_i\, m_j}{\omega(k, m_i)\, \omega(k, m_j)} \frac{1}{E - \omega(k, m_i) - \omega(k, m_j) + i\epsilon}$$

✓ Generalized Kadyshevsky propagator of NN scattering *V. Kadyshevsky, NPB (1968)*

✓ SELF-CONSISTENTLY obtained in TOPT

- **Successfully applied to the NN and YN interactions**

V. Baru, E. Epelbaum, J. Gegelia, XLR, Phys. Lett. B 798, 134987 (2019)

XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101, 034001 (2020)

XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

XLR, et al., arXiv:2510.xxxx

At physical world —>

Leading order studies

XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner,
Eur. Phys. J. C80 (2020) 406; Eur. Phys. J. C81 (2021) 582

Leading order potential

Chiral effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LO}} = & \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle \\
& -\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\,\langle \bar{B}\gamma_\mu[V^\mu, B]\rangle
\end{aligned}
$$

Time ordered diagrams

(a) (b) (c) (d)

- **Vector mesons included as explicit degrees of freedom**
  - ✓ Vector meson exchange potential instead of the Weinberg-Tomozawa term
  - ✓ Improve the UV behaviour without changing the low-energy physics

LO potential in TOPT

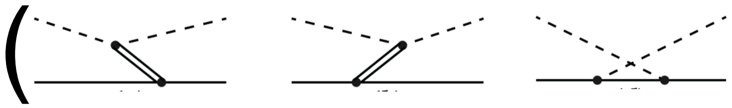
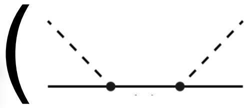
$$
\begin{aligned}
V^{(a+b)}_{M_jB_j,M_iB_i} = & -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right) \\
& \times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)} + \frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]
\end{aligned}
$$

$$
V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.
$$

$$
V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.
$$

Subtractive renormalization

- LO potential: **one-baryon irreducible and reducible parts**

$$V_{\rm LO} = V_I\Big($$  $$\Big) + V_R\Big($$  $$\Big)$$

- LO T-matrix

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G\, T_{\rm LO} \Longrightarrow \begin{cases} T_{\rm LO} = T_I + (1+T_I\, G)\, T_R\, (1 + G\, T_I) \\ T_I = V_I + V_I\, G\, T_I \\ T_R = V_R + V_R\, G\, (1 + T_I\, G)\, T_R \end{cases}$$

- Irreducible part: $T_I \xrightarrow{\Lambda \sim \infty}$ Finite
- Reducible part: $T_R \xrightarrow{\Lambda \sim \infty}$ Divergent

✓ **Potential can be rewritten as separable form**

$$V_R(p', p; E) = \xi^T(p')\, C(E)\, \xi(p) \qquad \text{C(E): constant} \qquad \xi^T(q) := (1, q)$$

✓ T_R can be rewritten as $\quad T_R(p', p; E) = \xi^T(p')\, \chi(E)\, \xi(p) \quad \chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G\, T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

✓ Using **subtractive renormalization,** replacing Green function $\quad G^{Rn} = G(E) - G(m_B)$

