南京大學
NANJING UNIVERSITY

Relativistic mean field approach with Chiral-scale Density Counting (CSDC) Rules for Nuclear Matter Properties

Presenter: Yao Ma (马垚)

In collaboration with Jia-Ying Xiong (熊佳颖), Dr. Bing-Kai Sheng(盛炳开) and Prof. Yong-Liang Ma (马永亮)

Nanjing Normal University, Nanjing, Oct. 20th

Motivation

- The past studies on nuclear matter properties around saturation density or beyond are always serving the phenomenological purpose, .e.g. Walecka-type models:

$$\mathscr{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi + \ldots$$

- The direct application of effective models/theories, e.g. chiral perturbation theory and linear sigma model, to finite densities and/or tens of MeV temperatures are not systematically established with the considerations of thermal effects, and parameter space haven't been discussed rigorously.

誠樸雄偉 勵學敦行

Phenomenons at finite densities and temperatures

$T = 0$

Nuclear matter properties around saturation point

$T \neq 0$

Nuclear matter gas-liquid phase transition

Equation of state of a system

Thermal potential

$$U(V, S, N)$$

Particle number

Internal energy

Volume

Entropy

$$\Phi_G(V, T, \mu)$$

Landau potential

Entropy

Chemical potential

Pressure

Legendre transformation

$$\mathrm{d}U = -P\mathrm{d}V + T\mathrm{d}S + \mu\mathrm{d}N \longleftrightarrow \mathrm{d}\Phi_G = -P\mathrm{d}V - S\mathrm{d}T - N\mathrm{d}\mu$$

For a infinite homogeneous system (EOS)

$$P = -\Omega = -\Phi_G/V\,, \quad n = N/V = -\left.\frac{\partial\Omega}{\partial\mu}\right|_T\,, \quad s = S/V = -\left.\frac{\partial\Omega}{\partial T}\right|_\mu\,, \quad \epsilon = U/V = \Omega + \mu n + Ts$$

Nuclei structures ($T = 0$)

- Hadron interactions around saturation density $n_0 \simeq 0.16\mathrm{fm}^{-3}$ are crucial to nuclei structures, e.g. $^{24}\mathrm{Mg}$, $^{90}\mathrm{Zr}$, $^{116}\mathrm{Sn}$ and ^{208} Pb

$$E(n,\delta) = \frac{\epsilon}{n} \simeq E\left(n_0,0\right) + E_{\mathrm{sym}}\left(n_0\right)\delta^2 + L\left(n_0\right)\delta^2\chi + \frac{K\left(n_0\right)}{2!}\chi^2 + \frac{J\left(n_0\right)}{3!}\chi^3 + \mathcal{O}\left(\delta^4\right) + \mathcal{O}\left(\chi^4\right) + \mathcal{O}\left(\delta^2\chi^2\right)$$

$$\simeq E_0 + \frac{\delta^2}{2}\frac{\partial^2 E\left(n_0,\delta\right)}{\partial\delta^2}\bigg|_{\delta=0} + \delta^2\chi\left(3n\frac{\partial E_{\mathrm{sym}}(n)}{\partial n}\right)\bigg|_{n=n_0} + \frac{\chi^2}{2!}\left(9n^2\frac{\partial^2 E_0(n)}{\partial n^2}\right)\bigg|_{n=n_0}$$

$$+\frac{\chi^3}{3!}\left(27n^3\frac{\partial^3 E_0(n)}{\partial n^3}\right)\bigg|_{n=n_0} + \mathcal{O}\left(\delta^4\right) + \mathcal{O}\left(\chi^4\right) + \mathcal{O}\left(\delta^2\chi^2\right)$$

$$n = n_n + n_p$$

$$\delta = \left(n_n - n_p\right)/n$$

$$\chi \equiv \left(n - n_0\right)/3n_0$$

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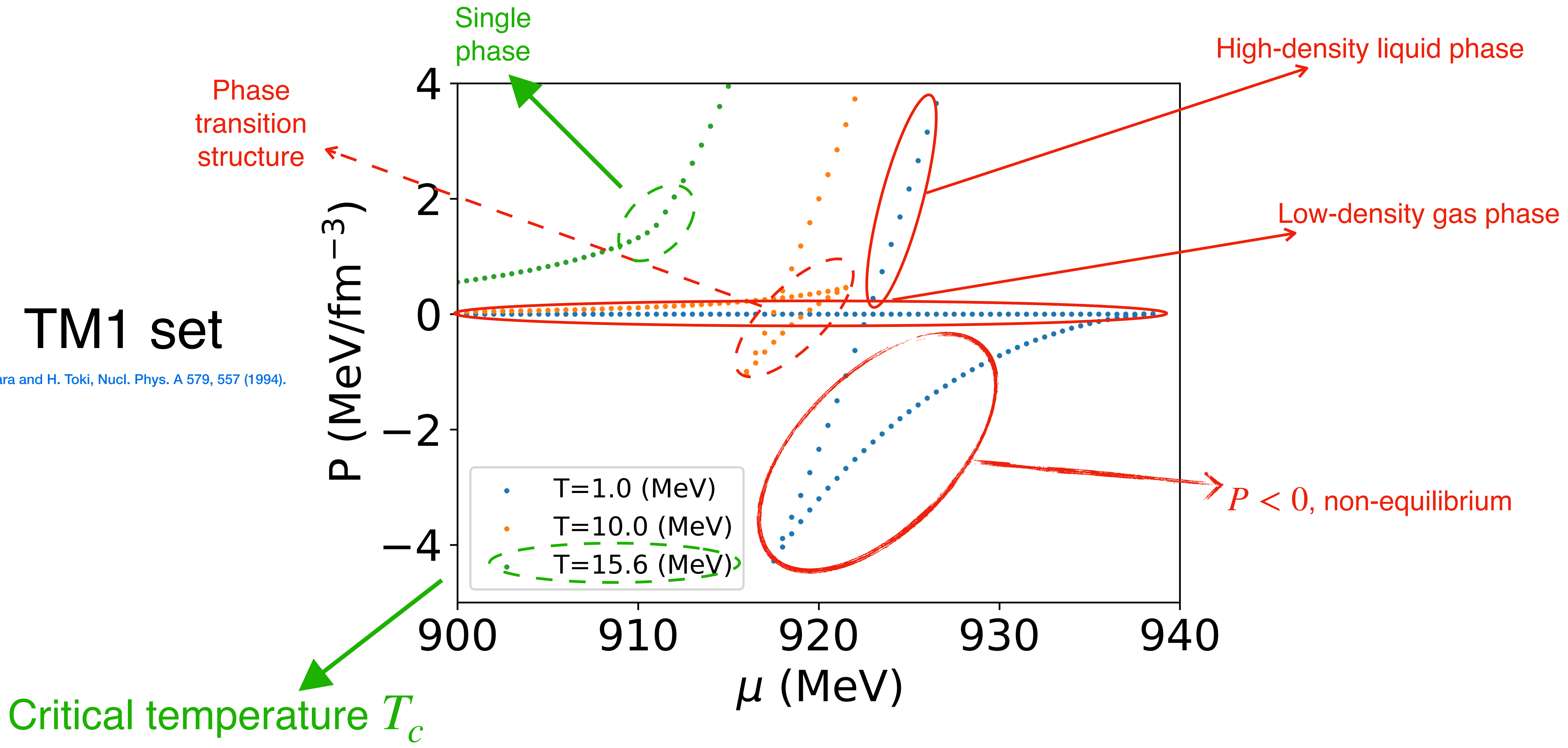
Grams, G., Somasundaram, R., Margueron, J., *et al.*, *Phys. Rev. C* 106, 044305 (2022).

$$n_0 \to 0.155 \pm 0.050\left(\mathrm{fm}^{-3}\right)$$
$$E_0(n_0) \to -16.0 \pm 1.0(\mathrm{MeV})$$
$$E_{\mathrm{sym}}(n_0) \to 31.7 \pm 3.2(\mathrm{MeV})$$
$$K_0 \to 250 \pm 50(\mathrm{MeV})$$
$$L_0 \to 58.7 \pm 28.1(\mathrm{MeV})$$
$$J_0 \to -200 \pm 600(\mathrm{MeV})$$

Gas-liquid phase transition for nuclear matter

TM1 set

Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994).



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A chiral-scale effective field theory

A low energy NG boson set: π, κ, σ for SU(3), with HLS: ρ, ω

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R. J. Crewther and L. C. Tunstall, Phys. Rev. D 91, 034016 (2015).
O. Catà, et.al., Phys. Rev. D 100, 095007 (2019).

M. Bando, et.al., Phys. Rev. Lett. 54, 1215 (1985).

$$\theta^{\mu}_{\mu} = \frac{\beta\left(\alpha_s\right)}{4\alpha_s} G^{a}_{\mu\nu} G^{a\mu\nu} + \left(1 + \gamma_m\left(\alpha_s\right)\right) \sum_{q=u,d,s} m_q \bar{q} q$$

