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Perturbative Renormalization of chiral nuclear forces at subleading order

彭　锐 (Rui Peng)

Peking University

合作者：许甫荣　龙炳蔚

outline

1 Chiral Effective Field theory

2 Chiral Nuclear Forces and Renormalization

3 Pertubative Renormalization at subleading order

4 Summary and outlook

Chiral Effective Field theory

Energy Resolution

~1 GeV Λ_χ	QCD	Quarks Gluons
~140 MeV m_π	**Chiral EFT**	**Pions Nucleons**
~45 MeV Q_D	Pionless EFT	Nucleons
	Halo/Cluster EFT	Nucleons Nucleus
~1 MeV vibration	EFT for nuclear vibrations deformed nuclei	

Degrees of freedom

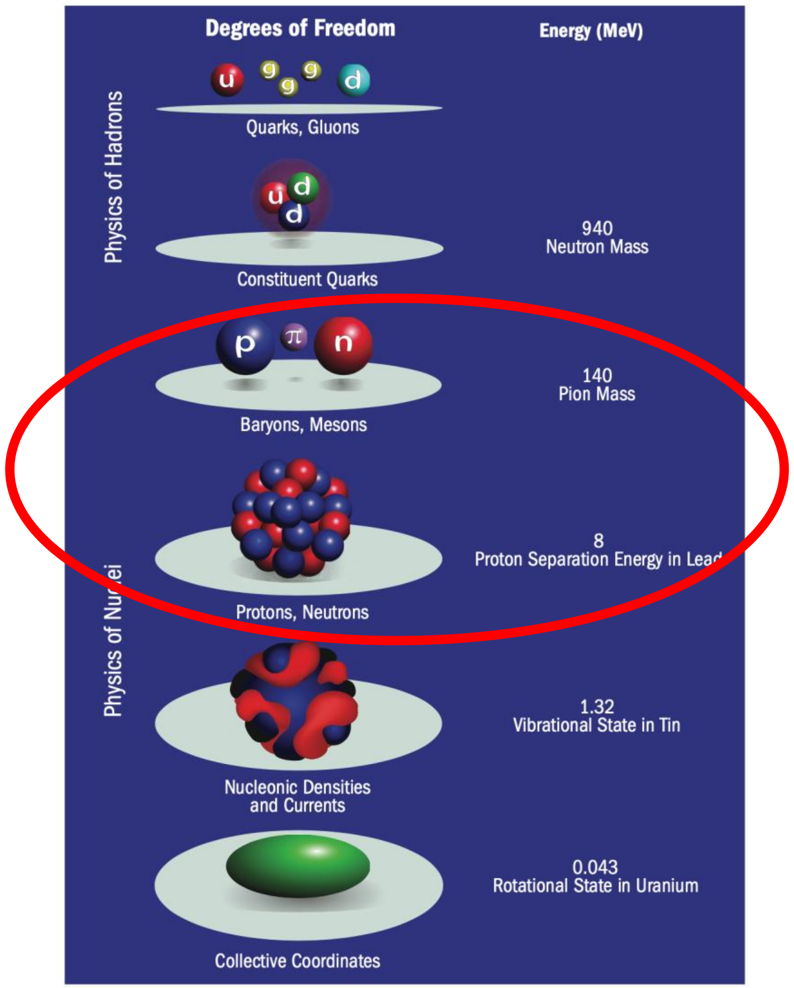
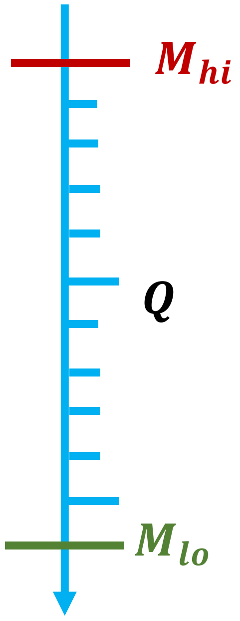


Fig.: Bertsch, Dean, Nazarewicz (2007)

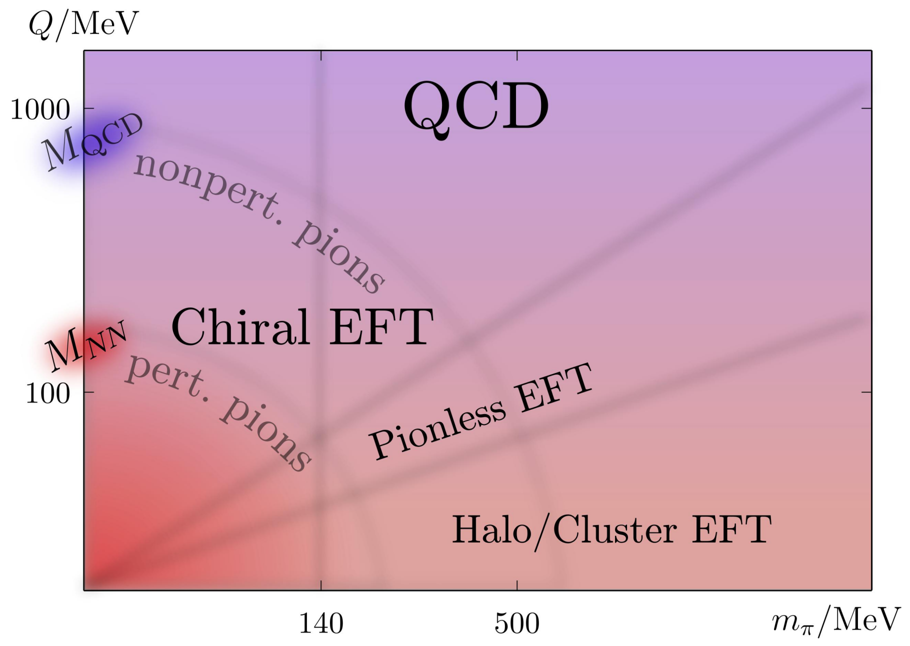
Effective Field theory(EFT)

breakdown scale: M_{hi} $(\mathbf{m}_\sigma\ ,\ \Delta)$

typical momentum: $Q \sim m_\pi$



$$\mathcal{L} = \sum_i c_i(M_{lo}, M_{hi}, \Lambda) O_i(\{\psi\})$$



H.-W. Hammer, et al. RevModPhys.92.025004 (2020)