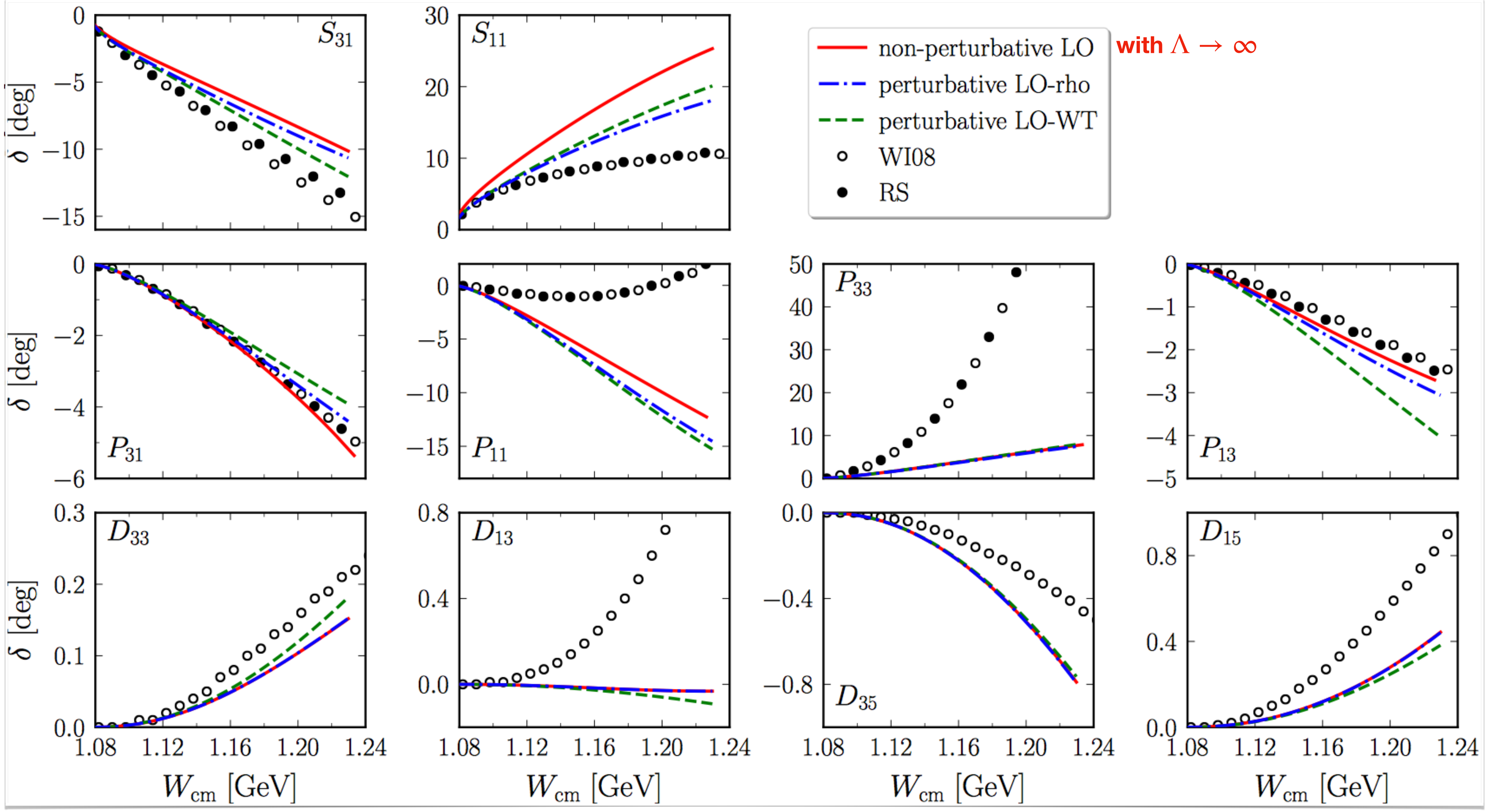
E. Epelbaum, et al., EPJA56(2020)152

Renormalized LO T-matrix

$$\boxed{T_{\rm LO}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)}$$

Pion–Nucleon scattering

- Description phase shifts of pion-nucleon scattering

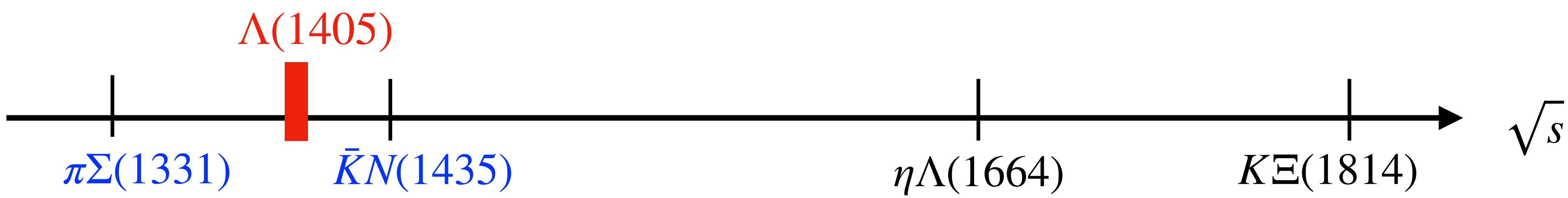


- Rho-meson-exchange contribution similar as WT term
- **Non-perturbative results slightly different from the perturbative results**
  - ✓ **Our non-perturbative treatment is valid**
  - ✓ BChPT has good convergence in SU(2) sector

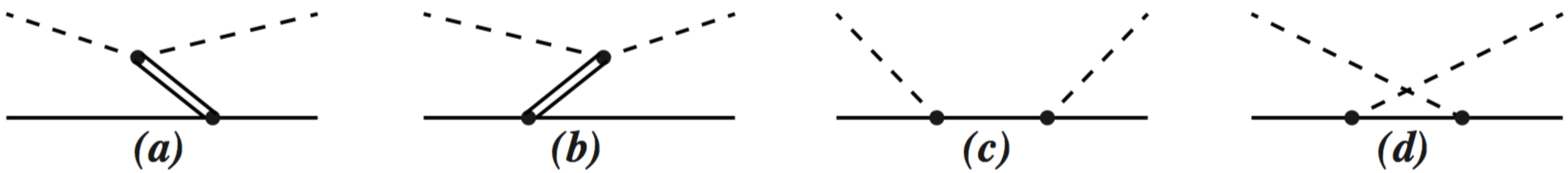
XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

S=−1 meson-baryon scattering

- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$ in isospin limit



- Focus on the S-wave potential



- Born term (p-wave) does not contribute
- Crossed-Born term $\sim$5% of VME contribution
- VME potential couplings

C^V	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$K\Xi$
$\pi\Sigma$	$C^\rho = -16$	$C^{K^*} = 2\sqrt{6}$	0	$C^{K^*} = -2\sqrt{6}$
$\bar{K}N$	attractive	$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$	$C^{K^*} = -6\sqrt{2}$	0
$\eta\Lambda$		attractive	0	$C^{K^*} = 6\sqrt{2}$
$K\Xi$				$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$

S=−1 meson-baryon scattering

P. W. scattering equation

$$T^{LJ}_{M_jB_j,M_iB_i}(p',p) = V^{LJ}_{M_jB_j,M_iB_i}(p',p) + \sum_{MB}\int \frac{dkk^2}{(2\pi)^3} V^{LJ}_{M_jB_j,MB}(p',k)\frac{1}{2\,\omega_M\omega_B}\,\frac{m_B}{E-\omega_M-\omega_B+i\epsilon}\,T^{LJ}_{MB,M_iB_i}(k,p)$$

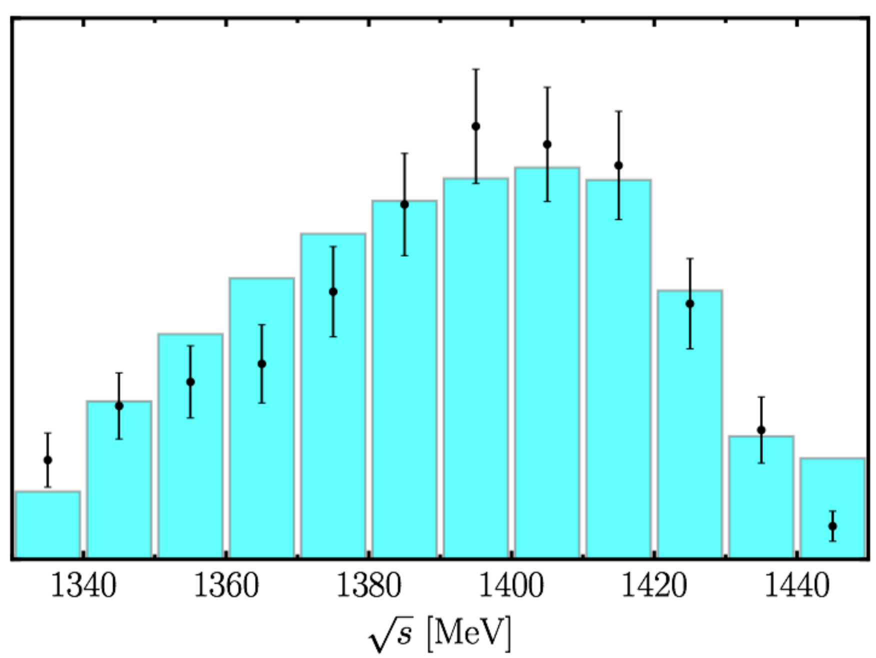
- Take into account **the off-shell effects of potential**
- Use **subtractive reormalization** to obtain the renormalized T-matrix
  - ➡ Cutoff-independent: $\Lambda \to \infty$

No free parameters needed to be fitted!

Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
(LO)	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
	Y. Ikeda,NPA(2012)	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
NLO	*M.Mai,EPJA(2015)-sol-2*	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA(2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582

Comes to the unphysical quark mass region —>

Quark mass dependence of $\Lambda(1405)$

XLR, Phys. Lett. B 855 (2024) 138802

$\Lambda(1405)$ from Lattice QCD

- First lattice study of $\Lambda(1405)$ pole positions

  Baryon Scattering Collaboration: PRL 132, 051901 (2024); PRD109,014511(2024)

  - $\pi\Sigma - \bar{K}N$ coupled channels (below $\pi\pi\Lambda$ threshold)
  - Pion and Kaon masses: $M_\pi \approx 200$ MeV, $M_K \approx 487$ MeV
  - Two pole structure of $\Lambda(1405)$

Virtual bound state

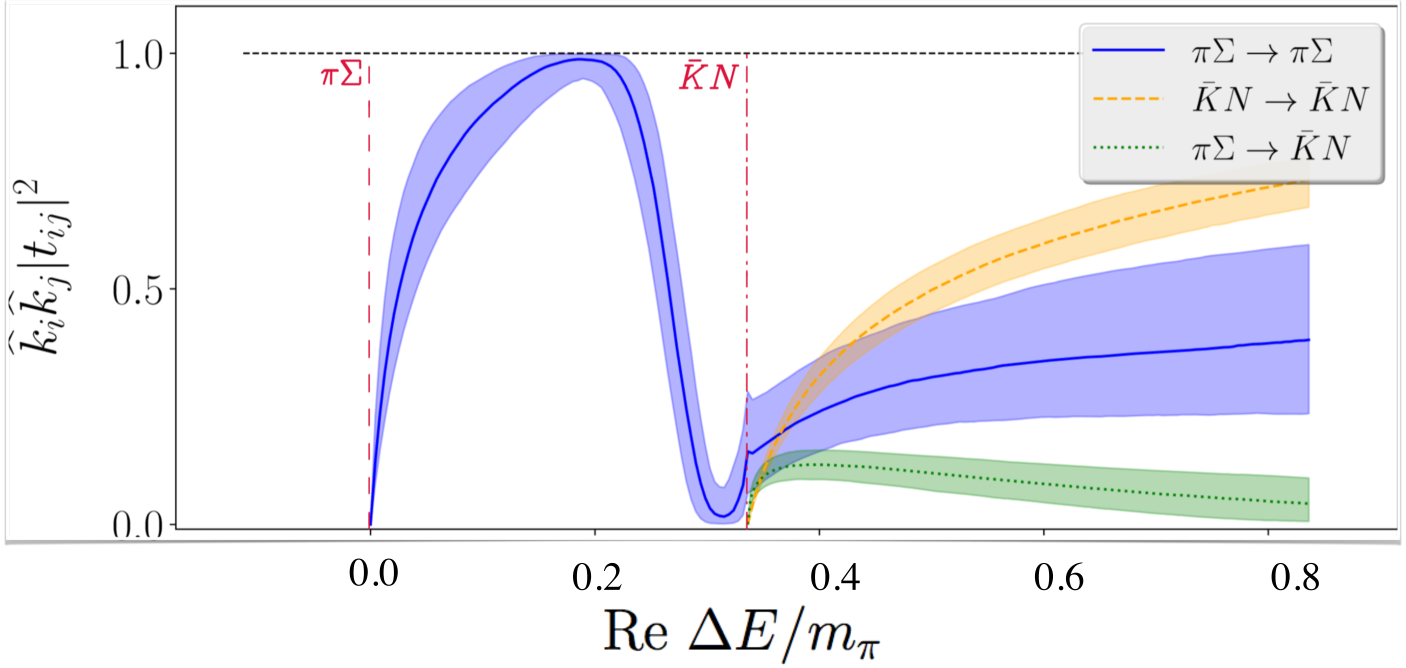
$$E_1 = 1392(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}\text{MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\rm stat}(6)_{\rm model}$$

Resonance

$$E_2 = \left[1455(13)_{\rm stat}(2)_{\rm model}(17)_{\rm a} - i11.5(4.4)_{\rm stat}(4.0)_{\rm model}(0.1)_{\rm a}\right]\text{MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2}_{\bar{K}N}}\right| = 0.53(9)_{\rm stat}(10)_{\rm model}$$



