第十届手征有效场论研讨会@南京师范大学

Productions of two-body DK and three-body $DK\bar{D}_s$ molecules in B meson decays

Reporter: Ming-Zhu Liu (刘明珠)

Collaborators: Li-Sheng Geng (耿立升)
Tian-Wei Wu(吴天伟)

Based on: Phys. Rev. D **109** (2024) 056014
Phys. Rev. Lett. **135** (2025) 031902

2025-10-20

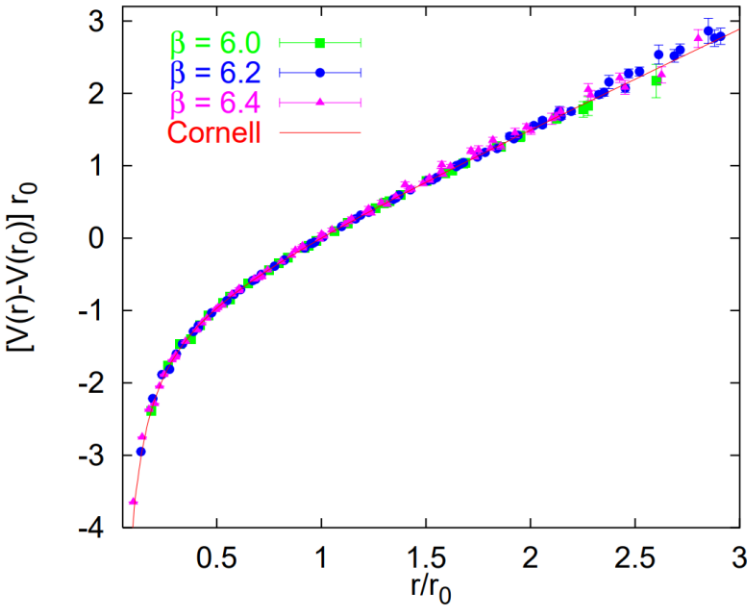
Outline

- Exotic states and hadron-hadron interactions
- Productions of the DK molecule in B decays
- Productions of the $DK\bar{D}_s$ molecule in B decays
- Summary

Hadron states for strong interaction

➢ Cornell model $V_0(r) = -\frac{4\alpha_s}{3r} + \sigma r$

E. Eichten et al., Phys. Rev. Lett. 34 (1975) 369-372



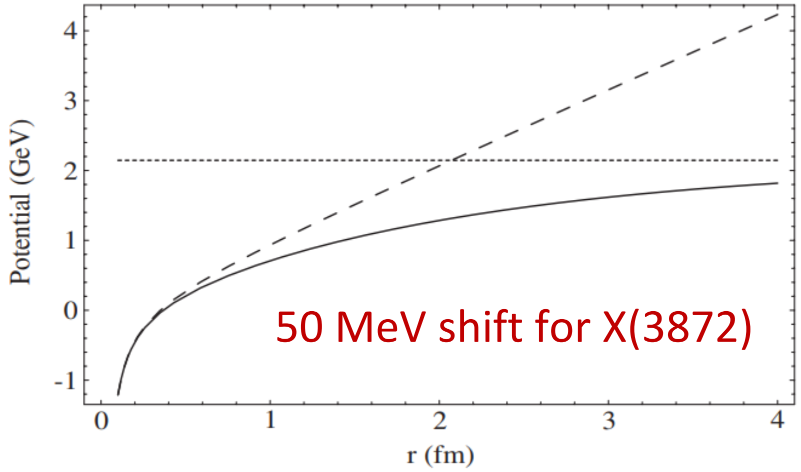
Gunnar S. Bali, Phys. Rept. 343 (2001) 1-136

➢ Godfrey-Isgur model

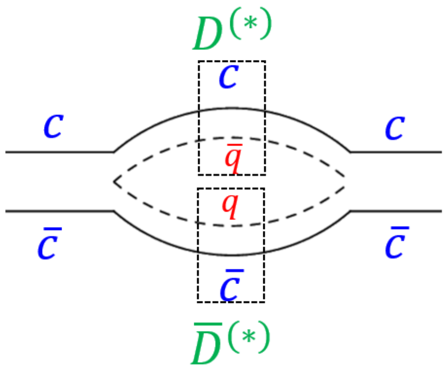
- Spin-Spin Term
- Spin-Orbit Term
- Tensor Term

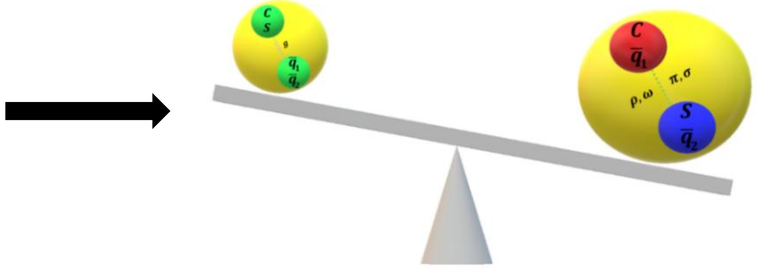
S. Godfrey et al., Phys. Rev. D32 (1985) 189-231

➢ Screen potential $br \to V^{\rm scr}(r) = \frac{b(1-e^{-\mu r})}{\mu}$



Bai-Qing Li, et al., Phys. Rev. D 79 (2009) 094004





The complex non-perturbative dynamics of QCD give rise to diverse structural formations in hadronic matter

Exotic state $D_{s0}^{*}(2317)$

➢ Experimental measurements of $D_{s0}^{*}(2317)$

$e^+e^- \to D_s^+\pi^0 X$

$e^+e^- \to D_s^{*+}(D_{s0}^{*-} \to D_s^-\pi^0)$

Inclusive process

2003

2017

Exclusive process

$B \to D_{s0}^{*}(\to D_s^+\pi^0)\bar{D}$

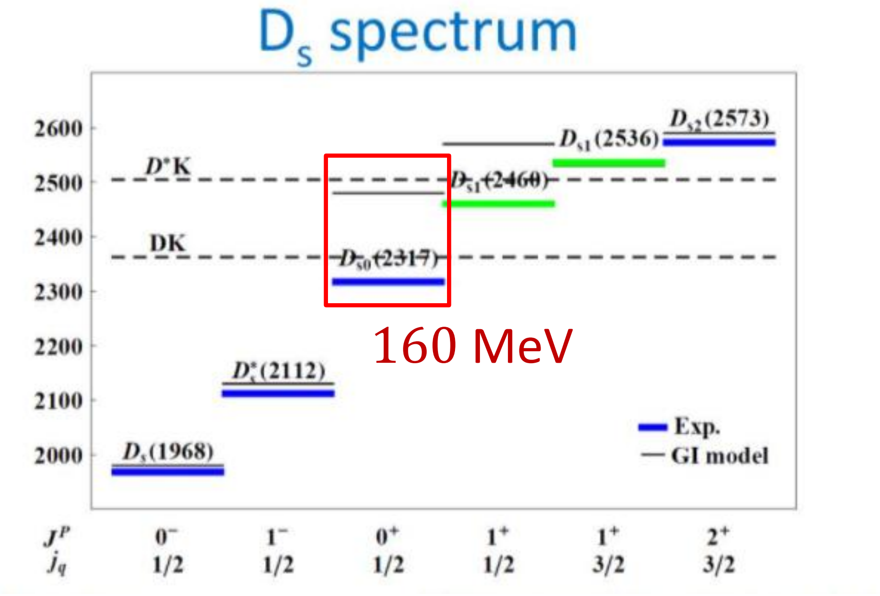
- **Mass and width**

$$D_{s0}^{*}(2317) = 2317.8 \pm 0.5 + \frac{i}{2} < 3.8$$

- **Decay Channel**

$$D_{s0}^{*-} \to D_s^-\pi^0$$

➢ Large mass deviation



➢ Branching fraction

$$Br(D_{s0}^{*-} \to D_s^-\pi^0) \approx 1$$

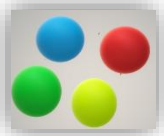
BESIII Collaboration, Phys. Rev. D 97, 051103 (2018)

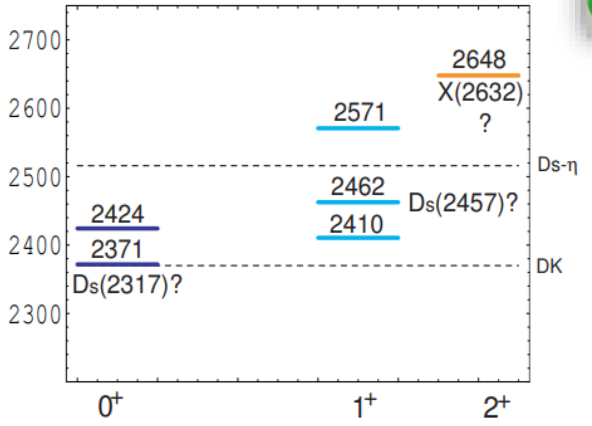
Disfavor the $D_{s0}^{*}(2317)$ as excited $\bar{c}s$ state

$D_{s0}^{*}(2317)$ is regarded as an exotic state!

