Prominent Enhancement of Axion Thermalization Rate from axion-kaon Interaction

based on: J. B. Wang, Z. H. Guo and H. Q. Zhou, Phys. Rev. D 112, no.3, L031701 (2025),

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- The thermal axion scenario, constrained by ΔN_{eff}, provides an important and independent bound on axion coupling.
  [Peccei and Quinn, 1977][Weinberg, 1978][Wilczek, 1978] [Di Luzio, 2020]

- Below the QCD crossover ($T_c \simeq 150$ MeV), the hadronic contribution to axion thermalization rate has long been estimated by only taking $a\pi \leftrightarrow \pi\pi$ reaction into account,
  [Chang and Choi, 1993][Hannestad et al., 2005][Giusarma et al., 2014][Di Luzio et al., 2022]

$$\Gamma_a(T) = \frac{1}{n_a^{eq}} \int \left[\prod_i \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i}\right] \left[\prod_j \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}\right] (2\pi)^4 \delta^4 \left(\sum_i p_i - \sum_j p_j\right) |\mathcal{M}_{\mathrm{reaction}}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)]$$

while the other channels, such as $aK \leftrightarrow \pi K$, $aN \leftrightarrow \pi N$, etc., are neglected because of Boltzmann suppression.

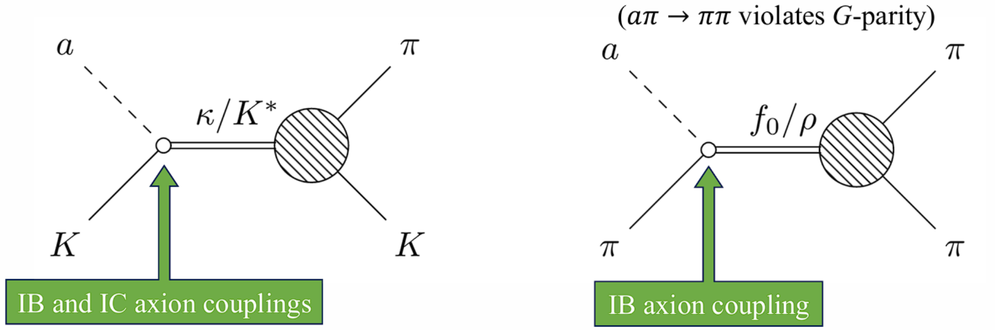
This approximation ignores the dynamical enhancement due to the reaction amplitudes.

- We reexamine this assumption in the KSVZ axion model, focusing on $aK \leftrightarrow \pi K$ (vs. $a\pi \leftrightarrow \pi\pi$).

  - Channels

$$\begin{bmatrix} aK^+ \to \pi^+ K^0, \pi^0 K^+ \\ aK^- \to \pi^- \bar{K}^0, \pi^0 K^- \\ aK^0 \to \pi^0 K^0, \pi^- K^+ \\ a\bar{K}^0 \to \pi^0 \bar{K}^0, \pi^+ K^- \end{bmatrix} \quad \text{vs.} \quad \begin{bmatrix} a\pi^0 \to \pi^+\pi^-, \pi^0\pi^0 \\ a\pi^\pm \to \pi^\pm \pi^0 \end{bmatrix}$$

  - Dominant contribution



IB=isospin-breaking, IC=isospin-conserving

- We calculated the LO scattering amplitudes for $a\pi \to \pi\pi$ and $aK \to \pi K$ processes in SU(3) ChPT.

$$\mathcal{M}^{(2)}_{a\pi^0\to\pi^+\pi^-} = \frac{C_3\left(s-M_\pi^2\right)}{2f_aF_\pi} - \frac{2C_S\Delta_I X_{\pi\eta}}{\sqrt{3}f_aF_\pi}\,,\quad\cdots$$

$$\mathcal{M}^{(2)}_{aK^+\to\pi^+K^0} = \frac{C_3\left(M_{K^0}^2 - M_{K^\pm}^2 + s - u\right)}{4\sqrt{2}f_aF_\pi} + \frac{\sqrt{\frac{3}{2}}C_8\left(t-M_\pi^2\right)}{4f_aF_\pi} + \frac{\sqrt{2}C_S}{f_aF_\pi}\,,\quad\cdots$$

$$\Delta_I \equiv B_0(m_d - m_u)\,,\qquad X_{\pi\eta} \equiv \frac{1}{\sqrt{3}\left(M_\pi^2 - M_\eta^2\right)}$$

