Three-body final state interactions in $B^+ \to D\bar{D}K^+$ decays

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Xin-Yue Hu, Jiahao He, Pengyu Niu, Qian Wang, Yupeng Yan, arXiv:2509.10039

Outline

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Introduction

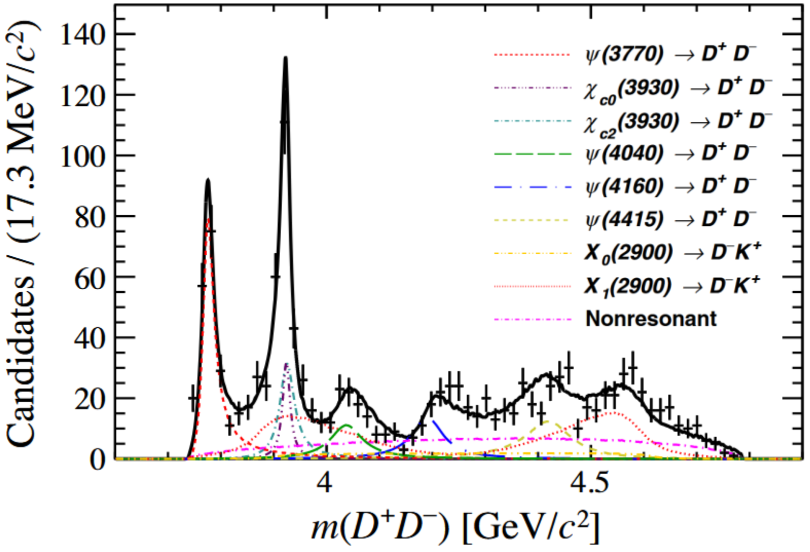
- LHCb observed $\chi_{c0}(3930)$ and $\chi_{c2}(3930)$ in the $B^+ \to D^+ D^- K^+$ decay.

Their experimental masses and widths are

$$M_{c0} = 3923.8 \pm 1.5 \pm 0.4 \text{ MeV} \qquad \Gamma_{c0} = 17.4 \pm 5.1 \pm 0.8 \text{ MeV}$$

$$M_{c2} = 3926.8 \pm 2.4 \pm 0.8 \text{ MeV} \qquad \Gamma_{c2} = 34.2 \pm 6.6 \pm 1.1 \text{ MeV}$$

R. Aaij et al. (LHCb), Phys. Rev. D 102, 112003 (2020).

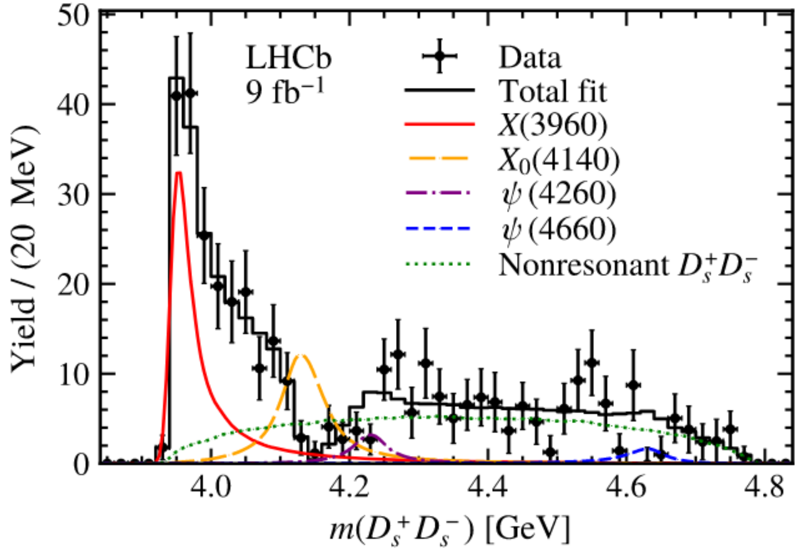


- LHCb observed only $X(3960)$ in $B^+ \to D_s^+ D_s^- K^+$.

The $J^{PC} = 0^{++}$ and the experimental masses and widths are

$$M = 3956 \pm 5 \pm 10 \text{ MeV} \qquad \Gamma = 43 \pm 13 \pm 8 \text{ MeV}$$

R. Aaij et al. (LHCb), Phys. Rev. Lett. 131, 071901 (2023).



Do we need three states?

Introduction

- How explain the three states, $X(3960)$, $\chi_{c0}(3930)$ and $\chi_{c2}(3930)$?
- Are they either charmonium states or molecular states?
- Is $X(3960)$ either $\chi_{c0}(3930)$ or $\chi_{c2}(3930)$?

