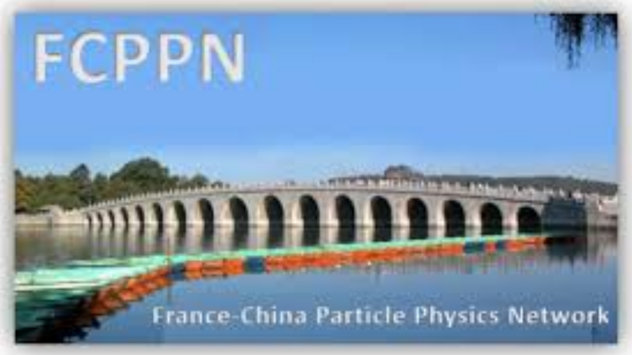
Quarkonium studies at the LHC and future facilities

Quarkonium4AFTER Franco-Chinese collaboration

Hua-Sheng Shao

16th FCPPN/L Workshop
Shandong University, Qingdao, China
21-25 July 2025



FCPPN/L Collaboration

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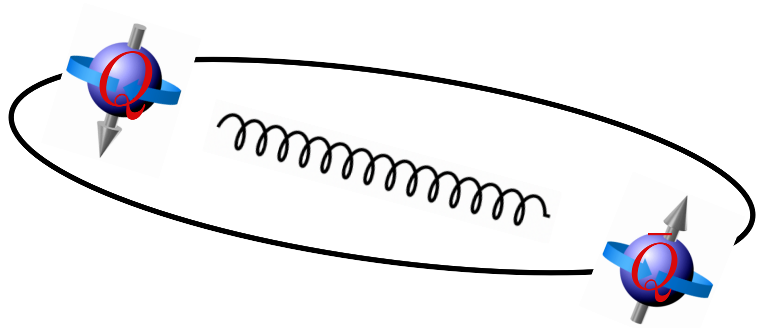
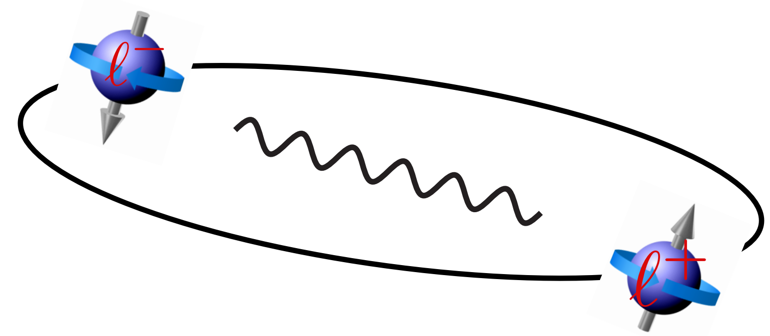
- **Theory and experiment of quarkonium studies**

	France	China
Leaders	H.-S. Shao (LPTHE)	Y.-Q. Ma (PKU)
Permanent	B. Fuks (LPTHE) C. Hadjidakis (IJCLab) J.-P. Lansberg (IJCLab) S. Wallon (IJCLab) L. Massacrier (IJCLab) M. Ozcelik (IJCLab)	K.-Q. Chao (PKU) Y. Gao (PKU) L. An (PKU) Z. Yang (PKU) Y. Zhang (PKU) J.-X. Wang (IHEP) B. Gong (IHEP) Y.-J. Zhang (Beihang) J. He (UCAS) Z. Tang (USTC) X. Bai (USTC) H.-F. Zhang (Guiyang) Y. Feng (Chongqing) K. Yi (Nanjing & THU) A.-P. Chen (Jiangxi)
Non-permanent	L. Maxia (LPTHE) L. Simon (LPTHE) G. Wang (LPTHE) M. Guicheneuy (LPTHE) C. Flett (IJCLab) K. Lynch (IJCLab) J. Bor (IJCLab) D. Daskalas (IJCLab) A. JOHNRUBESH RAJAN (IJCLab)	

- 32 members
- France (16) & China (16)
- 23 TH and 9 EXP
- 23 permanent, 4 postdocs, 5 PhD students

Quarkonium and Leptonium Simulations with MadGraph5_aMC@NLO





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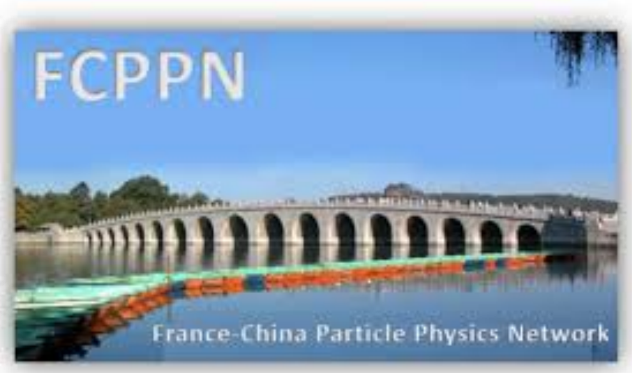


