Evaluation of Muon g-2 from $\tau \to \pi\pi\nu$ Data

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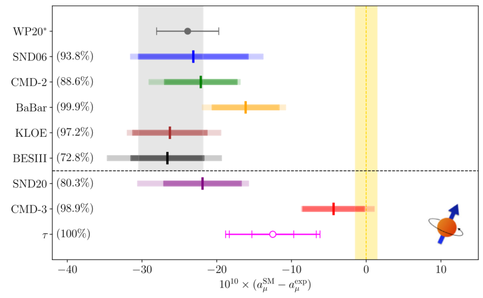
Introduction

- The anomalous magnetic moment of the muon, $a_\mu = (g-2)_\mu/2$, is one of the most precise tests of the Standard Model (SM). Among the various leading-order hadronic vacuum polarization (LO HVP) contributions to a_μ, the dominant part arises from the $\pi\pi$ channel, accounting for nearly three quarters of the total. However, resolving the inconsistencies among existing $e^+e^- \to \pi^+\pi^-$ measurements remains a highly nontrivial challenge, requiring comprehensive reanalyses and future high-precision experiments.



- Alternatively, the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay provides an independent approach to determine the $\pi\pi$ contribution to a_μ. This requires transforming the $\pi^-\pi^0$ states in τ decays into the corresponding $\pi^+\pi^-$ states observed in e^+e^- annihilation, with isospin-breaking (IB) effects properly taken into account, including the $\pi\pi$ form factors ($F_{+,0}$), kinematical factors ($\beta_{+,0}$), final-state radiation (FSR), electroweak short-distance correction (S_{EW}), and long-distance electromagnetic correction (G_{EM}).

Theoretical Framework

We concentrate on the updated evaluation of the EM corrected function $G_{EM}(t)$ in this work. The EM corrections to the $\tau^- \to \pi^-\pi^0\nu_\tau$ process consist of two parts, the virtual correction from the photon loops and the real photon radiative correction

$$G_{EM}(t) = 1 + G_{EM}^{(v)}(t) + G_{EM}^{(r)}(t)\,, \quad t = (p_{\pi^-} + p_{\pi^0})^2. \tag{1}$$

We directly take the complete $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$ amplitude and the phase-space receipt from Ref. [1] to calculate the full $G_{EM}^{(r)}(t)$

$$G_{EM}^{(r)}(t) = \frac{1}{\frac{d\Gamma_{\pi\pi}^{(0)}}{dt}\,32(2\pi)^{12}m_\tau^2}\int_{S_{\pi\pi}^-}^{S_{\pi\pi}^+} dS_{\pi\pi}\int_{S_{\nu\gamma}^-}^{S_{\nu\gamma}^+} dS_{\nu\gamma}\int_{S_{\pi\nu}^-}^{S_{\pi\nu}^+} dS_{\pi\nu}\int_{S_{\pi\nu\gamma}^-}^{S_{\pi\nu\gamma}^+}\frac{dS_{\pi\nu\gamma}}{\sqrt{-\Delta_4}}\overline{|\mathcal{M}|}^2_{\tau\to\pi\pi\nu_\tau\gamma}\,. \tag{2}$$

After combining the isospin breaking effects from other sources, we acquire the full set of isospin breaking correction function $R_{IB}(t)$,

$$R_{IB}(t) = \frac{FSR(t)}{G_{EM}(t)}\frac{\beta_{\pi^+\pi^-}^3(t)}{\beta_{\pi^-\pi^0}^3(t)}\frac{\left|F_{\pi\pi}^{(0)}(t)\right|^2}{\left|F_{\pi\pi}^{(-)}(t)\right|^2}, \tag{3}$$

one can also extract the $e^+e^- \to \pi^+\pi^-$ cross section using Eq. (4) from the experimental $\pi\pi$ invariant-mass distribution of τ decay, which can be conveniently recast as

$$\sigma_{e^+e^-\to\pi^+\pi^-}^0(t) = \frac{2\pi\alpha^2 m_\tau^2}{3|V_{ud}|^2 t}\frac{1}{\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)}\frac{\mathcal{B}_{\pi\pi}}{\mathcal{B}_{\tau_e}}\frac{1}{N_{\pi\pi}}\frac{dN_{\pi\pi}}{dt}\frac{R_{IB}(t)}{S_{EW}}. \tag{4}$$

Finally, we can evaluate the leading-order hadronic vacuum polarization (LO HVP) contribution to a_μ as

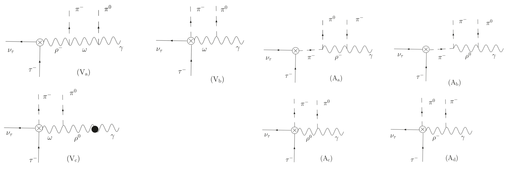
$$a_\mu^{HVP,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\,\sigma_{e^+e^-\to hadrons}^0(t)\,, \tag{5}$$

completing the calculation of a_μ^{SM} and allowing the determination of its deviation from experiment.

