Accurate nucleon iso-vector scalar and tensor charge at physical point

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Introduction

Experiments that use nucleons and nuclei as targets are important for testing the Standard Model and searching for new physics. To interpret their results, we need precise calculations of matrix elements—how the nucleon responds to certain currents or operators. These can only be reliably computed using first-principle lattice QCD. The isovector charges of the nucleon (axial, scalar, and tensor) are some of the simplest quantities to calculate. While the axial charge is well-determined experimentally, precise values of the scalar and tensor charges are in greater demand, as they play a key role in probing TeV-scale new physics. However, precise lattice QCD calculations are challenging due to excited-state contamination(ECS) and increasing noise at large time separations. Our work introduces a new calculation of the nucleon scalar and tensor charges (g_S and g_T), Using the blending method [1] and the idea of current-inspired interpolation field [2, 3] to better control systematic errors from excited states. This results in a significantly improved determination of g_S and g_T.

Simulation Setup

The nucleon matrix elements $g_{S/T}$ we consider are defined though the relation,

$$\langle N(p,s)|\mathcal{O}_X|N(p,s)\rangle = g_X \bar{u}(p,s)\Gamma_X u(p,s),$$

where the nucleon states $N(p,s)$ and spinors $u(p,s)$ have given momentum p and spin s, $O_X = \bar{u}\Gamma_X u - \bar{d}\Gamma_X d$ is the iso-vector singlet operator with $\Gamma_S = \mathbb{I}$ and $\Gamma_T = \sigma_{\mu\nu}$.

Within the framework of quantum field theory, the nu-cleon matrix element is extracted from the ratio of three-point (3pt) to two-point (2pt) correlation functions:

$$\mathcal{R}_X(t_f,t;\mathcal{N}) = \frac{\int \mathrm{d}^3x\mathrm{d}^3y\mathrm{d}^3z\langle \mathcal{N}(\vec{x},t_f)\mathcal{O}_X(\vec{y},t)\mathcal{N}^\dagger(\vec{z},0)\rangle}{\int \mathrm{d}^3x\mathrm{d}^3z\langle \mathcal{N}(\vec{x},t_f)\mathcal{N}^\dagger(\vec{z},0)\rangle} = \langle \mathcal{O}_X\rangle_N + \mathcal{O}(e^{-\delta m t}, e^{-\delta m(t_f-t)}, e^{-\delta m t_f}),$$

where $N(\vec{x},t) \equiv \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ denotes the nucleon interpolating operator and δm is the energy gap between the ground state and the first excited state.

our numerical tests indicate that the dominant ESC can beeliminated using a linear combination of matrix elementsof O_X within a basis with two interpolation fields, $H = \{N, NO_X\}$ where the second one is "current inspired",as suggested by Ref. [2, 3]. We do the a joint three-state fit to the two- and three-point functions for both interpolating fields N and $N + c_X NO_X$. In FIG. 1, we show the ratio ratios $R_X^{\mathrm{mid}}(t_f) \equiv R_X(t_f, t = t_f/2)$ and the Feynman-Hellman in-spired combination $R_X^{\mathrm{FH}}(t_f)$[4]

$$\mathcal{R}_X^{\mathrm{FH}}(t_f) \equiv \sum_{t=t_c}^{t_f+a-t_c} \mathcal{R}_X(t_f+a,t) - \sum_{t=t_c}^{t_f-t_c} \mathcal{R}_X(t_f,t) = \langle \mathcal{O}_X\rangle_N + \mathcal{O}(e^{-\delta m t_f}),$$

on the physical pion mass ensemble F64P14 at a = 0.078fm, for the distillated interpolation fields N and also $N + c_X NO_X$.

Our method is significantly more efficient than traditional approaches and also offers better control of excited-state contamination by providing more information across different source–sink separations and nucleon interpolating fields. As illustrated in Table I, we compare the costs at the physical point for different collaborations, using g_A as an example.

	Ensembles	L	T	a(fm)	m_π(MeV)	n_{cfg}	g_A^{u-d}	Propagators	Propagators for 1% error
CLQCD	F64P13	64	128	0.074	134	40	1.24(01)	0.34M	0.11M
CalLAT	a12m130	48	64	0.121	131	1000	1.29(03)	0.03M	0.15M
ETMC	cB211.072.64	64	128	0.080	139	750	1.29(02)	1.71M	5.5M
RQCD	D452	64	128	0.076	156	1000	1.19(25)	0.01M	5.2M
PNDME	a09m130	64	96	0.090	138	1290	1.32(03)	1.69M	11.2M

TABLE. I. Comparison of computational costs for determining g_A at the physical point across different collaborations. Our method demonstrates significantly improved efficiency over traditional approaches.