$\chi_{c0}(3930)$ can be well understood with $\chi_{c0}(2^3P_0)$ states in couple-channel model.

Q. Deng, R.-H. Ni, Q. Li, and X.-H. Zhong, Phys. Rev. D 110, 056034(2024).

$\chi_{c0}(3930)$ as molecular states in one-boson exchange model, and $\chi_{c2}(3930)$ need to be introduced to describe the distribution in the higher energy region.

Z.-M. Ding, Q. Huang, and J. He, Eur. Phys. J.C 84, 822 (2024).

X(3960) can not be understood with a pure $D_s^+D_s^-$ molecular, which more like a $c\bar{c}$ resonance with the contribution of couple-channel effect.

Y. Chen, H. Chen, C. Meng, H.-R. Qi, and H.-Q. Zheng, Eur. Phys. J. C 83, 381 (2023).

$\chi_{c0}(3930)$ has the same origin as the $X(3960)$, which is an S-wave $D_s^+D_s^-$ hadronic molecule.

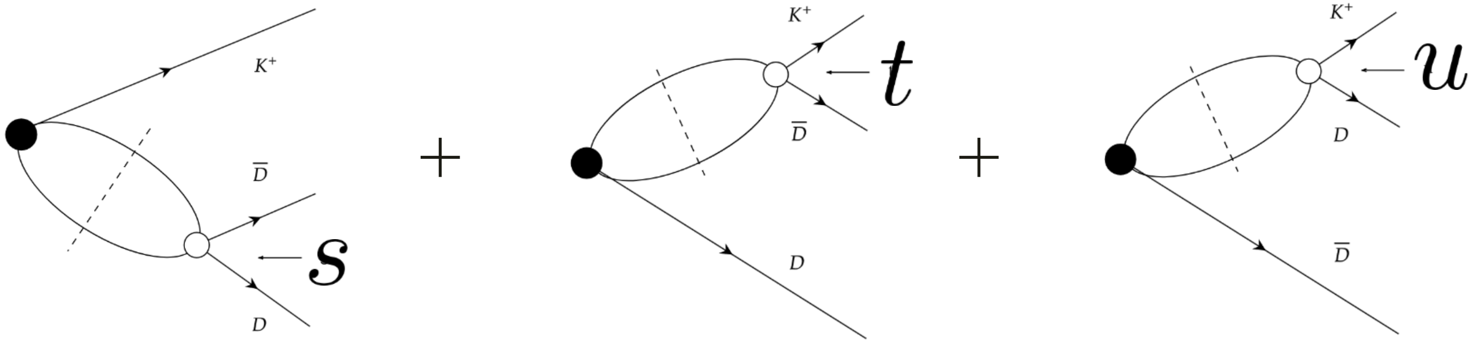
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, F.-K. Guo, Sci. Bull.68, 688 (2023).

Framework

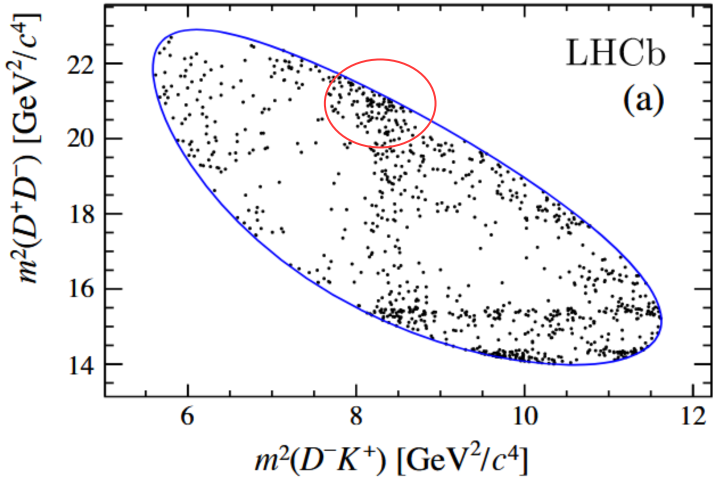
- Khuri–Treiman equation.

The $B^+ \to D\bar{D}K^+$ decay amplitude is given by

$$f(s,t,u) = A^{(s)}(s,t,u) + A^{(t)}(s,t,u) + A^{(u)}(s,t,u)$$



Two invariant mass distribution have overlap.