Results and Discussion

The EM corrected function $G_{EM}(t)$

The VJP and VVP Lagrangians are included to calculate the $\tau \to \pi\pi\nu_\tau\gamma$ decay amplitudes

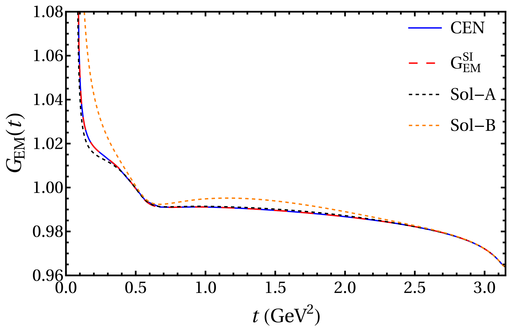


We further include the contributions from scalar resonances to the $\omega \to \pi^0\pi^0\gamma$ decay, in addition to the vector-meson resonance exchanges considered in Ref. [1], in order to give a more realistic determination of d_4. Two solutions for the parameter d_4 are obtained

$$d_4 = -0.42\,, \tag{6}$$
$$d_4 = 1.01\,. \tag{7}$$

and designate the negative solution of d_4 in Eq. (6) as Sol-A and the positive one in Eq. (7) as Sol-B.

Illustration of different results for EM corrected function $G_{EM}(t)$: Sol-A (black dashed line, with negative value of d_4), Sol-B (orange dashed line, with positive value of d_4), $G_{EM}^{SI}(t)$ (red long-dashed line, SI term only) and the result with the full amplitude from Ref. [2] (blue solid line, labeled as CEN).

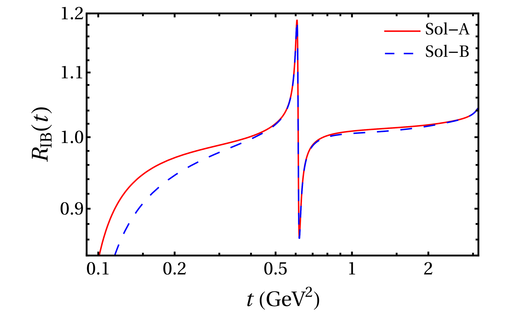


Full IB correction functions $R_{IB}(t)$

Under the factorization assumption, the various IB corrections can be introduced as products of different pieces [2]

$$R_{IB}(t) = \frac{FSR(t)}{G_{EM}(t)}\frac{\beta_{\pi^+\pi^-}^3(t)}{\beta_{\pi^-\pi^0}^3(t)}\frac{\left|F_{\pi\pi}^{(0)}(t)\right|^2}{\left|F_{\pi\pi}^{(-)}(t)\right|^2}, \tag{8}$$

Full IB correction functions $R_{IB}(t)$ with complete EM corrections $G_{EM}(t)$:



Updated evaluation of a_μ from tau data

We further calculate the relative shifts of a_μ caused by the IB corrections in the $\pi\pi$ channel via the following formula

$$\Delta a_\mu^{HVP,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{t_{max}} dt\, K(t)\left[\frac{K_\sigma(t)\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt}}{K_\Gamma(t)}\right]\times\left(\frac{R_{IB}(t)}{S_{EW}}-1\right), \tag{9}$$

The contributions of $G_{EM}(t)$ to the relative shifts of $\Delta a_\mu^{HVP,LO}$ (in units of 10^{-11}). The fourth column shows the results of Ref. [2], and the fifth to seventh columns present the outcomes from $F_V = \sqrt{2}F$ and $F_V = \sqrt{3}F$ at $\mathcal{O}(p^4)$, as well as those from $\mathcal{O}(p^6)$ in Ref. [3], in order.

$\Delta a_\mu^{HVP,LO}[\pi\pi]$ from $G_{EM}(t)$						
$[t_{min}, t_{max}]$	Sol-A	Sol-B	CEN	MR[$\mathcal{O}(p^4)$]	MR[$\mathcal{O}(p^6)$]	
$[4m_\pi^2, 1\,GeV^2]$	− 7.1	−44.9	−10.6	−10.4	−15.9	−63.2±16.5
$[4m_\pi^2, 2\,GeV^2]$	− 6.4	−44.5	−9.8	−9.6	−15.2	−58.1±12.2
$[4m_\pi^2, 3\,GeV^2]$	− 6.3	−44.4	−9.7	−9.5	−15.1	−67.8±17.5
$[4m_\pi^2, m_\tau^2]$	− 6.3	−44.4	−9.7	−9.5	−15.1	−64.9±13.4

Contributions to the relative shifts of $\Delta a_\mu^{HVP,LO}[\pi\pi]$ as defined in (9) from various IB sources (in units of 10^{-11}) by taking different values of t_{max}(in units of GeV^2)