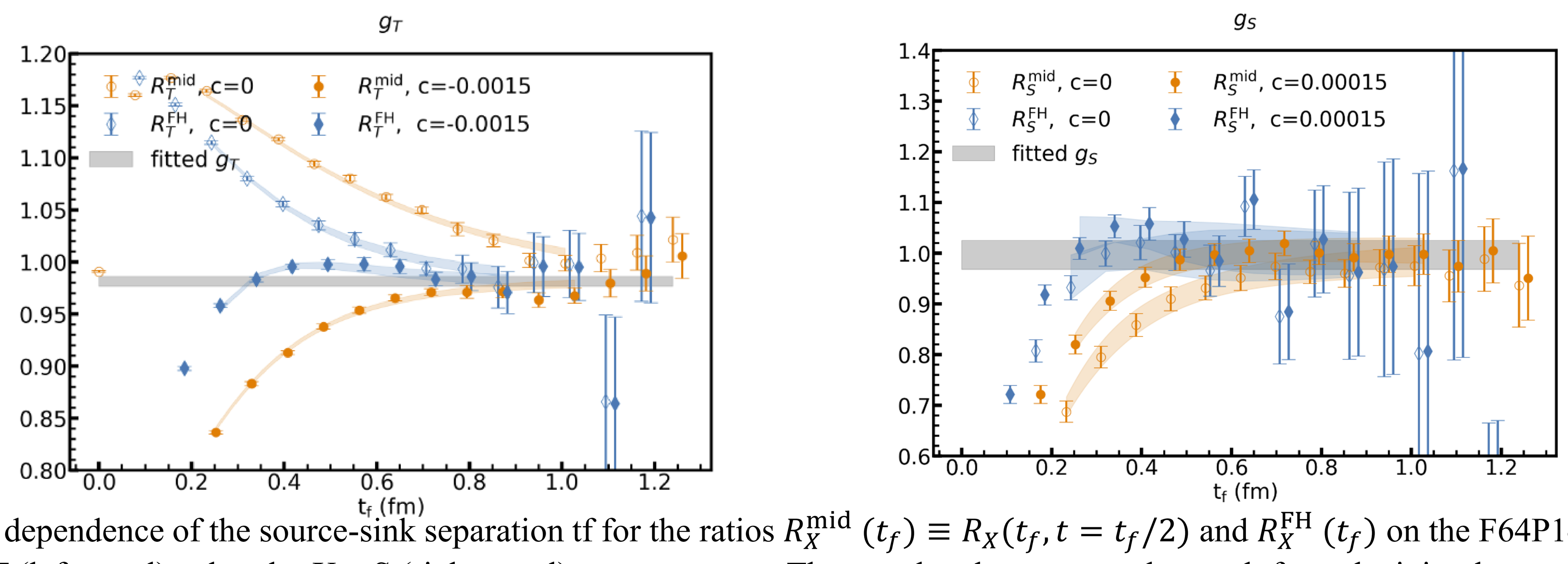

FIG. 1. The dependence of the source-sink separation tf for the ratios $R_X^{\mathrm{mid}}(t_f) \equiv R_X(t_f, t = t_f/2)$ and $R_X^{\mathrm{FH}}(t_f)$ on the F64P14 ensemble, for the tensor X = T (left panel)and scalar X = S (right panel) operator cases. The gray band represents the result from the joint three-state fit.

Results

To extract the results at the physical point in the continuum and infinite-volume limits, we employ thefollowing joint fit ansatz:

$$g_X(a,m_\pi,L) = g_X^{\mathrm{QCD}}\left(1 + \sum_{i=2,3} c_l^{(i)}(m_\pi^i - m_{\pi,\mathrm{phy}}^i)\right)(1 + c_V e^{-m_\pi L}) + c_a^{(1)}a^2,$$

where g_X^{QCD} is the target physical value. Our data strongly prefer a phenomenological exponential form $e^{-m_\pi L}$ for finite-volume effects over the functional form $m_\pi^2 e^{-m_\pi L}/\sqrt{m_\pi L}$ predicted by heavy baryon chiral perturbation theory (HBχPT)[5].

