Study of short-range physics in nuclei with the operator product expansion

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第十届手征有效场论研讨会
南京 2025.10.20

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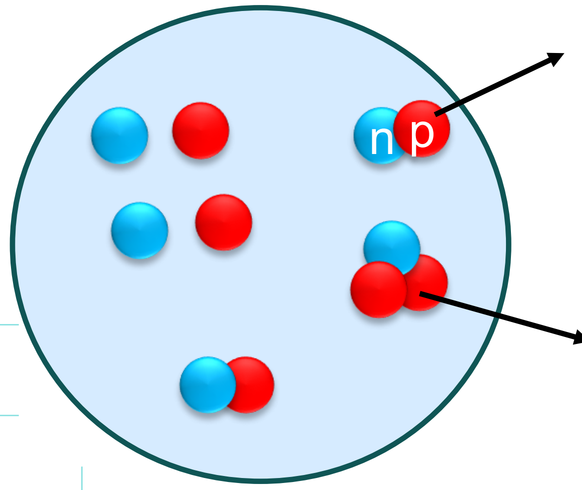
Short-range Correlation in Nuclei

- Small center-of-mass momentum
- Large relative momentum

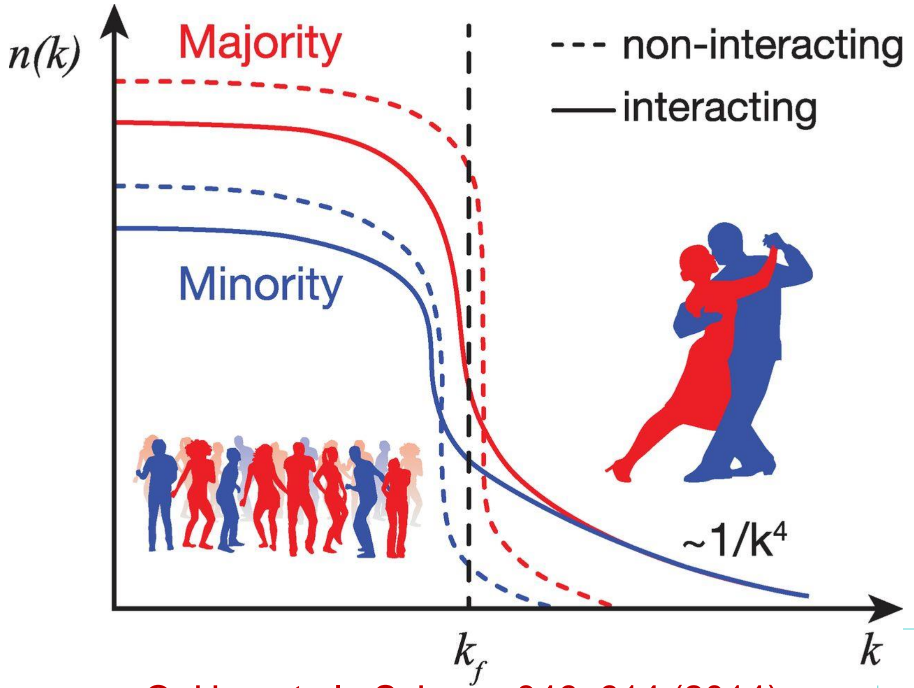


2N-SRC

3N-SRC

- Under investigation

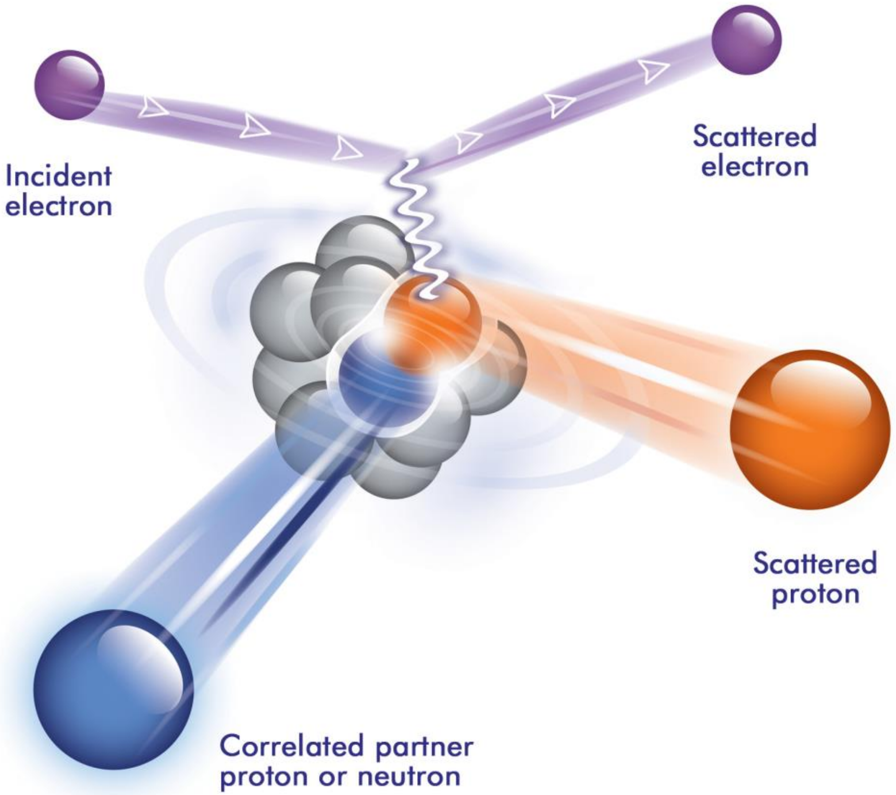
- Relation to the momentum distribution
  - Similar momentum tails for different nuclei



