

COMET Track Finding and Fitting

The 16th France-China Particule Physics Network/Laboratory Workshop (FCPPN/L 2025)

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● COMET (Coherent Muon to Electron Transition)

- 43 institutes, 17 countries, $\simeq 200$ collaborators
- Masaharu Aoki (spokesperson), Osaka University
- 6 Chinese institutes :

- Institute of High Energy Physics (IHEP), Beijing
- Institute of Modern Physics (IMP), Lanzhou
- Nanjing University, Nanjing
- Peking University, Beijing
- Zhengzhou University
- Sun Yat-sen University, Guangzhou

● 5 French institutes

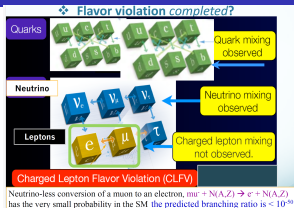
- Laboratory of Nuclear and High Energy Physics (LPNHE), Paris,
- Laboratoire de Physique Corpusculaire, Caen (LPC-Caen)
- LPC, Clermont-Ferrand,
- Institut de Physique des 2 Infinis de Lyon (IP2I),
- CC-IN2P3, Lyon,

● COMET Track Finding and Fitting (FCPPN Team)

- Haibo LI (IHEP)
- Ye YUAN (IHEP) (co-spokesperson)
- Yao ZHANG (IHEP)
- Zhaoke ZHANG (IHEP)
- Wilfrid da SILVA (LPNHE-Paris)
- Luigi DELBUONO (LPNHE-Paris)
- Patrice LEBRUN (IP2I-Lyon)
- Jean-Claude ANGELIQUE (LPC-Caen)

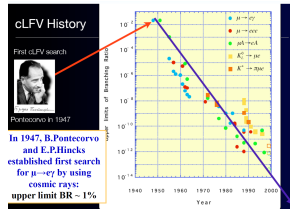
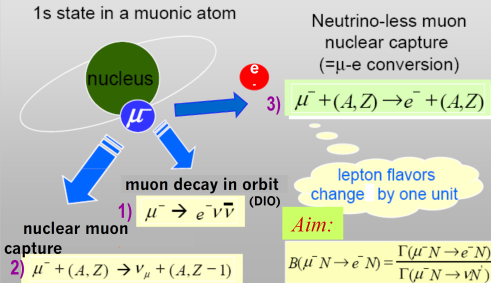


Principal Goal of COMET : search for the $\mu^- N \rightarrow e^- N$ process.



coherent neutrinoless conversion of muons to electrons ($\mu - e$ conversion)

➤ What's happen with a muon in *atom*?



$$B(\mu^- + Au \rightarrow e^- + Au) \leq 7 \cdot 10^{-13}$$

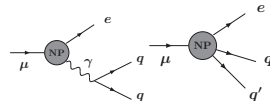
SINDRUM - II, PSI, 2006

COMET
pure/direct $\mu \rightarrow e$
conversion events.

Four order
improvement in BR
expected!

➤ Such extremely rare processes experimentally are non-detectable over the world.

❑ An observation of the conversion $\mu \rightarrow e$ would mean manifestation of **NP-Exotics** beyond the SM, and hence the results of the **COMET** could be of fundamental importance.



Left (right) : photonic (four-fermion/contact) interaction.

The shaded circles denote a BSM flavor-violating interaction.

● Violates the conservation of individual lepton flavours

Other experiences in the field : past, present and future

Current limits for CLFV $B(\mu^+ \rightarrow e^+ \gamma) < 1.5 \cdot 10^{-13}$ (90 % C.L.) (2025 MEG II experiment)

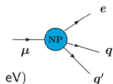
$$\mu \rightarrow e \gamma$$



$$\mu^+ \rightarrow e^+ e^+ e^+$$



$$\mu^+ (A, Z) \rightarrow e^+ (A, Z)$$



Collaboration	year	$BR(\mu \rightarrow e \gamma)$ 90% C.L.
PSI/MEG	2016	4.2×10^{-13}
PSI MEG II		4×10^{-14}

PSI/SINDRUM	1988	1.0×10^{-12}
PSI/PSI/Mu3e		$10^{-15} - 10^{-16}$

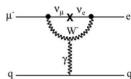
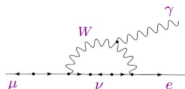
$CR(\mu - e, N)$ bound	material	year
4.3×10^{-12}	Ti	1993
4.6×10^{-11}	Pb	1996
7×10^{-13}	Au	2006

Experiment (material)	future sensitivity	year
Mu2e (Al)	3×10^{-17}	$\sim 20\text{xx}$
COMET (Al) - Phase I (II)	10^{-15} (10^{-17})	$\sim 20\text{yy}(zz)$
PRISM/PRIME (Ti)	10^{-18}	
DeeMe (SiC)	10^{-16}	

- Mu2e, the Coherent Muon to Electron Transition Fermilab experiment in many aspects is very similar to COMET.
- The projected sensitivity for :
- Mu2e after 690 days of operation is 3×10^{-17} at 90% CL.
- COMET Phase-II is 2.6×10^{-17} at 90% CL.

