第十届手征有效场论研讨会
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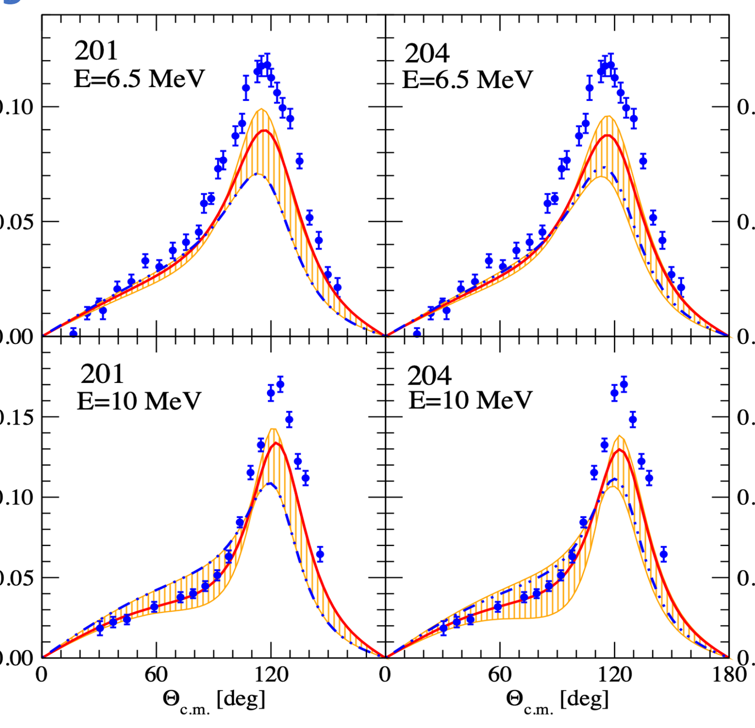
Covariant three-body scattering equation

Introduction

A_y PUZZLE

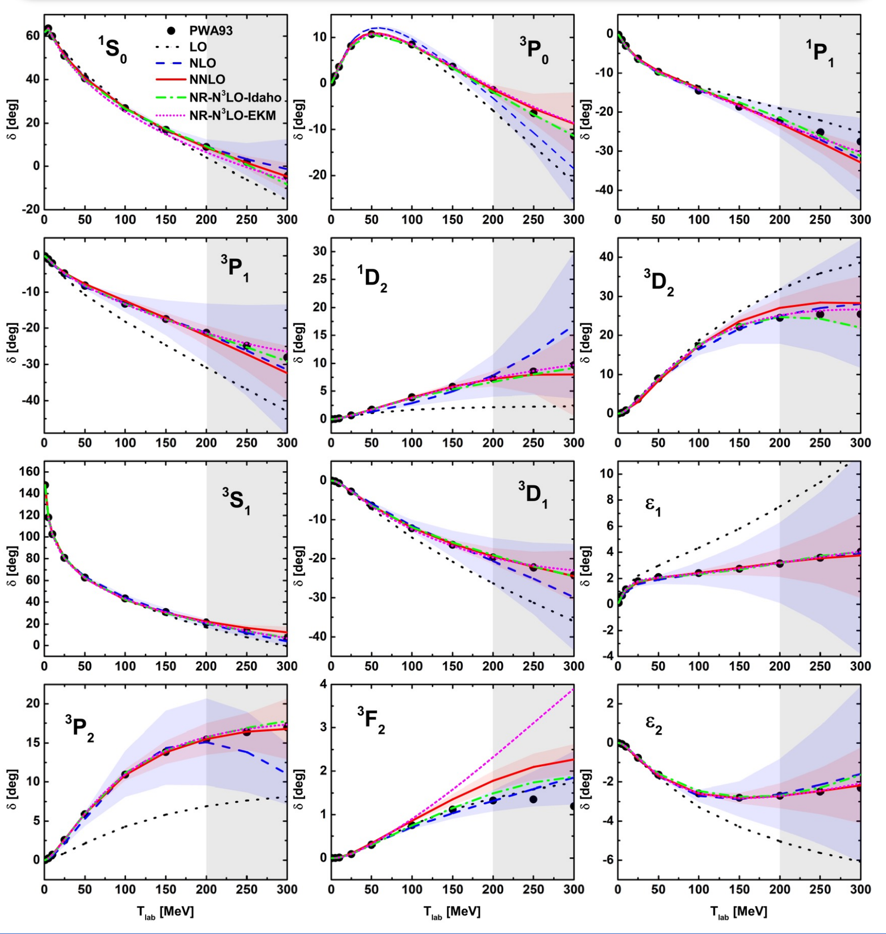
- Consider the lowest order of the three-body nuclear force – cannot solve the A_y puzzle [1];
- Higher order three-body contact terms – numerical difficulties & converge slow in terms of the chiral order.

