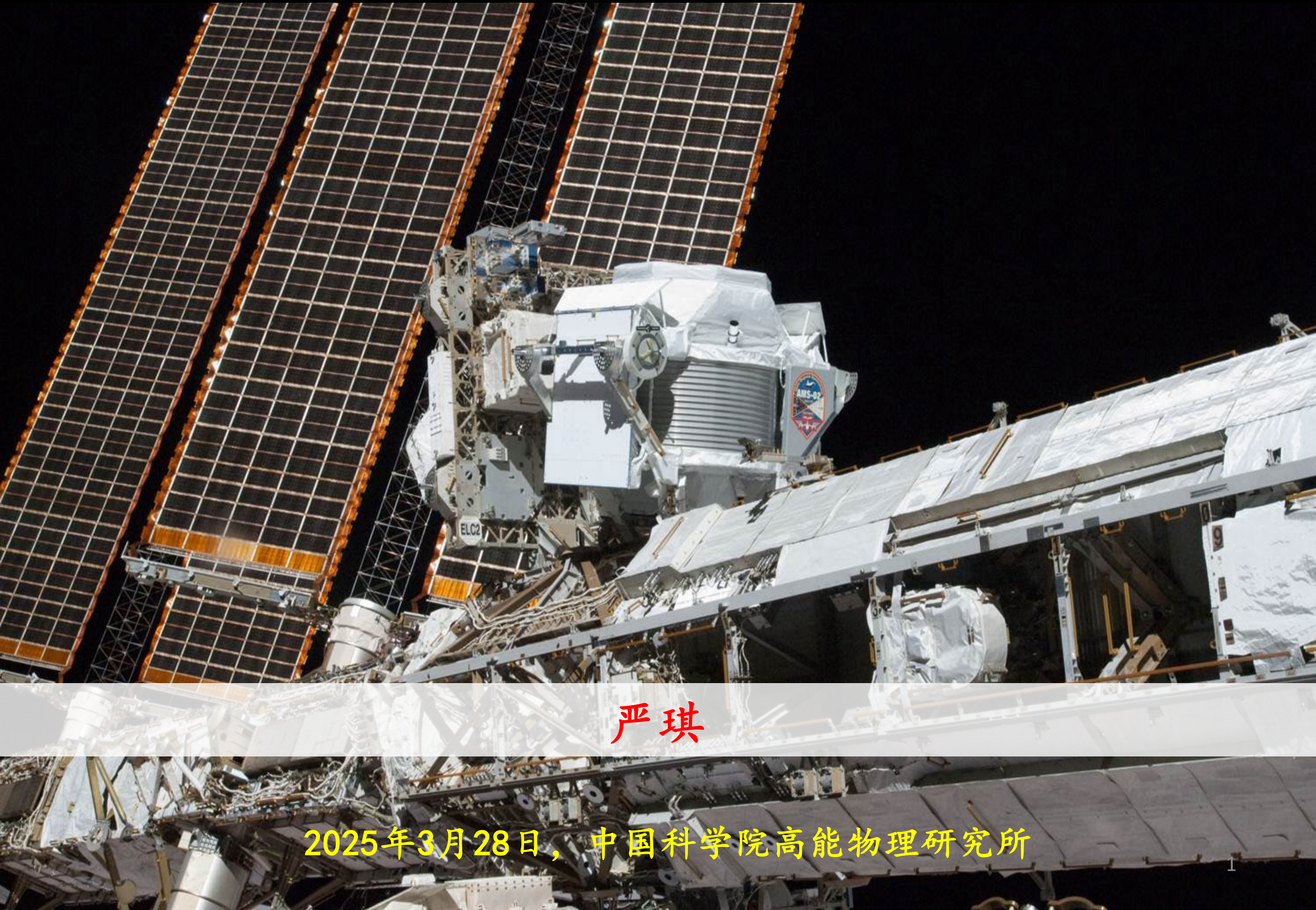
AMS硅径迹探测器

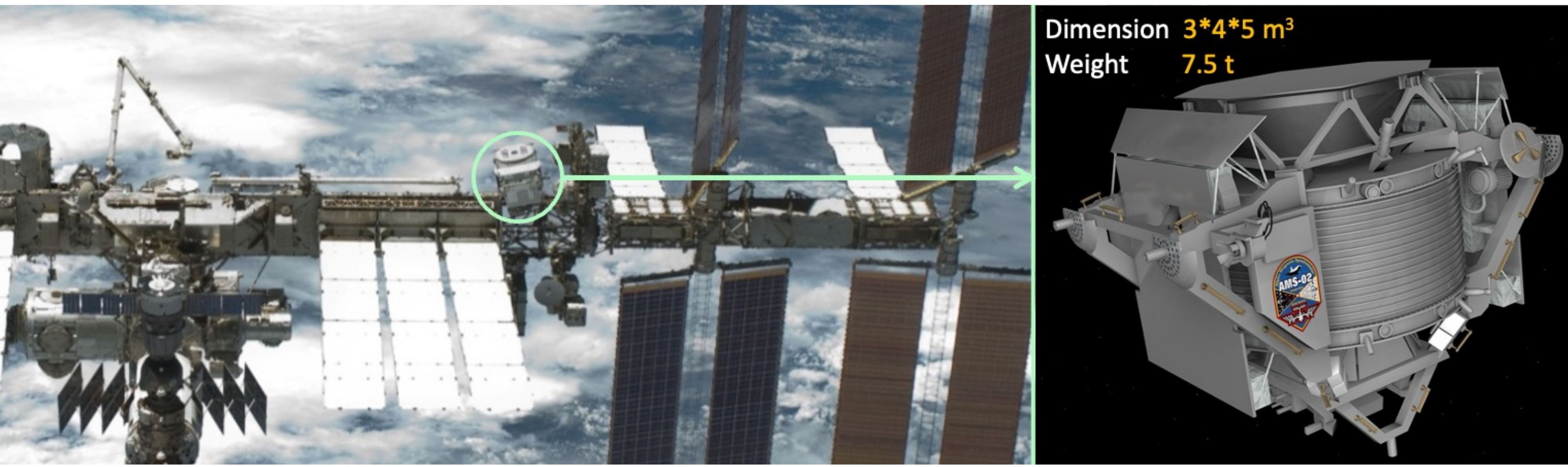


严琪

2025年3月28日，中国科学院高能物理研究所

个人简介

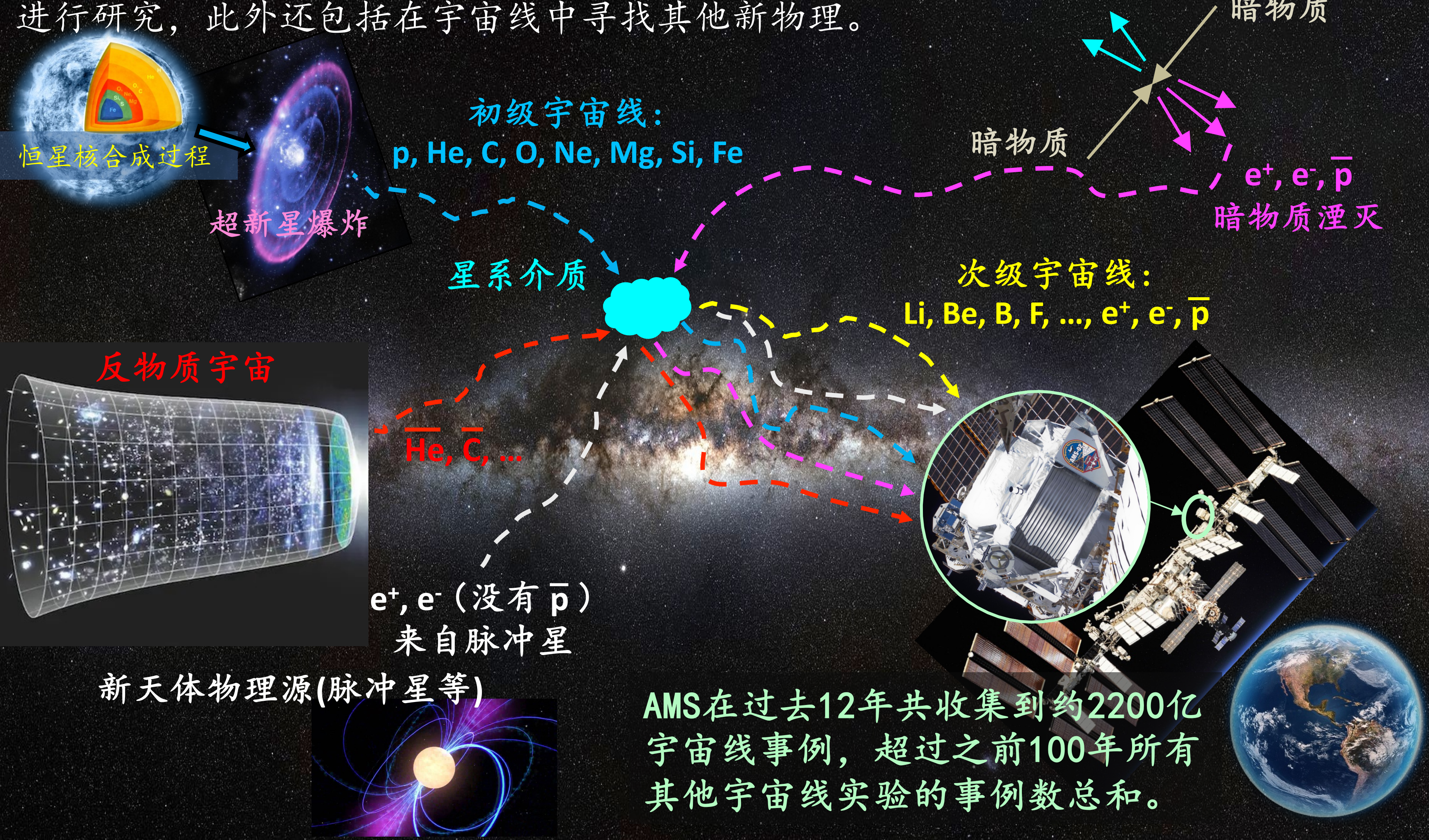
- 之前13年主要从事位于国际空间站上的阿尔法磁谱仪（AMS）的实验研究。2024年4月18日，从工作了10年的麻省理工学院离职(期间担任麻省理工学院首席研究科学家，在丁先生所在的MIT EMI组工作)，回国加入高能所。



