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How the quark mass and temperature influence the hadron spectrum

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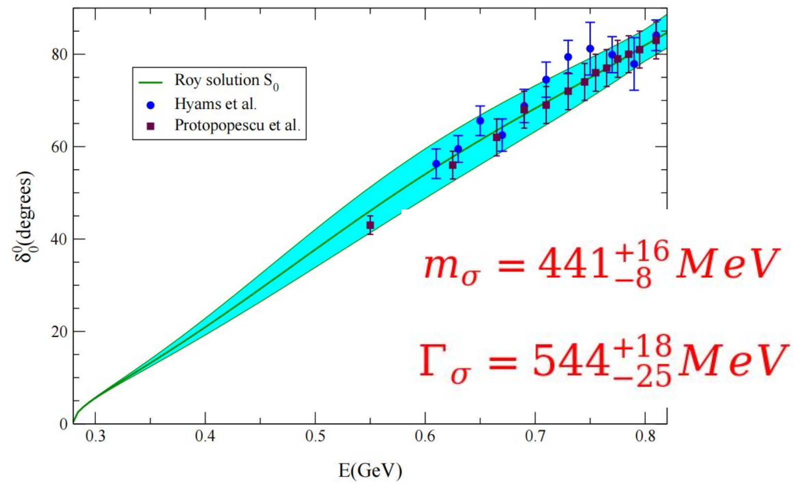
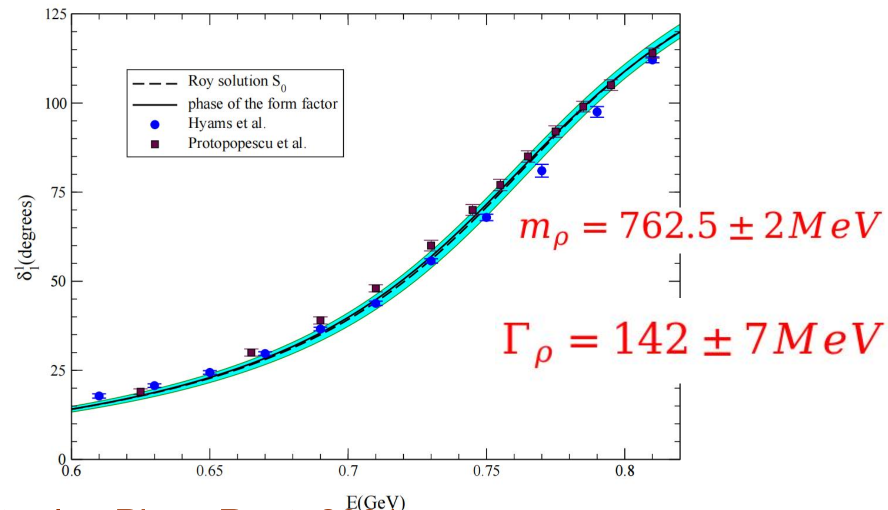
Outline:

1. Background & Introduction

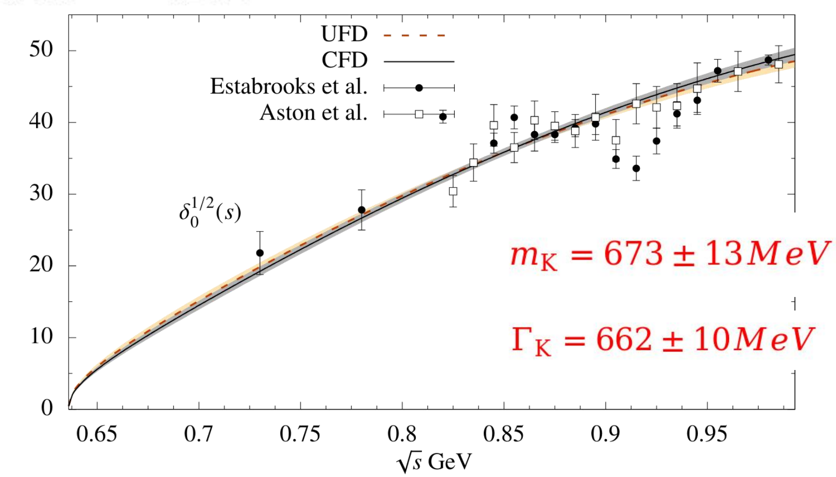
$\pi\pi \to \pi\pi$

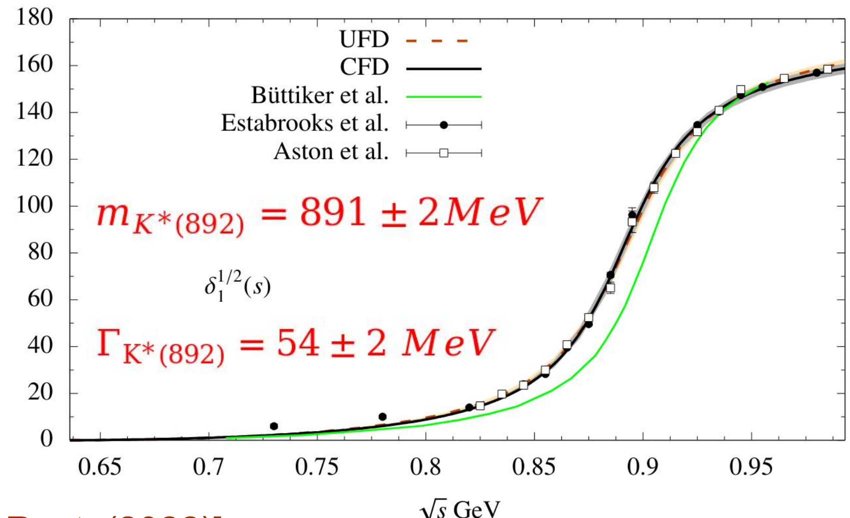




[Ananthanarayana, Colangelob, Gasserc, Leutwyler, Phys. Rept. 2001;
Caprini, Colangelo and Leutwyler, Phys. Rev. Lett. 2006]