$\Delta a_\mu^{HVP,LO}[\pi\pi]$ from various IB sources					
Parameters	t_{max}/GeV^2	S_{EW}	$\beta_{\pi^+\pi^-}^3/\beta_{\pi^-\pi^0}^3$	$\left\|F_{\pi\pi}^{(0)}/F_{\pi\pi}^{(-)}\right\|^2$	FSR
Sol-A	1.0	−112.7	−74.1	76.8	42.7
	2.0	−114.6	−74.3	76.7	43.3
	3.0	−114.7	−74.3	76.7	43.3
	m_τ^2	−114.7	−74.3	76.7	43.3
Sol-B	1.0	−113.6	−76.9	77.1	43.2
	2.0	−115.4	−77.1	77.0	43.8
	3.0	−115.5	−77.1	77.0	43.8
	m_τ^2	−115.5	−77.1	77.0	43.8

The IB-corrected $\pi\pi$ contribution to $a_\mu^{HVP,LO}$ in units of 10^{-10} by using various experimental $\tau^- \to \pi^-\pi^0\nu_\tau$ data:

Parameters	Experiments	$a_\mu^{HVP,LO}\vert_{\pi\pi,rdata}$
Sol-A	Belle	$516.7 \pm 2.1 \pm 7.9 \pm 2.2$
	ALEPH	$513.3 \pm 4.3 \pm 2.8 \pm 2.1$
	CLEO	$516.9 \pm 3.2 \pm 8.8 \pm 2.2$
	OPAL	$527.2 \pm 9.8 \pm 6.8 \pm 2.1$
Sol-B	Belle	$513.4 \pm 2.0 \pm 7.9 \pm 2.2$
	ALEPH	$510.2 \pm 4.2 \pm 2.8 \pm 2.1$
	CLEO	$513.7 \pm 3.2 \pm 8.8 \pm 2.2$
	OPAL	$523.5 \pm 9.5 \pm 6.8 \pm 2.1$

For the uncertainties in the third column, the first error is from the uncertainties of the experimental invariant-mass spectra, the second error arises from the branchings ratios of $\mathcal{B}_{\pi\pi}$ and $\mathcal{B}_{\tau e}$, and the third error comes from the IB correction function R_{IB}. .

We set the results by taking the parameter Sol-A with $d_4 = -0.42$ as our baseline in this work.

$$a_\mu^{HVP,LO|_{\pi\pi,rdata,\,Belle}} = 516.7 \pm 2.1_{Spec} \pm 7.9_{BR} \pm 2.2_{IB} \pm 3.3_{Sys}\,, \tag{10}$$
$$a_\mu^{HVP,LO|_{\pi\pi,rdata,\,ALEPH}} = 513.3 \pm 4.3_{Spec} \pm 2.8_{BR} \pm 2.1_{IB} \pm 3.2_{Sys}\,, \tag{11}$$
$$a_\mu^{HVP,LO|_{\pi\pi,rdata,\,CLEO}} = 516.9 \pm 3.2_{Spec} \pm 8.9_{BR} \pm 2.2_{IB} \pm 3.3_{Sys}\,, \tag{12}$$
$$a_\mu^{HVP,LO|_{\pi\pi,rdata,\,OPAL}} = 527.2 \pm 9.8_{Spec} \pm 6.8_{BR} \pm 2.1_{IB} \pm 3.7_{Sys}\,, \tag{13}$$

After performing the average of the central values in Eqs. (10)-(13) weighted by the experimental error bars of the invariant-mass spectra and branching ratios.

$$10^{10}\cdot a_\mu^{HVP,LO|_{\pi\pi,rdata}} = 516.0 \pm 2.9_{Spec+BR} \pm 4.0_{IB+Sys}\,, \tag{14}$$

By including the results from other hadronic channels in e^+e^- annihilation from Ref. [4], the full LO HVP contribution to a_μ reads

$$10^{10}\cdot a_\mu^{HVP,LO|_{rdata}} = 702.1 \pm 2.9_{Spec+BR} \pm 4.0_{IB+Sys} \pm 2.1_{Others}\,, \tag{15}$$

By further including the revised contributions from electromagnetic, weak and other hadronic interactions given in WP25 [5], the deviation between the Standard Model prediction and combined BNL+FNAL experimental result [5, 6] is

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (14.9 \pm 5.6) \times 10^{-10}\,, \tag{16}$$

leading to a 2.7σ discrepancy.

Conclusion

We have revisited the computation of the muon anomalous magnetic moment a_μ by using the experimental $\tau \to \pi\pi\nu_\tau$ data to assess the leading-order hadronic vacuum polarization from the $\pi\pi$ channel. The only unknown resonance coupling of the VVP type, namely d_4, is newly determined through the $\omega \to \pi^0\pi^0\gamma$ process by including both the vector and scalar resonances. We acquire the full set of isospin breaking correction function $R_{IB}(t)$, which is further exploited to calculate $a_\mu^{HVP,LO|_{\pi\pi}}$ by using the experimental $\tau \to \pi\pi\nu_\tau$ data from Belle, ALEPH, CLEO and OPAL. Our result agrees with the conservative estimation provided in the muon g − 2 White Paper 2025 within the uncertainties. The resulting deviation of a_μ between the current estimation and the updated world average in Ref. [7] lies at the level of 2.7 σ.

References

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