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Exotic Hadrons and Femtoscopic correlation functions

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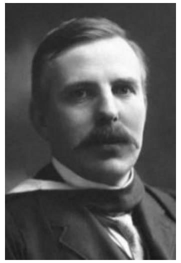
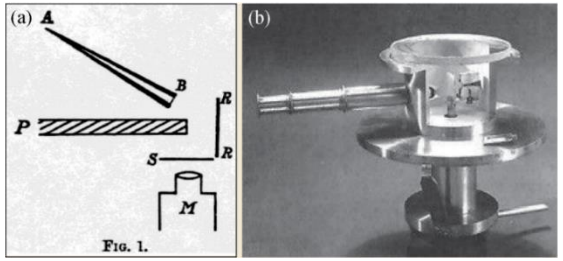
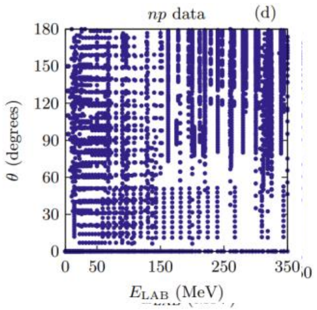
New probe—femtoscopic correlation functions

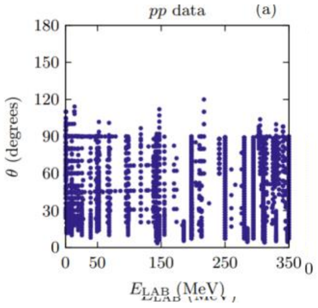
- For stable hadrons, scattering experiments are extremely valuable in extracting their interactions





Ernest Rutherford   Rutherford Scattering Experiment





- **NN scattering, 8125 data**
- Foundation of high-precision nuclear force

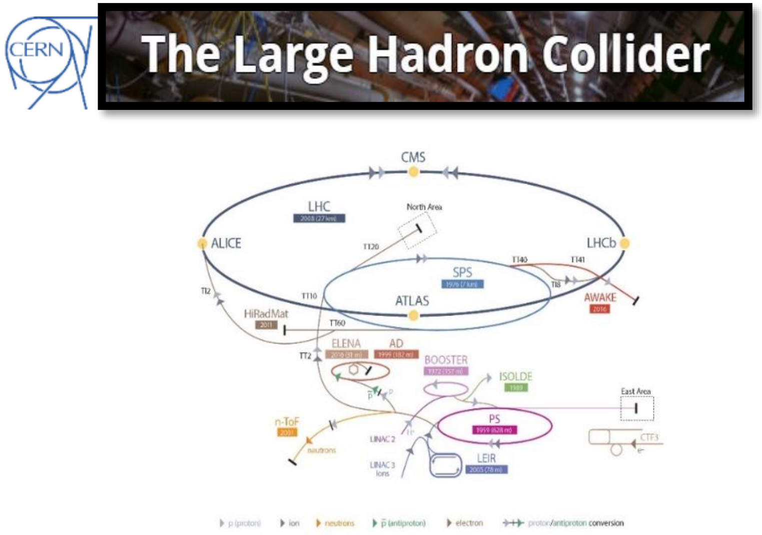
- **For unstable particles, direct scattering experiments are impossible!**

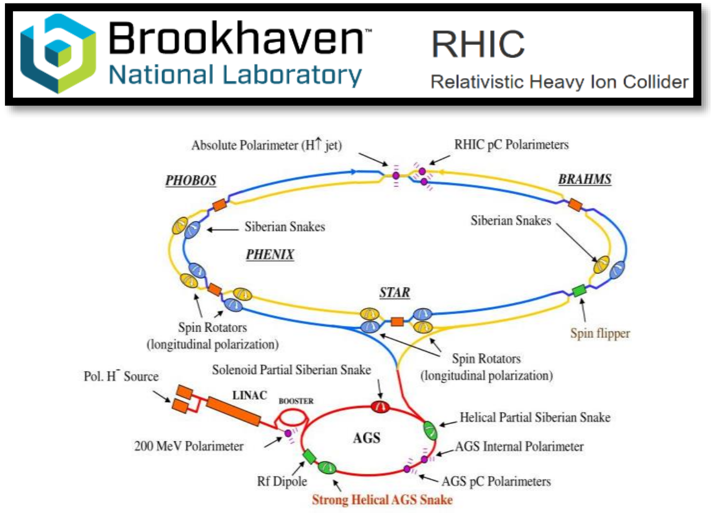
  - **Difficult to get large quantity of beam particles**
  - **No fixed targets available**

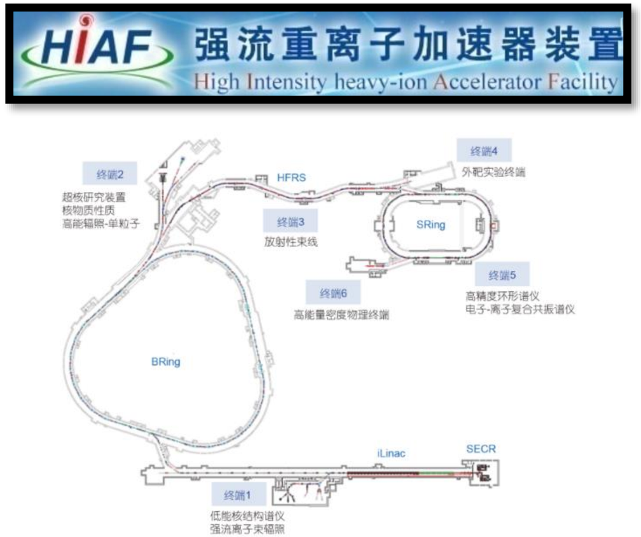
$\Lambda p \to \Lambda p$	$\Sigma^- p \to \Lambda n$	$\Sigma^+ p \to \Sigma^+ p$	$\Sigma^- p \to \Sigma^- p$	$\Sigma^- p \to \Sigma^0 n$
12	6	4	7	6

- **Hyperon-nucleon low energy scattering, 35 data**
- Hindering hyper-nuclear physics and neutron star studies

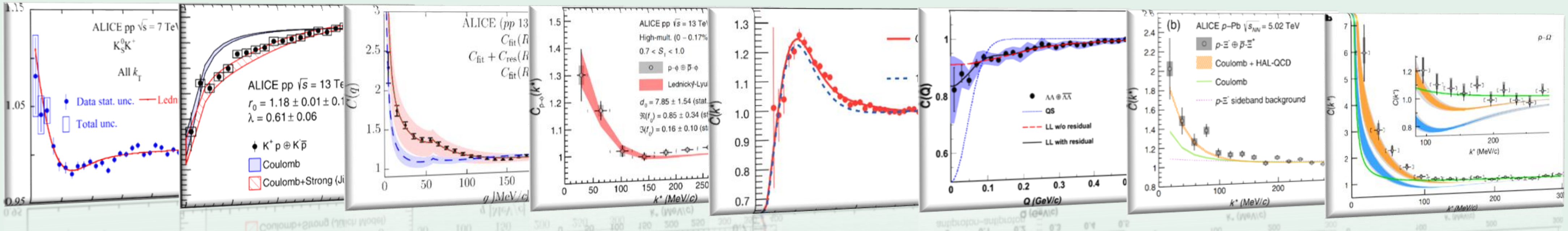
New probe—femtoscopic correlation functions







