Momentum resolution of a tracking system

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径迹测量在做什么？

位置测量空间

- 测量误差
- 物质的散射
- 测量点的分布（layout）和范围 L
- 磁场强度

→

~~径迹寻找~~

径迹拟合

→

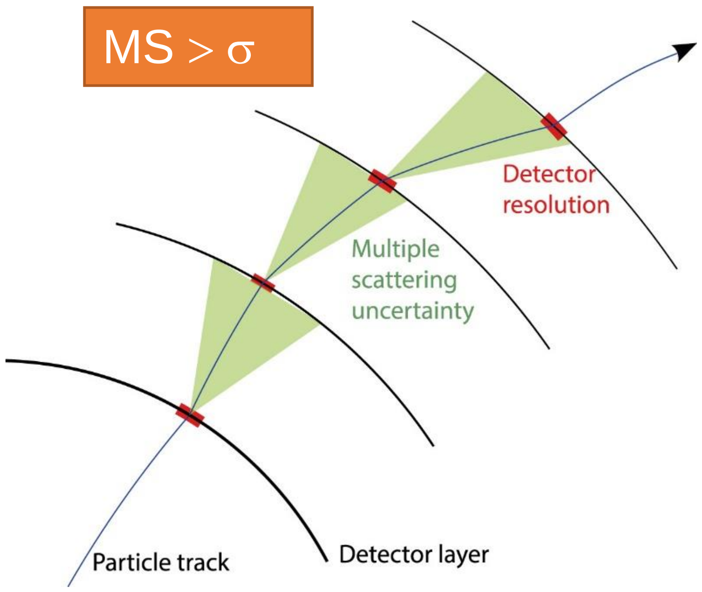
径迹参数空间

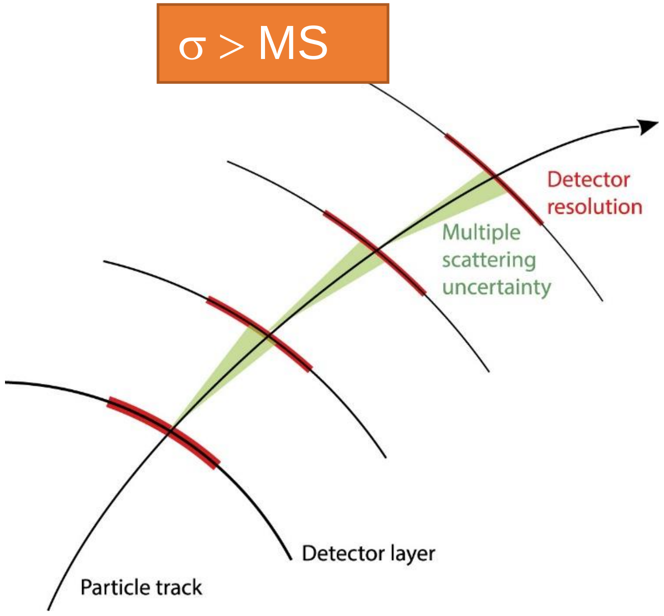
- 测量值
- 协方差矩阵

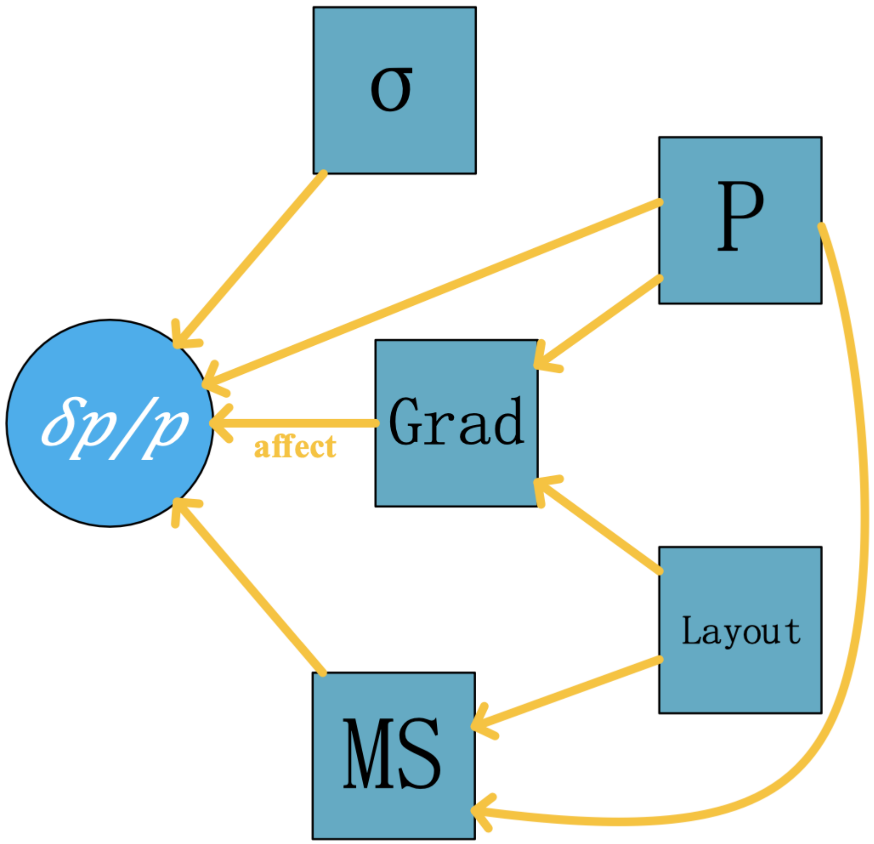
把空间测量结果变换到径迹参数空间

影响因素：空间测量本身、散射效应、变换矩阵

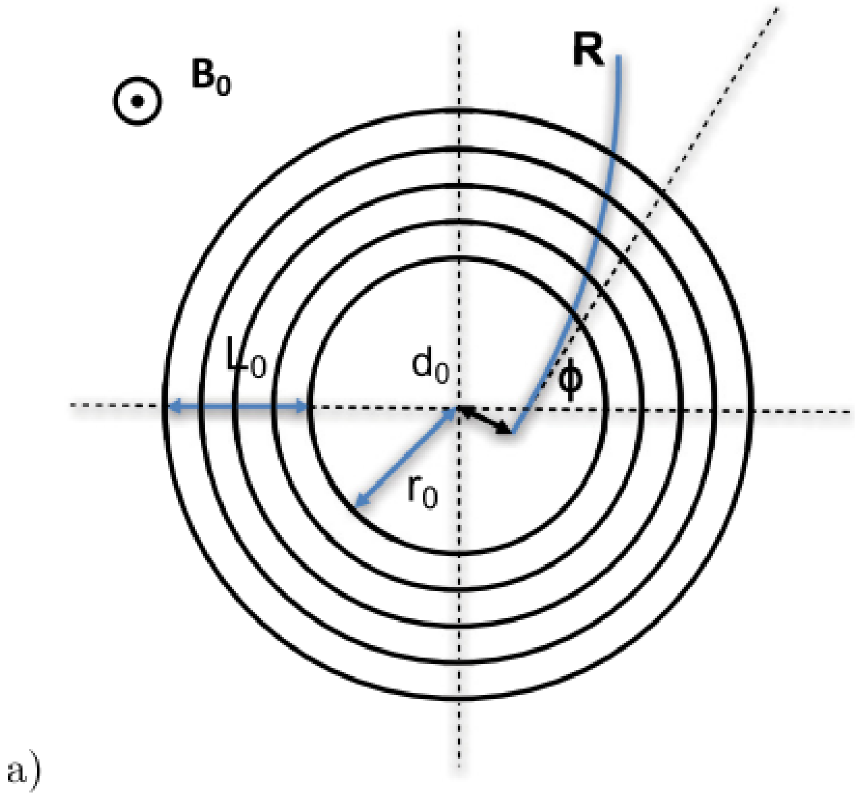
Momentum resolution

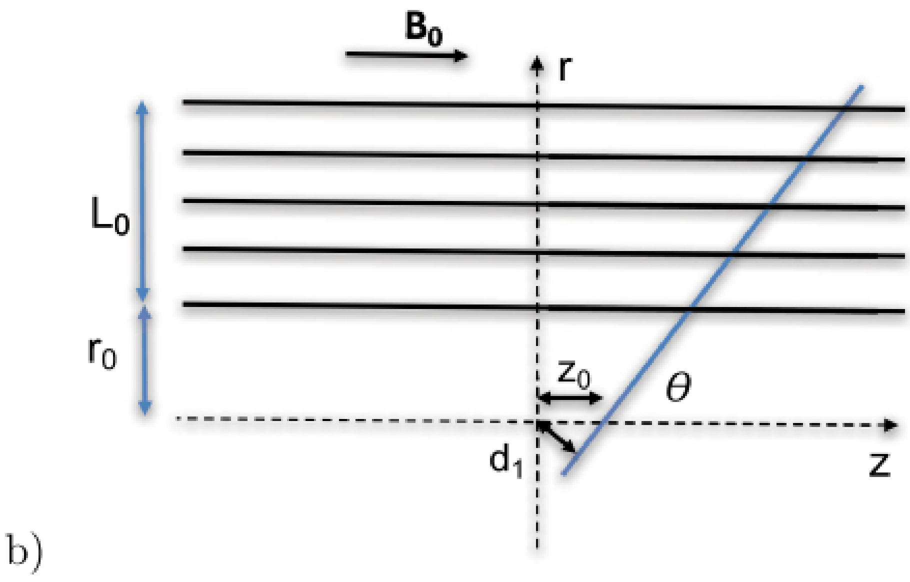






- ➢ **Complicated, when MS is taken into account**
- ➢ **Quite a few factors affect the momentum resolution**





$$\chi^2 = \sum_{m=0}^{N} \sum_{n=0}^{N} \left[y_m - \sum_{i=0}^{M} a_i g_i(x_m) \right] W_{mn} \left[y_n - \sum_{i=0}^{M} a_i g_i(x_n) \right]$$

$$(C_y)_{mn} = \sigma_n^2 \delta_{mn} + \sum_{j=0}^{\mathrm{Min}[m,n]-1} \sigma_{\alpha_j}^2 (x_m - x_j)(x_n - x_j)$$

$$\mathbf{C}_y = \mathbf{M}$$

$$= \frac{\sigma_\alpha^2 L^2}{N^2} \begin{pmatrix} \frac{N^2\sigma_0^2}{\sigma_\alpha^2 L^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & . \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ 0 & 2 & 5 & 8 & 11 & 14 & 17 & 