Theoretical interpretations for the $D_{s0}^*(2317)$

➢ Compact tetraquark





Maiani et al., Phys.Rev.D 71 (2005) 014028

➢ Molecular interpretation

✓ **Mass and mass splitting**

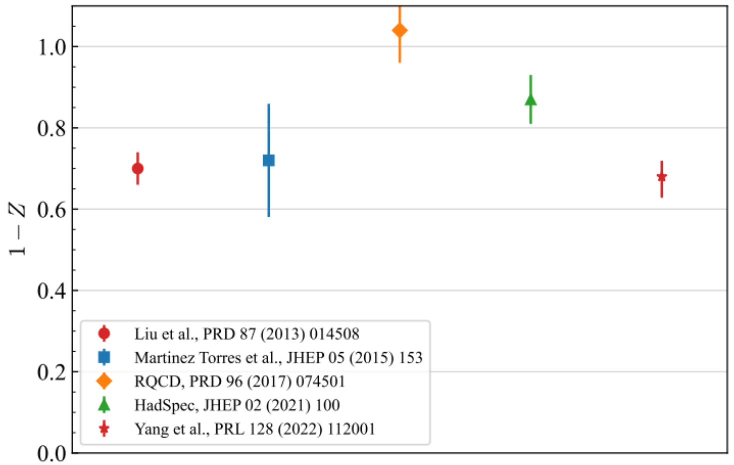
$D_{s0}(2317)$	$I(J^P)=0(0^+)$	DK
$D_{s1}(2460)$	$I(J^P)=0(1^+)$	D^*K

ChPT Guo et al., Phys.Lett.B 641 (2006) 278-285

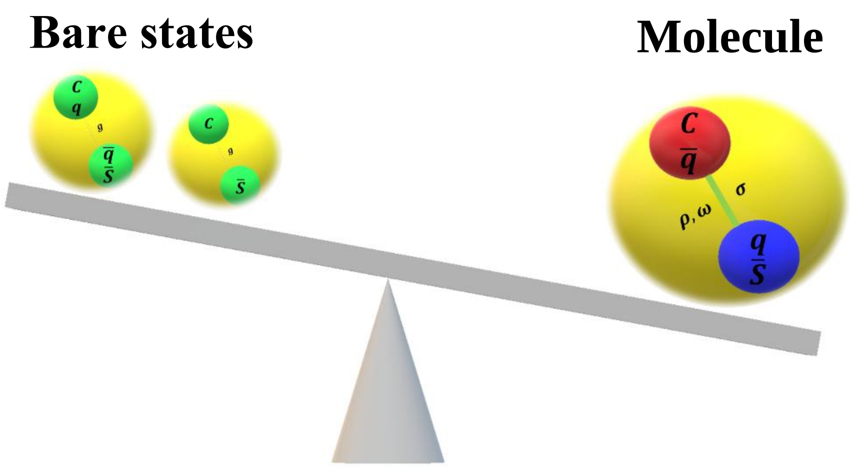
Lattice QCD Liu et al., Phys.Rev.D 87 (2013) 014508

✓ **Branching fraction** $Br(D_{s0}^{*-} \to D_s^- \pi^0) \approx 1$

➢ Mixture of molecular and other component



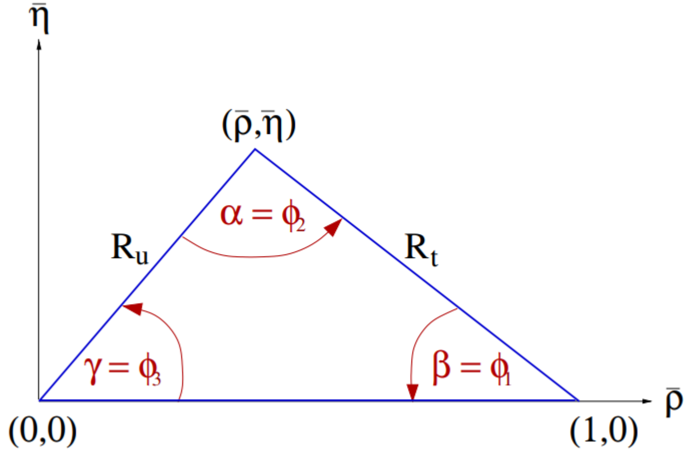
Guo, PoS LATTICE2022 (2023) 232

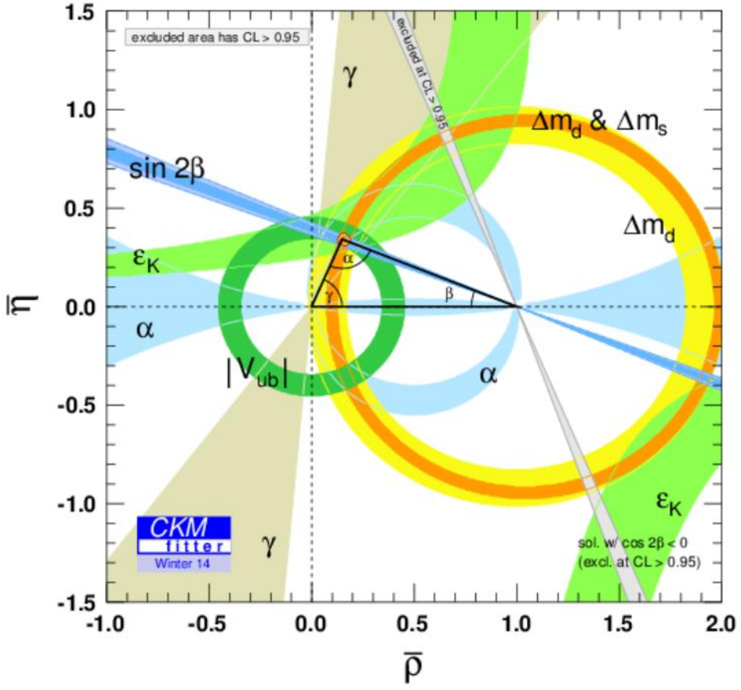


Molecular component more than 70%

Heavy flavor Physics

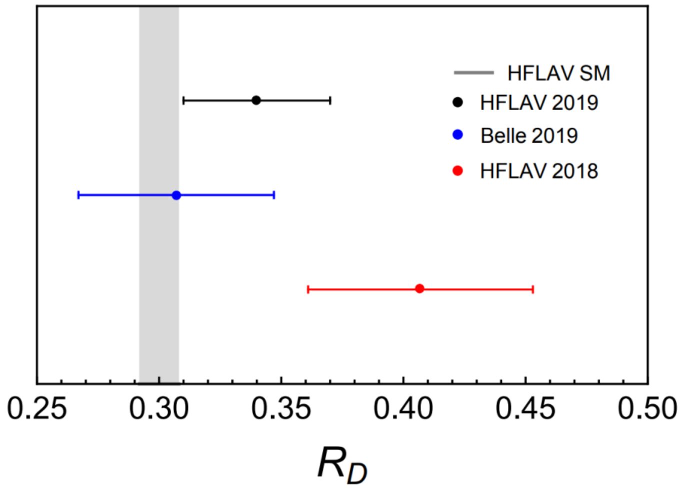
➢ Precise test for Standard Model

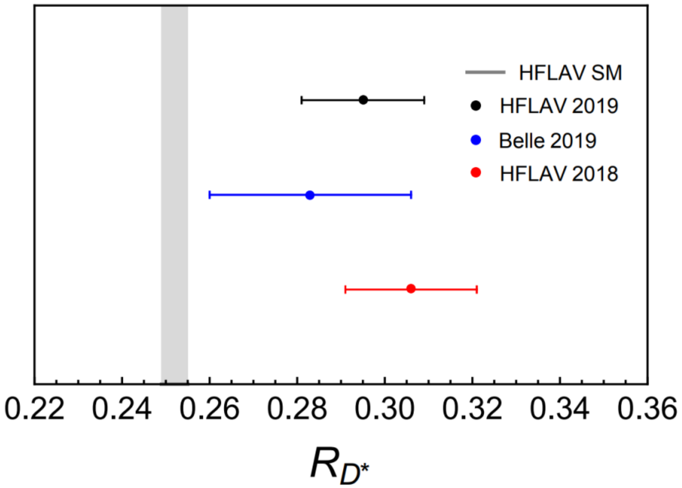




➢ Search for New Physics

$$R^{SM}_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)}\tau\bar{\nu})}{\Gamma(B \to D^{(*)}\ell\bar{\nu})}, \qquad (\ell = \mu, e)$$





