QCD Sum Rule Study of Topped Mesons within Heavy Quark Effective Theory

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Introduction

Specificity of Top Quark

- Large mass: ~173.57 GeV, setting it apart from all other quark flavors.
- Extremely short lifetime: ~5×10^{-25} s (Decay faster than hadronization $1/\Lambda_{\rm QCD}$ ~10^{-24} s)

The usual consensus is that **top quark cannot form bound states**, such as toponium ($t\bar{t}$), the topped mesons ($t\bar{q}$, with $\bar{q}=\bar{u},\bar{d},\bar{s}$) and the singly topped baryons (tqq, with $q=u,d,s$).

Discovery of Toponium

However, the CMS Collaboration recently reported an excess in the $t\bar{t}$ invariant mass spectrum near threshold. **This discovery challenges the traditional views**, initiating the exploration of an entirely new top-quark hadron spectrum.

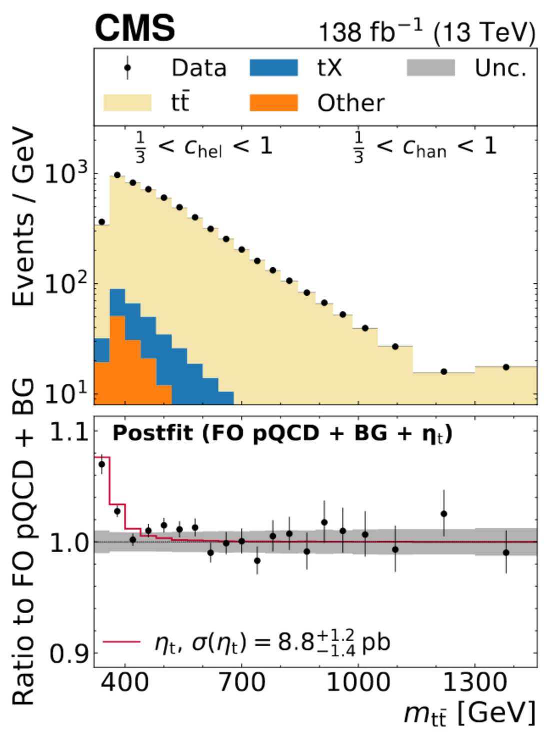

Fig.1 Discovery of Toponium. Ref [1]

Advantages of Topped Mesons

- **Longer lifetime:** Since only one top quark decays, their expected lifetimes are approximately twice that of toponium.
- **Simpler structure:** Only two types of ground states, namely pseudoscalar mesons (T and T_s) and vector mesons ($T^*$and T_s^*).
- **Easy to produce:** Only a single light antiquark involved, making it generally easier to produce than a singly topped baryon.
- **Decay products easy to observe:** Decay weakly into a Υ meson and a charmed meson.

Methodology

Heavy Quark Effective Theory

Since the mass of the top quark is much larger than $\Lambda_{\rm QCD}$, the employment of Heavy Quark Effective Theory (HQET) is very natural. In HQET, the top quark is treated as a static color source, allowing for a systematic classification of the light degrees of freedom—characterized by the light-quark spin $s_q=1/2$. Coupling the light component with the heavy-quark spin $s_Q=1/2$ yields two degenerate states with total angular momentum $J=s_Q\otimes s_q=0\oplus1$, forming an HQET doublet.

QCD Sum Rule

The interpolating currents are constructed as

$$J(x)=\bar{q}^a(x)\gamma_5 h_v^a(x),\ J_\mu(x)=\bar{q}^a(x)\gamma_\mu^t h_v^a(x)$$

And we consider the following two-point correlation function

$$\Pi_{\mu\nu}(\omega)=i\int d^4x\, e^{ik\cdot x}\langle 0|T[J_\mu(x)J_\nu^\dagger(0)]|0\rangle$$

Then, we calculate the two-point correlation function and perform the Boral transform at both the hadronic level and the QCD side. The sum rule equation are arrived as

$$\bar{\Lambda}(\omega_c,T)=\frac{1}{\Pi(\omega_c,T)}\cdot\frac{\partial\Pi(\omega_c,T)}{\partial(-2/T)},\ f^2(\omega_c,T)=\Pi(\omega_c,T)\cdot e^{2\bar{\Lambda}(\omega_c,T)/T}$$

These expressions involve two free parameters: the threshold value ω_c and the Borel mass T. We impose three standard QCD sum rule criteria to determine them.

- The convergence of OPE $\le 20\%$
- the pole contribution $\ge 40\%$
- the Borel stability condition

We extend our analysis to incorporate $\mathcal{O}(1/m_Q)$ corrections to evaluate the mass correction δm. Finally, the mass of the ground-state T_s^*meson is given by