Scale symmetry considerations

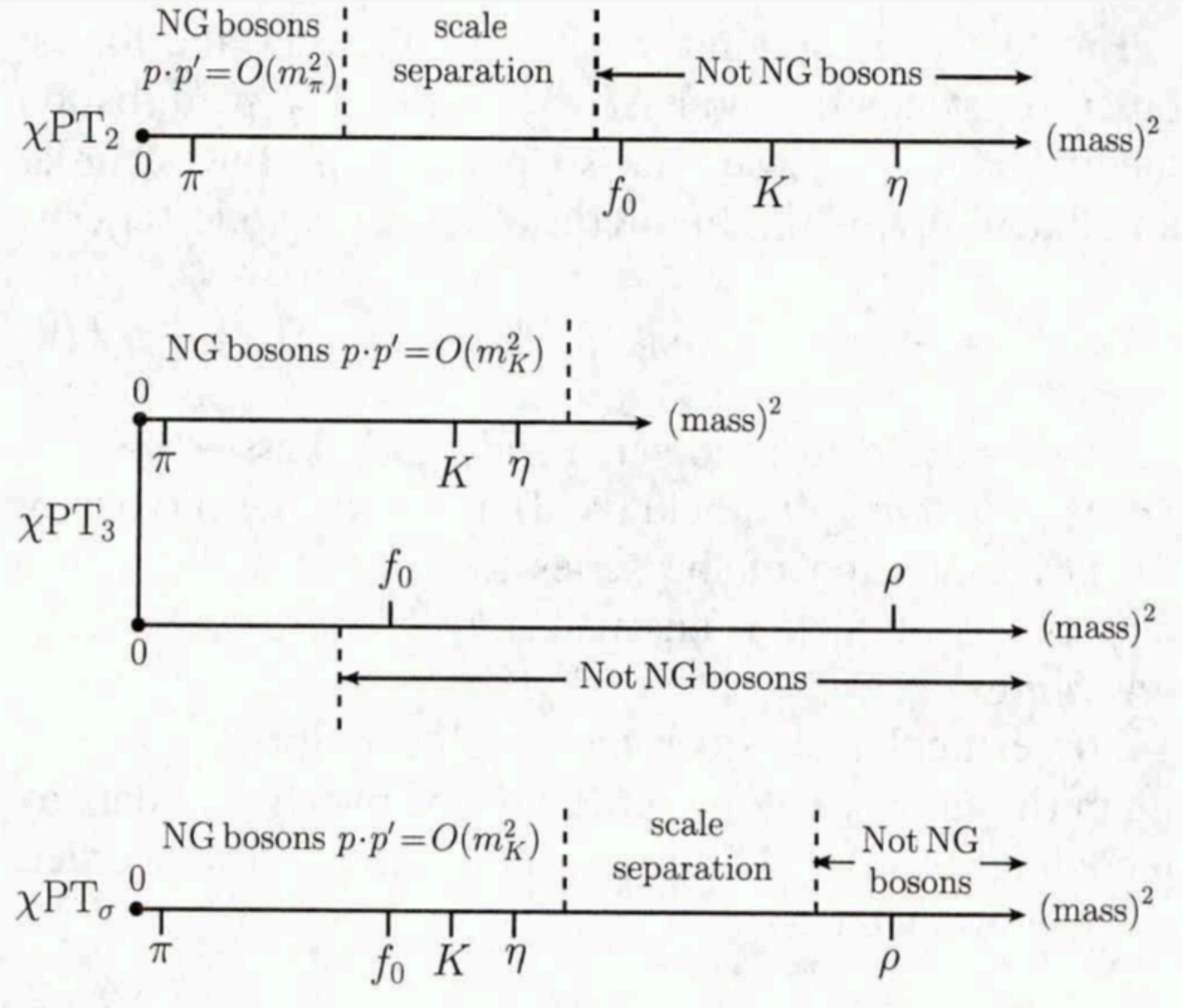


FIG. 4. Scale separations between NG sectors and other hadrons for each type of chiral perturbation theory χPT discussed in this paper. Note that scale separation in $\chi\mathrm{PT}_2$ [chiral $SU(2)_L \times SU(2)_R$, top diagram] is ensured by limiting extrapolations in momenta p, p' to $O(m_\pi)$ [not $O(m_K)$]. In conventional three-flavor theory $\chi\mathrm{PT}_3$ (middle diagram), there is no scale separation: the non-NG boson $f_0(500)$ sits in the middle of the NG sector $\{\pi, K, \eta\}$. Our three-flavor proposal $\chi\mathrm{PT}_\sigma$ (bottom diagram) for $O(m_K)$ extrapolations in momenta implies a clear scale separation between the NG sector $\{\pi, K, \eta, \sigma = f_0\}$ and the non-NG sector $\{\rho, \omega, K^*, N, \eta', \ldots\}$.

- Possible infrared fixed point of QCD at low energies

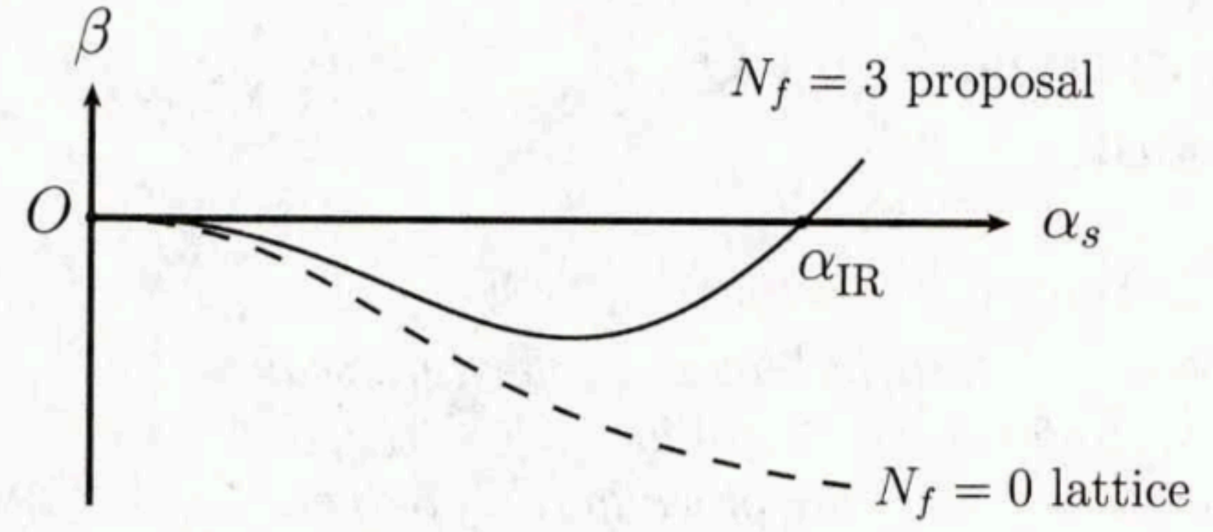


FIG. 1. The solid line shows a three-flavor β function (or better, a QCD version [20] of the Gell-Mann-Low Ψ function) with an infrared fixed point α_{IR} at which α_s freezes [21–26], but the manifest scale invariance of [13–15] is *avoided*. The existence of α_{IR} for small N_f values is not entirely settled. The dashed line shows the original lattice result [27] for $N_f = 0$ (no quarks) where β remains negative and becomes linear at large α_s.

$$\chi = f_\chi e^{\sigma/f_\chi} = f_\chi \Phi$$

$$\theta^\mu_\mu \Big|_{\alpha_s=\alpha_{\mathrm{IR}}} = \left(1 + \gamma_m\left(\alpha_{\mathrm{IR}}\right)\right)\left(m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s\right)$$

$\to 0, \quad \mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$ limit

$K_L \to \pi^0 \gamma\gamma$ power counting

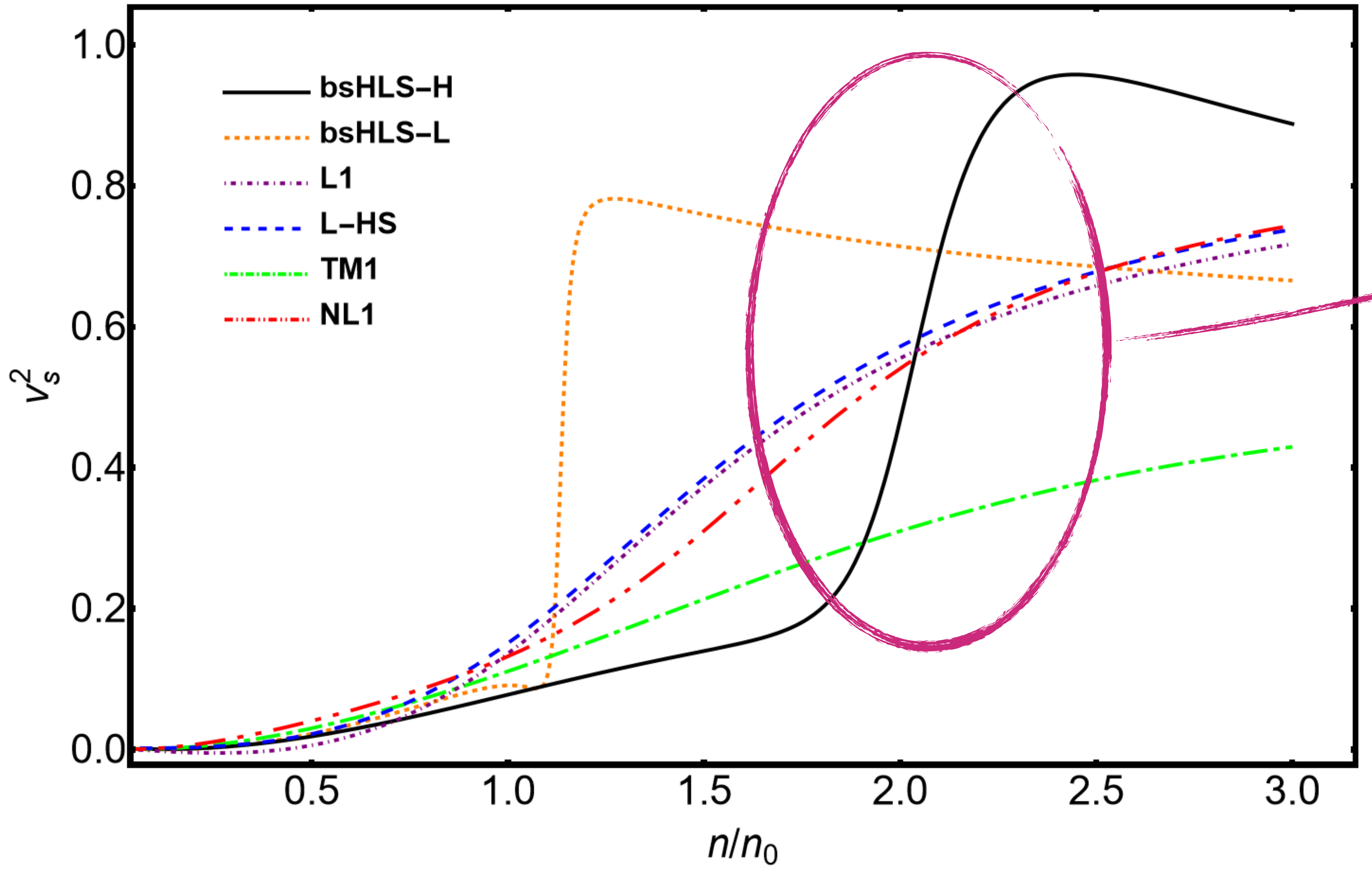
To keep chiral expansion consistent: $f_\pi \sim f_\chi$

R. J. Crewther and L. C. Tunstall, Phys. Rev. D 91, 034016 (2015).

A genuine peak in v_s^2 and neutron star M-R relation

Zhang, L.-Q., Ma, Y., Ma, Y.-L., *Phys. Rev. C* 112, L031001 (2025).
Zhang, L.-Q., Ma, Y., Ma, Y.-L., (2024), arXiv:2412.19023 [nucl-th].