Productions of exotic states in b-flavored hadron decays

- **A lot of exotic states were discovered in b-flavored hadron decays**

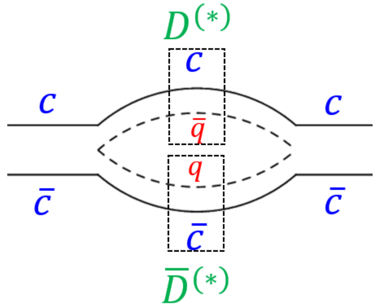
Decay modes	New states	Decay modes	New states
$\Xi_b \to J/\psi \Lambda K$	$P_{cs}(4459)$	$\Lambda_b \to J/\psi p K$	$P_c(4312)/P_c(4440)/P_c(4457)$
$\Lambda_b \to D p \pi$	$\Lambda_c(2940)$	$B^+ \to D_s{}^+ \pi^0 \bar{D}^0$	$D_{s0}(2317)$
$B_s \to J/\psi p \bar{p}$	$P_c(4337)$	$B^+ \to D_s{}^{*+} \pi^0 \bar{D}^0$	$D_{s1}(2460)$
$B \to J/\psi \Lambda \bar{p}$	$P_{cs}(4338)$	$B^+ \to D^- K^+ D^+$	$X_0(2900)/X_1(2900)$
$B^+ \to D_s{}^+ \pi^+ D^-$	$T_{c\bar{s}0}(2900)^{++}$	$B^+ \to J/\psi \phi K^+$	$X(4140)/X(4274)$
$B^0 \to D_s{}^+ \pi^- \bar{D}^0$	$T_{c\bar{s}0}(2900)^0$	$B^+ \to J/\psi \phi K^+$	$X(4500)/X(4700)$
$B^+ \to D^0 \bar{D}^{*0} K^+$	$X(3872)$	$B^+ \to J/\psi K^+ \phi$	$Z_{cs}(4000)/Z_{cs}(4220)$
$B^+ \to D_s{}^+ D_s{}^- K^+$	$X(3960)$	$B^0 \to \psi' \pi^\mp K^\pm$	$Z_c(4430)$
$B^+ \to J/\psi \omega K^+$	$X(3915)$	$B^0 \to \chi_{c1} \pi^\mp K^\pm$	$Z_c(4051)/Z_c(4248)$
$B^+ \to K^+ K^- K^+$	$f_0(980)$	$B^0 \to J/\psi \pi^\mp K^\pm$	$Z_c(4200)$
$B^+ \to \pi^0 \pi^0 K^+$	$f_0(980)$		

- **Providing rich physical observables to study the exotic states**
  - **Branching fraction**
  - **Invariant mass distribution**
  - **CP violation**
  - **Angular distributions**

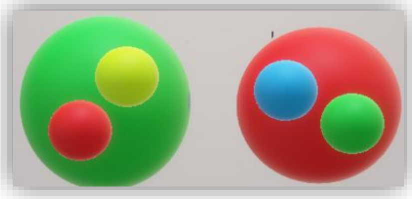
Hadronic molecule with new configurations

➢ Two-body hadronic molecules

L=1

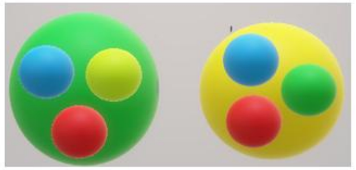


Meson- Meson

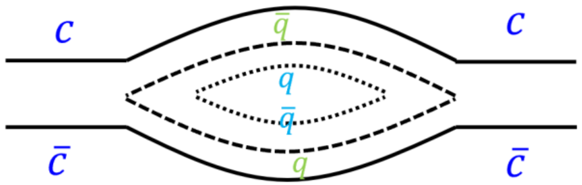


➢ Three-body hadronic molecules

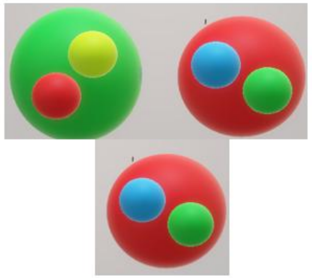
Baryon-Antibaryon



L=0/2

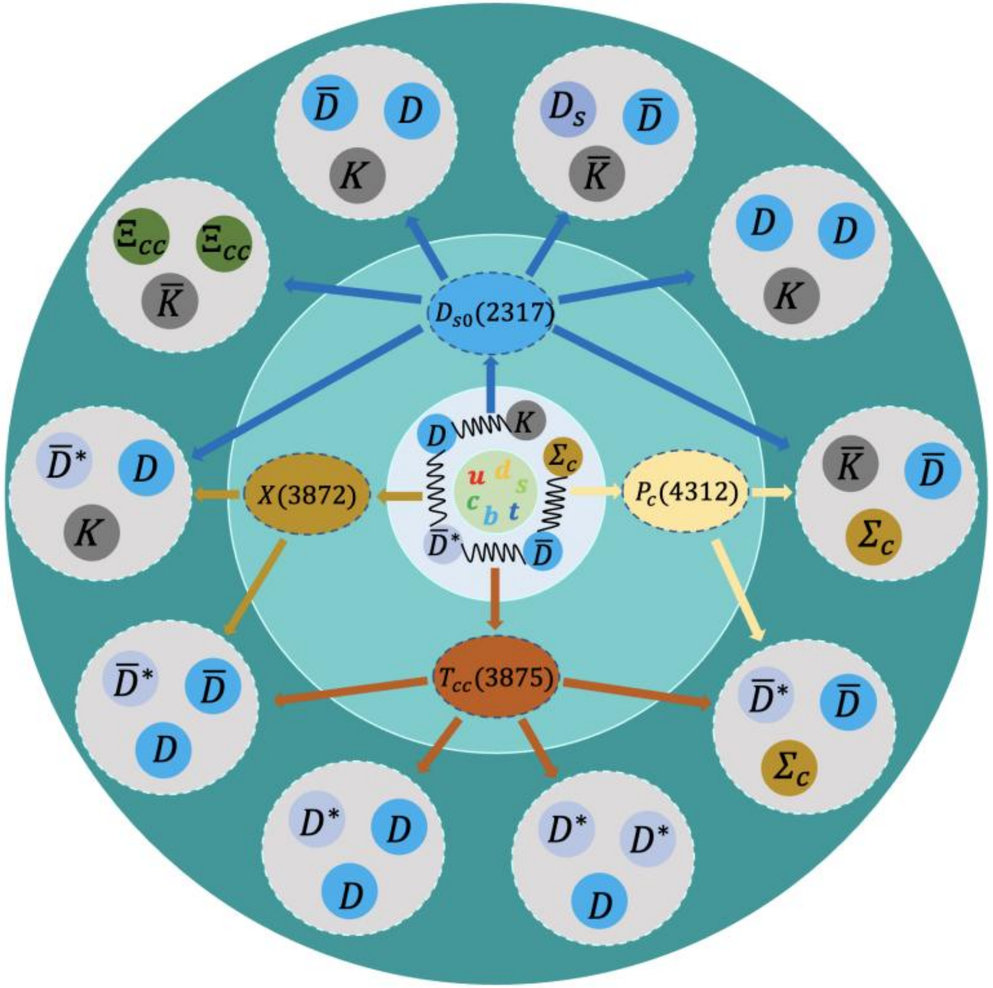


Three-body meson



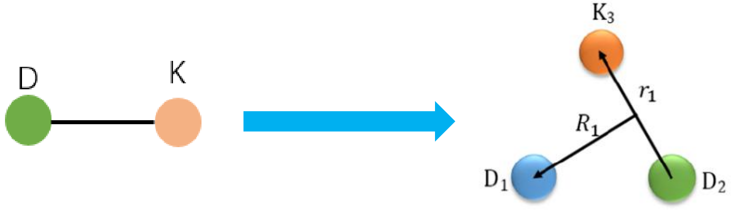
Three-body molecules can verify hadron-hadron interaction

➢ **Existence of three-body hadronic molecules**



Wu et al., Sci.Bull. 67 (2022) 1735-1738

➢ **D and K meson as the building block**



Liu et al., Phys. Rept. 1108 (2025) 1-108

- ✓ **New configuration states generated by hadron-hadron interactions**
- ✓ **The existence of three-body molecules can verify hadron-hadron interaction**