Amplitude expands in terms of Legendre polynomial,

$$f(s,t,u) = 16\pi \sum_{l=0}^{\infty} (2l+1) f_l(s) P_l(z_s)$$

orthogonality relation

$$f_l(s) = a_l^{(s)}(s) + b_l^{(s)}(s)$$

$a_l^{(s)}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s\ A^{(s)}(s,t,u) P_l(z_s)$ R.H.C

$b_l^{(s)}(s) = \sum_{m=t,u} \frac{2l'+1}{2} \int_{-1}^{1} dz_s\ a_{l'}^{(m)}(m) P_{l'}(z_m) P_l(z_s)$ L.H.C

Unitarity

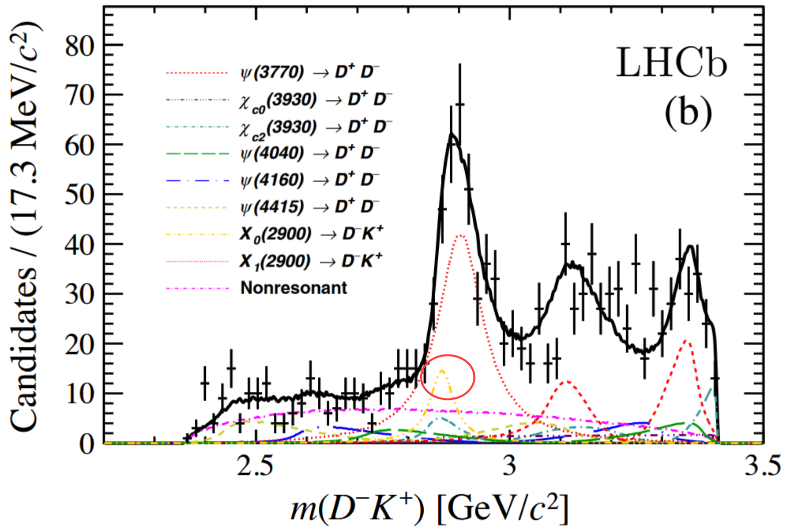
Dispersion relation

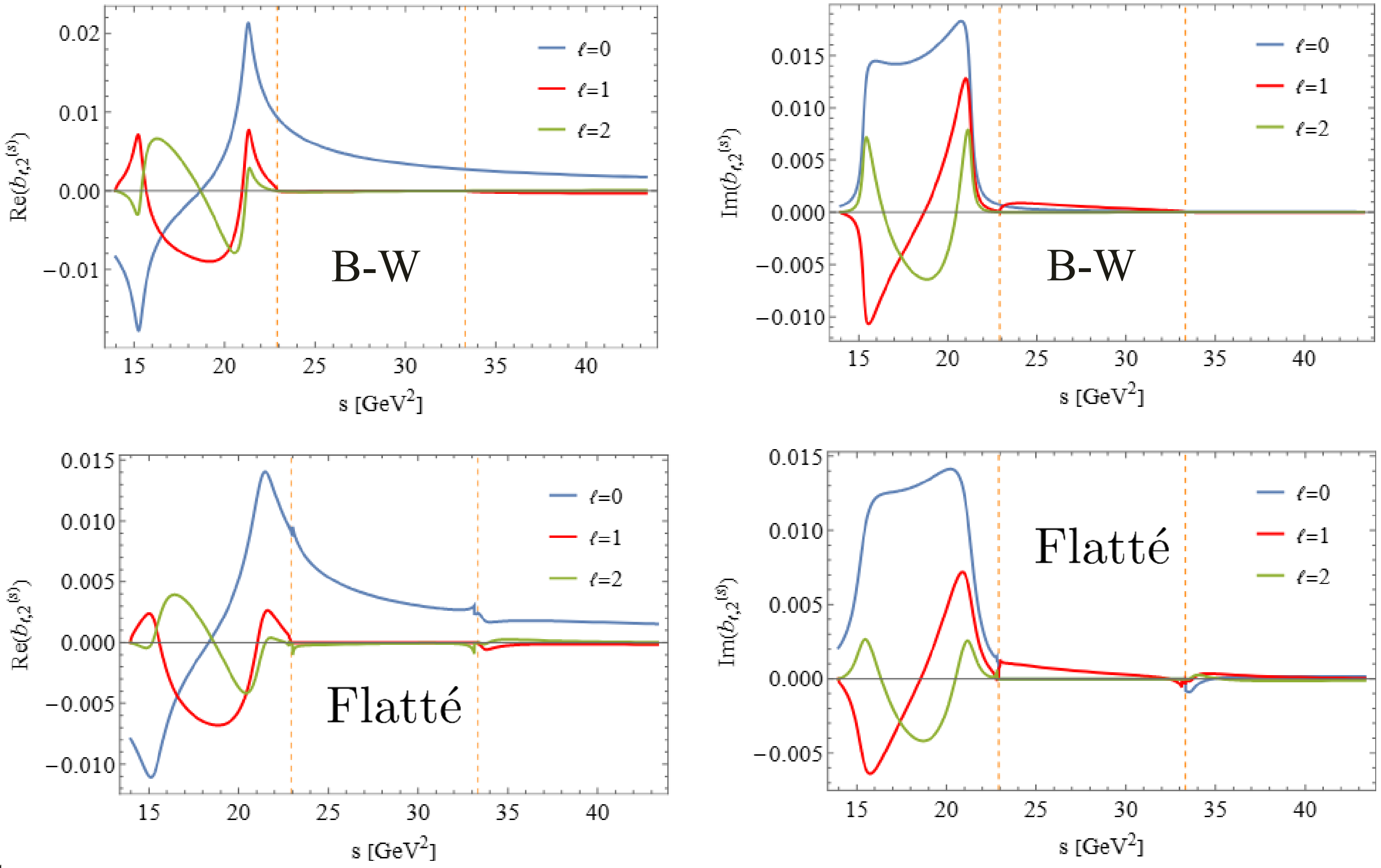
$$a_l^{(s)}(s) = t^l(s) \left(c_l + \frac{s}{\pi} \int_{s_{\rm th}}^{\infty} ds' \frac{\rho(s') b_l^{(s)}(s')}{s'(s'-s)} \right)$$