A peak at intermediate densities



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Sugahara, Y., Toki, H., *Nucl. Phys. A* 579, 557 (1994).
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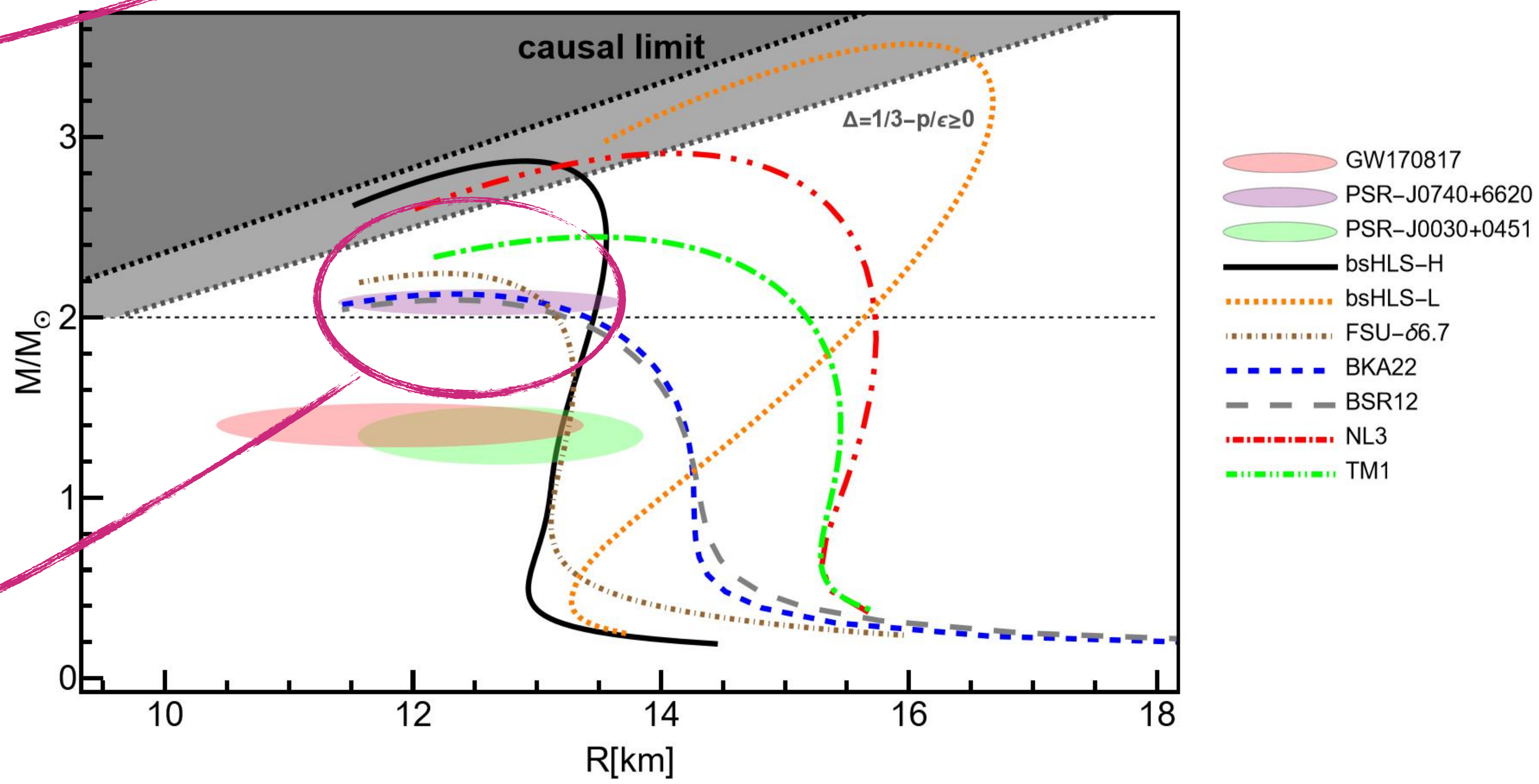
Dhiman, S. K., Kumar, R., Agrawal, B. K., *Phys. Rev. C* 76, 045801 (2007).



A larger mass than past studies

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Riley, T. E., et al., Astrophys. J. Lett. 918, L27 (2021).
Fujimoto, Y., Fukushima, K., McLerran, L. D., et al., Phys. Rev. Lett. 129, 252702 (2022).
Riley, T. E., et al., Astrophys. J. Lett. 887, L21 (2019).

誠樸雄偉 勵學敦行

Power-counting scheme

- Chiral-scale EFTs has the power counting by mesonic momentum, $\mathcal{O}(q^n)$
- The couplings can be assigned to a counting consistent with $\mathcal{O}(q^n)$
- Can density effects can be organized by a characteristic momentum?

At $T = 0$, with mean field approach

Baryon field

Fermi sphere with MFA

No-sea approximation

$$\psi_B(x) = \sum_s \int_0^{\boxed{\mathbf{k_F}}} \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2\boxed{E^*}}} \left[u(\mathbf{p}, s) e^{-ip\cdot x} \boxed{a(\mathbf{p}, s)}\right]$$

Effective energy at densities

Meson field

Induced background field

Nucleon source $\sim \mathcal{O}(k_{\mathrm{F}}^3)$

$$\boxed{\sigma(x)} \simeq \int_V \mathrm{d}^4 x' D_{\sigma,R}(x - x') \left(-g_{\sigma NN}^{\mathrm{OBE}} \boxed{\bar{N}(x') N(x')}\right)$$

Characteristic momentum expansion

- $\mathscr{O}(k_{\mathrm{F}}) \sim \mathscr{O}(p) \sim (0-500)\ \mathrm{MeV}$, for the typical density regions of a neutron star
- With assigning the consistent power counting of couplings and label the expansion factor as characteristic momentum $k_{\mathrm{c}} \sim$ hundreds of MeV, then the induced background field:

$$\sigma\ ,\quad \omega_\mu\ ,\quad \rho_\mu^i \sim \mathscr{O}\left(k_{\mathrm{c}}^2\right) \qquad \boxed{\pi^i \sim O\left(k_c^3\right)} \longrightarrow \text{Partial coupling}$$

Leading order: free Fermi gas, $\mathscr{H}_0 = \underbrace{\bar{N}\left(-i\gamma\cdot\nabla + m_N\right) N}_{\mathscr{O}\left(k_c^4\right)}$

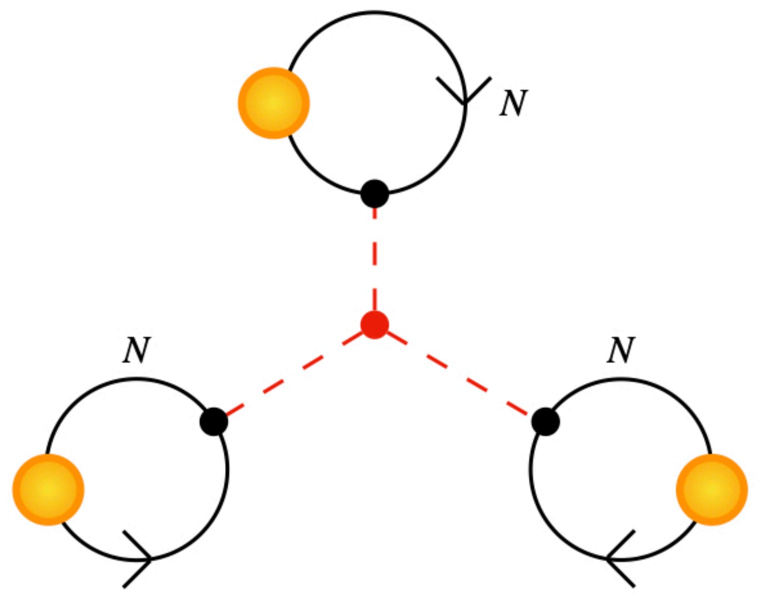
Fermion Loop expansion

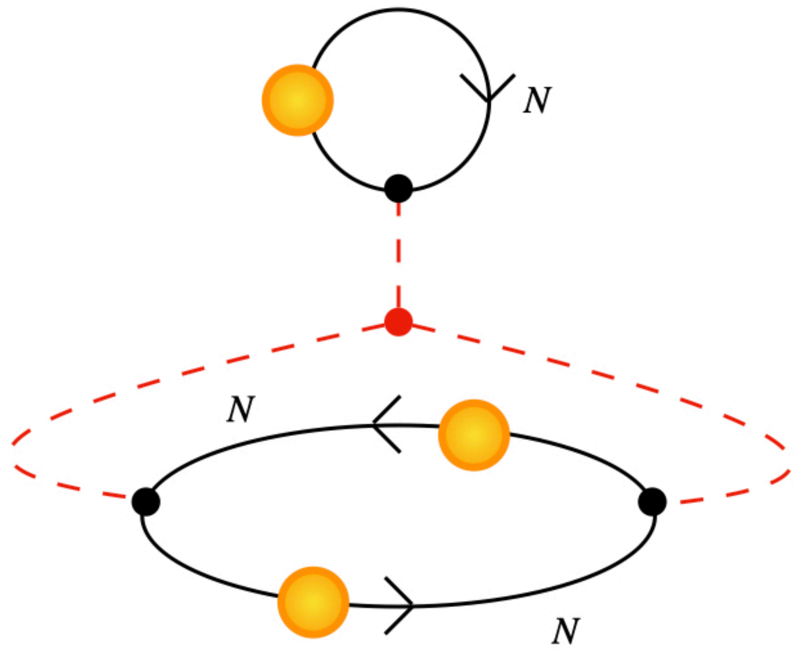
$$\mathscr{H}_I = \underbrace{g^{\rm OBE}_{\sigma NN}\bar{N}\sigma N + g^{\rm OBE}_{\omega NN}\bar{N}\gamma_\mu\omega^\mu N + g^{\rm OBE}_{\sigma NN}\bar{N}p^i\tau^i N}_{\mathcal{O}(k_c^6)} - \underbrace{\text{Free Lagrangian of } \rho, \omega, \sigma \text{ mesons}}_{\mathcal{O}(k_c^8)}$$