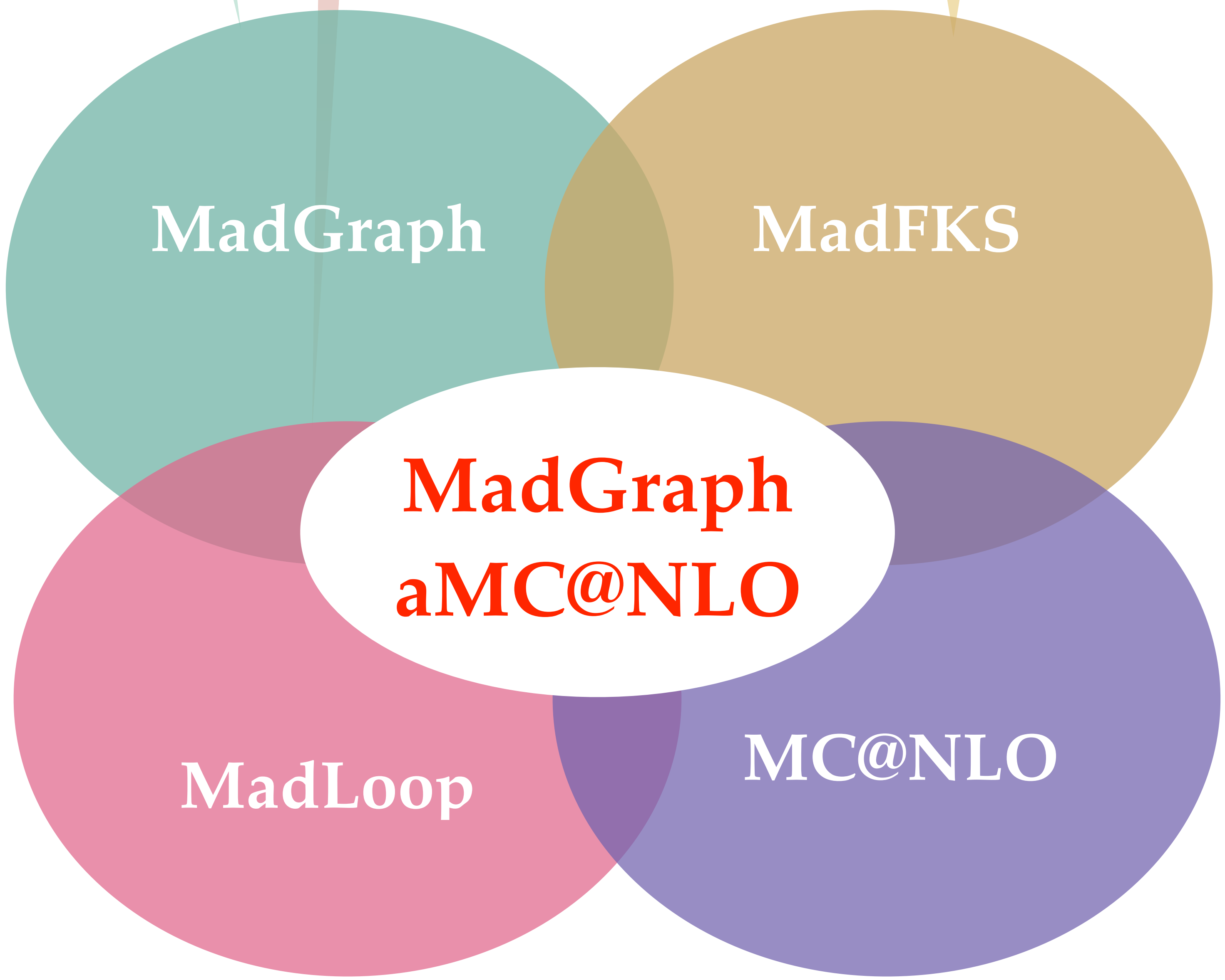
The Path to NLO Automation

$$\sigma_{\rm NLO} = \int d\phi_n \mathcal{B} + \int d\phi_n \mathcal{V} + \int d\phi_n \int d\phi_1 \mathcal{S} + \int d\phi_{n+1} \left(\mathcal{R} - \mathcal{S}\right)$$