Apply our framework to unphysical world

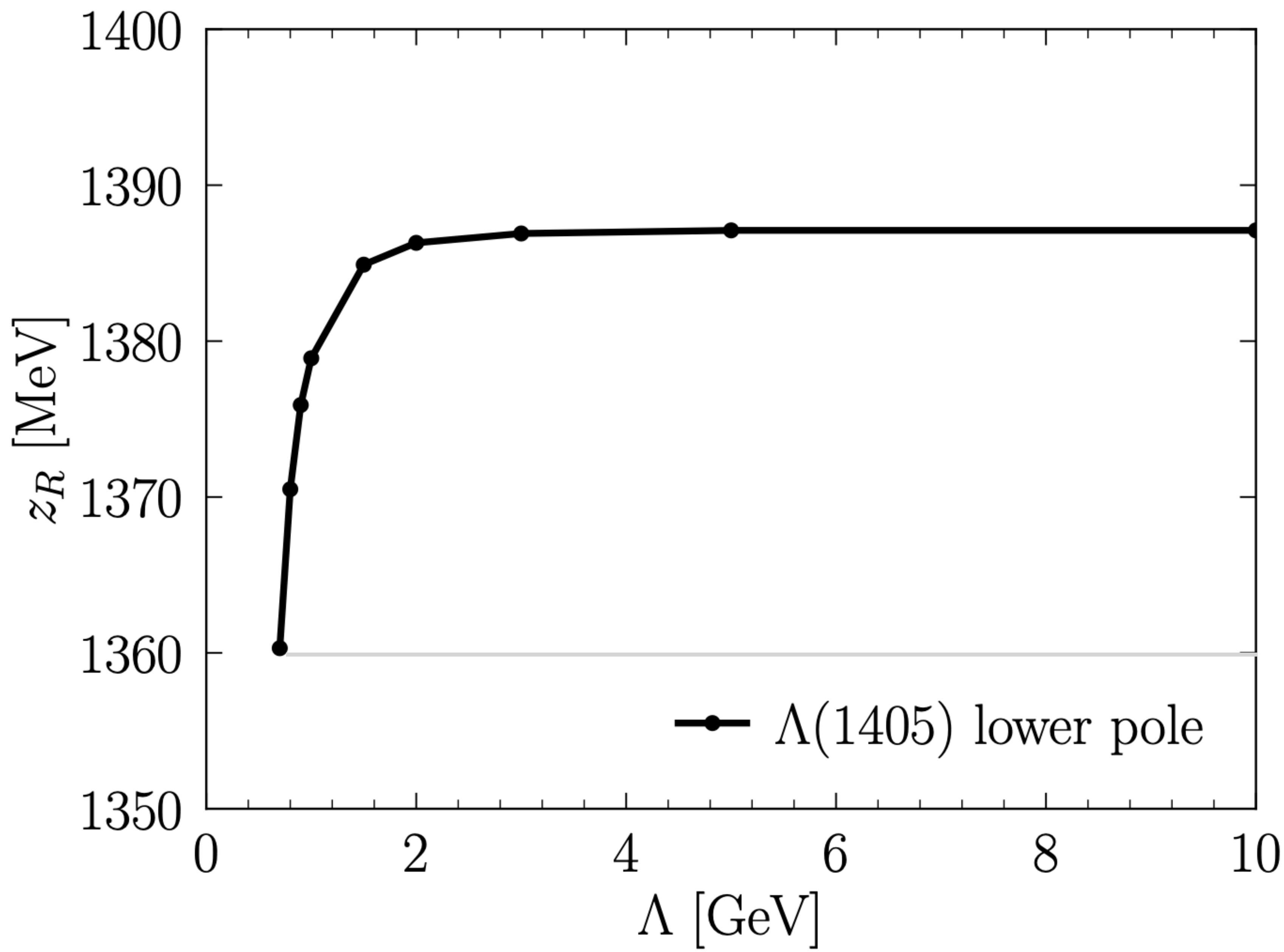
- BaSc results provide an ideal playground
  - **Check/verify the predictive power** of existing chiral unitary approaches
- We extend the calculation to the unphysical quark mass region
  - Take the **same** meson and baryon masses as the BaSc study
    - ✓ $M_\pi = 203.7$ MeV, $M_K = 486.4$ MeV, $m_N = 979.8$ MeV, $m_\Sigma = 1193.9$ MeV
    - ✓ $F_0 = F_\pi = 93.2$ MeV
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels
  - **Consistent with the BaSc results** $\Lambda \to \infty$, no free parameters

	BaSc [PRL2024]	This work			
$\Lambda(1405)$	z_R [MeV]	z_R [MeV]	$g_{\pi\Sigma}$	$g_{\bar{K}N}$	$\lvert g_{\pi\Sigma}\rvert / \lvert g_{\bar{K}N}\rvert$
Lower pole	**1392(18)**	1387.14	$0.021 + i1.87$	$0.017 + i1.55$	1.21
Higher pole	**1455(21) − i11.5(6.0)**	$1469.86 - i4.71$	$0.038 + i0.98$	$1.51 - i1.22$	0.50

  - A full calculation with $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ channels