20 & \dots \\ 0 & 3 & 8 & 14 & 20 & 26 & 32 & 38 & \dots \\ 0 & 4 & 11 & 20 & 30 & 40 & 50 & 60 & \dots \\ 0 & 5 & 14 & 26 & 40 & 55 & 70 & 85 & \dots \\ 0 & 6 & 17 & 32 & 50 & 70 & 91 & 112 & \dots \\ 0 & 7 & 20 & 38 & 60 & 85 & 112 & 140 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$V = \begin{pmatrix} \sigma_i^2 & 0 & 0 \\ 0 & \sigma_j^2 & 0 \\ 0 & 0 & \sigma_k^2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \left(z_j - z_i\right)^2 \delta\theta_i^2 & \left(z_k - z_i\right)\left(z_j - z_i\right)\delta\theta_i^2 \\ 0 & (z_k - z_i)(z_j - z_i)\delta\theta_i^2 & \left(z_k - z_i\right)^2 \delta\theta_i^2 + \left(z_j - z_i\right)^2 \delta\theta_j^2 \end{pmatrix}$$

Let`s recall least square fit

(Neglecting the correlations among hits)

$$\chi^2 = \sum \frac{(y - y_i)^2}{\sigma_i^2}$$

$$\frac{\partial \chi^2}{\partial p} = \sum \frac{\delta y_i}{\sigma_i^2} \frac{\partial y_i}{\partial p} = 0$$

It achieves the minimum when the derivatives are zeros

In textbook，the Fisher information (FI) is defined as

$$I = \sum I_i = \sum \frac{\left(\frac{\partial y_i}{\partial p}\right)^2}{\sigma_i^2}$$