阿尔法磁谱仪(AMS)是目前世界上唯一与地面加速器上使用的最先进探测器相类似的精密太空粒子磁谱仪，其造价高达20亿美金。AMS的物理目标包括测量宇宙线中的各种带电粒子和反粒子从而对暗物质、反物质、以及宇宙线的起源进行研究，此外还包括在宇宙线中寻找其他新物理。

AMS实验

位于国际空间站上的阿法尔磁谱仪(AMS)是目前世界上唯一与地面加速器上使用的最先进探测器相类似的精密太空粒子磁谱仪，其造价高达20亿美金。AMS的物理目标包括测量宇宙线中的各种带电粒子和反粒子从而对暗物质、反物质、以及宇宙线的起源进行研究，此外还包括在宇宙线中寻找其他新物理。



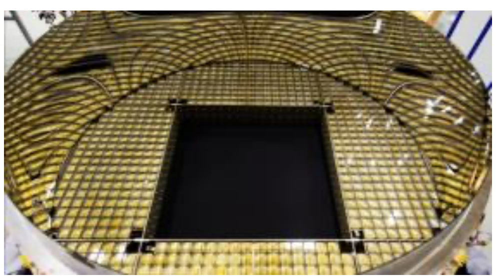
AMS在过去12年共收集到约2200亿宇宙线事例，超过之前100年所有其他宇宙线实验的事例数总和。