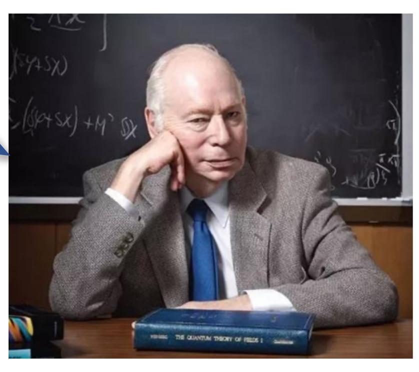


Red: NR N³LO chiral NN potential
Blue: NR N³LO chiral NN + full 3N potential

COVARIANT CHIRAL NUCLEAR FORCE

One has to write the most general Lagrangian consistent with the assumed symmetry principles, particularly the (broken) chiral symmetry of QCD.

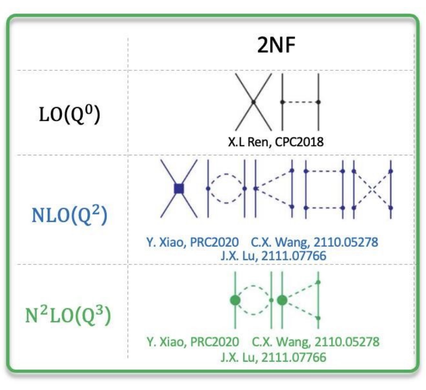




Steven Weinberg
(Nobel Prize 1979)

$\tilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$
$\tilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$
$\tilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$
$\tilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$

Covariant chiral nuclear force – LO

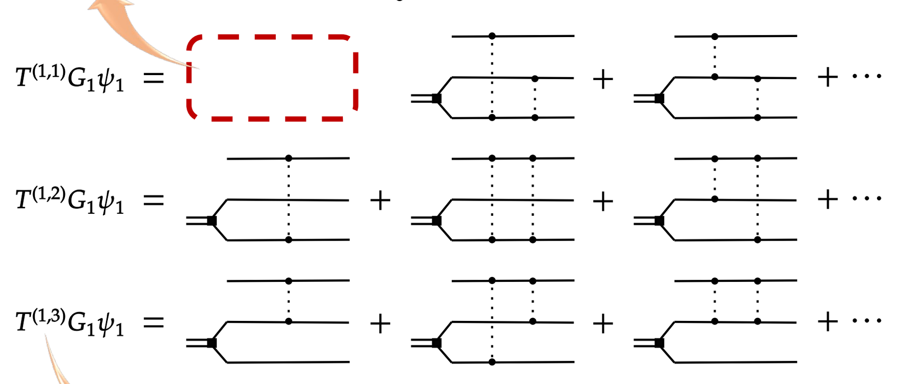


Theoretical Formalism

COVARIANT THREE-BODY SCATTERING EQUATION [3,4]

- Consider the break-up process of nd scattering, label the three particles 1,2 and 3 (top to bottom)

The deuteron wave-function has the form $\psi = vG\psi$, thus diagrams like are already taken into account.



superscript (1,3) means in the initial state particle 1 is free, and in the last interaction particle 3 is free.

$$\begin{cases} T^{(1,1)}G_1\psi_1 = t_1G_1\left(T^{(1,2)}+T^{(1,3)}\right)G_1\psi_1 \\ T^{(1,2)}G_1\psi_1 = t_2\tilde{G}_3\psi_1 + t_2G_2\left(T^{(1,1)}+T^{(1,3)}\right)G_1\psi_1 \\ T^{(1,3)}G_1\psi_1 = t_3\tilde{G}_2\psi_1 + t_3G_3\left(T^{(1,1)}+T^{(1,2)}\right)G_1\psi_1 \end{cases}$$

similar equations for the permutation of (123).

- Actually, the three nucleons are identical particles (with isospin 1/2)
  - Operators with different subscripts connected with permutation operator;
  - Antisymmetrize the wave fuction.