Automatic tree level matrix element generator

SM and BSM

Automation of real correction using FKS subtraction



Automation of virtual corrections

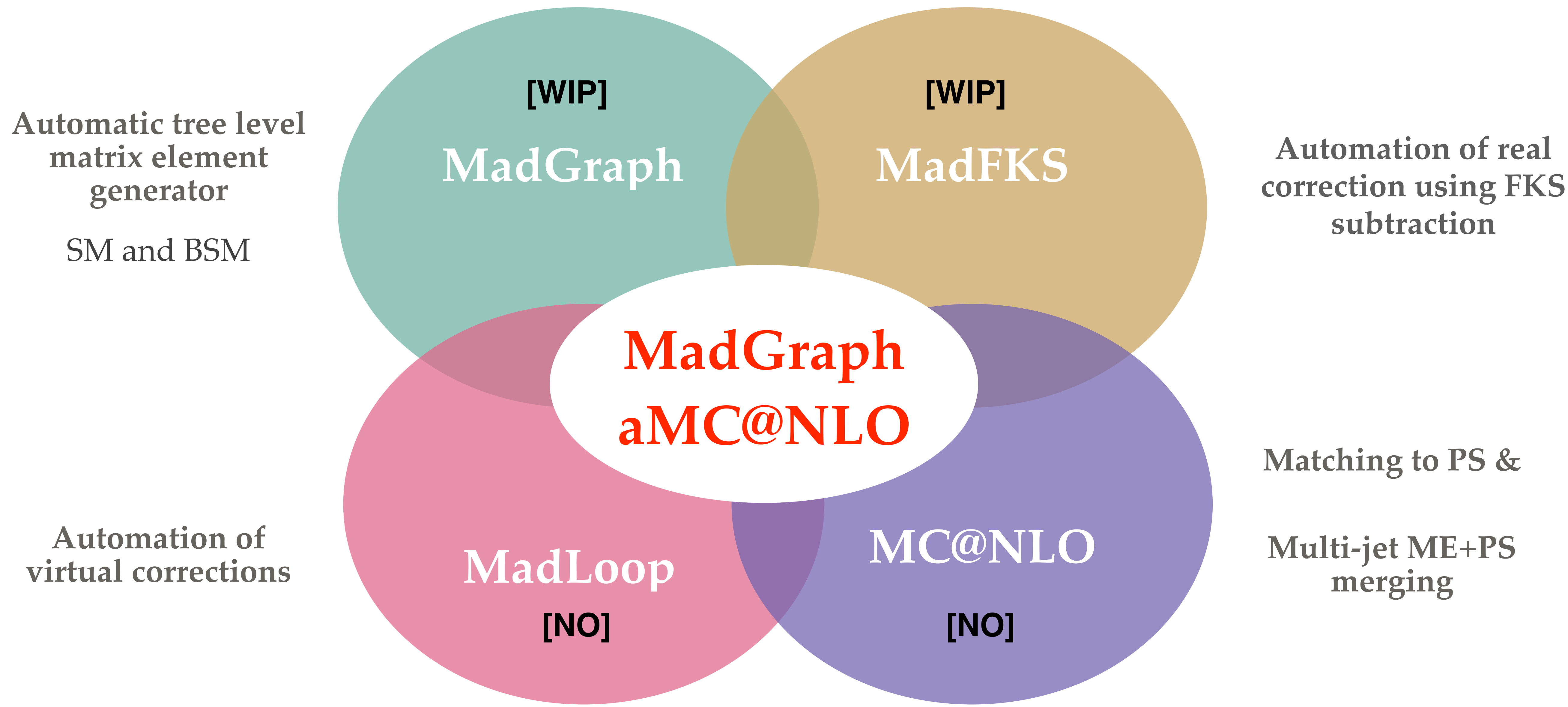
Matching to PS &

Multi-jet ME+PS merging

The Path to NLO Automation

- **Quarkonium requires many new TH developments**
  - Both on the conceptual and technical points of views

The status of last FCPPN/L Workshop in Bordeaux



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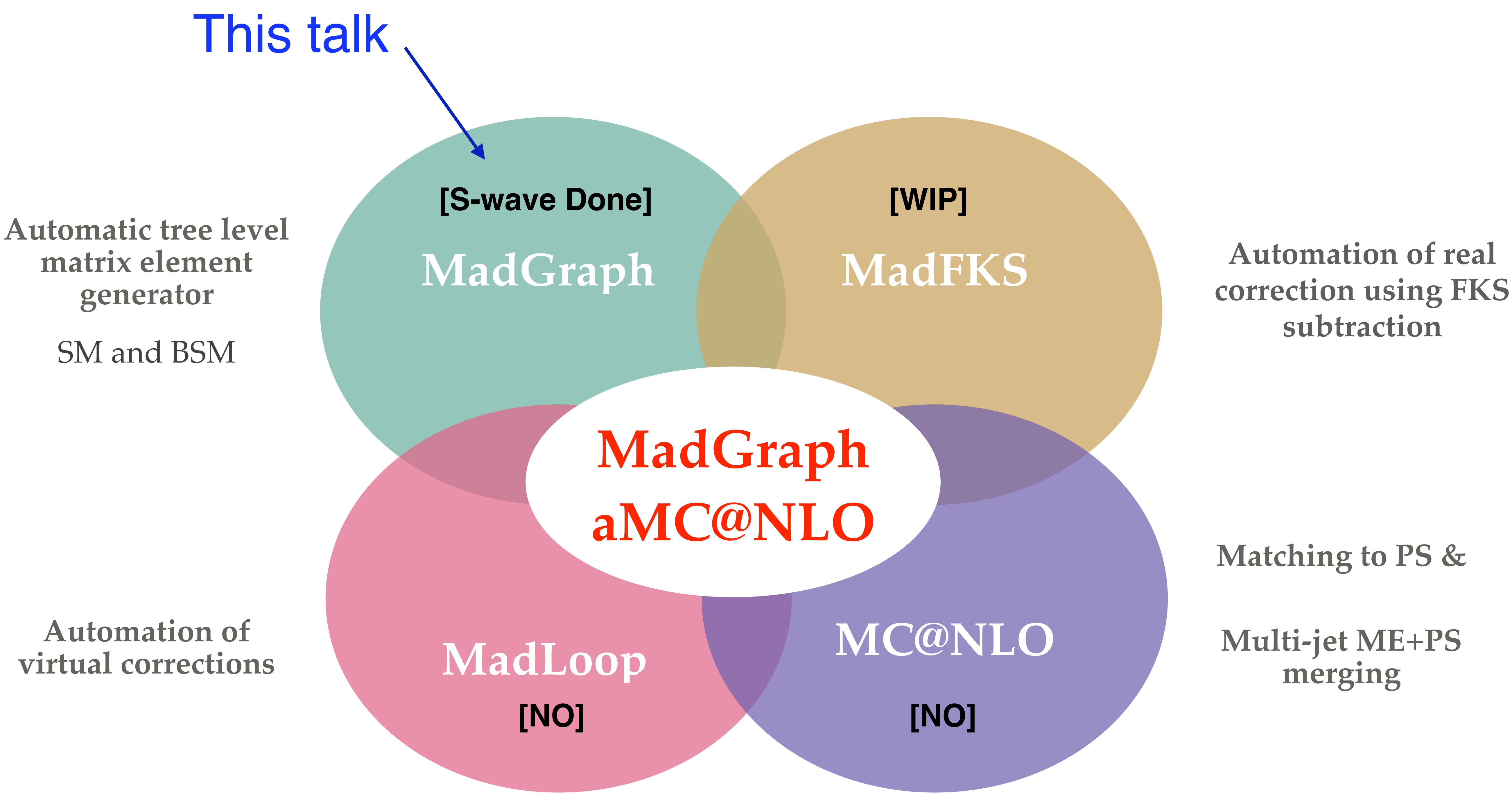