    Lower pole : $z_R = 1389.05$ MeV   Higher pole : $z_R = 1464.55 - i9.44$ MeV

Cutoff independence of pole position

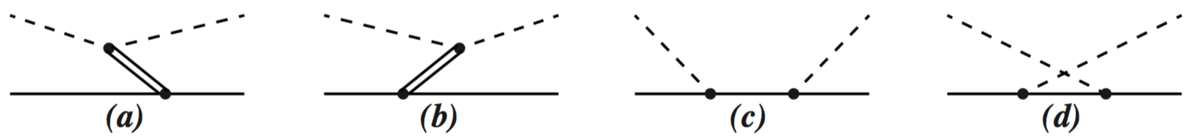


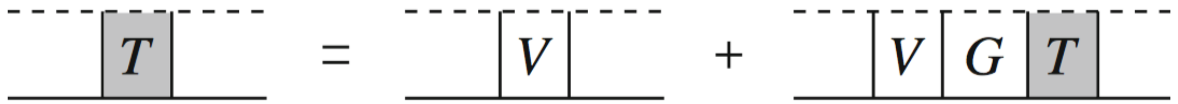
Finite-cutoff artifact
~ 30 MeV

XLR, Phys. Lett. B 855 (2024) 138802

Quark mass dependence

- Variables in our LO calculation

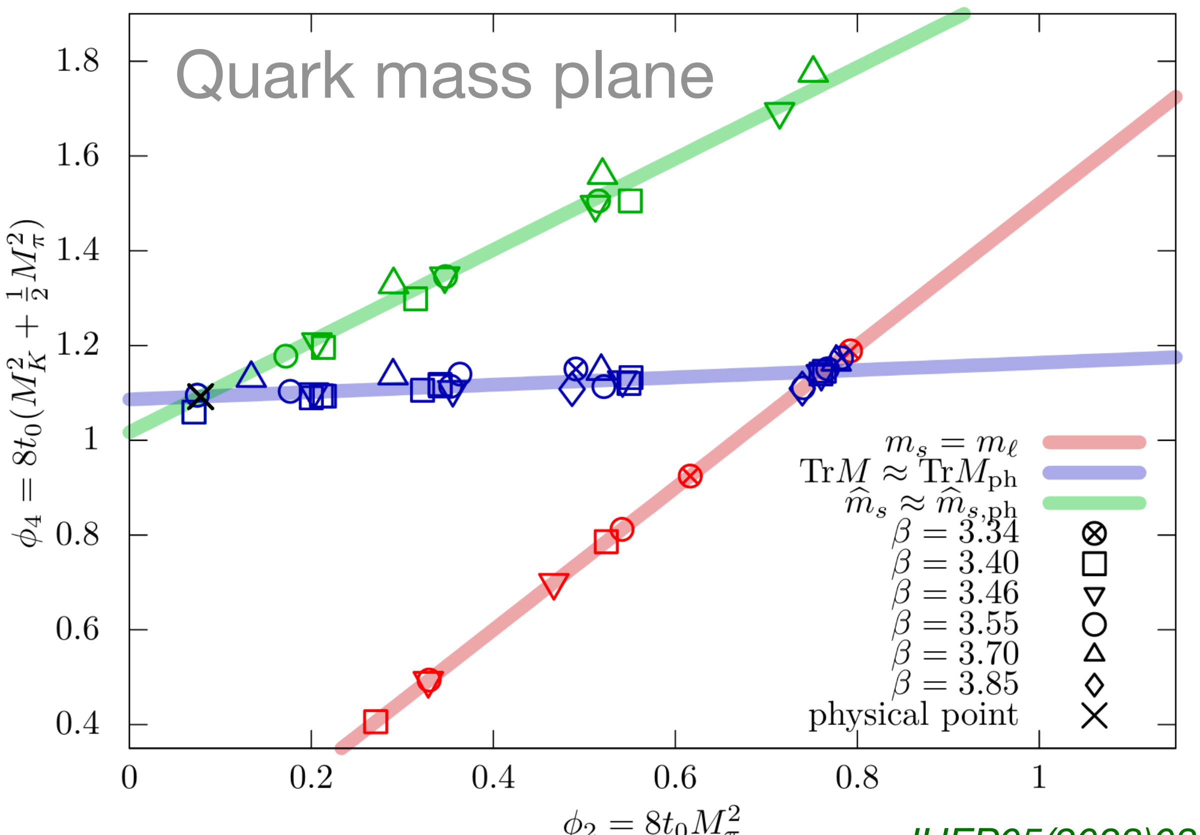




$$V_{\rm LO}(M_{\pi,K,\eta};\, m_{N,\Lambda,\Sigma,\Xi};\, F_0 \equiv F_\pi;\, M_{\rho,\omega,K^*,\phi})$$

$$G(M_{\pi,K,\eta};\, m_{N,\Lambda,\Sigma,\Xi})$$

- Fit **LQCD data** based on the CLS configuration
- with the quark mass trajectory $\mathrm{Tr}(M) = \text{const line}$