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}\phi_1 = \begin{pmatrix} t_1(P_{123}+P_{132}) \\ t_2(P_{132}+\mathbb{I}) \\ t_3(\mathbb{I}+P_{123}) \end{pmatrix}\frac{E_p}{m}(2\pi)^3\phi_1 + \begin{pmatrix} 0 & t_1 & t_1 \\ t_2 & 0 & t_2 \\ t_3 & t_3 & 0 \end{pmatrix} G_i \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}\phi_1$$

where $T_i\phi_1 \equiv \sum_k T^{(k,i)}\phi_k$, $\phi_i \equiv G_i\psi_i$.

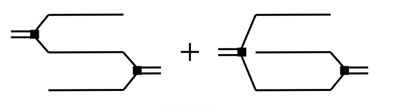
$$\begin{cases} t_2 = P_{123}t_1P_{123}^{-1} \\ t_3 = P_{132}t_1P_{132}^{-1} \end{cases}$$

three equations are not independent

$$T\phi = tP\frac{E_p}{m}(2\pi)^3\phi + tPGT\phi$$

- $\frac{E_p}{m}$ is a kinematic relativistic correction operator;
- $(2\pi)^3$ is from the convention of Feynman rules;
- $P \equiv P_{123} + P_{132}$ is the permutation operator;
- $\phi = Gv\phi$ the same form as in NR scheme.

- For elastic scattering
  - Set the final state as ϕ;
  - Add the nucleon-exchange term



$$\phi^\dagger U\phi = \phi^\dagger Pv\frac{E_q}{m}(2\pi)^3\phi + \phi^\dagger PtGU\phi$$

where $G = \frac{1}{(2\pi)^3}\frac{m^2}{E_p^2}\frac{1}{\sqrt{\left(\sqrt{s}-E_q\right)^2 - q^2 - 2E_p + i\epsilon}}$

- Difficulties

$$\langle pq\alpha|T|\phi\rangle = \langle pq\alpha|tP|\phi\rangle + \sum_{\alpha'}\sum_{\alpha''}\int_0^\infty dq'\, q'^2\int_{-1}^{+1}dx\, \frac{\tilde{t}_{\bar{\alpha}\bar{\alpha}'}(p,\pi_1,E-(3/4m)q^2)}{\pi_1^{l'}} \times G_{\alpha'\alpha''}(qq'x)\frac{1}{E+i\epsilon - q^2/m - q'^2/m - qq'x/m}\frac{\langle\pi_2 q'\alpha''|T|\phi\rangle}{\pi_2^{l''}}$$

logarithmic singularities

complex interpolation caused by permutation operator

WAVE-PACKET CONTINUUM DISCRETIZATION (WPCD) METHOD [5-9]

The continuous plane wave basis is coarse-grained into square-integrable basis that smooths out all singularities.

Wave-packet basis

- Choose a momentum cutoff $p_{\rm cut}$ and discretize continuous momenta – Chebyshev grid

$$p_n = p_s\tan\left(\frac{n}{N+N_{\rm add}+1}\frac{\pi}{2}\right), n = 0,1,2,\cdots,N,$$

$$p_s = \frac{p_{\rm cut}}{\tan\left(\frac{N}{N+N_{add}+1}\frac{\pi}{2}\right)},$$

set $p_{\rm cut} = 4.0$ GeV, $N_{\rm add} = 2$ and $N = 150$.

- Define a set of free wave-packet basis as

$$|p_i\rangle = \frac{1}{N_i}\int_{\mathcal{D}_i} p\mathrm{d}p\ f(p)|p\rangle$$

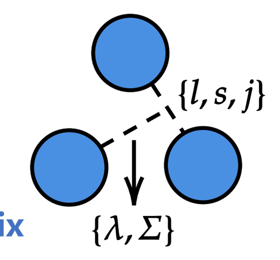
where N_i is the normalization factor and $\mathcal{D}_i = [p_{i-1}, p_i]$.
We use the energy wave-packet, i.e. $f(p) = \sqrt{\frac{p}{E_p}}$.

Partial-wave representation

- Work in the partial-wave (LS) representation

$$|p_m q_i\gamma\rangle \equiv |p_m q_i(ls)j(js_1)\Sigma(\lambda\Sigma)JM(tt_1)T\tau_T\rangle$$

we fixed $T = \frac{1}{2}$ and $\tau_T = -\frac{1}{2}$.