Minimal extension of the SM : Dirac masses for neutrinos are incorporated. Leptonic mixings are possible including CLFV.

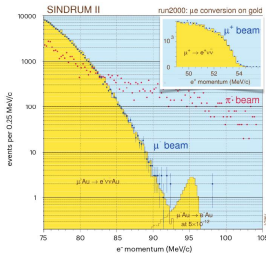
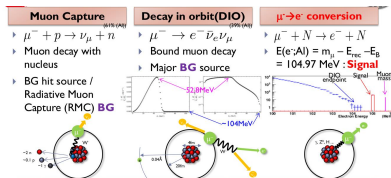
CLFV processes such as radiative decays ($\ell_i \rightarrow \ell_j \gamma$) yield extremely small rates



$$BR(\mu \rightarrow e \gamma) = \frac{3\alpha}{32\pi} \left| \sum_{j=1}^3 U_{ej} U_{\mu j}^* \frac{m_{\nu_j}^2}{M_W^2} \right|^2 \simeq 10^{-54}$$

Energy of the signal electron : $E_{\mu e} = m_{\mu} - B_{\mu} - E_{\text{recoil}}$
 $(B_{\mu} : \text{muon binding energy, } E_{\text{recoil}} \text{ nuclear recoil energy})$
 $= 104.97 \text{ MeV (aluminium)}$

- **Mono-energetic electron**
 \Rightarrow **simple signature**
- **Energy $e^{-} \sim 105 \text{ MeV}$**
 \Rightarrow well above the muon decay spectrum end-point energy $\sim 52.8 \text{ MeV}$.
- **Long lifetime (864 ns)**
 \Rightarrow backgrounds beam flash can be eliminated by waiting
- **Could improve sensitivity**
 by using a high muon rate



Momentum distributions for three different beam momenta and polarities: (i) 53 MeV/c negative muons, optimized for μ^{-} stops, (ii) 63 MeV/c negative pions, optimized for π^{-} stops, and (iii) 48 MeV/c positive muons, optimized for μ^{+} stops. The 63 MeV/c data were normalized to the same measuring time. The measurement with the stopped μ^{-} beam is compared with GEANT simulations of decay in orbit and μe conversion.

COMET will be conducted in 3 phases :

● Phase- α : In Feb-March 2022

- Run at low intensity (200 W) without pion capture solenoid

● Phase-I : Should start end 2026 - Physics running (2-3 years)

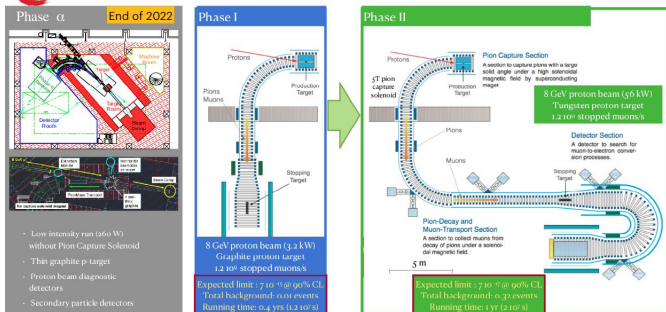
- Investigate the beam and backgrounds
- Branching ratio limit $B(\mu^- N \rightarrow e^- N) : 7 \times 10^{-15}$ at 90% confidence level

● Phase-II :

- Use the information gained in Phase-I
- Branching ratio limit $B(\mu^- N \rightarrow e^- N) : 2.6 \times 10^{-17}$ at 90% CL



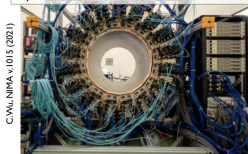
COMET, a 2(5) -stage experiment



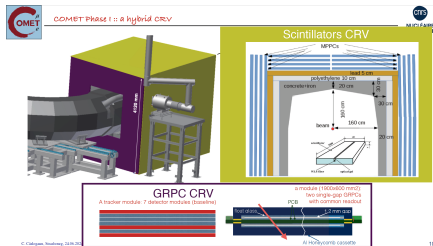
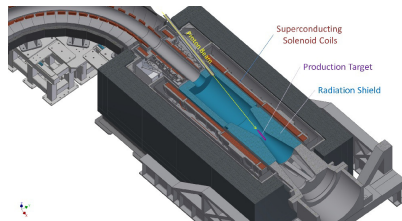
Some COMET requirements (need of lot of muons)

