Finite Volume Hamiltonian method: Connecting EFT and lattice study

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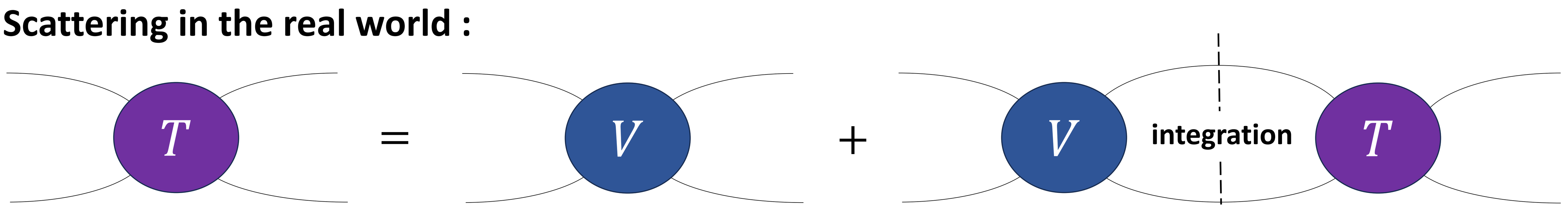
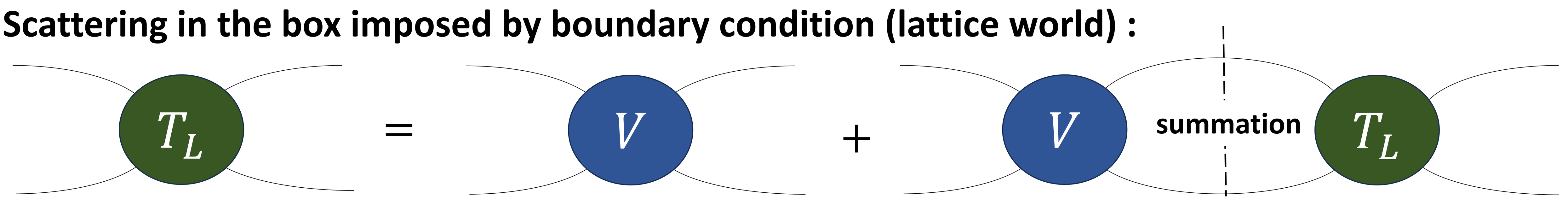
Unitarized EFT in the finite volume

Scattering in the real world :



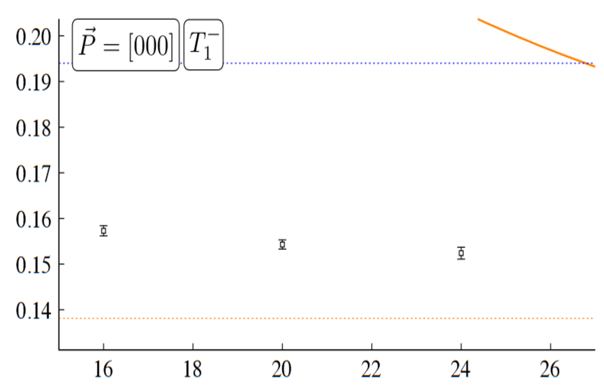
Scattering in the box imposed by boundary condition (lattice world) :



lattice spectra in the continuum limit = eigenvalues of finite volume Hamiltonian = poles of T_L

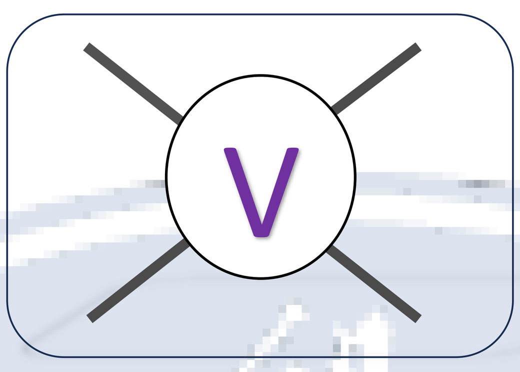
FVH method and Luscher Formula

$$T_{l'l}(p,k;E) = V_{l'l}(p,k) + \sum_{l''}\int q^2 dq V_{l'l''}(p,q) G(q;E) T_{l''l}(q,k;E) \quad , \quad G(q;E) = \left(E - \sqrt{q^2+m_1^2} - \sqrt{q^2+m_2^2}\right)^{-1}$$