O. Hen et al., Science 346, 614 (2014)

SRC Measurements

➢ Quasi-elastic scattering



O. Hen et al., Science 346, 614 (2014)

Inclusive measurements:

- Detect scattered electron
- Less final state interaction
- Unable to distinguish scattered hadrons

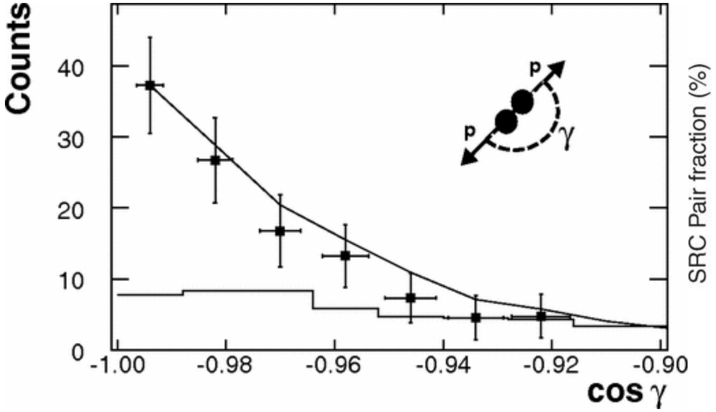
Exclusive measurements:

- Detect scattered proton and electron
- Strong final state interaction
- Enable to distinguish scattered hadrons

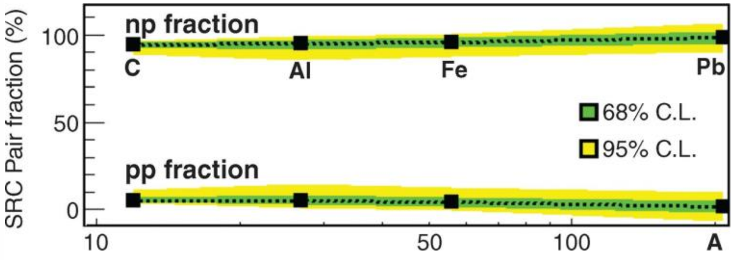
Introduction

SRC Measurements

- Exclusive measurements
  - Back to back nucleon pairs
  - Np dominance



R. Shneor et al., Phys. Rev. Lett.1399, 072501 (2007)



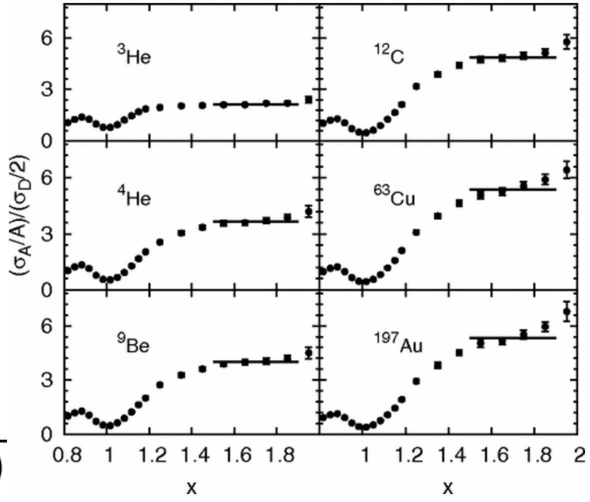
O. Hen et al., Science 346, 614 (2014)

- Inclusive measurements
  - 2N-SRC ($1.3 < x < 2$):

