第十届手征有效场论研讨会
2025 年 10 月

Study of Nuclear System using Relativistic Chiral Force

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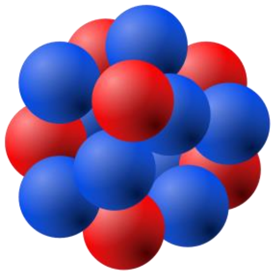
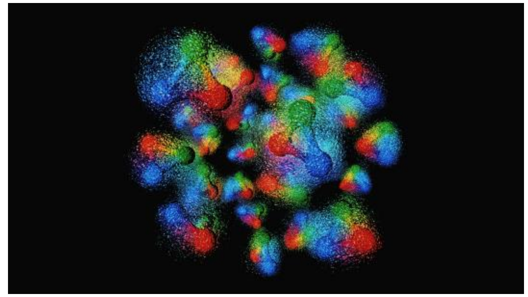
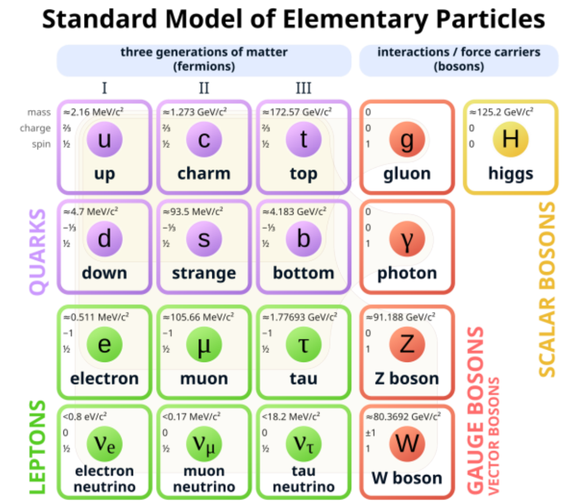






D. Drischler, W. Haxton, K. McElvain, E. Mereghetti, A. Nicholson, P. Vranas, and A. Walker-Loud, **Towards grounding nuclear physics in QCD**, Prop. Part. Nucl. Phys. 121 (2021) 103888

https://en.wikipedia.org/wiki/Standard_Model
https://www.quantamagazine.org/colliders-and-supercomputers-force-fresh-hints-in-quark-mystery-20220214/

Proper degrees-of-freedom (nucleons) and energy scale

- Ekström, C. Forssén, G. Hagen, G. R. Jansen, W. Jiang and T. Papenbrock, what is ab initio in nuclear theory? Front. Phys. 11:1129094 (2023)
- R. Machleidt, what is ab initio? Few-Body Syst (2023) 64

Do the same nuclear forces that explain ***free-space scattering*** *experiments also explain the properties of* ***finite nuclei*** *and* ***nuclear matter*** *when applied in nuclear many-body theory?*

R. Machleidt, what is ab initio? Few-Body Syst (2023) 64

Early Attempt

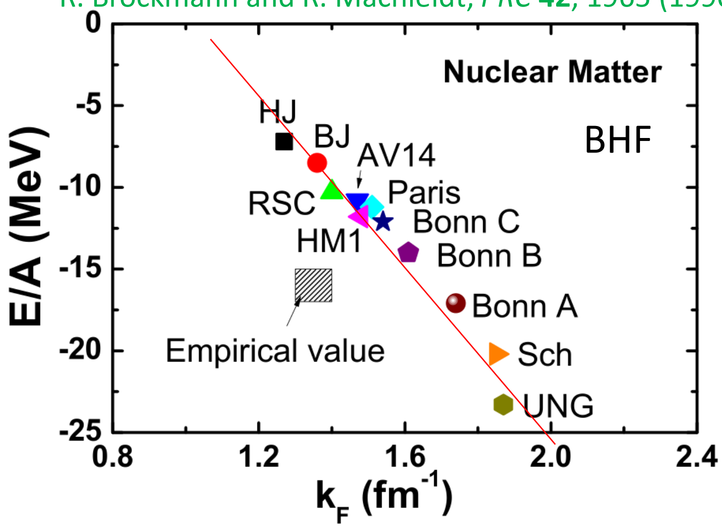
- Early phenomenological nuclear forces

R. Jastrow, Phys. Rev. 81 (1951) 165–170; J.L. Gammel, R.M. Thaler, Phys. Rev. 107 (1957) 291–298, T. Hamada, I. Johnston, Nuclear Phys. 34 (1962) 382–403; R.V. Reid, Ann. Phys., NY 50 (1968) 411–448; K. Erkelenz, Phys. Rep. 13 (1974) 191–258; ...

- Early many-body framework

- Brueckner theory K.A. Brueckner, C.A. Levinson, H.M. Mahmoud, Phys. Rev. 95, 217–228 (1954)
- Variation with correlation function R. Jastrow, Phys. Rev. 98 (1955) 1479–1484

R. Brockmann and R. Machleidt, *PRC* **42**, 1965 (1990) H. Müther, R. Brockmann, and R. Machleidt, *PRC* **42**, 1981 (1990)



^{16}O	Bonn C	Bonn B	Bonn A	Exp.
E/A (MeV)	-4.49	-5.35	-6.56	**-7.98**
r_c (fm)	2.465	2.380	2.291	**2.70**

Nuclear Physics **A328** (1979) 587–595 © *North-Holland Publishing Co., Amsterdam*

UPDATE ON THE CRISIS IN NUCLEAR-MATTER THEORY: A SUMMARY OF THE TRIESTE CONFERENCE

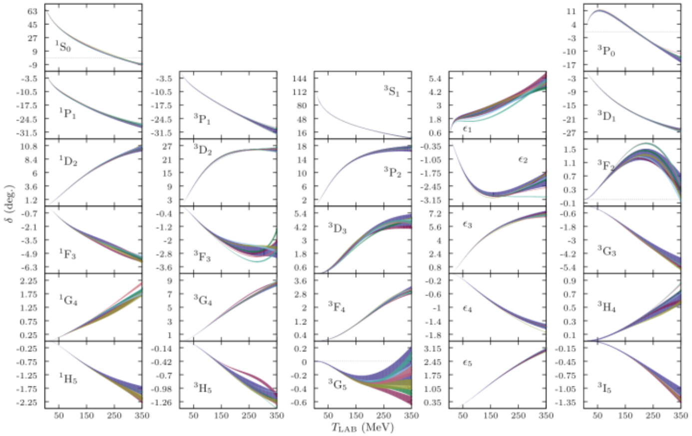
J. W. CLARK

*McDonnell Center for the Space Sciences and Department of Physics, Washington University, St. Louis, Missouri 63130**

Similar findings by variational method

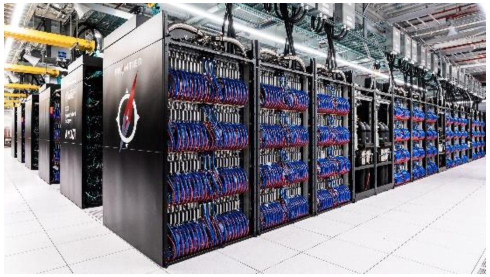
High-Precision Era

- PWA93 phase shift V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, J.J. de Swart, Phys. Rev. C 48 (1993) 792–815
- High-precision NN interaction in the 90s
  - Reid93 V.G.J. Stoks, et al., Phys. Rev. C 49 (1994) 2950
  - Argonne V18 R. Wiringa, et al., Phys. Rev. C 51 (1995) 38–51
  - CD-Bonn R. Machleidt, Phys. Rev. C 63 (2001) 024001



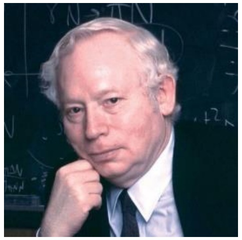
Arriola, E.R., Pérez, R.N. (2023). NN Experiments and NN Phase-Shift Analysis

- Large-scale many-body framework
  - Coupled-cluster theory G. Hagen, et al., Rep. Prog. Phys. 77, 096302 (2014)
  - Deep-neural-network Y. L. Yang and P. W. Zhao, Phys. Rev. C 107, 034320 (2023)
  - Gamow shell model Jianguo Li, et al., Physics 2021, 3, 977
  - In medium similarity renormalization group H. Hergert, et al., Phys. Rep. 621, 165 (2016)
  - Monte Carlo shell model T. Otsuka et al., Prog. Part. Nucl. Phys. 47, 319 (2001)
  - No core shell model B. R. Barrett, P. Navrátil, J. P. Vary, Prog. Part. Nucl. Phys. 69, 131 (2013)
  - Nuclear lattice effective field theory D. Lee, Prog. Part. Nucl. Phys. 63, 117 (2009)
  - Quantum Monte Carlo method J. Carlson, et al., Rev. Mod. Phys. 87, 1067 (2015)
  - Self-consistent Green’s function W. Dickhoff and C. Barbieri, Prog. Part. Nucl. Phys. 52, 377 (2004)
  - … …

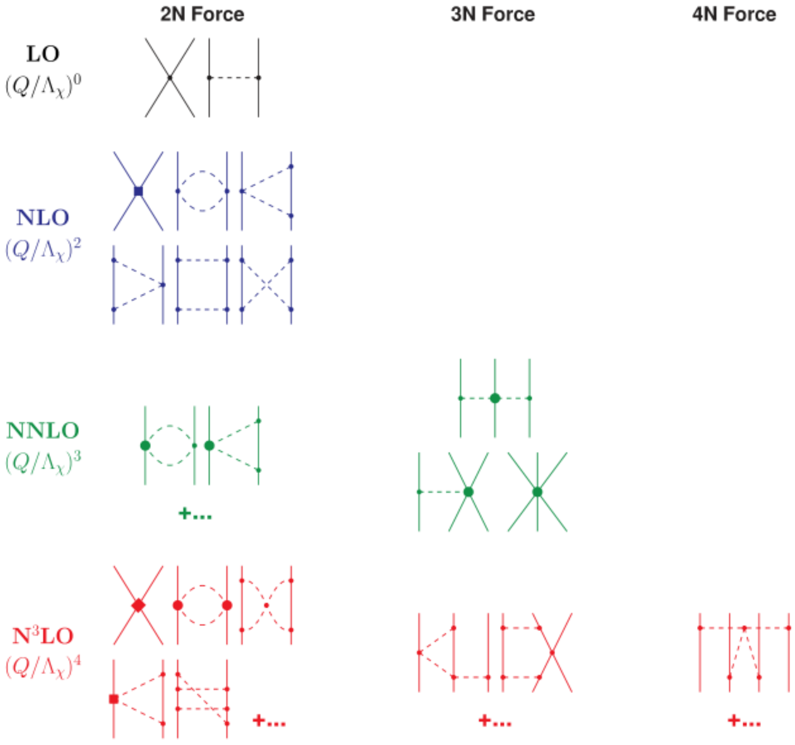


Chiral EFT Era

- Chiral effective field theory for the low-energy region of QCD
  S. Weinberg, Phys. Lett. B 251 (1990) 288–292; S. Weinberg, Nuclear Phys. B 363 (1991) 3–18