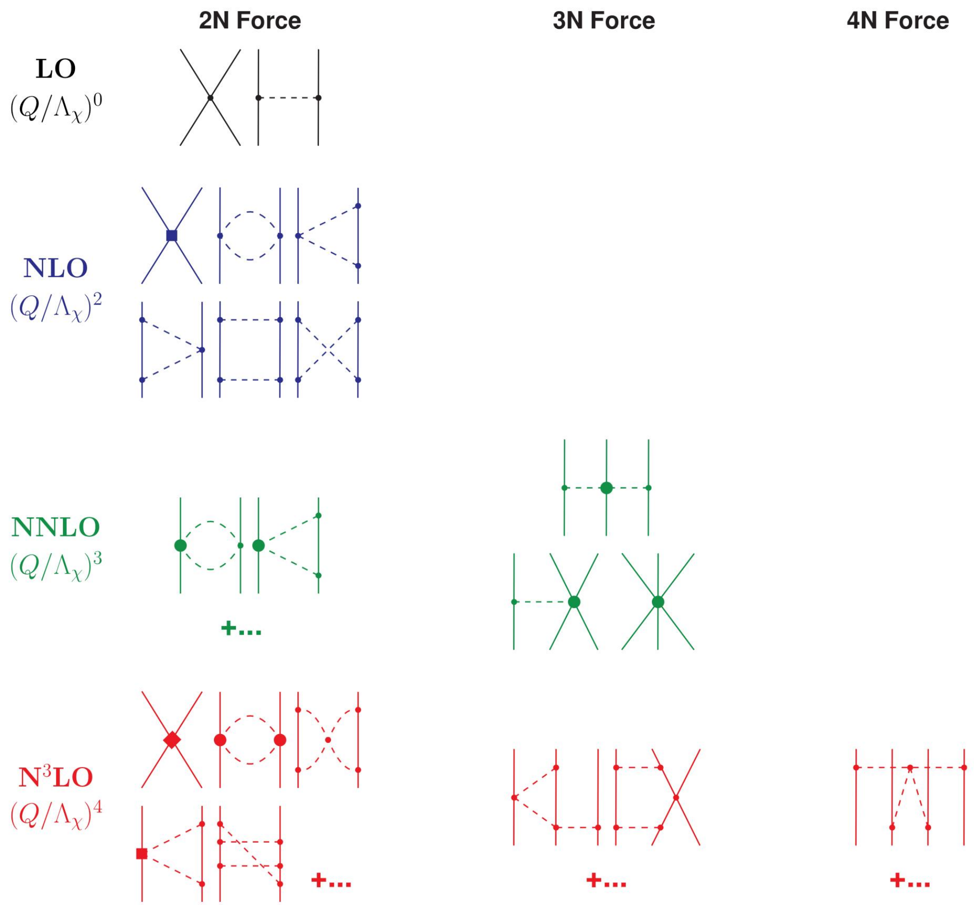
Power counting

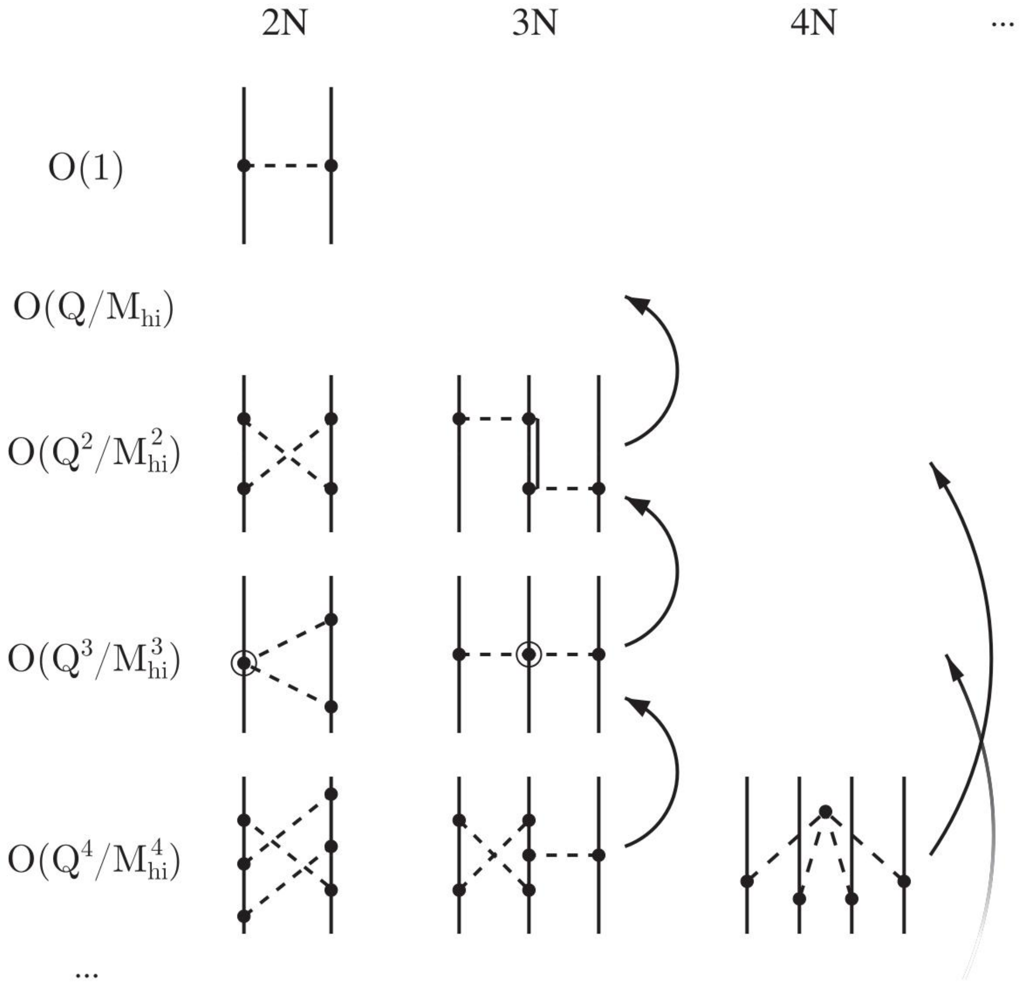
$$T(Q) \sim \sum_{\nu=0}^{\infty} \left(\frac{Q}{M_{hi}}\right)^{\nu} F^{(\nu)}\left(\frac{Q}{M_{lo}}, \frac{Q}{\Lambda}; \gamma^{\nu}\left(\frac{M_{lo}}{M_{hi}}, \frac{\Lambda}{M_{hi}}\right)\right)$$

$$\gamma^{\nu} = \mathcal{O}(1)$$

Power counting

$$N^{\nu}LO \quad \sim (\frac{Q}{M_{hi}})^{\nu}$$





R. Machleidt , D.R. Entem. Phys.Rept,503:1-75,2011

H.-W. Hammer, et al. RevModPhys.92.025004 (2020)

Lippmann-Schwinger equation

$$T(p', p; k) = V^{\Lambda}(p', p) + \int_0^{\infty} dp'' p''^2 V^{\Lambda}(p', p'') \frac{T(p'', p; k)}{k^2 + i\epsilon - p''^2}$$

regularize: $V^{\Lambda}(p', p) = f_R(\frac{p'}{\Lambda}) V(p', p) f_R(\frac{p}{\Lambda})$

regulator: $f_R(\mathrm{x}) = \mathrm{e}^{-\mathrm{x}^{\mathrm{n}}}; \mathrm{n} = 2,4, \cdots$

or $f_R(\mathrm{x}) = \theta(1 - \mathrm{x})$

Renormalization group invariance (RG-invariance):

$$T^{(0)}(k; \Lambda) = T^{(0)}(k; \infty)\left[1 + \mathcal{O}\left(\frac{k}{\Lambda}\right)\right].$$

Why perturbation ?

$$T = V + VGT$$

nonperturbation $\quad T = (V^{(0)} + V^{(1)} + V^{(2)} + \cdots) + (V^{(0)} + V^{(1)} + V^{(2)} + \cdots)GT$

perturbation

distorted-wave expansion

$V^{(0)} = 0$

plane-wave expansion

$$T^{(0)} = V^{(0)} + V^{(0)}GT^{(0)}$$

$$T^{(1)} = (1 + T^{(0)}G)\boxed{V^{(1)}}(1 + GT^{(0)})$$

$$T^{(2)} = (1 + T^{(0)}G)\boxed{(V^{(2)} + V^{(1)}GV^{(1)})}(1 + GT^{(0)})$$

$$\vdots$$

1. Linearly independent contributions from each term to the amplitude
   a. Clear identification of cancellations among terms.
   b. Reliable extraction of LECs.
2. Order-by-order preservation of **RG-invariance**

Chiral Nuclear Forces and Renormalization

PHYSICAL REVIEW C **72**, 054006 (2005)

Renormalization of one-pion exchange and power counting

A. Nogga,[1,*] R. G. E. Timmermans,[2,†] and U. van Kolck[3,‡]
[1]*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[2]*Theory Group, KVI, University of Groningen, The Netherlands*
[3]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*
(Received 2 June 2005; published 16 November 2005)

The renormalization of the chiral nuclear interactions is studied. In leading order, the cutoff dependence is related to the singular tensor interaction of the one-pion exchange potential. In *S* waves and in higher partial waves where the tensor force is repulsive this cutoff dependence can be absorbed by counterterms expected at that order. In the other partial waves additional contact interactions are necessary. The implications of this finding for the effective field theory program in nuclear physics are discussed.

DOI: 10.1103/PhysRevC.72.054006 PACS number(s): 21.30.Cb, 13.75.Cs, 21.45.+v

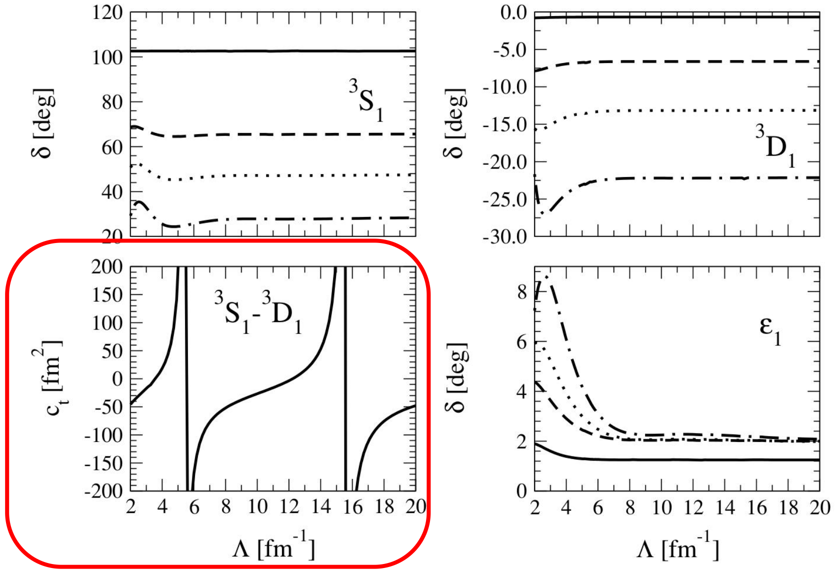
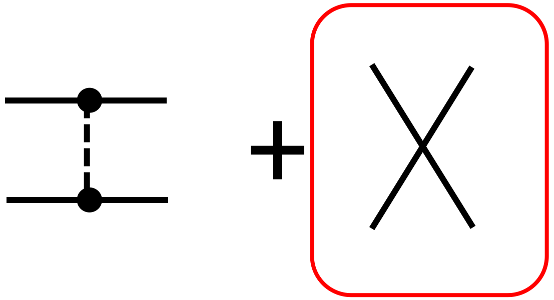
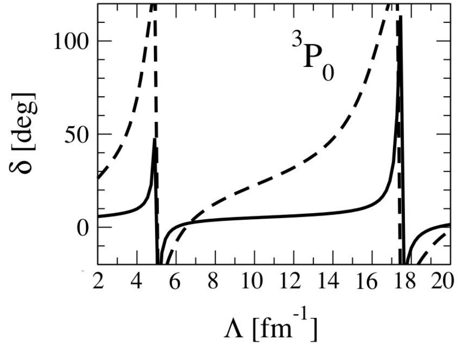
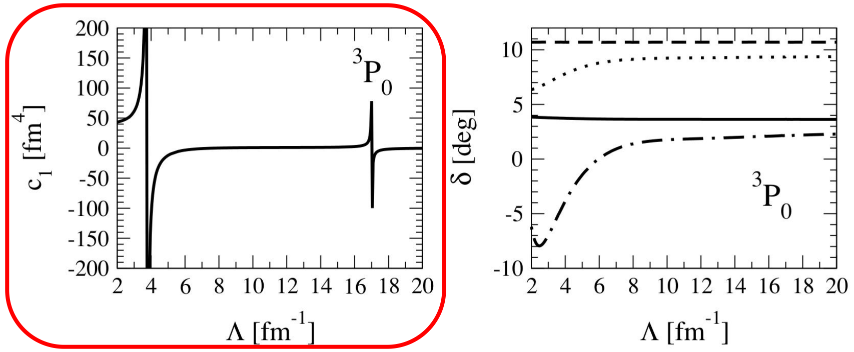