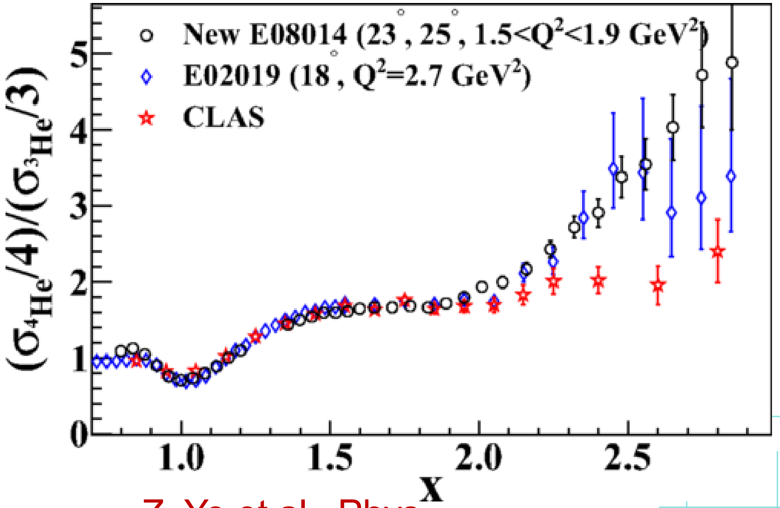
$$a_2(A, D) = \frac{a_2(A)}{a_2(D)} = \frac{2\sigma_A(x,Q^2)}{A\sigma_D(x,Q^2)}$$

  - 3N-SRC ($2 < x < 3$) (not found yet):

$$a_3(A, {}^3He) = \frac{a_3(A)}{a_3({}^3He)} = \frac{3\sigma_A(x,Q^2)}{A\sigma_{{}^3He}(x,Q^2)}$$



N. Fomin et al., Phys. Rev. Lett. 108, 092502 (2012)



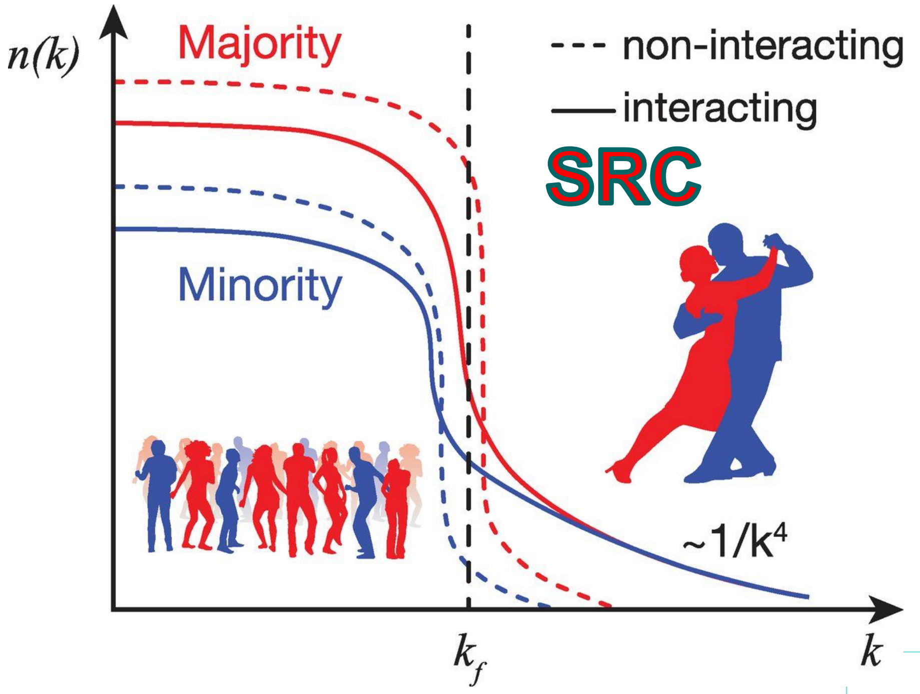
Z. Ye et al., Phys. Rev. C 97, 065204 (2018)

Large-momentum distribution in nuclei

- It reveals information about the short-range few-nucleon dynamics
- Connect to 2N-SRC(naive SRC model)

$$a_2(A) = \int_{k_f}^{\infty} dk k^2 n(k)$$

- Investigate potential 3N-SRC



Single-nucleon Momentum Distribution

- Nonlocal operator

$$\Omega(\vec{k}) \equiv \int d^3 r e^{-i\vec{k}\cdot\vec{r}} N^\dagger\left(-\tfrac{1}{2}\vec{r}\right) N\left(+\tfrac{1}{2}\vec{r}\right)$$

- Momentum distribution in nuclei A

$$\rho(k) \equiv \langle A|\Omega(\vec{k})|A\rangle$$

Hard scale Soft scale

Hard to handle two parts at the same time!

- Hard to find a “perfect” theory to cover two scales!
- “Perfect” means complexity and is hard to do the many-body calculation.