- Weighted sum of the squared derivatives
- Weights: inverse squared errors

Advantage of FI: related to variance directly

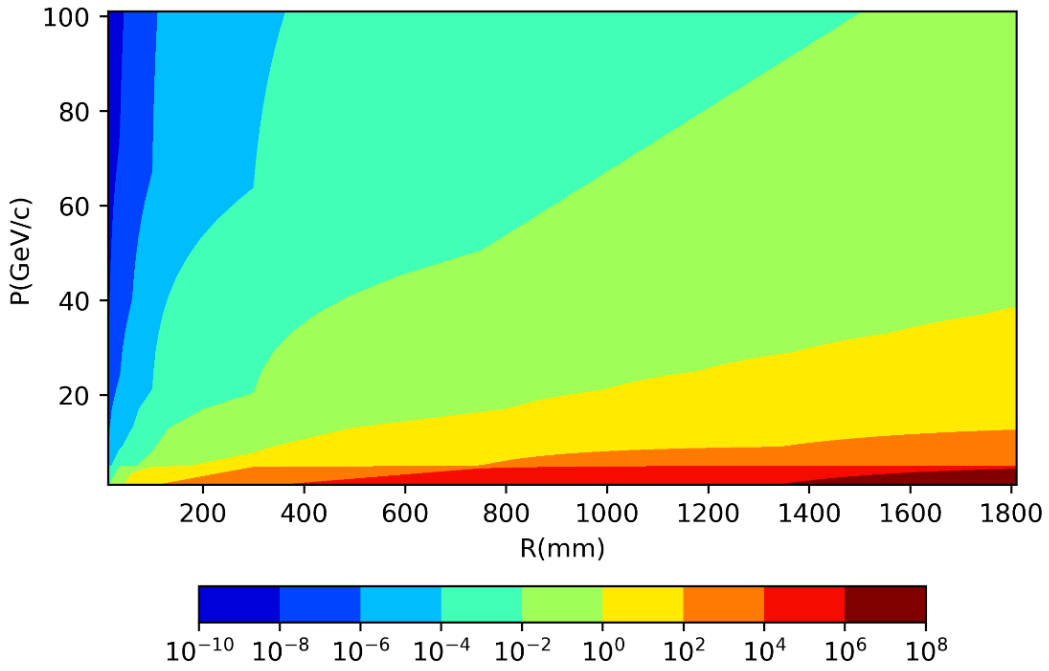
$$\sigma_p^2 \geq \frac{1}{I}$$

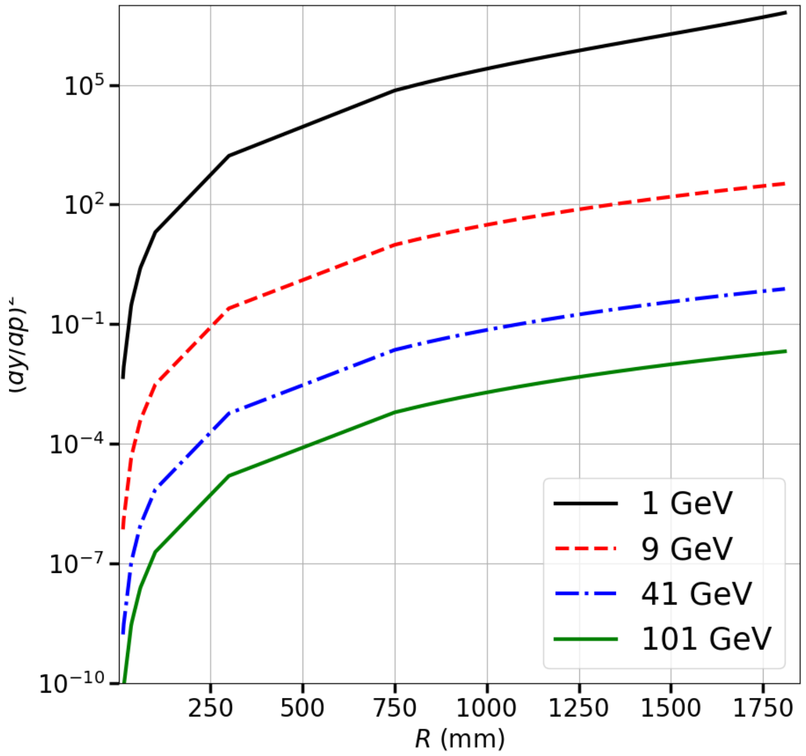
Two ingredients of FI

- Nominator : Derivatives of helix to measurements y_i
- Denominator: The σ_i including both spatial resolutions(SP) and MS

Nominator: Squared Derivatives (SD)

$$I_i = \frac{\left(\frac{\partial y_i}{\partial p}\right)^2}{\sigma_i^2}$$





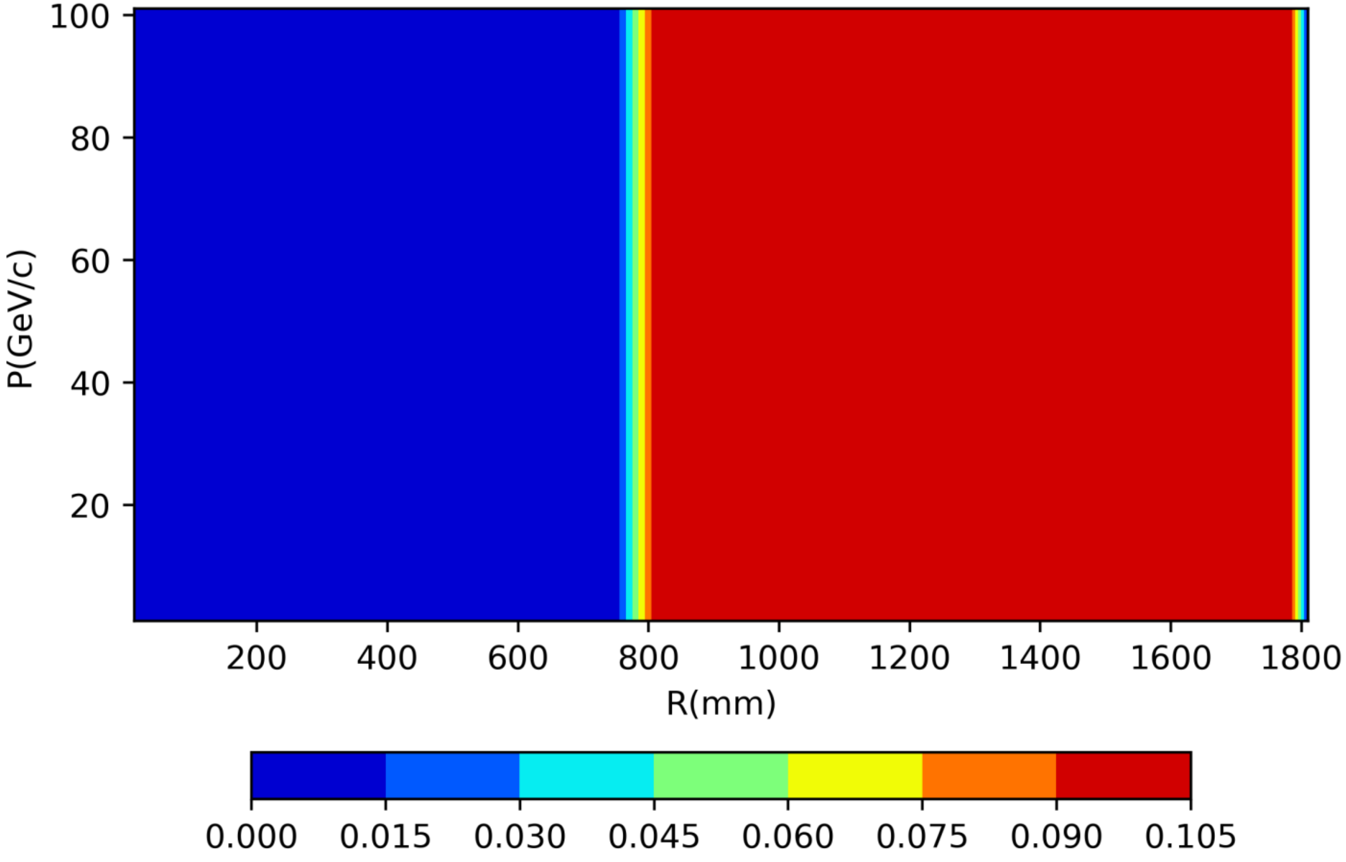
$\sim 10^8$

Varies in a huge range (vs. R & momenta)