FIG. 2. Fit result for the counterterm c_t as a function of the cutoff, and the resulting cutoff dependence of the $^3S_1$-$^3D_1$ phase shifts and the mixing angle ε_1 at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

$$V^{(0)} =$$







Chiral Nuclear Forces and Renormalization

PHYSICAL REVIEW C **84**, 057001 (2011)

Renormalizing chiral nuclear forces: A case study of $^3P_0$

Bingwei Long[1,*] and C.-J. Yang[2,3,†]

[1]*Excited Baryon Analysis Center (EBAC), Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

[3]*Institute of Nuclear and Particle Physics, Ohio University, Athens, Ohio 45701, USA*

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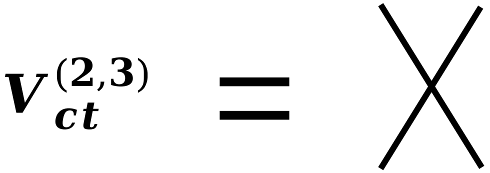
We discuss in this Brief Report the subleading contact interactions, or counterterms, in the $^3P_0$ channel of nucleon-nucleon scattering up to $O(Q^3)$, where, already at leading order, Weinberg's original power counting (WPC) scheme fails to fulfill renormalization group invariance due to the singular attraction of one-pion exchange. Treating the subleading interactions as perturbations and using renormalization group invariance as the criterion, we investigate whether WPC, although missing the leading order, could prescribe correct subleading counterterms. We find that the answer is negative and, instead, that the structure of counterterms agrees with a modified version of naive dimensional analysis. Using $^3P_0$ as an example, we also study the cutoffs where the subleading potential can be iterated together with the leading one.

DOI: 10.1103/PhysRevC.84.057001

PACS number(s): 21.30.Fe, 12.39.Fe



WPC(Weinberg's Power Counting)

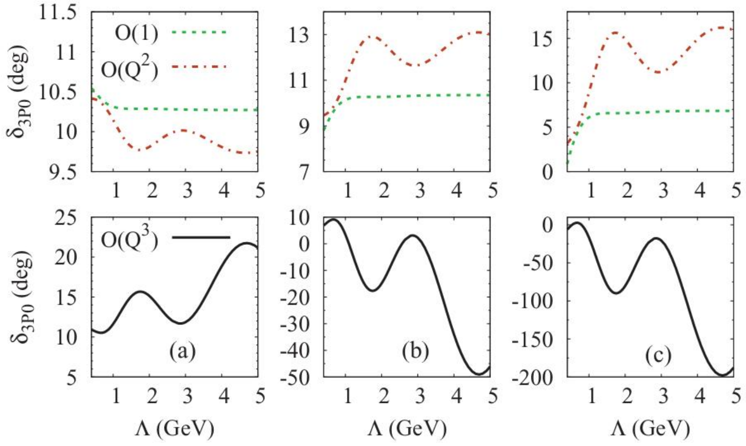
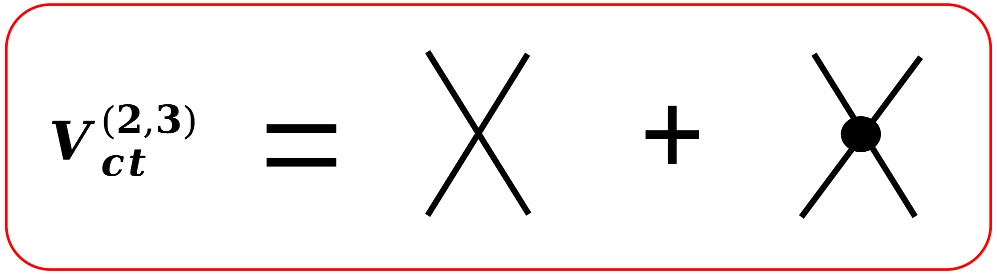


FIG. 1. (Color online) With the subleading counterterms (7), the $\mathcal{O}(1)$, $\mathcal{O}(Q^2)$ (upper row) and $\mathcal{O}(Q^3)$ (lower row) $^3P_0$ phase shifts as functions of the momentum cutoff at $T_{\rm lab} = 40$ (a), 80 (b), and 130 (c) MeV.



MMW(Minimal Modified Weinberg)

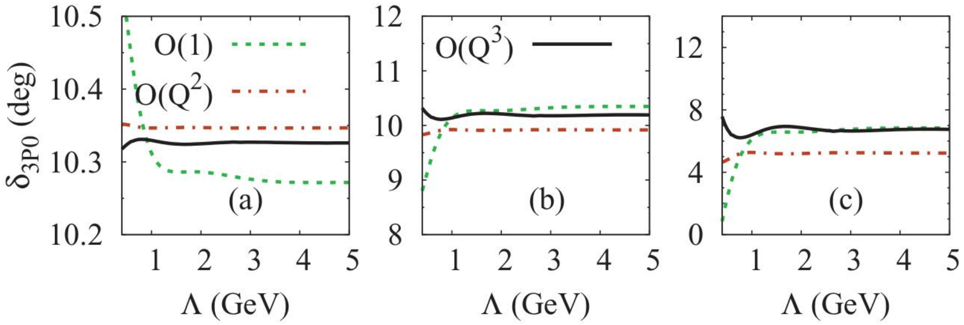


FIG. 2. (Color online) With the subleading counterterms (8), the $\mathcal{O}(1)$, $\mathcal{O}(Q^2)$, and $\mathcal{O}(Q^3)$ $^3P_0$ EFT phase shifts as functions of the momentum cutoff at $T_{\rm lab} = 40$ (a), 80 (b), and 130 (c) MeV.

PHYSICAL REVIEW C **85**, 034002 (2012)

Renormalizing chiral nuclear forces: Triplet channels

Bingwei Long[1,*] and C.-J. Yang[2,3,†]

[1] *Excited Baryon Analysis Center (EBAC), Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[2] *Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*
[3] *Institute of Nuclear and Particle Physics, Ohio University, Athens, Ohio 45701, USA*

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We discuss the subleading contact interactions, or counterterms, of the triplet channels of nucleon-nucleon scattering in the framework of chiral effective field theory, with S and P waves as the examples. The triplet channels are special in that they allow the singular attraction of one-pion exchange to modify Weinberg's original power-counting (WPC) scheme. With renormalization group invariance as the constraint, our power counting for the triplet channels can be summarized as a modified version of naive dimensional analysis in which, when compared with WPC, all of the counterterms in a given partial wave (leading or subleading) are enhanced by the same amount. More specifically, this means that WPC needs no modification in ${}^3S_1$-${}^3D_1$ and ${}^3P_1$, whereas a two-order enhancement is necessary in both ${}^3P_0$ and ${}^3P_2$-${}^3F_2$.

DOI: 10.1103/PhysRevC.85.034002 PACS number(s): 21.30.Fe, 12.39.Fe

$$\langle {}^3S_1\text{-}{}^3D_1 | V_S^{(2,3)} | {}^3S_1\text{-}{}^3D_1 \rangle = \begin{pmatrix} C_{{}^3S_1}^{(2,3)} + D_{{}^3S_1}^{(0,1)}(p'^2 + p^2) & E_{\text{SD}}^{(0,1)} p^2 \\ E_{\text{SD}}^{(0,1)} p'^2 & 0 \end{pmatrix}.$$