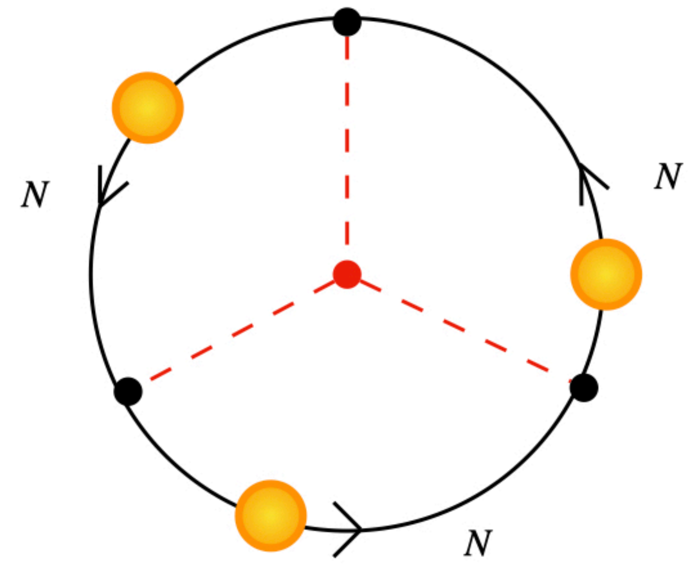
$$+ \underbrace{g^{\rm OBE}_{\pi NN}\bar{N}\gamma_5\gamma_\mu\partial^\mu\pi^i\tau^i N + g_{\sigma^3}\sigma^3 + g_{\sigma\rho^2}\sigma\rho^i_\mu\rho^{i,\mu} + g_{\sigma\omega^2}\sigma\omega_\mu\omega^\mu}_{\mathcal{O}(k_c^8)} - \underbrace{\text{Free Lagrangian of } \pi \text{ meson}}_{\mathcal{O}(k_c^8)}$$

$$+ \underbrace{g_{\sigma^2 NN}\bar{N}\sigma^2 N + g_{\sigma\omega NN}\bar{N}\sigma\omega^\mu\gamma_\mu N + g_{\sigma\rho N}\bar{N}\sigma\rho^{\mu a}\tau^a\gamma_\mu N}_{\mathcal{O}(k_c^8)} + \text{higher order terms} .$$

Six Fermion operators







At finite temperatures under RMF

Occupied state

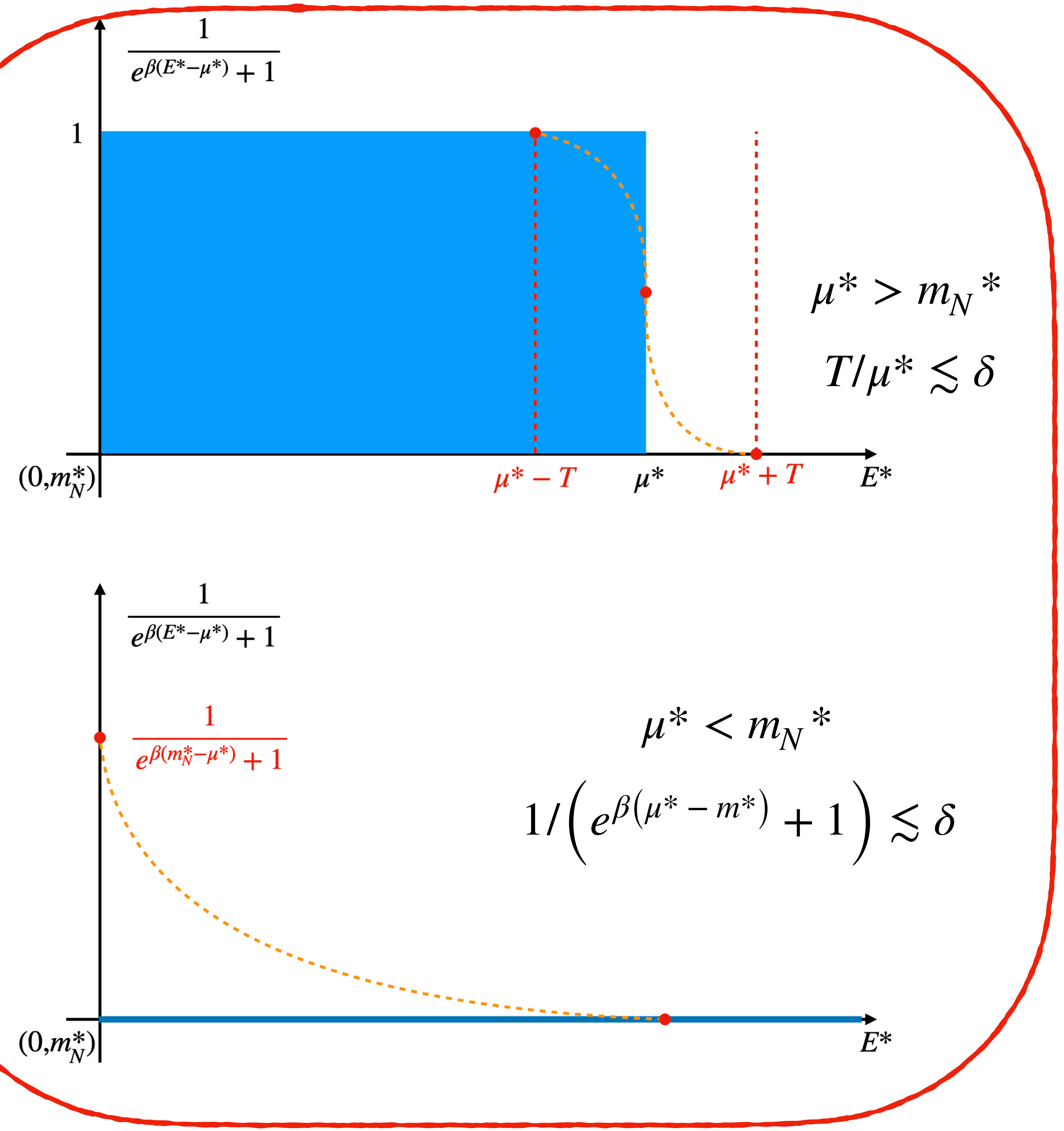
Unoccupied state

$$\tilde{k}_F = \sqrt{(\mu^* + T)^2 - m^{*2}} \lesssim m_N$$

$$\rho = 2\int \frac{d\mathbf{p}}{(2\pi)^3}\left[\frac{1}{e^{\beta\left(E^* - \mu^*\right)} + 1} - \frac{1}{e^{\beta\left(E^* + \mu^*\right)} + 1}\right]$$

$$T \to 0 \to 2\int \frac{d\mathbf{p}}{(2\pi)^3}\left[\theta\left(\mu^* - E^*\right) - \theta\left(-\mu^* - E^*\right)\right]$$