- Good J-PARC high-power proton beam
 - 3.2 kW (Phase I), 56 kW (Phase II)
- Good Efficient pion collection system
 - Surrounding the proton target : 5 T superconducting solenoid.
- **Good Cosmic Ray Veto (CRV)**
 - Georgia, France, IHEP, . . .
 - Has to identify cosmic-ray muons with an average inefficiency that is lower than 10^{-4}
 - The active veto system covering the CyDet is made of scintillator-based detectors/ glass resistive plate chambers (GRPC)
- Good track reconstruction in full stereo Central Drift Chamber

Cylindrical Drift Chamber, constructed in 2016



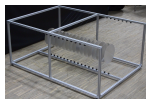
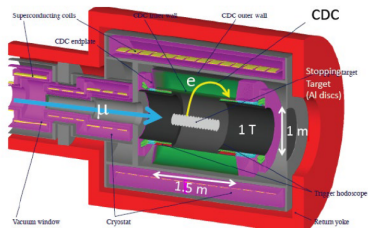
Protons \Rightarrow pions \Rightarrow muons \Rightarrow electron



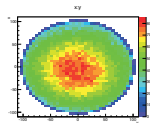
COMET Cylindrical Detector system (CyDet)

The Cylindrical Detector system (CyDet) is specially designed for Phase-I. Consists of:

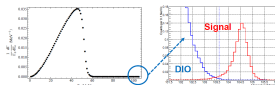
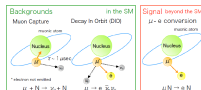
- **Cylindrical trigger hodoscope (CTH):**
 - Two layers: plastic scintillator for trigger time and Cerenkov counter for PID.
 - Finemesh PMT readout
 - 4-fold coincidence trigger
- **Cylindrical drift chamber (CDC):**
 - 20 stereo layers: z information with few layers' hits.
 - Helium based gas: minimize multiple scattering.
 - Large inner bore: to avoid beam flash and DIO electrons.
 - Momentum resolution: 200 keV/c (for $p=105$ MeV/c)
- **Stopping target**
 - Aluminum target with 17 disks
 - 100-mm radius, 0.2-mm thickness, 50-mm spacing.



Muon Stopping Target



Muon Stopping Target



Momentum distributions for the reconstructed μ -e conversion signals and reconstructed DIO events.

COMET Track finding in full stereo Central Drift Chamber (CDC)

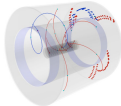
@IHEP team (Tianyu XING, Yao ZHANG, Ye YUAN, ...)

Challenges of tracking algorithm of CDC:

- All stereo layers which can't provide information in z axis directly
- No vertex constraint
- No seed from sub-detector could be used
- Tracks with low transverse momentum is circle inside detector
- Overlapping between tracks from different turns

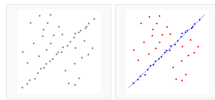
Major problem of tracking algorithm of CDC

- Calculate track parameters of seed
- Distinguish tracks from different turns
- High momentum tail of reconstructed tracks



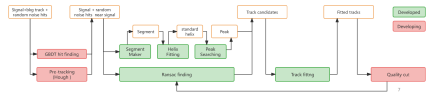
Random Sample Consensus (RANSAC):

1. Select a random subset of the original data (5/6 hits combination)
2. A model is fitted to the data set (helix fitting)
3. Points that fit the model well are considered as part of the set (pick-up signal hits)
4. The model is reasonably good if many points have been classified (track candidate)
5. The model may be improved by reestimating it using all sets (track fitting with genfit)



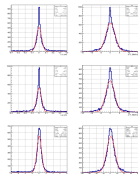
A data set with many outliers for which a line has to be fitted. Fitted line with RANSAC; outliers have no influence on the result.