Experimental search for the three-body molecule

➢ $J^{PC} = 0^{--}$ $\bar{D}_s DK$ molecule

✓ **Contain charge parity**

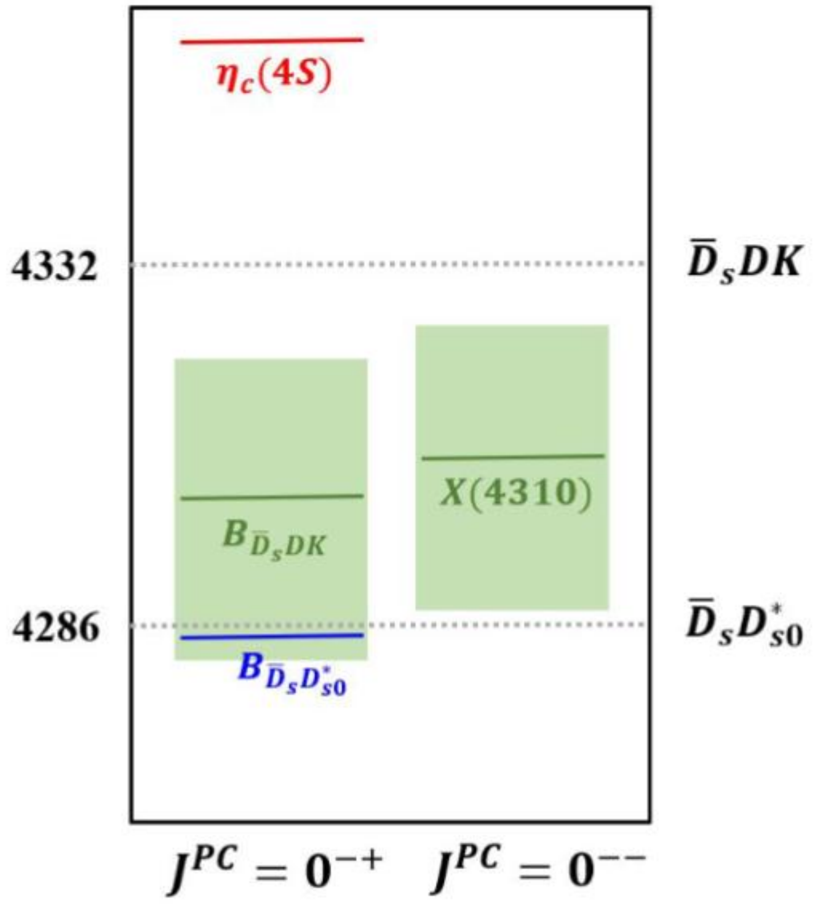
- This system with neutral charge

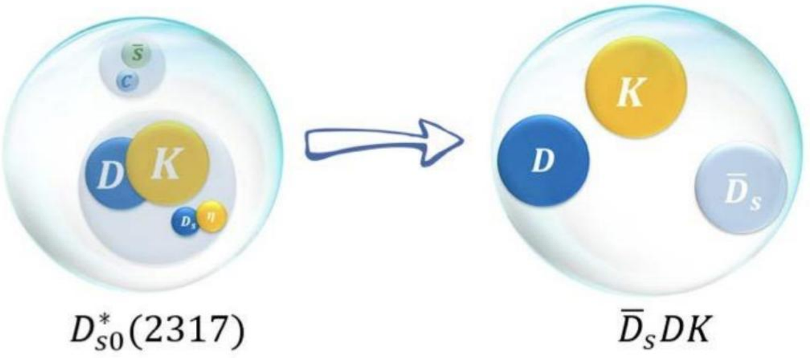
✓ **Impact of other non-perturbative effect is small**

- No mixture of conventional $c\bar{c}$
- Two-body subsystem $J^{PC} = 0^{--}$ $\bar{D}_s D_{s0}^*$ can not bind

✓ **Can be detected by experiment**

- Can be produced at e^+e^- and pp collisions





A good candidate of three-body molecule!

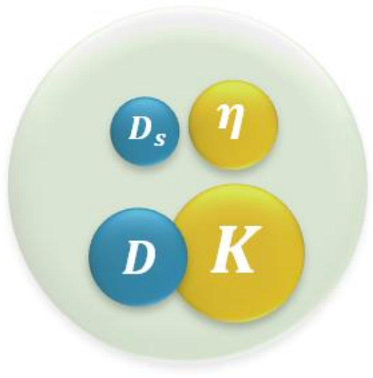
Outline

- Exotic states and hadron-hadron interactions
- Productions of the DK molecule in B decays
- Productions of the $DK\bar{D}_s$ molecule in B decays
- Summary

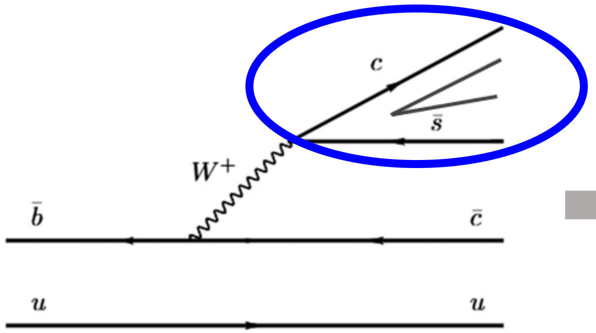
Productions of $D_{s0}^{*}(2317)$ in B decays

➢Explain these branching fractions

Decay modes	PDG 10^{-3}	BarBar 10^{-3}
$B^+ \to \bar{D}^0 D_{s0}^{*+}(2317)$	$0.80^{+0.16}_{-0.13}$	$1.0 \pm 0.3 \pm 0.1$
$B^0 \to D^- D_{s0}^{*+}(2317)$	$1.06^{+0.16}_{-0.16}$	$1.8 \pm 0.4 \pm 0.3$
$B^+ \to \bar{D}^{*0} D_{s0}^{*+}(2317)$	$0.90^{+0.70}_{-0.70}$	$0.9 \pm 0.6 \pm 0.2$
$B^0 \to D^{*-} D_{s0}^{*+}(2317)$	$1.50^{+0.60}_{-0.60}$	$1.5 \pm 0.4 \pm 0.2$

$D_{s0}^{*}(2317) \longrightarrow$ 

Productions of the DK molecule in B decays



Naïve factorization approach

$$\mathcal{A}\left(B^- \to D_{s0}^{*-}\bar{D}^{(*)0}\right) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_1 \boxed{\left\langle D_{s0}^{*-}|(s\bar{c})|0\right\rangle}\left\langle \bar{D}^{(*)0}|(c\bar{b})|B^-\right\rangle$$

- **Form factors**

$$\langle \bar{D}^{*0}|(c\bar{b})|B^+\rangle = \epsilon_\alpha^* \left\{ -g^{\mu\alpha}(m_{\bar{D}^{*0}} + m_{B^+})A_1\left(q^2\right) + P^\mu P^\alpha \frac{A_2\left(q^2\right)}{m_{\bar{D}^{*0}} + m_{B^+}} + i\varepsilon^{\mu\alpha\beta\gamma}P_\beta q_\gamma \frac{V\left(q^2\right)}{m_{\bar{D}^{*0}} + m_{B^+}} + q^\mu P^\alpha \left[\frac{m_{\bar{D}^{*0}} + m_{B^+}}{q^2}A_1\left(q^2\right) - \frac{m_{B^+} - m_{\bar{D}^{*0}}}{q^2}A_2\left(q^2\right) - \frac{2m_{\bar{D}^{*0}}}{q^2}A_0\left(q^2\right)\right]\right\}$$

$$\langle \bar{D}^0|(c\bar{b})|B^+\rangle = \left[P^\mu - \frac{m_{B^+}^2 - m_{\bar{D}^0}^2}{q^2}q_\mu\right]F_1(q^2) + \frac{m_{B^+}^2 - m_{\bar{D}^0}^2}{q^2}q_\mu F_0(q^2)$$

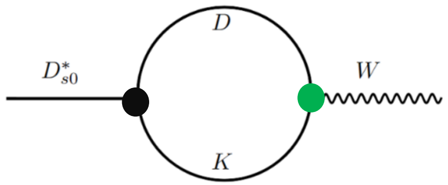
- **Decay constants**

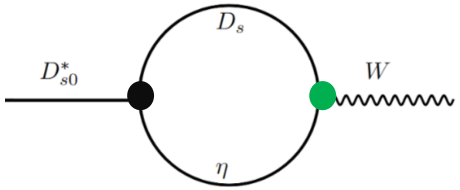
$$\boxed{\langle D_{s0}^{*+}|(s\bar{c})|0\rangle = f_{D_{s0}^{*+}}p^\mu_{D_{s0}^{*+}}}$$

Decay constants characterize the internal structure of exotic state

Calculating decay constant of the $D_{s0}^*(2317)$

➢ Effective Lagrangian approach





$$\boxed{\mathcal{L}_{D_{s0}^*DK} = g_{D_{s0}^*DK} D_{s0}^* DK, \quad \mathcal{L}_{D_{s0}^*D_s\eta} = g_{D_{s0}^*D_s\eta} D_{s0}^* D_s \eta}$$

$$\boxed{\mathcal{L}_{VDK} = f_1^{DK}(0) V^\mu (D\partial_\mu K - \partial_\mu DK), \quad \mathcal{L}_{VD_s\eta} = f_1^{D_s\eta}(0) V^\mu (D_s\partial_\mu \eta - \partial_\mu D_s\eta)}$$

➢ Loop integral

$$\mathcal{A}_a = g_{D_{s0}^*DK} f_1^{DK}(0) \int \frac{d^4q}{(2\pi)^4} \frac{1}{k_1^2 - m_1^2} \frac{1}{k_2^2 - m_2^2} (k_1^\mu - k_2^\mu) \varepsilon_\mu(V),$$

$$\mathcal{A}_b = g_{D_{s0}^*D_s\eta} f_1^{D_s\eta}(0) \int \frac{d^4q}{(2\pi)^4} \frac{1}{k_1^2 - m_1^2} \frac{1}{k_2^2 - m_2^2} (k_1^\mu - k_2^\mu) \varepsilon_\mu(V),$$

$$\langle D_{s0}^{*+} | (s\bar{c}) | 0 \rangle = f_{D_{s0}^{*+}} p^\mu_{D_{s0}^{*+}}$$

Extracting decay constant

$$f_{D_{s0}^*}^{m_1 m_2} = g_{D_{s0}^* m_1 m_2} f_1^{m_1 m_2}(0) \frac{1}{16\pi^2} \int_0^1 dx (2x-1) ln \frac{\Delta^2}{\mu^2}$$