- Using the spin-scattering matrix

$$\mathcal{M}_{m_dm_n,m_d'm_n'}(\theta) = \frac{2\pi i}{q}\sum_{J,P}\sum_{\substack{\lambda\Sigma \\ \lambda'\Sigma'}}\sum_{\substack{m_\Sigma,m_{\Sigma'} \\ m_\lambda,M}}\langle\lambda m_\lambda\Sigma m_\Sigma|JM\rangle\langle j_dm_ds_nm_n|\Sigma m_\Sigma\rangle$$

$$\langle\lambda'0\Sigma'm_{\Sigma'}|JM\rangle\langle j_dm_d's_nm_n'|\Sigma'm_{\Sigma'}\rangle\sqrt{\frac{2\lambda'+1}{4\pi}}Y_{\lambda m_\lambda}(\theta,0)\left(S^{J^P}_{\lambda\Sigma,\lambda'\Sigma'} - \delta_{\lambda\lambda'}\delta_{\Sigma\Sigma'}\right)$$

and $S^{J^P}_{\lambda\Sigma,\lambda'\Sigma'} = \delta_{\lambda\lambda'}\delta_{\Sigma\Sigma'} - i\pi\frac{q}{(2\pi)^3}\frac{8m^2}{m_d}\frac{E_q}{\sqrt{s}}U^{J^P}_{\lambda\Sigma,\lambda'\Sigma'}$,

we calculate the observables

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{6}\mathrm{Tr}(\mathcal{M}\mathcal{M}^\dagger) \quad A_y(n) = \frac{\mathrm{Tr}(\mathcal{M}\sigma_y\mathcal{M}^\dagger)}{\mathrm{Tr}(\mathcal{M}\mathcal{M}^\dagger)}$$

Operators under wave-packet basis

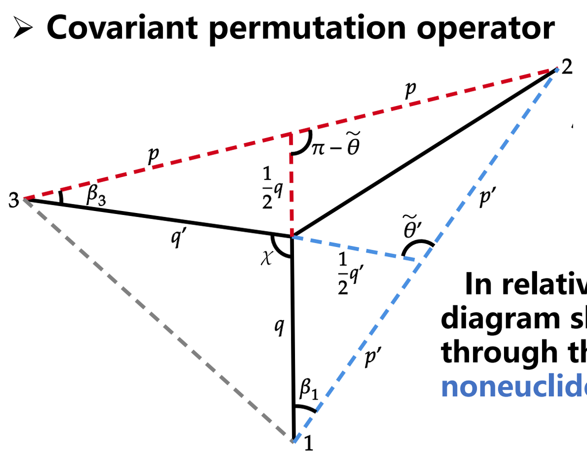
- Deuteron pole

$$\langle p_mq_i\gamma|t|p_nq_j\gamma'\rangle = \frac{1}{N}\int p\mathrm{d}p\, p'\mathrm{d}p'\, \mathrm{d}E_q\sqrt{\frac{pp'}{E_pE_{p'}}}\frac{\tilde{t}(p,p';\sqrt{s_2})}{\sqrt{s_2}-m_d+i\epsilon}$$

where $\sqrt{s_2} = \sqrt{s + m^2 - 2\sqrt{s}E_q}$.
Using the complex spectator momentum contour

