A first test on spooky actions between free-traveling charged lepton pairs
首次测试自由带电轻子对间的量子纠缠

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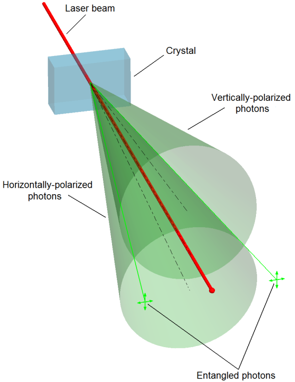
*https://lyazj.github.io/pkmuon-site/categories/activities/ and [1–4]

Outline

Outline



Laser beam
Crystal
Vertically-polarized photons
Horizontally-polarized photons
Entangled photons

Quantum entanglement measurements — history and today
量子纠缠测量的历史与现状

- As reviewed by C. N. Yang [5], the first experiment on quantum entanglement is the Wu-Shaknov Experiment published in 1950 [6] in which the angular correlation of two Compton-scattered **photons** arising from e^+e^- annihilation are measured
- The violation of Bell inequality was demonstrated in 1970s using entangled **photons** [7–9], confirming the non-locality of our universe
- Alain Aspect, John Clauser and Anton Zeilinger won the Nobel Prize in Physics in 2022 for demonstrating the potential to investigate and control particles (**photons**) that are in entangled states [10]

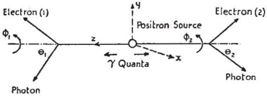


FIG. Angular correlation effects [11] demonstrated by the Wu-Shaknov Experiment

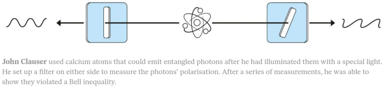


FIG. Clauser's photon entanglement experiment [10]

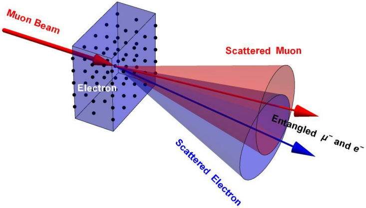
The current absence of free-traveling leptons in QE measurements
史上尚未实现自由轻子量子纠缠的测量

- The ATLAS and CMS Collaborations recently observed quantum entanglement involving **top quarks** at a center-of-mass energy of 13 TeV, marking the highest energy measurements of quantum entanglement to date [12–15]
- Most studies on charged lepton QE have concentrated on the decaying **tau** leptons [16–22], while less attention has been given to electrons and muons
- Solid-state quantum computation was established in 2005 with **electron pairs confined in semiconductor quantum dots** [23]: entangled states were prepared, coherently manipulated, and measured
- **No similar experiment has been done with free-traveling electrons** as measuring the spin of a single traveling electron poses a significant challenge due to interference from its orbital motion [24]

Our proposal

Conduct a **first** measurement of the polarization correlation between charged lepton beams through joint measurements of their individual polarization-sensitive scatterings off two separate targets.

Outline


Muon Beam
Electron
Scattered Muon
Entangled μ⁻ and e⁻
Scattered Electron

Theory: Concurrence, CHSH inequality, and the kinematic approach
理论：纠缠度、CHSH 不等式与运动学方法

- Entanglement can be quantified by *concurrence* [25–27], defined as

$$\mathcal{C}(\rho_f) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \in [0,1] \tag{1}$$

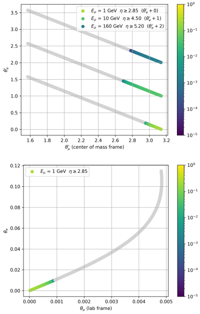
  for a two-qubit system, where λ_i $(\lambda_i \geq \lambda_j,\ \forall i < j)$ are the square roots of the eigenvalues of the matrix $\rho_f(\sigma_2 \otimes \sigma_2)\rho_f^*(\sigma_2 \otimes \sigma_2)$. If $\mathcal{C} > 0$, the two-qubit system is entangled.
- The *CHSH inequality*, $I_2 \leq 2$ [28], is the Bell inequality for a two-qubit system. The optimal (maximal) I_2 [29] evaluates to

$$I_2 = 2\sqrt{\lambda_1 + \lambda_2}, \tag{2}$$

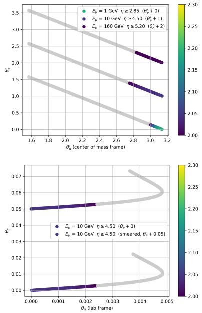
  where λ_1 and λ_2 are the two largest eigenvalues of the matrix $C^{\mathrm{T}}C$, and C is the correlation matrix calculated by $C_{ij} = \mathrm{Tr}\left(\rho_f\left(\sigma_i \otimes \sigma_j\right)\right)$. $I_2 = 2\sqrt{2}$ is the upper limit of the quantum mechanics.
- In addition to the *decay approach* used for decaying particles, the *kinematic approach* [30, 31] can reconstruct quantum states from production kinematics, applicable to stable particles produced in simple QED scatterings.