- **Unitarization recipe** accounting for the hadron resonances in both $a\pi \to \pi\pi$ and $aK \to \pi K$:

[Oller, 1997]

meson-meson PW scattering matrix

$$T^{\rm uni}_{IJ} = T^{(2)}_{IJ}\cdot\left[T^{(2)}_{IJ} - T^{(4){\rm LECs}}_{IJ} - T^{(2)}_{IJ}\cdot\mathcal{G}\cdot T^{(2)}_{IJ}\right]^{-1}\cdot T^{(2)}_{IJ}\,,$$

$aP_1 \to P_2P_3$ PW scattering amplitudes

$$\vec{M}^{\rm uni}_{IJ} = T^{(2)}_{IJ}\cdot\left[T^{(2)}_{IJ} - T^{(4){\rm LECs}}_{IJ} - T^{(2)}_{IJ}\cdot\mathcal{G}\cdot T^{(2)}_{IJ}\right]^{-1}\cdot \vec{M}^{(2)}_{IJ}\,,$$

$T^{(4){\rm LECs}}_{IJ}$: polynomial parts with $\mathcal{O}(p^4)$ LECs important for P-wave sector [Truong, 1988][Dobado, 1993]

Two-point one-loop function matrix $\mathcal{G} = {\rm diag}\left(G_{n_1}, G_{n_2}, \cdots\right)$ $\quad {\rm Im}G(a_{\rm sc}, s+i\varepsilon, m_1, m_2) = \Theta(s-(m_1+m_2)^2)\rho(s+i\varepsilon, m_1, m_2)\,.$

- Coupled channels

$a\pi^0 \to \pi^+\pi^-, \pi^0\pi^0$

$(\pi\pi)_{IJ=00}$: $\Leftarrow (K\bar{K})_{IJ=00}$

$(\pi\pi)_{IJ=20}$: elastic $\pi\pi$

$a\pi^+ \to \pi^+\pi^0$:

$(\pi\pi)_{IJ=11}$: $\Leftarrow (K\bar{K})_{IJ=11}$

$(\pi\pi)_{IJ=20}$: elastic $\pi\pi$

$aK^+ \to \pi^+K^0, \pi^0K^+$

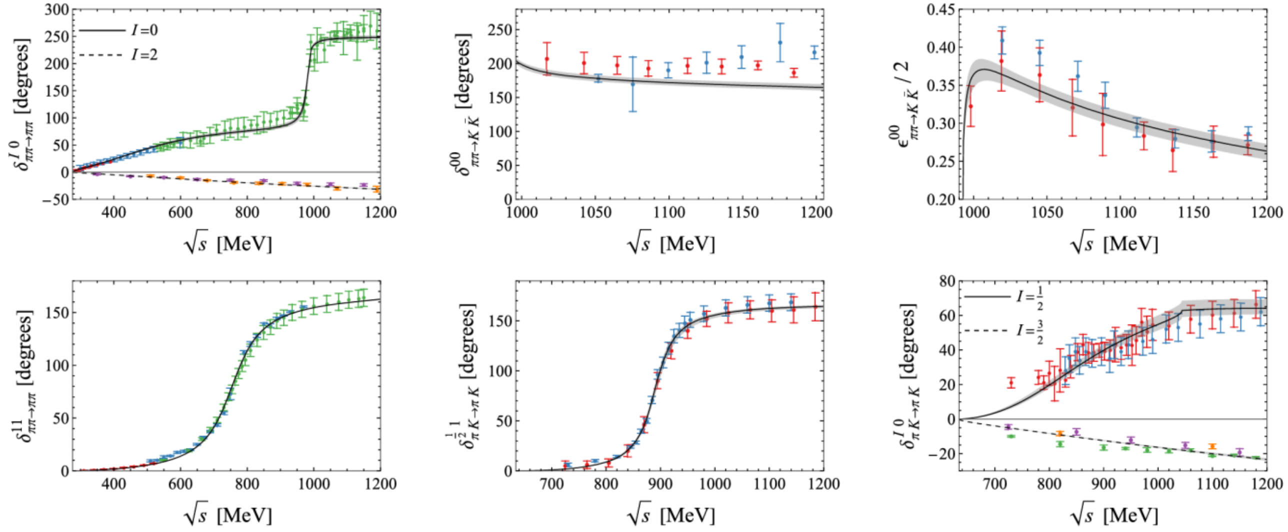
$(\pi K)_{IJ=\frac{1}{2}0,\frac{1}{2}1}$: $\Leftarrow (\eta K)_{IJ=\frac{1}{2}0,\frac{1}{2}1}$