Quarkonium production in NRQCD

- **Consider a single quarkonium H production**

$$d\sigma(AB \to H + X) = \sum_n \left(\sum_{a,b,X} \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) d\hat{\sigma}(ab \to Q\bar{Q}'[n] + X) \right) \langle \mathcal{O}_n^H \rangle$$

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PDF: parton distribution function

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(Assumed) perturbative & process dependent & rich physics

Building amplitudes for quarkonium

- **We need to start from open Q and $\bar{Q}'$ production amp.**

$$a(k_1)b(k_2) \to Q(k_3)\bar{Q}'(k_4) + \ldots$$

$$\boxed{\mathcal{A} = \bar{u}_{\lambda_Q}(k_3)\Gamma v_{\lambda_{\bar{Q}'}}(k_4)}$$

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↓ Color Projection

$$\mathcal{A}_{\{[C]\}} = \sum_{c_3,c_4} \mathbb{P}_C \mathcal{A}$$

Color Projectors:

$$\mathbb{P}_{C=1} = \frac{\delta_{c_4 c_3}}{\sqrt{N_c}}$$

$$\mathbb{P}_{C=8} = \sqrt{2} t^{c_{34}}_{c_4 c_3}$$

Building amplitudes for quarkonium

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↓ Spin Projection

$$\mathcal{A}_{\{[C],S\}} = \sum_{\lambda_Q,\lambda_{\bar{Q}'}} \mathbb{P}_S \mathcal{A}_{\{[C]\}}$$

Spin Projectors:

$$\mathbb{P}_{S=0} = \frac{\bar{v}_{\lambda_{\bar{Q}'}}(k_4)\gamma_5 u_{\lambda_Q}(k_3)}{2\sqrt{2m_Q m_{\bar{Q}'}}}$$

$$\mathbb{P}_{S=1} = \frac{\bar{v}_{\lambda_{\bar{Q}'}}(k_4)\not{\epsilon}^*_{\lambda_s}(K)u_{\lambda_Q}(k_3)}{2\sqrt{2m_Q m_{\bar{Q}'}}}$$

$$k^\mu_{3,4} = \frac{m_Q}{m_Q + m_{\bar{Q}'}}K^\mu \pm q^\mu$$

Building amplitudes for quarkonium

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$$\mathcal{A}_{\{[C],S\}} = \sum_{\lambda_Q,\lambda_{\bar{Q}'}} \mathbb{P}_S \mathcal{A}_{\{[C]\}}$$

↓ Orbital angular momentum expansion

$$\mathcal{A}_{\{[C],S,L\}} = \left[\left(\varepsilon^{\mu,*}_{\lambda_l}(K)\frac{d}{dq^\mu}\right)^L \mathcal{A}_{\{[C],S\}}\right]_{q=0}$$

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Building amplitudes for quarkonium

- **We need to start from open Q and $\bar{Q}'$ production amp.**

$$a(k_1)b(k_2) \to Q(k_3)\bar{Q}'(k_4) + \dots$$

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↓ Spin Projection

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↓ Orbital angular momentum expansion

$$\mathcal{A}_{\{[C],S,L\}} = \left[\left(\varepsilon^{\mu,*}_{\lambda_l}(K)\frac{d}{dq^\mu}\right)^L \mathcal{A}_{\{[C],S\}}\right]_{q=0}$$