once subtraction

Framework

For t-channel and u-channel isobar amplitude, we focus the contribution from S-wave.





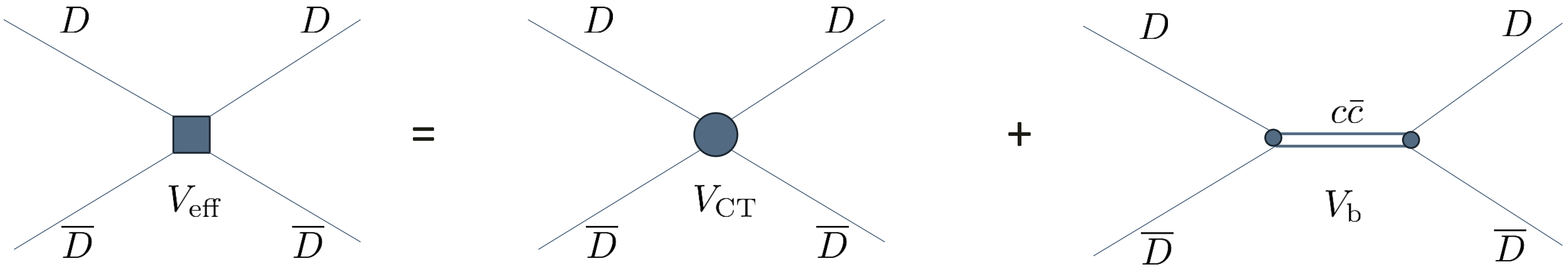
Isolate and narrow resonance.

Using a B-W to parameterize the $a_0^{(t)}(t)$.

$$a_0^{(t)}(t) = \frac{g^2}{M^2 - iM\Gamma - t} \Longrightarrow \text{Broken Unitarity}$$

Framework

- The scattering for $D\overline{D}$.



- Lippmann-Schwinger equation.