$\pi K \to \pi K$

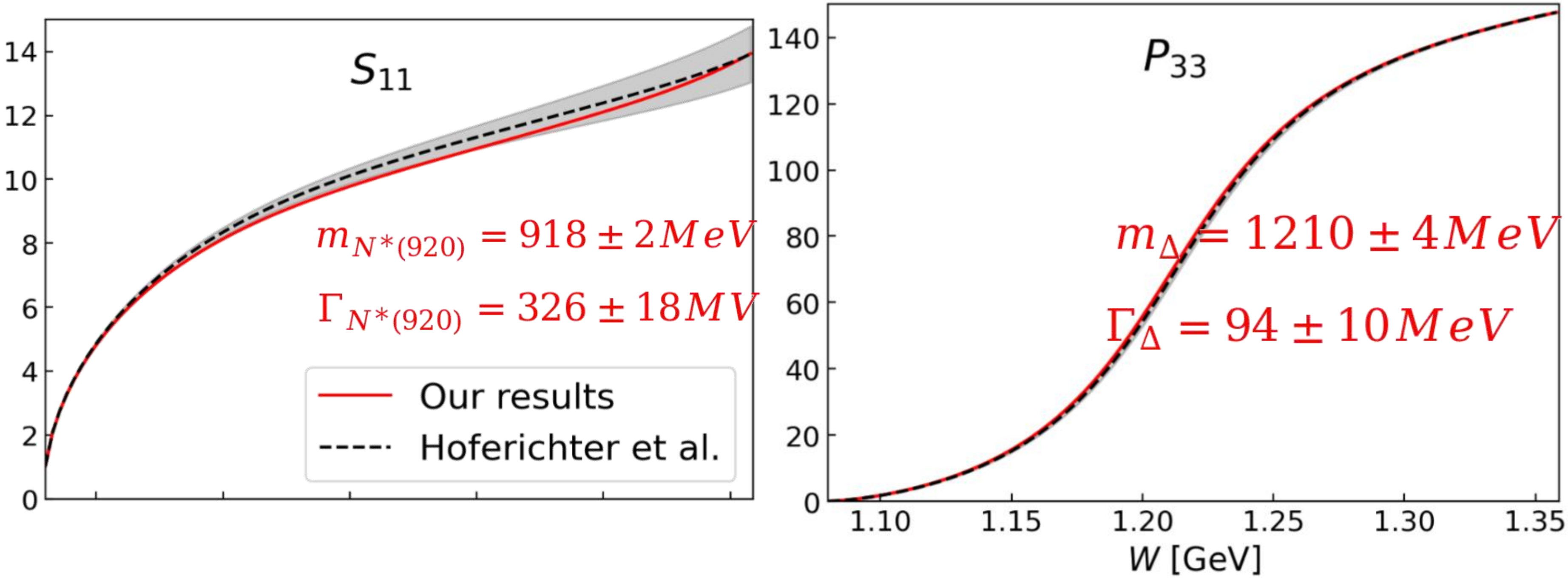




[José R. Peláez and Arkaitz Rodas, Phys. Rept. (2022)]

$\pi N \to \pi N$

X.H. Cao, QZL, and H.Q. Zheng, JHEP (2022)



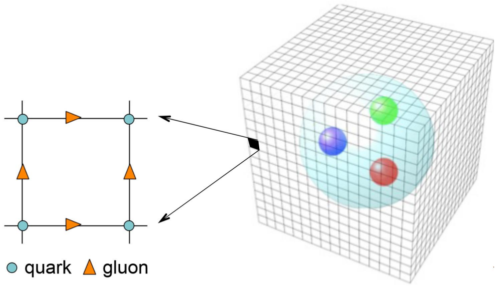
- m_π increasing ⟹ σ, ρ , $K^*(892)$ become bound states
- temperature increasing ⟹ restoration of chiral symmetry and degeneracy of the parity doublet (σ and π)

2. Quark masses dependences

Quark masses

- lattice QCD

$$\det\left(1+\mathrm{i}\mathcal{T}\left(1+\mathrm{i}\mathcal{M}^{\mathrm{P}}\right)\right)=0$$



- chiral perturbation theory + unitarization methods

★ SU(2)χPT, SU(3)χPT, BχPT

★ K-matrix, N/D method, PKU, IAM, ...

- models

★ O(N) model: solvabel (leading order)

★ linear σ model (renormalizable) + unitarization methods

$$\mathcal{L}=\bar{\Psi}_0 i\gamma^\mu\partial_\mu\Psi_0-g_0\bar{\Psi}_0\left(\sigma_0+i\gamma_5\vec{\tau}\cdot\boldsymbol{\pi}_0\right)\Psi$$
$$+\frac{1}{2}\left(\partial_\mu\sigma_0\partial^\mu\sigma_0+\partial_\mu\boldsymbol{\pi}_0\cdot\partial^\mu\boldsymbol{\pi}_0\right)-\frac{\mu_0^2}{2}\left(\sigma_0^2+\boldsymbol{\pi}_0^2\right)-\frac{\lambda_0}{4!}\left(\sigma_0^2+\boldsymbol{\pi}_0^2\right)^2+C\sigma_0$$

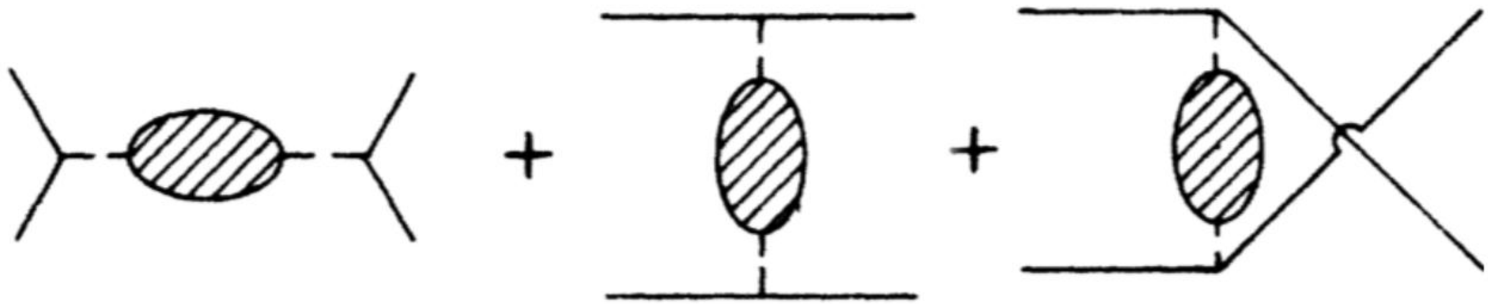
- Roy/Roy-Steiner equations
  - ★ unitarity, analyticity and crossing symmetry
  - ★ experimental data for physical pion mass
  - ★ lattice data for unphysical pion mass

Finite temperature

- thermal field theory
  - ★ imaginary formalism, real-time formalism, thermofiled dynamics
- chiral perturbation theory + unitarization methods
  - ★ SU(2)χPT, SU(3)χPT
  - ★ thermal unitarization conditions
  - ★ K-matrix, N/D method, IAM, ...
- models
  - ★ O(N) model, NJL model

- O(N) model leading order ππ amplitud [Coleman,et.al.,PRD10,2491]