$(\pi K)_{IJ=\frac{3}{2}0}$: elastic πK

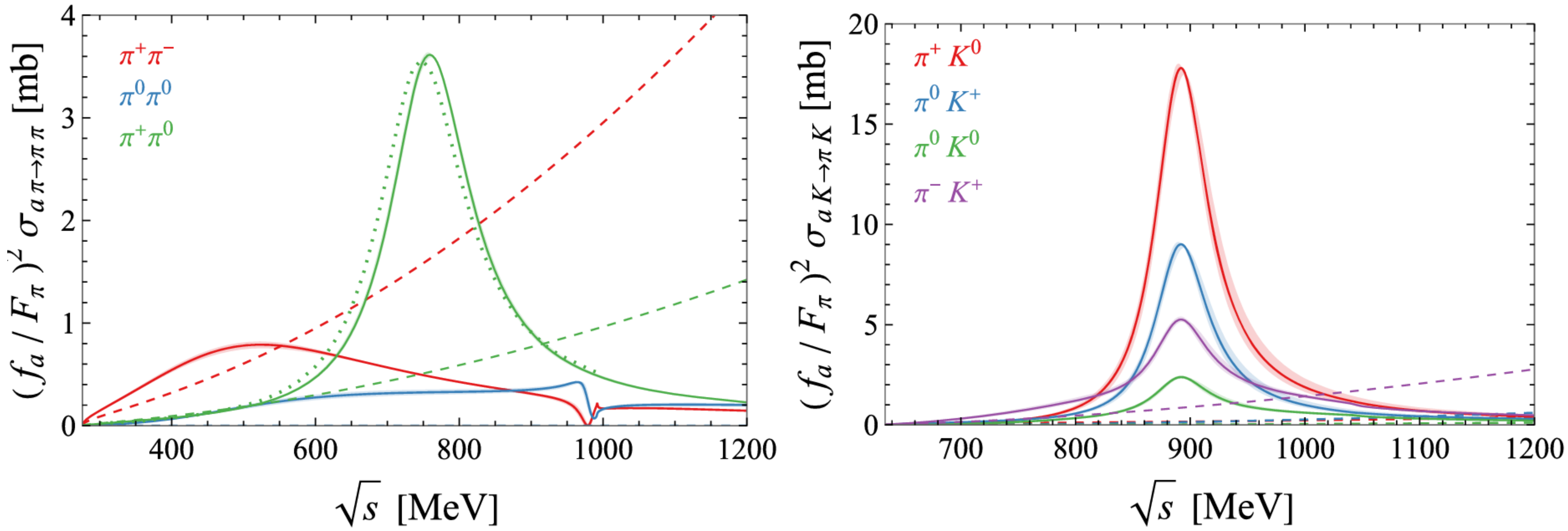
$(\pi K)_{IJ=\frac{3}{2}1}$: rather weak, neglected

(similar for aK^0 channels)

- Fix the meson-meson FSIs using the scattering data up to $\sqrt{s} = 1.2$ GeV.