- Random noise hits added to signal samples
 - Assuming Hough algorithm has been developed
- RANSAC track finder (RTF) and track fitting are developed and tested
- Pre-tracking and quality cut are still developing



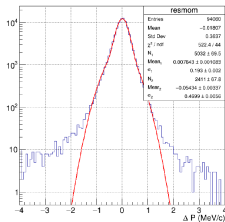
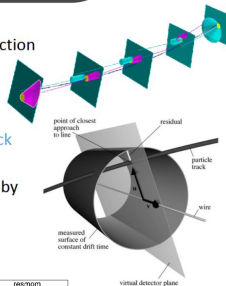
Track finding

- With ~30% random noise hits:
- For double turn events:
 - Peak searching efficiency: 98.9%
 - Resolution:
 - $\sigma_x = 0.35 \text{ mm}, \sigma_y = 0.35 \text{ mm}, \sigma_z = 5.0 \text{ mm}$
 - $\sigma_{px} = 0.81 \text{ MeV}, \sigma_{py} = 0.81 \text{ MeV}, \sigma_{pz} = 5.7 \text{ MeV}$
- For double turn events:
 - Peak searching efficiency: 89.3%
 - Resolution:
 - $\sigma_x = 0.37 \text{ mm}, \sigma_y = 0.35 \text{ mm}, \sigma_z = 6.8 \text{ mm}$
 - $\sigma_{px} = 2.5 \text{ MeV}, \sigma_{py} = 2.4 \text{ MeV}, \sigma_{pz} = 11.8 \text{ MeV}$
- Almost same worse resolution



Track Fitting

- Kalman fitter is widely used in reconstruction algorithm
- Based on GenFit
(<https://github.com/GenFit/GenFit>)
- An experiment-independent **generic track fitting** framework
- Official track fitting for BelleII, also used by PANDA, CEPC, BESIII, GEM-TPC etc



⇒ Good resolution, single and double turn fitting resolution still under study

Classical "Hough Transform" for the Track finding in stereo drift chamber and link to Apollonius De Pergas's Problem

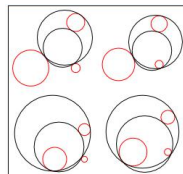
Apollonius's problem : construct circles that are tangent to three given circles in a plane
 Greek Apollonius of Perga (200 BC)
 Compass constructions by French François Viète (1600)
 Algebraic solution (cyclographic model) :

$$(-x_1 + x_s)^2 + (-y_1 + y_s)^2 = (r_s - r_1)^2$$

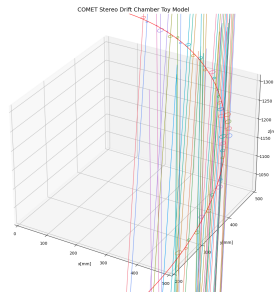
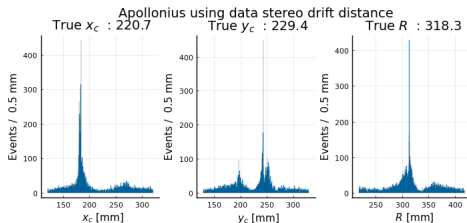
$$(-x_2 + x_s)^2 + (-y_2 + y_s)^2 = (r_s - r_2)^2$$

$$(-x_3 + x_s)^2 + (-y_3 + y_s)^2 = (r_s - r_3)^2$$

with $s_{1,2,3} = \pm 1$



Radius r_i are well defined in a Axial Drift Chamber (radius = axial drift distance) but not in a Stereo Drift Chamber



⇒ Not bad ! but to many wrong answers ! Can we do better ! i.e estimate all helix parameters

Stereo Drift Distance and Apollonius De Pergas's Problem

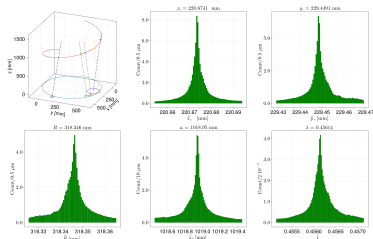
Just rewrite the stereo drift distance standard equation for an helix in the Apollonius Formalism :

$$(x_i^{\text{Ax.}} - x_c)^2 + (y_i^{\text{Ax.}} - y_c)^2 - (R + d_i^{\text{Ax.}})^2 = 0$$

$$d_i^{\text{Ax.}} = d_i^{\text{St.}} + f^{\text{Ax.}}(d_i^{\text{St.}}, x_i^{\text{Ax.}}, y_i^{\text{Ax.}}, \phi_{si}, \tan \tau_i, x_c, y_c, R, z_0, \lambda, n_i^{\text{Turn}}),$$

$x_i^{\text{Ax.}}, \dots$ wire parameters, $d_i^{\text{St.}}$ drift distance and $f^{\text{Ax.}}$ know exactly

⇒ Can now estimate all helix parameters ($x_c, y_c, R, z_0, \lambda$) from data



⇒ very good results on perfect track helix toy model

Hit Finding Approach

- Apollonius' problem is extended to **Full Stereo Drift Chamber**
- The method and the mathematical description are in the ArXiv <https://arxiv.org/abs/2401.04576>