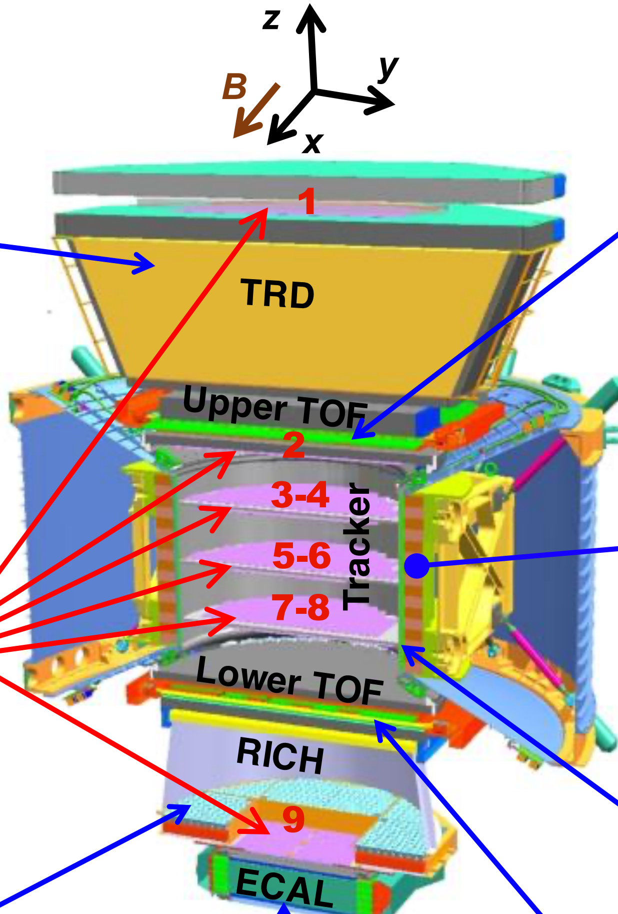
AMS：太空中的TeV精密磁谱仪

穿越辐射探测器
鉴别e+, e−

径迹探测器
测量电荷（Z）和动量（P）

环形成像切伦科夫探测器
鉴别e+, e−



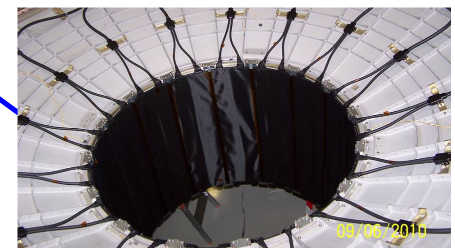


电磁量能器测量e+, e−能量

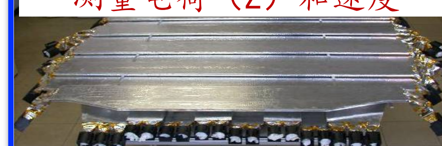
上层飞行时间探测器
测量电荷（Z）和速度

磁铁
分辨电荷正负和动量

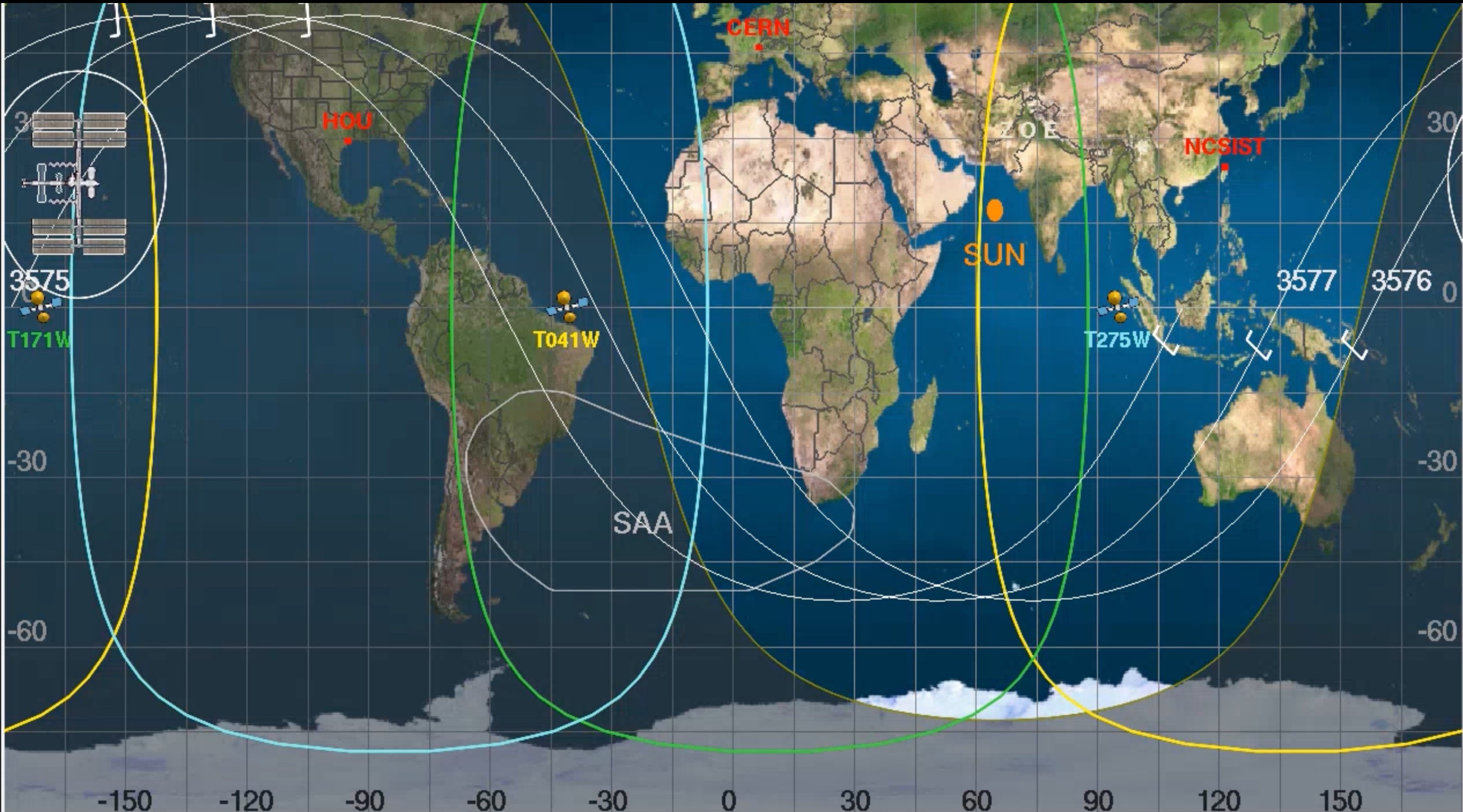
反符合探测器
过滤掉从侧面进入的粒子



下层飞行时间探测器
测量电荷（Z）和速度



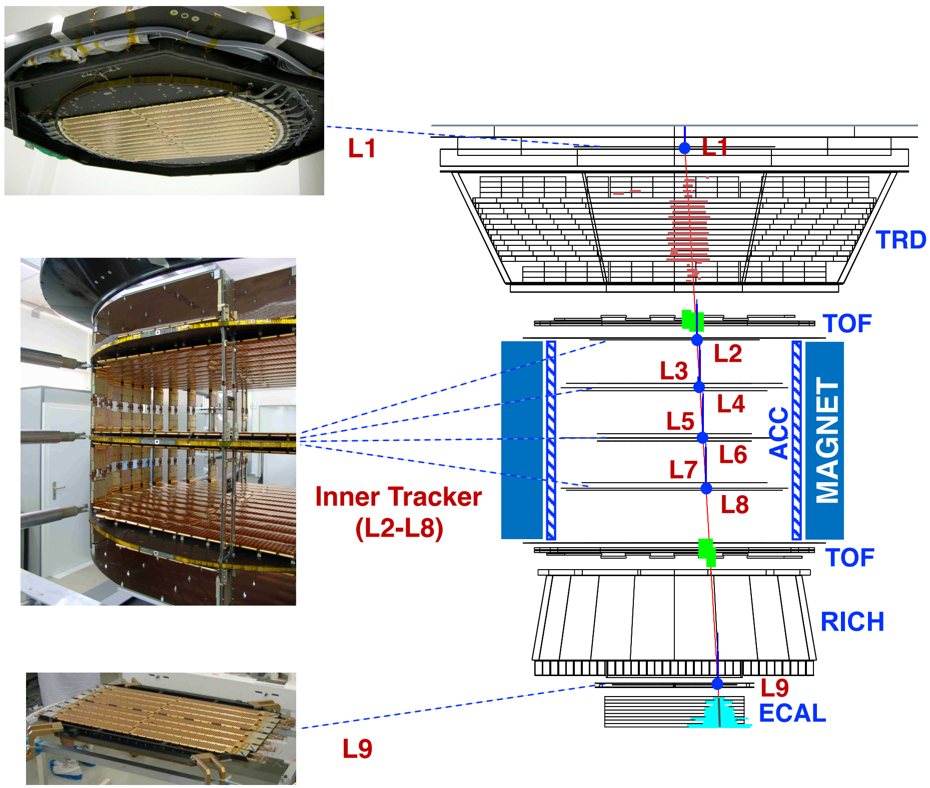
2011-2023: AMS在轨不间断取数



220,182,212,893

AMS在太空中的运行环境