- Hierarchy of nuclear forces in ChPT



- Systematically improvable
- Theoretical uncertainty can be determined
- 3N, 4N, … many-body forces appear naturally in the same hierarchy

E. Epelbaum, H.W. Hammer, U.G. Meißner, Rev. Mod. Phys. 81 (2009) 1773–1825
R. Machleidt, D. Entem, Phys. Rep. 503 (2011) 1–75

Challenges

PRL 117, 052501 (2016) PHYSICAL REVIEW LETTERS week ending 29 JULY 2016

Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,[1,*] V. Somà,[1] C. Barbieri,[2] H. Hergert,[3] J. D. Holt,[4] and S. R. Stroberg[4]
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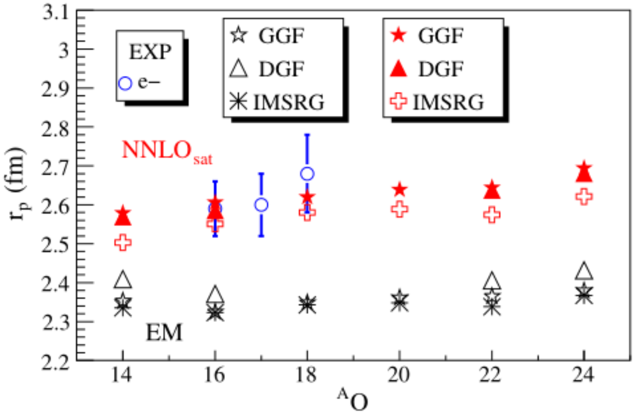
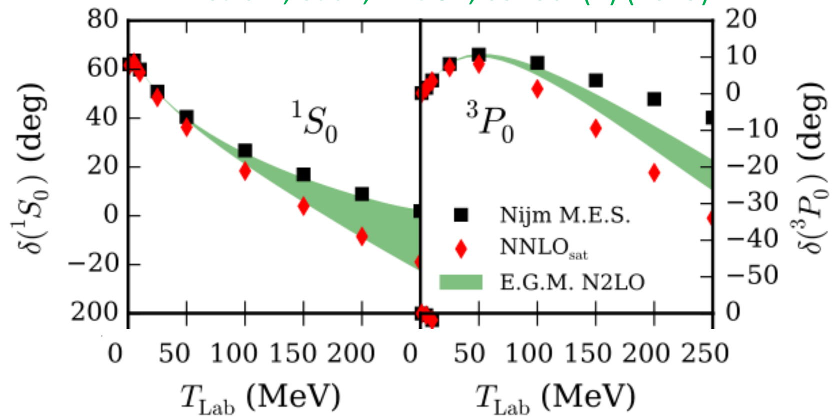


A. Ekström, et al., PRC **91**, 051301(R) (2015)



Article

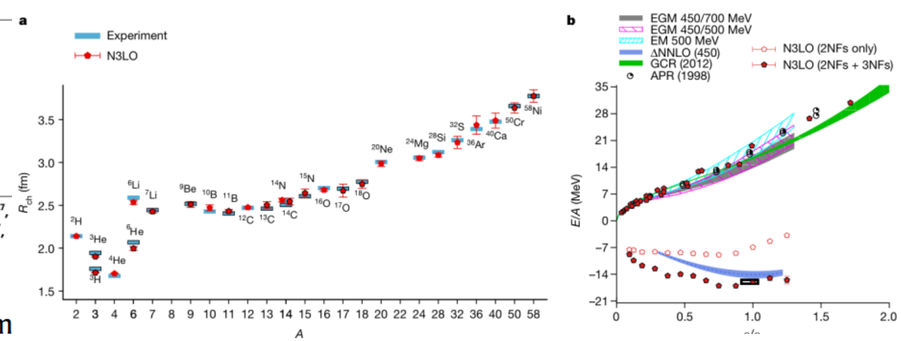
Wavefunction matching for solving quantum many-body problems

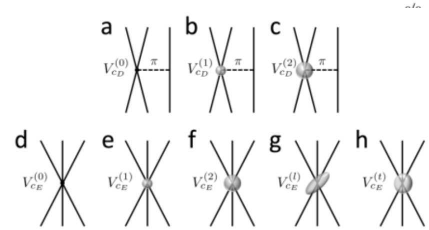
Nature **volume 630**, 59–63 (2024)

https://doi.org/10.1038/s41586-024-07422-z
Received: 23 November 2022
Accepted: 15 April 2024

Serdar Elhatisari[1,2], Lukas Bovermann[3], Yuan-Zhuo Ma[4,5], Evgeny Epelbaum[3], Dillon Frame[6,7], Fabian Hildenbrand[6,7], Myungkuk Kim[8], Youngman Kim[8], Hermann Krebs[3], Timo A. Lähde[6,7], Dean Lee[4,✉], Ning Li[9], Bing-Nan Lu[10], Ulf-G. Meißner[2,6,7,11], Gautam Rupak[12], Shihang Shen[6,7], Young-Ho Song[13] & Gianluca Stellin[14]

data. These results are accompanied by insights on the nuclear interactions that m help to resolve long-standing challenges in accurately reproducing nuclear binding energies, charge radii and nuclear-matter saturation in ab initio calculations[19,20].





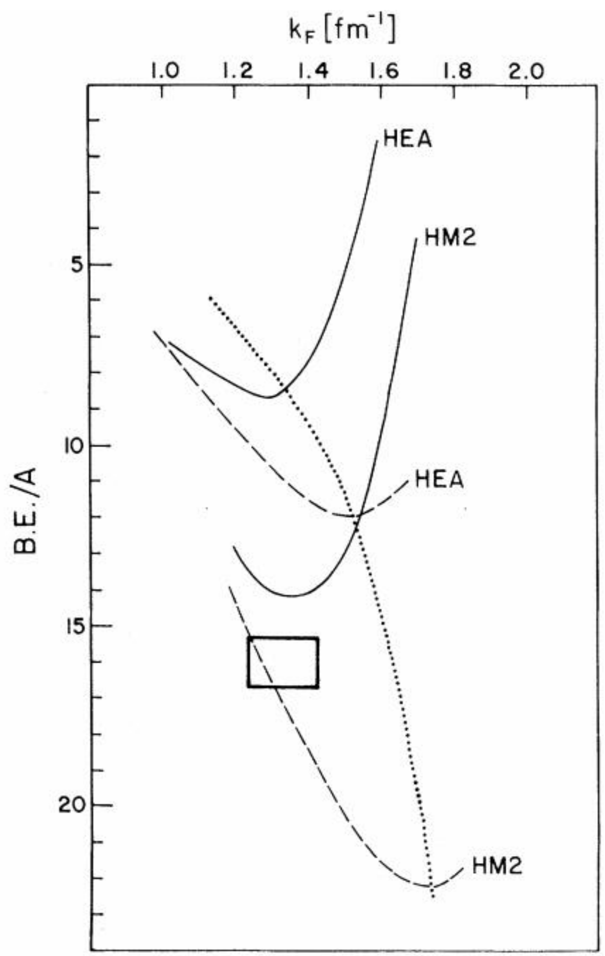
Still a challenge to achieve a universal description

Relativistic Brueckner-Hartree-Fock Theory for Nuclear Matter

VOLUME 45, NUMBER 26 PHYSICAL REVIEW LETTERS 29 DECEMBER 1980

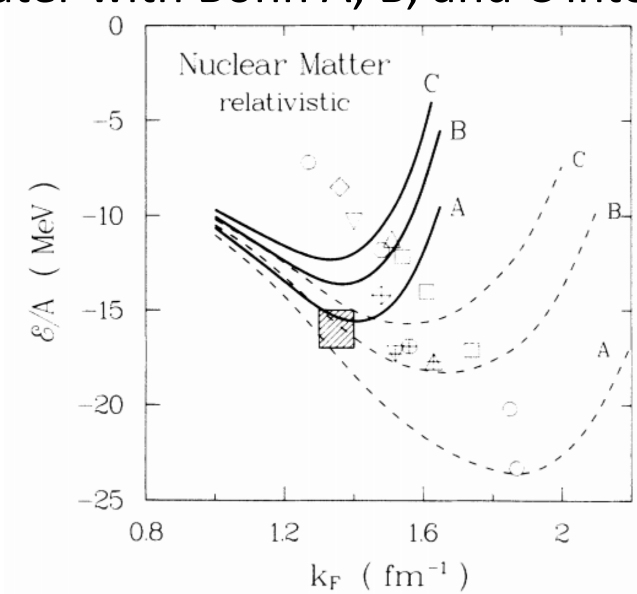
Nuclear Saturation as a Relativistic Effect

M. R. Anastasio, L. S. Celenza, and C. M. Shakin
Institute for Nuclear Theory and Department of Physics, Brooklyn College of the City University of New York, Brooklyn, New York 11210



M. R. Anastasio, L. S. Celenza, and C. M. Shakin, PRL **45**, 2096 (1980)
M. R. Anastasio, L. S. Celenza, and C. M. Shakin, PRC **23**, 569 (1981)
M. R. Anastasio, L. S. Celenza, and C. M. Shakin, PRC **23**, 2258 (1981)
M. R. Anastasio, L. S. Celenza, and C. M. Shakin, PRC **23**, 2273 (1981)
M. R. Anastasio, L. S. Celenza, and C. M. Shakin, PRC **23**, 2606 (1981)
M. Anastasio, L. Celenza, W. Pong, C. Shakin, Phys. Rep. **100**, 327 (1983)