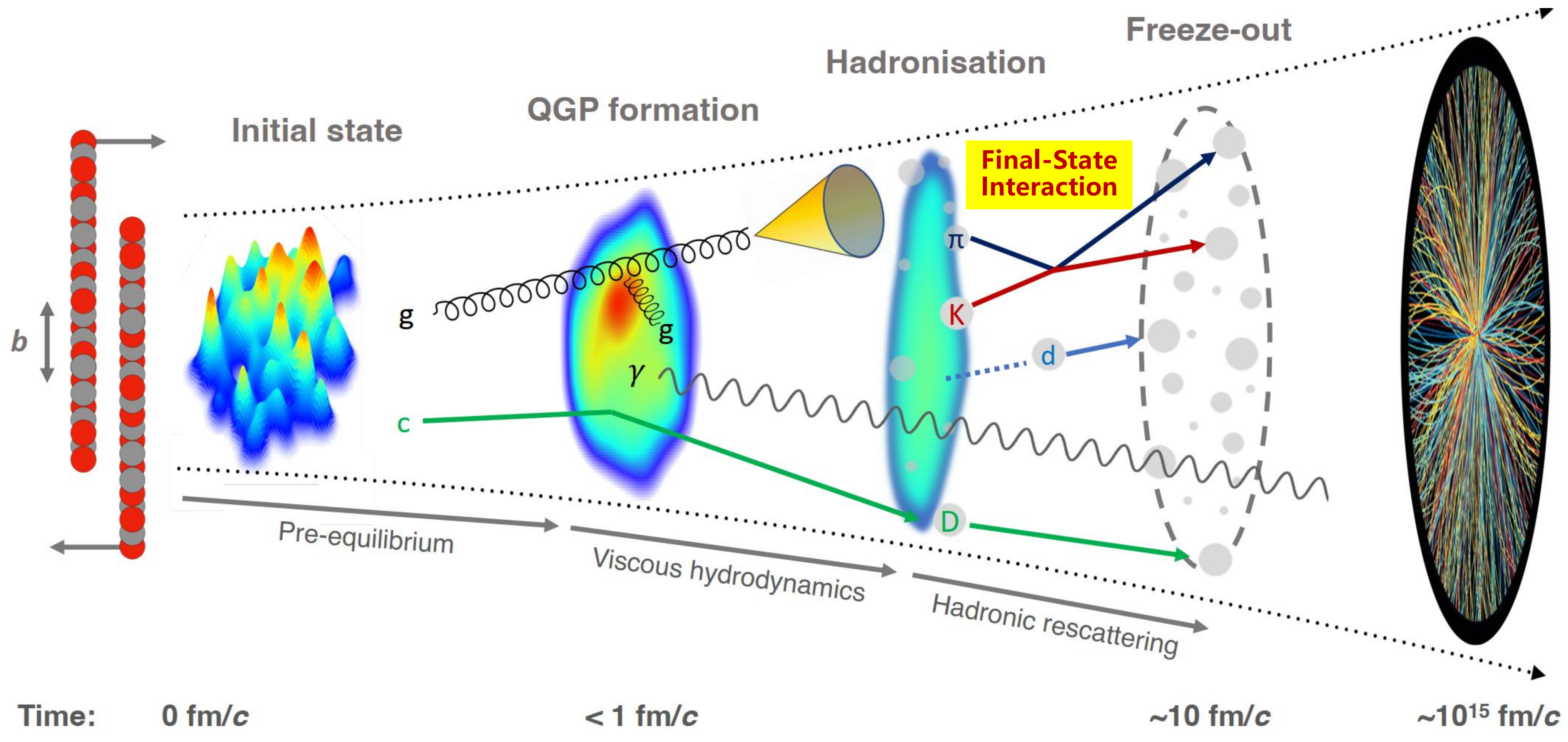
$K_S^0K^{\pm}$ | $K^{\pm}p$ | K^-p | ϕp | $\overline{p}\overline{p}$ | $\Lambda\Lambda$ | Ξ^-p | Ω^-p



PLB 2019 | PRL 2020 | PRL 2020 | PRL 2021 | Nature 2015 | PRL 2015 | PRL 2019 | Nature 2020

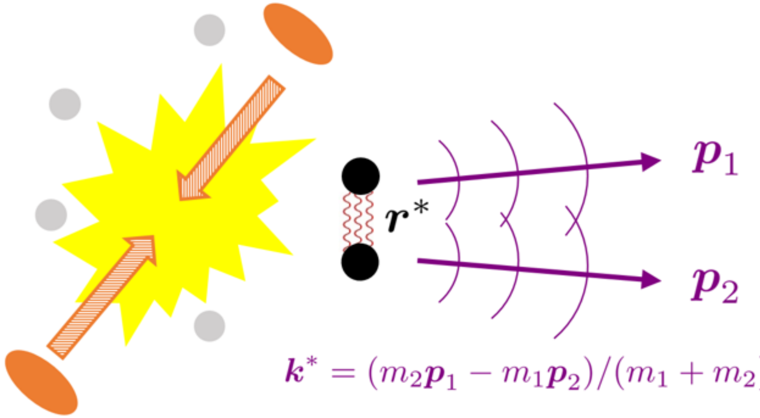
K^-d
Σ^0p
$\Lambda\Xi$
$\Xi\Xi$
$\Omega\Omega$
pD
ppp
$pp\Lambda$

Abundant particles produced in AA, pA, and pp collisions



Femtoscopic correlation functions (CFs)

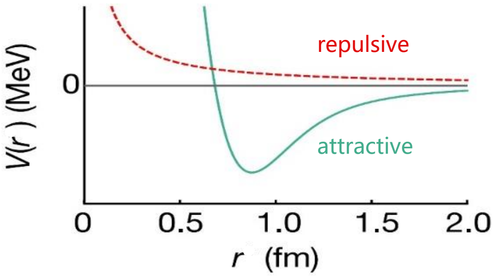
$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{\mathrm{P}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{\mathrm{P}(\boldsymbol{p}_1)\cdot\mathrm{P}(\boldsymbol{p}_2)}$$



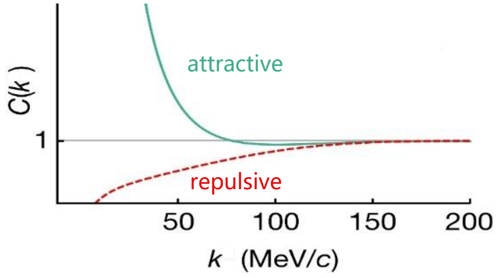
$\boldsymbol{k}^* = (m_2\boldsymbol{p}_1 - m_1\boldsymbol{p}_2)/(m_1+m_2)$

Emission source $S_{12}(\boldsymbol{r}^*)$ **Two-particle wavefunction $\psi(\boldsymbol{k}^*,\boldsymbol{r}^*)$**

Interacting potential



Correlation function



↓

Schrödinger equation

↓

Two-particle wavefunction $\psi(\boldsymbol{k}\ ,\boldsymbol{r}\ )$

Exp. measurement

mixed-event technique

$$C(k) = \xi(k)\frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

N_{same}: the same event distributions

N_{mixed}: the mixed event distributions

ξ: the corrections for experimental effects

Theo. description

Koonin–Pratt formula

$$C(k) = \int S_{12}(\boldsymbol{r})\ |\psi(\boldsymbol{k},\boldsymbol{r})|^2\ \mathrm{d}\boldsymbol{r}$$

spacial structure

final-state interactions

quantum statistics effects

coupled-channel effects

Basic Properties

$$C(k)\begin{cases} >1 \text{ if the interaction is attractive} \\ =1 \text{ if there is no interaction} \\ <1 \text{ if the interaction is repulsive} \end{cases}$$

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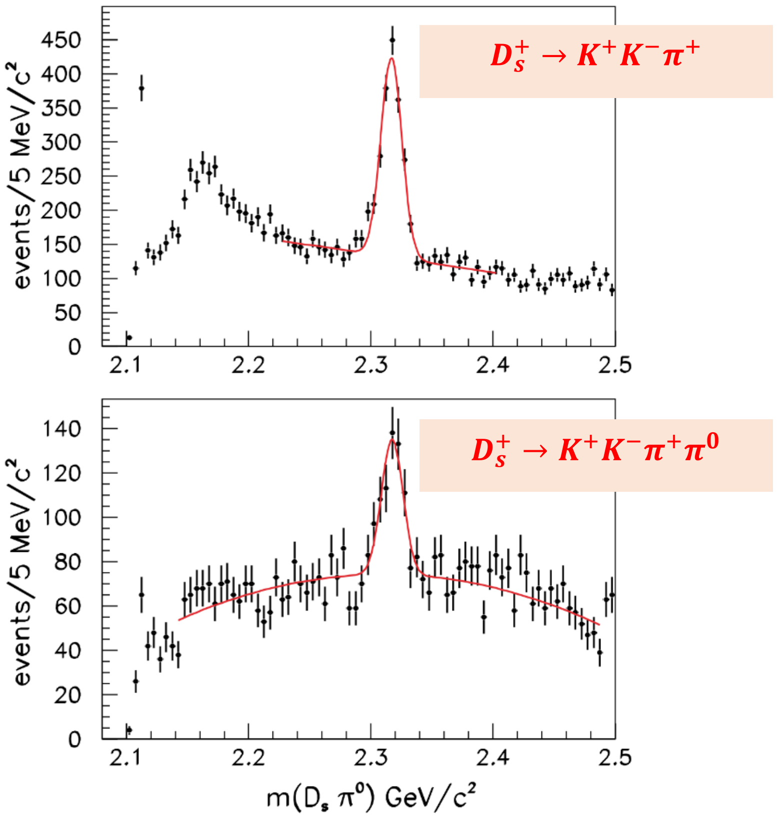
- ☞ **Summary and outlook**

Mysterious exotic hadron $D_{s0}^*(2317)$

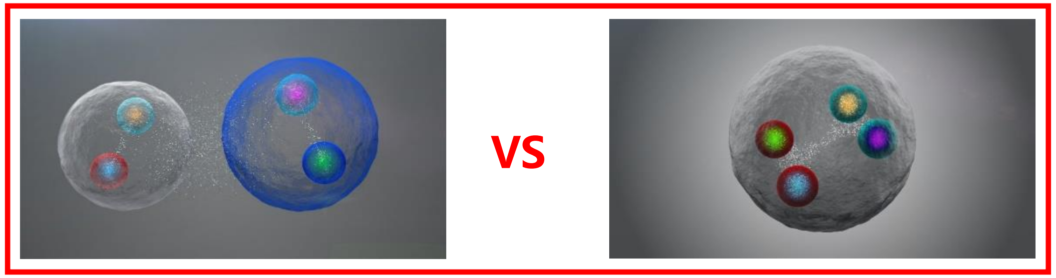
$\mathrm{M} = 2317.8 \pm 0.6$ and $\Gamma < 3.8$ MeV



BABAR, Phys. Rev. Lett. 90 (2003) 242001

- **160 MeV lower than quark model predictions – difficult to be understood as conventional charm-strange mesons**
- $m(D_{s1}) - m(D_{s0}^*) \approx m(D^*) - m(D)$
  F. K. Guo, C. Hanhart, U.-G. Meißner, Phys. Rev. Lett. 102 (2009) 242004
- **DK scattering length from LQCD matches molecular scenario**
  L. Liu, K. Orginos, F. K. Guo, C. Hanhart, U.-G. Meißner, Phys. Rev. D 87 (2013) 014508

……



How to verify the DK molecular picture ?