↓ Total angular momentum projection

$$\mathcal{A}_{\{[C],S,L,J\}} = \sum_{\lambda_s,\lambda_l} \mathbb{P}_J \mathcal{A}_{\{[C],S,L\}}$$

Clebsch-Gordan coefficient:

$$\mathbb{P}_J = \langle J, \lambda_j | L, \lambda_l; S, \lambda_s \rangle$$

Quarkonium in NRQCD: velocity expansion

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- **Upto a given order in v, we have finite number of Fock states**

$$|J/\psi\rangle = \underbrace{|c\bar{c}[^3S_1^{[1]}]\rangle}_{\mathcal{O}(1)} + \underbrace{|c\bar{c}[^3P_J^{[8]}] + g\rangle}_{\mathcal{O}(v)} + \underbrace{|c\bar{c}[^1S_0^{[8]}] + g\rangle}_{\mathcal{O}(v^{\frac{3}{2}})} + \underbrace{|c\bar{c}[^3S_1^{[8]}] + gg\rangle}_{\mathcal{O}(v^2)} + \ldots$$

Fock states $c\bar{c}[^{2S+1}L_J^{[C]}]$ are treated as elementary particles

Physical states are a collection of Fock states

$$d\sigma(AB \to H + X) = \sum_n \left(\sum_{a,b,X} \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) d\hat{\sigma}(ab \to Q\bar{Q}'[n] + X) \right) \langle \mathcal{O}_n^H \rangle$$

$H = J/\psi$ $\quad n = {}^3S_1^{[1]}, {}^3P_J^{[8]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}$ up to $\mathcal{O}(v^7)$

Quarkonium in NRQCD: velocity expansion

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Fock states $c\bar{c}[^{2S+1}L_J^{[C]}]$ are treated as elementary particles

→ Physical states are a collection of Fock states

Power counting	η_Q	ψ, Υ	h_Q	χ_{QJ}
v^3	$^1S_0^{[1]}$	$^3S_1^{[1]}$	—	—
v^5	—	—	$^1P_1^{[1]}, {}^1S_0^{[8]}$	$^3P_J^{[1]}, {}^3S_1^{[8]}$
v^7	$^1S_0^{[8]}, {}^3S_1^{[8]}, {}^1P_1^{[8]}$	$^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}$	—	—

Existing tools for quarkonium

- **MadOnia** P. Artoisenet, F. Maltoni, T. Stelzer (JHEP'08)
  - Based on obsolete MadGraph/MadEvent v4
  - Processes with single quarkonium only
  - The code is dead (no maintenance & new developments)
- **HELAC-Onia** HSS (CPC'13, CPC'16)
  - Based on HELAC-Phegas (recursion relations)
  - Processes with any number of S- or P-wave quarkonia (in principle)
  - No plan to NLO with HELAC-NLO (I am not familiar with)
- **My natural choice of realizing NLO+PS for arbitrary quarkonium processes is MadGraph5_aMC@NLO**
  - How to deal with IR divergences is known Ajjath, Simon, HSS (JHEP'24)
  - Possibly to include quarkonium analogous too (leptonium, squarkonium, gluinonium, darkonium etc)

Colpani Serri, Flett, Lansberg, Mattelaer, HSS, Simon (in prep)