JHEP05(2023)035

$\mathrm{Tr}(M) = \text{const line}$

BaSc: "D200" ensemble

$M_{\pi,K}$ JHEP05(2023)035

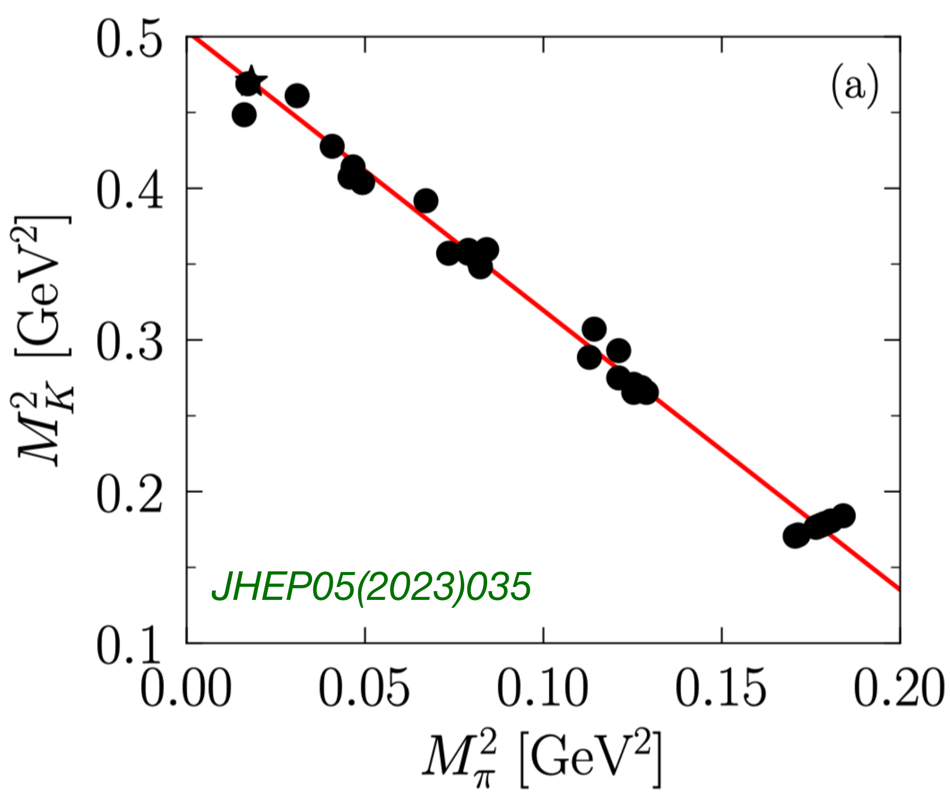
$m_{N,\Lambda,\Sigma,\Xi}$

F_π JHEP 08 (2022) 220

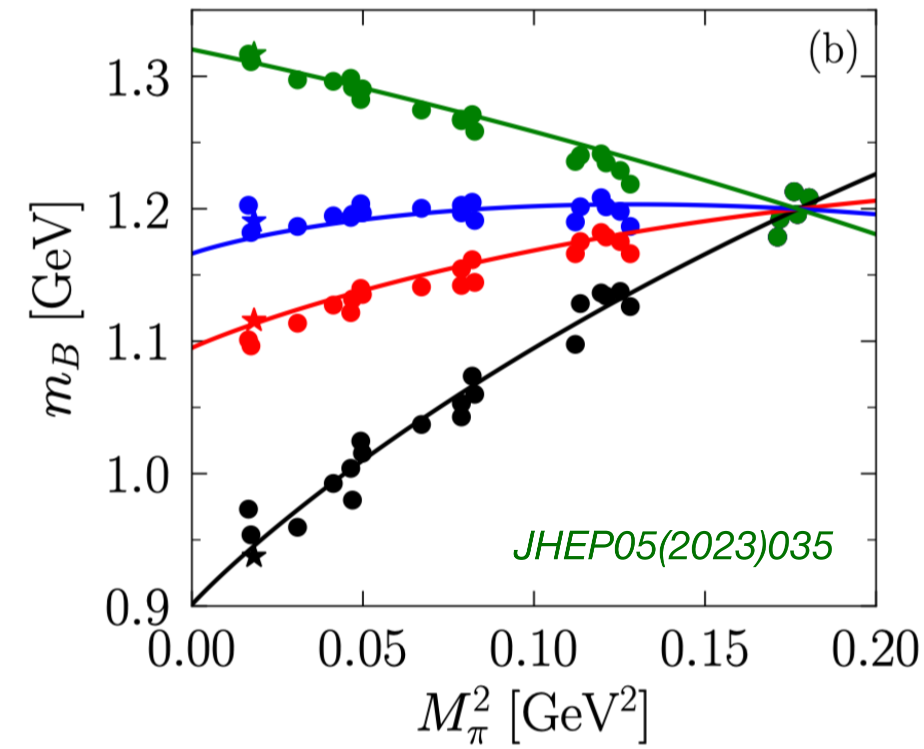
M_ρ PRD 106(2022)114502

Quark mass dependence

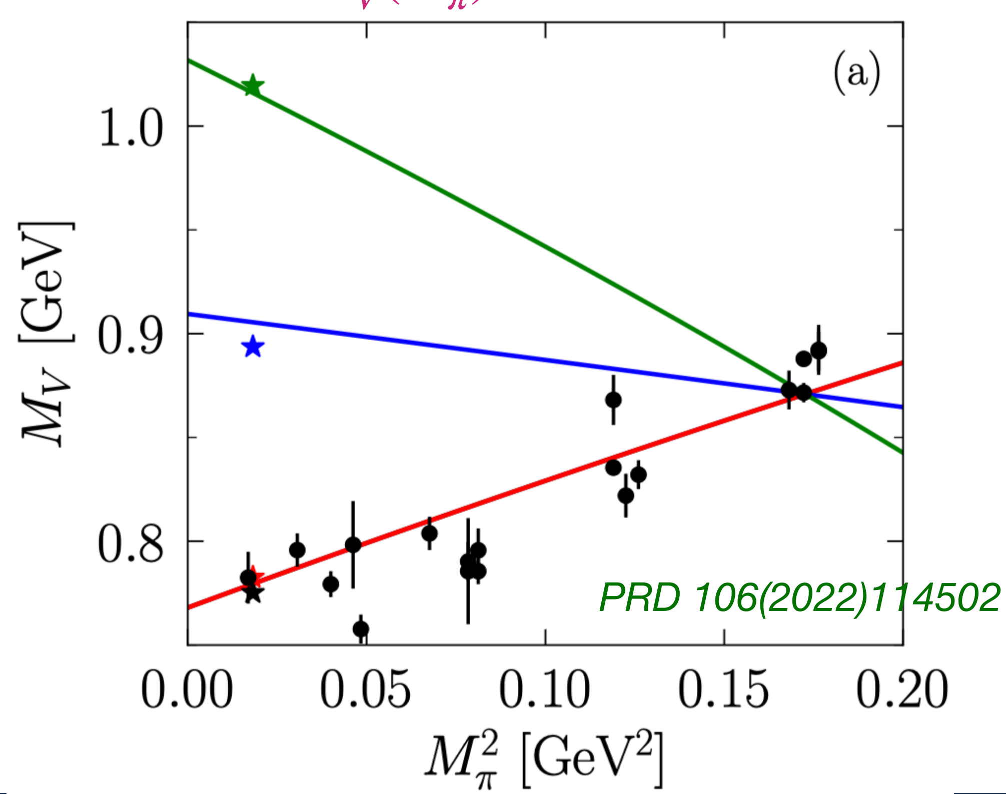
$M_K^2 = a + bM_\pi^2$ $\quad M_\eta^2 = (4M_K^2 - M_\pi^2)/3$