- **From DK 2-body system to DDK 3-body molecules**
  T. W. Wu, Y. W. Pan, M. Z. Liu, and LSG, Sci. Bull. 67 (2022) 1735*
- **Production yields of $D_{s0}^*(2317)$**
  T. C. Wu and LSG, Phys. Rev. D 108 (2023) 014015*

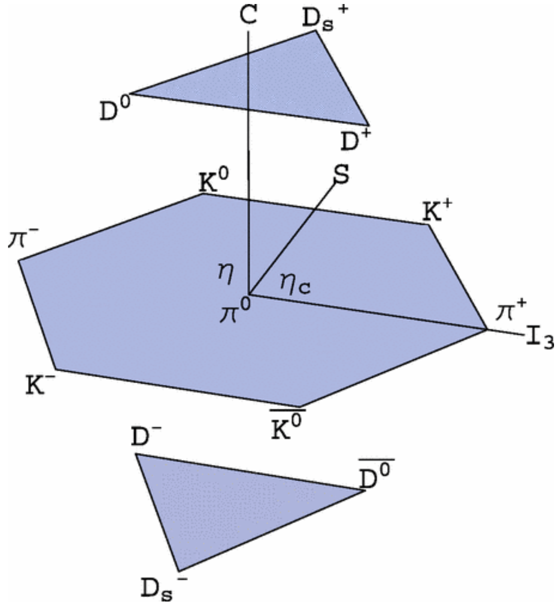
Weinberg-Tomozawa Interaction (leading order)

□ LO interaction between a NGB and a heavy pseudoscalar boson

$$\mathcal{L} = \frac{1}{4f_\pi^2}\left(\partial^\mu P[\Phi, \partial_\mu \Phi]P^\dagger - P[\Phi, \partial_\mu \Phi]\partial^\mu P^\dagger\right)$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$P = \left(D^0, D^+, D_s^+\right)$$

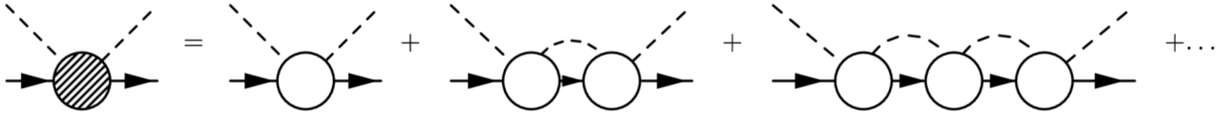


□ Weinberg-Tomozawa (WT) potential – parameter free

$$V_{\nu'\nu} = \frac{C_{\nu'\nu}}{4f_0^2}\left[(p_1+p_2)^2 - (p_1-p_4)\right] \qquad p_{1(3)} = (E_{1(3)}, \boldsymbol{p}^{(\prime)}), \quad p_{2(4)} = (\sqrt{s} - E_{1(3)}, -\boldsymbol{p}^{(\prime)})$$

Scattering wave function

□ Coupled-channel scat. eq.



$$T_{\nu'\nu}(k',k) = V_{\nu'\nu}\cdot f_{\Lambda_F}(k',k) + \sum_{\nu''}\int_0^\infty \frac{\mathrm{d}k''k''^2}{8\pi^2}\frac{V_{\nu'\nu''}\cdot f_{\Lambda_F}(k',k'')\cdot T_{\nu''\nu}(k'',k)}{E_{P,\nu''}E_{\Phi,\nu''}(\sqrt{s}-E_{P,\nu''}-E_{\Phi,\nu''}+i\epsilon)}$$

Dynamically generated state — singularities of unitarized amplitude *T*

$$f_{\Lambda_F}(k',k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2 - \left(\frac{k}{\Lambda_F}\right)^2\right]$$

$M_{D^*_{s0}} = 2317.8$ MeV → $\Lambda_F = 1107$ MeV

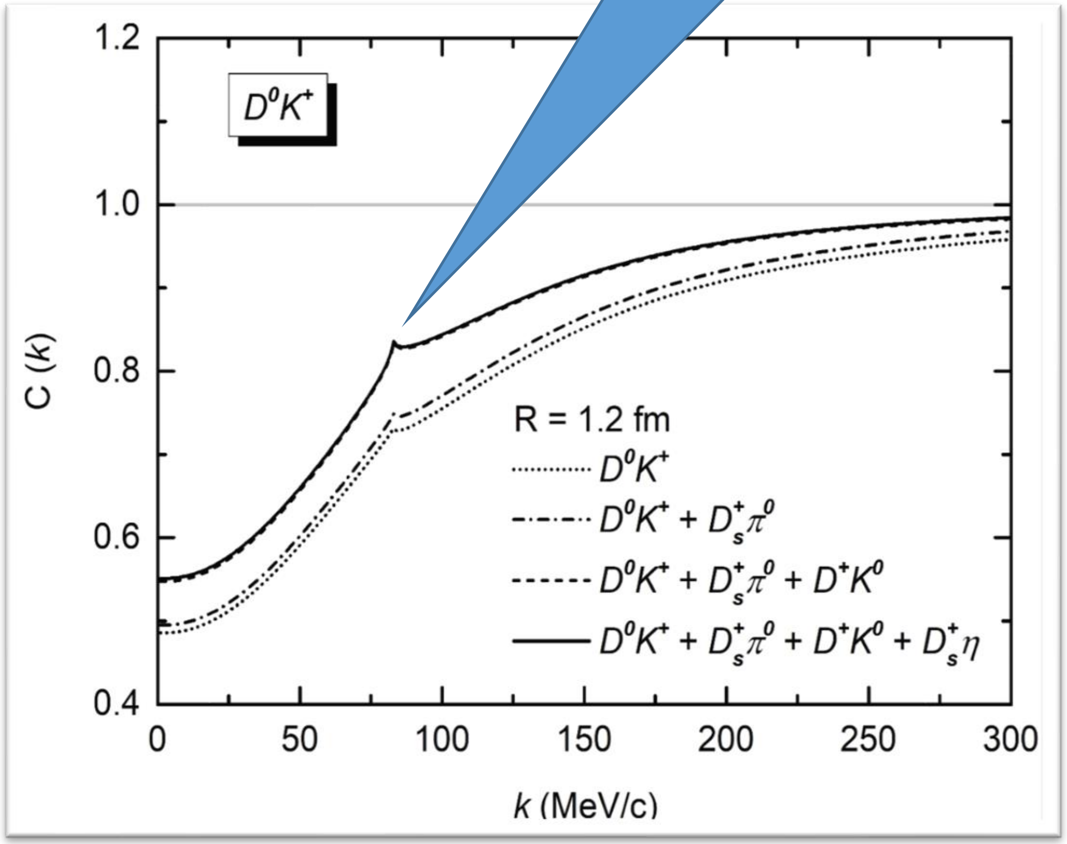
□ S-wave scattering wave function (including off-shell effect)