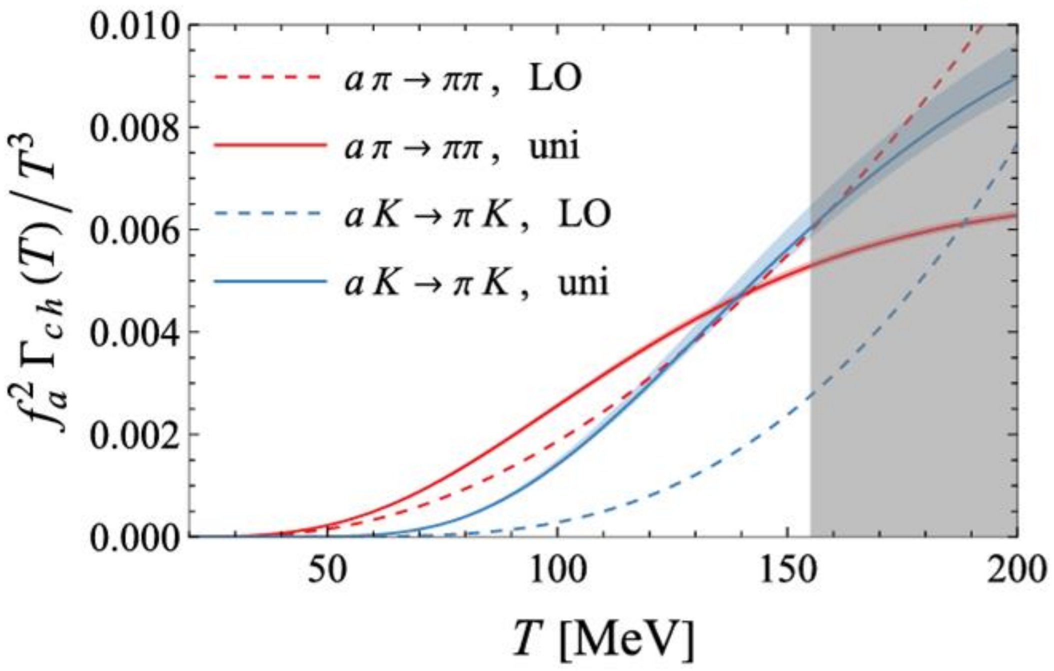


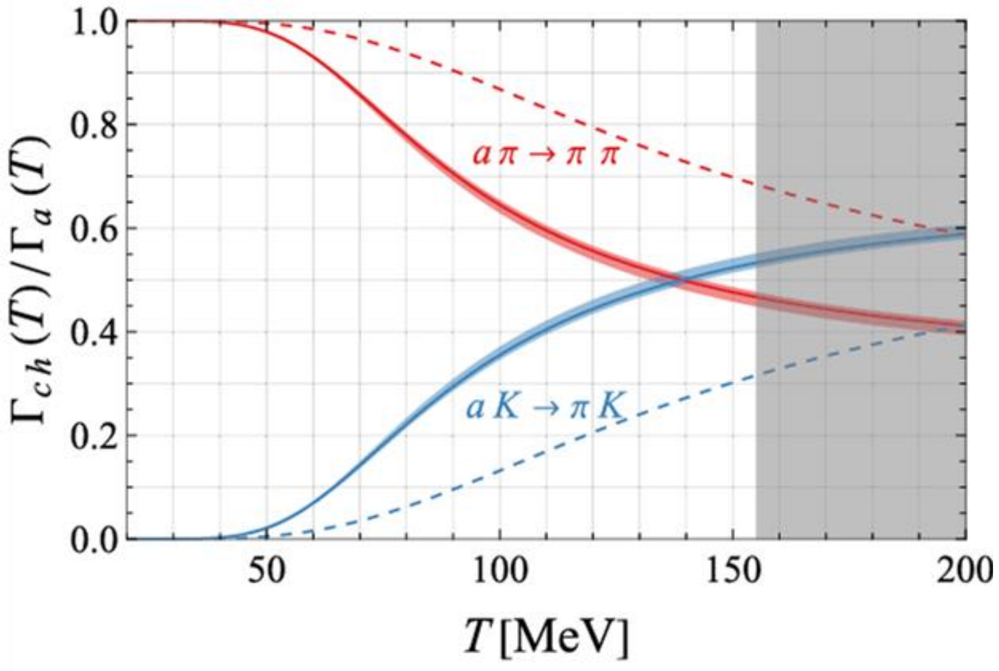
- Cross sections of axion reaction processes



The $aK^+ \to \pi^+ K^0$ process exhibits a particularly large cross section — about four times greater than that of $a\pi^+ \to \pi^+\pi^0$, challenging the assumption that $a\pi \to \pi\pi$ process is the dominant hadronic axion thermalization channel below T_c.