Later with Bonn A, B, and C interactions



R. Brockmann and R. Machleidt, *PRC* **42**, 1965 (1990)

Recent Development in RBHF Nuclear Matter

- Angle-averaged center-of-mass momentum approximation is removed

  H. Tong, et al., Phys. Rev. C 98 (5) (2018) 054302

- Full Dirac space has been developed

  S. Wang, et al., Phys. Rev. C 103, 054319 (2021)

- High-precision charge-dependent potentials

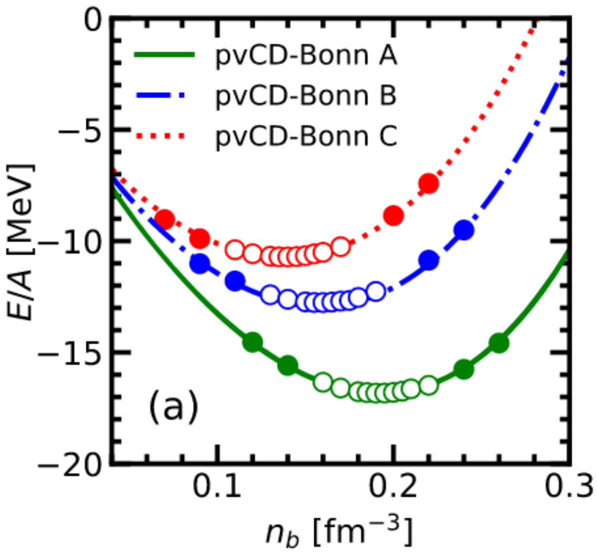
  Chencan Wang, Jinniu Hu, Ying Zhang and Hong Shen, J. Phys. G 47 (2020) 105108

- Neutron star properties

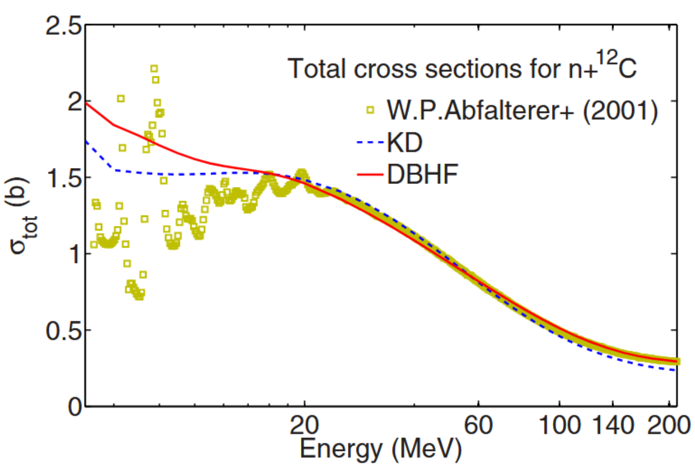
  Delaney Farrell and Fridolin Weber, ApJ 969:49, 2024
  S. Wang, et al., Phys. Rev. C 106, L021305 (2022)
  H. Tong, C. Wang, and S. Wang, ApJ 930:137, 2022

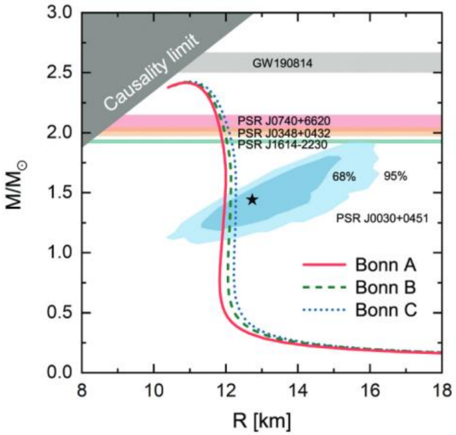
- Optical potentials

  P. Qin, et al., Phys. Rev. C 109, 064603 (2024)
  R. Xu, et al., Phys. Rev. C 94, 034606 (2016)



C Wang, et al., J. Phys. G 47 (2020) 105108)





S. Wang, et al., Phys. Rev. C 106, L021305 (2022)

Relativistic Nucleon-Nucleon Interaction

- One boson exchange model: Bonn interaction R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989), pvCD-Bonn C. Wang, J. Hu, Y. Zhang, H. Shen, *Chin. Phys. C* **43**, 114107 (2019) , ...
- Relativistic chiral interaction

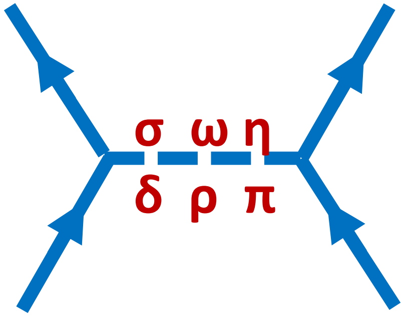
  X.L. Ren, K.W. Li, L.S. Geng, B. Long, P. Ring, J. Meng, Chin. Phys. C 42 (2018) 014103
  K.W. Li, X.L. Ren, L.S. Geng, B.W. Long, Chin. Phys. C 42 (2018) 014105
  J.-X. Lu, C.-X. Wang, Y. Xiao, L.-S. Geng, J. Meng, P. Ring, Phys. Rev. Lett. 128, 142002 (2022)

- Take the Bonn interaction as an example

$$\mathscr{L}_{NNpv} = -\frac{f_{ps}}{m_{ps}}\bar{\psi}\gamma^5\gamma^\mu\psi\partial_\mu\varphi^{(ps)},$$

$$\mathscr{L}_{NNs} = g_s\bar{\psi}\psi\varphi^{(s)},$$

$$\mathscr{L}_{NNv} = -g_v\bar{\psi}\gamma^\mu\psi\varphi^{(v)}_\mu - \frac{f_v}{4M}\bar{\psi}\sigma^{\mu\nu}\psi\left(\partial_\mu\varphi^{(v)}_\nu - \partial_\nu\varphi^{(v)}_\mu\right) .$$



- Bosons to be exchanged include σ, δ (scalar); ω, ρ (vector); η, π (pseudovector).
- A monopole-type form factor is attached to each vertex.

$$\frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + \mathbf{q}^2}$$

Relativistic Hamiltonian

- Hamiltonian in second quantized form

$$H = \sum_{k'k} \langle k'|T|k\rangle a^\dagger_{k'} a_k + \frac{1}{2} \sum_{k'l'kl} \langle k'l'|V|kl\rangle a^\dagger_{k'} a^\dagger_{l'} a_l a_k,$$

- Matrix elements are defined as

$$\langle k'|T|k\rangle = \int d^3r\, \bar{\psi}_{k'}(\mathbf{r})\,(-i\boldsymbol{\gamma}\cdot\nabla + M)\,\psi_k(\mathbf{r}),$$

$$\langle k'l'|V_\phi|kl\rangle = \int d^3r_1 d^3r_2\, \bar{\psi}_{k'}(\mathbf{r}_1)\Gamma^{(1)}_\phi \psi_k(\mathbf{r}_1) D_\phi(\mathbf{r}_1,\mathbf{r}_2)\bar{\psi}_{l'}(\mathbf{r}_2)\Gamma^{(2)}_\phi \psi_l(\mathbf{r}_2).$$

- Interaction vertex

$$\Gamma_s = g_s, \qquad \Gamma_{pv} = \frac{f_{ps}}{m_{ps}}\gamma^5\gamma^i\partial_i, \qquad \Gamma^\mu_v = g_v\gamma^\mu + \frac{f_v}{2M}\sigma^{i\mu}\partial_i.$$

- Propagator

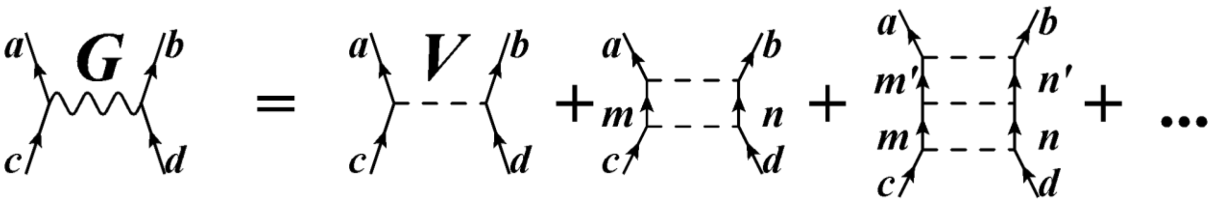
$$D_\phi(x_1,x_2) = \pm\int \frac{d^4q}{(2\pi)^4}\frac{1}{m_\phi^2 - q^2} e^{-iq\cdot(x_1-x_2)},$$

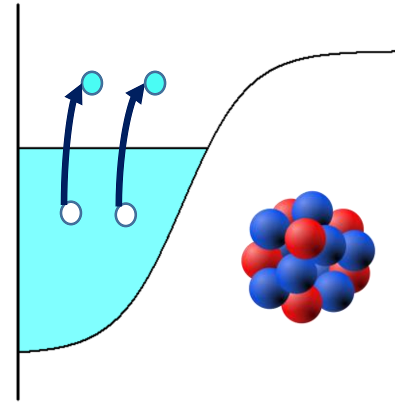
in Bonn interaction the retardation effect is ignored

$$D_\phi(\mathbf{r}_1,\mathbf{r}_2) = \pm\int \frac{d^3q}{(2\pi)^3}\frac{1}{m_\phi^2 + \mathbf{q}^2} e^{i\mathbf{q}\cdot(\mathbf{r}_1-\mathbf{r}_2)} = \pm\frac{1}{4\pi}\frac{e^{-m_\phi|\mathbf{r}_1-\mathbf{r}_2|}}{|\mathbf{r}_1-\mathbf{r}_2|}.$$

Effective Interaction: G-Matrix

- Bethe-Goldstone (BG) equation





$$\langle ab|G(W)|cd\rangle = \langle ab|V|cd\rangle + \sum_{mn} \langle ab|V|mn\rangle \frac{Q(m,n)}{W - e_m - e_n} \langle mn|G(W)|cd\rangle$$

- Q is the Pauli operator which forbids the states being scattered below Fermi surface.

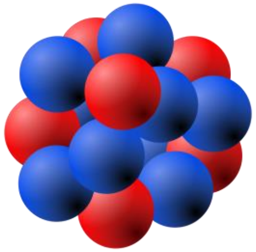
$$Q = \begin{cases} 1, & e_m, e_n > e_F \\ 0, & e_m \leq e_F \text{ or } e_n \leq e_F \end{cases}$$

- W is the so-called starting energy.