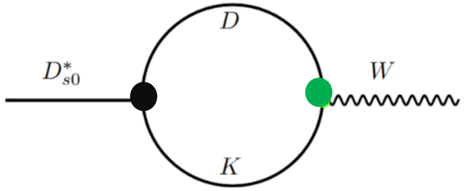
➢ Unknown parameters

- Renormalization energy scale
  - Dimensional Regularization Scheme
- Coupling constants

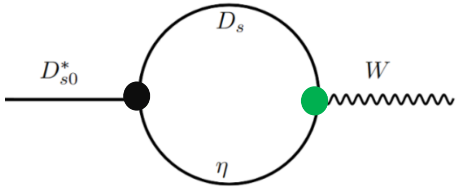
$$\int \frac{d^4k_1}{(2\pi)^4} \frac{k_1^\mu - k_2^\mu}{(k_1^2 - m_1^2)[(p-k_1)^2 - m_2^2]} = \frac{p^\mu}{16\pi^2} \int_0^1 dx (2x-1) ln \frac{\Delta^2}{\mu^2}$$

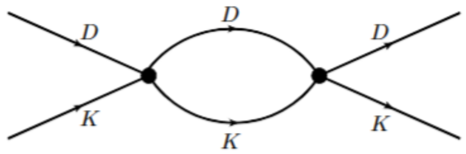
Calculating decay constant of $D_{s0}^*(2317)$

➢ Coupling constants



decay modes	Exp [34]	decay modes	Exp [34]
$D^+ \to \bar{K}^0 e^+ \nu_e$	8.72 ± 0.09	$D_s^+ \to \eta e^+ \nu_e$	2.32 ± 0.08





$$\langle \vec{k}' | T | \vec{k} \rangle = \langle \vec{k}' | T | \vec{k} \rangle + \int \frac{d^3\vec{q}}{(2\pi)^3} \langle \vec{k}' | V | \vec{q} \rangle G(s) \langle \vec{q} | T | \vec{k} \rangle$$

$$V_{DK-D_s\eta}^{J^P=0^+} = \begin{pmatrix} -2C_a & \sqrt{3}C_a \\ \sqrt{3}C_a & 0 \end{pmatrix}$$

$$G(\sqrt{s})^{D_{s0}^*} = \frac{1}{16\pi^2} \int_0^1 dx ln \frac{\Delta^2}{\mu^2}$$

$$g_i g_j = \lim_{\sqrt{s} \to \sqrt{s_0}} (\sqrt{s} - \sqrt{s_0}) T_{ij}(\sqrt{s})$$

$$f_{D_{s0}^*}^{m_1 m_2} = g_{D_{s0}^* m_1 m_2} f_1^{m_1 m_2}(0) \frac{1}{16\pi^2} \int_0^1 dx (2x-1) ln \frac{\Delta^2}{\mu^2}$$

Similar regularization approach

Calculating the decay constant of $D_{s0}^*(2317)$

➢ Strong decay constants of $D_{s0}^*(2317)$

Couplings	$\mu = 1.00$	$\mu = 1.50$	$\mu = 2.00$
$g_{D_{s0}^* DK}$	11.75	11.92	11.95
$g_{D_{s0}^* D_s \eta}$	8.13	7.47	7.32

➢ Decay constant of $D_{s0}^*(2317)$

Decay Constants	$\mu = 1000$	$\mu = 1500$	$\mu = 2000$
$f_{D_{s0}^*(2317)}$	59.36	58.74	58.59

Decay constant of $D_{s0}^*(2317)$ is almost independent on the renormalization energy scale

Production rates of $D_{s0}^*(2317)$ in b-flavored hadron decays

➢ Production rates of $D_{s0}^*(2317)$ in B decays

10^{-3}

Decay modes	Ours	Experiments [29]
$B^+ \to \bar{D}^0 D_{s0}^{*+}(2317)$	0.48	$0.80^{+0.16}_{-0.13}$
$B^+ \to \bar{D}^{*0} D_{s0}^{*+}(2317)$	0.39	$0.90^{+0.70}_{-0.70}$

➢ Decay constant of $D_{s0}^*(2317)$ as a bare state

Hadronic molecule

$$f_{D_{s0}^*(2317)} = 59 \text{ MeV}$$

$$f_{D_{s0}^*(2317)} = 71 \text{ MeV}$$

Physical state

70%

Bare state

$$f_{D_{s0}^*(2317)} = 116 \text{ MeV}$$

Quark model

$$f_{D_{s0}^*(2317)} = 110 \text{ MeV}$$

Phys.Rev.D 54 (1996) 6803-6810

Relativistic Bether-Salpeter method

$$f_{D_{s0}^*(2317)} = 112 \text{ MeV}$$

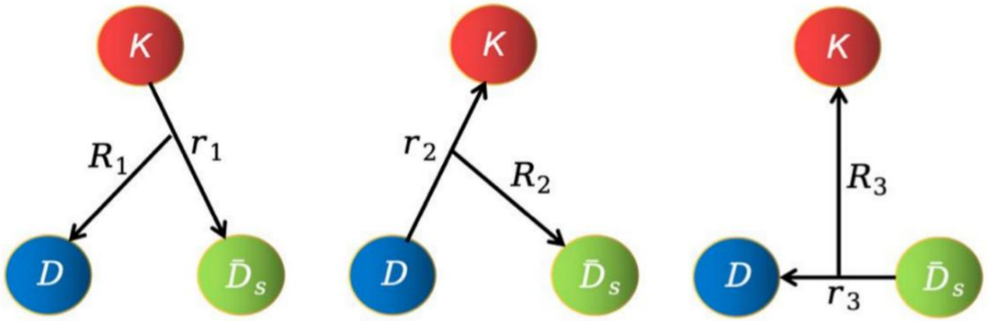
Phys.Lett.B 650 (2007) 15-21

From B decay, the D_{s0}^* can not be explained as a pure hadronic molecule

Outline

- Exotic states and hadron-hadron interactions
- Productions of the DK molecule in B decays
- Productions of the $DK\bar{D}_s$ molecule in B decays
- Summary

Spectrum of the $DK\bar{D}_s$ molecule



- **Wave function of $DK\bar{D}_s$ system**

$$\Psi^C = \frac{1}{\sqrt{2}}(\Psi_{\bar{D}_s DK} + C\Psi'_{D_s\bar{D}\bar{K}}) \quad C = \pm 1$$

- **Effective Hamiltonian of $DK\bar{D}_s$ system**

$$H = T + T' + V + V' + V^C$$

$$H = T + V + V_C$$

$$\langle\Psi^C|(H-E)|\Psi^C\rangle = 0$$

- **Two-body hadron-hadron potential**

$$V(r) = C_a \frac{e^{-(r/R_c)2}}{\pi^{3/2}R_c^3}$$

✓ **$D^*_{s0}(2317)$ and SU(3)-flavor symmetry**

$$V = \begin{pmatrix} C_a + \frac{\alpha}{\sqrt{s}-m_{bare}} & -\frac{\sqrt{3}}{2}C_a \\ -\frac{\sqrt{3}}{2}C_a & 0 \end{pmatrix}$$

$$V_{\bar{D}_s K}(r) = \tfrac{1}{2}V_{DK}(r)$$

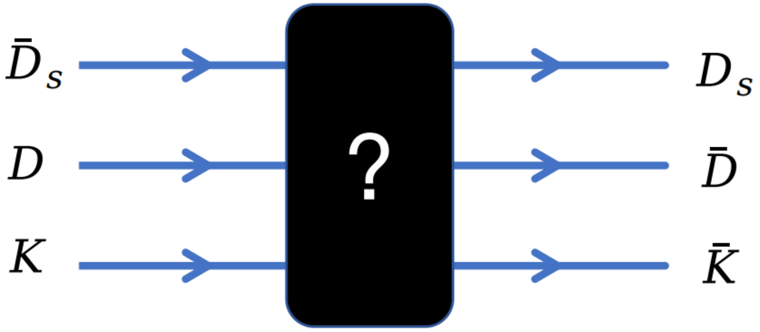
✓ **$X(3872)$ with SU(3)-flavor and HQSS symmetry**

$$V_{D_s\bar{D}} = C_a$$

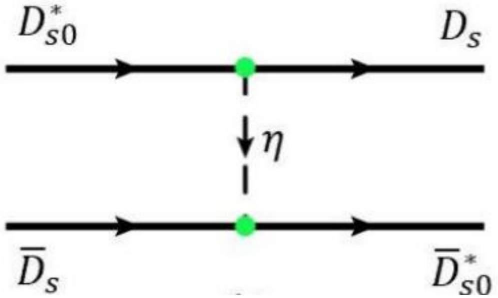
$$C_a^{DK} : C_a^{\bar{D}_s K} : C_a^{\bar{D}_s D} \approx 1 : 0.5 : 0.1$$