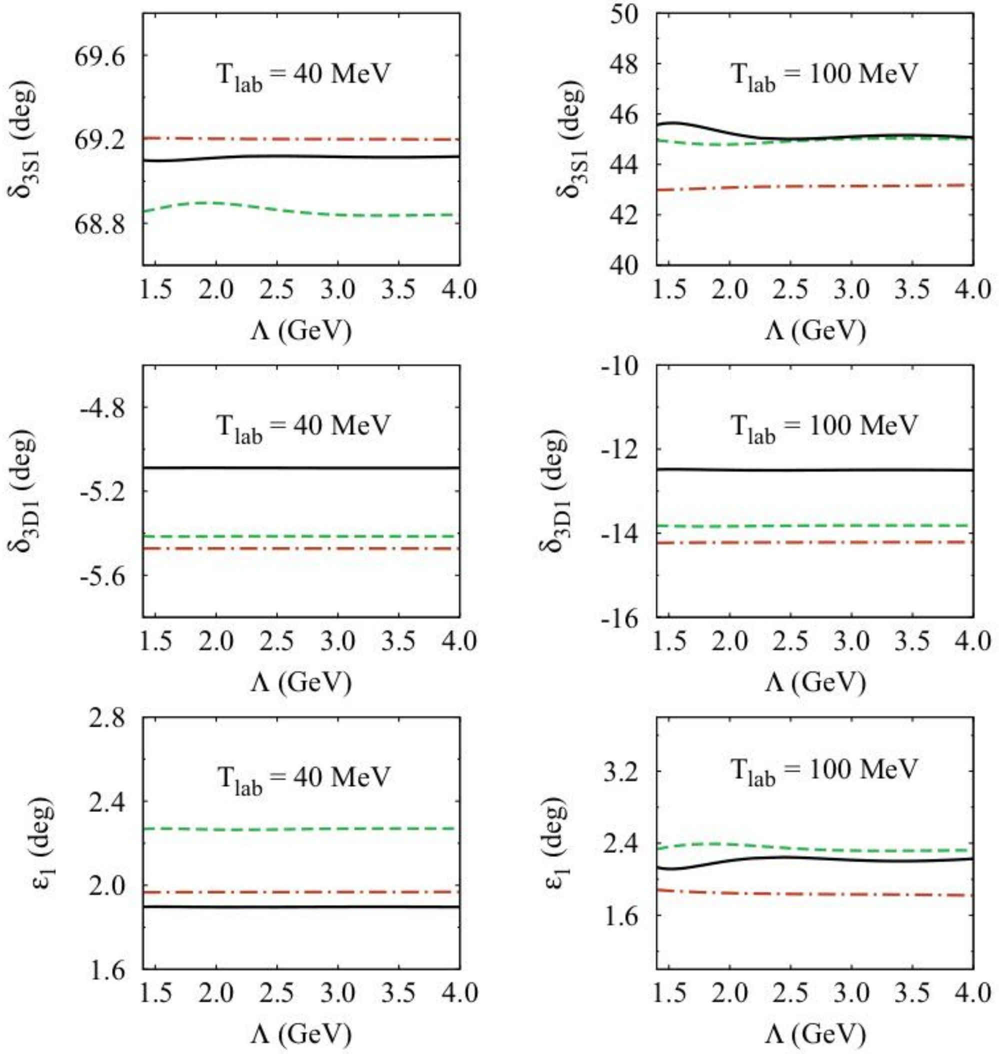


FIG. 3. (Color online) With the SCTs (25), the ${}^3S_1$, ${}^3D_1$ phase shifts and the mixing angle ϵ_1 at $T_{\text{lab}} = 40$ and 100 MeV, as functions of the momentum cutoff. The dashed, dot-dashed, and solid lines are $O(1)$, $O(Q^2)$, and $O(Q^3)$, respectively.

PHYSICAL REVIEW C **107**, 034001 (2023)

"Renormalization-group-invariant effective field theory" for few-nucleon systems is cutoff dependent

A. M. Gasparyan* and E. Epelbaum†
Ruhr-Universität Bochum, Fakultät für Physik und Astronomie, Institut für Theoretische Physik II, D-44780 Bochum, Germany

(Received 10 November 2022; accepted 10 March 2023; published 28 March 2023)

We consider nucleon-nucleon scattering using the formulation of chiral effective field theory which is claimed to be renormalization group invariant. The cornerstone of this framework is the existence of a well-defined infinite-cutoff limit for the scattering amplitude at each order of the expansion, which should not depend on a particular regulator form. Focusing on the $^3P_0$ partial wave as a representative example, we show that this requirement can in general not be fulfilled beyond the leading order, in spite of the perturbative treatment of subleading contributions to the amplitude. Several previous studies along these lines, including the next-to-leading order calculation by B. Long and C. J. Yang [Phys. Rev. C **84**, 057001 (2011)] and a toy model example with singular long-range potentials by B. Long and U. van Kolck [Ann. Phys. **323**, 1304 (2008)], are critically reviewed and scrutinized in detail.

DOI: 10.1103/PhysRevC.107.034001

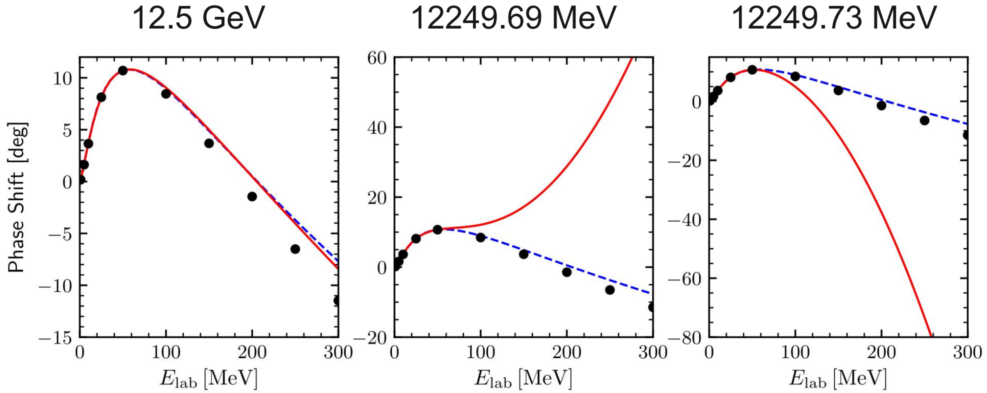
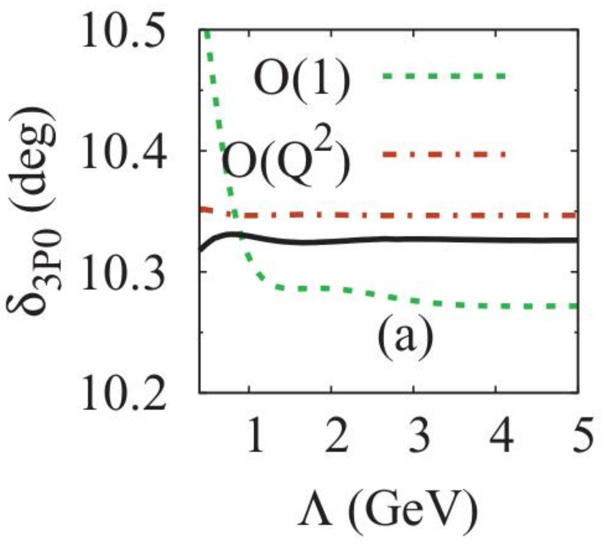


FIG. 6. The $^3P_0$ phase shifts calculated using the approach of Ref. [30] at LO (dashed lines) and NLO (solid lines). The left panel shows the solution for a typical cutoff ($\Lambda = 12.5$ GeV), whereas the middle and right panels correspond to the cutoff values slightly below ($\Lambda =$ 12 249.69 MeV) and above ($\Lambda =$ 12 249.73 MeV) the second exceptional point.



Bingwei Long, C.-J. Yang.
PhysRevC.84.057001(2011)

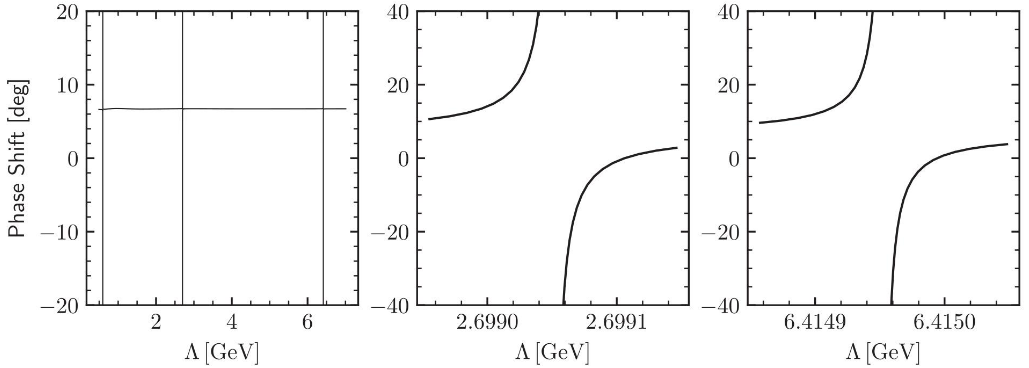


FIG. 5. Cutoff dependence of $^3P_0$ phase shift calculated at the fixed laboratory energy of $T_{\rm lab} = 130$ MeV using the approach of Ref. [30] at NLO. The middle and right panels show zoomed regions in the vicinity of two exceptional cutoffs.