AMS运行在严酷的太空环境中，在发射过程中承受剧烈应力，运行于真空环境，并经历轨道上超过 ±10° C 的持续温度变化。这些因素共同影响探测器的性能。为了实现精确实验，针对太空中的磁谱仪开发了专门的探测器重建方法。



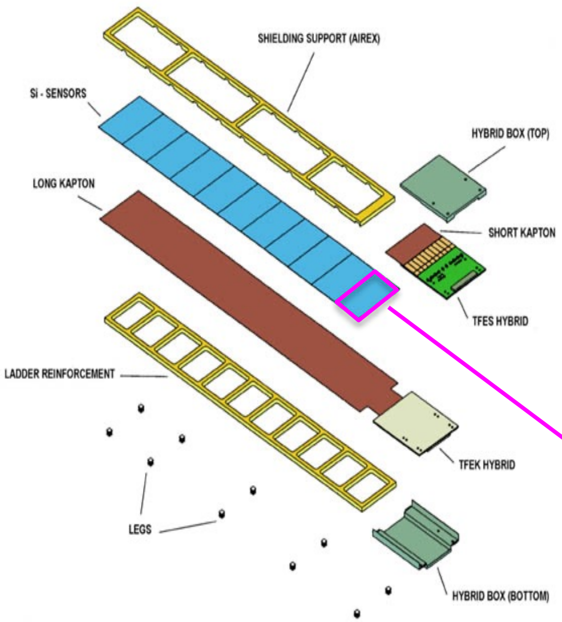
AMS 硅径迹探测器

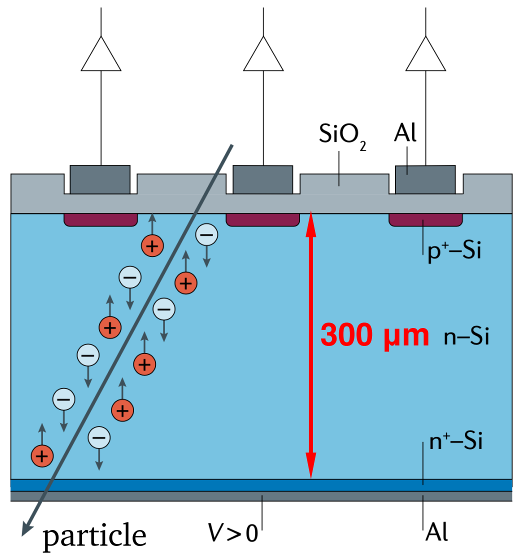
AMS精密的硅径迹探测器，结合永磁体（0.14特斯拉），通过对带电粒子沿其径迹的多个位置测量来确定其刚度（动量/电荷）。

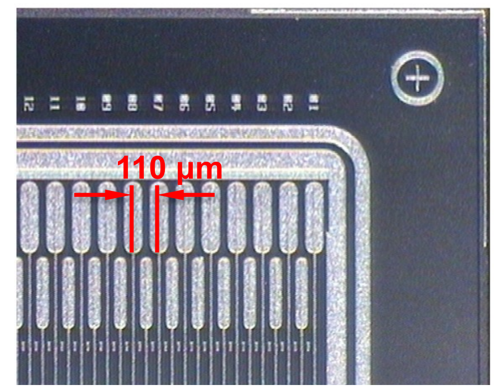
硅径迹探测器是AMS中最重要且最复杂的子探测器，其高性能对AMS任务至关重要。

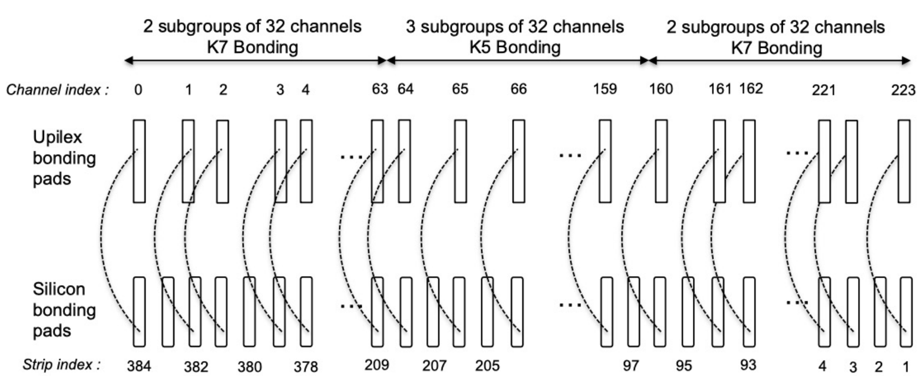
AMS硅传感器

Dimension	Thickness	Y Strip Pitch Readout (implantation)	X Stirp Pitch Readout (implantation)
4.1 cm×7.2cm	300 μm	110 (27.5) μm	208/104 (104) μm