Spectrum of the $DK\bar{D}_s$ molecule

➢ Three-body interaction



✓ **Two-body subsystem** $\bar{D}_s D^*_{s0} - D_s \bar{D}^*_{s0}$



One eta exchange potential

$$V^{C=\pm} = \mp \frac{2}{3}\frac{k^2}{f_\pi^2} q_0^2 \left(\frac{e^{-mr} - e^{-\Lambda r}}{4\pi r} - \frac{\Lambda^2 - m^2}{8\pi\Lambda} e^{-\Lambda r}\right)$$

$$\Lambda = \alpha\Lambda_{QCD} + m_{eff}$$

➢ Results of $DK\bar{D}_s$ molecule

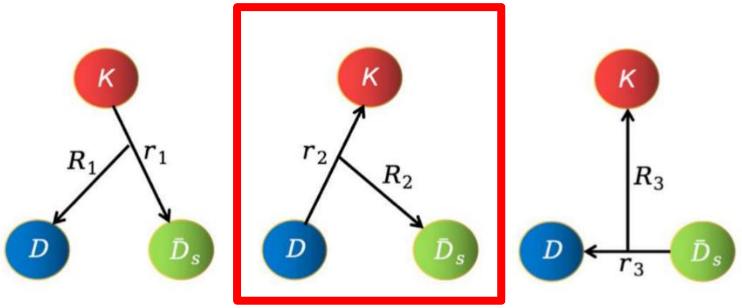
Scenarios	B.E.(0^{--})	$P_{\bar{D}_s K - D}$	$P_{DK - \bar{D}_s}$	$P_{\bar{D}_s D - K}$
$\alpha = 1$	22^{+23}_{-14}	11^{+1}_{-1} %	78^{-1}_{+2} %	11^{+0}_{-1} %
$\alpha = 2$	20^{+22}_{-13}	10^{+1}_{-1}%	80^{-1}_{+2} %	10^{+0}_{-1} %
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T \rangle$
$\alpha = 1$	$1.6^{-0.4}_{+0.9}$	$1.2^{-0.3}_{+0.5}$	$1.4^{-0.3}_{+0.8}$	177^{+81}_{-74}
$\alpha = 2$	$1.7^{-0.4}_{+1.4}$	$1.2^{-0.3}_{+0.7}$	$1.6^{-0.5}_{+1.3}$	169^{+82}_{-78}
Scenarios	$\langle V_{D_s \bar{K}} \rangle$	$\langle V_{DK} \rangle$	$\langle V_{D_s \bar{D}} \rangle$	$\langle V^{C=-}_{D_s \bar{D}^*_{s0}} \rangle$
$\alpha = 1$	-40^{-25}_{+20}	-147^{-73}_{+62}	-14^{-7}_{+6}	2^{+0}_{-1}
$\alpha = 2$	-37^{-25}_{+22}	-143^{-73}_{+64}	-13^{-6}_{+7}	3^{+2}_{-1}

✓ **$X(4310)$ is predicted in our model**

✓ **DK-$\bar{D}_s$ configuration is dominant**

$J^{PC} = 0^{--}$ $\bar{D}_s DK$

Strong decays of the $DK\bar{D}_s$ molecule

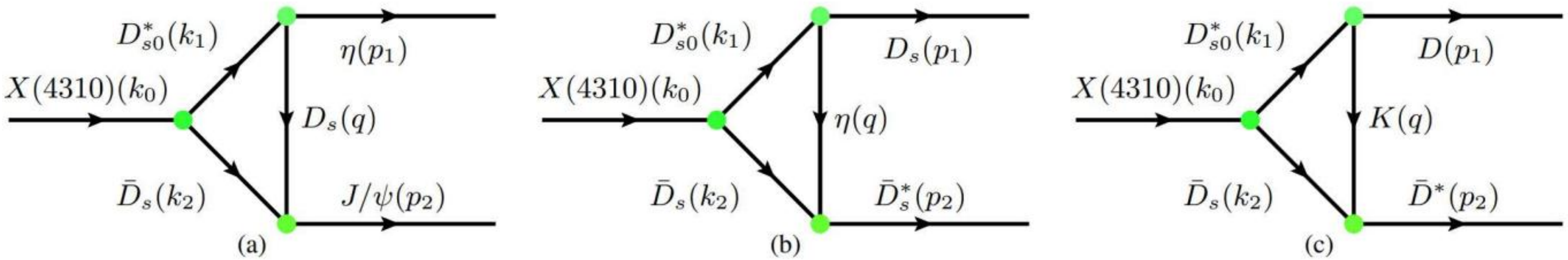


$$\boxed{1-Z} = \boxed{g^2 \frac{\partial G(s)}{\partial s}}$$

80%

Two-body subsystem

- **Final states interaction approach**



$$i\mathcal{M}_a = g_{XD_{s0}^*\bar{D}_s} g_{D_{s0}^*D_s\eta} g_{\psi\bar{D}_sD_s} \int \frac{d^4q}{(2\pi)^4}(k_2^\mu - q^\mu)\frac{1}{k_1^2 - m_{D_{s0}^*}^2}\frac{1}{k_2^2 - m_{\bar{D}_s}^2}\frac{1}{q^2 - m_{D_s}^2}\varepsilon_\mu(p_2)F(q^2),$$

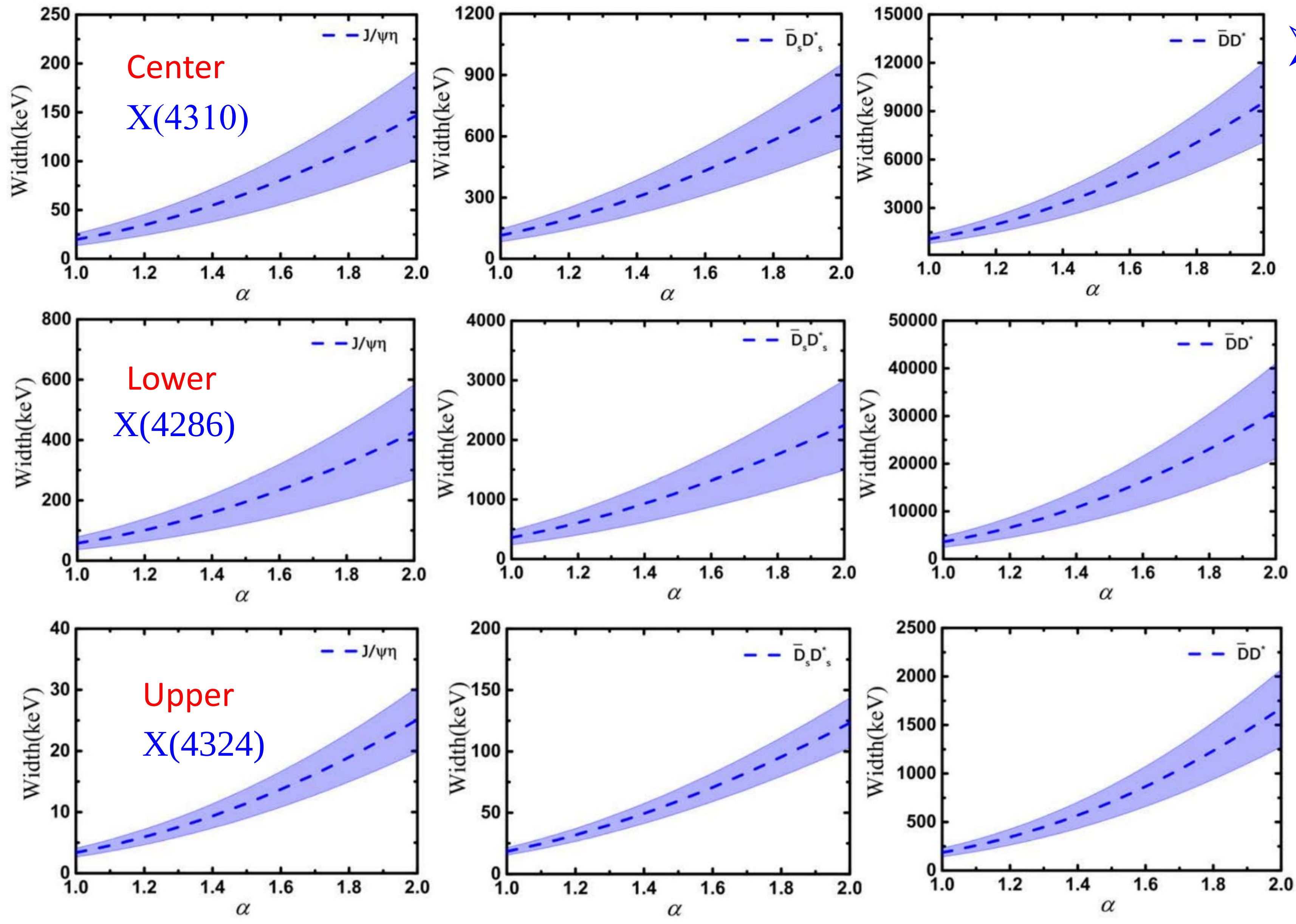
$$i\mathcal{M}_b = g_{XD_{s0}^*\bar{D}_s} g_{D_{s0}^*D_s\eta} g_{\bar{D}_sD_s^*\eta} \int \frac{d^4q}{(2\pi)^4}q^\mu\frac{1}{k_1^2 - m_{D_{s0}^*}^2}\frac{1}{k_2^2 - m_{\bar{D}_s}^2}\frac{1}{q^2 - m_\eta^2}\varepsilon_\mu(p_2)F(q^2),$$

$$F(q,\Lambda,m) = \left(\frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2}\right)^2$$

$$i\mathcal{M}_c = g_{XD_{s0}^*\bar{D}_s} g_{D_{s0}^*DK} g_{\bar{D}_sD^*K} \int \frac{d^4q}{(2\pi)^4}q^\mu\frac{1}{k_1^2 - m_{D_{s0}^*}^2}\frac{1}{k_2^2 - m_{\bar{D}_s}^2}\frac{1}{q^2 - m_K^2}\varepsilon_\mu(p_2)F(q^2),$$

$$\boxed{\Lambda = \alpha\Lambda_{QCD} + m_E}$$

Strong decays of the $DK\bar{D}_s$ molecule



- **Select dominant decay channel**

Center	Lower	Upper
$\Gamma(X\to\bar{D}^*D)/\Gamma(X\to\bar{D}^*{}_sD_s)$		
9.3~12.6	9.9~13.6	8.7~11.8
$\Gamma(X\to\bar{D}^*{}_sD_s)/\Gamma(X\to J/\psi\eta)$		
5.1~5.7	5.2~6.1	4.9~5.4