Hartree-Fock Equation

➢ Wave function (spherical symmetry) $|a\rangle = \frac{1}{r}\begin{pmatrix} F_{n_a\kappa_a}(r)\Omega^{l_a}_{j_a m_a}(\theta,\varphi) \\ iG_{n_a\kappa_a}(r)\Omega^{\tilde{l}_a}_{j_a m_a}(\theta,\varphi)\end{pmatrix}$

$$\Omega^{l}_{jm}(\theta,\varphi) = \sum_{m_l m_s} C^{jm}_{lm_l\frac{1}{2}m_s} Y_{lm_l}(\theta,\varphi)\chi_{m_s}.$$



➢ Single-particle equation of motion (Dirac equation)

$$\sum_l (T_{kl}+U_{kl})\,D_{la} = e_a D_{ka}. \qquad |a\rangle = \sum_k D_{ka}|k\rangle,$$

➢ Kinetic energy

$$\langle k'|T|k\rangle = \int d^3r\,\bar{\psi}_{k'}(\mathbf{r})\,(-i\boldsymbol{\gamma}\cdot\nabla + M)\,\psi_k(\mathbf{r}),$$

➢ Single-particle potential

$$\langle a|U^{\mathrm{RBHF}}|b\rangle = \begin{cases} \frac{1}{2}\sum_{c=1}^{A}\langle ac|\bar{G}(e_a+e_c)+\bar{G}(e_b+e_c)|bc\rangle, & e_a, e_b \le e_F \\ \sum_{c=1}^{A}\langle ac|\bar{G}(e_a+e_c)|bc\rangle, & e_a \le e_F,\ e_b > e_F \\ \sum_{c=1}^{A}\langle ac|\bar{G}(e'+e_c)|bc\rangle, & e_a, e_b > e_F, \end{cases}$$

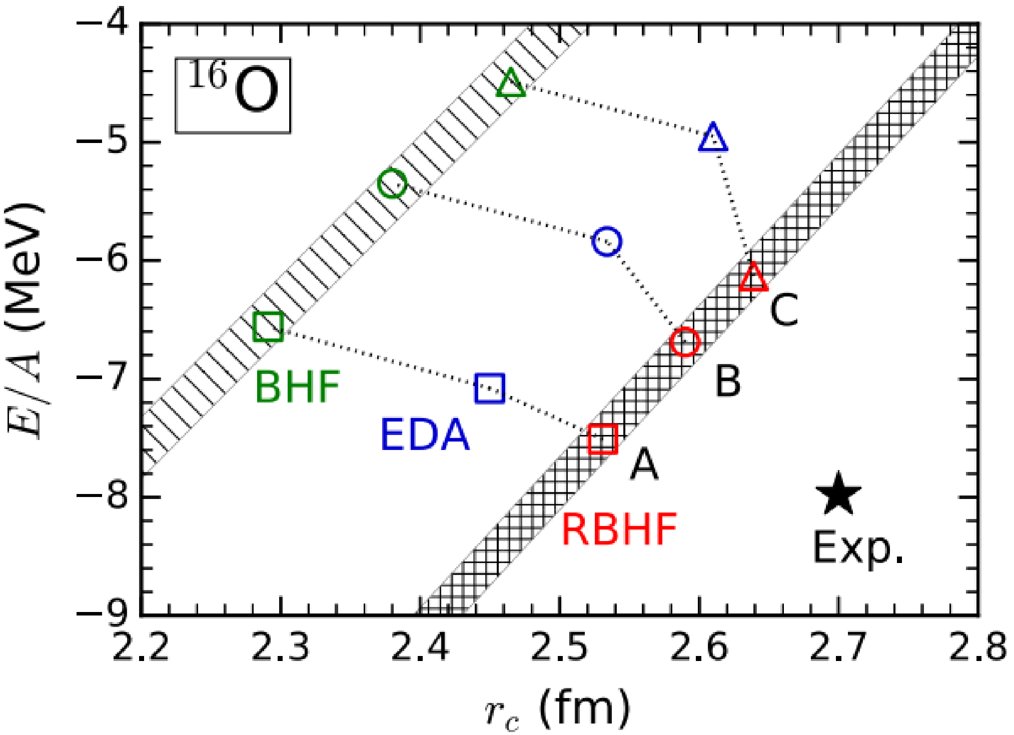
$$U_{k'k} = \sum_{c=1}^{A}\sum_{l'l}\bar{G}_{k'l'kl}(W)D^{*}_{l'c}D_{lc}.$$

$$\langle ab|G(W)|cd\rangle = \langle ab|V|cd\rangle + \sum_{mn}\langle ab|V|mn\rangle\frac{Q(m,n)}{W-e_m-e_n}\langle mn|G(W)|cd\rangle$$

RBHF for ^{16}O

- Total energies and charge radii of ^{16}O calculated by RBHF, BHF, and BHF+EDA with Bonn A, B, and C interactions.

SS, J. Hu, H. Liang, J. Meng, P. Ring, and S. Zhang, Chin. Phys. Lett. 33, 102103 (2016)
SS, H. Liang, J. Meng, P. Ring, and S. Zhang, Phys. Rev. C 96, 014316 (2017)



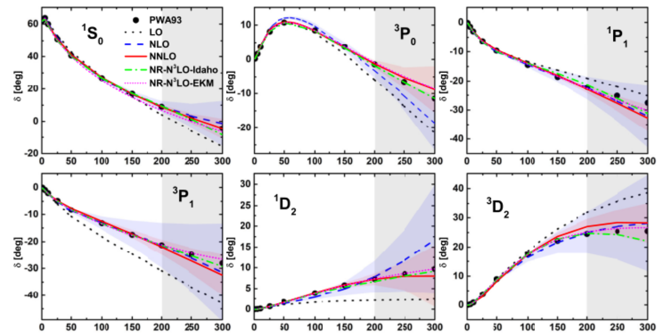
For BHF and EDA: H. Müther, R. Brockmann, and R. Machleidt, *PRC* **42**, 1981 (1990)

RBHF with Relativistic Chiral Interaction

PHYSICAL REVIEW LETTERS **128**, 142002 (2022)

Accurate Relativistic Chiral Nucleon-Nucleon Interaction up to Next-to-Next-to-Leading Order

Jun-Xu Lu,[1,2] Chun-Xuan Wang,[2] Yang Xiao,[2,3] Li-Sheng Geng,[2,4,5,*] Jie Meng,[6] and Peter Ring[7]



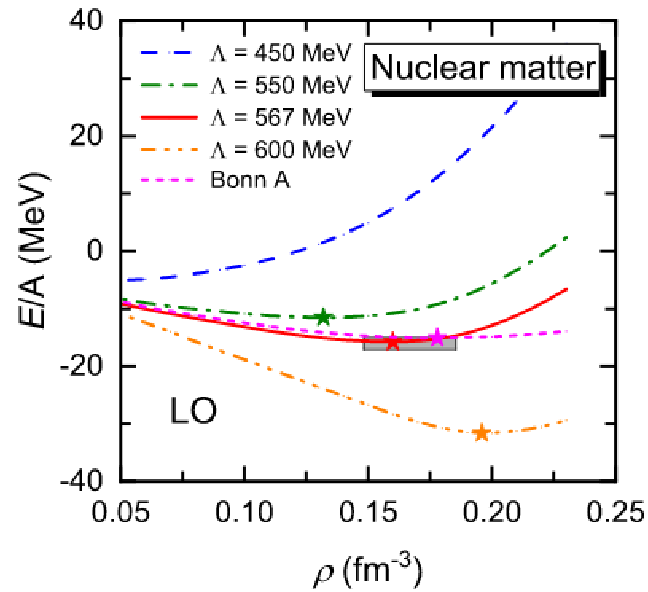
Phys. Lett. B 854 (2024) 138732

Saturation of nuclear matter in the relativistic Brueckner-Hatree-Fock approach with a leading order covariant chiral nuclear force

Wei-Jiang Zou[a], Jun-Xu Lu[b,c], Peng-Wei Zhao[a], Li-Sheng Geng[c,d,e,f,*], Jie Meng[a,*]

$$
\begin{aligned}
V_{\mathrm{CTP}}(\boldsymbol{p}',\boldsymbol{p}) = {} & C_S\left[\bar{u}(\boldsymbol{p}',\lambda_1')u(\boldsymbol{p},\lambda_1)\right]\left[\bar{u}(-\boldsymbol{p}',\lambda_2')u(-\boldsymbol{p},\lambda_2)\right] \\
& +C_V\left[\bar{u}(\boldsymbol{p}',\lambda_1')\gamma_\mu u(\boldsymbol{p},\lambda_1)\right]\left[\bar{u}(-\boldsymbol{p}',\lambda_2')\gamma^\mu u(-\boldsymbol{p},\lambda_2)\right] \\
& +C_{AV}\left[\bar{u}(\boldsymbol{p}',\lambda_1')\gamma_\mu\gamma_5 u(\boldsymbol{p},\lambda_1)\right]\left[\bar{u}(-\boldsymbol{p}',\lambda_2')\gamma^\mu\gamma_5 u(-\boldsymbol{p},\lambda_2)\right] \\
& +C_T\left[\bar{u}(\boldsymbol{p}',\lambda_1')\sigma_{\mu\nu} u(\boldsymbol{p},\lambda_1)\right]\left[\bar{u}(-\boldsymbol{p}',\lambda_2')\sigma^{\mu\nu} u(-\boldsymbol{p},\lambda_2)\right], \quad (12)
\end{aligned}
$$