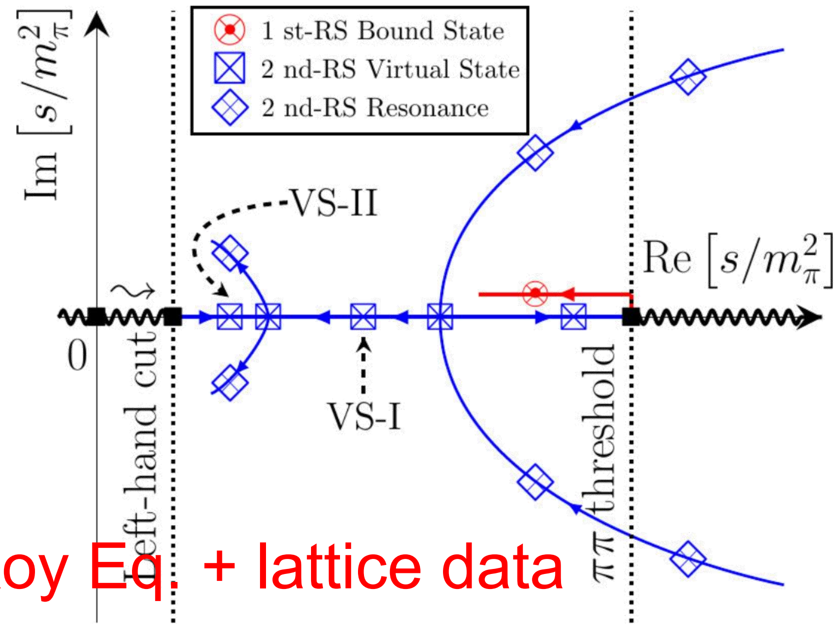
$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a - \frac{1}{2}\mu_0^2 \phi_a \phi_a - \frac{\lambda_0}{8N}(\phi_a \phi_a)^2 + \alpha \phi_N$$



- O(N) amplitude for I=J=0:

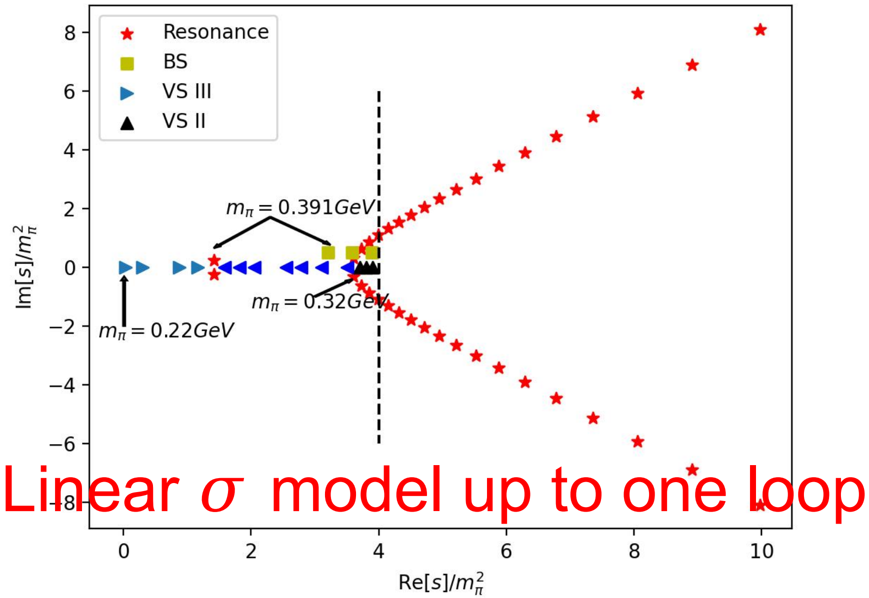
$$\mathcal{T}_{00}^{LO}(s) = \frac{iND_{\tau\tau}(s)}{32\pi} = \frac{N}{32\pi}\mathcal{A}(s) \qquad \mathcal{A}(s) = \frac{m_\pi^2 - s}{(s - m_\pi^2)B(s, m_\pi, M) - f_\pi^2/N}$$

- σ pole : $(s - m_\pi^2)B^{II}(s, m_\pi, M) - \frac{f_\pi^2}{N} = 0$



Roy Eq. + lattice data

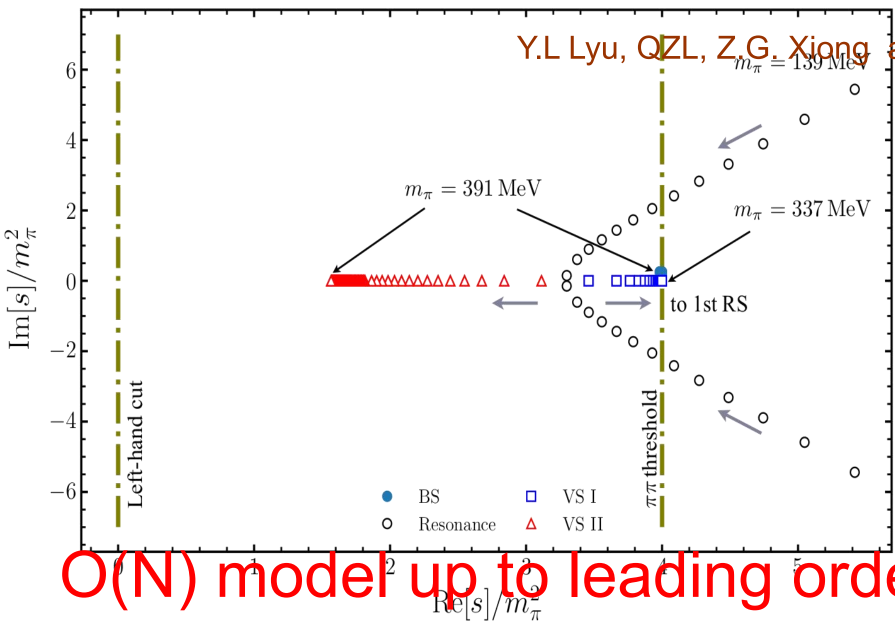
X.H. Cao,QZL, Z.H. Guo and H.Q. Zheng,Phys. Rev. D(2023)



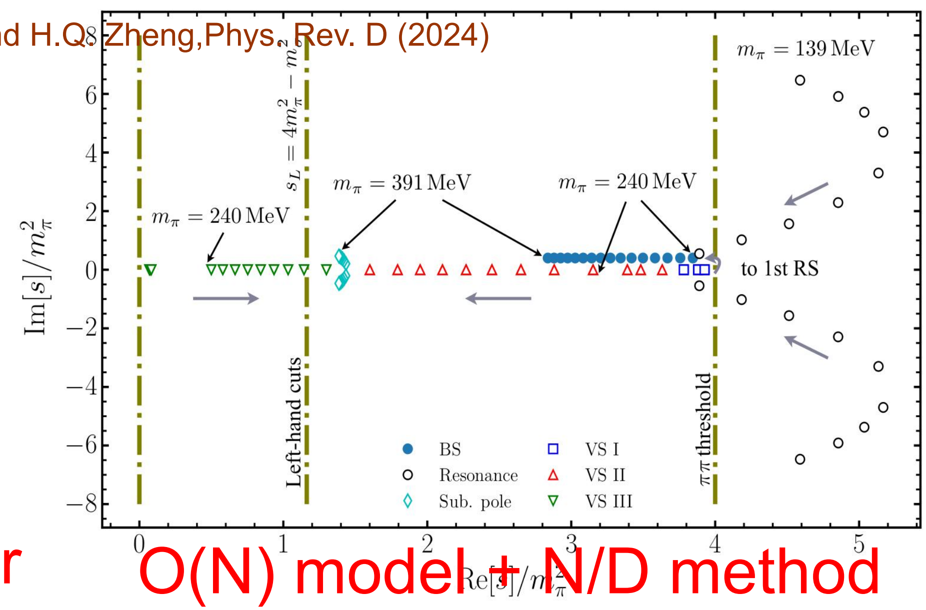
Linear σ model up to one loop

QZL, Z.G. Xiao and H.Q. ZhengChin.Phys.C (2025)