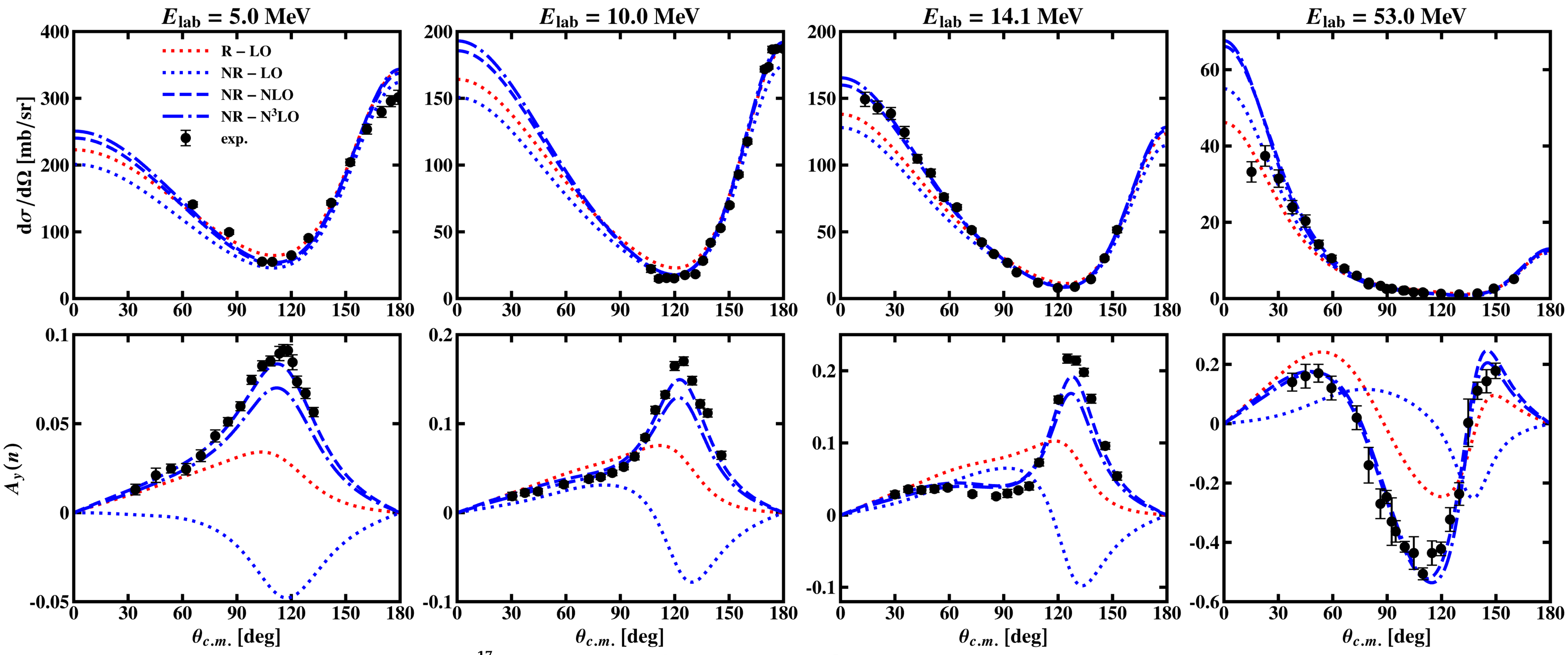
$$\Gamma_{SMC}(E_q) = E_q + iV_0\left(1 - e^{\frac{E_{q_{i-1}}-E_q}{w}}\right)\left(1 - e^{\frac{E_q - E_{q_i}}{w}}\right)$$

- Covariant permutation operator



In relativistic scheme, this diagram should be viewed through the lens of noneuclidean geometry.

Results



- Truncate the angular momentum as $j \le 2$ and $J \le \frac{17}{2}$ [10]. Dimension of the matrices ~ 10^5, thus we employ the epsilon algorithm [11] to accelerate the Neumann series;
- We calculate the nd elastic scattering in the nonrelativistic scheme with EKM chiral nuclear force at different order as a benchmark;
- We do the same calculation using our equation with the covariant NN forces at LO. The results show that covariant LO NN force describes the three-body observables much better than the NR one;
- Although the $\tilde\chi^2/\mathrm{d.o.f.}$ of R-LO is comparable to those of NR-NLO, they provide a noticeably poorer description of the three-body observables – the results may be sensitive to certain partial waves like $^3S_1 - {}^3D_1$ and $^3P_2$, which will be studied in future work.

$\tilde\chi^2 = \sum_i\left(\delta^i - \delta^i_{\rm PWA93}\right)^2$ of different chiral forces ($E_{\rm lab} \le 200$ MeV, $j \le 2$).
For LO and NR-LO, the fit was performed up to $E_{\rm lab} \le 100$ MeV and $j \le 1$.

	Total	$\tilde\chi^2/\mathrm{d.o.f.}$	$^1S_0$	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	$^3D_1$	ϵ_1	$^1D_2$	$^3D_2$	$^3P_2$	$^3F_2$	ϵ_2
LO	841.49	8.77	47.36	10.41	108.51	248.87	8.59	85.65	25.41	46.42	33.02	219.72	0.002	7.52
NR-LO	2666.9	27.78	674.01	391.53	434.08	288.28	166.02	60.55	56.10	51.46	70.52	474.30	0.01	0.03
NR-NLO	392.13	4.08	11.64	126.26	56.50	53.03	0.67	3.41	3.69	32.04	99.57	3.76	0.01	1.55

Summary & Outlook

- ✓ We constructed the covariant three-body scattering equation, which can be reduced to the well-known Faddeev-AGS equation in the nonrelativistic limit;
- ✓ In nd elastic scattering, covariant NN forces at LO can reproduce the experimental data better than the NR one;

- ☐ The same calculation with covariant NN forces at higher order need to be completed – NN forces without energy in c.m.s;
- ☐ Check if the $^3S_1 - {}^3D_1$ channel plays an important role;
- ☐ Breakup process & pd scattering.

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