$$t(s) = [1 - V_{\mathrm{eff}} G_\Lambda(s)]^{-1} V_{\mathrm{eff}}$$

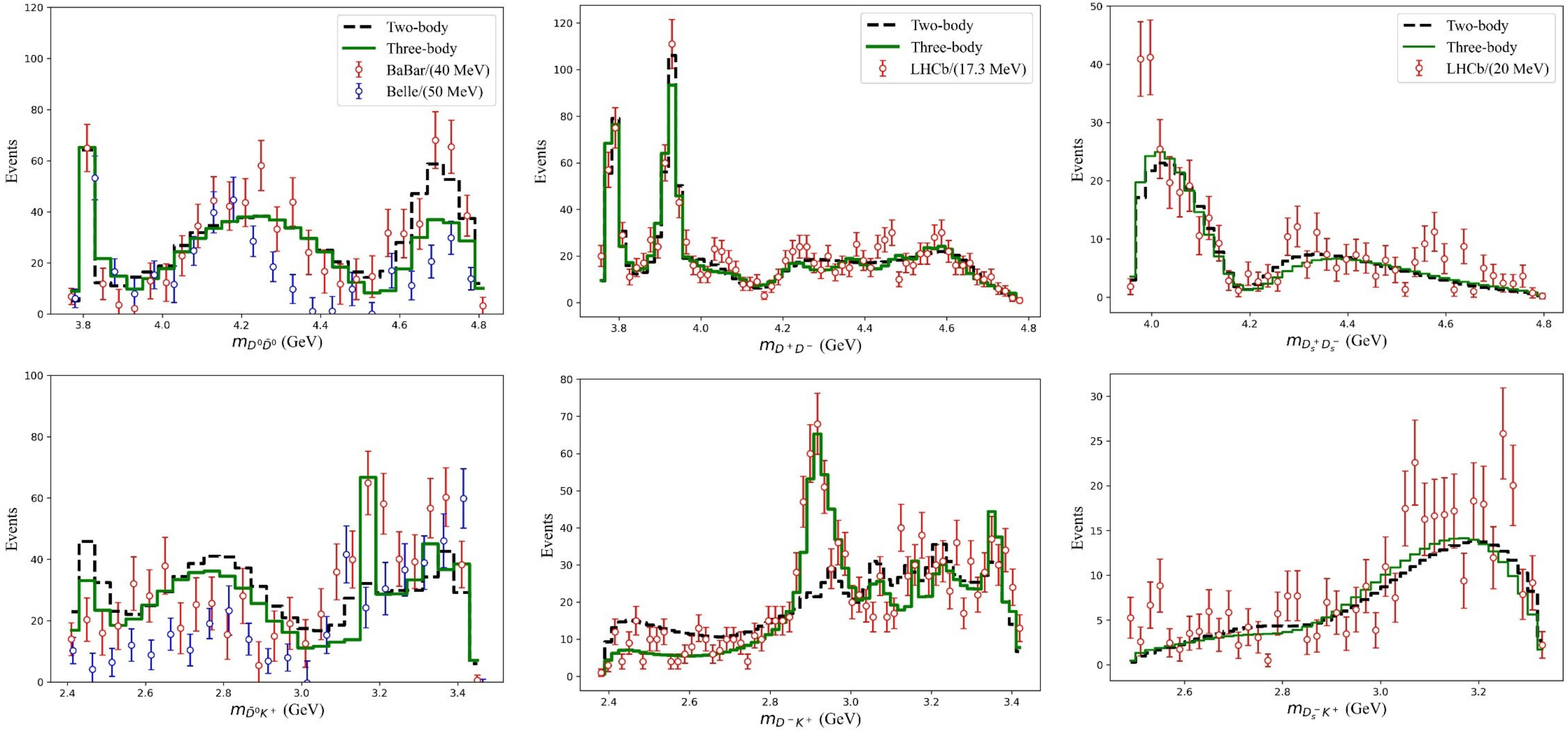
Taking single-particle normalization, the two point correlation functions are given by

$$G^S(s) = -\frac{\mu\Lambda}{(2\pi)^{3/2}} + \frac{\mu k}{2\pi} e^{-2k^2/\Lambda^2}\left[\operatorname{erfi}\left(\frac{\sqrt{2}k}{\Lambda}\right) - i\right]$$

$$G^P(s) = -\frac{\mu\Lambda}{(2\pi)^{3/2}}\left(k^2 + \frac{\Lambda^2}{4}\right) + \frac{\mu k^3}{2\pi} e^{-2k^2/\Lambda^2}\left[\operatorname{erfi}\left(\frac{\sqrt{2}k}{\Lambda}\right) - i\right]$$

Results and Discussions

- Two scheme to fit the experiments and extract the interaction for $D\overline{D}$.



$\chi^2/\text{d.o.f} = 1.758$

$\chi^2/\text{d.o.f} = 2.065$

Results and Discussions

- Poles analysis

Poles appear in $\mathrm{Det}[1 - V^l_{\mathrm{eff}}(s)G^l(s)] = 0$

$$t^l(s) \propto (1 - V^l_{\mathrm{CT}}G^l)^{-1}\left((s - m_l^2 + i m_l \Gamma_l) - (1 - V^l_{\mathrm{CT}}G^l)^{-1} g_l^2 G^l\right)^{-1}$$

Two situations: $\mathrm{Det}[1 - V^l_{\mathrm{CT}}G^l(s)] = 0$ Originating from contact potential

$$\mathrm{Det}[s - m_l^2 + i m_l \Gamma_l - (1 - V^l_{\mathrm{CT}}G^l)^{-1} g_l^2 G^l] = 0$$

Originating from the renormalized bare pole contribution.