Y.L Lyu, QZL, Z.G. Xiong and H.Q. Zheng,Phys. Rev. D (2024)

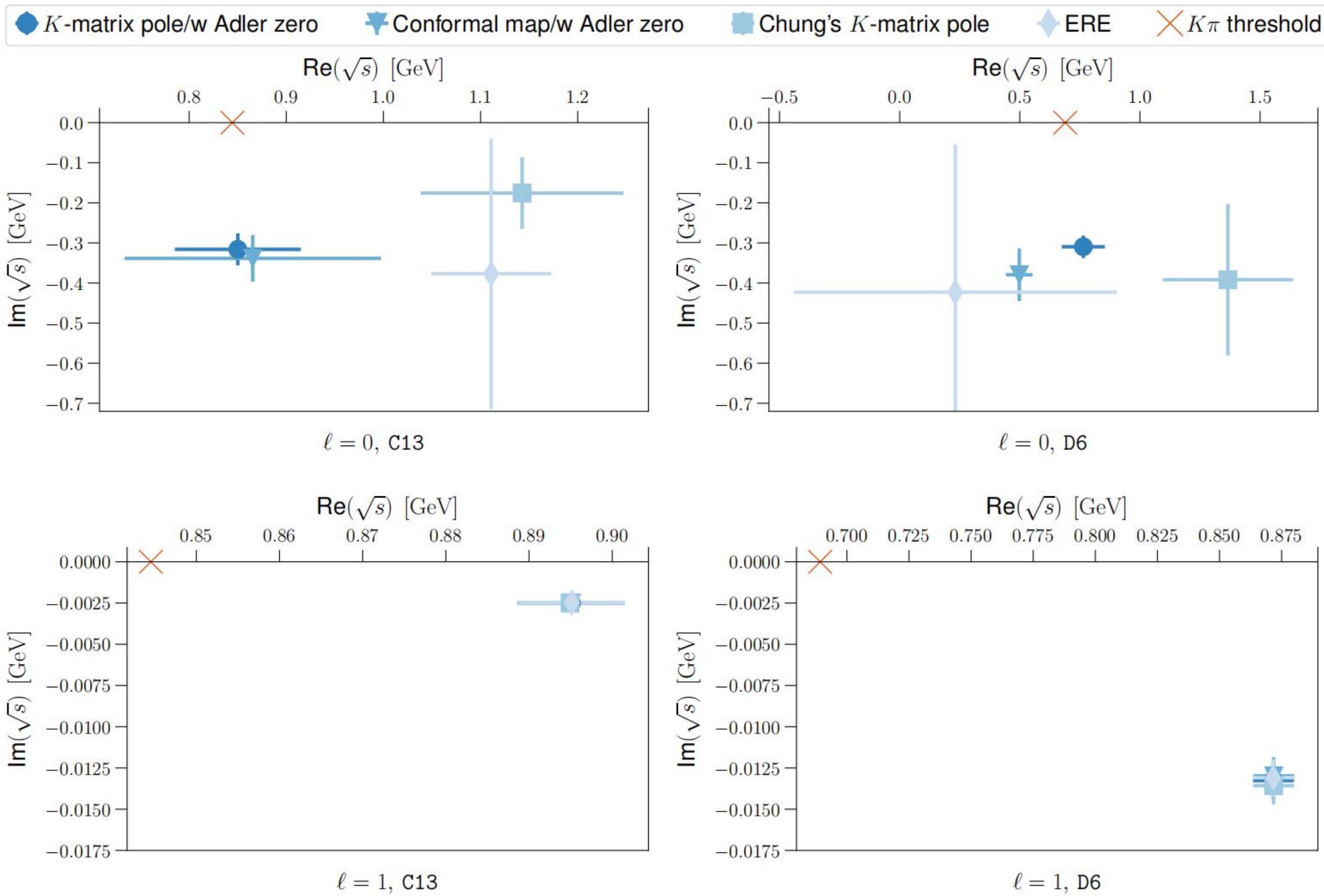


O(N) model up to leading order

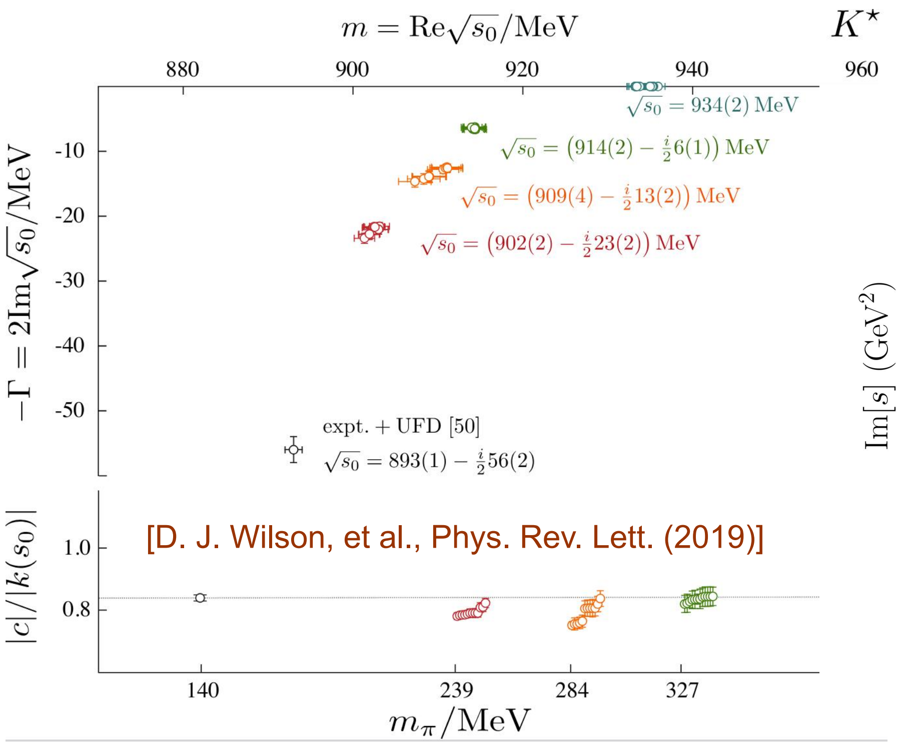


O(N) model + N/D method

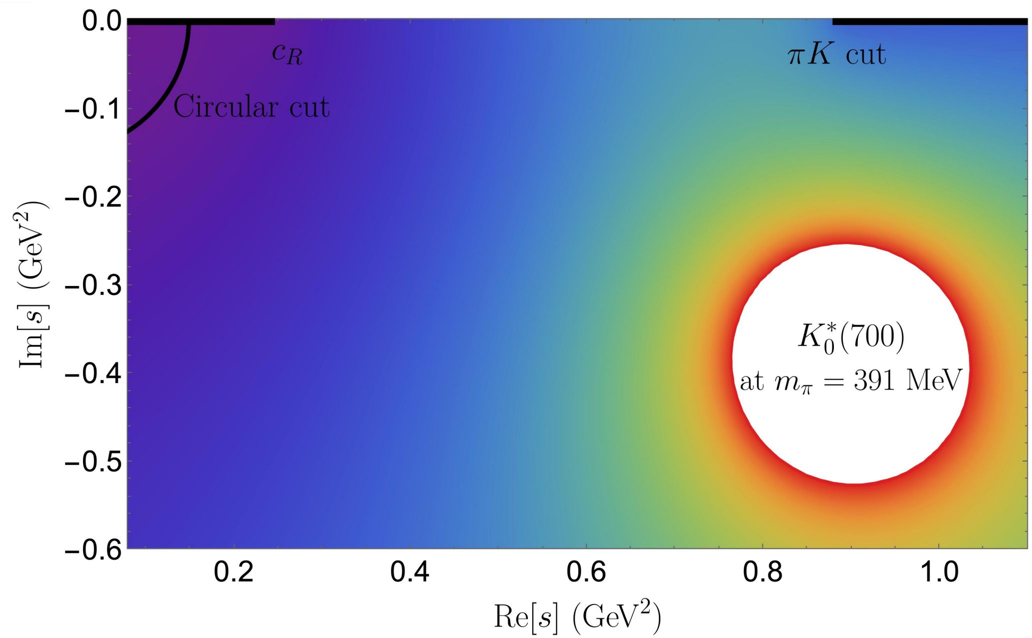