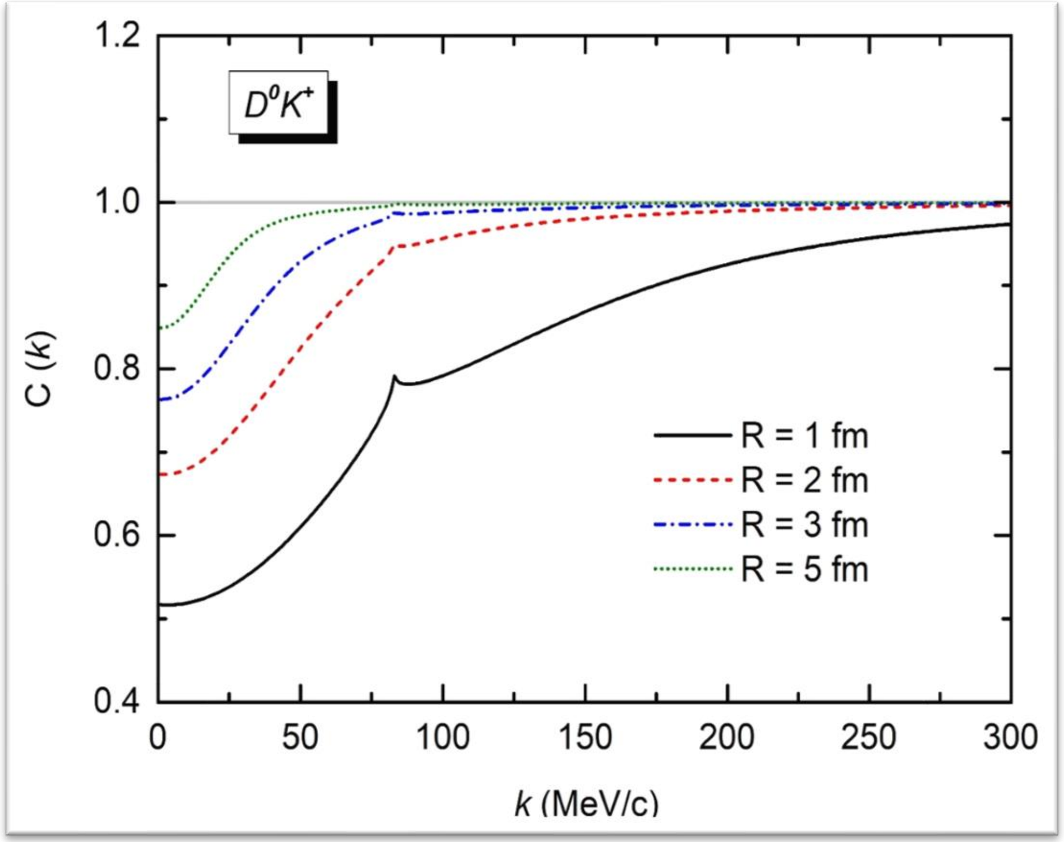
$$\psi_{\nu'\nu}(k,r) = \delta_{\nu'\nu}j_0(kr) + \int_0^\infty \frac{\mathrm{d}k'k'^2}{8\pi^2}\frac{T_{\nu'\nu}(k',k)\cdot j_0(k'r)}{E_{P,\nu'}E_{\Phi,\nu'}(\sqrt{s}-E_{P,\nu'}-E_{\Phi,\nu'}+i\epsilon)}$$

DK CFs and its source size dependence

$D^+K^0 \to D^0K^+$

Typical feature of deeply bound states





ZWL, Jun-Xu Lu and Li-Sheng Geng, PRD 107, 074019 (2023)*

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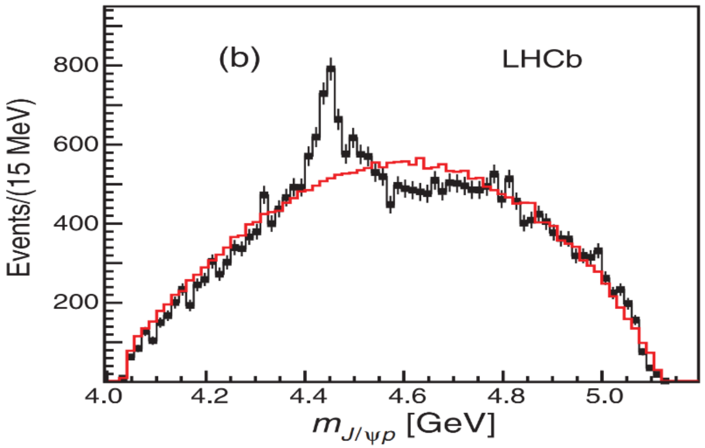
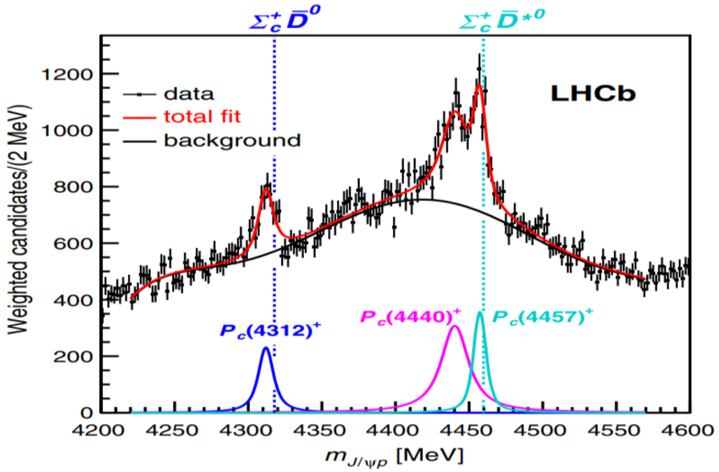
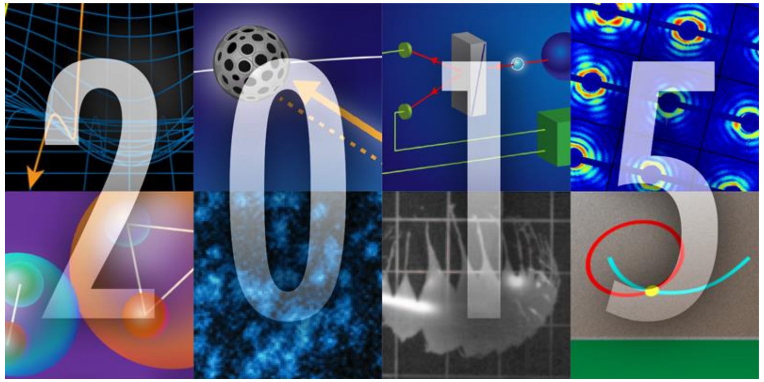
Pentaquark states $P_c(4440)$ & $P_c(4457)$



LHCb, Phys. Rev. Lett. 115 (2015) 072001



LHCb, Phys. Rev. Lett. 122 (2019) 222001



Pentaquark states • 2015 APS Highlights

How to distinguish the spins of $P_c(4440)$ and $P_c(4457)$?

Masses, invariant mass distributions, decays, magnetic momenta, production rates

M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sánchez S, LSG, A. Hosaka, M. P. Valderrama, Phys. Rev. Lett. 122 (2019) 242001*
M. L. Du, V. Baru, F. K. Guo, C. Hanhart, U. G. Meißner, J. A. Oller, Q. Wang, Phys. Rev. Lett. 124 (2020) 072001
Y. H. Lin, B. S. Zou, Phys. Rev. D 100 (2019) 056005
M. W. Li, Z. W. Liu, Z. F. Sun and R. Chen, Phys. Rev. D 104 (2021) 054016
Q. Wu, D. Y. Chen, Phys. Rev. D 100 (2019) 114002

Heavy antiquark diquark symmetry

Y. W. Pan, M. Z. Liu, F. Z. Peng, M. S. Sánchez, LSG, M. P. Valderrama, Phys. Rev. D 102 (2020) 011504*

Neural network-based approach

Z. Zhang, J. Liu, J. Hu, Q. Wang, U.-G. Meißner, Sci. Bull. 68 (2023) 981

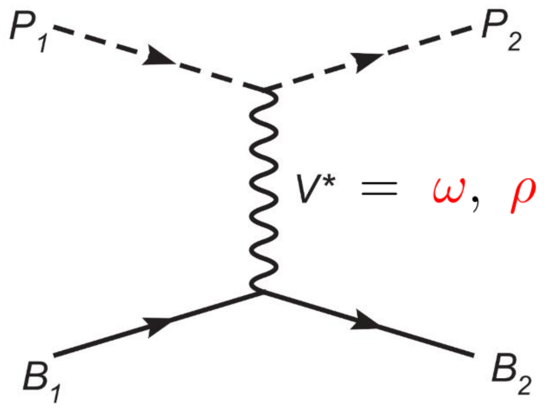
OTHER

Light vector meson exchange interaction

- Interaction in hidden local symmetry approach – **parameter free**

$$V^{I=\frac{3}{2}}_{\Sigma_c \bar{D}^{(*)}} = 2M_{\Sigma_c} M_{\bar{D}^{(*)}} \widetilde{\beta}_1 \widetilde{\beta}_2 g_V^2 \left(\frac{1}{m_\omega^2} + \frac{1}{m_\rho^2} \right)$$

$$V^{I=\frac{1}{2}}_{\Sigma_c \bar{D}^{(*)}} = 2M_{\Sigma_c} M_{\bar{D}^{(*)}} \widetilde{\beta}_1 \widetilde{\beta}_2 g_V^2 \left(\frac{1}{m_\omega^2} - \frac{2}{m_\rho^2} \right)$$