$g_l \to 0$

The position of the pole come back to the position of the bare pole.

$$s - m_l^2 + i m_l \Gamma_l = 0$$

Results and Discussions

- Riemann Sheets

$$\mathrm{RS}_+ : k_i = \sqrt{2\mu_i(\sqrt{s} - m_{D^i} - m_{\overline{D}^i})} \qquad \mathrm{RS}_- : k_i = -\sqrt{2\mu_i(\sqrt{s} - m_{D^i} - m_{\overline{D}^i})}$$

Three channel, eight Riemann Sheets

We redefined the "momentum" and mapped the eight RSs on the $\sqrt{s}$ plane onto z plane. $q_i = \sqrt{\frac{\epsilon_i}{\mu_i}} k_i$

$$q_1 = \frac{\sqrt{\varepsilon_2^2-\varepsilon_1^2}}{2}\left[\frac{\gamma}{SN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})} + \frac{SN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})}{\gamma}\right]$$

$$\gamma \equiv \frac{\sqrt{\epsilon_3^2-\epsilon_1^2}+\sqrt{\epsilon_3^2-\epsilon_2^2}}{\sqrt{\epsilon_2^2-\epsilon_1^2}}$$

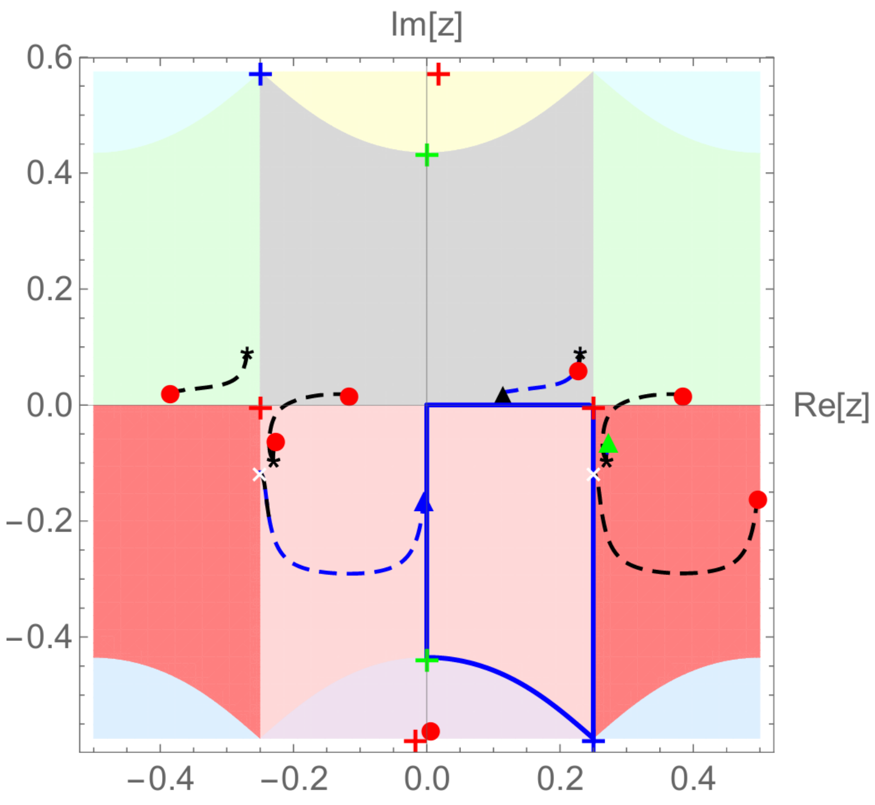
$$q_2 = \frac{\sqrt{\varepsilon_2^2-\varepsilon_1^2}}{2}\left[\frac{\gamma}{SN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})} - \frac{SN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})}{\gamma}\right]$$

$$q_3 = \frac{\sqrt{\varepsilon_2^2-\varepsilon_1^2}}{2}\frac{\gamma CN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})DN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})}{SN(4K(\frac{1}{\gamma^2})z,\frac{1}{\gamma^2})}$$