Eventually we predict $g_{T,S}^{QCD}$ with statistical and systematic errors to be:

$$g_T^{\mathrm{QCD}} = 1.0253[99]_{\mathrm{tot}}(55)_{\mathrm{stat}}(46)_a(59)_{\mathrm{FV}}(13)_\chi(34)_{\mathrm{ex}},$$
$$g_S^{\mathrm{QCD}} = 1.103[41]_{\mathrm{tot}}(32)_{\mathrm{stat}}(04)_a(26)_{\mathrm{FV}}(01)_\chi(01)_{\mathrm{ex}},$$

where the mark "stat" denotes the statistical error, while a, FV, χ and "ex" represent the systematic errors of the continuum, infinite volume, chiral extrapolation and excited state contamination, respectively. "tot" is the total error combining the statistical and systematic ones. We achieve a statistical precision for $g_{T/S}$ that is improved by a factor of three or more over all previous works, with the most substantial gains on physical-point ensembles. Our final value has a total uncertainty 1/3 smaller than the current $N_f = 2 + 1$ FLAG average[6].

Using $m_d - m_u$=2.35(12) MeV (from the FLAG average) and a QED correction of $-1.00(7)(14)$ MeV[7], we predict the neutron-proton mass difference as

$$m_n - m_p = 1.59[0.23]_{\mathrm{tot}}(0.10)_{g_S}(0.13)_{\mathrm{ISB}}(0.16)_{\mathrm{QED}}.$$

which agrees with the experimental value (1.293 MeV) within 1.3σ. However, using a newer QED correction of $-0.58(16)$ MeV[8] yields a prediction roughly 3σ higher than experiment. This discrepance underscores the importance of an up-dated direct lattice QCD+QED calculation of the QED correction.

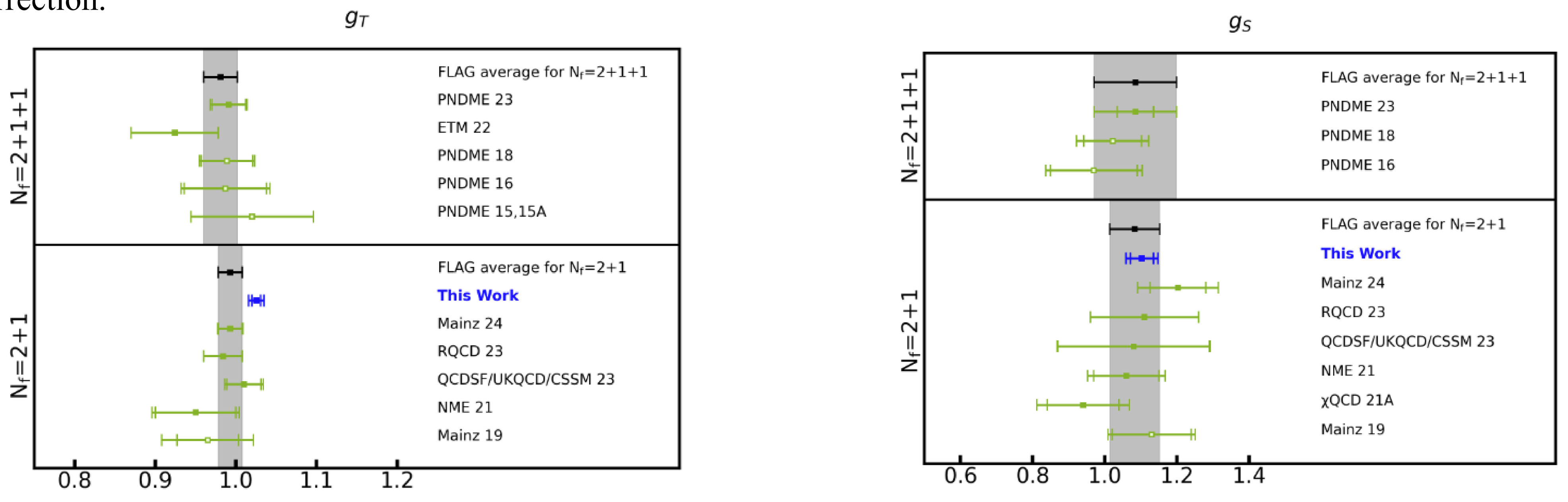

FIG. 2. Comparison of gT (left panel) and gS (right panel)from this work, other collaboration and also the FLAG averages [6].

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