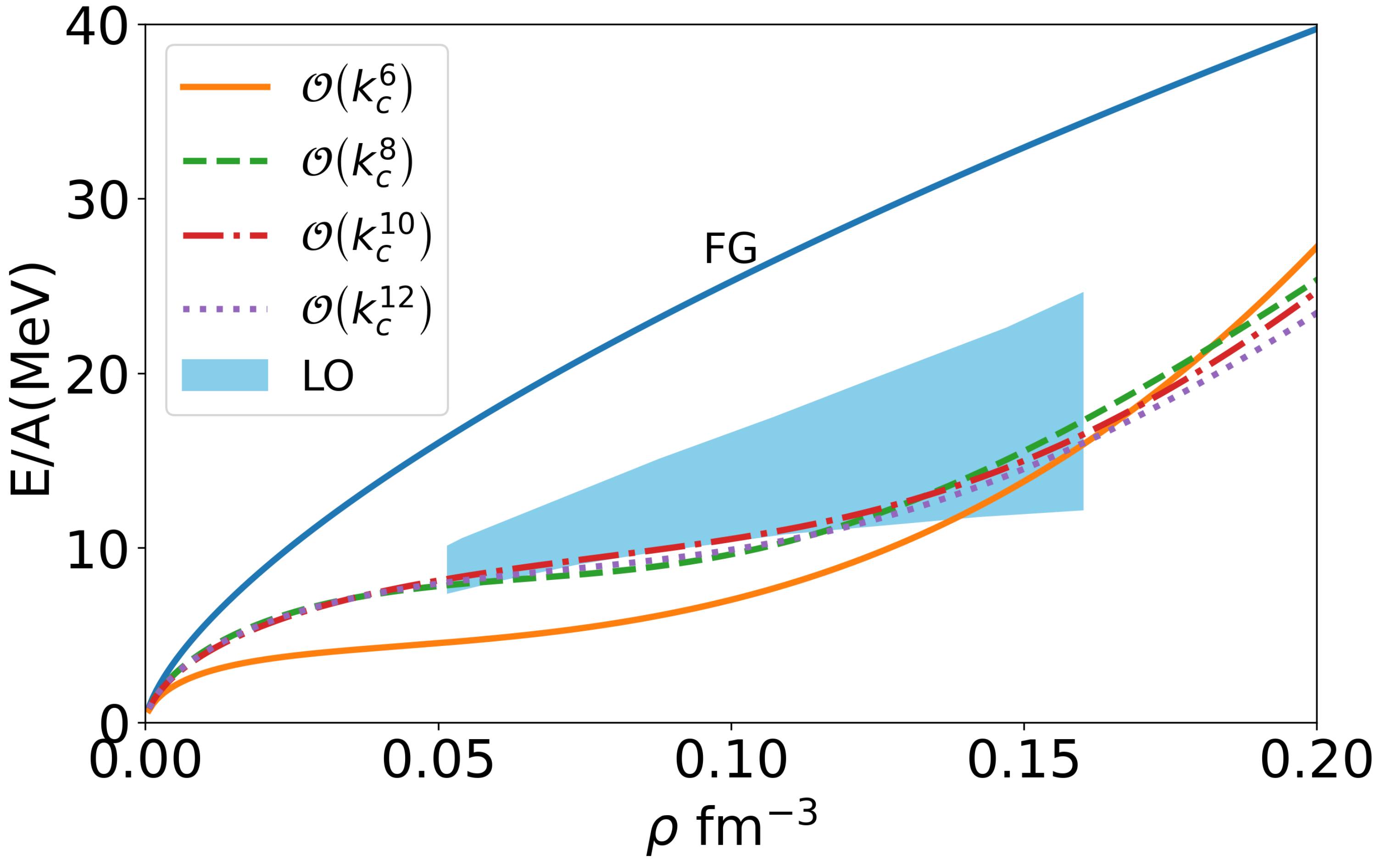
T effects



Nuclear matter properties

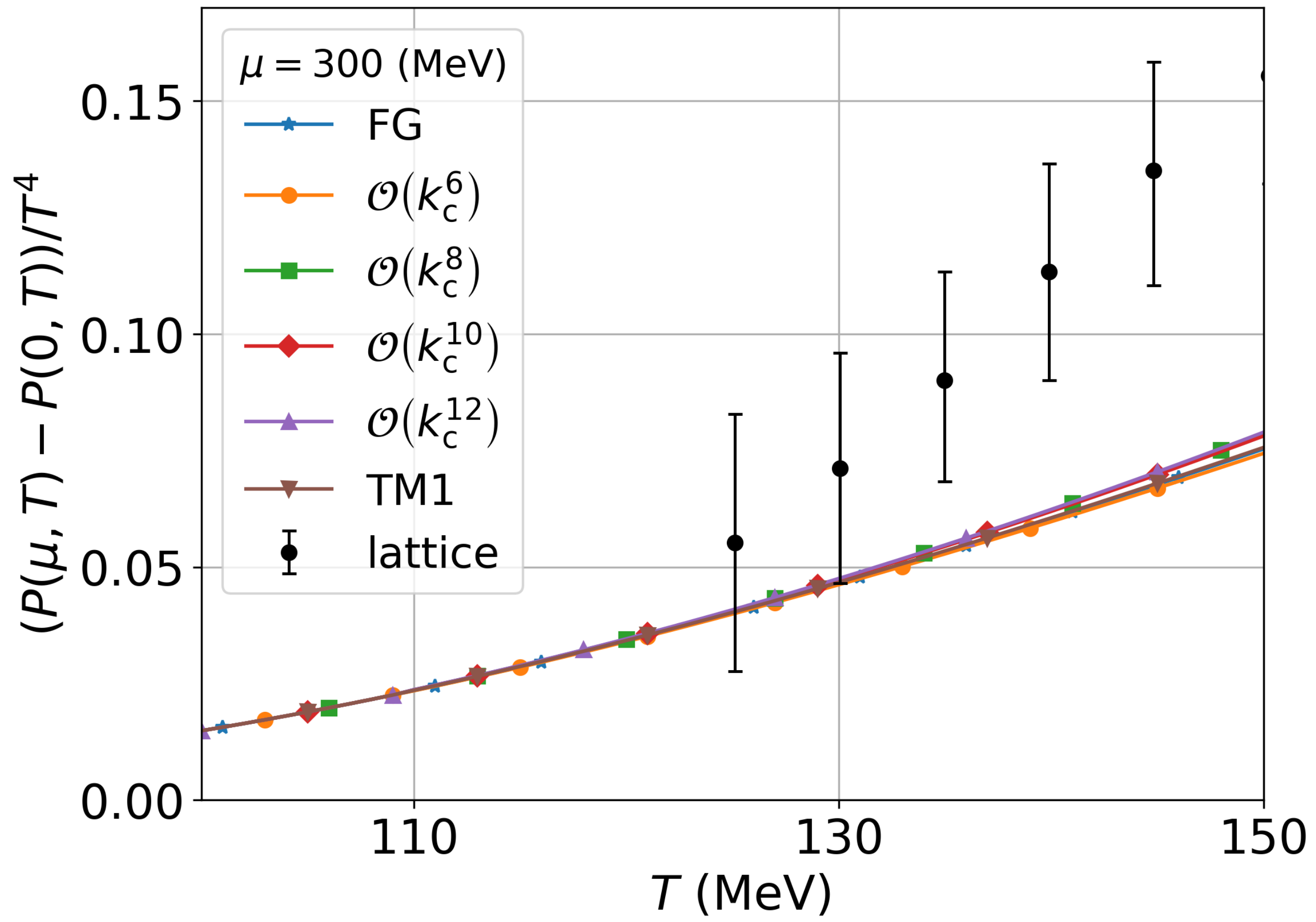
	Empirical	$\mathcal{O}(k_c^6)$	$\mathcal{O}(k_c^8)$	$\mathcal{O}(k_c^{10})$	$\mathcal{O}(k_c^{12})$
n_0 (fm^{-3})	0.155 ± 0.050	0.160	0.160	0.160	0.158
$E(n_0)$ (MeV)	-15.0 ± 1.0	-16.3	-15.6	-16.1	-15.4
$E(1.5n_0)$ (MeV)	-13.3 ± 0.5	-6.92	-12.0	-8.99	-11.2
$K(n_0)$ (MeV)	230 ± 30	566	366	572	391
$E_{\rm sym}(n_0)$ (MeV)	30.9 ± 1.9	31.6	32.1	31.8	32.2
$L(n_0)$ (MeV)	52.5 ± 17.5	104	83.2	75.7	77.8
T_c (MeV)	20.0 ± 3.0	19.0	26.5	24.0	22.5

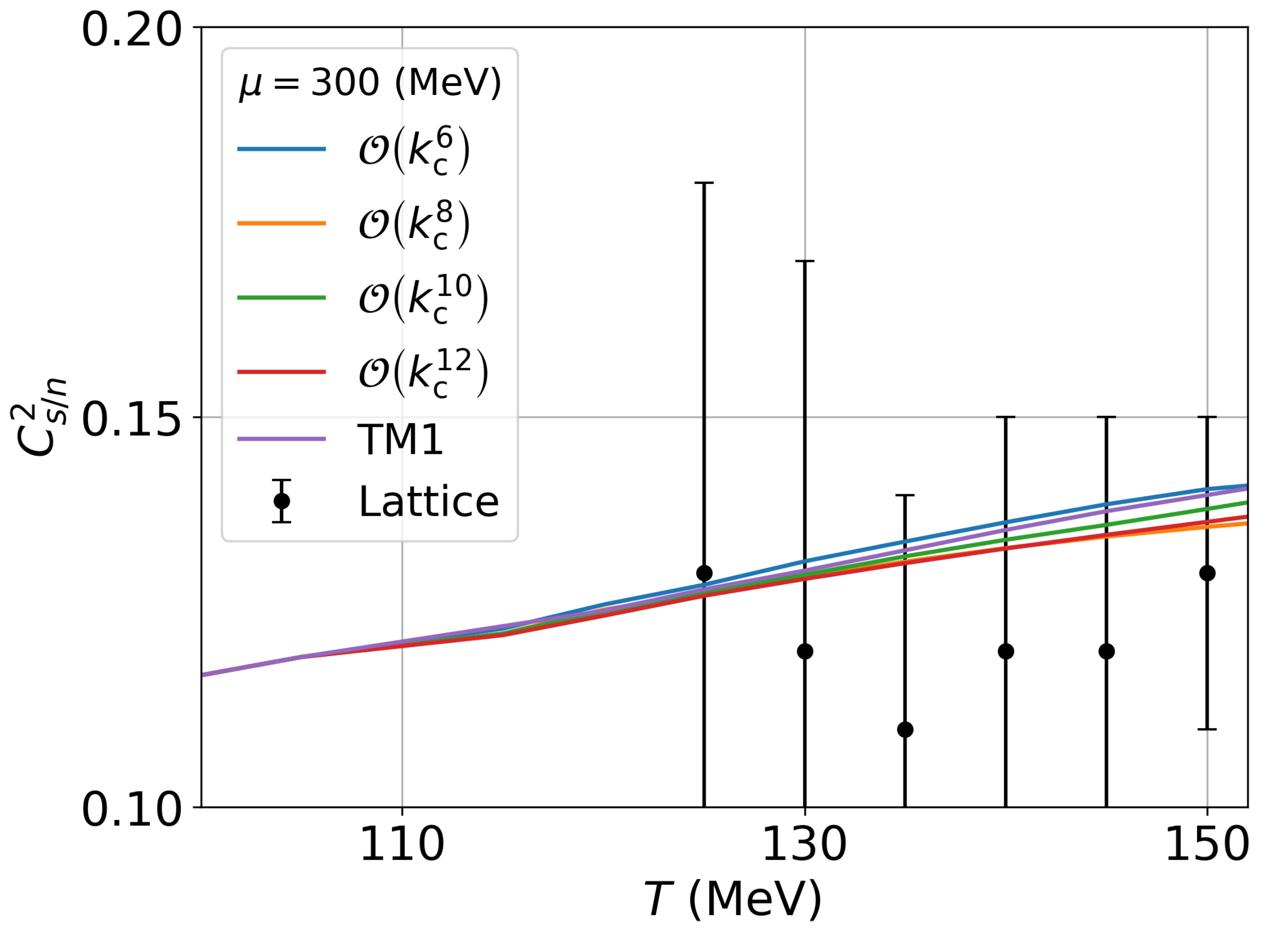
A. Sedrakian, J.-J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023).
Y. Leifels, A. LeFevre, W. Reisdorf, J. Aichelin, and C. Hartnack, EPJ Web Conf. 88, 00018 (2015).
M. Dutra, O. Lourenco, J. S. Sa Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, Phys. Rev. C 85, 035201 (2012).
J. M. Lattimer and Y. Lim, The Astrophysical Journal 771, 51 (2013).
V. A. Karnaukhov et al., Phys. Rev. C 67, 011601 (2003).