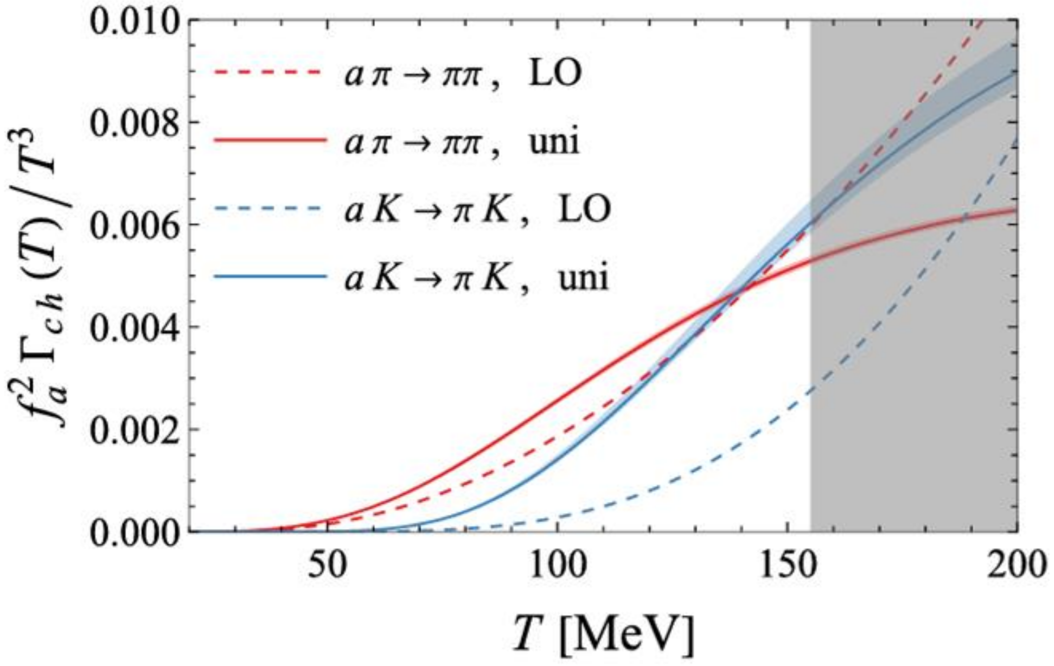
$a\pi \leftrightarrow \pi\pi$ channel contribution **VS.** $aK \leftrightarrow \pi K$ channel contribution (large amplitudes with Boltzmann-suppressed)

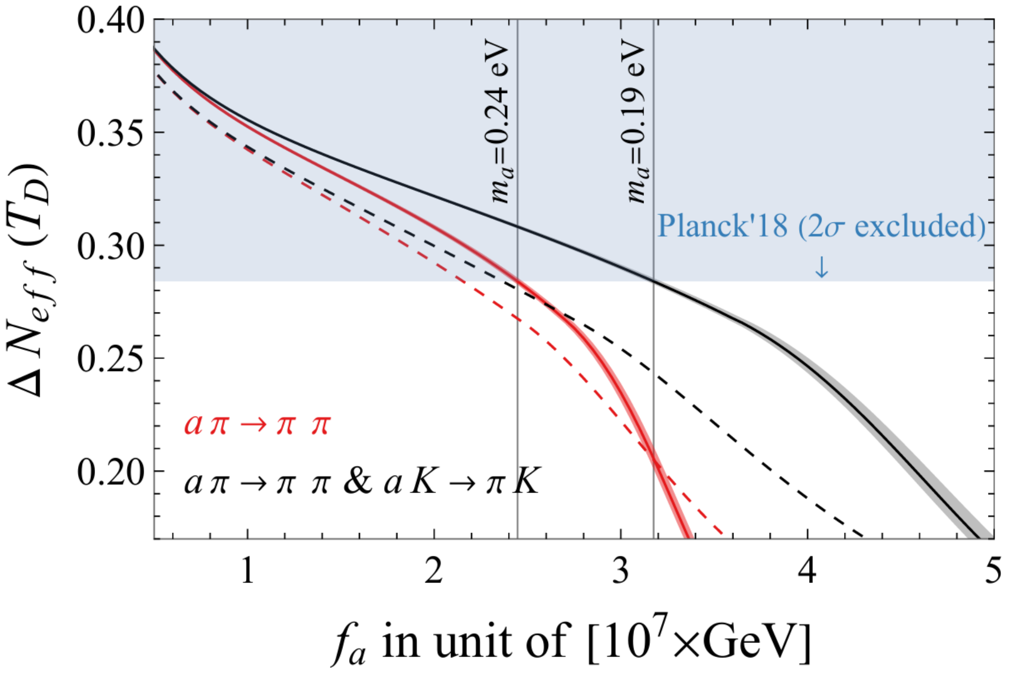




- The $aK \leftrightarrow \pi K$ channel
  - begins to account for more than 40% of the total rate at temperatures above 110 MeV
  - surpasses the $a\pi \leftrightarrow \pi\pi$ contribution around 130 MeV

$a\pi \leftrightarrow \pi\pi$ channel contribution **VS.** $aK \leftrightarrow \pi K$ channel contribution (large amplitudes with Boltzmann-suppressed)





✓ Confronting to the current upper limit on ΔN_{eff}, the lower bound on f_a is tightened by approximately 30% after $aK \leftrightarrow \pi K$ channel is included.

Thank you for your attention