Lattice researches on πK scatterings

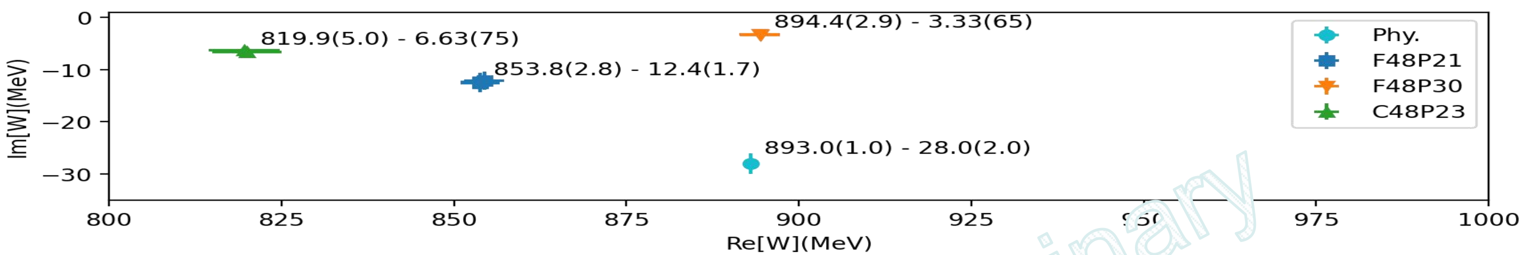


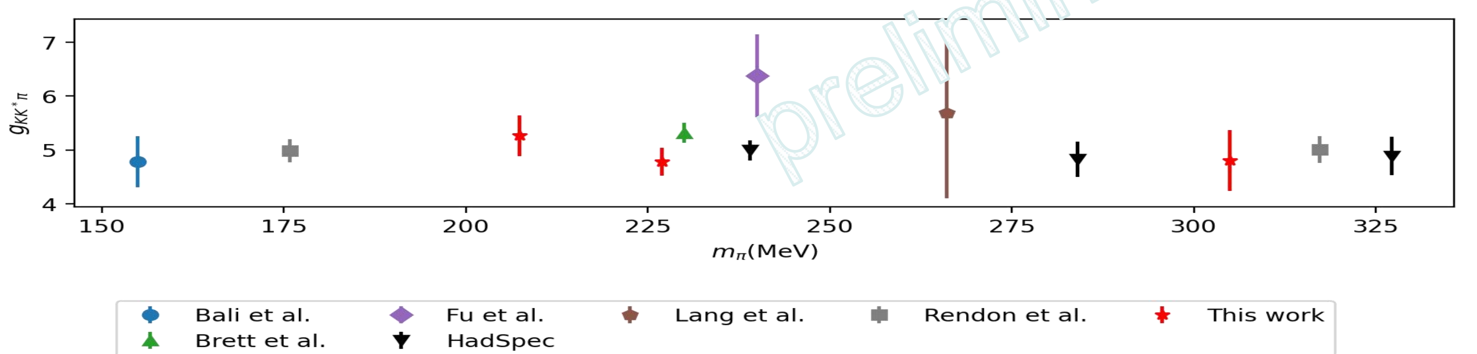
[G.Rendon, et al., Phys. Rev. D (2020)]



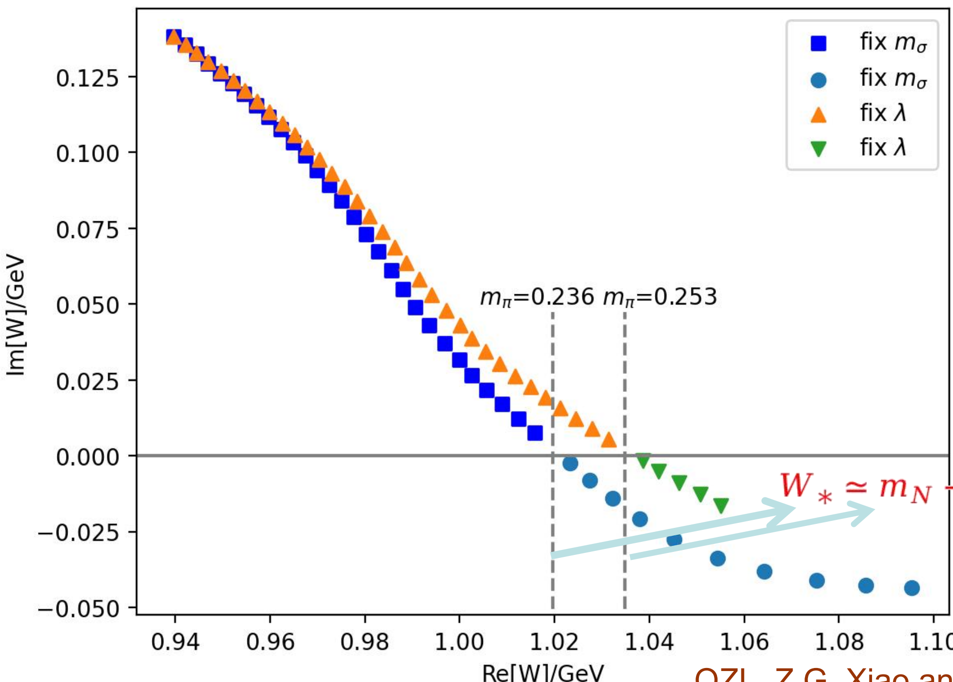
[X.H. Cao, F.K. Guo, Z.H. Guo and QZL, Phys.Rev.D 112 (2025)]