Electron-muon entanglement sources via muon on-target experiments
缪子打靶实现可控电子-缪子纠缠源

Concurrence $\mathcal{C}(\rho_f)$:



Optimal I_2:



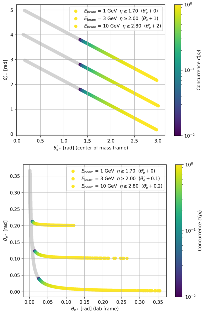
- Simulated by MG5_aMC@NLO 3.5.5 [32] in tree-level QED with non-zero lepton masses
- The light gray regions depict $\mathcal{C}(\rho_f) = 0$ or $I_2 \leq 2$
- Assuming a **1-day** run with a **10 GeV** muon beam of flux $\mathbf{10^5}$**/s** on 10 cm Al targets, the expected number of events with $\mathcal{C}(\rho_f) > 0$ is $\mathbf{2.6 \times 10^4}$

$E_{\rm beam}$/GeV	$E_{\rm COM}$/GeV	$\mathcal{C}(\rho_f)_{\rm max}$	$\theta_{\mu,\rm max}$/mrad	$\theta_{e,\rm max}$/mrad	$E_{\mu,\rm min}$/GeV	$E_{e,\rm min}$/GeV	$\sigma_{\rm E}$/μb	$\sigma_{{\rm E},\,\theta\geq 0.5\,\rm mrad}$/μb
1	0.111	**0.22**	0.9	10.2	0.92	0.08	0.56	0.56
10	0.146	**0.044**	**2.8**	**3.3**	**5.2**	**4.5**	**0.39**	**0.39**
160	0.418	0.0014	4.6	0.5	10	145	0.027	0.022

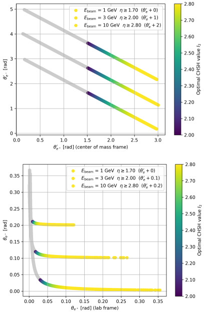
Electron-positron entanglement sources via positron on-target experiments

正电子打靶实现可控电子-正电子纠缠源

Concurrence $\mathcal{C}(\rho_f)$:



Optimal I_2:

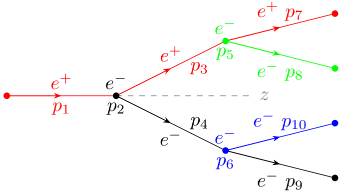


- The angular ranges exhibiting $\mathcal{C}(\rho_f) > 0$ in the center-of-mass frame are significantly broader
- The theoretical upper limits for both $\mathcal{C}(\rho_f)$ and I_2 in quantum mechanics are nearly reached as θ'_{e^+} approaches 3
- Assuming a **1 GeV** positron beam with a flux of $\mathbf{10^{12}/s}$ directed at a 10 cm thick Al target, the expected entangled event rate is $\mathbf{1.9 \times 10^9/s}$
- A golden region for measurements:
  - $E_{\text{beam}} = \mathbf{1\ GeV}$, $0.05\ \text{rad} \leq \theta_{e^+} \leq 0.1\ \text{rad}$
  - 23.4% of all events with $\mathcal{C}(\rho_f) > 0$
  - $E \geq \mathbf{0.094\ GeV}$, $\theta \geq \mathbf{0.0103\ rad}$
  - $\mathcal{C}(\rho_f)$ reaching up to $\mathbf{0.953}$ and I_2 up to $\mathbf{2.8281}$

E_{beam}/GeV	E_{COM}/GeV	$\mathcal{C}^{\max}(\rho_f)$	$I_2^{\max}$	$E_{e^+}^{\min}$/GeV	$E_{e^-}^{\min}$/GeV	$\theta_{e^+}^{\min}$/rad	$\theta_{e^-}^{\min}$/rad	σ_{E}/μb
1	0.032	0.9996	2.8281	0.008	0.389	0.0255	0.0028	243.6
3	0.055	0.9997	2.8282	0.023	1.166	0.0147	0.0016	82.1
10	0.101	0.9997	2.8282	0.074	3.890	0.0081	0.0009	26.5

Outline

1. 量子纠缠测量史中自由轻子的缺失
2. 可控纠缠轻子对源的建立
3. **对自由轻子纠缠对间相关性的首次测量**
4. 总结、参考文献和备用页面



A first electron-positron beam correlation measurement proposal
电子-正电子束相关性测量的首个提案

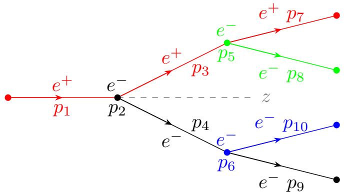


FIG. Proposed cascade experiment for measuring polarization correlations of the primary products