Comparison with the leading order chiral nuclear force results of PNM

C. Drischler, J. W. Holt, and C. Wellenhofer, Ann. Rev. Nucl. Part. Sci. 71, 403 (2021).

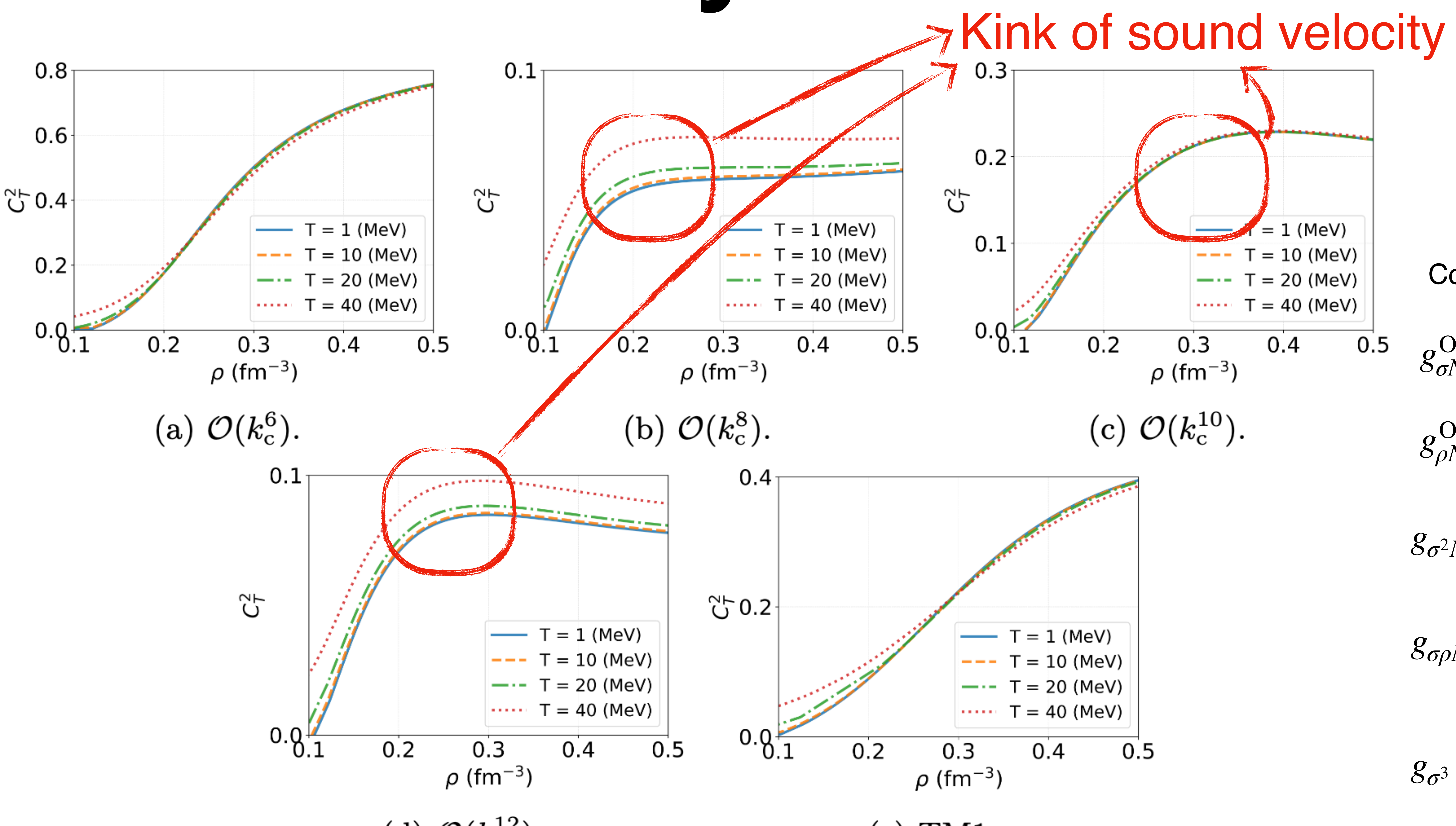




Comparison with the Lattice QCD results

S. Borsanyi, et. al., JHEP 08, 053 (2012).

Sound velocity



(a) $\mathcal{O}(k_c^6)$. (b) $\mathcal{O}(k_c^8)$. (c) $\mathcal{O}(k_c^{10})$.

(d) $\mathcal{O}(k_c^{12})$. (e) TM1.

Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994).

Coupling relations from scale compensator:

$$g_{\sigma NN}^{\mathrm{OBE}} = \frac{m_N}{f_\chi}, g_{\omega NN}^{\mathrm{OBE}} = g_{\omega N}$$

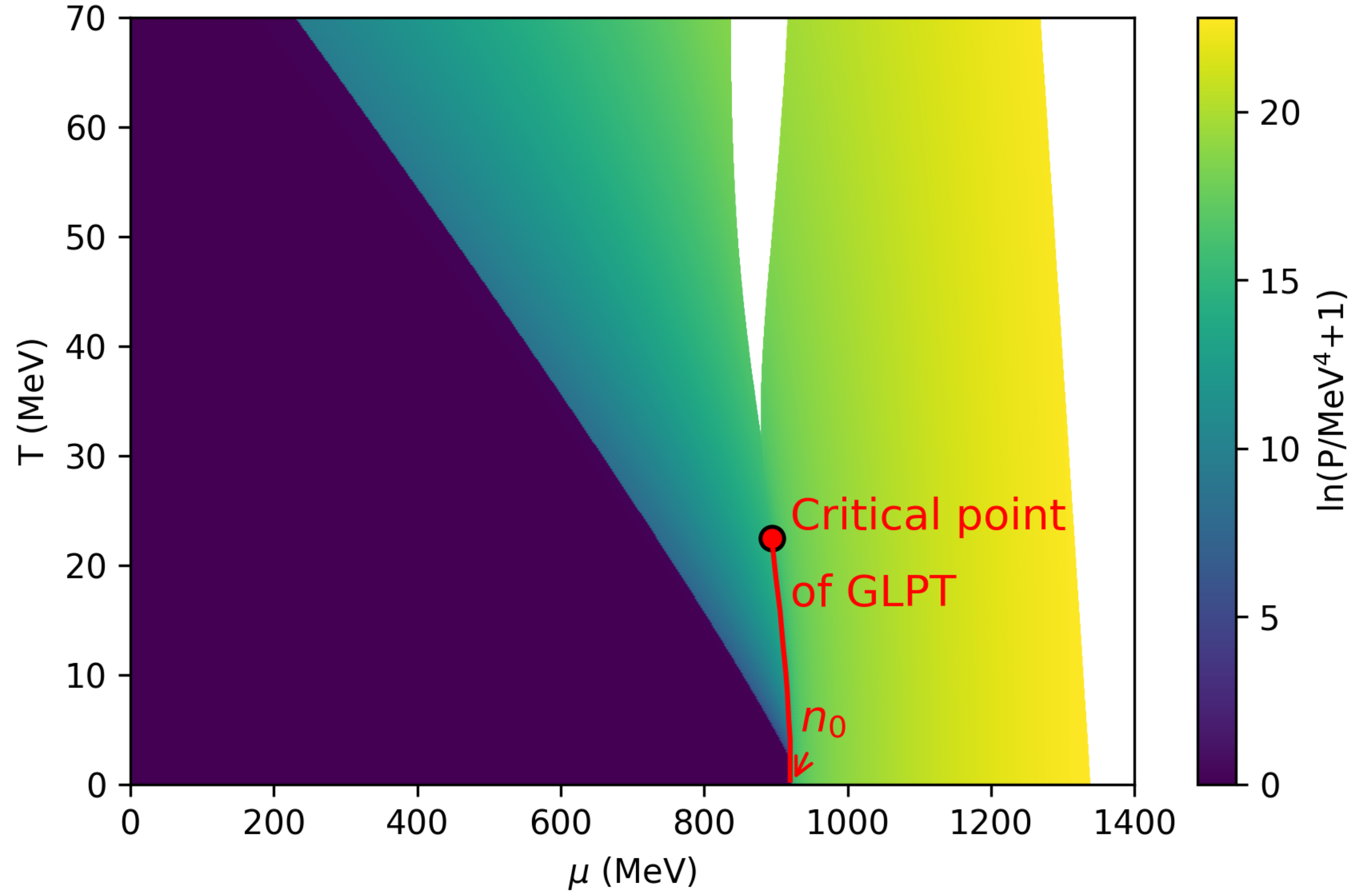
$$g_{\rho NN}^{\mathrm{OBE}} = g_{\rho NN} + g_{\rho NN}^{SSB}, g_{\pi NN}^{\mathrm{OBE}} = g_{\pi NN} + g_{\pi N}^{SSB}$$

$$g_{\sigma^2 NN} = \frac{m_N}{2f_\chi^2}, \quad g_{\sigma\omega NN} = g_{\omega NN}^{SSB}\frac{\beta'}{f_\chi},$$

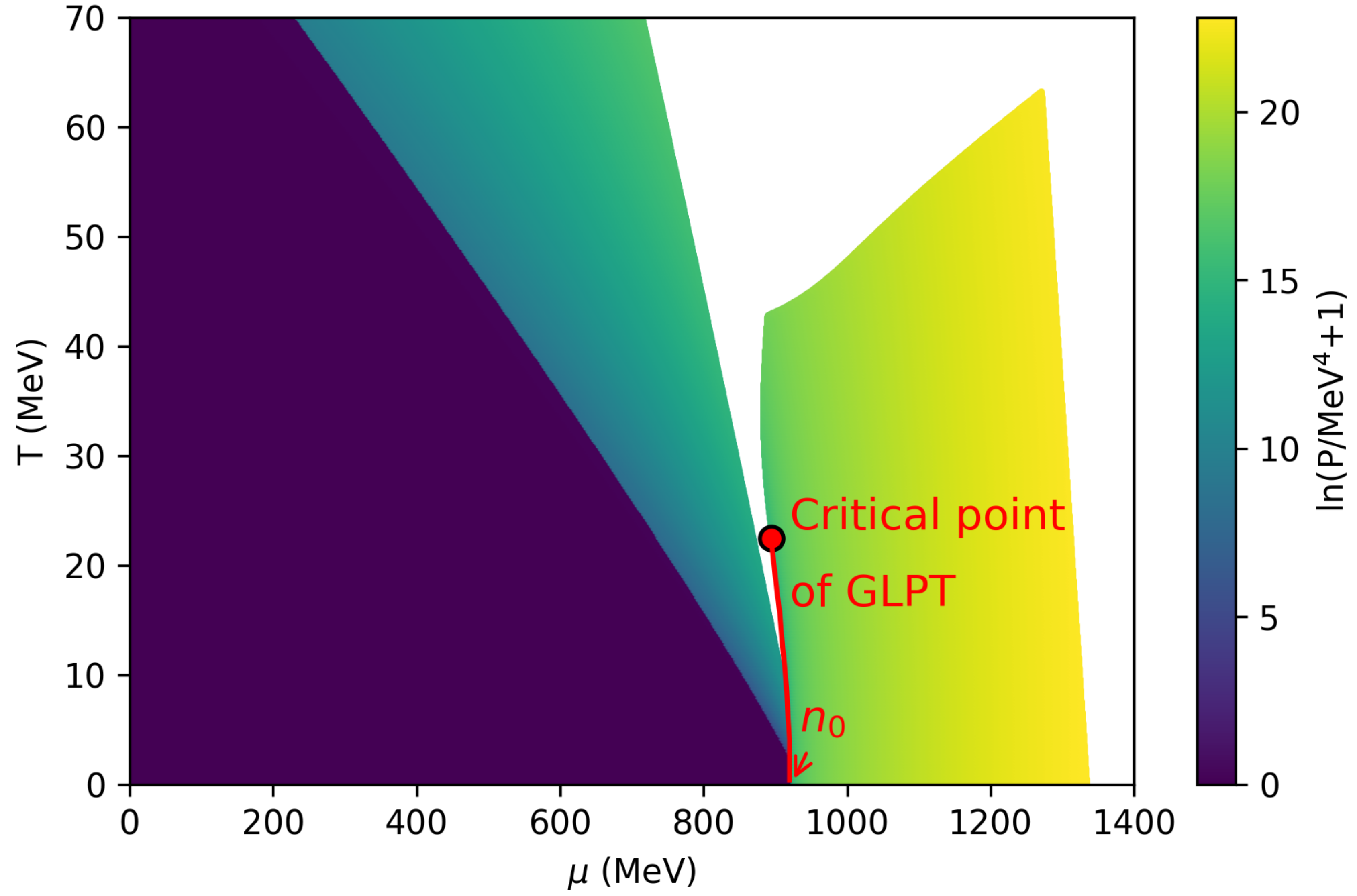
$$g_{\sigma\rho NN} = g_{\rho NN}^{SSB}\frac{\beta'}{f_\chi},$$

$$g_{\sigma^3} = -\frac{32h_5}{3f_\chi^3} - \frac{(4+\beta')^3 h_6}{6f_\chi^3} - m_\pi^2 f_\pi^2 \frac{4}{3f_\chi^3},$$