Chiral Nuclear Forces and Renormalization

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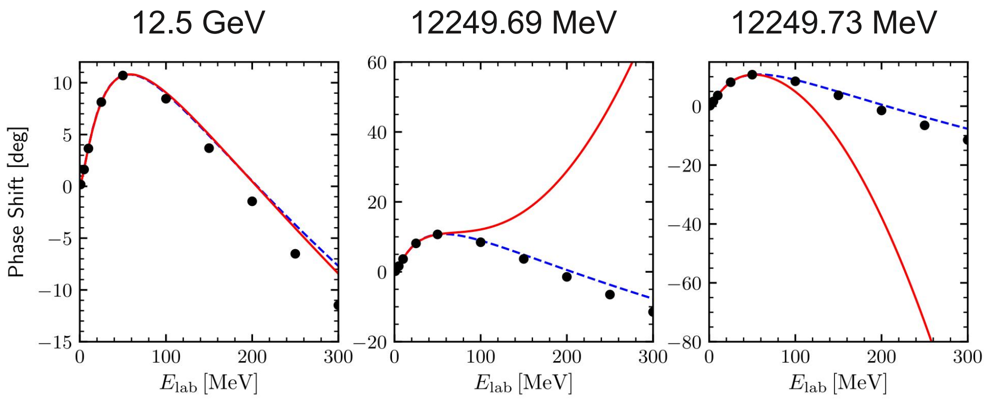

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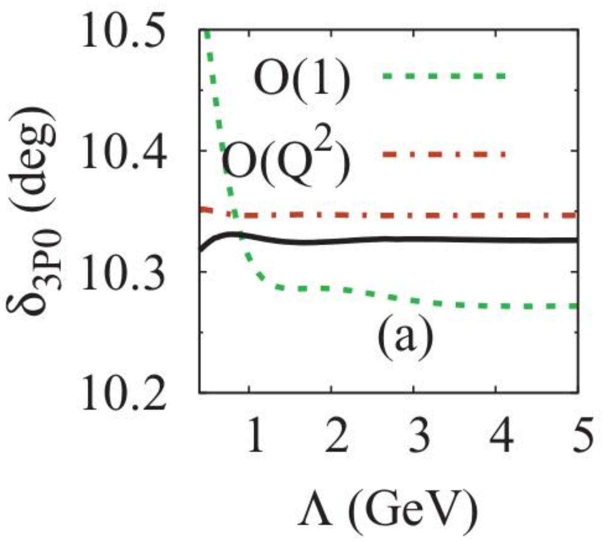

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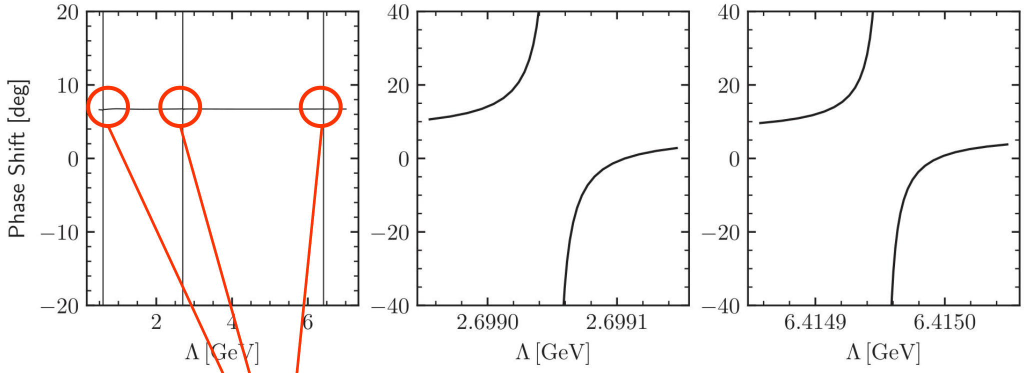

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genuine exceptional cutoffs(GECs)

Chiral Nuclear Forces and Renormalization

PHYSICAL REVIEW C **107**, 034001 (2023)

"Renormalization-group-invariant effective field theory" for few-nucleon systems is cutoff dependent

A. M. Gasparyan* and E. Epelbaum†
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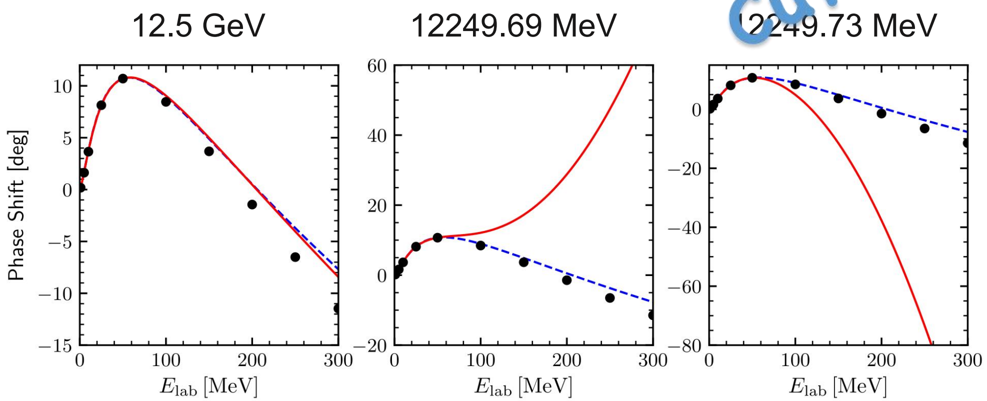
Cutoff dependent? ?



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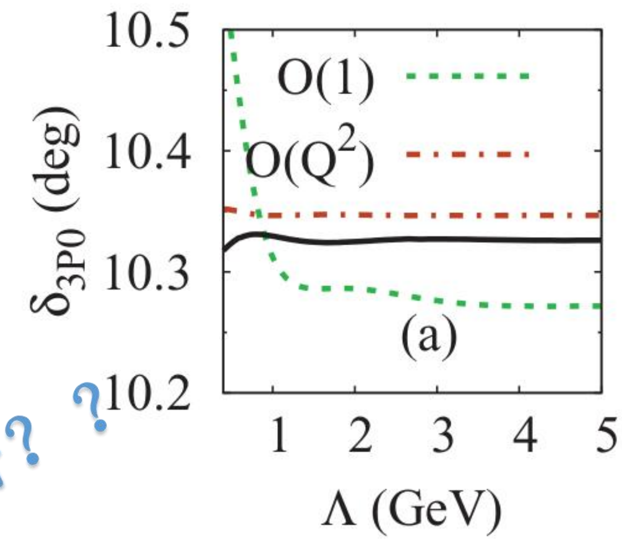
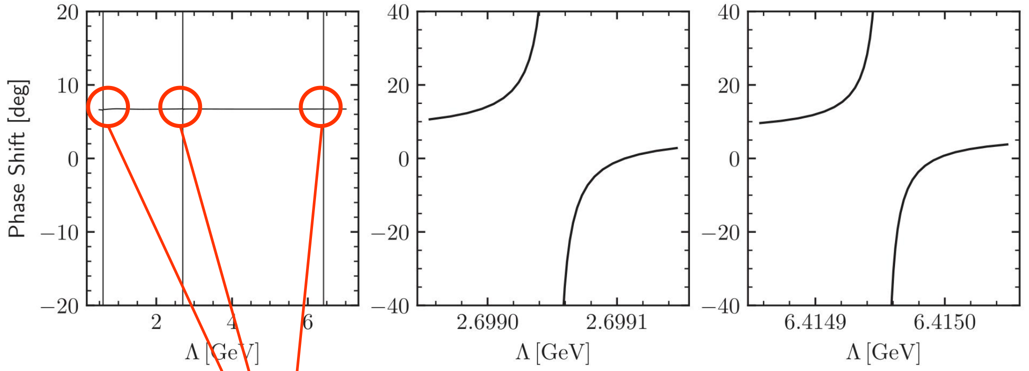


Bingwei Long， C.-J. Yang. PhysRevC.84.057001(2011)



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genuine exceptional cutoffs(GECs)

Rui Peng , Bingwei Long , and Fu–Rong Xu，PhysRevC.110.054001(2024)

3P0 channel

$$V^{(0)} = V_\pi + C^{(0)} pp'$$

$$V^{(1)} = 0$$

$$V^{(2)} = V_{2\pi} + C^{(2)} pp' + D^{(0)} pp'(p'^2 + p^2)$$

$$T^{(2)} = (1 + T^{(0)} G_0) V^{(2)} (1 + G_0 T^{(0)})$$

$$\delta^{(2)}(k) = -\frac{\pi}{2} k e^{-2i\delta^{(0)}} T^{(2)}(k)$$

$$\delta_{\mathrm{PWA}}(k_1) - \delta^{(0)}(k_1) = C^{(2)} \boxed{\theta_C(k_1)} + D^{(0)} \boxed{\theta_D(k_1)} + \theta_{2\pi}(k_1),$$

$$\delta_{\mathrm{PWA}}(k_2) - \delta^{(0)}(k_2) = C^{(2)} \boxed{\theta_C(k_2)} + D^{(0)} \boxed{\theta_D(k_2)} + \theta_{2\pi}(k_2).$$

$$\boxed{\tilde{g}(k; \Lambda, \delta_{\mathrm{In}})} \equiv \Lambda^2 \left. \frac{\theta_C(k)}{\theta_D(k)} \right|_0^k$$

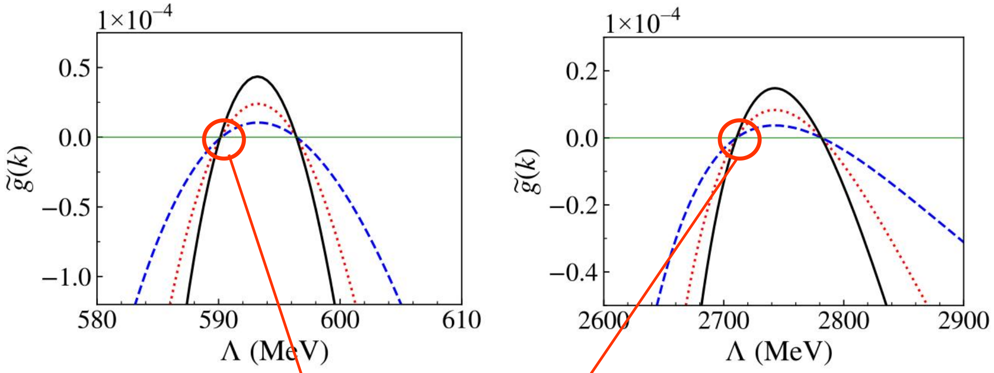

FIG. 4. $\tilde{g}(k;\Lambda)$ as functions of Λ at various values of k. The dashed, dotted, and solid curves correspond to $k = 100$, 150, and 200 MeV, respectively.