Isospin basis

↕

Charge basis

$$\left| \Sigma_c \bar{D}^{(*)}, I = \frac{3}{2}, I_3 = \frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| \Sigma_c^{++} D^{(*)-} \right\rangle + \sqrt{\frac{2}{3}} \left| \Sigma_c^{+} \bar{D}^{(*)0} \right\rangle$$

$$\left| \Sigma_c \bar{D}^{(*)}, I = \frac{1}{2}, I_3 = \frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}} \left| \Sigma_c^{++} D^{(*)-} \right\rangle - \sqrt{\frac{1}{3}} \left| \Sigma_c^{+} \bar{D}^{(*)0} \right\rangle$$

Two different spin assignments

☐ Interaction strengths

$$f_{\Lambda_F}(k',k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2 - \left(\frac{k}{\Lambda_F}\right)^2\right]$$

$\Lambda_F = 1067\ \mathrm{MeV}$

deep bound state of $\Sigma_c\bar{D}^*$

$\Lambda_F = 860\ \mathrm{MeV}$

shallow bound state of $\Sigma_c\bar{D}^*$

itation: R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 upda

$P_c(4440)^+$ Status: *

$P_c(4440)^+$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$4440.3 \pm 1.3^{+4.1}_{-4.7}$	AAIJ	19W	LHCB	pp at 7, 8, 13 TeV

$P_c(4457)^+$ Status: *

$P_c(4457)^+$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$4457.3 \pm 0.6^{+4.1}_{-1.7}$	AAIJ	19W	LHCB	pp at 7, 8, 13 TeV
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$4449.8 \pm 1.7 \pm 2.5$	[1] AAIJ	15P	LHCB	Repl. by AAIJ 19W

[1] Considering $P_c(4440)$ and $P_c(4457)$ as a single resonance.

CF for the shallow bound state is **significantly larger** than that for the deep bound

☐ Experimental CFs – spin-averaged

Scenarios A

$\mathbf{P_c(4440) : J^P = (1/2)^-}$ $\mathbf{P_c(4457) : J^P = (3/2)^-}$

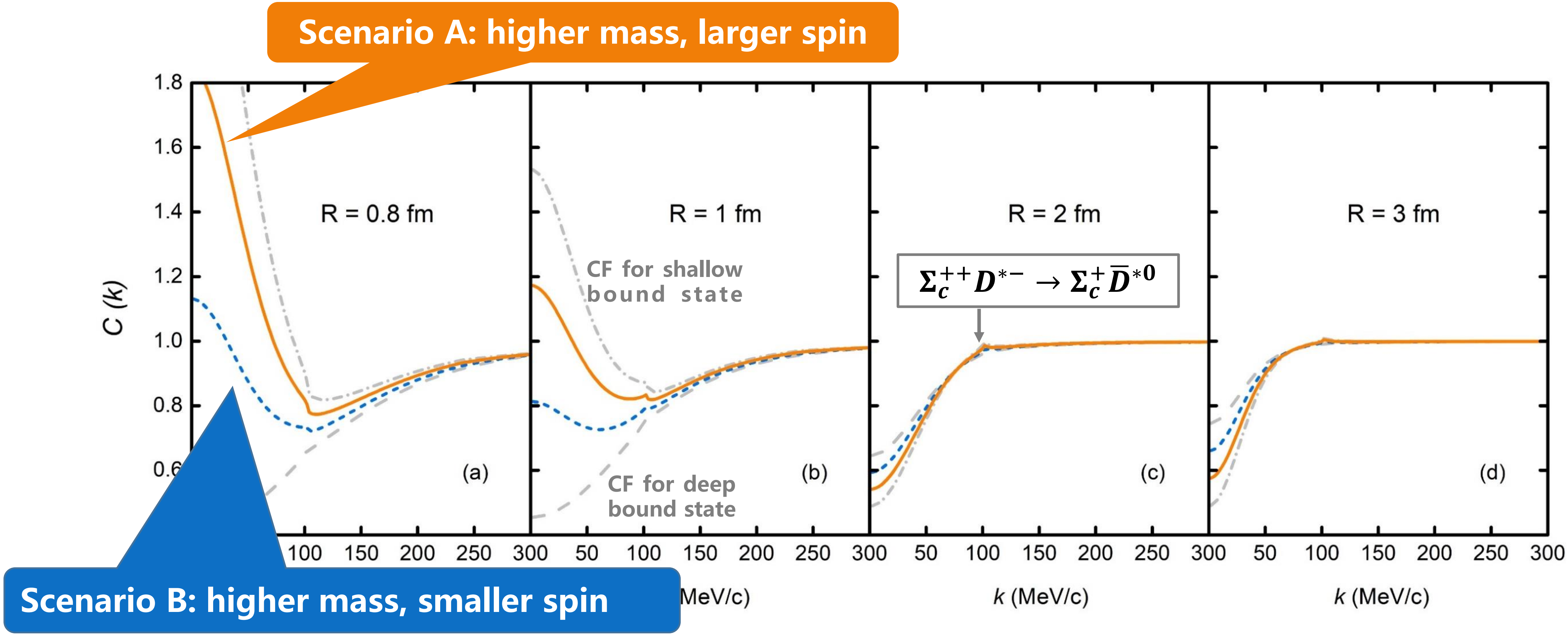
$$\bar{\mathbf{C}} = \mathbf{1/3 \cdot C_{deep} + 2/3 \cdot C_{shallow}}$$

Scenarios B

$\mathbf{P_c(4440) : J^P = (3/2)^-}$ $\mathbf{P_c(4457) : J^P = (1/2)^-}$

$$\bar{\mathbf{C}} = \mathbf{2/3 \cdot C_{deep} + 1/3 \cdot C_{shallow}}$$

Spin-averaged $\Sigma_c\bar{D}^*$ CFs



ZWL, Ming-Zhu Liu, Jun-Xu Lu and Li-Sheng Geng, PRD 108, L031503 (2023)*

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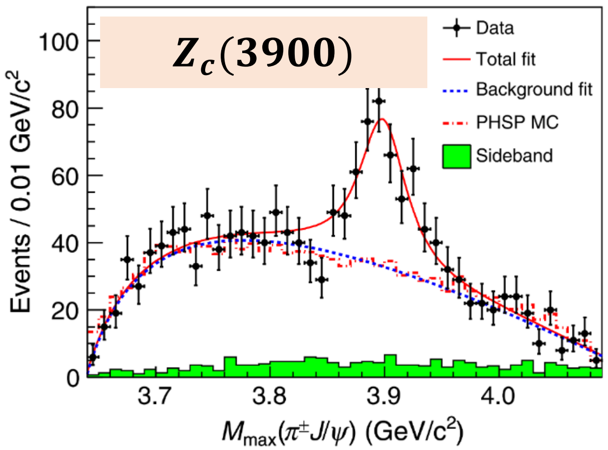
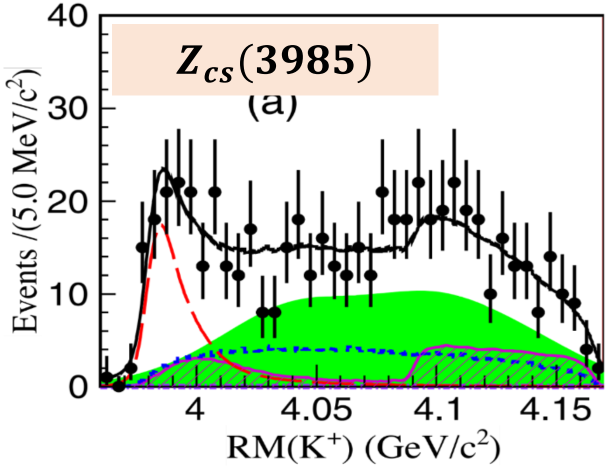
☞ **Summary and outlook**

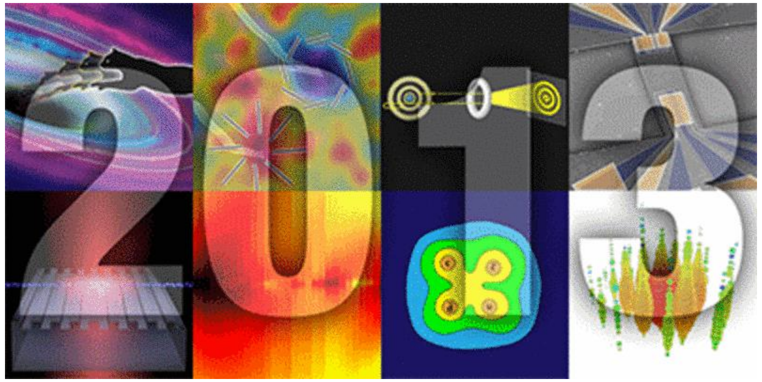
Tetraquark states $Z_c(3900)$ & $Z_{cs}(3985)$