OPE & Matching

Operator Product Expansion

➢ Separation of two scales

$$N^\dagger\left(-\tfrac{1}{2}\vec{r}\right) N\left(+\tfrac{1}{2}\vec{r}\right) = \sum_n W_n(r)\mathcal{O}_n(0) \quad \text{for } r \to 0$$

➢ Momentum distribution in OPE

$$\rho(k) = \sum_n \underbrace{\widetilde{W}_n(k)}_{\text{Hard scale}} \underbrace{\langle A|\mathcal{O}_n(0)|A\rangle}_{\text{Soft scale, Calculated in the EFT}}$$

Hard scale

Soft scale
Calculated in the EFT

$\widetilde{W}_n(k)$ are independent of the sates

- Universal for different nuclei
- Obtained in scattering states

➢ OPE-EFT

- Many-body parameters calculated in the EFT
- $\widetilde{W}_n(k)$ obtained from matching the underlying theory and the EFT in scattering states

Local Operator

- How to determine local operators

$$N^\dagger\left(-\tfrac{1}{2}\vec{r}\right)N\left(+\tfrac{1}{2}\vec{r}\right)=\sum_n W_n(r)\mathcal{O}_n(0)$$

- ChPT, Pionless EFT, …

$$N,\ \pi,\ \ldots$$

- The same quantum number as the nonlocal operator
  - ✓ Spin 0
  - ✓ Isospin 0
  - ✓ …

- Local operators
  - One-body operators

$$\phi_0 \equiv N^\dagger N\,,$$

$$\phi_2 \equiv N^\dagger \overleftrightarrow{\nabla}^2 N + \text{h.c.}\,,$$

$\Rightarrow$ Wilson coefficient $\delta^{(3)}(k)$

  - Two-body operators (related to the 2N-SRC)

$$\mathcal{O}_0(\alpha)=\left[N^T P_i(\alpha) N\right]^\dagger\left[N^T P_i(\alpha) N\right]$$

$$\mathcal{O}_2(\alpha)=\left[N^T P_i(\alpha) N\right]^\dagger\left[N^T P_i(\alpha)\overleftrightarrow{\nabla}^2 N\right]+\text{h.c.}$$

$P_i(\alpha)$: projection matrix

α :

Spin-isospin space: ${}^3S_1$, ${}^1S_0$, …

Flavour space: nn, np, pp

Matching For Wilson Coefficients In Scattering

- Matching

$$\Omega(\vec{k}) = \sum_n \boxed{\widetilde{W}_n(k)} \mathcal{O}_n(0)$$

Solvable

$$\boxed{\langle \psi_p^- | \Omega(\vec{k}) | \psi_p^+ \rangle}$$

Calculated in the underlying theory (AV18)

$$\boxed{\langle \psi_p^- | \mathcal{O}_n | \psi_p^+ \rangle}$$

Calculated in the EFT

$|\psi_p^\pm\rangle$:Two-body scattering in (out) sate

p : soft matching momentum (available in the EFT)

The hard scale information can be extracted in the two-body calculation!
Much easier for the underlying theory

OPE-EFT of Deuteron

Underlying theory: AV18
EFT: Pionless

- Matching in $^3S_1$ states(2 local operators in, NLO Pionless):

$$\mathcal{A}(k,p) = \widetilde{W}_0^{\mathrm{LO}}(k)\mathcal{M}_0^{\mathrm{LO}}(p) + \widetilde{W}_0^{\mathrm{NLO}}(k)\mathcal{M}_0^{\mathrm{LO}}(p) + \widetilde{W}_0^{\mathrm{LO}}(k)\mathcal{M}_0^{\mathrm{NLO}}(p) + \widetilde{W}_0^{\mathcal{O}_2}(k)\mathcal{M}_0^{\mathrm{LO}}(p) + \widetilde{W}_2(k)\mathcal{M}_2^{\mathrm{LO}}(p) + \cdots$$

Calculated in AV18

$$\mathcal{A}(k,p) = \langle \psi_p^- | \Omega(\vec{k}) | \psi_p^+ \rangle$$ — **AV18**

$$\mathcal{M}_n(p) = \langle \psi_p^- | \mathcal{O}_n | \psi_p^+ \rangle$$ — **Pionless**

$$\mathcal{O}_0(^3S_1) = \left[N^T P_i(^3S_1)N\right]^\dagger \left[N^T P_i(^3S_1)N\right]$$

$$\mathcal{O}_2(^3S_1) = \left[N^T P_i(^3S_1)N\right]^\dagger \times \left[N^T P_i(^3S_1)\overleftrightarrow{\nabla}^2 N\right] + \mathrm{h.c.}$$

- $\widetilde{W}_0^{\mathcal{O}_2}$ refer to the "operator mixing" of $\mathcal{O}_0$ and $\mathcal{O}_2$