~ 8 orders of magnitudes difference @ R= 1.8 m

Denominator: spatial resolution(SP)

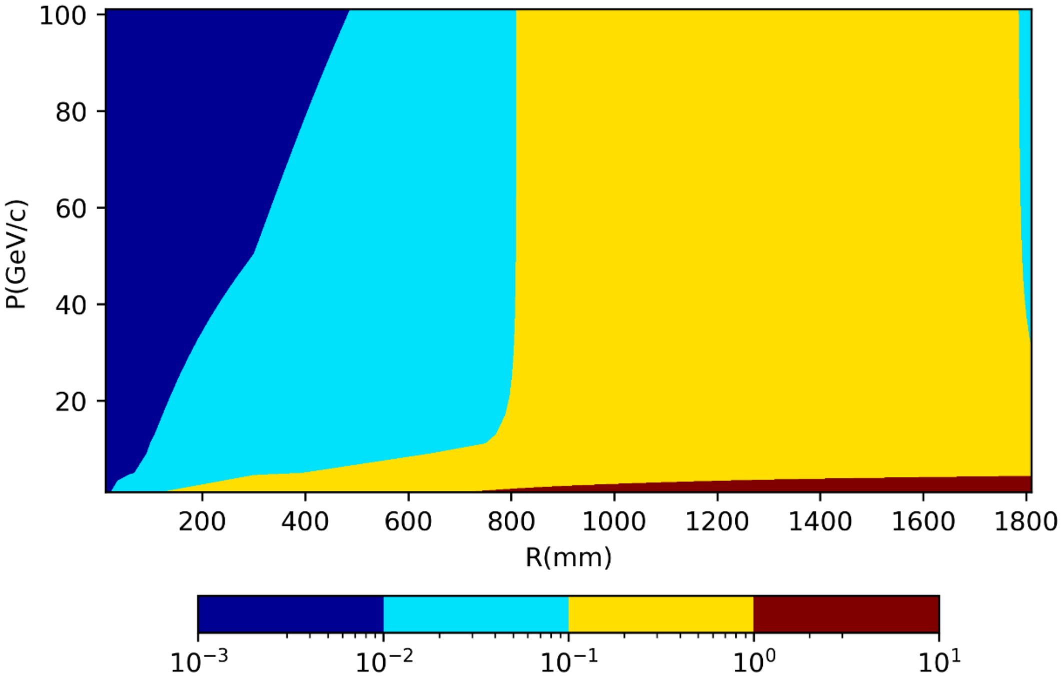
$$I_i = \frac{\left(\frac{\partial y_i}{\partial p}\right)^2}{\sigma_i^2}$$

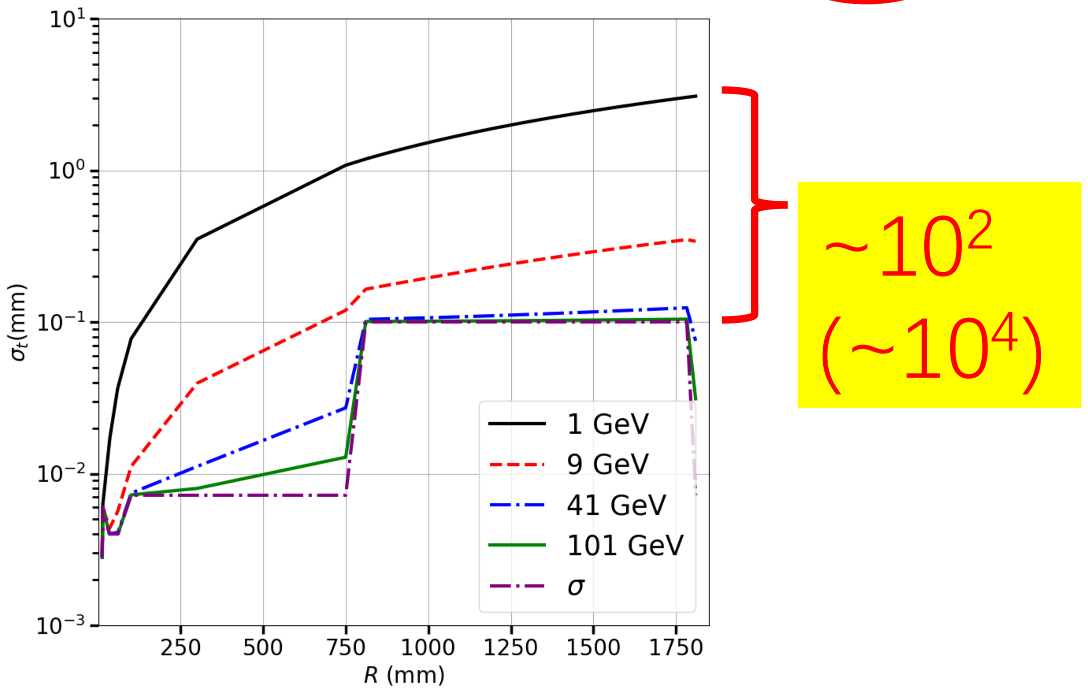


- Independent on p
- (while MS depends on p)