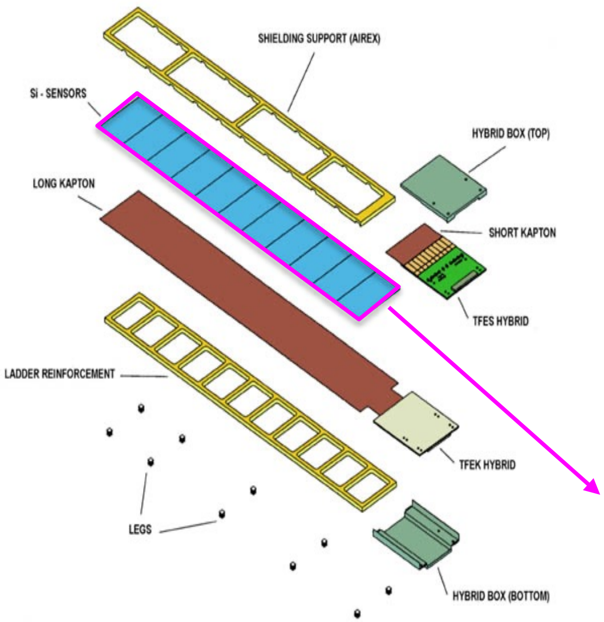




K7 X-side routing scheme

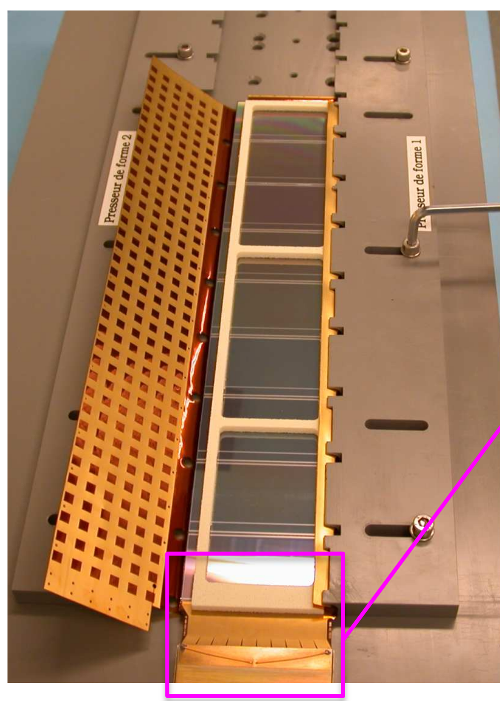
AMS 采用两种类型硅传感器：K5和K7。在X方向（非弯曲方向），K5和K7传感器的读出方案不同。对于X微条的读出间距，K5传感器为208μm，而 K7传感器则包括 208μm 和104μm 两种间距

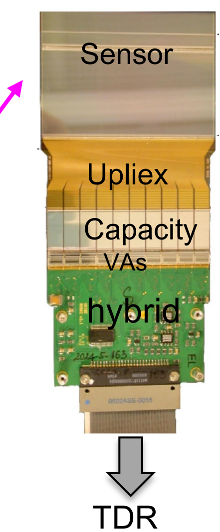
AMS硅径迹探测器结构（1）

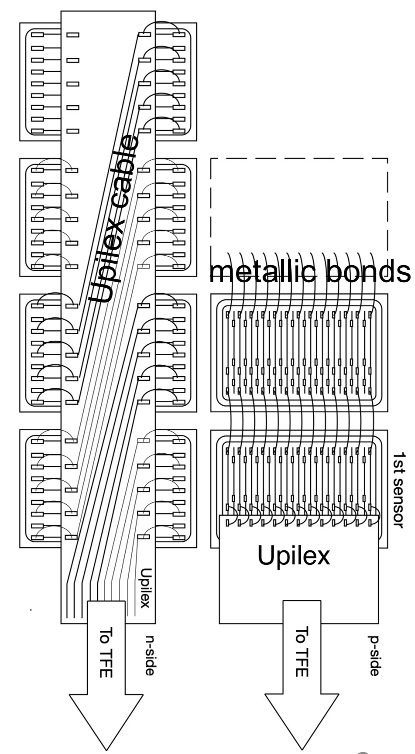


1) 9至15个传感器被组装成一个机械和读出单元，称为“梯形结构”（ladder）。传感器在梯形结构中的组装精度在X和Y方向均约为 6 μm。梯形结构的前端电子学实现微条与数字化系统的耦合。