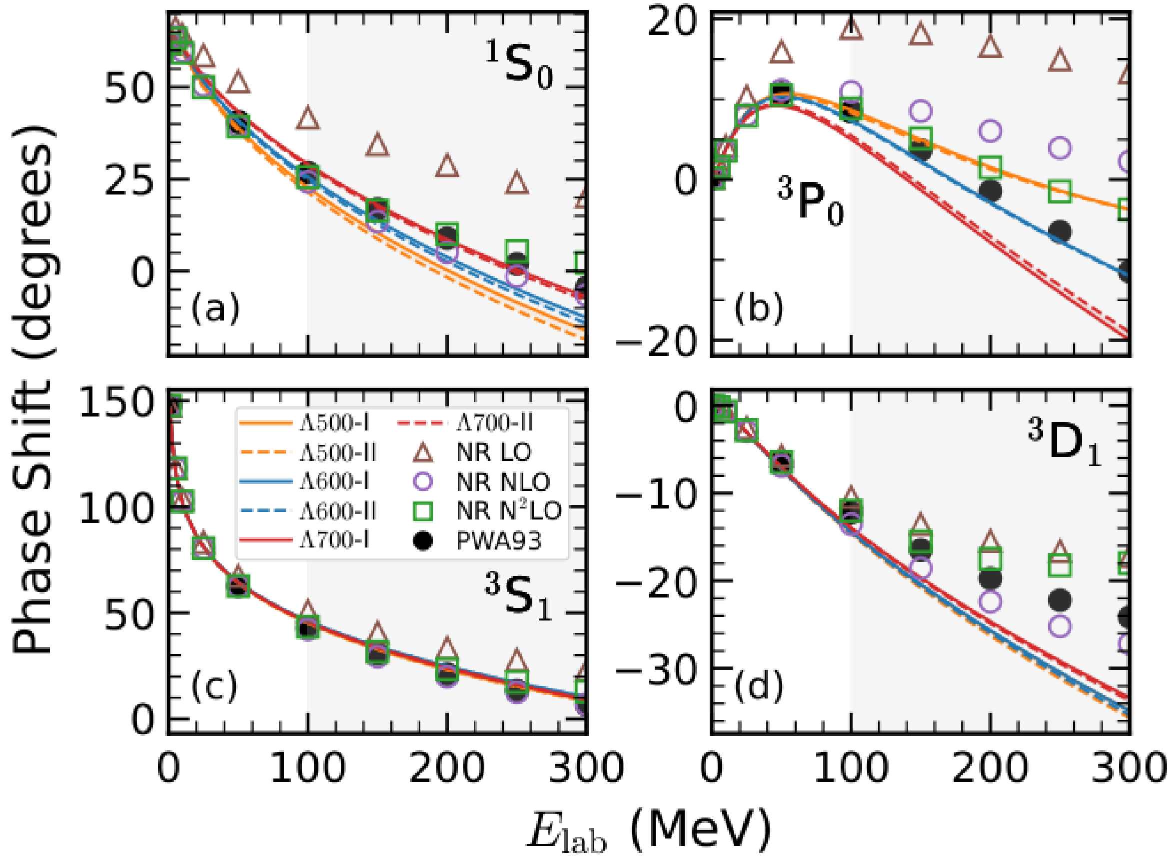
$$
V_{\mathrm{OPEP}}(\boldsymbol{p}',\boldsymbol{p}) = \frac{g_A^2}{4f_\pi^2}\frac{\left[\bar{u}(\boldsymbol{p}',\lambda_1')\boldsymbol{\tau}_1\gamma^\mu\gamma_5 q_\mu u(\boldsymbol{p},\lambda_1)\right]\left[\bar{u}(-\boldsymbol{p}',\lambda_2')\boldsymbol{\tau}_2\gamma^\nu\gamma_5 q_\nu u(-\boldsymbol{p},\lambda_2)\right]}{(E_{p'}-E_p)^2-(\boldsymbol{p}'-\boldsymbol{p})^2-m_\pi^2}, \quad (13)
$$



Already at LO one could achieve a remarkable description for **both NN scattering** and **Nuclear Matter**, with only **4 low-energy constants**!

Local LO Relativistic Chiral Interaction

SS, J.-X. Lu, L.-S. Geng, J. Meng, and W.-J. Zou, arXiv:2507.01257

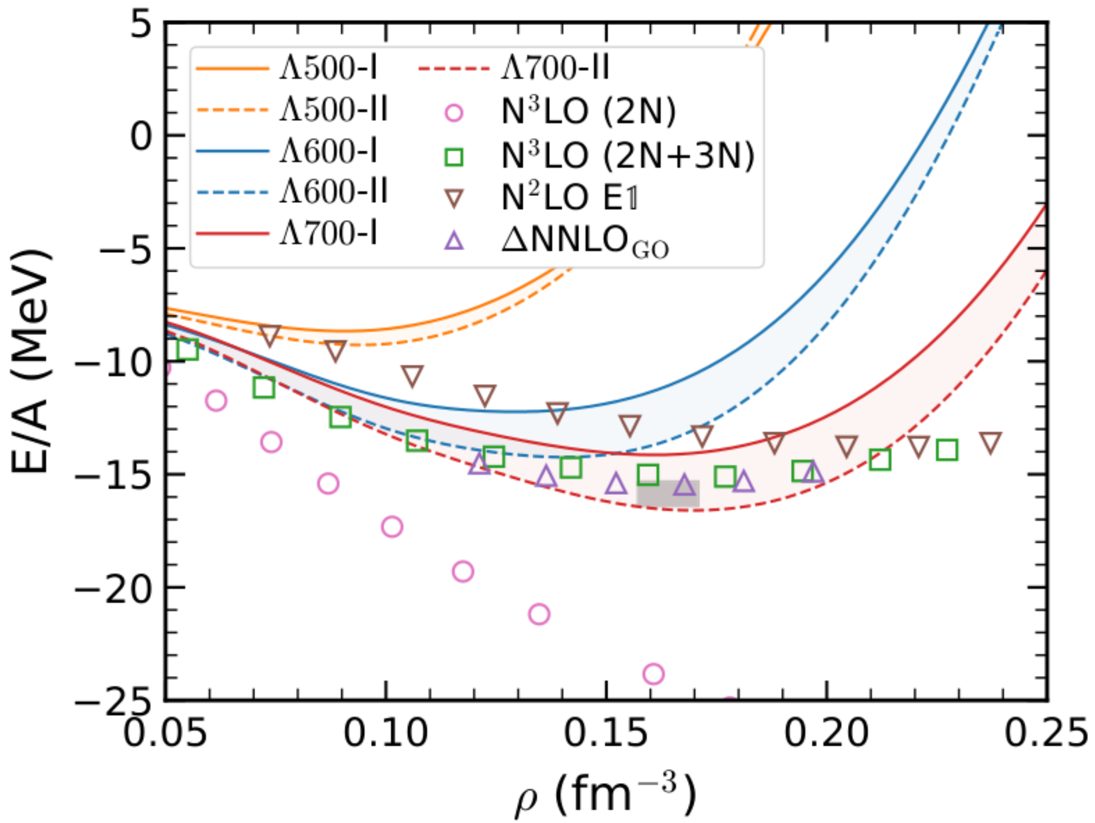


Interaction	C_S	C_V	C_{AV}	C_T	$\tilde{\chi}^2/N$
Λ500-I	-691.26	603.38	-187.96	-87.56	7.60
Λ500-II	-749.77	664.46	-217.41	-101.72	7.41
Λ600-I	-672.28	616.35	-194.20	-87.52	4.18
Λ600-II	-711.64	659.47	-216.41	-98.15	4.10
Λ700-I	-656.98	614.94	-122.34	-51.84	3.59
Λ700-II	-674.85	637.83	-145.61	-62.93	3.32

W.-J. Zou, Y.-L. Yang, SS, and J. Meng, Chin. Phys. Lett. 42, 080101 (2025)

Local LO Interaction for Nuclear Matter

- Nuclear matter equation-of-state calculated by RBHF using local relativistic LO chiral interaction, in comparison with nonrelativistic results.