GECs

Pertubative Renormalization at subleading order

Rui Peng , Bingwei Long , and Fu-Rong Xu，PhysRevC.110.054001(2024)

3P0 channel

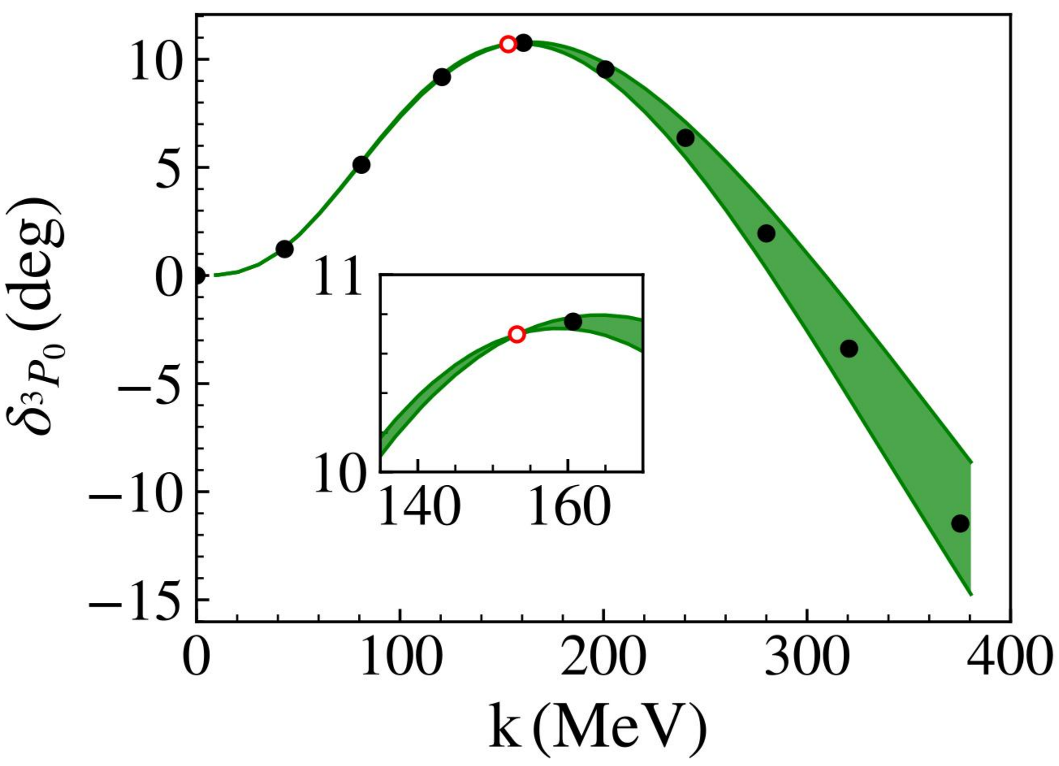

FIG. 3. The LO $^3P_0$ phase shifts as a function of k. The solid circles are the PWA phase shifts, and in particular the red empty circle is the input to determine $C^{(0)}$. The band is produced by varying Λ from 800 to 3200 MeV.

$$\delta_{\rm PWA}\left(\frac{\mu_0}{M_{\rm hi}}\right)^2 \approx 2^\circ$$

$$\delta_{\rm In}(\Lambda) = \begin{cases} \delta_{\rm PWA} + \Delta\delta_{\rm In}, & \Lambda \in \boxed{[588, 592] \cup [2707, 2716],} \\ \delta_{\rm PWA}, & \text{the rest.} \end{cases} \quad (34)$$

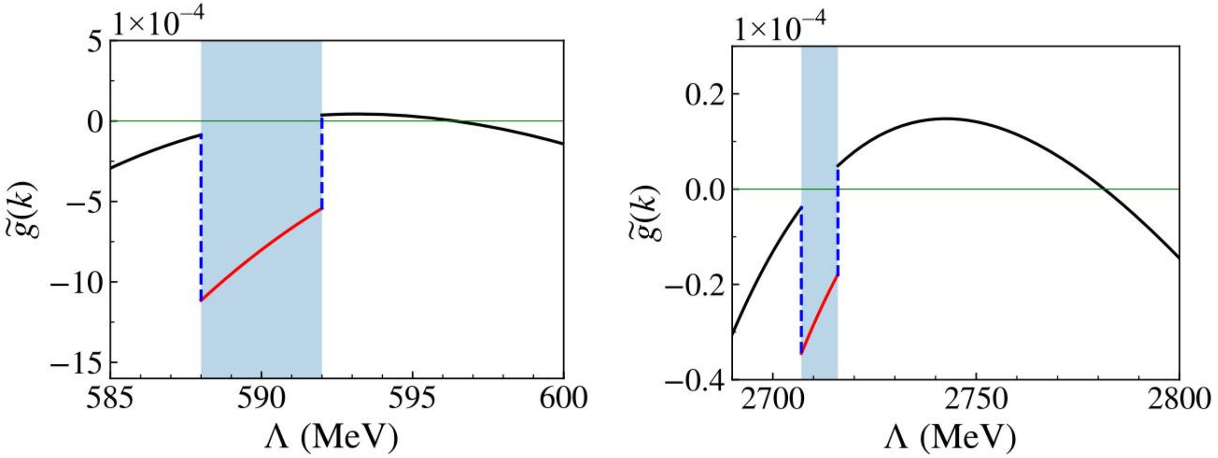

FIG. 6. $\widetilde{g}(k;\Lambda)$ as a function of Λ at $k = 200$ MeV. The bands indicate the intervals of Λ where $\delta_{\rm In}$ takes an alternative value as defined in Eq. (34).

Pertubative Renormalization at subleading order

Rui Peng , Bingwei Long , and Fu-Rong Xu，PhysRevC.110.054001(2024)

3P0 channel

NNLO phase shifts at GECs

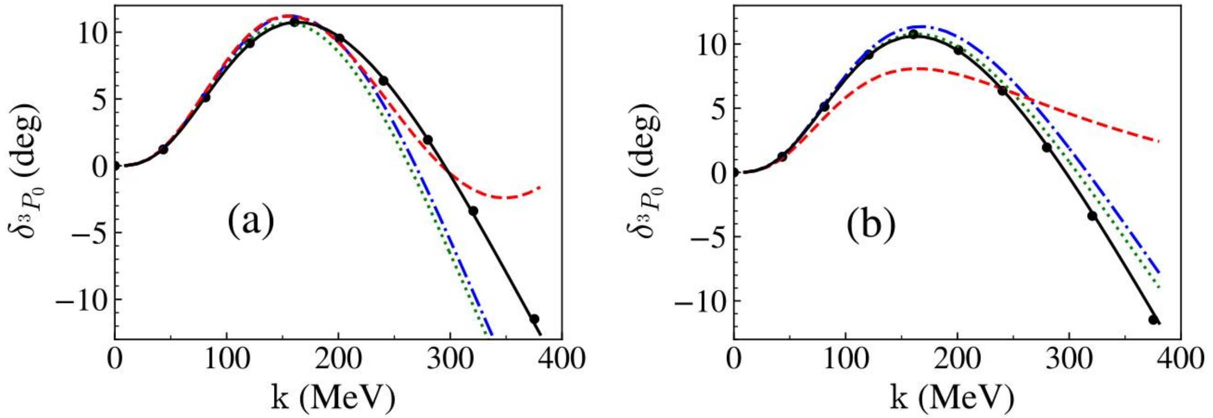

FIG. 8. The $^3P_0$ phase shifts as a function of k for $\Lambda = 590$ MeV (a) and for $\Lambda = 2710.54$ MeV (b). The dash-dotted (LO) and the solid ($\mathrm{N^2LO}$) curves use an alternative δ_{In} for the LO input while the dotted (LO) and dashed ($\mathrm{N^2LO}$) curves use the unchanged δ_{In}. The solid dots are from the PWA.

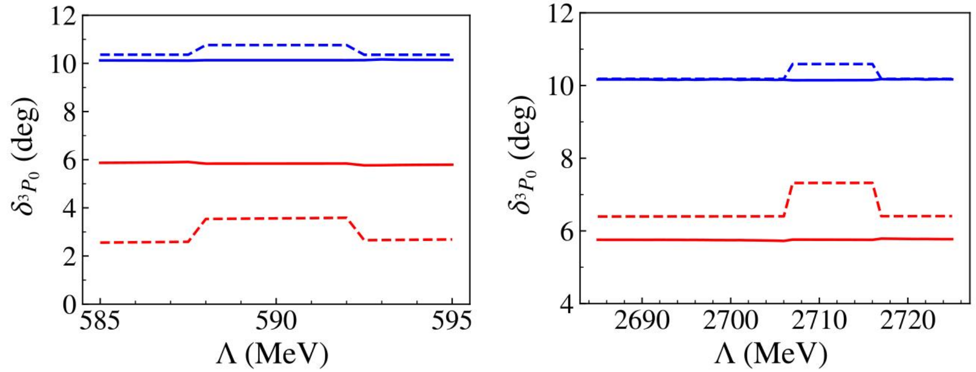

FIG. 9. The $^3P_0$ phase shifts as a function of Λ at a given k. The dashed and solid curves correspond to the LO and $\mathrm{N^2LO}$ phase shifts, respectively. Different colors indicate the value of k used: $k = 137$ MeV (blue) and $k = 247$ MeV (red).