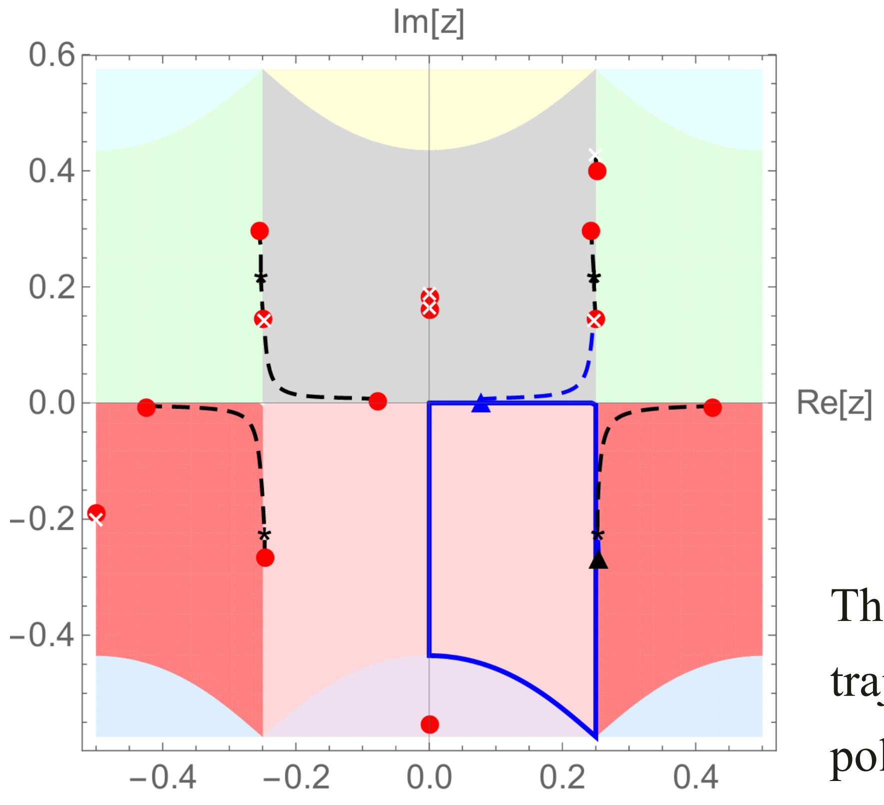
Using Jacobi Elliptic function represent q_i as a single-value function of z.

W. A. Yamada, O. Morimatsu, and T. Sato, Phys. Rev. Lett. 129, 192001 (2022)

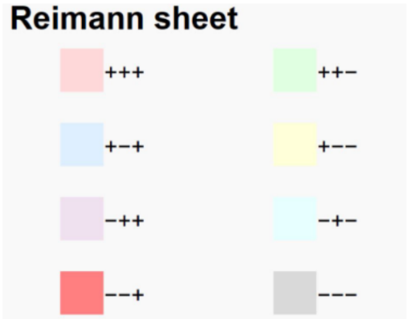
Results and Discussions



S-wave



P-wave



The dashed lines represent the trajectories along which the poles move when $g_l \to 0$

RS	$+++$	$--+$	$---$
S-wave	$3.591^{+0.044}_{-0.045} - 0.010^{+0.004}_{-0.004}i$	$3.913^{+0.009}_{-0.009} - 0.018^{+0.007}_{-0.007}i$	$4.151^{+0.044}_{-0.033} - 0.126^{+0.029}_{-0.037}i$
P-wave		$3.764^{+0.003}_{-0.003} - 0.002^{+0.000}_{-0.000}i$	$4.579^{+0.060}_{-0.058} - 0.122^{+0.013}_{-0.016}i$

Results and Discussions

- Extracting the scattering length and effective range from Effective-Range-Expansion (ERE).

$$t^0(s) = -\frac{2\pi}{\mu}\frac{1}{-1/a_0+1/2r_0k^2-ik+\mathcal{O}(k^4)}$$

$$a_0 = \frac{\mu}{2\pi}t^0(s)\Big|_{s\to s_{th}} \qquad r_0 = -\frac{2\pi}{\mu}\mathrm{Re}\left[\frac{d(t^0(\sqrt{s}))^{-1}}{d\sqrt{s}}\right]\Big|_{\sqrt{s}\to\sqrt{s_{th}}}$$

The effective range includes the corrections from coupled channels, and we extract the corrections through choose the limit of $\mathrm{SU}(3)_f$.

$$\Delta r = -\frac{2\pi}{\mu}\mathrm{Re}\left[\frac{d(t_{11}^{-1}-t_{11}^{-1}|_{\Delta\to 0})}{d\sqrt{s}}\right]\Big|_{\sqrt{s}\to\sqrt{s_{th}}}$$

Δ are the threshold gap between different couple channels.

a_0	r_0	$r' = r_0 - \Delta r$
$0.9998 - 0.0007i$	-6.9722	3.5934

Weinberg criterion is used to characterize the compositeness of bound, virtual and resonance.

$$\overline{X} = \frac{1}{\sqrt{1+2\left|\frac{r'}{\mathrm{Re}[a_0]}\right|}} = 0.3495$$

Summary

- We use a simplified three-body FSI model to analyze the $B^+ \to D\overline{D}K^+$decay.
- One 0^{++} states with pole $3.913^{+0.009}_{-0.009} - 0.018^{+0.007}_{-0.007}i$ is sufficient to describe the $\chi_{c0}(3930)$, $\chi_{c2}(3930)$, and $X(3960)$.
- The compositeness of $\chi_{c0}(3930)$ is 35%. $\Longrightarrow$ A large $c\bar{c}$ component.