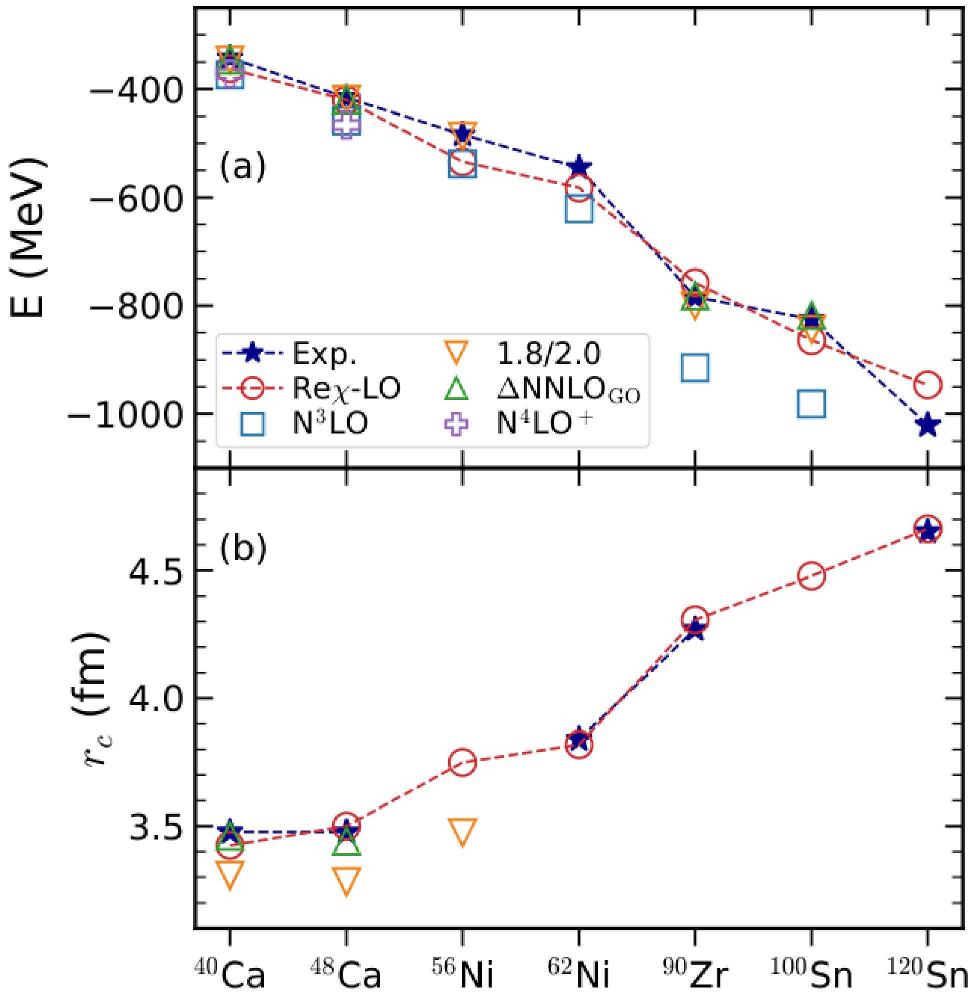


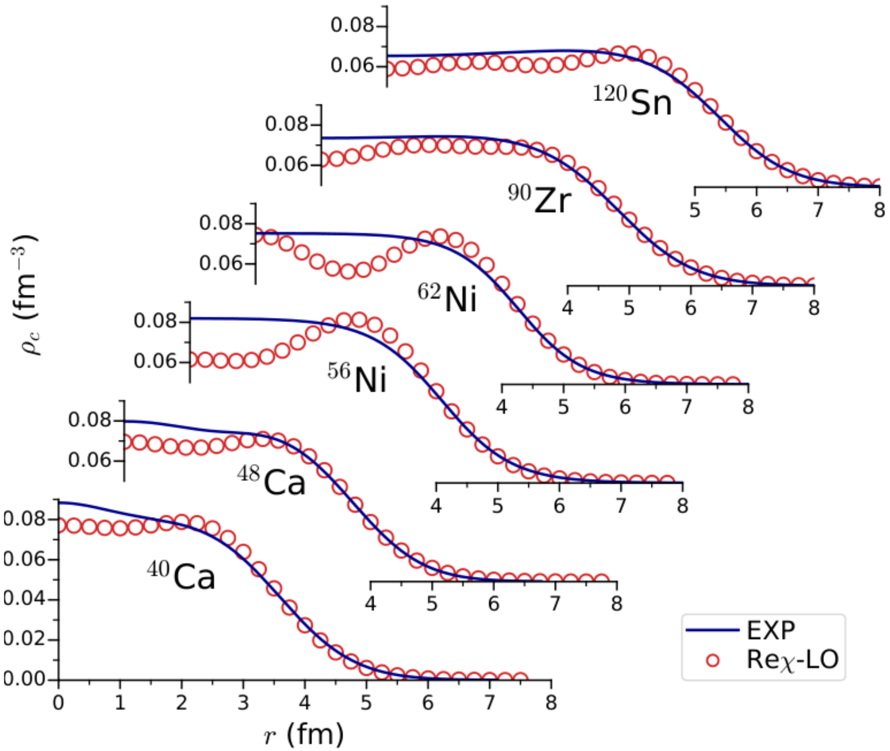
F. Sammarruca and R. Millerson, Phys. Rev. C 102, 034313 (2020)
D. Lonardoni, I. Tews, S. Gandolfi, and J. Carlson, Phys. Rev. Res. 2, 022033 (2020)
W. G. Jiang, A. Ekström, C. Forssén, G. Hagen, G. R. Jansen, and T. Papenbrock, Phys. Rev. C 102, 054301 (2020)
W.-J. Zou, Y.-L. Yang, SS, and J. Meng, Chin. Phys. Lett. 42, 080101 (2025)

Medium-mass nuclei

- Energy, charge radius, and charge density calculated by RBHF using relativistic LO chiral interaction, in comparison with nonrelativistic ab initio studies.

SS, J.-X. Lu, L.-S. Geng, J. Meng, and W.-J. Zou, arXiv:2507.01257





Nuclear matter and finite nuclei

- Results of nuclear matter and ^{40}Ca calculated by RBHF using relativistic LO chiral interaction, in comparison with Bonn interaction and nonrelativistic study

LO with only 4 LECs

No three-body force

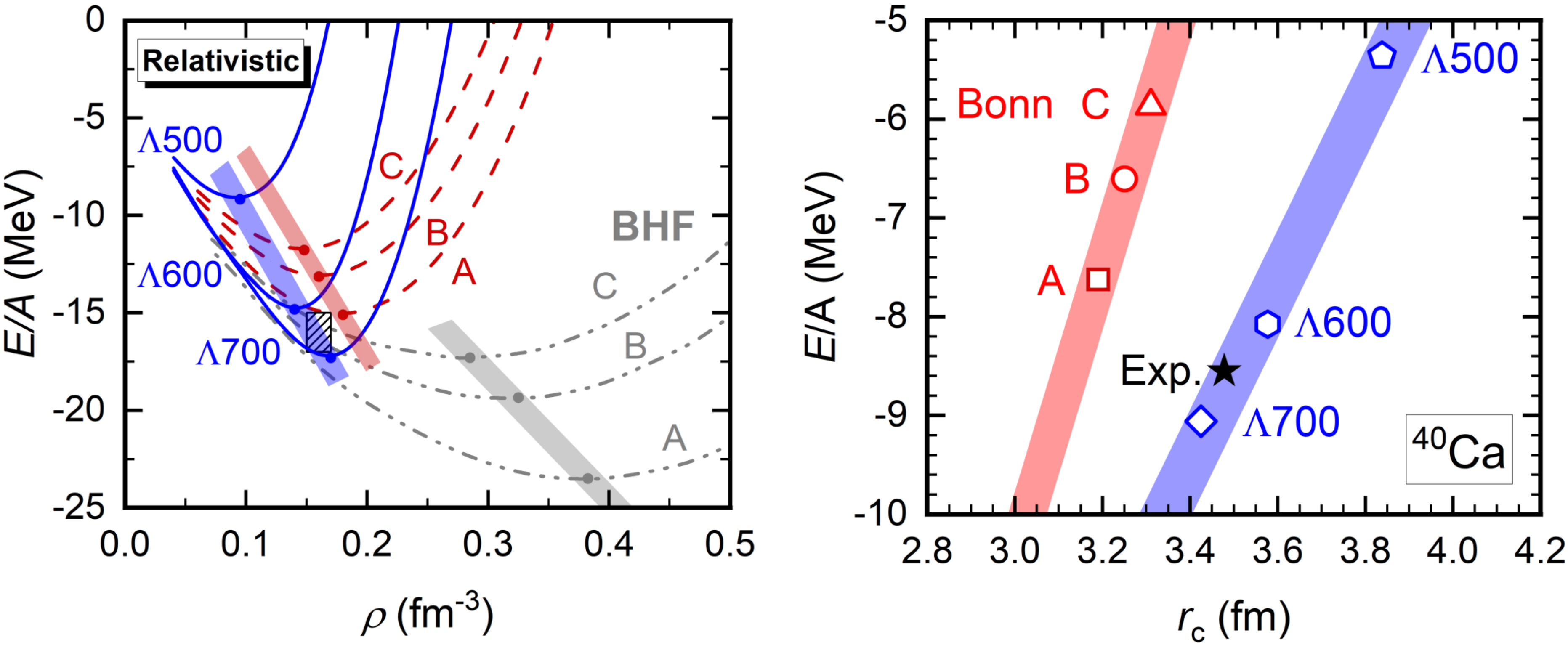


Figure in courtesy of Weijiang Zou

Summary and perspective

- We have developed the **self-consistent relativistic *ab initio* framework**, relativistic Brueckner-Hartree-Fock theory, **for finite nuclei** study.
- RBHF gives impressive **improvement** for the **simultaneous description** of **nuclear matter** saturation, **finite nuclei energies**, and **radii** (density distribution).
- We developed **relativistic LO chiral interactions with local regulator** and applied to **finite nuclei** study.
- With only 4 low-energy constants, relativistic LO chiral interactions could give **good descriptions** for **overall properties of finite nuclei**.

? Relativistic NLO interaction

? Further optimization such as V_{lowk} towards A ~ 200

? Open shell nuclei

? Deformed nuclei

? … …

THANK YOU!

Local Interaction for Finite Nuclei

- For the calculation of two-body interaction matrix element in finite nuclei

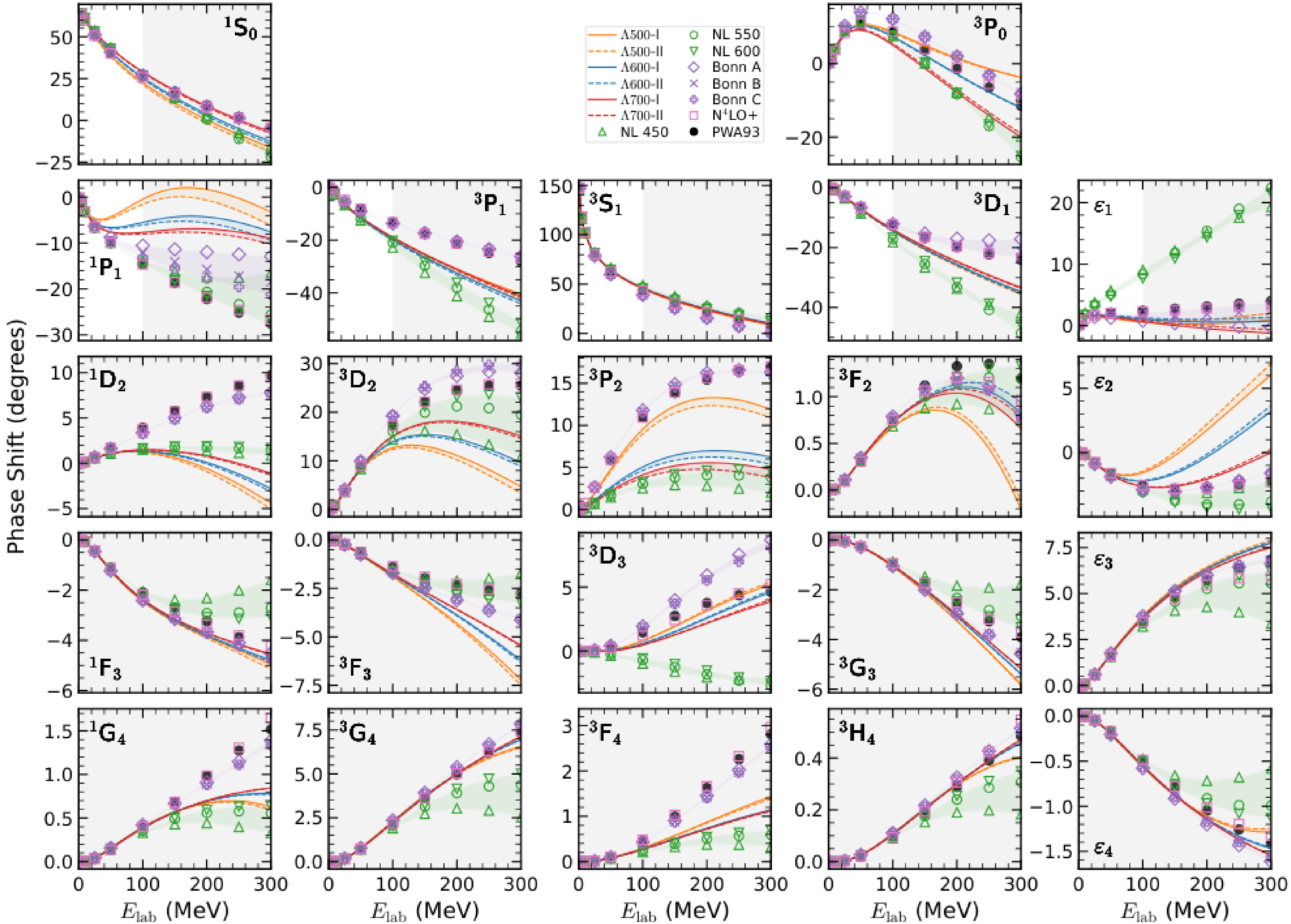
$$\langle ab|V_\alpha|cd\rangle = \int \frac{d^3q}{(2\pi)^3}\frac{1}{m_\alpha^2+q^2}\langle a|\gamma^0\Gamma_\alpha(1)\, e^{i\mathbf{q}\cdot\mathbf{r}_1}|c\rangle\langle b|\gamma^0\Gamma_\alpha(2)e^{-i\mathbf{q}\cdot\mathbf{r}_2}|d\rangle.$$

It is easier to work with local interaction that only depends on the relative momentum transfer q.