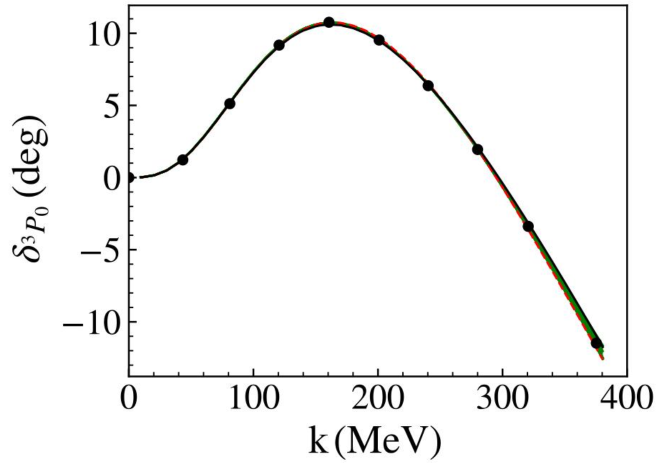

FIG. 10. The $\mathrm{N^2LO}$ $^3P_0$ phase shifts as a function of k. The green band is produced by varying Λ from 800 to 3200 MeV with an unchanged δ_{In} and the exceptional cutoff windows are ignored. The red dashed and the black solid lines correspond to the $\mathrm{N^2LO}$ results for $\Lambda = 590$ MeV and for $\Lambda = 2710.54$ MeV, respectively, where the alternative δ_{In} was used. The solid dots are from the PWA.

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3S1–3D1 channel

$$V^{(2)} = V_{2\pi} + \begin{pmatrix} C^{(2)} + D^{(0)}(p'^2 + p^2) & E^{(0)}p^2 \\ E^{(0)}p'^2 & 0 \end{pmatrix}$$

$$\begin{aligned} \delta_1^{\mathrm{PWA}}(k_1) - \delta_1^{(0)}(k_1) - \theta_{2\pi}(k_1) &= C^{(2)}\theta_C(k_1) + D^{(0)}\theta_D(k_1) + E^{(0)}\theta_E(k_1)\,, \\ \varepsilon^{\mathrm{PWA}}(k_2) - \varepsilon^{(0)}(k_1) - \mathcal{E}_{2\pi}(k_2) &= C^{(2)}\mathcal{E}_C(k_2) + D^{(0)}\mathcal{E}_D(k_2) + E^{(0)}\mathcal{E}_E(k_2)\,, \\ B_{\mathrm{emp}} - B^{(0)} - \beta_{2\pi} &= C^{(2)}\beta_C + D^{(0)}\beta_D + E^{(0)}\beta_E\,. \end{aligned} \tag{22}$$

$$\mathcal{G}(\Lambda; k_1, k_2, B^{(0)}) \equiv \Lambda^{-7} \times \begin{vmatrix} \theta_C(k_1) & \theta_D(k_1) & \theta_E(k_1) \\ \mathcal{E}_C(k_2) & \mathcal{E}_D(k_2) & \mathcal{E}_E(k_2) \\ \beta_C & \beta_D & \beta_E \end{vmatrix}\,, \tag{23}$$

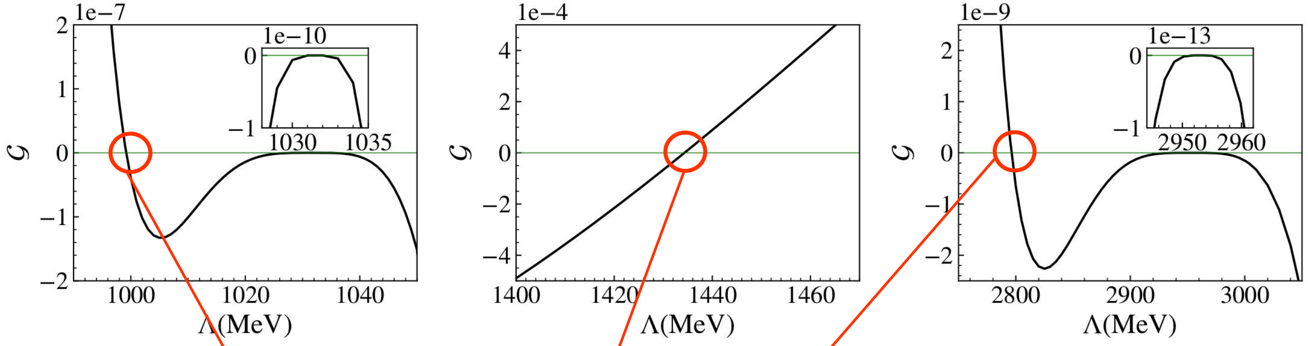

FIG. 1: $\mathcal{G}(\Lambda; k_1, k_2, B^{(0)})$ as a function of Λ using Scheme I everywhere.

GECs

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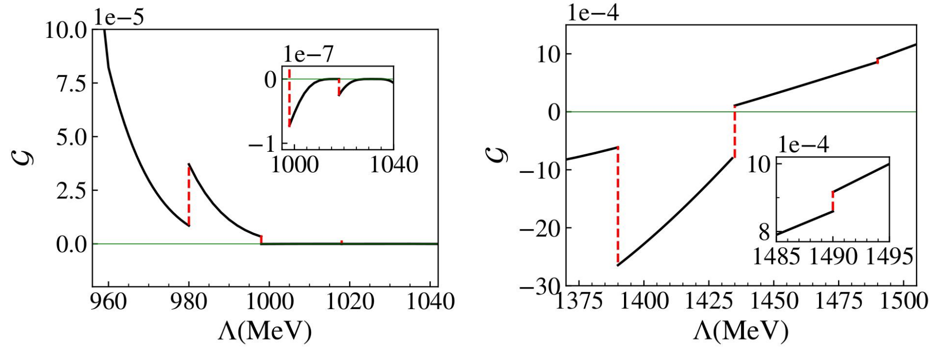
3S1–3D1 channel

TABLE I: The modified fitting strategy is composed of several schemes. Each scheme is defined by $B^{(0)}$, k_1, and k_2. The right-most column specifies which cutoff region each scheme is applies. $\Delta B = 0.5$ MeV.

Scheme	$B^{(0)}$(MeV)	k_1(MeV)	k_2(MeV)	Λ(MeV)
I	$B_{\rm emp}$	300.2	201.0	the rest
II	$B_{\rm emp} + \Delta B$	300.2	201.0	[980,998]
III	$B_{\rm emp} - \Delta B$	300.2	201.0	[998,1018]
IV	$B_{\rm emp}$	350.7	300.2	[1390,1435]
V	$B_{\rm emp}$	350.7	160.7	[1435,1490]



$\mathcal{G}(\Lambda; k_1, k_2, B^{(0)})$ as a function of Λ using the modified fitting strategy defined in Table I.

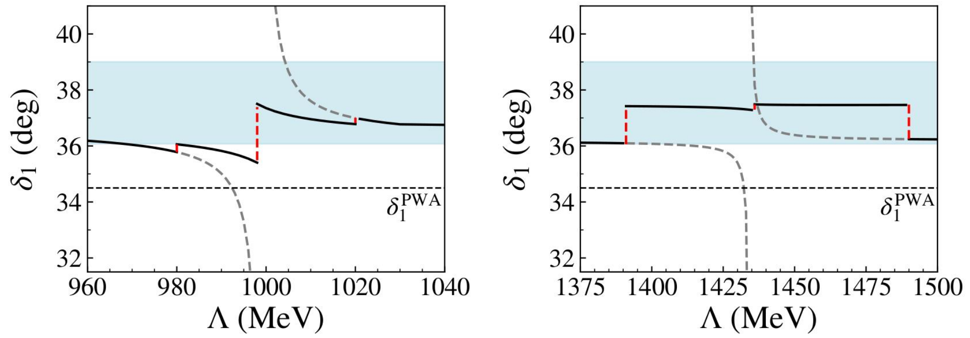


FIG. 3: The N^{2}LO $^3S_1$ phase shifts at $k = 250$ MeV as a function of Λ. The solid lines correspond to the modified fitting strategy defined in Table I, and the dashed lines correspond to the original fitting strategy. The shaded bands are the background cutoff variations. See the text for more explanations.