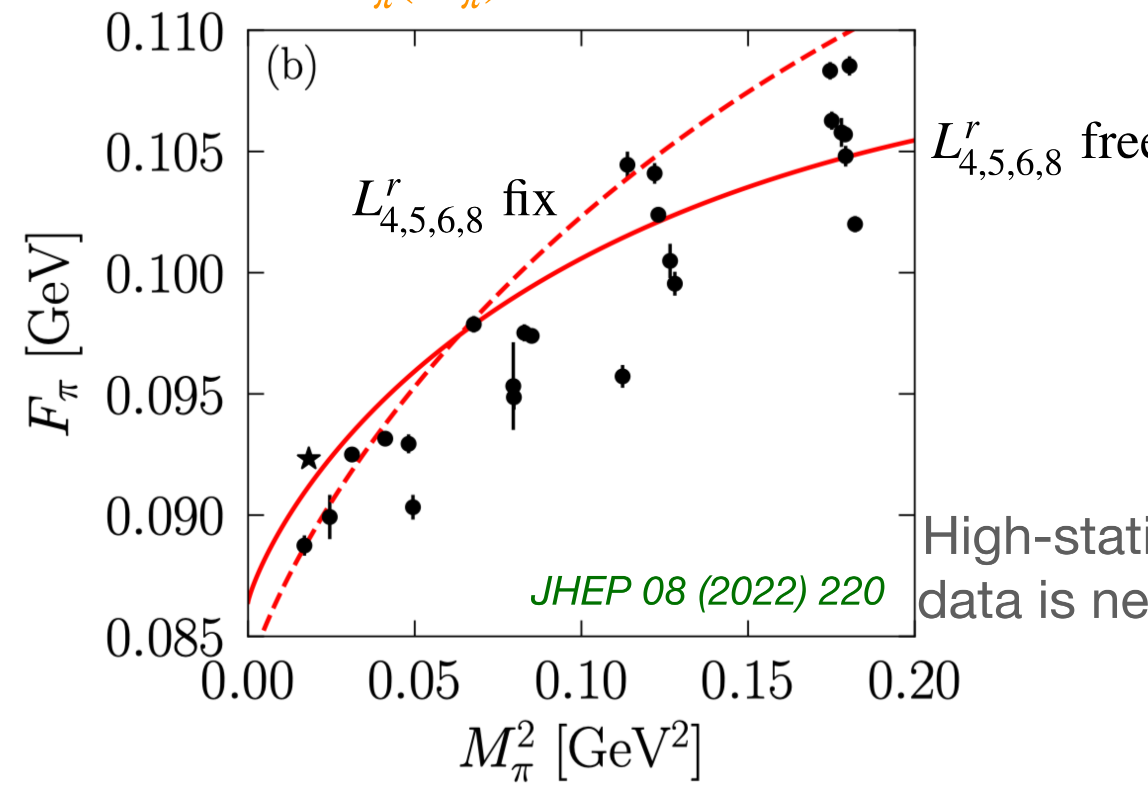
$$m_B = m_0 + \sum_{P=\pi,K} \xi_{PB}^{(2)} M_P^2 + \frac{1}{(4\pi F_0)^2} \sum_{P=\pi,K,\eta} \xi_{PB}^{(3)} H_B\left(\frac{M_P}{m_0}\right)$$



$M_V(M_\pi)$: LO ChPT



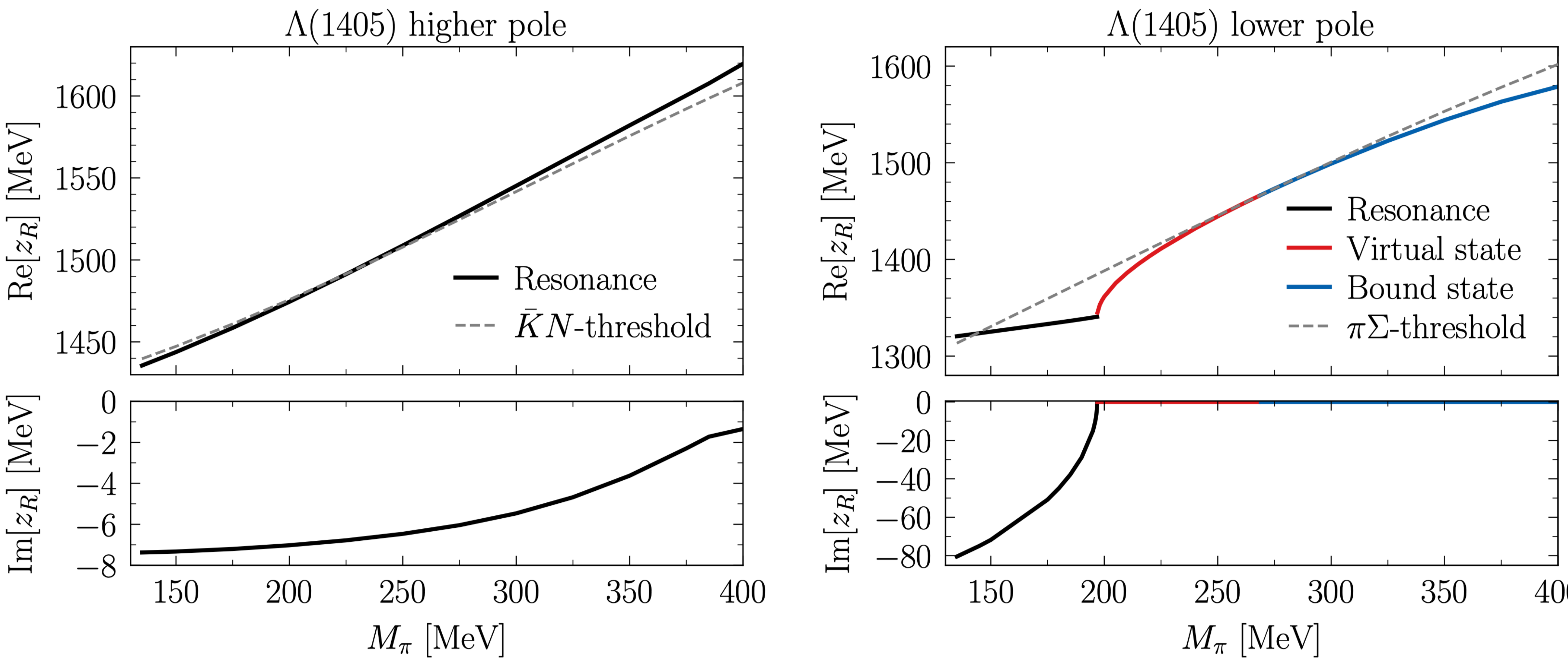
$F_\pi(M_\pi)$: NLO ChPT



High-statistics data is needed

Light-quark mass dependence of $\Lambda(1405)$

- Full calculation with $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ channels



With the increase of pion mass

- $\bar{K}N$ interaction gradually strengthens
- $\pi\Sigma$ interaction changes rapidly and is enhanced sufficiently

XLR, Phys. Lett. B 855 (2024) 138802

Next-to-leading order studies

XLR, et al., In preparation

Beyond leading order

- **Maintain the scattering T-matrix renormalizable**
  - Take LO potential non-perturbatively
  - Higher order corrections are perturbatively included