BESIII, PRL110 (2013) 252001



BESIII, PRL126 (2021) 102001



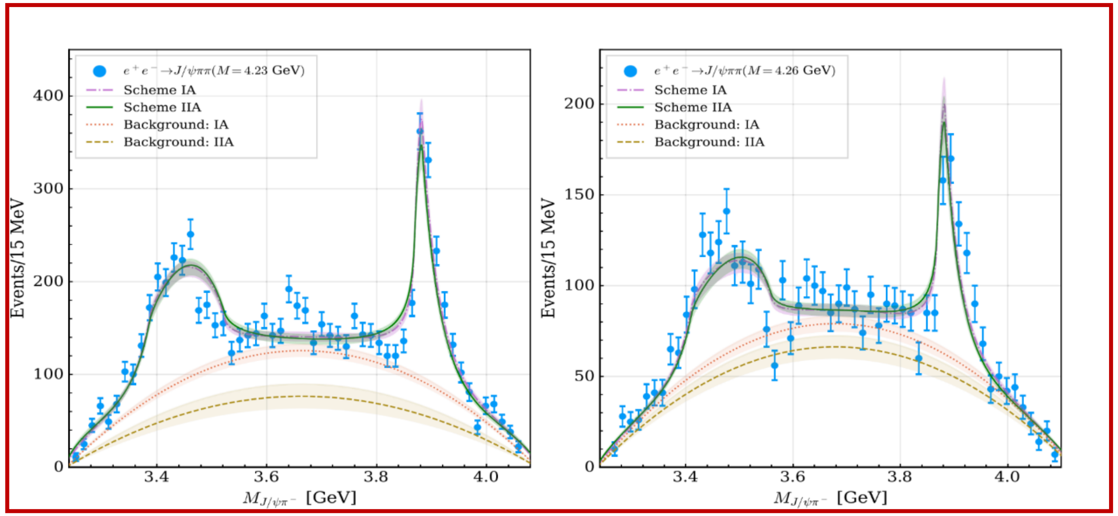
Tetraquark states • 2013 APS Highlights

$Z_c(3900)$ & $Z_{cs}(3985)$: Resonant VS Virtual states

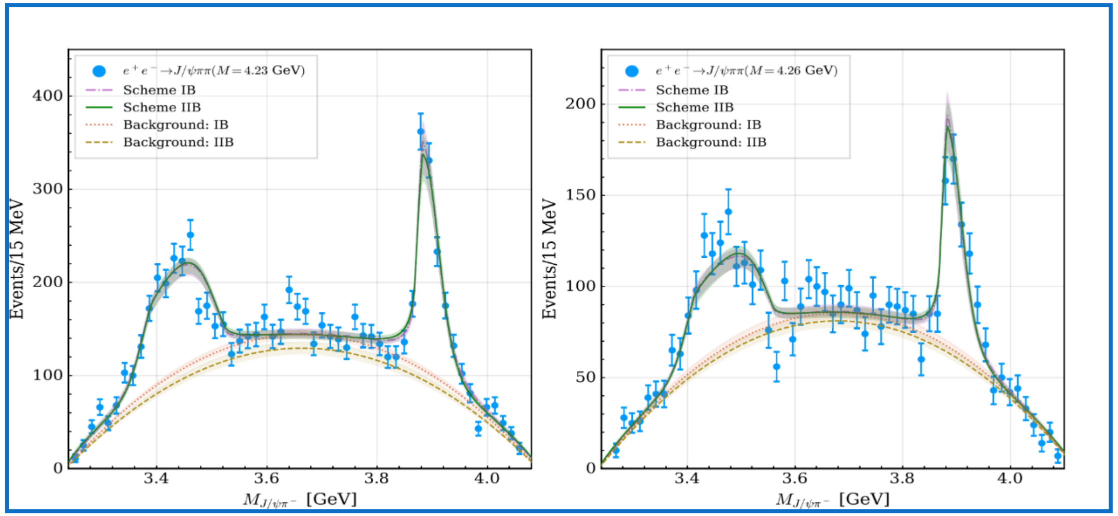
Particle Data Group, PTEP 2022 (2022) 083C01
M.-L. Du, M. Albaladejo, F.-K. Guo and J. Nieves, PRD 105 (2022) 074018
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, F.-K. Guo and J. Nieves, PRD106 (2022) 094002
L.-W. Yan, Z.-H. Guo, F.-K. Guo, D.-L. Yao and Z.-Y. Zhou, PRD109 (2024) 014026

Invariant mass distributions fail to distinguish vir. or res.

Virtual state scenario



Resonant state scenario



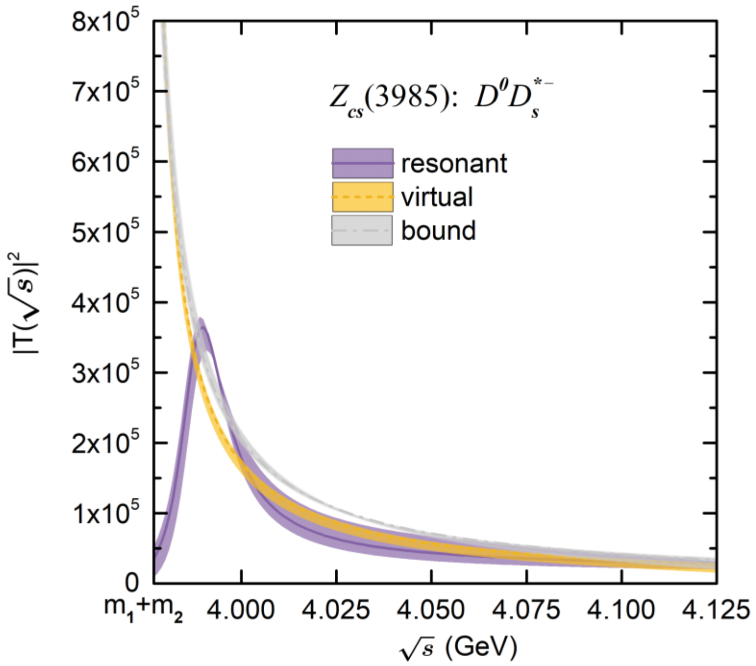
M.-L. Du, M. Albaladejo, F.-K. Guo, and J. Nieves, PRD105(2022)074018

Data are compatible with either a resonant or virtual state.

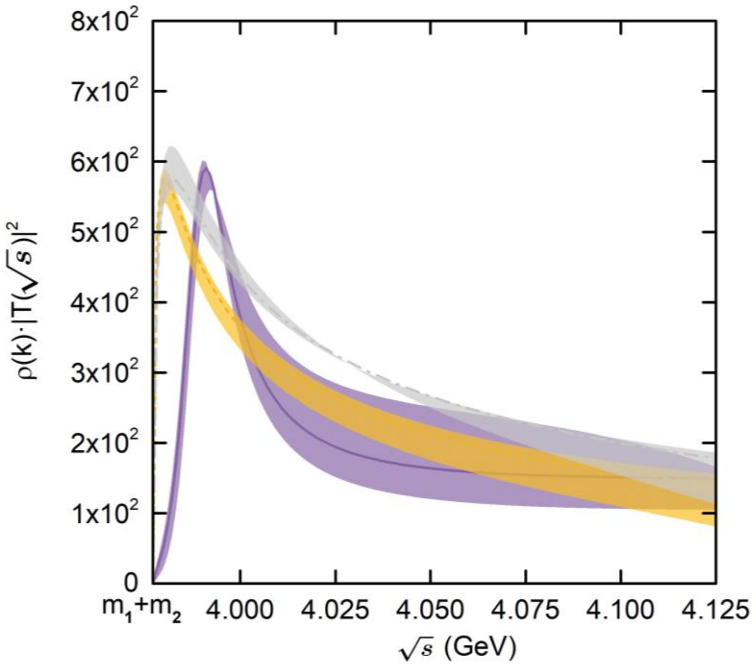
How to tell which is reality?