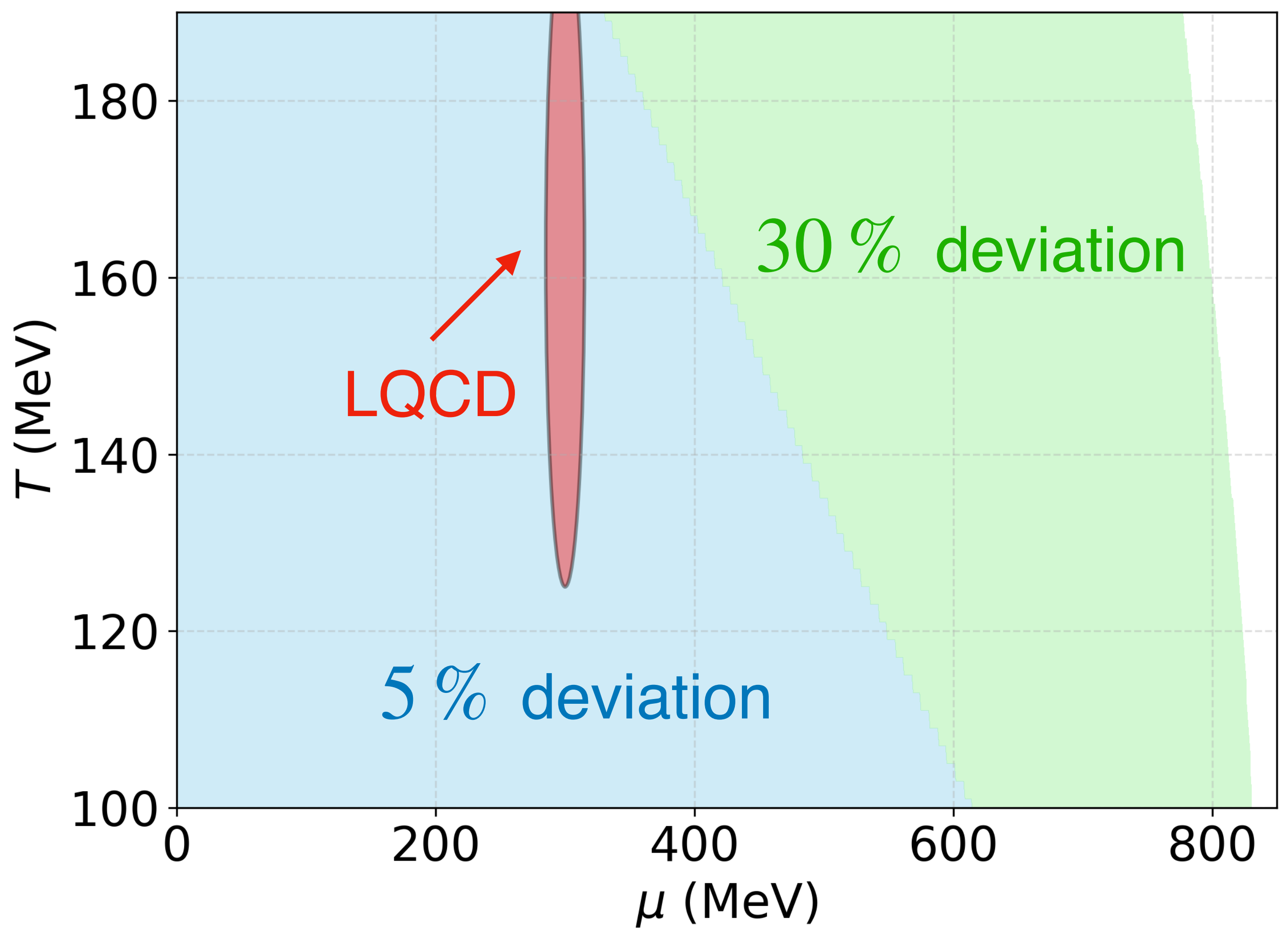
The effects of T on CSDC expansions



$30\,\%$ deviation



$5\,\%$ deviation



Summary and outlook

嚴謹求實
勤奮創新

I. CSDC can capture the physics with appropriate choice of the order, and this scheme may help we understand low energy QCD at finite temperatures and densities;

II. In a near future, a proton-neutron star study will be performed, which has a much wider region in $T-\mu$ plane, and a more systematic analysis of parameters will be carried out with the power of Bayesian analysis and deep learning.

III. A full version of Hartree-Fock calculation will be also tried and see the effects of Fock terms.

Thank you!

Backup

LO chiral-scale Lagrangian

$$\mathscr{L}_M = f_\pi^2\Phi^2\,\mathrm{Tr}\left(\hat{\alpha}_\perp^\mu\hat{\alpha}_{\mu\perp}\right) + f_{\sigma\rho}^2\Phi^2\,\mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu\hat{\alpha}_{\mu\parallel}\right) + f_0^2\Phi^2\,\mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu\right)\mathrm{Tr}\left(\hat{\alpha}_{\mu\parallel}\right) -$$
$$\frac{1}{2g_\rho^2}\mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu}\right) - \frac{1}{2g_0^2}\mathrm{Tr}\left(V_{\mu\nu}\right)\mathrm{Tr}\left(V^{\mu\nu}\right) +$$
$$\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{f_\pi^2}{4}\Phi^{3-\gamma_m}\mathrm{Tr}\left(\mathscr{M}U^\dagger + U\mathscr{M}^\dagger\right) + h_5\Phi^4 + h_6\Phi^{4+\beta'}$$

$$\mathscr{L}_B = \bar{N}i\gamma_\mu D^\mu N - m_N\Phi\bar{N}N$$
$$+\left[g_A C_A + g_A\left(1 - C_A\right)\Phi^{\beta'}\right]\bar{N}\hat{\alpha}_\perp^\mu\gamma_\mu\gamma_5 N + \left[g_{V_\rho}C_{V_\rho} + g_{V_\rho}\left(1 - C_{V_\rho}\right)\Phi^{\beta'}\right]\bar{N}\hat{\alpha}_\parallel^\mu\gamma_\mu N$$
$$+\frac{1}{2}\left[g_{V_0}C_{V_0} + g_{V_0}\left(1 - C_{V_0}\right)\Phi^{\beta'}\right]\mathrm{Tr}\left[\hat{\alpha}_\parallel^\mu\right]\bar{N}\gamma_\mu N$$

H. K. Lee, W.-G. Paeng, and M. Rho, Phys. Rev. D 92, 125033 (2015)
Y. L. Li, Y. L. Ma, and M. Rho, Phys. Rev. D 95, 114011 (2017)

Lagrangians under CSDC

$$\mathscr{L}_{\mathrm{LO}} = \bar{N}\left(i\gamma_\mu\partial^\mu - m_N\right)N,$$

$$\mathscr{L}_{\mathrm{NLO}} = \mathscr{L}_{\mathrm{LO}} - \bar{N}\left(g^{\mathrm{OBE}}_{\omega NN}\gamma_0\omega + g^{\mathrm{OBE}}_{\rho NN}\gamma_0\rho\tau_3 + g^{\mathrm{OBE}}_{\sigma NN}\sigma\right)N + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2}m_\sigma^2\sigma^2,$$

$$\mathscr{L}_{\mathrm{N^2LO}} = \mathscr{L}_{\mathrm{NLO}} - \bar{N}\left(g_{\sigma^2 NN}\sigma^2 - g_{\sigma\omega NN}\sigma\omega\gamma^0 - g_{\sigma\rho NN}\sigma\rho\gamma^0\tau_3\right)N - g_{\sigma_3}\sigma^3 - g_{\sigma\omega^2}\sigma\omega^2 - g_{\sigma\rho^2}\sigma\rho^2,$$

$$\mathscr{L}_{\mathrm{N^3LO}} = \mathscr{L}_{\mathrm{N^2LO}} - \bar{N}\left(g_{\sigma^3 NN}\sigma^3 - g_{\sigma^2\omega NN}\sigma^2\omega\gamma^0 - g_{\sigma^2}\rho NN\sigma^2\rho\gamma^0\tau_3\right)N - g_{\sigma^4}\sigma^4 - g_{\sigma^2}\omega^2\sigma^2\omega^2 - g_{\sigma^2\rho^2}\sigma^2\rho^2,$$

$$\mathscr{L}_{\mathrm{N^4LO}} = \mathscr{L}_{\mathrm{N^3LO}} - \bar{N}\left(g_{\sigma^4 NN}\sigma^4 - g_{\sigma^3\omega NN}\sigma^3\omega\gamma^0 - g_{\sigma^3\rho NN}\sigma^3\rho\gamma^0\tau_3\right)N - g_{\sigma^5}\sigma^5 - g_{\sigma^3\omega^2}\sigma^3\omega^2 - g_{\sigma^3\rho^2}\sigma^3\rho^2\,.$$