- Nonlocal to local regulator

$$f(\mathbf{p}',\mathbf{p}) = \exp\left(\frac{-\mathbf{p}^4-\mathbf{p}'^4}{\Lambda^4}\right) \to \exp\left(\frac{-(\mathbf{p}-\mathbf{p}')^2}{\Lambda^2}\right), \quad \mathbf{q}=\mathbf{p}-\mathbf{p}'.$$

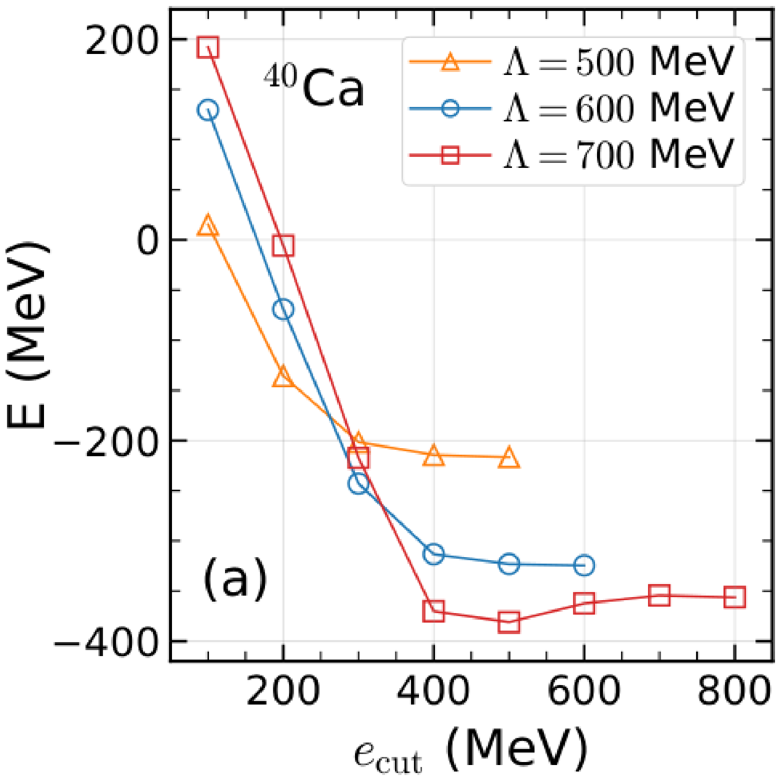
P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A (2018) 54: 86



Λ500-I
Λ500-II
Λ600-I
Λ600-II
Λ700-I
Λ700-II
NL 450
NL 550
NL 600
Bonn A
Bonn B
Bonn C
N4LO+
PWA93
1S0
3P0
1P1
3P1
3S1
3D1
ε1
1D2
3D2
3P2
3F2
ε2
1F3
3F3
3D3
3G3
ε3
1G4
3G4
3F4
3H4
ε4
Phase Shift (degrees)
E_lab (MeV)

Convergence of RBHF

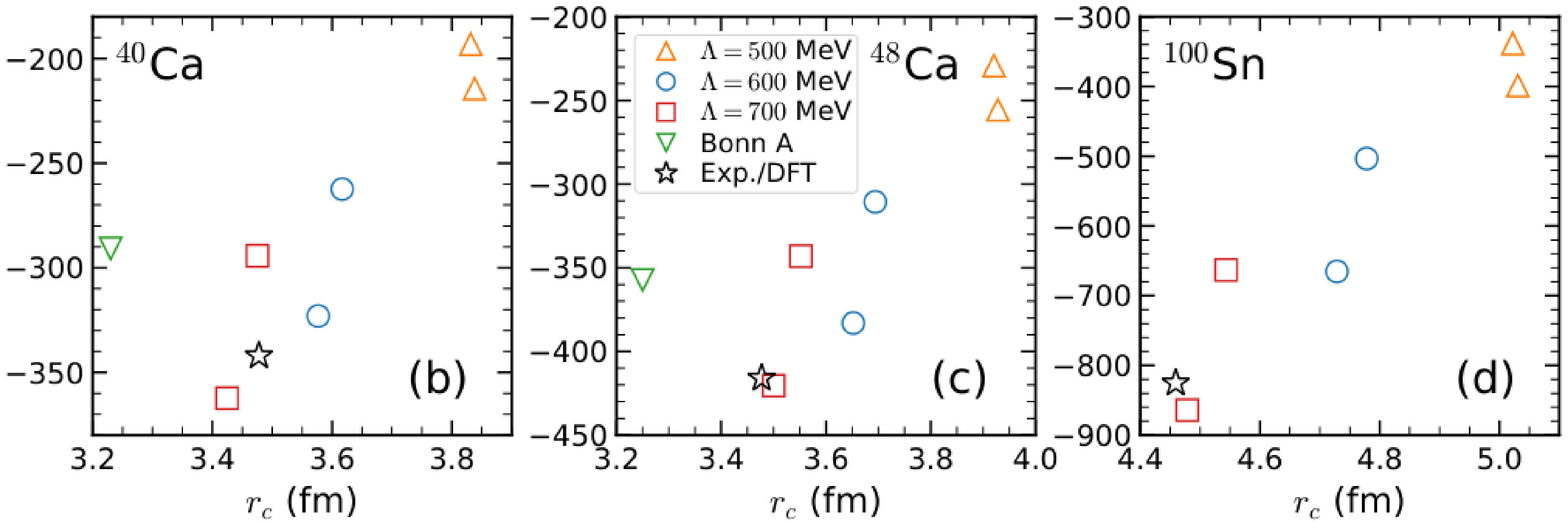
➢ Total energy of ^{40}Ca calculated by RBHF using local relativistic LO chiral interaction as a function of single-particle basis cut-off.



- Lower resolution cutoff scale of chiral interaction gives an advantage of smaller basis required

Energy and radius

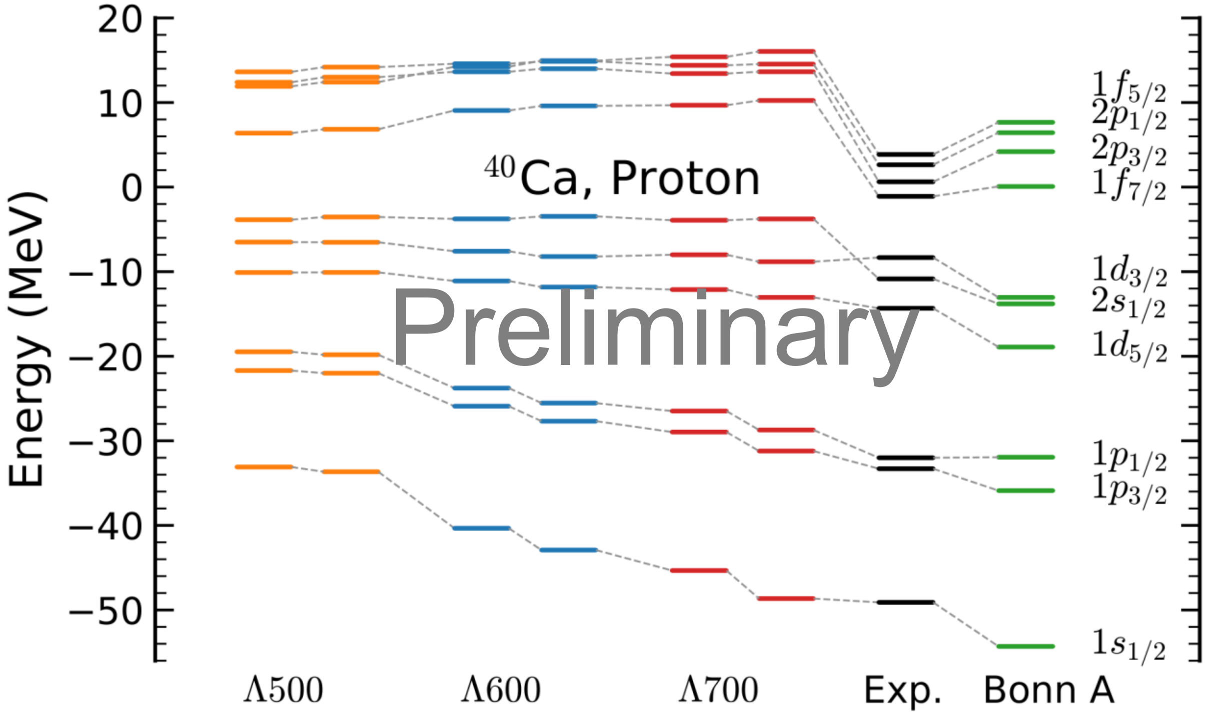
➢ Total energy and charge radii of ^{40}Ca, ^{48}Ca, and ^{100}Sn calculated by RBHF using local relativistic LO chiral interaction, in comparison with Bonn interaction.



- Relativistic LO chiral interaction improves the 'coester band' of finite nuclei further

Single-particle levels

- Proton single-particle levels of ^{16}O and ^{40}O calculated by RBHF using local relativistic LO chiral interaction, in comparison with Bonn interaction.