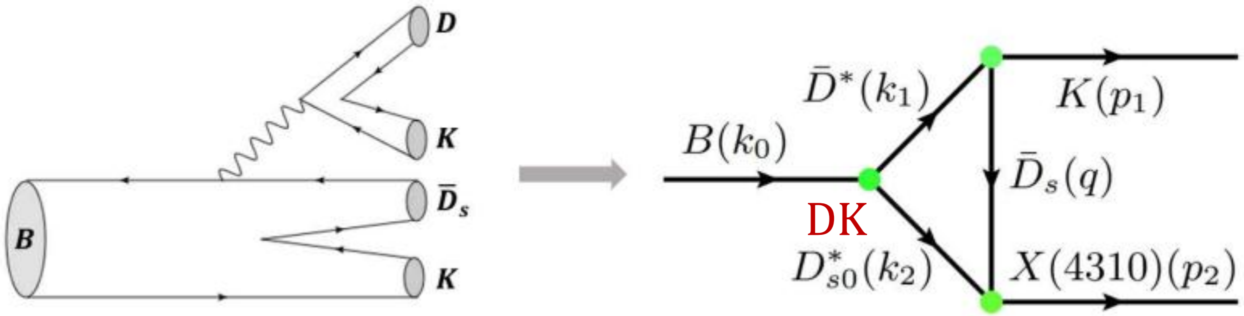
$\Gamma(X\to\bar{D}^*D):\Gamma(X\to\bar{D}^*{}_sD_s):\Gamma(X\to J/\psi\eta)$

50 : 5 : 1

$X\to\bar{D}^*D$

Productions of the $DK\bar{D}_s$ molecule in B decays

➢ Final states interaction approach



(a) (b)

➢ Weak interaction vertices

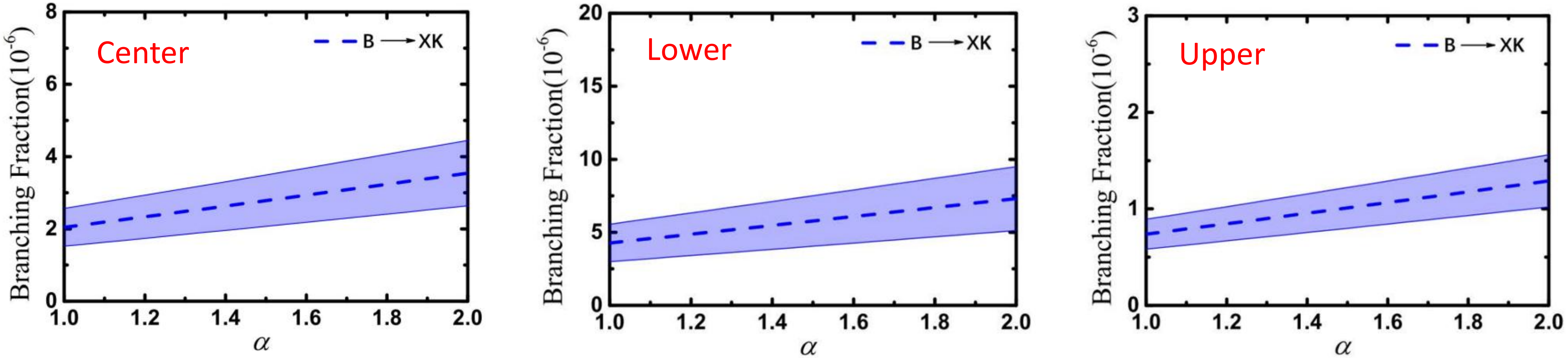
$$\mathcal{A}(B \to D^*_{s0}\bar{D}^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1 f_{D^*_{s0}} \{ -q_1 \cdot \varepsilon(q_2)(m_{D^*} + m_B) A_1\left(q_1^2\right) + (k_0 + q_2) \cdot \varepsilon(q_2) q_1 \cdot (k_0 + q_2)$$

$$\frac{A_2\left(q_1^2\right)}{m_{D^*} + m_B} + (k_0 + q_2) \cdot \varepsilon(q_2)[(m_{D^*} + m_B) A_1(q_1^2) - (m_B - m_{D^*}) A_2(q_1^2) - 2 m_{D^*} A_0(q_1^2)]\}$$

Decay constant of $D^*_{s0}(2317)$ → **Molecular picture**

Decay Constants	$\mu = 1000$	$\mu = 1500$	$\mu = 2000$
$f_{D^*_{s0}(2317)}$	59.36	58.74	58.59

Productions of the $DK\bar{D}_s$ molecule in B decays



$\mathcal{B}(B \to X(4310)K) \approx 10^{-6}$ $\quad$ $\mathcal{B}\left(B \to D_{s0}^*(2317)\bar{D}^{(*)}\right) \approx 10^{-3}$

- **The ratio of the yield of the three-body molecule to that of the two-body molecule**

$$\mathcal{B}(B \to \bar{D}_s DK\ X)/\mathcal{B}\left(B \to D_{s0}^*(2317)\bar{D}^{(*)}\right) \approx 10^{-3}$$

$$Y(e^+e^- \to \bar{D}DK\ X)/\mathcal{B}(e^+e^- \to D_{s0}^*(2317)X) \approx 10^{-3}$$

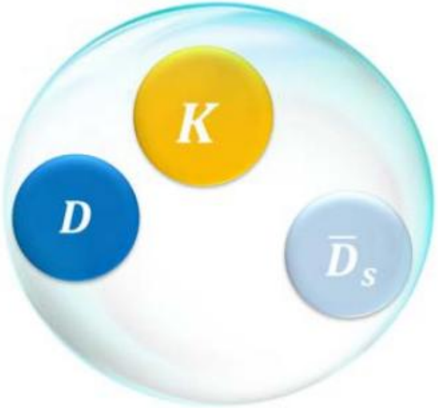
Tian-Chen Wu et al., Phys. Rev. D 108 (2023) 014015

Such a ratio indicates that it's independent on the short-range interaction, but dependent on the long-range interaction.

Event number of the $DK\bar{D}_s$ molecule at LHCb experiment

➢ Estimate the event number

$\mathcal{B}(B \to X(4310)K) \approx 2 \times 10^{-6}$



$\bar{D}_s DK$

✓ $\boldsymbol{B \to D^{(*)}\bar{D}K}$

$$\mathcal{B}\left(X(4310) \to D^{*\pm}D^{\mp}\right) \approx 0.25$$

$$\mathcal{B}\left(B^{+} \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^{+}\right) \approx 5 \times 10^{-7}$$

$$\mathcal{B}\left(B^{+} \to D^{*\pm}D^{\mp}K^{+}\right) \approx 6 \times 10^{-4}$$

$$\mathcal{B}\left(B^{+} \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^{+}\right)/\mathcal{B}\left(B^{+} \to D^{*\pm}D^{\mp}K^{+}\right) \approx 10^{-3}$$

LHCb Collaboration 9 fb^{-1} $\quad \mathrm{N}\left(B^{+} \to D^{*\pm}D^{\mp}K^{+}\right) \approx 2 \times 10^{3}$

$$\mathrm{N}\left(B^{+} \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^{+}\right) \approx 2$$

LHCb Collaboration 50 fb^{-1} $\quad \mathrm{N}\left(B^{+} \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^{+}\right) \approx 10$

LHCb Collaboration 350 fb^{-1} $\quad \mathrm{N}\left(B^{+} \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^{+}\right) \approx 100$

Outline

- Exotic states and hadron-hadron interactions
- Productions of the DK molecule in B decays
- Productions of the $DK\bar{D}_s$ molecule in B decays
- Summary

Summary

- ✓ From the perspective of B decays, the $D_{s0}^{*}(2317)$ can not be explained as the pure hadronic molecule instead of mixture of molecule and other bare components.