Parameter definition

$$g_{\omega NN} = \frac{1}{2}\left(g_{V_p}C_{V_p} + g_{V_0}C_{V_0} - 1\right) g_{\omega},$$

$$g_{\rho NN} = \frac{1}{2}\left(g_{V_p}C_{V_p} - 1\right) g_{\rho},$$

$$g_{\pi NN} = -\frac{\left(g_A C_A\right)}{2f_\pi},$$

$$g_{\omega NN}^{SSB} = \frac{1}{2}\left[g_{V_\rho}\left(1 - C_{V_\rho}\right) + g_{V_0}\left(1 - C_{V_0}\right)\right] g_{\omega},$$

$$g_{\rho NN}^{SSB} = \frac{1}{2}\left[g_{V_\rho}\left(1 - C_{V_\rho}\right)\right] g_{\rho},$$

$$g_{\pi NN}^{SSB} = -\frac{\left[g_A\left(1 - C_A\right)\right]}{2f_\pi};$$

$$4h_5 + \left(4 + \beta'\right) h_6 + 2m_\pi^2 f_\pi^2 = 0$$

$$12h_5 + \left(4 + \beta'\right)\left(3 + \beta'\right) h_6 + 2m_\pi^2 f_\pi^2 = -m_\sigma^2 f_\chi^2$$

$$g_{\sigma NN}^{\mathrm{OBE}} = \frac{m_N}{f_\chi}, \quad g_{\omega NN}^{\mathrm{OBE}} = g_{\omega NN} + g_{\omega NN}^{SSB}, \quad g_{\rho NN}^{\mathrm{OBE}} = g_{\rho NN} + g_{\rho NN}^{SSB}, \quad g_{\pi NN}^{\mathrm{OBE}} = g_{\pi NN} + g_{\pi NN}^{SSB}$$

$$g_{\sigma^2 NN} = \frac{m_N}{2f_\chi^2}, \quad g_{\sigma\omega NN} = g_{\omega NN}^{SSB}\frac{\beta'}{f_\chi}, \quad g_{\sigma\rho NN} = g_{\rho NN}^{SSB}\frac{\beta'}{f_\chi}$$

$$g_{\sigma^3} = -\frac{32h_5}{3f_\chi^3} - \frac{\left(4+\beta'\right)^3 h_6}{6f_\chi^3} - m_\pi^2 f_\pi^2 \frac{4}{3f_\chi^3}, \quad g_{\sigma\rho^2} = -\frac{m_\rho^2}{f_\chi^2}, \quad g_{\sigma\omega^2} = -\frac{m_\omega^2}{f_\chi^2}$$

$$g_{\sigma^3 NN} = \frac{m_N}{6f_\chi^3}, \quad g_{\sigma^4 NN} = \frac{m_N}{24f_\chi^4}, \quad g_{\sigma^3\omega NN} = \frac{\beta'^2 g_{\sigma\omega NN}}{6f_\chi^2}, \quad g_{\sigma^3\rho NN} = \frac{\beta'^2 g_{\sigma\rho NN}}{6f_\chi^2},$$

$$g_{\sigma^4} = -\frac{4^4 h_5 + \left(4+\beta'\right)^4 h_6 + 2^4 m_\pi^2 f_\pi^2}{24f_\chi^4}, \quad g_{\sigma^2\omega\omega} = -\frac{m_\omega^2}{f_\chi^2}, \quad g_{\sigma^2\rho\rho} = -\frac{m_\rho^2}{f_\chi^2},$$

$$g_{\sigma^5} = -\frac{4^5 h_5 + \left(4+\beta'\right)^5 h_6 + 2^5 m_\pi^2 f_\pi^2}{120f_\chi^5}, \quad g_{\sigma^3\omega\omega} = -\frac{2m_\omega^2}{3f_\chi^3}, \quad g_{\sigma^3\rho\rho} = -\frac{2m_\rho^2}{3f_\chi^3}.$$

Effective mass and chemical potential at each order under RMF

$\mathcal{O}(k_c^6)$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma$$

$$\mu_p^* = \mu_p - g_{\omega NN}^{\mathrm{OBE}}\omega - g_{\rho NN}^{\mathrm{OBE}}\rho$$

$$\mu_n^* = \mu_n - g_{\omega NN}^{\mathrm{OBE}}\omega + g_{\rho NN}^{\mathrm{OBE}}\rho$$

$\mathcal{O}(k_c^8)$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma + g_{\sigma^2 NN}\sigma^2$$

$$\mu_p^* = \mu_p - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma\right)\omega - \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma\right)\rho$$

$$\mu_n^* = \mu_n - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma\right)\omega + \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma\right)\rho$$

$\mathcal{O}(k_{\mathrm{c}}^{10})$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma + g_{\sigma^2 NN}\sigma^2 + g_{\sigma^2\sigma NN}\sigma^3$$

$$\mu_p^* = \mu_p - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2\right)\omega - \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2\right)\rho$$

$$\mu_n^* = \mu_n - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2\right)\omega + \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2\right)\rho$$

$\mathcal{O}(k_\mathrm{c}^{12})$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma + g_{\sigma^2 NN}\sigma^2 + g_{\sigma^3 NN}\sigma^3 + g_{\sigma^4 NN}\sigma^4$$

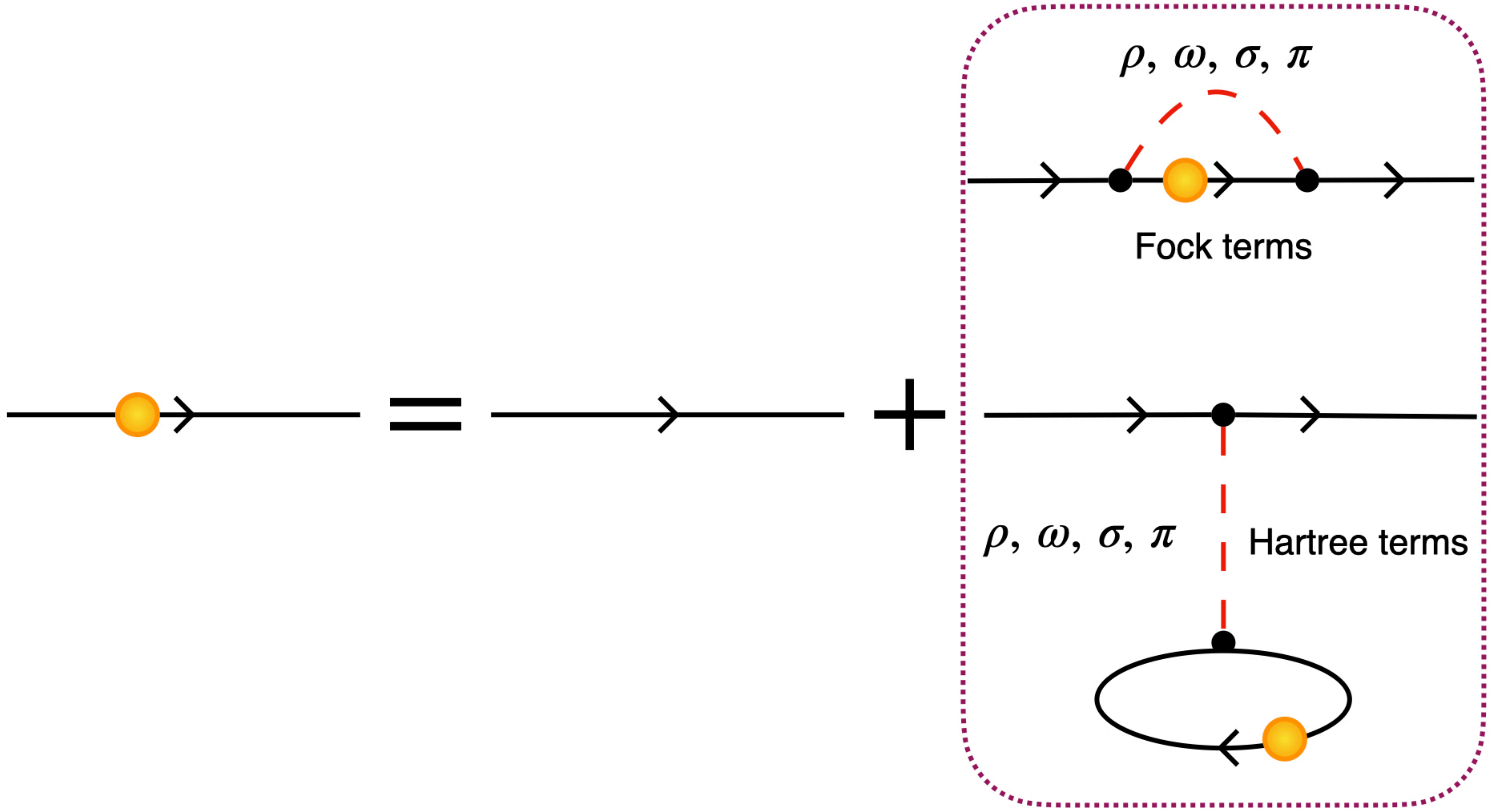
$$\mu_p^* = \mu_p - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2 + g_{\sigma^3\omega NN}\sigma^3\right)\omega$$
$$-\left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2 + g_{\sigma^3\rho NN}\sigma^3\right)\rho$$

$$\mu_n^* = \mu_n - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2 + g_{\sigma^3\omega NN}\sigma^3\right)\omega$$
$$+\left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2 + g_{\sigma^3\rho NN}\sigma^3\right)\rho$$

MFA Dirac equation

$$\left(\gamma^\mu p_\mu - m_B - \Sigma(\gamma^\mu p_\mu)\right)\psi_B = 0, \quad p^{*\mu} = p^\mu - \Sigma^\mu, m_B^* = m_B + \Sigma_s,$$

$$\left[\gamma \cdot \mathbf{p}^* + m_B^*\right] u(\mathbf{p}, s) = \gamma_0 E^* u(\mathbf{p}, s),$$



Thermal quants computation

$$\langle \Omega | : \hat{\mathscr{H}} : | \Omega \rangle = \langle 0 | T \left\{ : \hat{\mathscr{H}} \exp \left[-i \int_{-\infty}^{+\infty} \mathrm{d}t' \hat{H}_I(t') \right] : \right\} | 0 \rangle_c$$

$$\begin{aligned}\hat{\mathscr{H}}(x) = \bar{N}(x)\big(-i\gamma \cdot \nabla + m_N\big)\, N(x) \\ +\bar{N}(x)\left(g_{\sigma NN}^{\mathrm{OBE}} \int_V \mathrm{d}^4x' D_\sigma(x-x') \left(-g_{\sigma NN}^{\mathrm{OBE}} \bar{N}(x') N(x')\right) \right) N(x) + \cdots,\end{aligned}$$

$$\Phi_{\mathrm{G}} = -\, T \ln Z$$

$$Z = \int \left[\mathrm{d}\bar{N}_p\right] \left[\mathrm{d}N_p\right] \left[\mathrm{d}\bar{N}_n\right] \left[\mathrm{d}N_n\right] \exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left(\mathscr{L}_{\mathrm{RMF}} + \mu_p N_p^\dagger N_p + \mu_n N_n^\dagger N_n \right) \right)$$