2) AMS 硅轨迹探测器由192个梯形结构组成，总计包含2284个传感器和196608个读出通道。

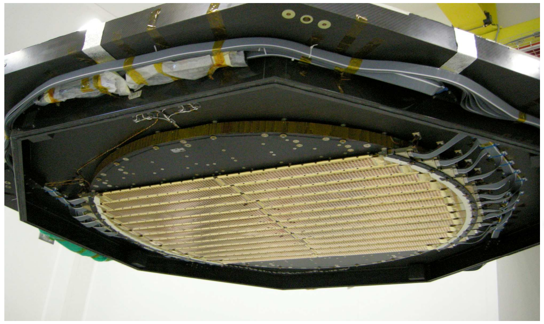




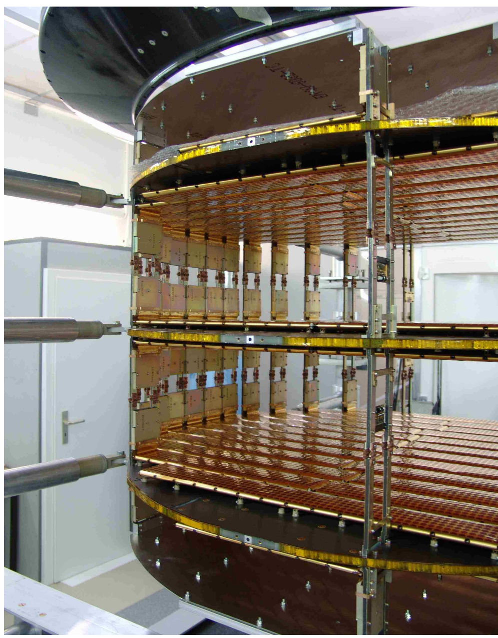


AMS硅径迹探测器结构（2）

L1



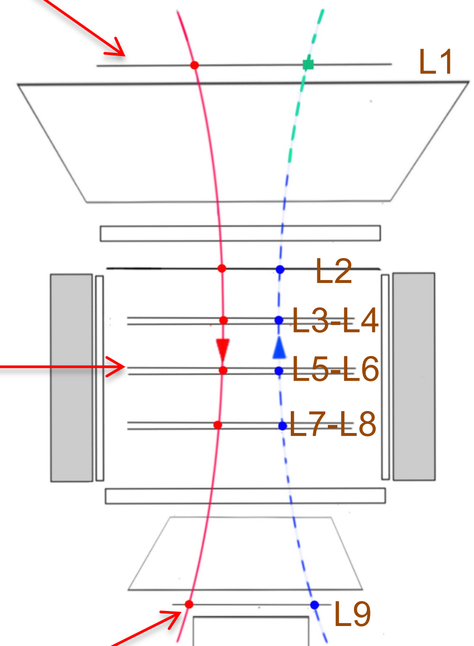
Inner Tracker L2-L8



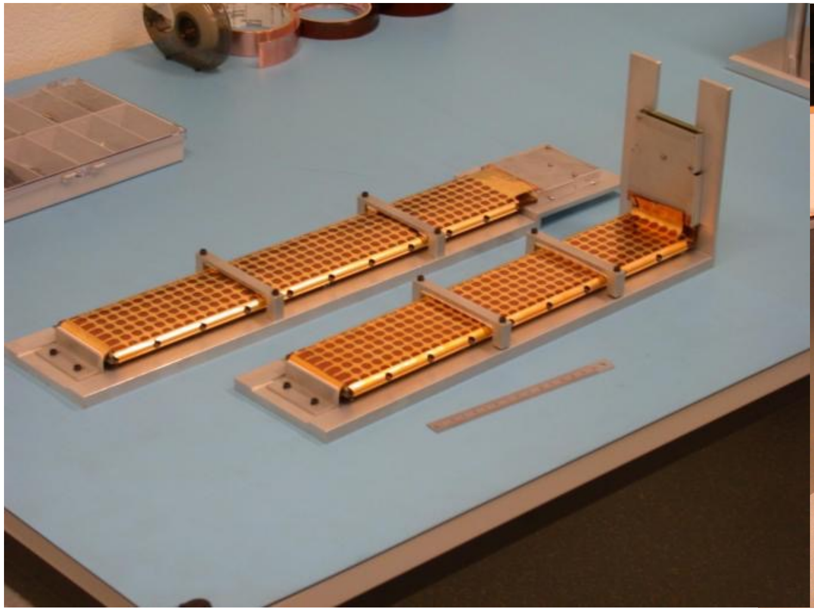
L9

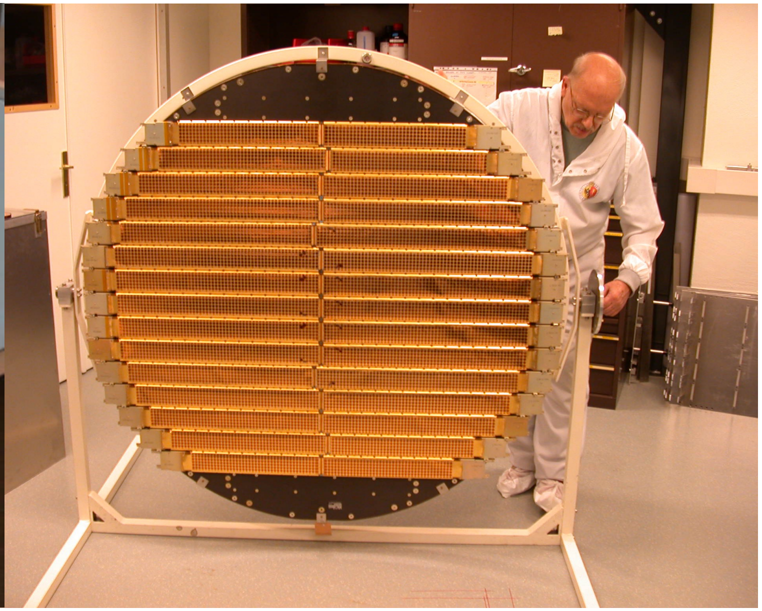
9个径迹探测器层：

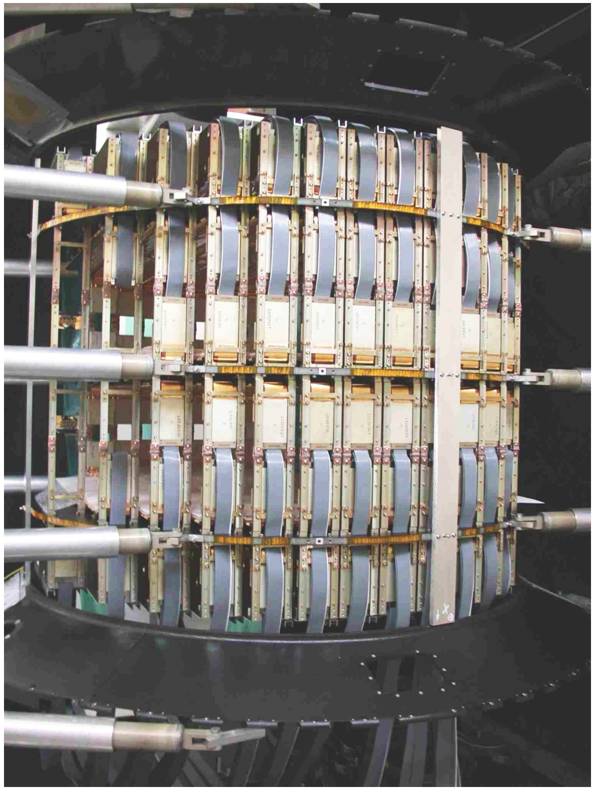
AMS径迹探测器由9层（L1-L9）组成，每层安装16至26个梯形结构（ladders）。梯形结构在层内的安装精度约为60μm。