Phase-space suppression effect

Modulus squared of T-matrix



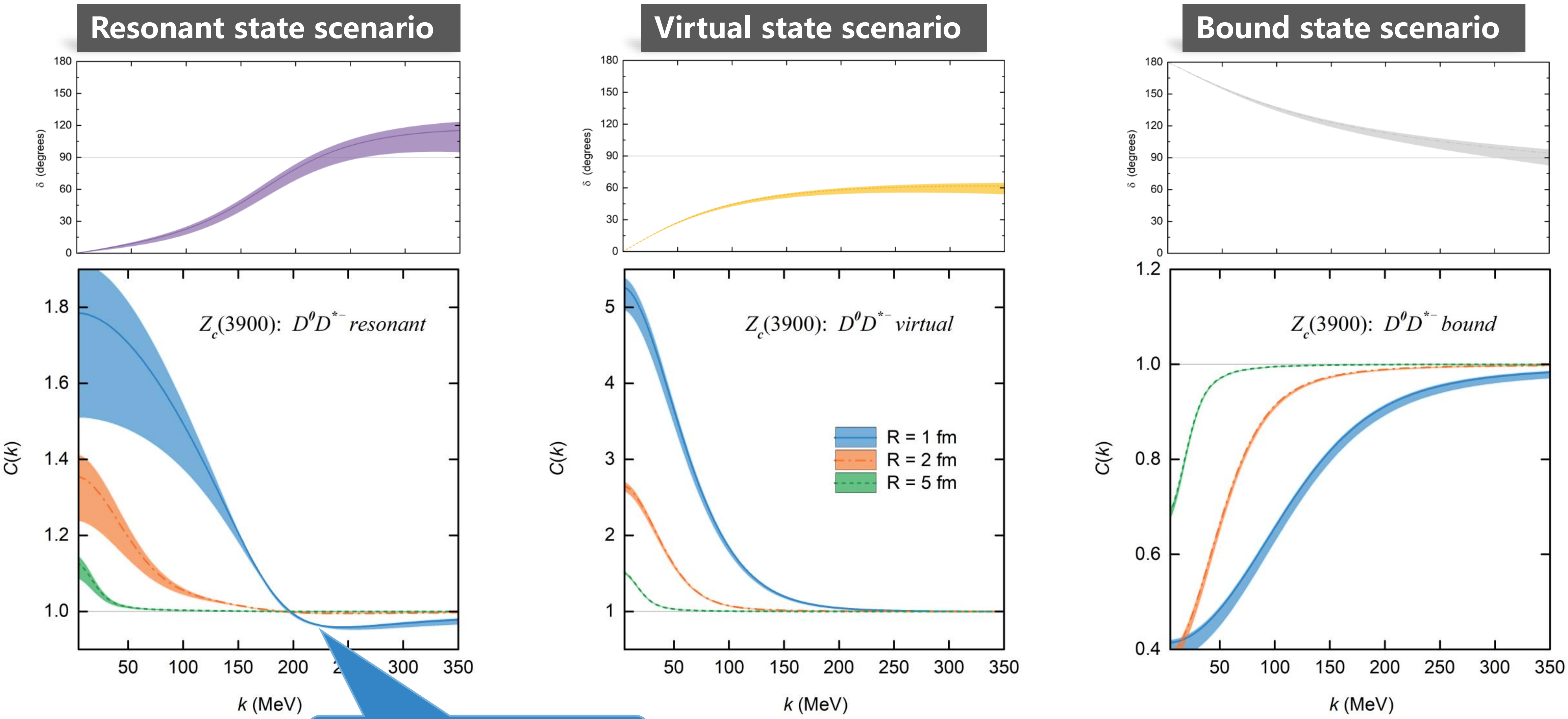
Line shape of invariant mass spectra



Phase-space suppression makes near-threshold states hard to distinguish using invariant mass spectra alone. ***ZWL**, Jun-Xu Lu*, Ming-Zhu Liu and Li-Sheng Geng*, Sci. Bull. in press*

D^0D^{*-} CFs for $Z_c(3900)$

ZWL, Jun-Xu Lu*, Ming-Zhu Liu and Li-Sheng Geng*, Sci. Bull. in press



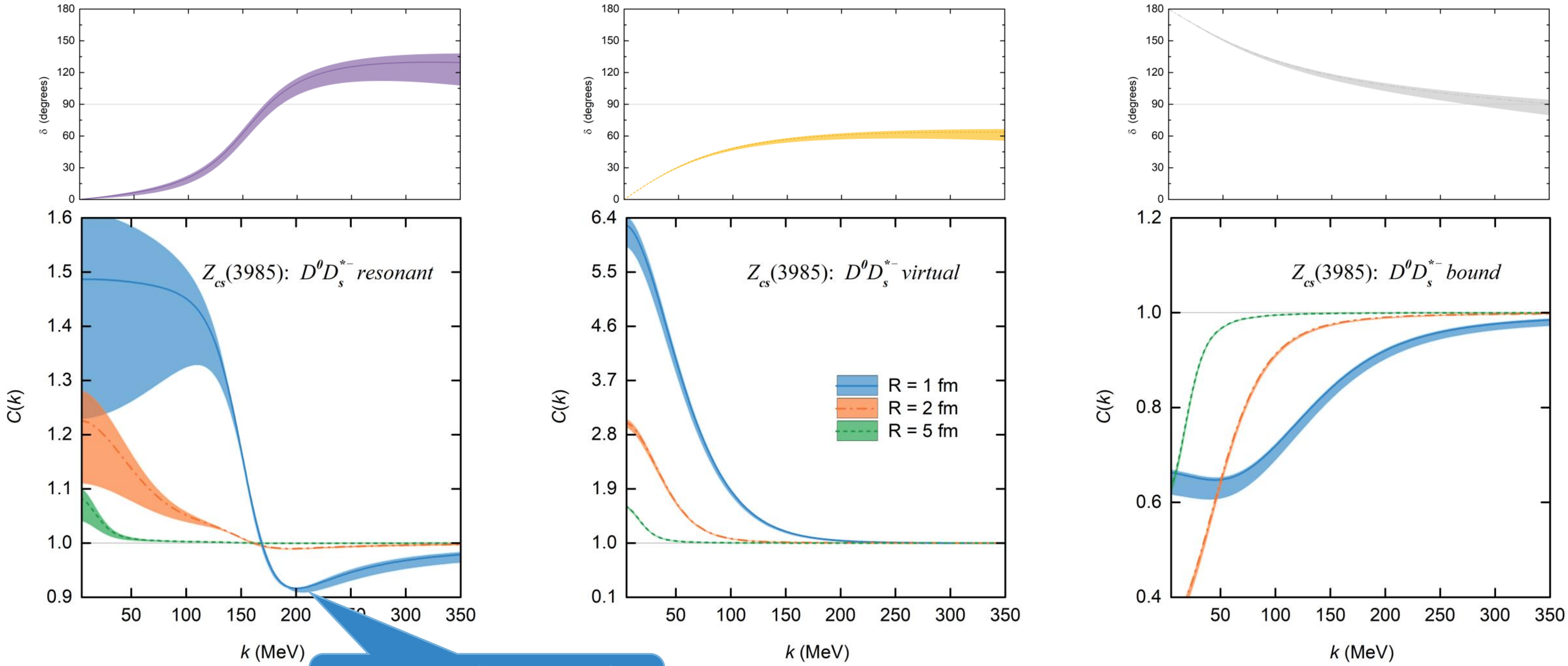
Typical feature of broad resonant state

$$C(k) = 1 + \mathcal{F}_1 \sin^2\delta + \mathcal{F}_2 \sin\delta\cos\delta, \qquad \mathcal{F}_2 > 0$$

$D^0 D_s^{*-}$ CFs $Z_{cs}(3985)$

ZWL, Jun-Xu Lu, Ming-Zhu Liu and Li-Sheng Geng*, Sci. Bull. in press*

Resonant state scenario | **Virtual state scenario** | **Bound state scenario**



Typical feature of narrow resonant state

$$C(k) = 1 + \mathcal{F}_1 \sin^2\delta + \mathcal{F}_2 \sin\delta\cos\delta, \qquad \mathcal{F}_2 > 0$$

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- ☞ **Summary and outlook**

Summary and outlook

- ☐ **Femtoscopy offers high-precision tests of the strong interaction between pairs of (un)stable particles and can be valuable to decipher the nature of the many exotic hadrons discovered so far.**
  - ✓ $D_{s0}^{*}(2317)$, $P_c(4440)$ **and** $P_c(4457)$, $Z_c(3900)$ **and** $Z_{cs}(3985)$

- ☐ **With more data from LHC Run3, very promising:**
  - Pp data taking: event count increase: ~3000×
  - Pb-pb data taking: event count increase: ~70×

- ☐ **Many questions remain unanswered, at least not satisfactorily**
  - ✓ **Is the factorization of the KP formula well justified**
  - ✓ **Can one define a universal source function**
  - ✓ **How to study three-body correlation functions**
  - ✓ **…**

Thanks a lot for your attention!

(Image: CERN)

Off-shell ambiguities

2505.13433v1.

