One-Loop Renormalization of GraviChPT with Heat-Kernel Technique

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What is GraviChPT? Why Important?

- ChPT in Curved Spacetime[1,2]
- Incorporate Also Interactions between Pion, Nucleon and Gravitational Field

$$\mathcal{L}_{\pi N}^{(2)} = \frac{c_8}{8} R\bar{\Psi}\Psi + \frac{ic_9}{m} R^{\mu\nu}\left[\bar{\Psi}\gamma_\mu\left(\vec{\nabla}_\nu - \overleftarrow{\nabla}_\nu\right)\Psi\right]$$

$$\begin{aligned}\mathcal{L}_{\pi N}^{(3)} &= \tilde{d}_{g1} R\bar{\Psi}\gamma^\mu\gamma_5 u_\mu \Psi + \tilde{d}_{g2} R^{\mu\nu}\bar{\Psi}\gamma_\mu\gamma_5 u_\nu\Psi \\ &+\tilde{d}_{g3} R^{\alpha\beta\mu\nu}\left[\bar{\Psi}\sigma_{\alpha\beta}\gamma_5 u_\mu\left(\vec{\nabla}_\nu + \overleftarrow{\nabla}_\nu\right)\Psi\right] \\ &+\tilde{d}_{g4}\nabla_\nu R^{\alpha\beta\mu\nu}\left[\bar{\Psi}\sigma_{\alpha\beta}\gamma_5\left(\vec{\nabla}_\mu - \overleftarrow{\nabla}_\mu\right)\Psi\right]\end{aligned}$$

- New LECs Present in Nucleon's GFFs Even in Flat Spacetime

$$A(t) = 1 - \frac{2c_9}{m_N}t + \cdots \quad J(t) = \frac{1}{2} - \frac{c_9}{m_N}t + \cdots \quad D(t) = c_8 m_N + \cdots$$

How to Renormalize at One-Loop Level?

✗ Feynman-Diagram Technique	✓ Heat-Kernel Technique
$\mathrm{div}[W_{L=1}] = \mathrm{div}\left[\sum_{\text{One-Loop 1PI}} \text{Feynman Diagram}\right]$ Compute One-Loop 1PI Feynman Diagrams	$\mathrm{div}[W_{L=1}] \propto \frac{1}{\epsilon}\int d^d x\sqrt{g}\,\mathrm{str}\left[\boldsymbol{A}^{(2)}(x,x)\right]$ Compute Second-Order Heat-Kernel Expansion Coefficient
Cumbersome	Algebraic, Systematic

- *The left panel of the figure below illustrates the general workflow of the heat-kernel technique[3,4].*
- *The right panel details the calculation of the heat-kernel expansion coefficients, for which we propose a novel iterative method in momentum space.*

Step 1: Start with Action

$$S = \int d^4x\sqrt{g}\mathcal{L}(\phi)$$

Step 2: Split Fields into Classical and Quantum Parts

$$\phi = \phi_{\mathrm{cl}} + \xi \ , \quad \left.\frac{\delta S}{\delta\phi}\right|_{\phi_{\mathrm{cl}}} = 0$$

Step 3: Perform Path Integral at One-Loop Level

$$e^{iW_{L=1}} = \int \mathcal{D}[\xi]e^{iS_{\mathrm{bil}}}$$

$$S_{\mathrm{bil}} = \int d^4x\sqrt{g}\frac{1}{2}\xi^T\boldsymbol{M}\xi \ , \boldsymbol{M} = -\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}(g^{\mu\nu}\boldsymbol{I} + \boldsymbol{V}^{\mu\nu})\partial_\nu - \boldsymbol{X}^\mu\partial_\mu + \boldsymbol{Q}$$

Step 4: Apply Gaussian Quadrature Formula

$$W_{L=1} = \frac{i}{2}\ln\{\mathrm{Sdet}[\boldsymbol{M}]\} = \frac{i}{2}\mathrm{Str}\{\ln[\boldsymbol{M}]\}$$

Step 5: Apply Proper-Time Representation

$$W_{L=1} = -\frac{i}{2}\int_0^\infty \frac{d\tau}{\tau}\mathrm{Str}\left[e^{-\tau\boldsymbol{M}}\right] = -\frac{i}{2}\int_0^\infty\frac{d\tau}{\tau}\int d^4x\sqrt{g}\,\mathrm{str}\left[\langle x|e^{-\tau\boldsymbol{M}}|x\rangle\right]$$