- Up to NNLO
  - Potential: $V = V_{\rm LO} + V_{\rm NLO} + V_{\rm NNLO}$
  - T-matrix: $T = T_{\rm LO} + T_{\rm NLO} + T_{\rm NNLO}$

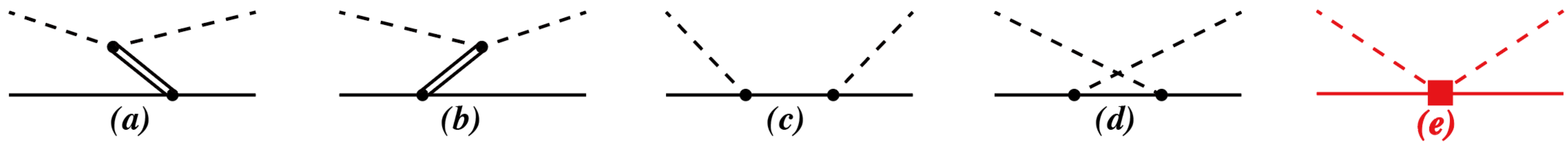
$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO}$$ (non-perturbative)

$$T_{\rm NLO} = V_{\rm NLO} + V_{\rm LO} G T_{\rm NLO} + V_{\rm NLO} G T_{\rm LO}$$

$$T_{\rm NNLO} = V_{\rm NNLO} + V_{\rm LO} G T_{\rm NNLO} + V_{\rm NLO} G T_{\rm NLO} + V_{\rm NNLO} G T_{\rm LO}$$

  - Use **the subtractive renormalization** to remove divergent terms and power-counting breaking terms

πN scattering at NLO



- Chiral effective Lagrangian

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

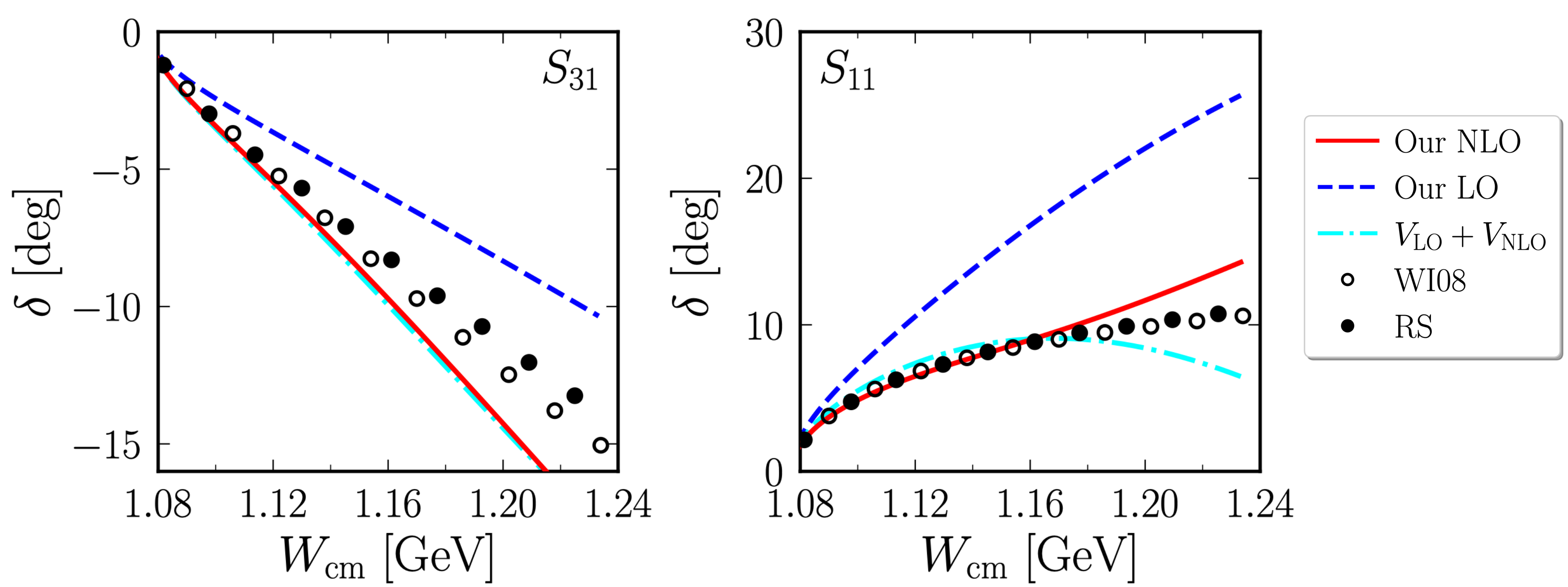
  - Fix $c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \text{GeV}^{-1}$

D. Siemens, et al., PLB770 (2017) 27-34

- NLO potential

$$\boxed{\begin{aligned} V = &\quad V_{\text{LO}} \quad + \quad V_{\text{NLO}} \\ = &\ V^{(a+b+c+d)}\big|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c+d)}\big|_{u=u_1\sim\mathcal{O}(p)} + V^{(e)}\big|_{u=u_0\sim(1,0)^\dagger} \end{aligned}}$$

- Prediction for the πN phase shifts



Summary

- **A renormalized framework** for MB scattering is proposed
  - Time-ordered perturbation theory + Covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Employ the subtractive renormalization
    - ✓ **Achieve T-matrix is cutoff-independent**
- Leading order study
  - πN scattering; $\bar{K}N$ scattering with coupled channels
  - Obtain the **two-pole structure** of $\Lambda(1405)$
  - **Predictive power:** consistent with LQCD results
- Higher order studies in progress
  - Higher order corrections are perturbatively included
  - πN scattering: improve the description of phase shifts

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  - Higher order corrections are perturbatively included
  - πN scattering: improve the description of phase shifts
  - **Plan: extend to $\bar{K}N$ scattering, $\Lambda(1405)$, and other resonances**

Thank you for your attention!

Back up

A Feynman diagram represents the sum of **all** time orderings