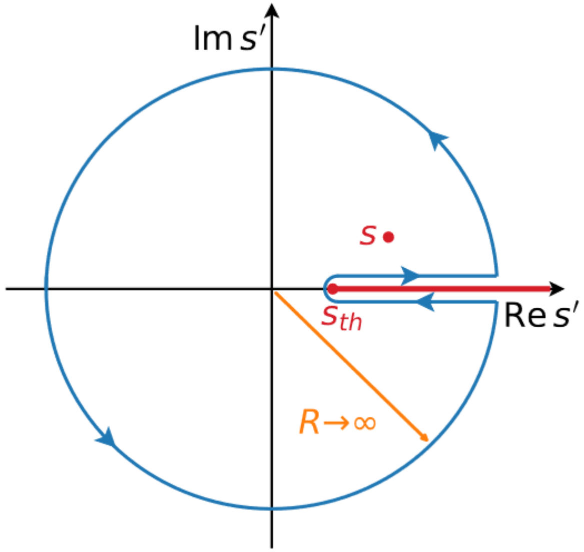
Thank you for your attention!

Partial-wave unitarity relations

$$\text{Disc } f_l(s) = f_l(s)\rho(s)t_l(s)\theta(\sqrt{s} - m_1 - m_2)$$

$$\text{Disc } a_l^{(s)}(s) = \text{Disc } f_l(s)$$

Dispersion relation



$$f(s) = \frac{1}{2\pi i}\oint dz \frac{f(z)}{z-s}$$

Constructing a function $m_l(s) = \frac{a_l^{(s)}(s)}{t^l(s)}$ $\Downarrow$

$$\text{Disc } m_l(s) = 2i\rho(s)b_l^{(s)}(s)\theta(\sqrt{s} - m_1 - m_2)$$

$$m_l(s) = \frac{1}{\pi}\int_{sth}^{\infty} ds' \frac{\rho(s')b_l^{(s)}(s')}{s'-s}$$ When $\lim_{s\to\infty} m_l(s) = 0$

We need take subtraction if $\lim_{s\to\infty} m_l(s) \neq 0$

$$a_l^{(s)}(s) = t^l(s)\left(c_l + \frac{s}{\pi}\int_{s_{\mathrm{th}}}^{\infty} ds' \frac{\rho(s')b_l^{(s)}(s')}{s'(s'-s)}\right)$$ first subtraction

$B^+ \to D\overline{D}K^+$

- The interaction between D and $\overline{D}$.

Taking NR approximation, the LO Lagrangian in HQSS

$$\mathcal{L}_{4H} = -\frac{1}{4}\,\mathrm{Tr}\left[H^{a\dagger}H_b\right]\mathrm{Tr}\left[\overline{H}^c\overline{H}_d^\dagger\right]\left(F_A\,\delta_a^b\delta_c^d + F_A^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)+\frac{1}{4}\,\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m\right]\mathrm{Tr}\left[\overline{H}^c\overline{H}_d^\dagger\sigma^m\right]\left(F_B\,\delta_a^b\delta_c^d + F_B^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)$$

$\Rightarrow$ S-wave contact potential $V_{\mathrm{CT}}^S = \begin{pmatrix} F_A+\frac{4}{3}F_A^\lambda & 2F_A^\lambda & 2F_A^\lambda \\ 2F_A^\lambda & F_A+\frac{4}{3}F_A^\lambda & 2F_A^\lambda \\ 2F_A^\lambda & 2F_A^\lambda & F_A+\frac{4}{3}F_A^\lambda \end{pmatrix} \xRightarrow{\mathrm{SU}(3)_f\ \text{violation}} V_{\mathrm{CT}}^S = \begin{pmatrix} v_{11}^0 & v_{12}^0 & v_{13}^0 \\ v_{12}^0 & v_{22}^0 & v_{23}^0 \\ v_{13}^0 & v_{23}^0 & v_{33}^0 \end{pmatrix}$

NLO Lagrangian $\Rightarrow$ P-wave contact potential $V_{\mathrm{CT}}^P = \begin{pmatrix} v_{11}^1 & v_{12}^1 & v_{13}^1 \\ v_{12}^1 & v_{22}^1 & v_{23}^1 \\ v_{13}^1 & v_{23}^1 & v_{33}^1 \end{pmatrix}(\vec{p}_1-\vec{p}_2)(\vec{k}_1-\vec{k}_2)$

- The interaction between $D\overline{D}$ and charmonium.

$$V_{\mathrm{b}}^S = \frac{g_0^2}{s-m_0^2-+im_0\Gamma_0} \qquad V_{\mathrm{b}}^P = \frac{g_1^2(\vec{p}_1-\vec{p}_2)(\vec{k}_1-\vec{k}_2)}{s-m_1^2+im_1\Gamma_1}$$