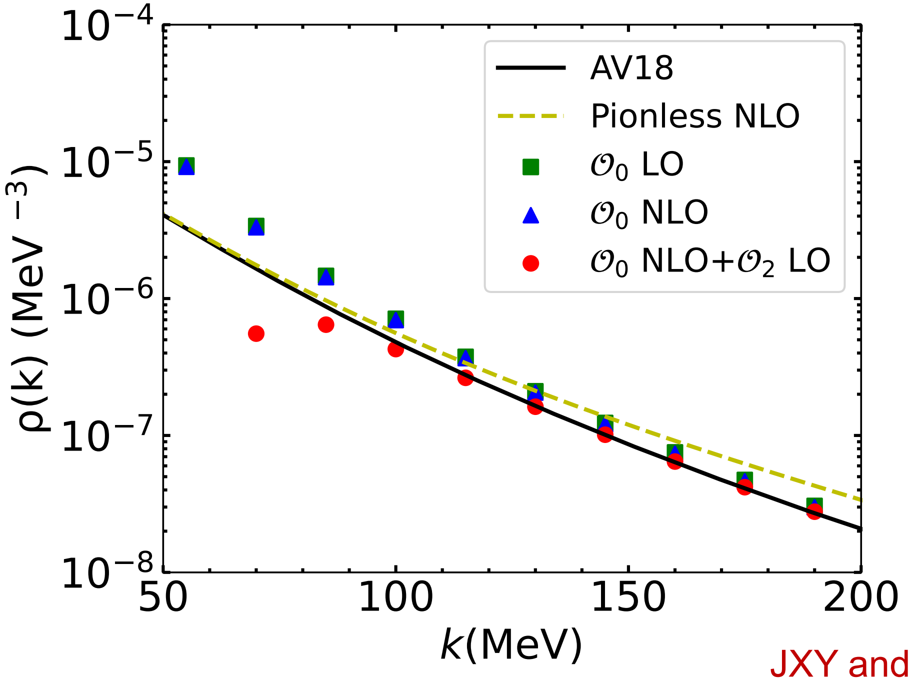
- Large-momentum distribution in the deuteron

$$\rho_d(k) = \left[\widetilde{W}_0^{\mathrm{LO}}(k) + \widetilde{W}_0^{\mathrm{NLO}}(k) + \widetilde{W}_0^{\mathcal{O}_2}(k)\right]\langle d|\mathcal{O}_0|d\rangle_{\mathrm{LO}} + \widetilde{W}_0^{\mathrm{LO}}(k)\langle d|\mathcal{O}_0|d\rangle_{\mathrm{NLO}} + \widetilde{W}_2^{\mathrm{LO}}(k)\langle d|\mathcal{O}_2|d\rangle_{\mathrm{LO}} + \cdots$$

Two distinct expansion parameters:
- $1/k$ (OPE)
- $1/M_{\mathrm{hi}}$ (EFT)

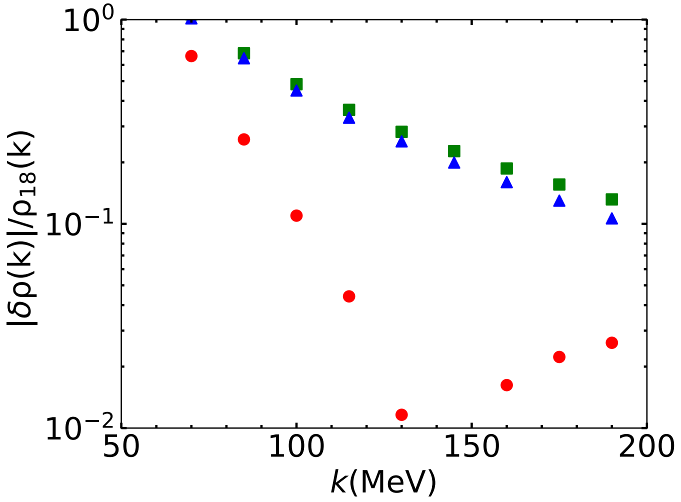
Results

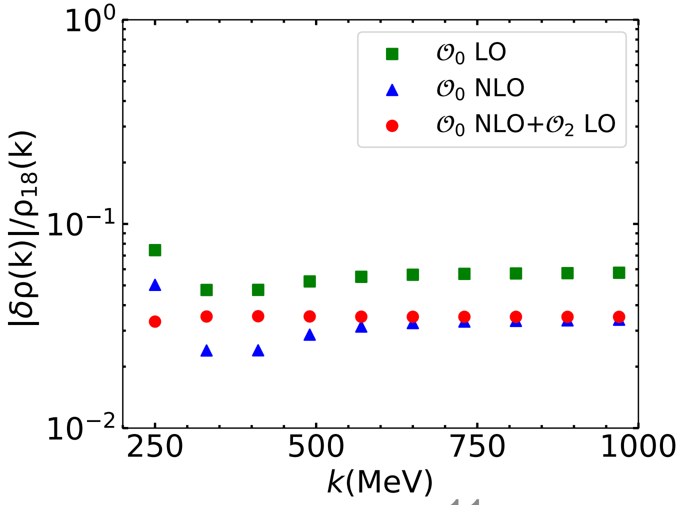
- Deuteron single-nucleon momentum distribution
  - Matching momentum $p_1 = 4$ MeV and $p_2 = 5$ MeV



- Difference of each orders and AV18

$$\frac{|\delta\rho(k)|}{\rho_{18}(k)} \equiv \frac{|\rho_{\rm OPE}(k) - \rho_{18}(k)|}{\rho_{18}(k)}$$





JXY and B. Long, Phys. Rev. C 112, 014005 (2025).