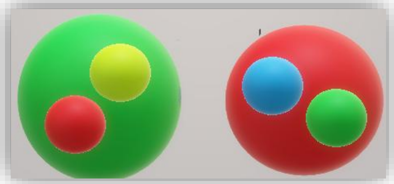
- ✓ With the $D_{s0}^{*}(2317)$ being predominantly a DK molecule, we predicted a compelling three-body hadronic molecule with quantum numbers $J^{PC}=0^{--}$ $\bar{D}_s DK$, which can not be mixed with the conventional charmonium state or two-body hadronic molecule.

- ✓ The mass of the $J^{PC}=0^{--}$ $\bar{D}_s DK$ molecule is estimated to be around 4310 MeV, and it dominantly decays to $D^{(*)}\bar{D}$. We estimate that its production events number at the LHCb Collaboration would be around 10 for an integrated luminosity of 50 fb^{-1} and about 100 for 350 fb^{-1}.

Thanks for your attention!

Backup

➢ Meson-meson



- **With all light quark**

$f_0(980)$	$I^G(J^{PC}) = 0^+(0^{++})$	$\bar{K}K$
$a_0(980)$	$I^G(J^{PC}) = 1^-(0^{++})$	$\bar{K}K$
$\Lambda(1405)$	$I(J^P) = 0(1/2^-)$	$N\bar{K}$

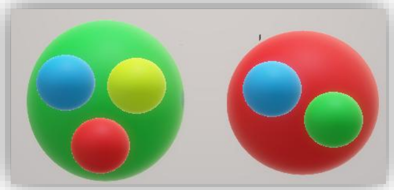
- **With a pair of $c\bar{c}$**

$X(3872)$	$I^G(J^{PC}) = 0^+(1^{++})$	$\bar{D}D^*$
$Y(4260)$	$I^G(J^{PC}) = 0^-(1^{--})$	$\bar{D}_1D^*$

- **With a pair of cc**

$T_{cc}(3985)$	$I(J^P) =?\,(1^+)$	D^*D

➢ Meson-baryon



- **With a charm quark**

$D_{s0}(2317)$	$I(J^P) = 0(0^+)$	DK
$D_{s1}(2460)$	$I(J^P) = 0(1^+)$	D^*K
$\Sigma_c(2800)$	$I(J^P) = 1(?^?)$	Dp
$\Lambda_c(2940)$	$I(J^P) = 0(3/2^-)$	D^*p

- **With a pair of $c\bar{c}$**

$P_c(4312)$	$I(J^P) =?\,(?^?)$	$\bar{D}\Sigma_c$
$P_c(4440)$	$I(J^P) =?\,(?^?)$	$\bar{D}^*\Sigma_c$
$P_c(4457)$	$I(J^P) =?\,(?^?)$	$\bar{D}^*\Sigma_c$

Backup

Component	$J^{P(C)}$	Fit fraction [%] $B^+ \to D^{*+}D^-K^+$	Fit fraction [%] $B^+ \to D^{*-}D^+K^+$	Branching fraction $[10^{-4}]$
$\mathrm{EFF}_{1^{++}}$	1^{++}	$10.9^{+2.3}_{-1.2}{}^{+1.6}_{-2.1}$	$9.9^{+2.1}_{-1.0}{}^{+1.4}_{-1.9}$	$0.74^{+0.16}_{-0.08}{}^{+0.11}_{-0.14} \pm 0.07$
$\eta_c(3945)$	0^{-+}	$3.4^{+0.5}_{-1.0}{}^{+1.9}_{-0.7}$	$3.1^{+0.5}_{-0.9}{}^{+1.7}_{-0.6}$	$0.23^{+0.04}_{-0.07}{}^{+0.13}_{-0.05} \pm 0.02$
$\chi_{c2}(3930)^\dagger$	2^{++}	$1.8^{+0.5}_{-0.4}{}^{+0.6}_{-1.2}$	$1.7^{+0.5}_{-0.4}{}^{+0.6}_{-1.1}$	$0.12^{+0.03}_{-0.03}{}^{+0.04}_{-0.08} \pm 0.01$
$h_c(4000)$	1^{+-}	$5.1^{+1.0}_{-0.8}{}^{+1.5}_{-0.8}$	$4.6^{+0.9}_{-0.7}{}^{+1.4}_{-0.7}$	$0.35^{+0.07}_{-0.05}{}^{+0.10}_{-0.05} \pm 0.03$
$\chi_{c1}(4010)$	1^{++}	$10.1^{+1.6}_{-0.9}{}^{+1.3}_{-1.6}$	$9.1^{+1.4}_{-0.8}{}^{+1.2}_{-1.4}$	$0.69^{+0.11}_{-0.06}{}^{+0.09}_{-0.11} \pm 0.06$
$\psi(4040)^\dagger$	1^{--}	$2.8^{+0.5}_{-0.4}{}^{+0.5}_{-0.5}$	$2.6^{+0.5}_{-0.4}{}^{+0.4}_{-0.5}$	$0.19^{+0.04}_{-0.03}{}^{+0.03}_{-0.03} \pm 0.02$
$h_c(4300)$	1^{+-}	$1.2^{+0.2}_{-0.5}{}^{+0.2}_{-0.2}$	$1.1^{+0.2}_{-0.5}{}^{+0.2}_{-0.2}$	$0.08^{+0.01}_{-0.03}{}^{+0.02}_{-0.01} \pm 0.01$
$T^*_{\bar{c}\bar{s}0}(2870)^{0,\dagger}$	0^+	$6.5^{+0.9}_{-1.2}{}^{+1.3}_{-1.6}$	$\cdots$	$0.45^{+0.06}_{-0.08}{}^{+0.09}_{-0.10} \pm 0.04$
$T^*_{\bar{c}\bar{s}1}(2900)^{0\dagger}$	1^-	$5.5^{+1.1}_{-1.5}{}^{+2.4}_{-1.6}$	$\cdots$	$0.38^{+0.07}_{-0.10}{}^{+0.16}_{-0.11} \pm 0.03$
$\mathrm{NR}_{1^{--}}(D^{*\mp}D^\pm)$	1^{--}	$20.4^{+2.3}_{-0.6}{}^{+2.1}_{-2.6}$	$18.5^{+2.1}_{-0.5}{}^{+1.9}_{-2.3}$	$1.39^{+0.16}_{-0.04}{}^{+0.14}_{-0.17} \pm 0.12$
$\mathrm{NR}_{0^{--}}(D^{*\mp}D^\pm)$	0^{--}	$1.2^{+0.6}_{-0.1}{}^{+0.7}_{-0.6}$	$1.1^{+0.6}_{-0.1}{}^{+0.6}_{-0.5}$	$0.08^{+0.04}_{-0.01}{}^{+0.05}_{-0.04} \pm 0.01$
$\mathrm{NR}_{1^{++}}(D^{*\mp}D^\pm)$	1^{++}	$17.8^{+1.9}_{-1.4}{}^{+3.6}_{-2.6}$	$16.1^{+1.7}_{-1.3}{}^{+3.3}_{-2.3}$	$1.21^{+0.13}_{-0.10}{}^{+0.24}_{-0.17} \pm 0.11$
$\mathrm{NR}_{0^{-+}}(D^{*\mp}D^\pm)$	0^{-+}	$15.9^{+3.3}_{-1.2}{}^{+3.3}_{-3.3}$	$14.5^{+3.0}_{-1.1}{}^{+3.0}_{-3.0}$	$1.09^{+0.23}_{-0.08}{}^{+0.22}_{-0.23} \pm 0.09$

LHCb Collaboration, Phys. Rev. Lett. 133 (2024) 131902

$\mathcal{B}(B \to X(4310)K) \approx 2 \times 10^{-6}$

Backup

- **The weight of Jacobi**

$$\hat{P}_i^G = \sum_j |\Phi_i\rangle S_{ij}^{-1} \langle \Phi_j|,$$

$$P_i = \langle \Psi | \hat{P}_i^G | \Psi \rangle = \langle \Psi | \Phi_i \rangle \langle \tilde{\Phi}_i | \Psi \rangle$$

$$\langle \tilde{\Phi}_i | = \sum_j S_{ij}^{-1} \langle \Phi_j |$$

$$\langle \tilde{\Phi}_i | \Phi_j \rangle = \delta_{ij}$$

$$\begin{aligned} P_i &= \langle \Psi | \hat{P}_i^G | \Psi \rangle \\ &= \langle \Psi | \Phi_i \rangle \sum_j S_{ij}^{-1} \langle \Phi_j | \Psi \rangle \\ &= \sum_{i'} S_{i'i} \sum_j S_{ij}^{-1} \sum_{j'} S_{jj'} \\ &= \sum_{i'} S_{i'i} \sum_{j'} \mathcal{I}_{ij'} \\ &= \sum_{j'} S_{j'i}. \end{aligned}$$

$$P_i = \sum_{j'} S_{j'i} = \langle \Psi | \Phi_i \rangle$$

$$\begin{pmatrix} isS_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} = \begin{pmatrix} 1.71\% & 7.15\% & 1.27\% \\ 7.15\% & 65.63\% & 7.31\% \\ 1.27\% & 7.31\% & 1.21\% \end{pmatrix}$$

Backup

➢ **Three-body force**