Denominator: combined MS and SP

$$I_i = \frac{\left(\frac{\partial y_i}{\partial p}\right)^2}{\sigma_i^2}$$



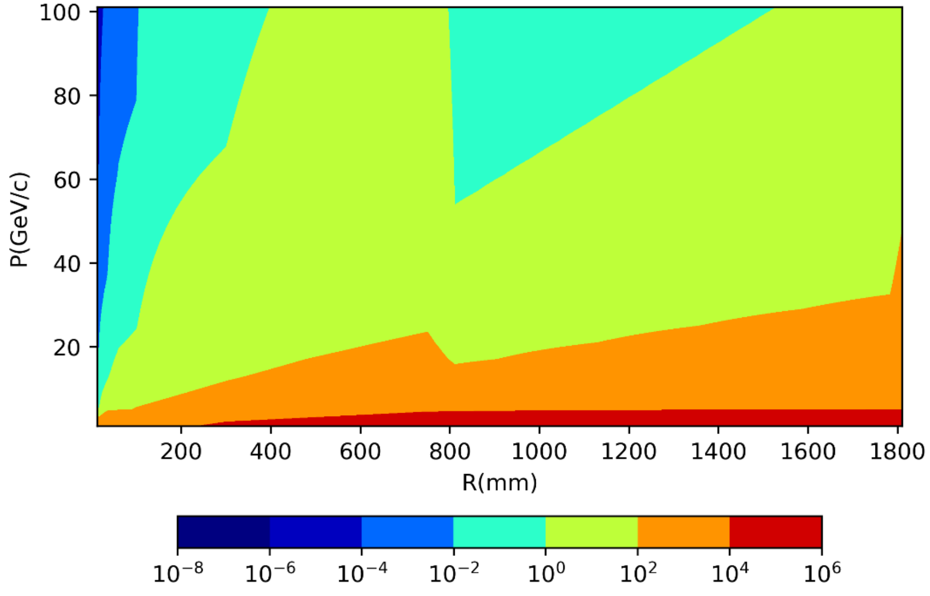


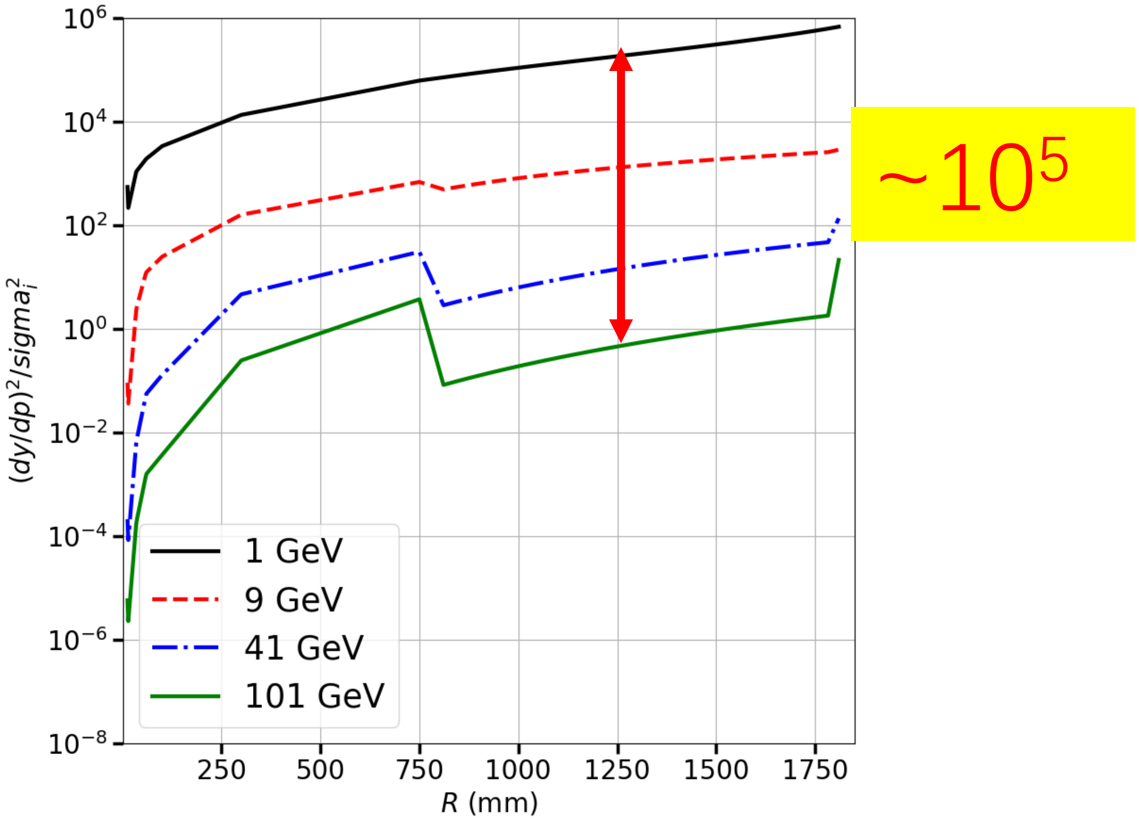
Complicated but understandable

- MS: negligible for sufficiently high pt (>10 GeV)
- MS: ~3 order of magnitudes between 1 and 100 GeV at R = 1.8 m

FI= SD/(SP^2+MS^2)