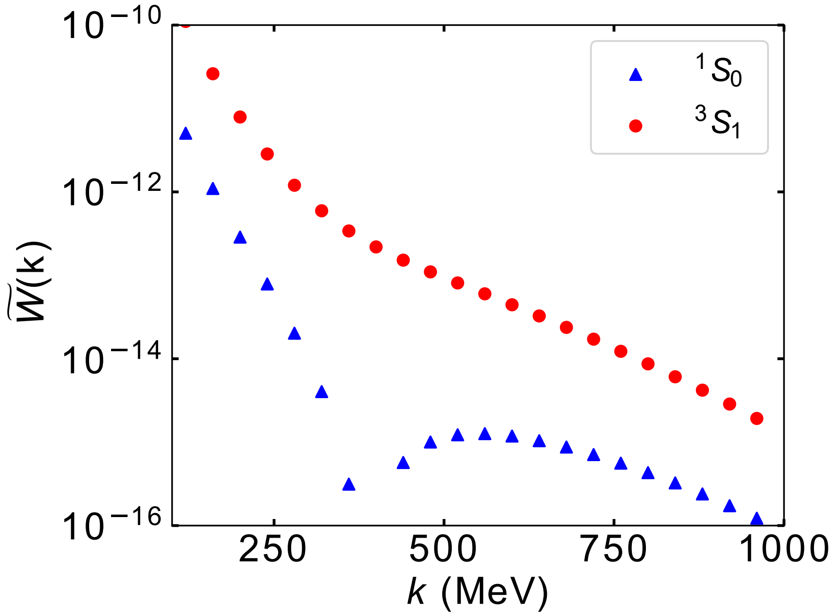
Large-momentum Distribution of Nuclei

- Local operator (dominance)

$$\mathcal{O}_0(^3S_1) = \left[N^T P_i(^3S_1) N\right]^\dagger \left[N^T P_i(^3S_1) N\right]$$

$$\mathcal{O}_0(^1S_0) = \left[N^T P_i(^1S_0) N\right]^\dagger \left[N^T P_i(^1S_0) N\right]$$

  - *S*-wave
  - The lowest dimension



Underlying theory: AV18
Low energy theory: LO Chiral force

- Large-momentum distribution of nuclei A

$$\rho(k) = \widetilde{W}_0(k; {}^1S_0)\langle A|\mathcal{O}_0(^1S_0)|A\rangle + \widetilde{W}_0(k; {}^3S_1)\langle A|\mathcal{O}_0(^3S_1)|A\rangle + \cdots$$

The ellipsis denotes the contribution of higher-order operators.

Many-body Parameter

$$\langle A|\mathcal{O}_n|A\rangle$$

- Constants determined by the nuclei
  - Calculated in the EFT (not done yet)
  - Fitted

$$\rho(k) = \widetilde{W}_0(k;{}^1S_0)\underline{\langle A|\mathcal{O}_0({}^1S_0)|A\rangle} + \widetilde{W}_0(k;{}^3S_1)\underline{\langle A|\mathcal{O}_0({}^3S_1)|A\rangle} + \cdots$$