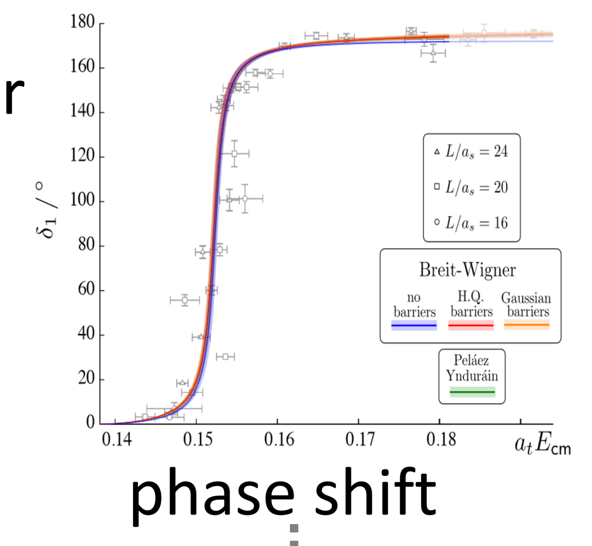


lattice spectra

Eigenvalues



Lippmann-Schwinger Equation



phase shift

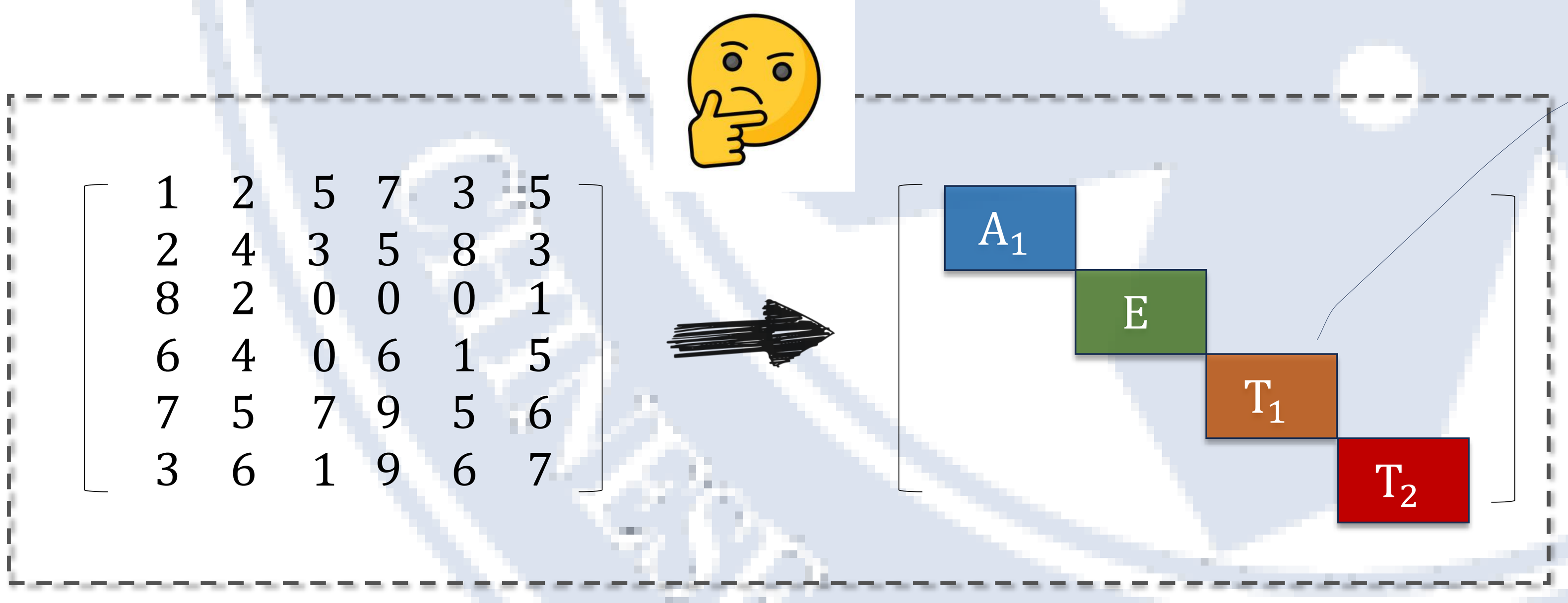
Luscher Formula

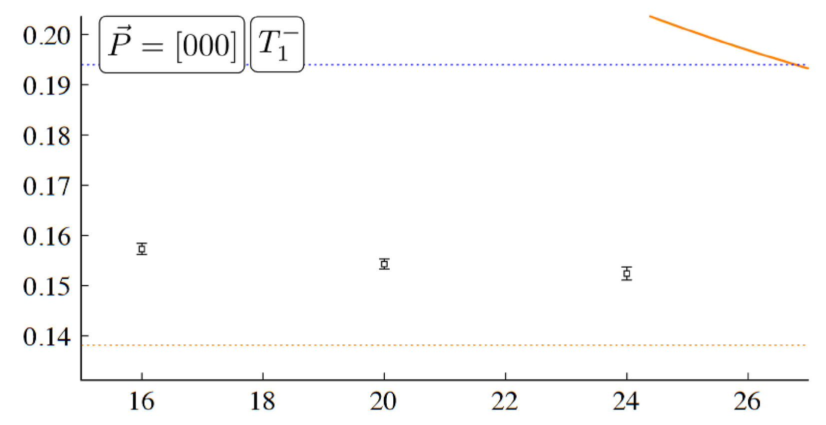
standard Luscher formula only work for short-range system ; FVH method works for all cases.

unprojected finite volume Hamiltonian matrix element $\sim \left(\frac{2\pi}{L}\right)^3 \cdot \begin{pmatrix} V\left(\frac{2\pi\vec{n}_1}{L}, \frac{2\pi\vec{n}_1}{L}\right) & V\left(\frac{2\pi\vec{n}_1}{L}, \frac{2\pi\vec{n}_2}{L}\right) & \cdots \\ V\left(\frac{2\pi\vec{n}_2}{L}, \frac{2\pi\vec{n}_1}{L}\right) & V\left(\frac{2\pi\vec{n}_2}{L}, \frac{2\pi\vec{n}_2}{L}\right) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$

irrep decomposition of FVH

Since lattice data are always extracted by operators furnishing a certain irrep, we need to project the finite volume Hamiltonian at first in order to compare with lattice spectra.





Taking spinless rest system, which is the simplest case, as an illustrative example. The workflow :