- **Helicity amplitudes checked against with HELAC-Onia**

process	MadGraph5_aMC@NLO HELAC-Onia	rel. ratio
$gg \to J/\psi\left[{}^3S_1^{[8]}\right] g$	0.004132917971335 0.004132917971659	$7.8 \cdot 10^{-11}$
$gg \to 2\,J/\psi\left[{}^3S_1^{[8]}\right] g$	$7.885077765242083 \cdot 10^{-11}$ GeV^{-2} $7.885077766678929 \cdot 10^{-11}$ GeV^{-2}	$1.8 \cdot 10^{-10}$
$gg \to J/\psi\left[{}^3S_1^{[1]}\right] \Upsilon\left[{}^3S_1^{[1]}\right] g$	$1.156767080922911 \cdot 10^{-15}$ GeV^{-2} $1.156765946402274 \cdot 10^{-15}$ GeV^{-2}	$9.8 \cdot 10^{-7}$
$gg \to J/\psi\left[{}^3S_1^{[1]}\right] \Upsilon\left[{}^3S_1^{[1]}\right] J/\psi\left[{}^3S_1^{[8]}\right]$	$4.332491835139833 \cdot 10^{-20}$ GeV^{-2} $4.332487584607067 \cdot 10^{-20}$ GeV^{-2}	$9.8 \cdot 10^{-7}$
$gg \to J/\psi\left[{}^3S_1^{[1]}\right] e^+e^- g$	$1.104536809365942 \cdot 10^{-15}$ GeV^{-4} $1.104524494048194 \cdot 10^{-15}$ GeV^{-4}	$1.1 \cdot 10^{-5}$
$d\bar{d} \to J/\psi\left[{}^3S_1^{[1]}\right] H$	$1.659846131267172 \cdot 10^{-10}$ $1.659837876796703 \cdot 10^{-10}$	$5.0 \cdot 10^{-6}$
$gg \to \eta_c\left[{}^1S_0^{[1]}\right] ggg$	$5.306389168052851 \cdot 10^{-11}$ GeV^{-4} $5.306385508474058 \cdot 10^{-11}$ GeV^{-4}	$6.9 \cdot 10^{-7}$
$gg \to \eta_c\left[{}^1S_0^{[8]}\right] ggg$	$7.538826464024039 \cdot 10^{-13}$ GeV^{-4} $7.53882646469246 \cdot 10^{-13}$ GeV^{-4}	$8.9 \cdot 10^{-11}$
$\gamma g \to \eta_c\left[{}^1S_0^{[1]}\right] ggg$	$2.168626920441142 \cdot 10^{-14}$ GeV^{-4} $2.168625104725573 \cdot 10^{-14}$ GeV^{-4}	$8.4 \cdot 10^{-7}$
$u\bar{u} \to \eta_c\left[{}^1S_0^{[1]}\right] c\bar{c}$	$1.559730896106226 \cdot 10^{-12}$ GeV^{-2} $1.559730081676616 \cdot 10^{-12}$ GeV^{-2}	$5.2 \cdot 10^{-7}$
$e^+e^- \to J/\psi\left[{}^1S_0^{[8]}\right] c\bar{c}$	$1.304158214898919 \cdot 10^{-15}$ GeV^{-2} $1.304157872301759 \cdot 10^{-15}$ GeV^{-2}	$2.6 \cdot 10^{-7}$
$e^+e^- \to J/\psi\left[{}^3S_1^{[1]}\right] gg$	$1.843805235534046 \cdot 10^{-14}$ GeV^{-2} $1.843803158649872 \cdot 10^{-14}$ GeV^{-2}	$1.1 \cdot 10^{-6}$
$u\bar{u} \to B_c^{*+}\left[{}^3S_1^{[1]}\right] b\bar{c}$	$1.675806738379629 \cdot 10^{-11}$ GeV^{-2} $1.675811485415695 \cdot 10^{-11}$ GeV^{-2}	$2.8 \cdot 10^{-6}$
$gg \to B_c^+\left[{}^1S_0^{[1]}\right] b\bar{c}$	$3.484406228037889 \cdot 10^{-12}$ GeV^{-2} $3.484415672339772 \cdot 10^{-12}$ GeV^{-2}	$2.7 \cdot 10^{-6}$

A new implementation in MG5_aMC

Colpani Serri, Flett, Lansberg, Mattelaer, HSS, Simon (in prep)

- **Cross sections checked against with HELAC-Onia too**

process	MadGraph5_aMC@NLO HELAC-Onia	pull
$u\bar{u} \to J/\psi\left[{}^3S_1^{[8]}\right] + H$	42.055(2) yb 42.056(3) yb	0.28
$gg \to J/\psi\left[{}^3S_1^{[8]}\right] + H$	1.8530(7) ab 1.8523(7) ab	0.71
$gg \to J/\psi\left[{}^3S_1^{[8]}\right] + HH$	15.927(3) yb 15.93(2) yb	0.15
$gg \to J/\psi\left[{}^3S_1^{[8]}\right] + HHH$	1.9802(5) rb 1.967(4) rb	3.27
$gg \to J/\psi\left[{}^3S_1^{[8]}\right] + g$	8.9215(7) μb 8.927(2) μb	2.60
$gg \to J/\psi\left[{}^3S_1^{[1]}\right] + J/\psi\left[{}^3S_1^{[1]}\right]$	8.921(2) nb 8.916(4) nb	1.12
$gg \to J/\psi\left[{}^3S_1^{[8]}\right] + J/\psi\left[{}^3S_1^{[8]}\right]$	86.240(7) pb 86.27(2) pb	1.42
$gg \to \eta_c\left[{}^1S_0^{[8]}\right] + \eta_b\left[{}^1S_0^{[8]}\right]$	195.984(9) fb 195.987(9) fb	0.24
$u\bar{u} \to \eta_c\left[{}^1S_0^{[1]}\right] + J/\psi\left[{}^3S_1^{[8]}\right]$	152.79(1) fb 152.73(6) fb	0.99
$u\bar{u} \to \eta_c\left[{}^1S_0^{[1]}\right] + \Upsilon\left[{}^3S_1^{[1]}\right]$	212.90(2) zb 212.9(1) zb	0.00
$u\bar{u} \to B_c^+\left[{}^1S_0^{[1]}\right] + B_c^{*-}\left[{}^3S_1^{[1]}\right]$	2.7920(5) pb 2.7925(7) pb	0.58
$e^+e^- \to J/\psi\left[{}^3S_1^{[1]}\right] + Z$	1.61586(9) fb 1.61584(8) fb	0.17

A new implementation in MG5_aMC

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Colpani Serri, Flett, Lansberg, Mattelaer, HSS, Simon (in prep)