L1	Plane 1	L1	26 ladders
Inner Tracker L2-L8	Plane 2	L2	22 ladders
	Plane 3	L3	22 ladders
		L4	22 ladders
	Plane 4	L5	20 ladders
		L6	20 ladders
	Plane 5	L7	22 ladders
		L8	22 ladders
L9	Plane 6	L9	16 ladders





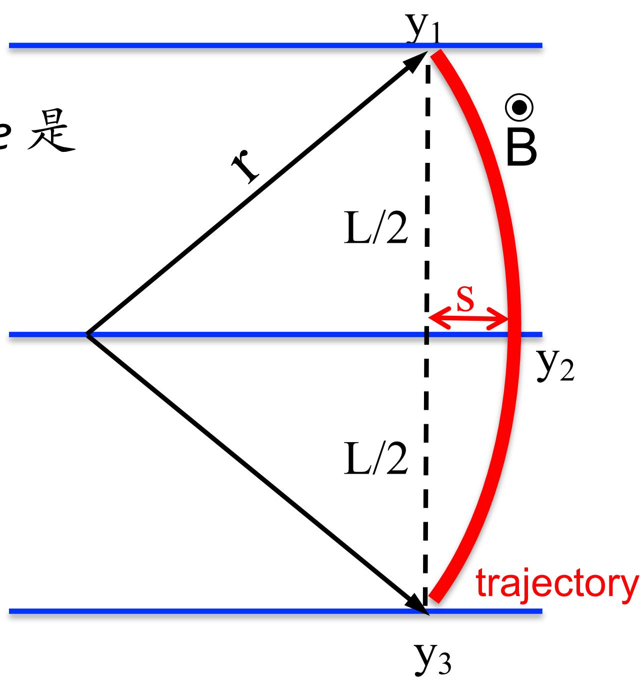


磁场中粒子刚度（R）的测量

$\vec{F}=q\vec{v}\times\vec{B}=Ze\vec{v}\times\vec{B}$

p_T[GeV/c]=R_T[GV]Ze/c=0.3ZB[T] r[m]

p_T是粒子的横向动量(⊥ to B); Ze是粒子电荷; $R_T=p_Tc/Ze$ 是粒子的横向刚度.



sagitta, s, 用于描述粒子轨迹偏离直线的大小，由以下公式给出：

$$s=r-\sqrt{r^2-\left(\frac{L}{2}\right)^2}=y_2-\frac{y_1+y_3}{2}\approx\frac{L^2}{8r}=\frac{0.3BL^2c}{8R_Te}\quad (s<<r)$$

由粒子位置的测量精度对sagitta的精度影响(σ_s):

$\sigma_s=\sqrt{3/2}\,\sigma_y$

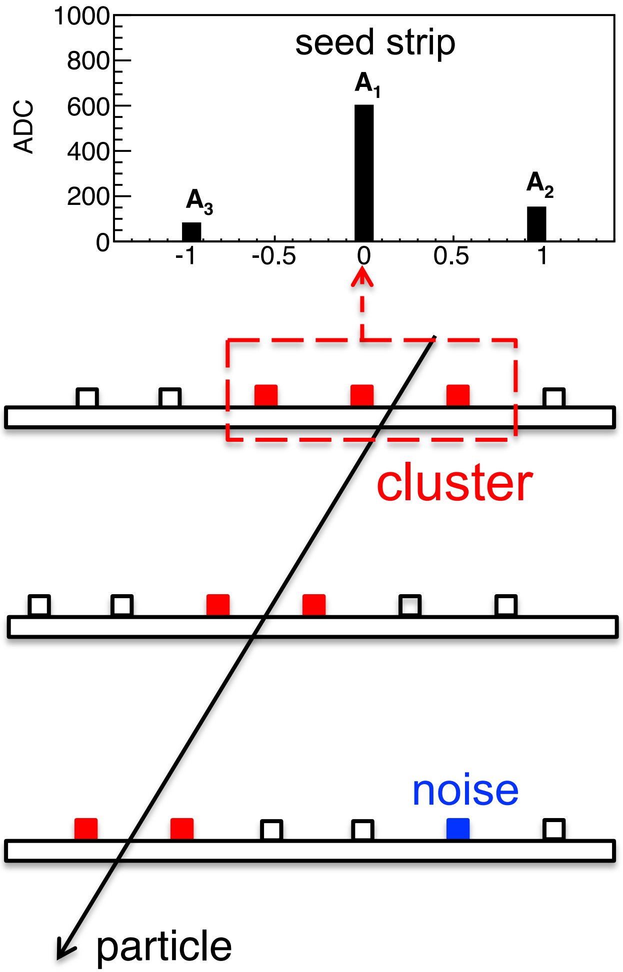
因此，由坐标测量分辨率引起的横向刚度分辨率为：

$$\frac{\sigma_{R\perp}}{R_\perp}=\frac{\sigma_s}{s}=\frac{\sqrt{3/2}\,\sigma_y}{0.3BL^2/(8R_Te/c)}=\frac{\sqrt{96}(R_Te)\sigma_y}{0.3BL^2c}$$

For the measurement points n>>3 distributed uniformly along L with the same coordinate resolution σ_y, the formula to describe the resolution of the transverse rigidity is:

$$\frac{\sigma_{R\perp}}{R_\perp}=\frac{\sigma_s}{s}=\frac{\sqrt{720/(n+4)}(R_Te)\sigma_y}{0.3BL^2c}$$