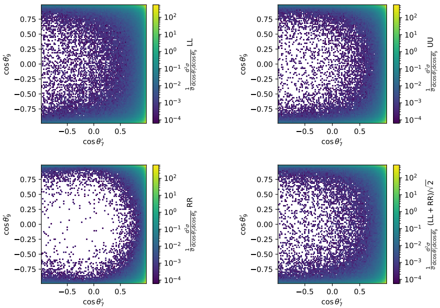


FIG. Joint angular distribution densities of the two secondary scattering processes

Simulation setup:

- $0.05\ \mathrm{rad} \leq \theta_3 \leq 0.1\ \mathrm{rad}$ in a 1 GeV positron on-target experiment
- The spins of target electrons 5 and 6 are aligned with the beam direction
- Consider the main component of the primary state, $(LL + RR)/\sqrt{2}$

Assuming the two secondary targets are $10\ \mathrm{cm}$ thick iron, the event rate in $\cos\theta_7' \leq 0.5 \wedge -0.75 \leq \theta_9' \leq 0.75$ is $\mathbf{1.4 \times 10^2}$**/s** for the state $(LL + RR)/\sqrt{2}$.

Future prospects: Scattering-based simplified state tomography

展望：基于散射的简化的量子态解析

Take $0.05 \text{ rad} \leq \theta_3 \leq 0.1 \text{ rad}$ in a 1 GeV positron on-target experiment as an example:

- The state of the primary products is approximately 1% $(RL + LR)/\sqrt{2}$, 1% $(RL - LR)/\sqrt{2}$, 7% $(RR - LL)/\sqrt{2}$, and 90% $(RR + LL)/\sqrt{2}$ in the lab frame
- The optimized ratio of the yields of $(LL + RR)/\sqrt{2}$ to UU is $1.29 \pm 0.03(\text{MC})$, corresponding to 4.4×10^3 post-optimization efficient signal event counts and an expected signal yield over a **27-second** run; the result for $(LR + RL)/\sqrt{2}$ is $0.78 \pm 0.02(\text{MC})$ in comparison
- Other uncertainties, such as those from process modeling and background suppression, may dominate the real experimental analysis
- For the 20% polarized targets, the ratios are 1.010 ± 0.009 and 0.986 ± 0.009 generated from 25 times the number of Monte Carlo events, corresponding to 2.5×10^4 efficient event counts accumulated in **680 seconds**
- The high event rate can help mitigate the decline in resolving power associated with low target polarization purities in real-world applications
- **A simplified state tomography can be performed assuming prior knowledge from the primary scattering**

Outline

Summary

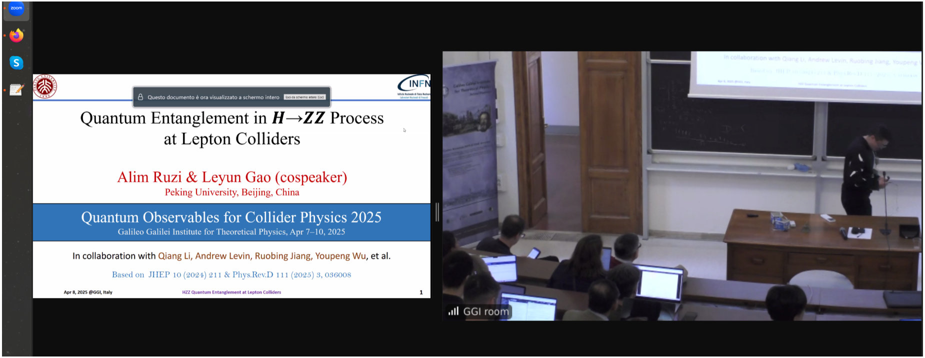
- GeV-scale muon and positron on-target experiments are examined as **controllable entangled lepton pair sources** through the kinematic approach
- Quantum entanglement and the CHSH inequality violation are present in the primary scattering products
- A **first measurement of the correlation between entangled free-traveling lepton pairs** is proposed to verify the entanglement
- The electron-positron beam polarization correlation measurement can be conducted with **a high event rate at many domestic positron beam facilities**

Process	Incident flux	Primary event rate	Secondary coincidence rate
$\mu^- e^- \to \mu^- e^-$	$10^5/\text{s}$	$2.6 \times 10^4/\text{d}$	(not estimated)
$e^+ e^- \to e^+ e^-$	$10^5/\text{s}$	$1.9 \times 10^2/\text{s}$	$4.4 \times 10^2/\text{y}$
$e^+ e^- \to e^+ e^-$	$10^{12}/\text{s}^*$	$\mathbf{1.9 \times 10^9/s}$	$\mathbf{1.4 \times 10^2/s}$

*Possibly from the beam dump of the STCF.

Relevant activities

- 20'+10' oral report in *Quantum Observables for Collider Physics 2025*
- 20' oral report in *Workshop on Quantum Entanglement at the Energy Frontier* (scheduled on Apr 26)
- 30' oral report in *Muon4Future 2025* (scheduled on May 26–30)



Thanks for your attention!

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