Extracted by Variational Monte Carlo (VMC) data

https://www.phy.anl.gov/theory/research/momenta2/

Two-nucleon Momentum Distribution

- The OPE of the two-nucleon momentum distribution

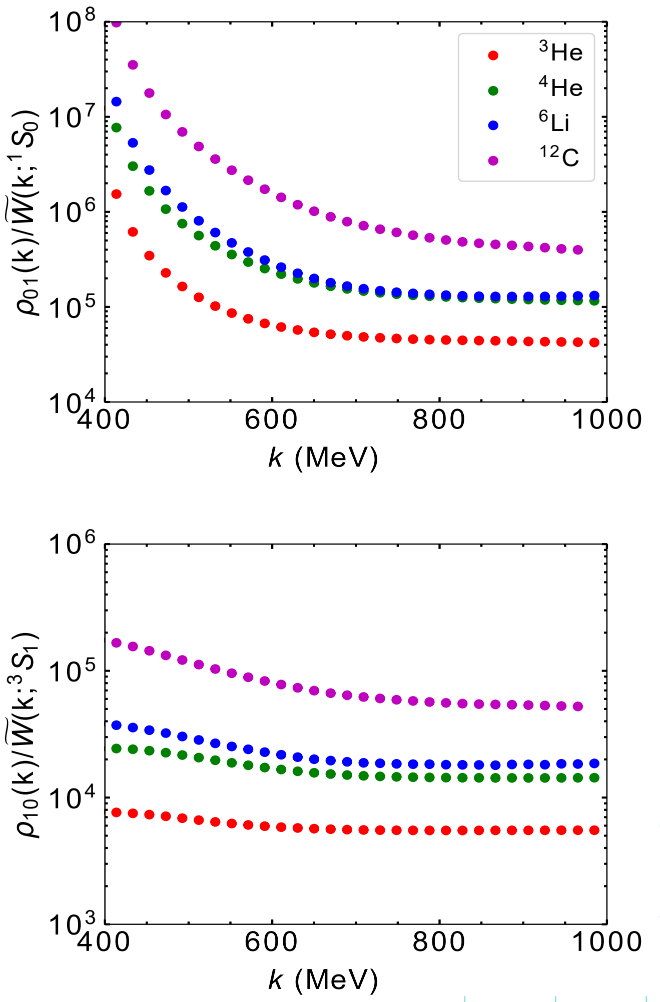
$$\rho_{01}(k) = \widetilde{W}_0^{(2N)}(k;{}^1S_0)\langle A|\mathcal{O}_0({}^1S_0)|A\rangle + \cdots ,$$

$$\rho_{10}(k) = \widetilde{W}_0^{(2N)}(k;{}^3S_1)\langle A|\mathcal{O}_0({}^3S_1)|A\rangle + \cdots .$$