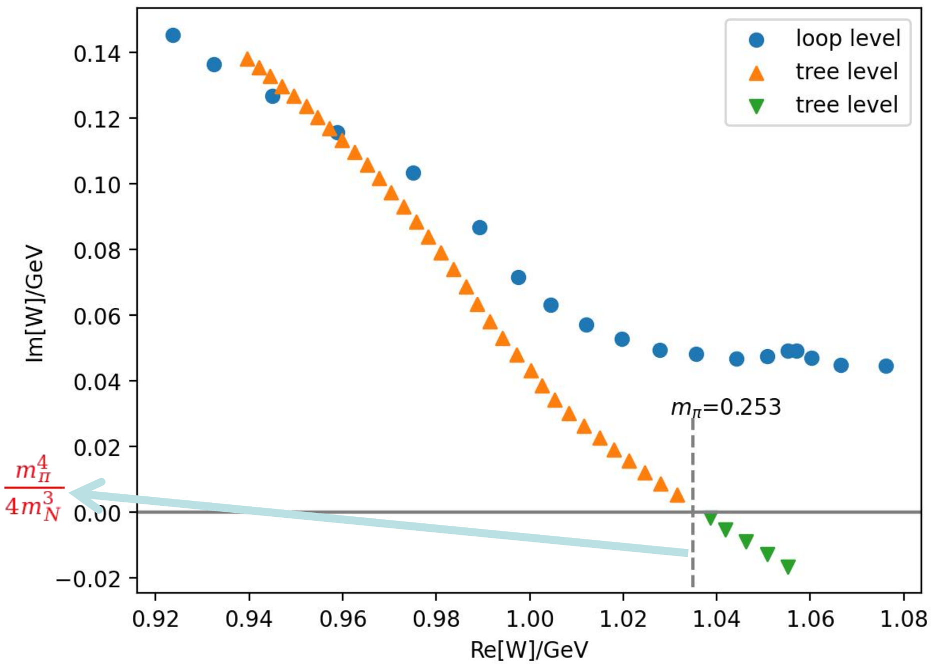




Linear σ model on $N^*(920)$ in πN scatterings

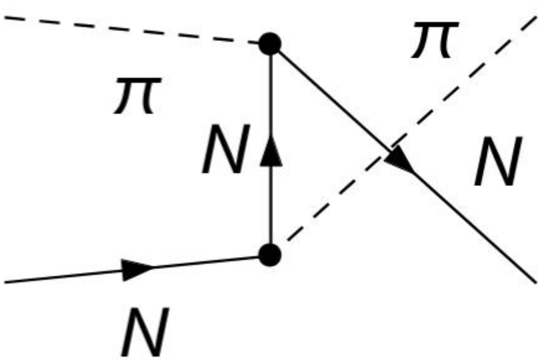


$$W_* \simeq m_N - \frac{m_\pi^4}{4m_N^3}$$



QZL, Z.G. Xiao and H.Q. ZhengChin.Phys.C (2025)

$$(m_N - m_\pi - W)(m_N + m_\pi - W)[m_N(m_N - W)(m_N + W)^2 - m_\pi^4] = 0, \quad W \equiv \sqrt{s}$$



partial-wave projection

$$\ln\left(\frac{s(s-c_R)}{m_N^2(s-c_L)}\right)$$

$$\ln\left(\frac{s(s-c_R)}{m_N^2(s-c_L)}\right) \pm 2\pi i$$

adjacent Riemann sheets

3. Finite temperature

Matsubara Fromalism

- Partion function

$$Z = Tre^{-\beta H} =< \phi|e^{-\beta H}|\phi >= \int D\phi e^{-S_E}$$

$$S_E = \int_0^{\beta} d\tau \int d^3 x L_E$$

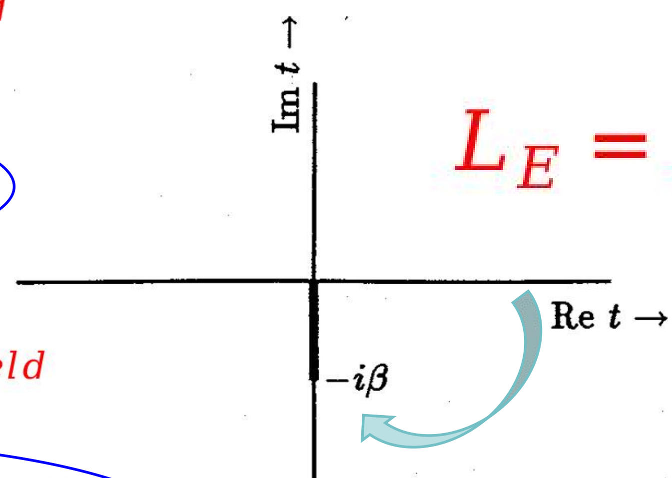
- Boundary conditions

★ $\phi(0, x) = \phi(\beta, x)$, *bosonic field*

$$\omega = \frac{2n\pi}{\beta}, n = 0, \pm 1, \pm 2, \cdots$$

★ $\psi(0, x) =- \psi(\beta, x)$, *fermionic field*

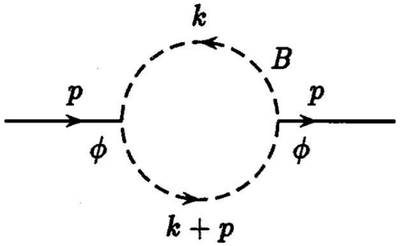
$$\omega = \frac{(2n+1)\pi}{\beta}, n = 0, \pm 1, \pm 2, \cdots$$



$$L_E = L(t \to - i\tau)$$

Matsubara frequencies

$$\int \frac{dk^0}{(2\pi)} \to \frac{1}{\beta} \sum_n$$



$$\Pi^T(\boldsymbol{p}, p^0) = \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\frac{4n^2\pi^2}{\beta^2}+\boldsymbol{k}^2+M^2} \frac{1}{(\frac{2n\pi}{\beta}+p^0)^2+(\boldsymbol{k}+\boldsymbol{p})^2+M^2}$$