- **Also an event generator for leptonium (NEW !)**

Positronium

process	σ	process	σ
$pp \xrightarrow{\gamma\gamma} p\ Ps_0\ p$	14.30(4) nb	$pp \to Ps_1$	7.5304(6) yb
$pp \to Ps_0 + \gamma$	3.3348(6) yb	$pp \to Ps_1 + j$	9.483(2) fb
$e^+e^- \to Ps_0 + \gamma$	3.300(1) ab	$e^+e^- \to Ps_1 + \gamma$	8.420(2) ab

True tauonium

process	σ	process	σ
$pp \xrightarrow{\gamma\gamma} p\ \mathcal{T}_0\ p$	108.055(7) ab	$pp \to \mathcal{T}_1$	9.1906(4) fb
$pp \to \mathcal{T}_0 + \gamma$	3.4522(3) ab	$pp \to \mathcal{T}_1 + j$	6.055(1) fb
$e^+e^- \to \mathcal{T}_0 + \gamma$	110.001(3) zb	$e^+e^- \to \mathcal{T}_1 + \gamma$	9.576(2) ab

True muonium

process	σ	process	σ
$pp \xrightarrow{\gamma\gamma} p\ TM_0\ p$	87.953(5) fb	$pp \to TM_1$	47.743(4) ab
$pp \to TM_0 + \gamma$	143.03(3) zb	$pp \to TM_1 + j$	9.460(2) fb
$e^+e^- \to TM_0 + \gamma$	438.16(1) yb	$e^+e^- \to TM_1 + \gamma$	8.422(2) ab

Checked against with

d'Enterria, HSS (PLB'23)