1. Decompose Hilbert space several G-orbits, e.g. span$\{g|\vec{n}_{\text{ref}}\rangle\}$ ($\vec{n}_{\text{ref}}$ is reference momentum)
2. construct irrep basis in each orbit $|\vec{n}_{\text{ref}}; T_1^-, \mu=1, r=1\rangle \propto \sum_{g\in O_h}\sum_{\nu=1}^{3} [c^{r=1}]_\nu D^{T_1^-}_{\mu=1,\nu}(g)\, |g\vec{n}_{\text{ref}}\rangle$

   where $[c^r]$ is the r-th eigenvectors of $I_{\nu'\nu} = \sum_{g\in LG(\vec{n}_{\text{ref}})} D^{T_1^-}(g)$ associated to non-zero eigenvals
3. the projected Hamiltonian element reads

$$\left\langle \vec{n}'_{\text{ref}}, r' \left| V_L^{T_1^-} \right| \vec{n}_{\text{ref}}, r \right\rangle = \left(\frac{2\pi}{L}\right)^3 \sqrt{\frac{|LG(\vec{n}_{\text{ref}})|}{|LG(\vec{n}'_{\text{ref}})|}} \sum_{g\in LC(\vec{n}_o)} \underbrace{\left[c^{r'}_{\vec{n}'_{\text{ref}}}\right]\cdot D^{T_1^-}(g)\cdot\left[c^{r}_{\vec{n}_{\text{ref}}}\right]}_{\text{“kinematical”}}\ \underbrace{V(\vec{n}', g\vec{n})}_{\text{dynamical}}$$