- The occupied levels are in reasonable agreement with the data
- Spin-orbit splitting appears naturally
- A clear under attraction of 2s state is observed, which could be an indication for the necessity of Q2 order in the chiral expansion

RBHF for ^{16}O

- Energy, charge radius, matter radius, and π1p spin-orbit splitting in ^{16}O calculated by RBHF with Bonn A/B/C, in comparison with data, BHF with Bonn A H. Müther, R. Brockmann, and R. Machleidt, *PRC* **42**, 1981 (1990) and with AV18 B. Hu, et al., *PRC* **95**, 034321 (2017), No Core Shell Model (NCSM) R. Roth, et al., *PRL* 107, 072501 (2011), Coupled-Cluster (CC) G. Hagen, et al., *PRC* 80, 021306 (2009), and Nuclear Lattice Effective Field Theory (NLEFT) T. Lähde, et al., PLB 732, 110 (2014).

	E (MeV)	r_c (fm)	r_m (fm)	$\Delta E_{\pi 1p}^{ls}$ (MeV)
Exp.	**-127.6**	**2.70**	**2.54**	**6.3**
RBHF, Bonn A	-120.2	2.53	2.39	5.3
RBHF, Bonn B	-107.1	2.59	2.45	4.5
RBHF, Bonn C	-98.0	2.64	2.50	3.9
BHF, Bonn A	-105.0	2.29	-	7.5
BHF, AV18	-134.2	-	1.92	13.0
NCSM, N^3LO	-119.7(6)	-	-	-
CC, N^3LO	-121.0	-	2.30	-
NLEFT, N^2LO	-121.4(5)	-	-	-

- The results of RBHF are comparable to other state-of-the-art ab initio results.

RBHF for ^{40}Ca and ^{48}Ca

- Energies, charge radii, matter radii, and π1d spin-orbit splittings of ^{40}Ca and ^{48}Ca calculated by RBHF with Bonn A, comparing with data, CC with N^{3}LO G. Hagen, et al., *PRC* **82**, 034330 (2010) and with AV18 G. Hagen, et al., *PRC* **76**, 044305 (2007), BHF B. Hu, et al., *PRC* **95**, 034321 (2017), NCSM R. Roth, et al., *PRL* **99**, 092501 (2007).

	^{40}Ca				^{48}Ca		
	E (MeV)	r_c (fm)	r_m (fm)	$\Delta E_{\pi 1d}^{ls}$ (MeV)	E (MeV)	r_c (fm)	$\Delta E_{\pi 1d}^{ls}$ (MeV)
Exp.	**-342.1**	**3.48**	**-**	**6.6 ± 2.5**	**-416.1**	**3.48**	**4.7**
RBHF, Bonn A	-306.1	3.22	3.10	5.9	-357.3	3.25	2.7
CC, N^{3}LO	-345.2	-	-	-	-396.5	-	-
CC, AV18	-502.9	-	-	-	-	-	-
BHF, AV18	-552.1	-	2.20	24.9	-	-	-
NCSM, AV18	-461.8	-	2.27	-	-	-	-

For RBHF

^{40}Ca — Storage: 1100 GB; CPU time: 1720 h

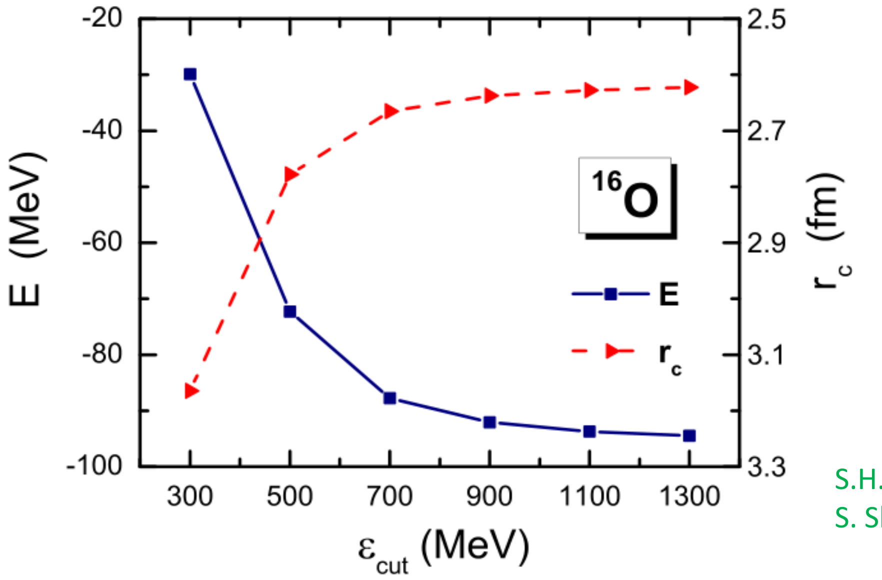
^{48}Ca — Storage: 1800 GB; CPU time: 4900 h

- Results for ^{40}Ca and ^{48}Ca given by RBHF are similar as for ^{16}O.
- CC with N^{3}LO reproduce the binding energy well, while other non-relativistic calculations give too much binding and too small radii.

What's the Difficulty?

- Total energy and charge radius of ^{16}O as a function of energy cut-off. (Center-of-mass motion was not removed)

$$\langle ab|G(W)|cd\rangle = \langle ab|V|cd\rangle + \sum_{mn} \langle ab|V|mn\rangle \frac{Q(m,n)}{W - \varepsilon_m - \varepsilon_n} \langle mn|G(W)|cd\rangle.$$



Rough estimation of the dimension of |ab> :

$(20 \times 2 \times 20)^2$ = 640,000 (symmetry)→ ≈ 60,000

l s n

For one RBHF calculation:

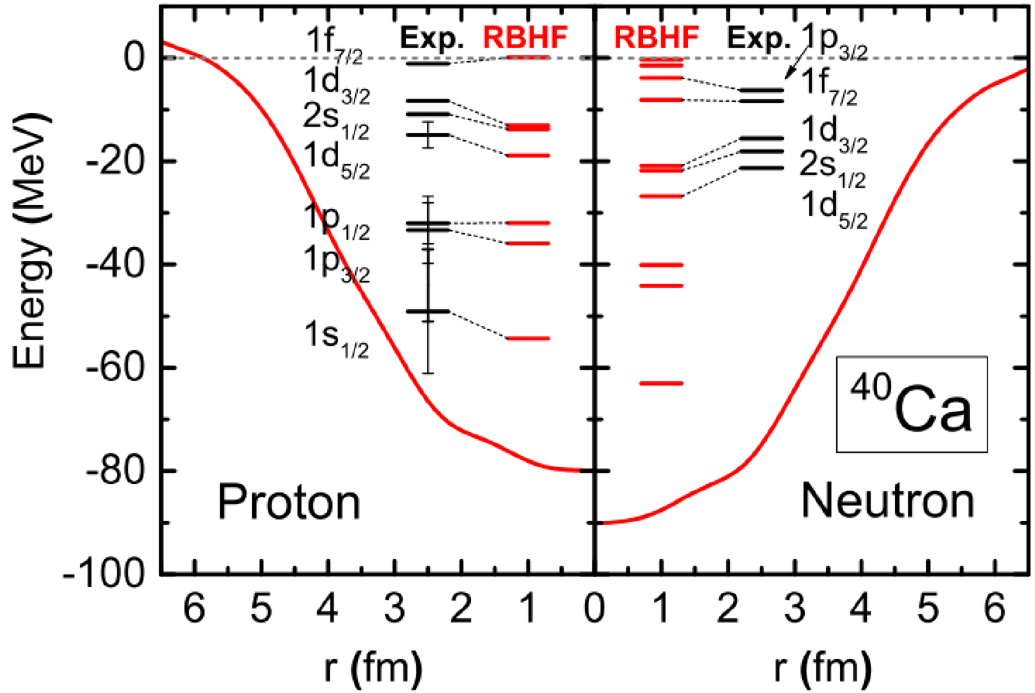
Storage: 256 GB CPU time: 224 h

S.H. Shen, J.N. Hu, H.Z. Liang, J. Meng, P. Ring, S.Q. Zhang, CPL 33 (2016) 102103
S. Shen, H. Liang, J. Meng, P. Ring, S. Zhang, Phys. Rev. C 96 (2017) 014316

- Convergence of solving Bethe-Goldstone equation is achieved! Though as high as > 1GeV!

RBHF for ^{40}Ca

- Figure: single-particle spectrum and localized potential by RBHF with Bonn A.
- Table: single-particle energies by BHF with AV18 (v-low k).



renormalized

		^{40}Ca Neutron			Proton		
Orbital		BHF	RBHF	Expt.	BHF	RBHF	Expt.
$0s_{1/2}$	ε	−137.71	−133.94		−137.14	−133.38	−49.1±12 −77±14
	P	1.00	0.99		1.00	0.99	
$0p_{3/2}$	ε	−92.34	−88.94		−91.84	−88.46	−33.3±6.5
	P	1.00	0.99		1.00	0.99	
$0p_{1/2}$	ε	−74.62	−72.21		−74.16	−71.77	−32±4
	P	1.00	0.99		1.00	0.99	
$0d_{5/2}$	ε	−53.13	−50.24	−21.30	−52.70	−49.82	−14.9±2.5 −13.8±7.5
	P	1.00	0.98		1.00	0.98	
$1s_{1/2}$	ε	−43.33	−41.06	−18.104	−42.88	−40.64	−10.850
	P	1.00	0.97		1.00	0.97	
$0d_{3/2}$	ε	−28.15	−26.38	−15.635	−27.80	−26.04	−8.328
	P	1.00	0.95		1.00	0.95	

$$\begin{pmatrix} M+\Sigma(r) & -\frac{d}{dr}+\frac{\kappa}{r} \\ \frac{d}{dr}+\frac{\kappa}{r} & -M+\Delta(r) \end{pmatrix} \begin{pmatrix} F_a(r) \\ G_a(r) \end{pmatrix} = e_a \begin{pmatrix} F_a(r) \\ G_a(r) \end{pmatrix}$$

$$\Sigma_a(r) = e_a - M + \frac{\frac{dG_a(r)}{dr} - \frac{\kappa}{r} G_a(r)}{F_a(r)}.$$

B. Hu, et al., *PRC* **95**, 034321 (2017)

Shell-model potential!