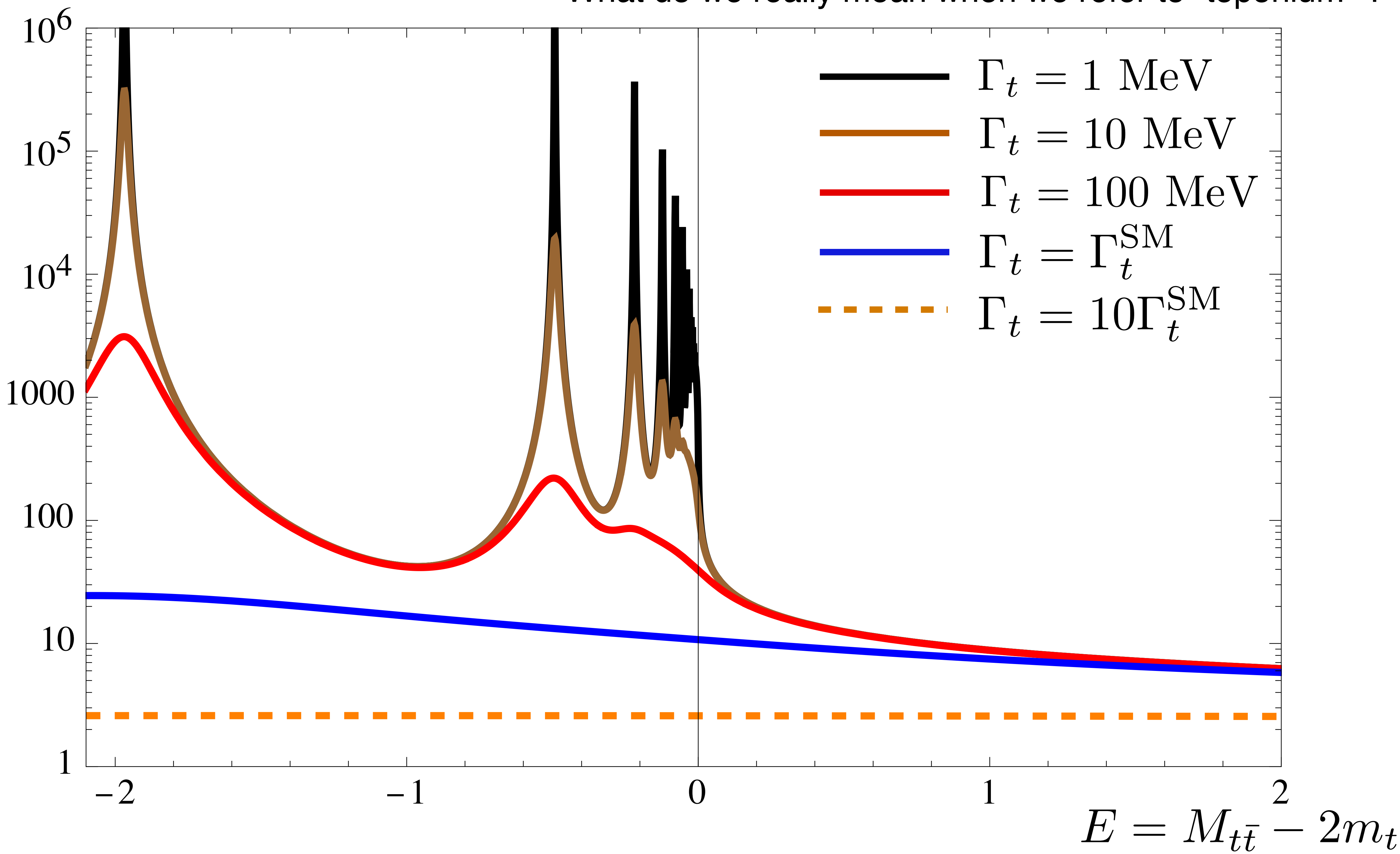
Conclusion & Outlook

- Quarkonium physics is theoretically hard since quarkonium is
  - neither sufficiently heavy (that perturbation theory alone is sufficient)
  - nor sufficiently light (that lattice QCD easily applies)
- But it is also the reason why quarkonium physics is interesting
  - related physics is rich & probing perturbative and non-perturbative regimes simultaneously
- Understanding quarkonium requires dedicated efforts both on the theory and experiment sides.
- Steady (though slow) progress in Monte Carlo tools development
- Next steps:
  - P-wave (requiring dedicated construction in MG5aMC)
  - Onium-like states in BSM (squarkonium, gluinonium, darkonium ...)
  - NLO -> NLO+PS (improving many steps needed)

Conclusion & Outlook

- BUT … new complications may arise (such as toponium)

What do we really mean when we refer to "toponium" ?



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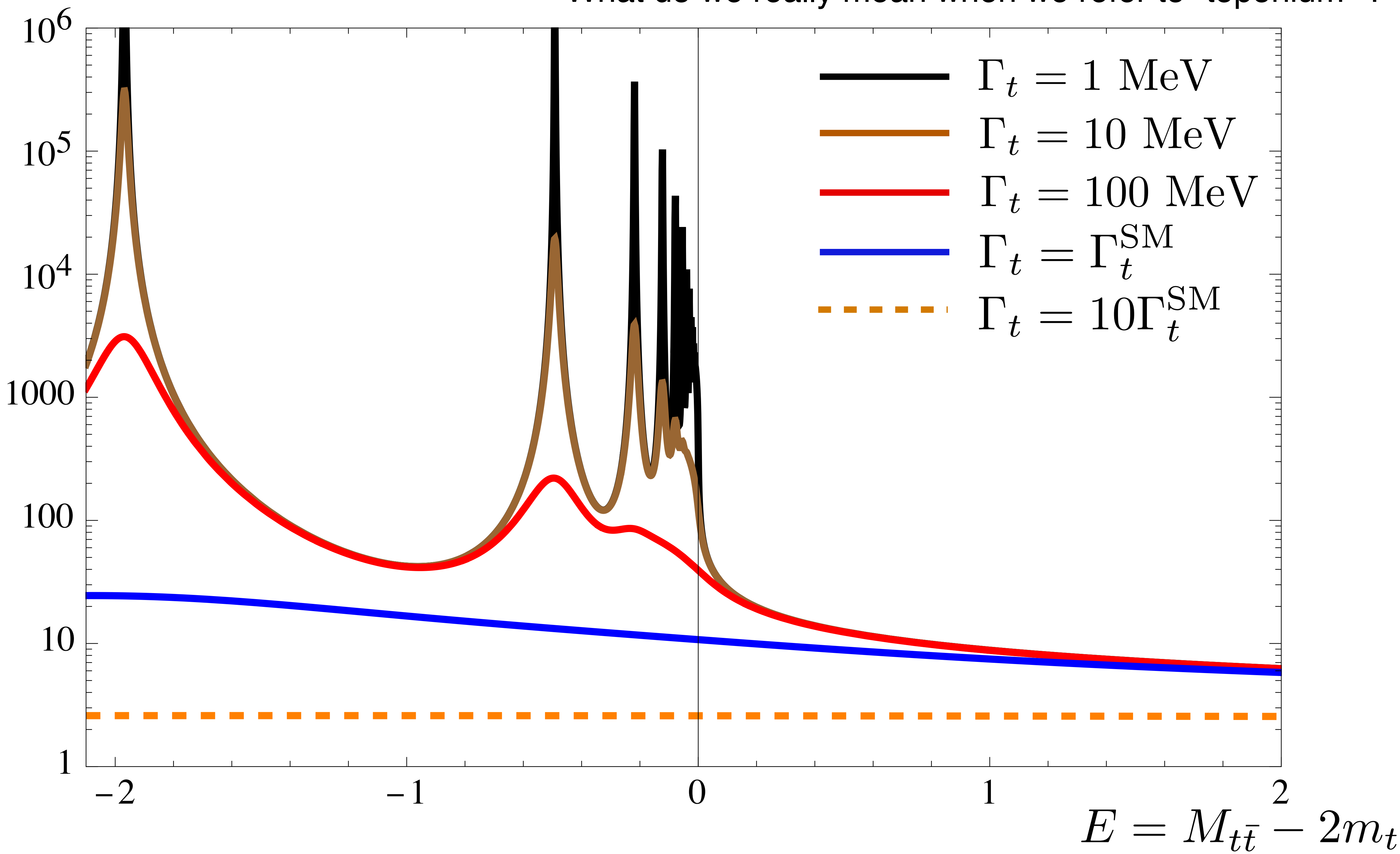

Thank you for your attention !