Given a stereo wire (anchored i defined by the stereo angle τ_i , the intersection coordinate $x_i^{\text{Ax.}}, y_i^{\text{Ax.}}$ of the stereo wire to the stereo transverse plane and the wire projection angle ϕ_{si} in the plane, the signed drift distance $d_i^{\text{St.}}$ to this wire is outside the equation (2) (see Appendix A).

$$(x_i^{\text{Ax.}} - x_c)^2 + (y_i^{\text{Ax.}} - y_c)^2 - (R + d_i^{\text{Ax.}})^2 = 0 \quad (3)$$

where the expression of $d_i^{\text{Ax.}}$, function of the stereo wire i and the helix parameters, is given in the equation (4.6.6). The absolute value of constructed signed drift distance $d_i^{\text{Ax.}}$ can be interpreted as the radius of a circle with the center coordinate $(x_i^{\text{Ax.}}, y_i^{\text{Ax.}})$ in the stereo transverse plane. This circle is tangent to the helix and therefore can be seen as the Apollonius' problem [2].

Wilfrid da Silva, Patrice Lebrun, Jean-Claude Angeli, Luigi Del Buono

GPU-accelerated Arithmetic to solve the Apollonius Problem applied to a Stereo Drift Chamber

Wilfrid da Silva, Patrice Lebrun, Jean-Claude Angeli, Luigi Del Buono

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Aim : develop/test a COMET track finding software package using Apollonius's Problem on realistic COMET simulation with Julia language

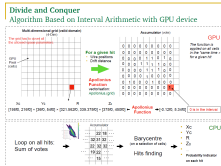
realistic COMET simulation given/produced by IHEP team

Tracking Apollonius Software

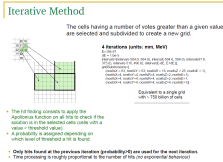
- Deal: Search for tracks and hits in a (very) noisy environment with the **lowest possible processing time** and using **common computing hardware** like GPUs
 - Improve efficiency of other methods.
- Written in **Julia**, a high-level, high-performance dynamic language for scientific and technical computing.
 - Allows easy use of GPU with the availability of powerful math packages.
 - Julia performance is identical to C/C++**
- Two Main Packages Used:
 - [CUDA.jl](#) (Nvidia)
 - [IntervalArithmetic.jl](#)

- Jupyter (JuliaPythonR) notebook is used to analyse output and for coding development (testing).
- Gitlab: Tracking_Apollonius Projects
- Apptainer (Singularity)
 - tested successfully with a container having alma linux 9, iRods client, julia and all julia packages needed by the project installed. Its Size is ~1.8 GB (mainly occupied by CUDA)

- Helix parameters given by Hough Transform
- Hough Transform based on arithmetic Interval



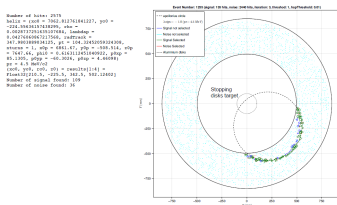
- Using Divide and Conquer algorithm



- GPU straight forward implementation with Julia
- Run on CC-IN2P3 GPU

Some preliminary results ($\simeq 80$ signal hits and $\simeq 2500$ noise hits (50 % CDC occupancy))

Apollonius Helix and Hits



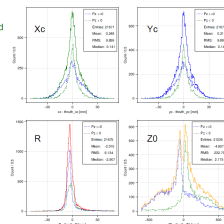
Apollonius Helix Results

The true values are evaluated using the momentum and the position of the hit **having the highest momentum**, unfortunately introducing fluctuation and systematic effects too.

Results are given by the third iteration with cell size of $1 \times 1 \times 5 \text{ mm}^4$

Distributions are done with $p_x \geq 0$ and $p_x < 0$

P_z distribution will be discussed at the end of this presentation



- Apollonius's algorithm is currently tested
- Very good behavior at high CDC occupancy 50 % (very similar result as standart algorithm at low CDC occupancy 5-10 %)
- Currently including correction due to non uniform COMET magnetic field
- Currently Considering the "Hough transform" output (noise or signal hits (helix parameters, drift distance sign and turn number) as input of denoising diffusion neural net.

- COMET Phase-I (commisioning) should start end 2026.
- Apollonius's algorithm is currently tested (show very good result at high CDC occupancy)
- Julia programming language can make algorithm development easier (GPU, ...)
- Consider correction due to non uniform COMET magnetic field
- Consider using denoising diffusion neural net.

Thanks you very much