$$I_i = \frac{\left(\frac{\partial y_i}{\partial p}\right)^2}{\sigma_i^2}$$

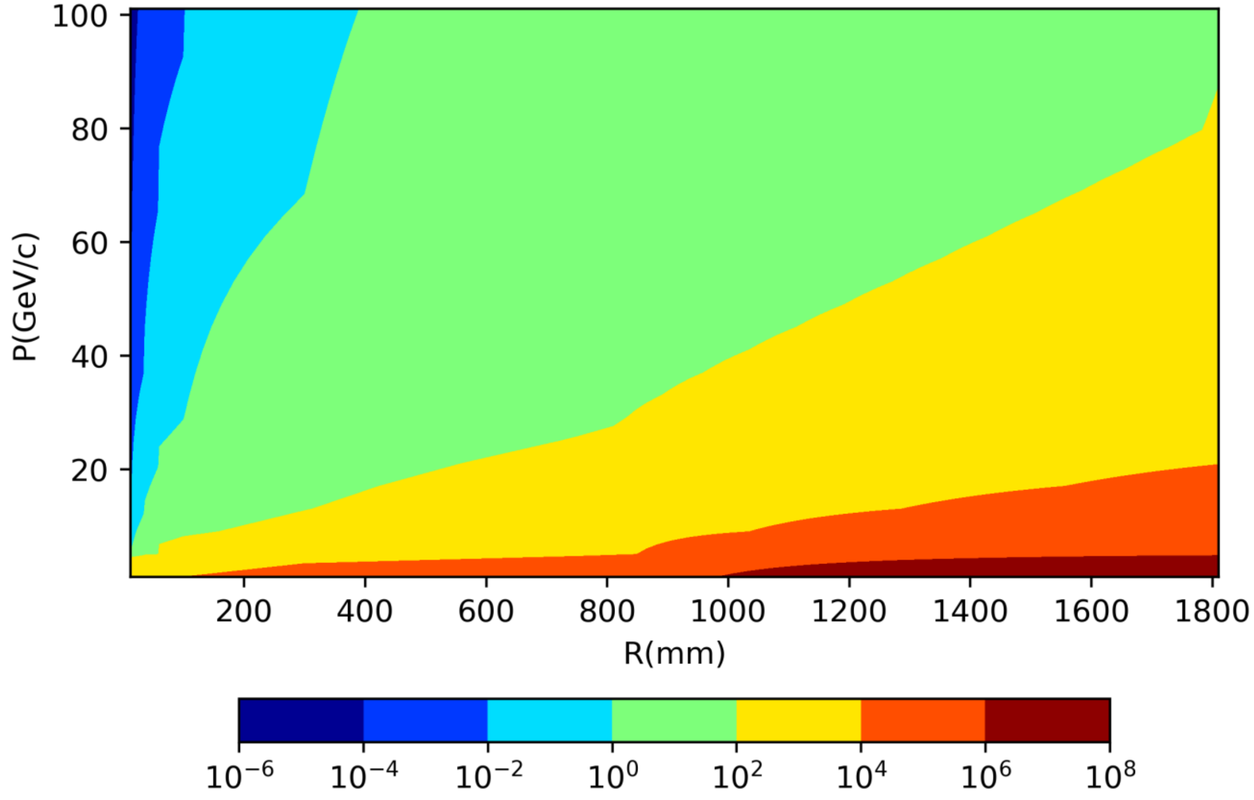


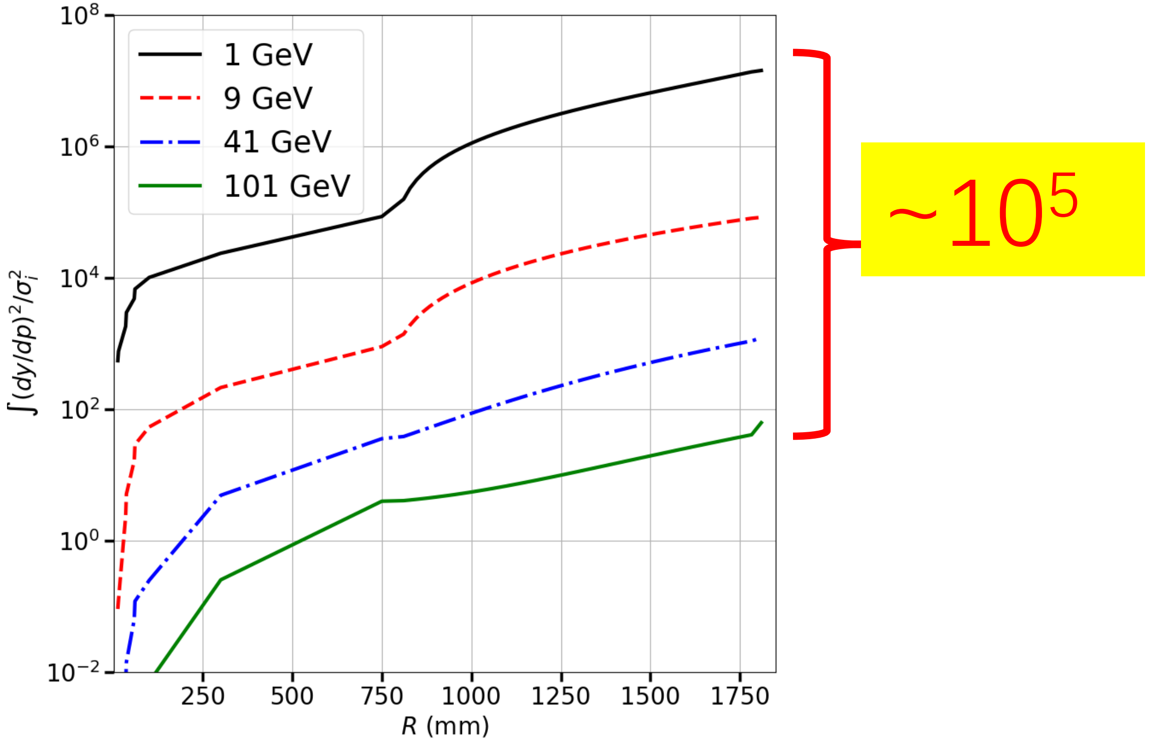


Though taking MS into account, DC contributes more information to low pt tracks → derivates dominant.