Step 6: Apply Heat Kernel Expansion

$$\langle x|e^{-\tau\boldsymbol{M}}|y\rangle \equiv \boldsymbol{K}(x,y|\tau) = \frac{1}{(4\pi\tau)^{d/2}}e^{-\frac{\sigma^2(x,y)}{4\tau}}\sum_{n=-\infty}^{\infty}\tau^n\boldsymbol{A}^{(n)}(x,y)$$

$$W_{L=1} = -\frac{i}{2(4\pi)^{d/2}}\int_0^\infty\frac{d\tau}{\tau^{d/2+1}}\int d^dx\sqrt{g}\sum_{n=-\infty}^{\infty}\tau^n\mathrm{str}\left[\boldsymbol{A}^{(n)}(x,x)\right]$$

Step 7: Extract Divergent Part

$$\mathrm{div}[W_{L=1}] = -\frac{i}{(4\pi)^{d/2}}\frac{4}{d(d-2)}\frac{\Gamma(2-d/2)}{\mu^{4-d}}\int d^dx\sqrt{g}\,\mathrm{str}\left[\boldsymbol{A}^{(2)}(x,x)\right]$$

Step 6.1: Start with Heat Equation and Boundary Condition

$$\partial_\tau\boldsymbol{K}(x,y|\tau) = \left\{(\eta^{\mu\nu}\boldsymbol{I} + \boldsymbol{V}^{\mu\nu})\partial_\mu\partial_\nu + \boldsymbol{X}^\mu\partial_\mu - \boldsymbol{Q}\right\}\boldsymbol{K}(x,y|\tau)$$

$$\boldsymbol{K}(x,y|\tau=0) = \delta^{(d)}(x-y)\boldsymbol{I}$$

Step 6.2: Heat Kernel Expansion in Coordinate Space

$$\boldsymbol{K}(x,y|\tau) = \frac{1}{(4\pi\tau)^{d/2}}e^{-\frac{(x-y)^2}{4\tau}}\sum_{n=-\infty}^{\infty}\tau^n\boldsymbol{A}^{(n)}(x,y)$$

Step 6.3: Fourier Transform to Momentum Space

$$\boldsymbol{K}(x,y|\tau) = \int\frac{d^dk}{(2\pi)^d}e^{ik\cdot(x-y)}e^{-\tau k^2}\sum_{r=0}^{\infty}\tau^r\boldsymbol{B}^{(r)}(k;x,y)$$

Step 6.4: Iterative Solution

$$\boldsymbol{B}^{(r)}(k;x,y) = \sum_{s=0}^{r+1}k_{\mu_1}\cdots k_{\mu_s}\boldsymbol{B}^{(r,s)\mu_1\cdots\mu_s}(x,y) + \mathcal{O}(V^2)$$

Step 6.5: Perform Momentum Integral and Take Coincidence Limit

$$\lim_{x\to y}\int\frac{d^dk}{(2\pi)^d}e^{ik\cdot(x-y)}e^{-\tau k^2}k_{\mu_1}\cdots k_{\mu_s} = \begin{cases} 0 & \text{if } s \text{ is odd} \\ \eta_{\mu_1\mu_2}\cdots\eta_{\mu_{s-1}\mu_s} + \cdots & \text{if } s \text{ is even}\end{cases}$$

Step 6.6: Match from Momentum to Coordinate Space

$$\boldsymbol{B}^{(r)}(k;x,y) \xrightarrow{\text{contribute to}} \begin{cases}\boldsymbol{A}^{(r-\mathrm{floor}[r+1/2])}(x,x) \\ \vdots \\ \boldsymbol{A}^{(r)}(x,x)\end{cases}$$

$$\left.\begin{matrix}\boldsymbol{B}^{(2)}(k;x,y) \\ \vdots \\ \boldsymbol{B}^{(5)}(k;x,y)\end{matrix}\right\} \xrightarrow{\text{contribute to}} \boldsymbol{A}^{(2)}(x,x)$$

Preliminary Results

$$c_8 = c_8^R - \frac{15g_A^2 m}{2}\Lambda \quad c_9 = c_9^R + \frac{25g_A^2m}{864F^2}\Lambda \quad \tilde{d}_{g1} = \tilde{d}_{g1}^R + \left(\frac{10g_A^3}{23F^2} - \frac{g_A}{3F^2}\right)\Lambda \quad \tilde{d}_{g2} = \tilde{d}_{g2}^R + \frac{g_A^3}{72F^2}\Lambda \quad \tilde{d}_{g3} = \tilde{d}_{g3}^R \quad \tilde{d}_{g4} = \tilde{d}_{g4}^R$$

$$\Lambda = \frac{1}{32\pi^2}\left(\frac{2}{d-4} - 1 + \gamma - \ln(4\pi)\right)$$

References

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