$$\frac{1}{\beta}\sum_n f(\omega_n; K) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega f(\omega, K) + i\sum_{Im\omega_i>0} \frac{R_f(\bar{\omega}_i)}{e^{-i\beta\bar{\omega}_i}-1} - i\sum_{Im\bar{\omega}_i<0} \frac{R_f(\bar{\omega}_i)}{e^{i\beta\bar{\omega}_i}-1}$$

center-of-mass frame

$$\omega_k = \sqrt{k^2 + m_\pi^2}$$

$$\Pi^T(s) = \Pi(s) + \int_0^\infty \frac{dk\,k^2}{8\pi^2\omega_k^2} n_B(\omega_k) \left(\frac{1}{\sqrt{s}/2+2\omega_k} - \frac{1}{\sqrt{s}/2-2\omega_k}\right)$$

$$n_B(E) = \frac{1}{e^{E/T}-1}$$

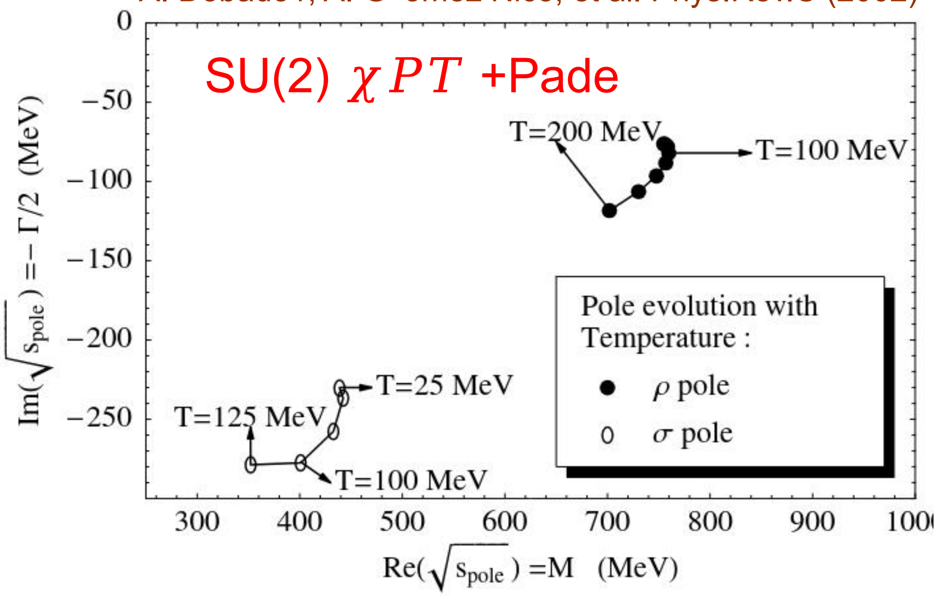
particle distribution function

- Thermal unitarity contion

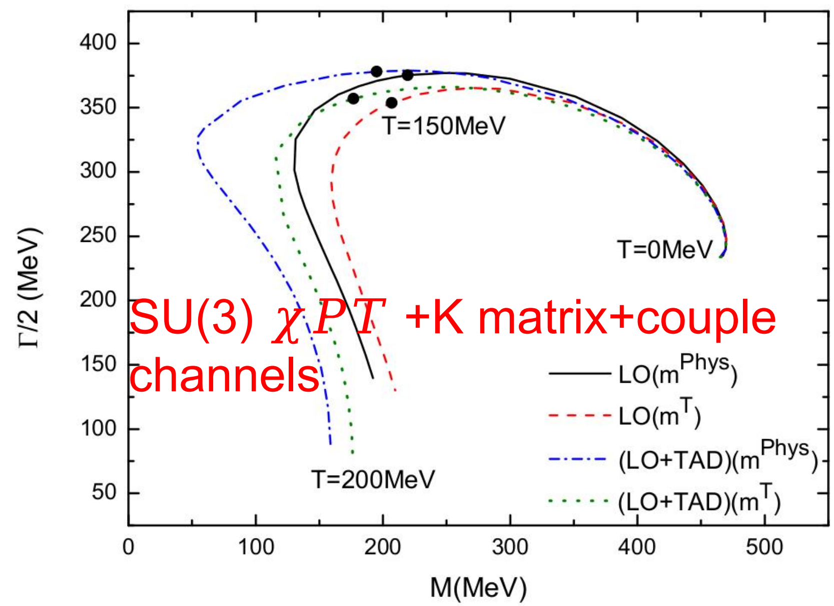
$$ImM(s) = \rho(s, m, m)(1 + 2n_B(\sqrt{s}/2))$$ equal masses

$$ImM(s) = \rho(s, m_1, m_2)(1 + n_B(E_1) + n_B(E_2))$$ unequal masses

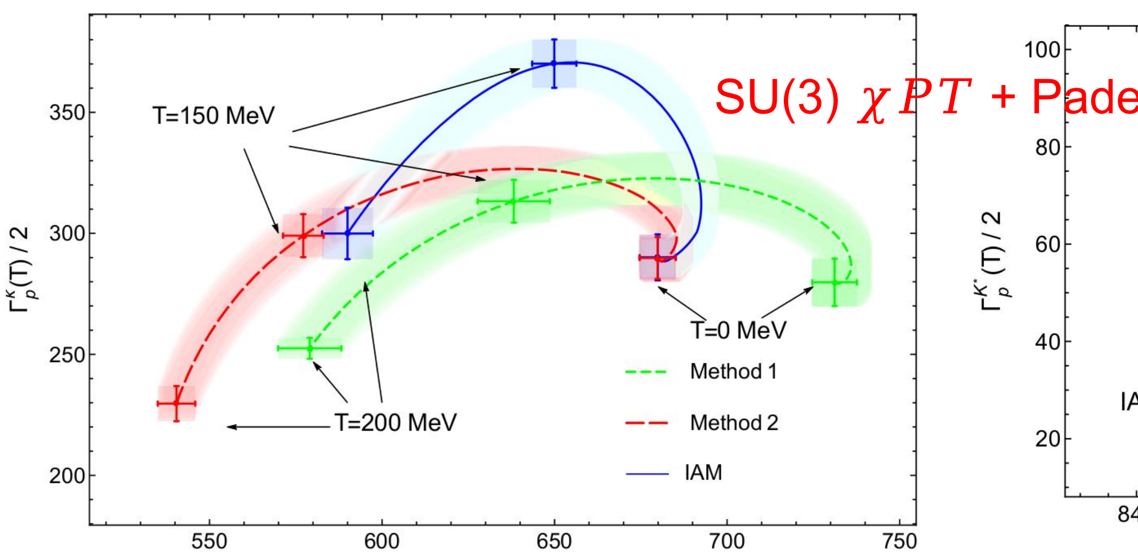
A. Dobado1, A. G´omez Nico, et al. Phys.Rev.C (2002)