Accumulated FI: sum of the first *n* layers

$$I = \sum I_i$$



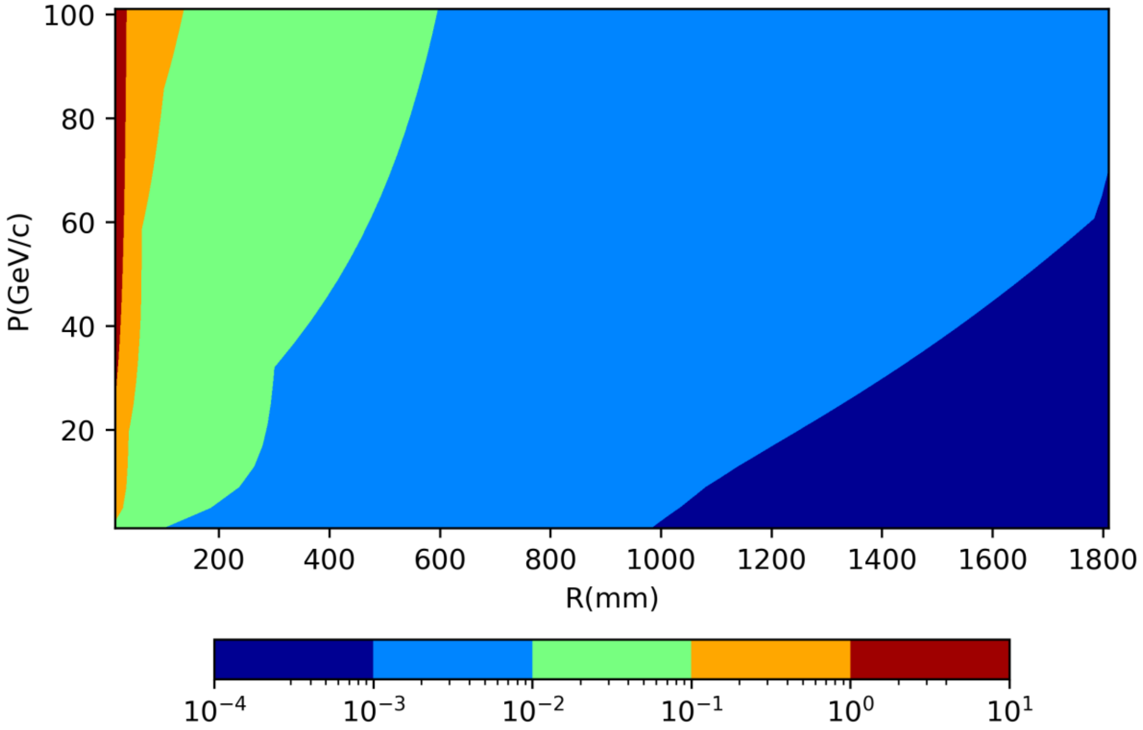


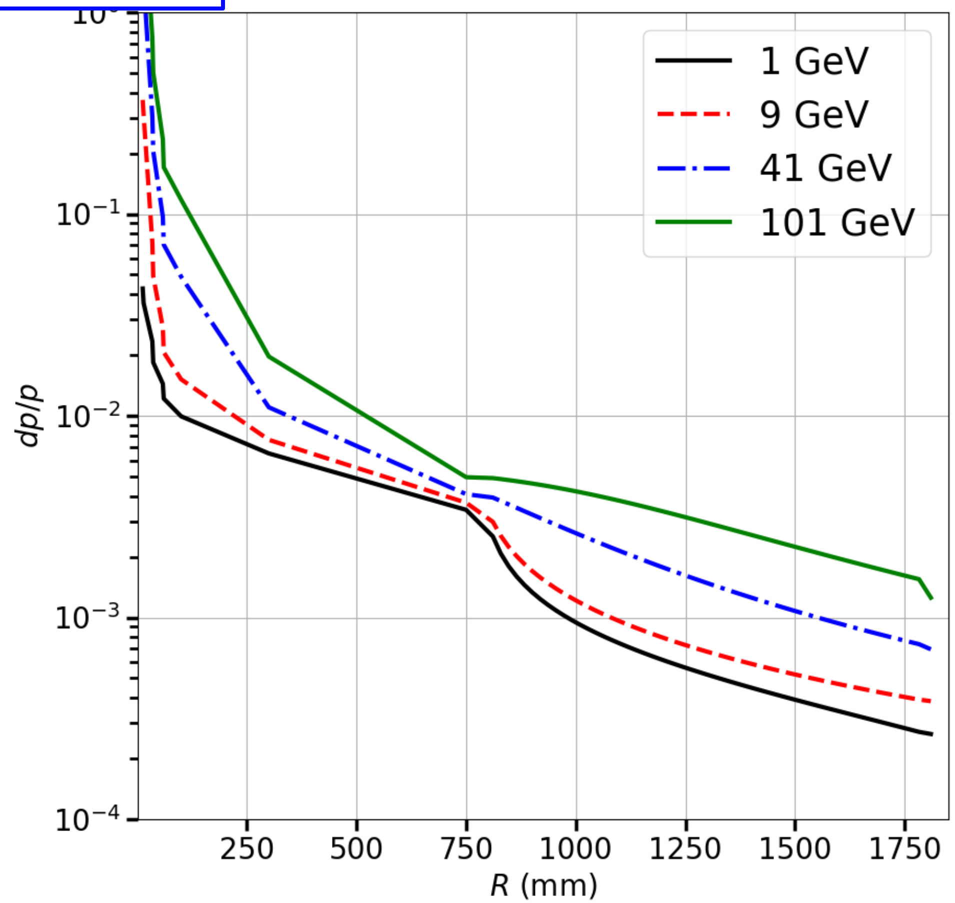
Same conclusion

DC does play an important role for low pt due to large derivatives

P_t resolution

$$\sigma_p = \frac{1}{\sqrt{I}}$$

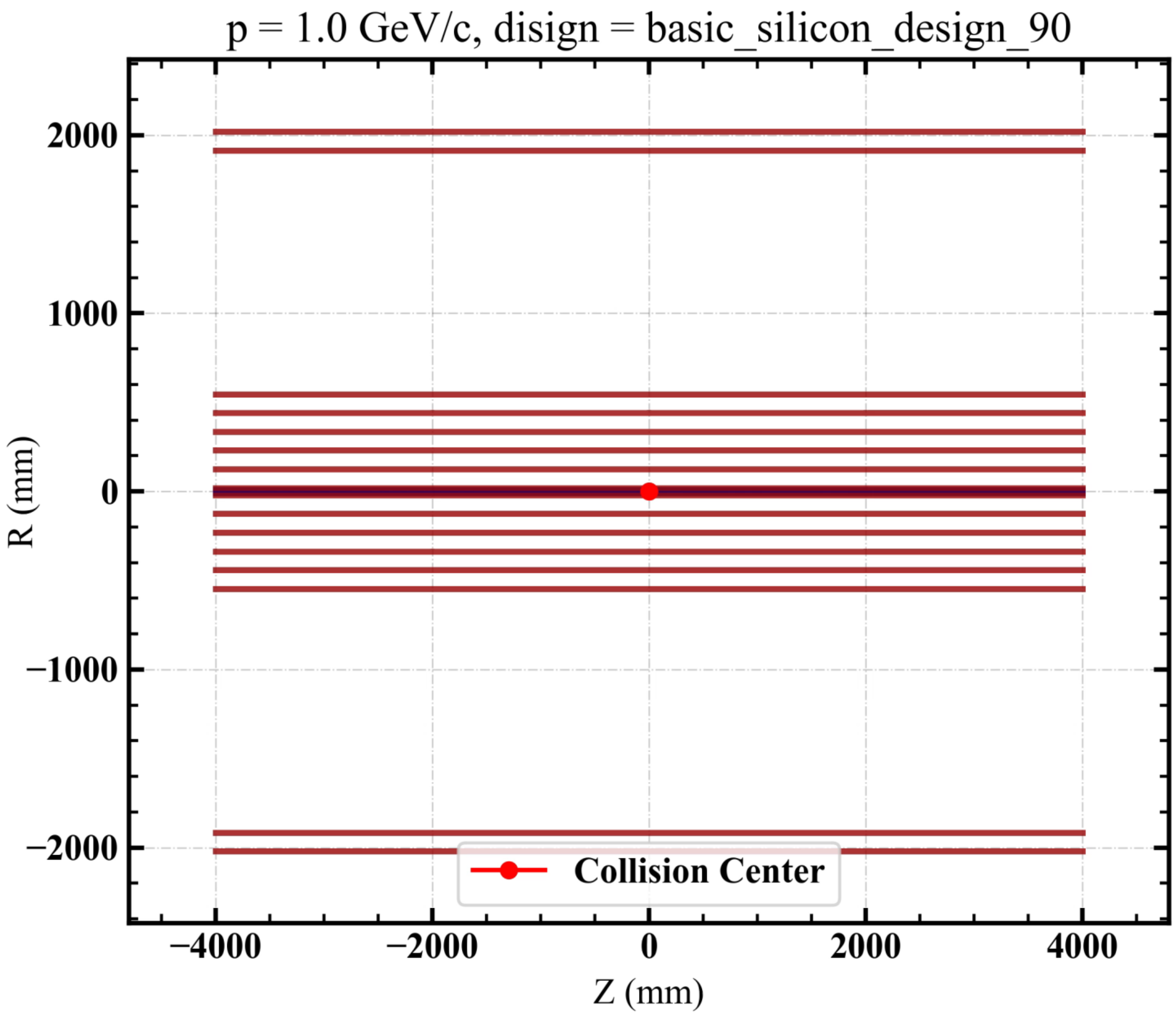


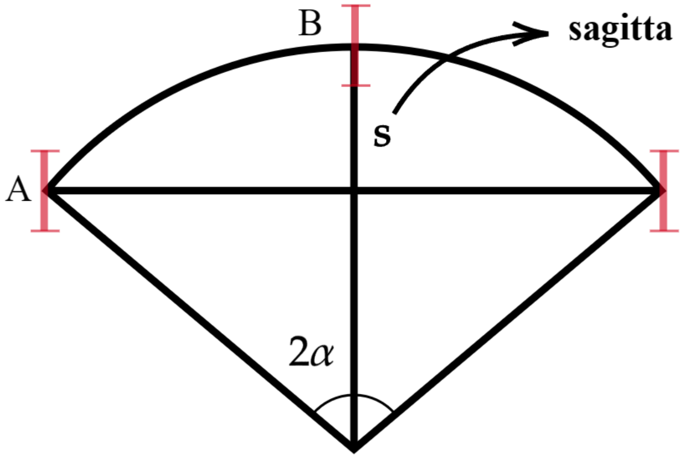


Low pt tracks gain more information from DC

~ 10 times difference because that it is square root and relative resolution

Optimal layout of a toy silicon tracker with Multiple-scattering

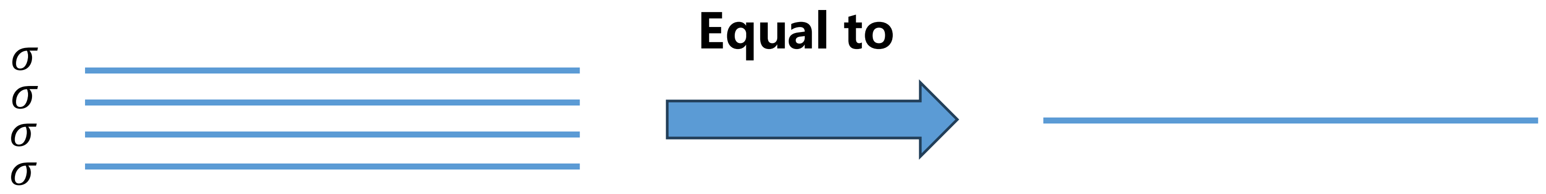




$$s = y_B - \frac{y_A + y_C}{2}$$

$$\sigma_s^2 = \sigma_B^2 + \frac{1}{4}\left(\sigma_A^2 + \sigma_C^2\right)$$