Pertubative Renormalization at subleading order

Rui Peng , Bingwei Long , and Fu-Rong Xu，arXiv:2508.06838v1(PRC under reivew)

3S1–3D1 channel

NNLO phase shifts at GECs

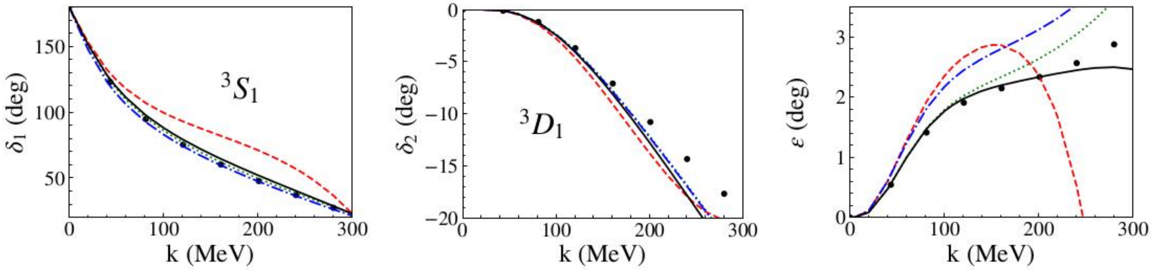


FIG. 6: The $^3S_1 - {}^3D_1$ phase shifts as functions of k for $\Lambda = 1000$ MeV. The green dotted (LO) and the red dashed ($\mathrm{N^2LO}$) curves use the original fitting strategy while the blue dash-dotted (LO) and black solid ($\mathrm{N^2LO}$) curves use the modified one. The solid dots are from the PWA.

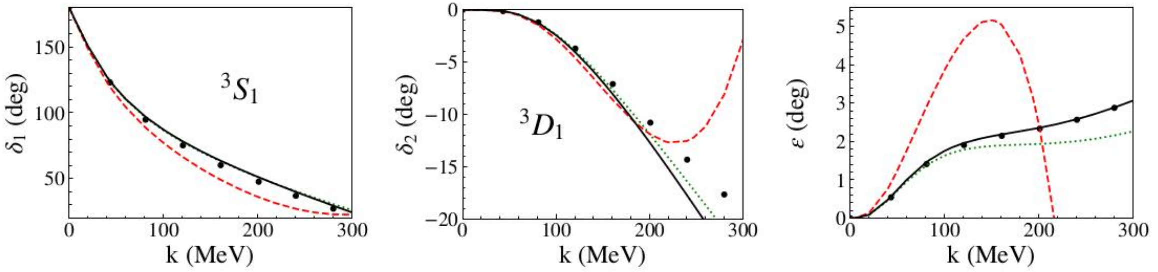


FIG. 7: The $^3S_1 - {}^3D_1$ phase shifts as functions of k for $\Lambda = 1434$ MeV. The green dotted curves are the LO. At $\mathrm{N^2LO}$, the red dashed represent the original fitting strategy and the black solid curves use the modified one. The solid dots are from the PWA.

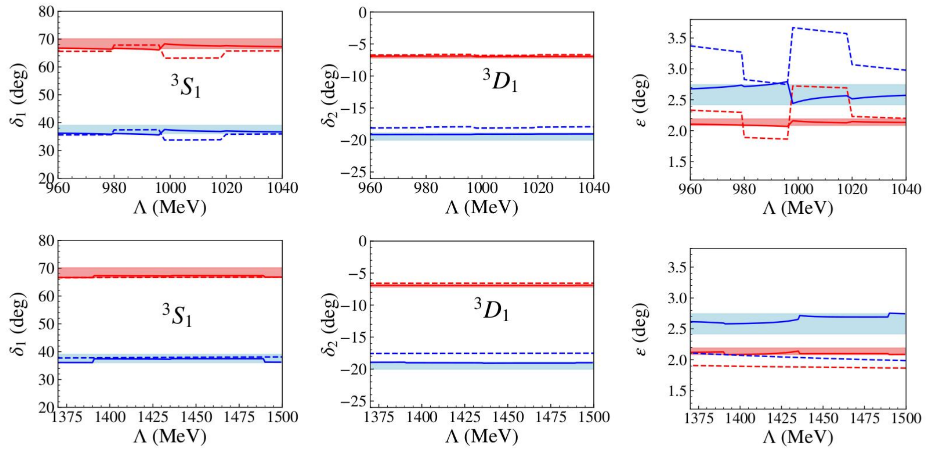


FIG. 8: The $^3S_1 - {}^3D_1$ phase shifts as functions of Λ at a given k using the modified fitting strategy. The dashed and solid curves correspond to the LO and $\mathrm{N^2LO}$ phase shifts, respectively. Different colors correspond to different values of k used: blue for $k = 150$ MeV and red for $k = 250$ MeV. The bands are the background cutoff variations.

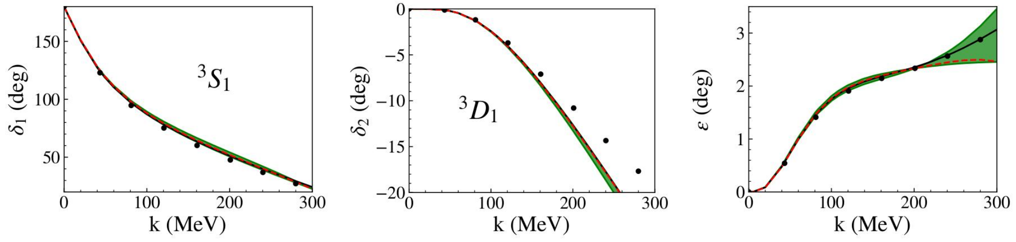


FIG. 9: The $\mathrm{N^2LO}$ $^3S_1 - {}^3D_1$ phase shifts as functions of k. The red dashed and the black solid lines represent the $\mathrm{N^2LO}$ using the modified fitting strategy for $\Lambda = 1000$ MeV and $\Lambda = 1434$ MeV, respectively. The green bands are the background variations. The solid dots are from the PWA.

Summary and outlook

The criterion for any modifications on the fitting strategy is that they introduce only small cutoff variations to the phase shifts that are comparable with the EFT uncertainty expected at this order. By trial and error, we found a fitting strategy that satisfies this criterion, as tabulated in Table I. Not only this modified strategy offers mild cutoff variations, it shows a much better agreement with the PWA near the GECs than the original fitting strategy does.

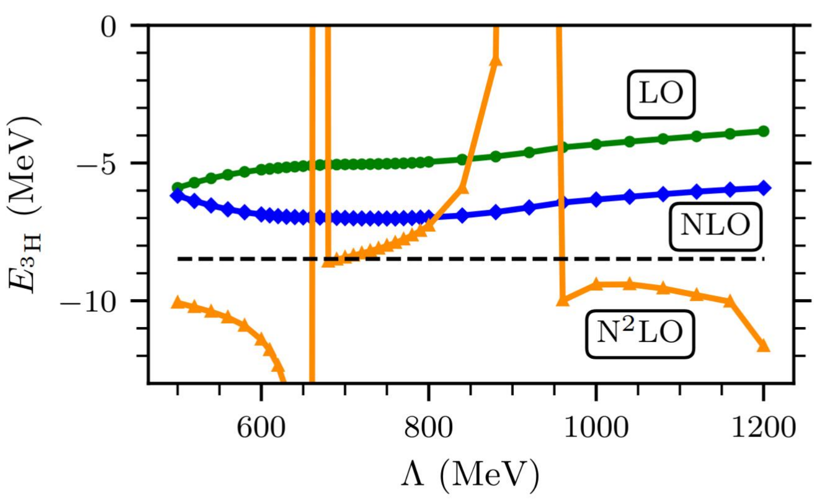


FIG. 10. The ground-state energy of the triton up to N^{2}LO as a function of the momentum cutoff, Λ, computed in the NCSM. The HO frequency of the LO variational minimum for $N_{\max} = 46$ is used at all orders, see Fig. 9. The black dashed line shows the experimental value [51].

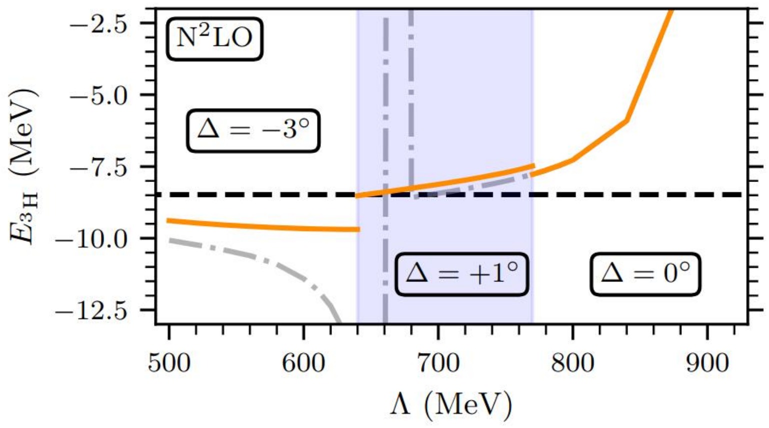


FIG. 13. The triton ground-state energy at N^{2}LO as a function of the momentum cutoff. The colored solid lines in each cutoff region correspond to shifts $\Delta = -3^\circ, +1^\circ$ and 0°, respectively, as defined in Eq. (29) for the $^3P_0$ channel. The gray dashed-dotted line shows the $\Delta = 0^\circ$ result across all cutoffs. The black dashed line shows the experimental triton ground-state energy [52]. All calculations use $N_{\max} = 46$ and the HO frequency, ω, is chosen as the LO variational minimum at all orders.

Oliver Thim, et al., arXiv:2510.12207v1

1. Probe the power counting in few-body systems ($^3\mathrm{H}$ and $^4\mathrm{He}$).

2. Determine the LECs of three-body forces

3. Calculate nuclear many-body properties with perturbative chiral forces.

Thanks for your attention !