$$m_{T_s^*}=m_t+\bar{\Lambda}+\delta m$$

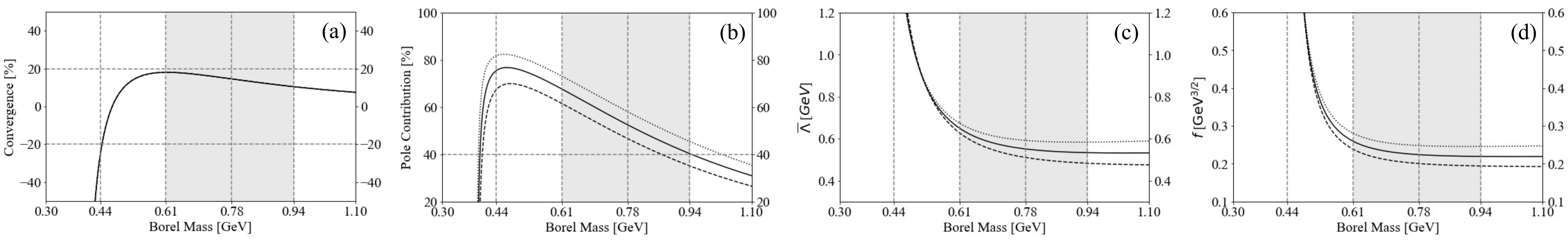

Fig.2 Variations of (a) the convergence parameter, (b) the pole contribution, (c) the residual mass $\bar{\Lambda}$ and (d) the decay constant f with respect to the Borel mass T

Results

- The masses of these mesons lie around 173.1GeV, approximately 0.5~0.6 GeV above the top quark's pole mass.

For comparison, ground-state bottom mesons are typically located about 0.5~0.6 GeV above the bottom quark pole mass, whereas charmed mesons usually lie 0.2 ~0.4 GeV above the charm quark pole mass.

- A topped meson is anticipated to decay weakly into a Υ meson and a charmed meson, as shown in Fig.3. Several possible decay channels are given in the last column of Table 1.

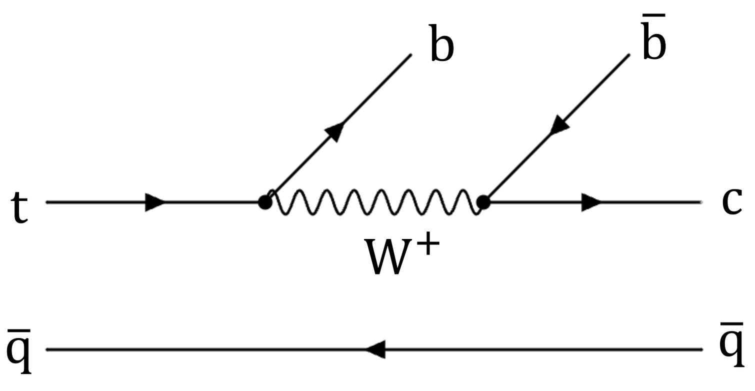

Fig.3 Feynman diagram of topped meson decay

Table 1. Parameters of ground-state topped mesons calculated using QCD sum rules within the HQET framework. The states $T^{(*)}$ and $T_s^{(*)}$ contain the quark contents $t\bar{q}(\bar{q}=\bar{u}/\bar{d})$ and $t\bar{s}$, respectively.

	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	Meson (J^P)	Mass (GeV)	Difference (MeV)	f ($\mathrm{GeV}^{3/2}$)	Decay channels
$T^{(*)}$	1.65	$0.79\le T\le0.89$	$0.46^{+0.07}_{-0.07}$	$T\ (0^-)$	$173.03^{+0.30}_{-0.30}$	$0.12^{+0.04}_{-0.04}$	$0.19^{+0.02}_{-0.02}$	$\Upsilon D^{(*)}$, $\bar{B}_c^{(*)}B^{(*)}$, $B_s^{(*)}D^{(*)}$, $D_s^{(*)}B^{(*)}$
				$T^*\ (1^-)$	$173.03^{+0.30}_{-0.30}$			
$T_s^{(*)}$	1.85	$0.61\le T\le0.94$	$0.55^{+0.12}_{-0.07}$	$T_s\ (0^-)$	$173.12^{+0.31}_{-0.30}$	$0.11^{+0.04}_{-0.04}$	$0.22^{+0.04}_{-0.03}$	$\Upsilon D_s^{(*)}$, $\bar{B}_c^{(*)}B_s^{(*)}$, $B_s^{(*)}D_s^{(*)}$
				$T_s^*\ (1^-)$	$173.12^{+0.31}_{-0.30}$			

Discussions and Conclusion

A key aspect of the mass of topped meson analysis is the choice of renormalization scheme for the top quark mass. Unlike light quarks, the gap between the two schemes of top quark becomes significantly larger:

$$m_t^{\overline{\rm MS}}=162.5^{+2.1}_{-1.5}\ \mathrm{GeV},\ \ m_t^{\rm pole}=172.57^{+0.29}_{-0.29}\ \mathrm{GeV}$$

We adopt the pole mass of top quark in our final evaluation after considering the following reasons:

- This result highlights the robustness of the HQET framework even in the extreme mass regime of the top quark.
- Once electroweak corrections are included, the $\overline{\rm MS}$ top-quark mass approaches the pole mass.

In summary, this work presents the first application of QCD sum rules within the HQET framework to mesonic systems containing a top quark. The extremely short lifetime places topped hadrons in a transitional regime where weak decay competes directly with strong binding, providing a unique setting to probe the onset of hadron formation at the extreme heavy-quark mass scale.

Reference

1. A. Hayrapetyan, et al. [The CMS Collaboration] Observation of a pseudoscalar excess at the top quark pair production threshold. Rept. Prog. Phys. 2025, 88, 087801.
2. S.W. Zhang, W.H. Tan, X. Luo, H.X. Chen, Topped baryons from QCD sum rules, arXiv.2507.05895.