Nucleon-nucleon correlation functions from different interactions in comparison

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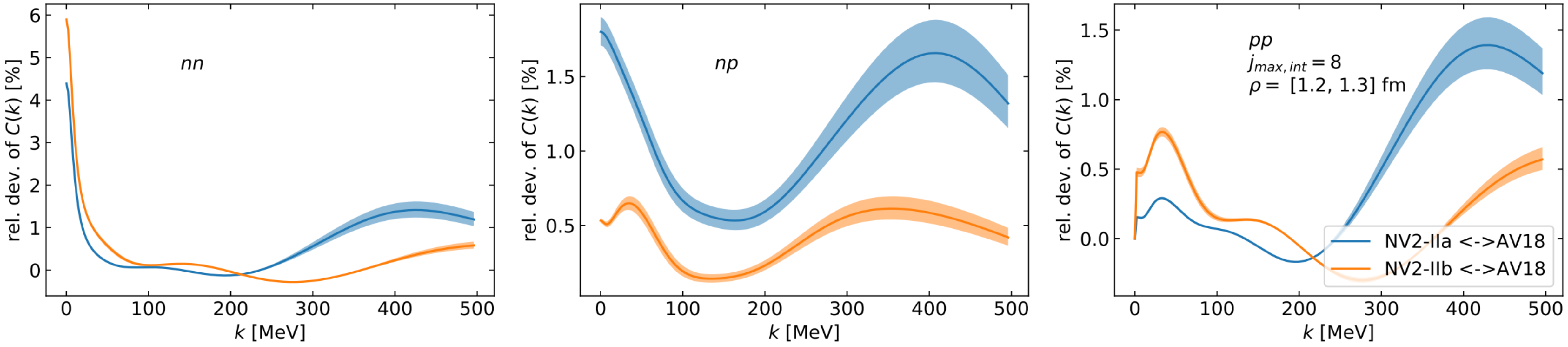
"The analysis of the differences between the correlations of the various interactions shows that for momenta between 0 and 500 MeV there are variations of up to 5.9 % for the nn system, of up to 1.8 % for the np system, and of 1.4 % for the pp system."



Off-shell ambiguities

Off-shell ambiguities in correlation functions: strategies to minimize them

2506.03669v1.

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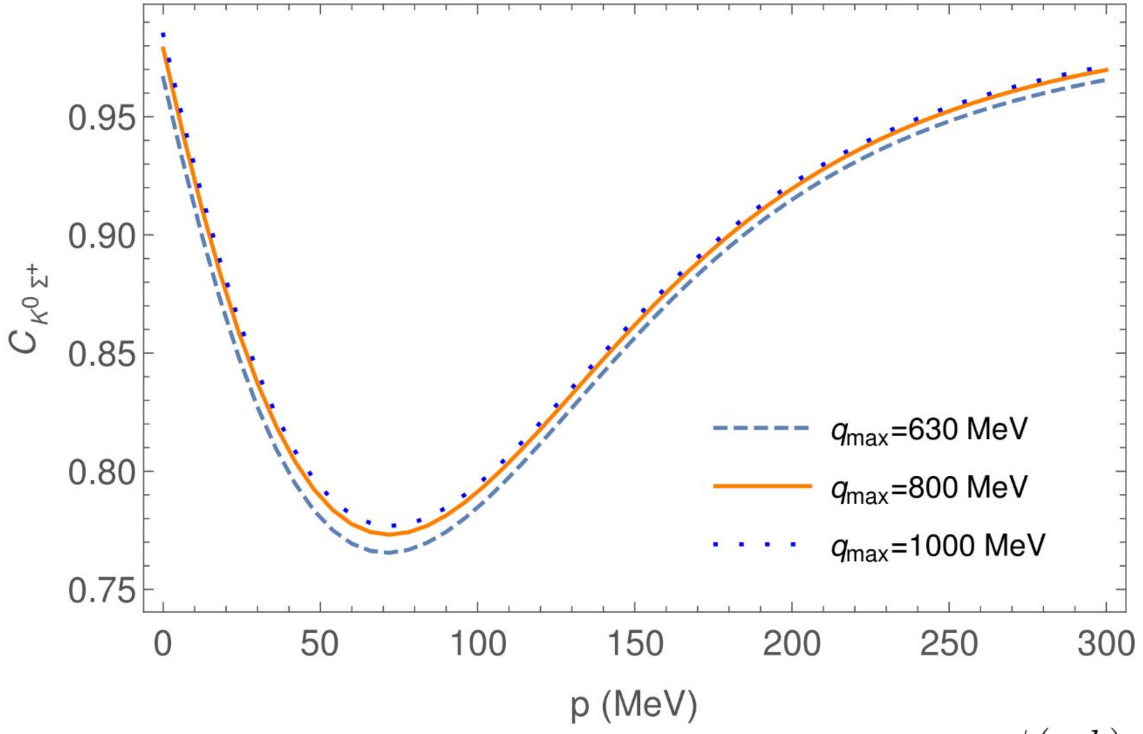


"We find that using realistic interactions based on chiral dynamics the uncertainties are small, of the order of 2 - 3 %, but could be much bigger if other methods are used."

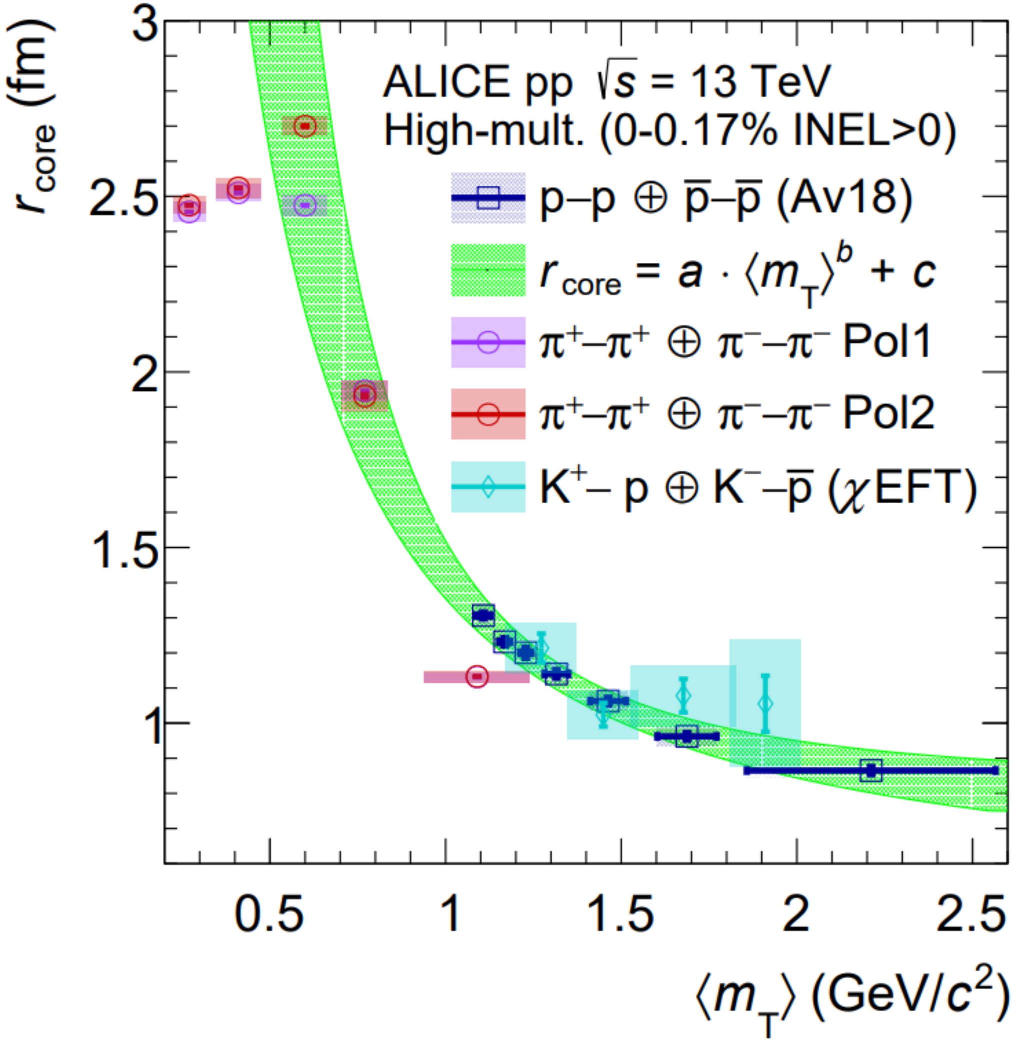
$$\mathcal{V}(\sqrt{s}; k, k') = V(\sqrt{s}) \cdot \theta(q_{\rm max} - k) \cdot \theta(q_{\rm max} - k'), \quad (1)$$

$$\mathcal{T}(\sqrt{s}; k, k') = T(\sqrt{s}) \cdot \theta(q_{\rm max} - k) \cdot \theta(q_{\rm max} - k'), \quad (2)$$

$$\psi(r,k) = j_0(kr) + T(\sqrt{s}) \cdot \theta(q_{\rm max} - k) \cdot \int_0^\infty \frac{{\rm d}^3 k'}{(2\pi)^3} \frac{E_1' + E_2'}{2E_1' E_2'} \frac{\theta(q_{\rm max} - k') \cdot j_0(k'r)}{s - (E_1' + E_2')^2 + i\varepsilon}, \quad (3)$$

Constraining the source function

2311.14527v1.



… "providing compelling evidence for the presence of a common emission source for all hadrons in small collision systems at the LHC …"