VMC

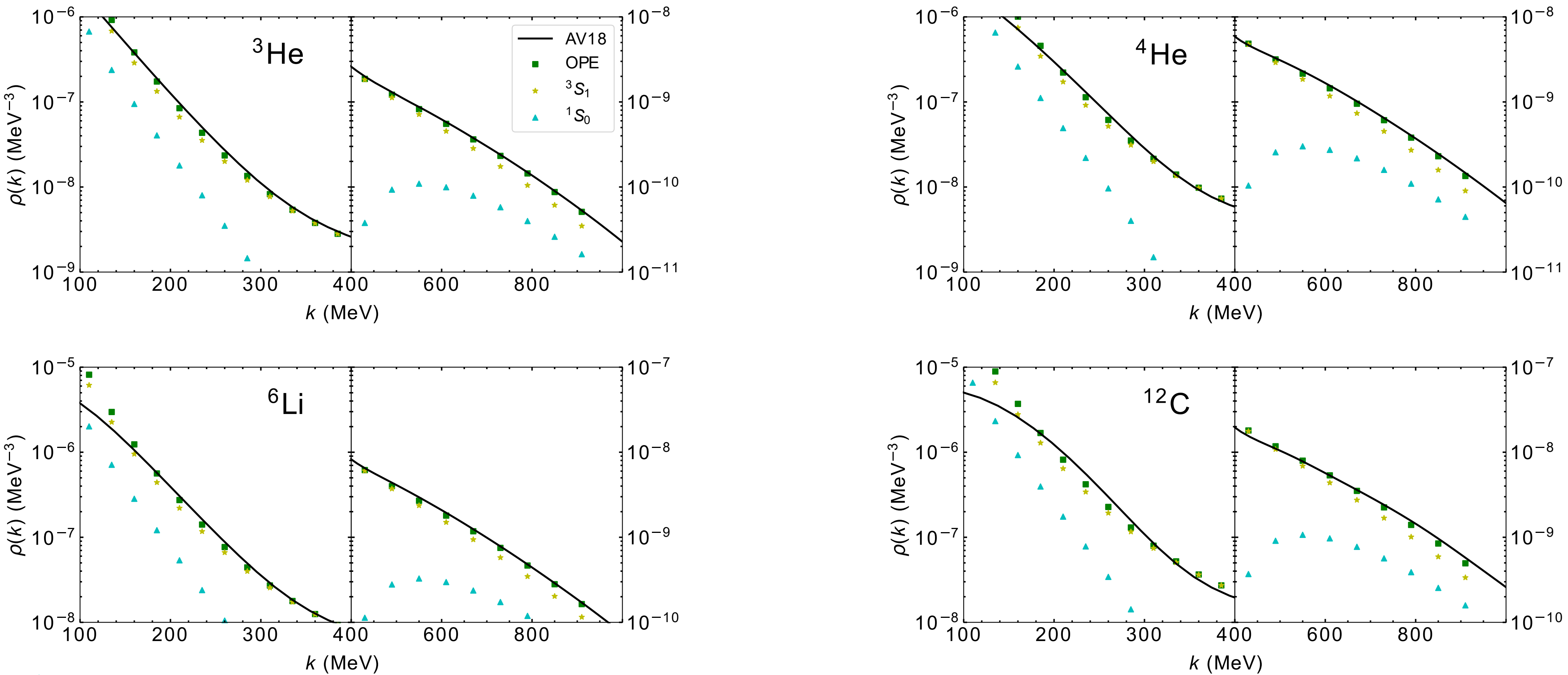
Extracted from two-body scattering

Output for the single-nucleon OPE after the normalization transformation

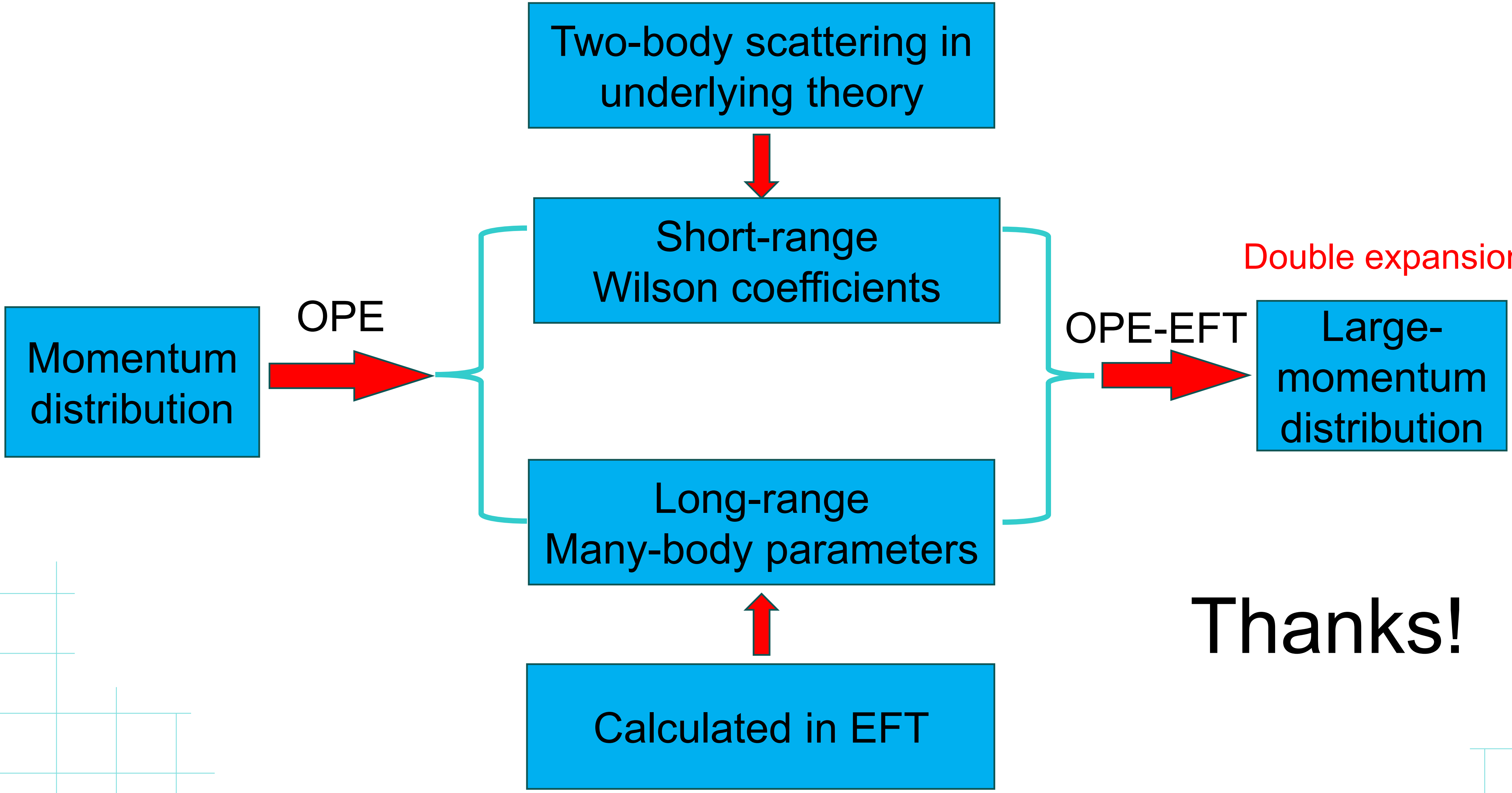


Large-momentum distribution of nuclei

$$\rho(k) = \widetilde{W}_0(k; {}^1S_0)\langle A|\mathcal{O}_0({}^1S_0)|A\rangle + \widetilde{W}_0(k; {}^3S_1)\langle A|\mathcal{O}_0({}^3S_1)|A\rangle + \cdots$$



Summary



Thanks!