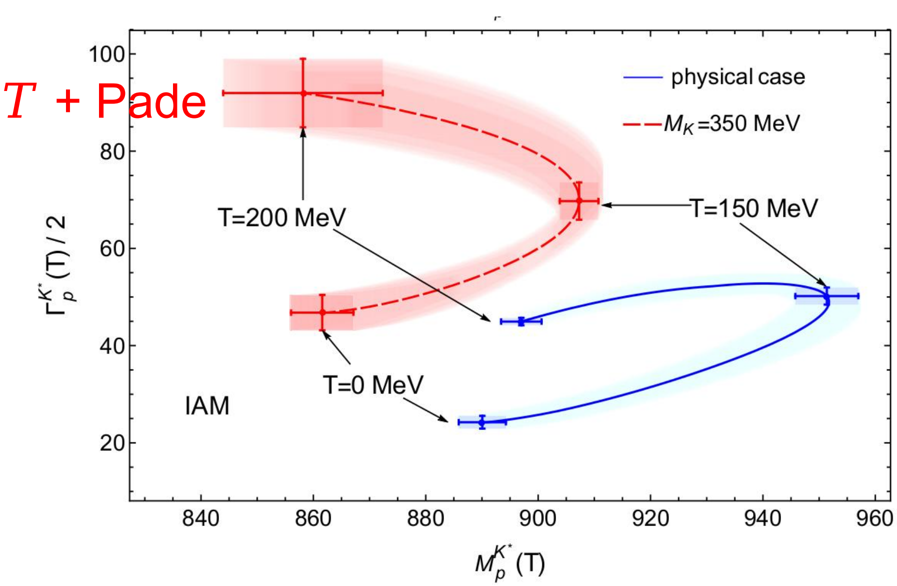


R. Gao, Z.H. Guo and J.Y. Pang Phys.Rev.D (2019)



A. Gómez Nicola, J. Ruiz de Elvira, and A. Vioque-Rodríguez, JHEP 08 (2023)

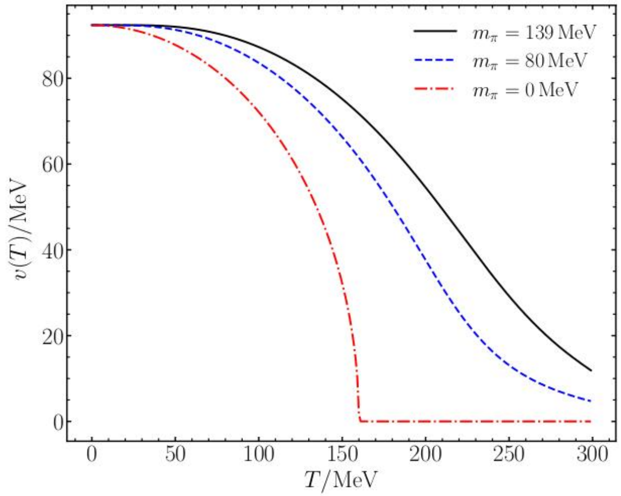


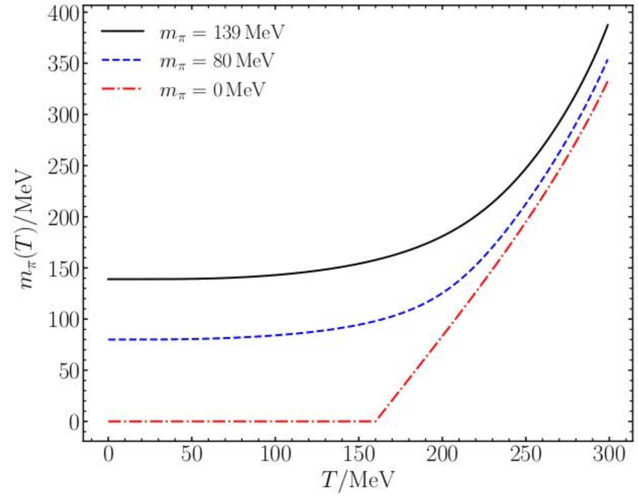


O(N) model

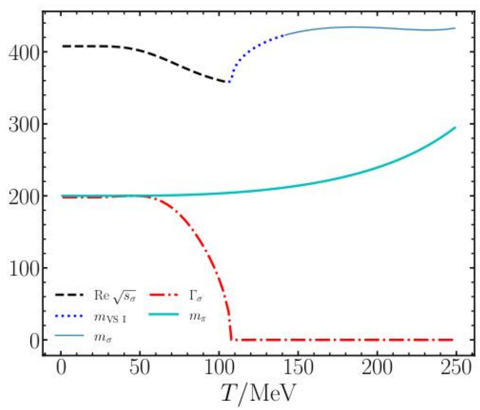
$$v^2(T) = f_\Pi + \frac{N}{16\Pi^2}\left(m_\Pi \log\frac{m_\Pi^2}{M^2} - m_\Pi^2(T)\log\frac{m_\Pi^2(T)}{M^2}\right) - N A^{T\neq 0}(m_\Pi^2(T))$$

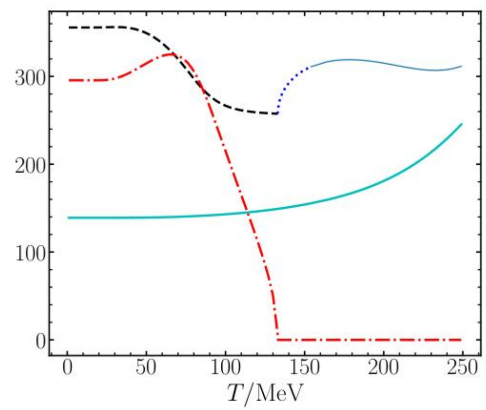
$$\alpha = v(T) m_\Pi^2(T) \qquad A^{T\neq 0}(m_\Pi^2(T)) = \int_0^\infty \frac{dk}{2\Pi}\frac{k^2 n_B(\omega_k)}{\omega_k}$$

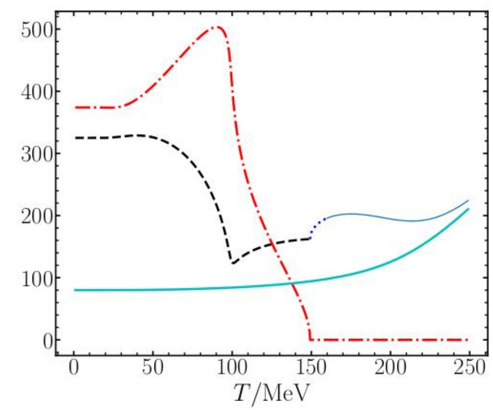




$$\mathcal{T}_{00}^{T}(s) = -\frac{1}{32\Pi}\frac{s - m_\Pi^2(T)}{(s - m_\Pi^2(T))B^T(s, m_\Pi(T), M) - v^2(T)/N}$$







4. Conclusions

1. Studying the dependence on quark masses provides a new perspective on hadron properties.

2. The hadron spectrum features a parity doublet with the restoration of chiral symmetry as temperature increases.

3. The dependence of hadronic properties on quark mass and temperature deserves further study.


Thanks for
your attention