order of little group of $\vec{n}_{\text{ref}}$

left-coset of little group

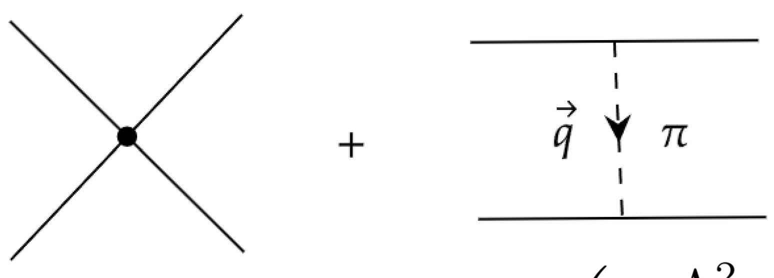
NN scattering as an example

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$
$$V = V^{(0)} + V^{(1)} + V^{(2)} + \cdots$$

effective potential → **FVH method** → **finite volume spectra**

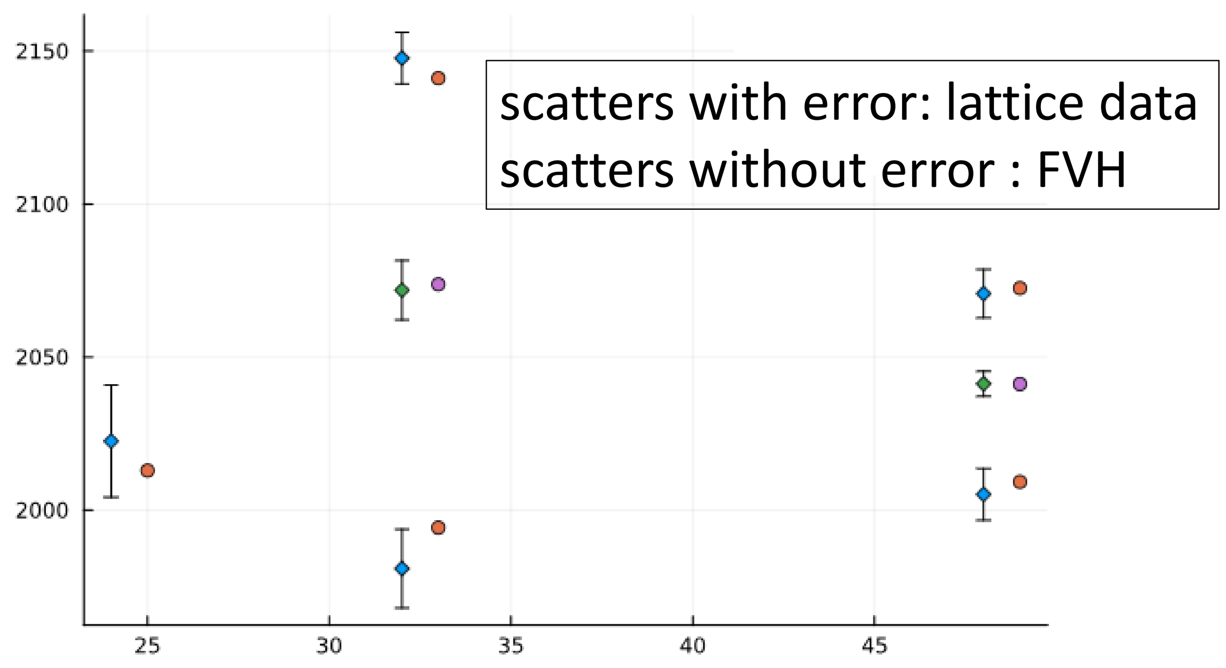
numerical package in the future

isovector NN scattering at LO:



$$V_{\text{short}} = C^{I=1}(\delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'} - \delta_{\sigma_1\sigma_2'}\delta_{\sigma_2\sigma_1'})\left(\frac{\Lambda_1^2}{p^2+\Lambda_1^2}\right)^2\left(\frac{\Lambda_1^2}{k^2+\Lambda_1^2}\right)^2$$

$$V_{\text{OPE}} = -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\vec{q}_t\cdot\vec{\sigma}_1)(\vec{q}_t\cdot\vec{\sigma}_2)}{\vec{q}_t^2+m_\pi^2} e^{-|\vec{q}_t|^2/\Lambda_{\text{long}}^2} - \ \vec{q}_t \to \vec{q}_u$$



For more details, see ***JHEP04(2025)108***