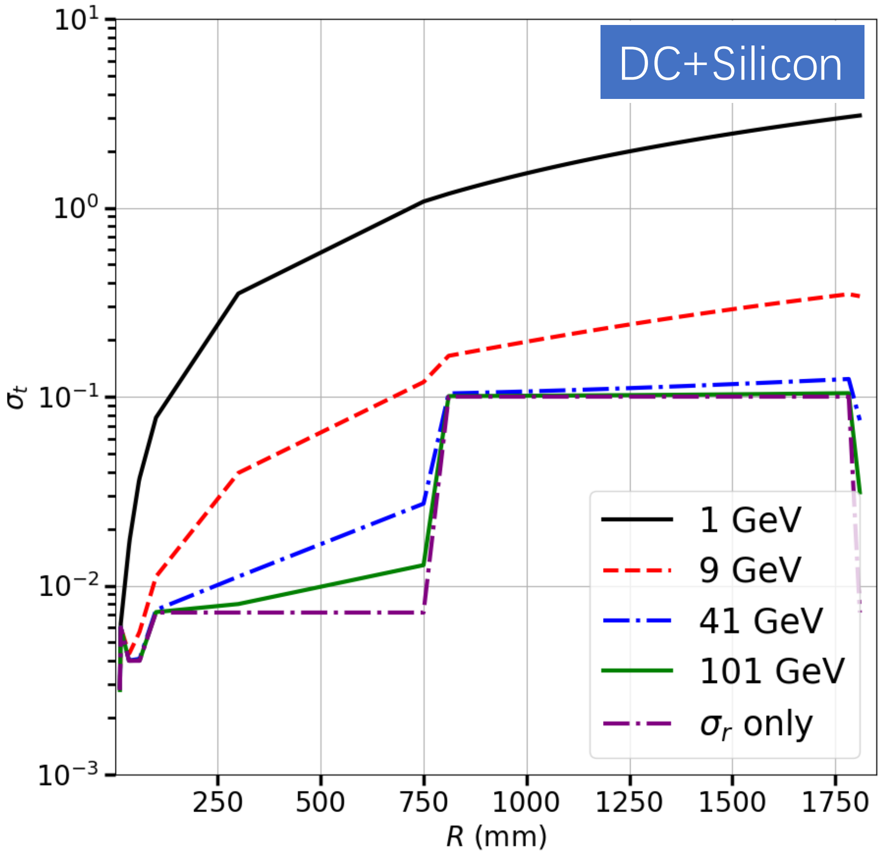
$$\delta s \sim \delta p \,/\, p$$



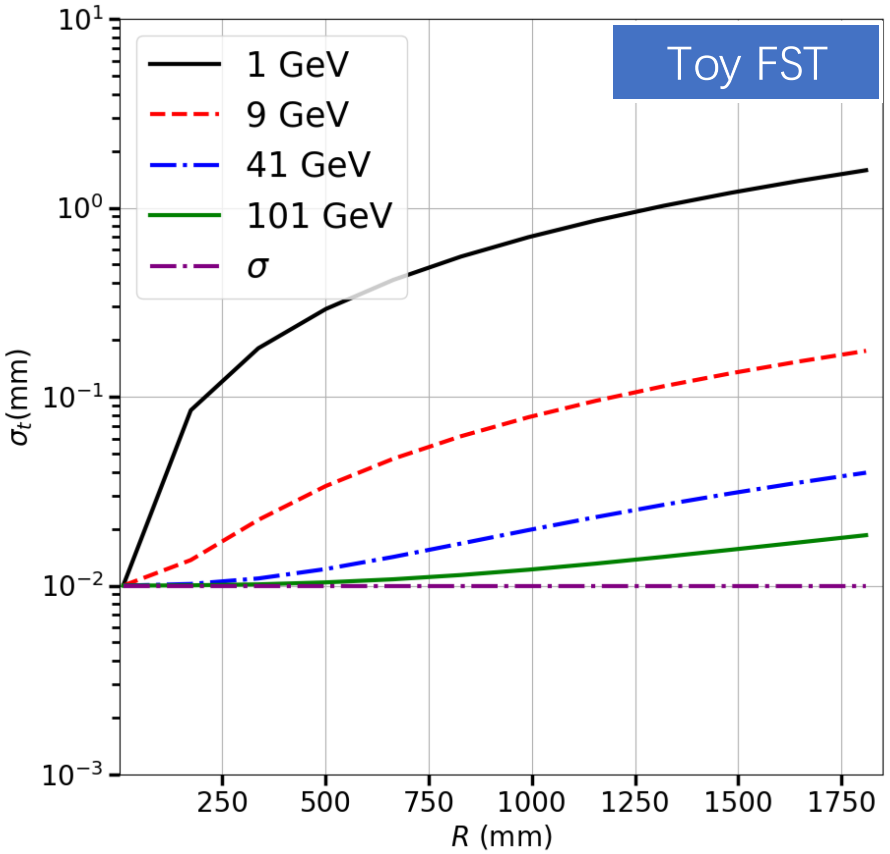
$$\frac{\sigma}{sqrt(4)}$$

Summary

- Mathematical test shows that various factors affects the resolutions of track parameters
  - L
  - B
  - X0
  - Layout
  - Derivatives
  - …



DC+Silicon
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
σ_t
250 500 750 1000 1250 1500 1750
R (mm)
1 GeV
9 GeV
41 GeV
101 GeV
σ_r only



Toy FST
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
σ_t(mm)
250 500 750 1000 1250 1500 1750
R (mm)
1 GeV
9 GeV
41 GeV
101 GeV
σ

Our tracker vs a toy FST

(uniformly distributed 12 layers of silicon with 10 microns σ & 0.002X0)