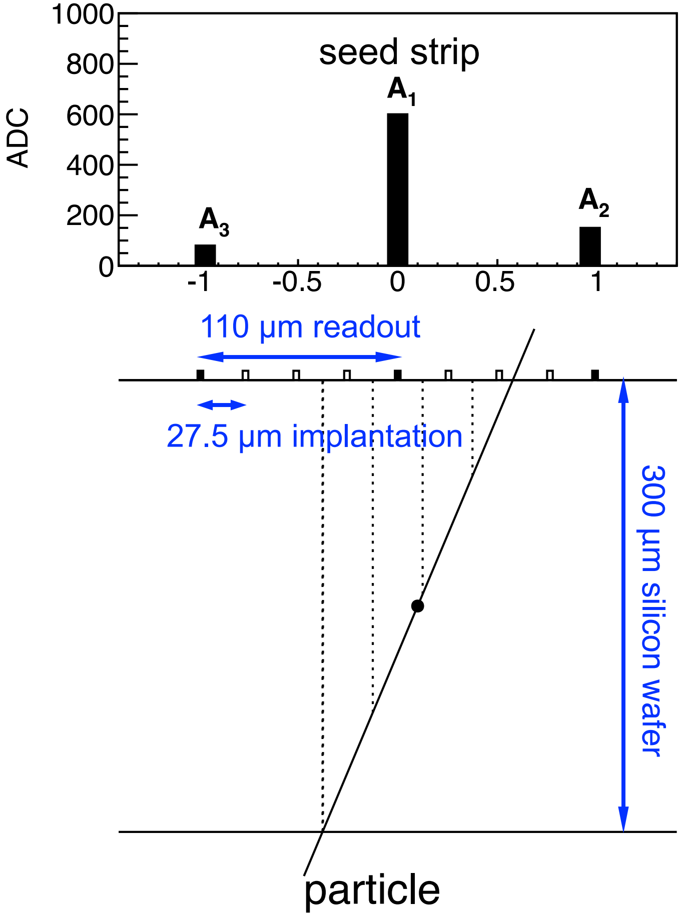
粒子簇（击中）重建



重建粒子径迹的第一步是确定粒子在每个经过的探测器传感器的位置。

- 对于硅微条形探测器，在径迹“簇”重建过程中，局部最高幅度的微条叫作为“簇种子”，这些微条需要超过一定的信噪比阈值，以区分噪声。
- 微条及其相邻的信号微条构成一个簇。径迹簇是重建粒子冲击位置的基本单元，并且用于区分不同粒子和噪声的信号。
- 在线标定是簇重建过程中的重要步骤。AMS每个读出通道的标定每23分钟进行一次，记录信号减去基线的偏置，计算噪声，并标记“失效”的微条。

粒子击中位置的测量

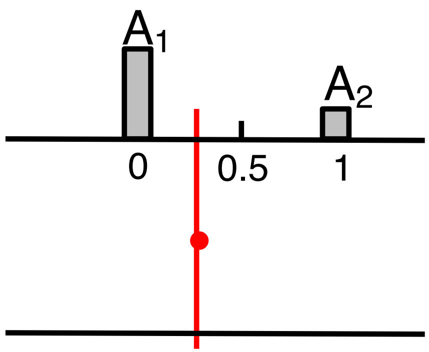


Y方向（弯曲方向）相邻读出微条之间的间距为110 μm，X方向的间距为208/104 μm。

为了获得更好的空间分辨率，目标达到10 μm的测量精度，利用作为簇组合的微条信号幅度进行坐标重建。为了达到最佳分辨率，邻近微条之间的电荷分布应与粒子击中位置呈线性关系：

1. 在硬件方面，引入了非读出中间微条，利用其与种子微条之间的电容耦合来改善电荷分配的线性。对于Y方向，在每两个读出微条之间，设置了3个非读出的中间微条，间距为27.5 μm。
2. 在AMS上，我开发了一种精确的粒子位置重建方法，用于修正由以下因素导致的非线性：
    a) 晶圆中电荷载流子的固有扩散机制与电容耦合效应；
    b) 读出电子学（前置放大器）的非线性增益。

在传统算法中，粒子位置通过加权平均邻近的两个最高幅度微条的位置来重建，权重为它们的信号幅度，计算公式如下

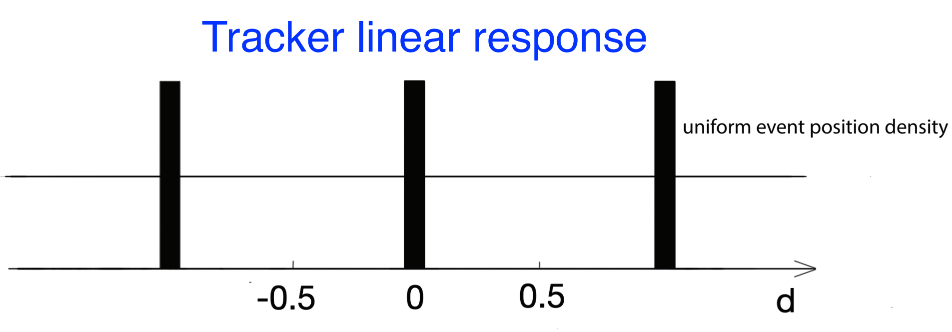


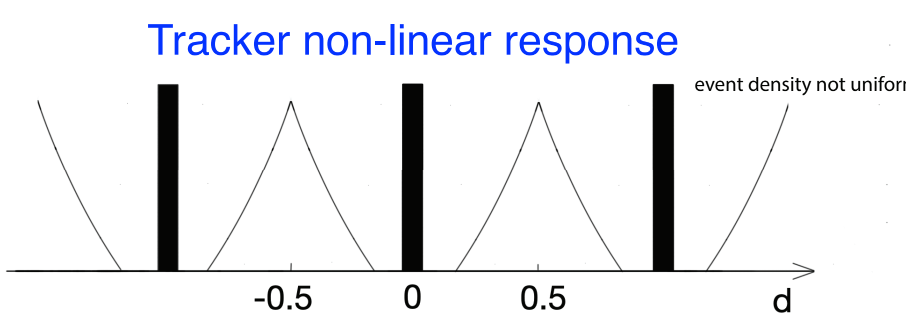
$$X_{\mathrm{LIN}} = \hat{s}\frac{A_2}{A_1 + A_2}$$

其中，A_1和A_2分别是“种子”微条（最高幅度微条）和“第二”微条（第二高幅度微条）的幅度；s^ 为+1 或 −1，当第二微条位于种子微条的右侧时为 +1，位于左侧时为 −1。

然而，当微条之间的电荷分配非线性，那么在应用该算法时，重建的粒子位置将会产生偏差。

我开发的重建方法的核心思想是利用宇宙线（或束流）空间均匀性这一特性。对于一个理想的径迹探测器，在没有非线性的情况下，我们应该看到重建位置没有偏差，事件位置的密度分布是均匀的；而在非线性情况下，事件密度会发生失真。期望通过优化的位置重建算法恢复均匀性，从而获得最佳的位置分辨率。



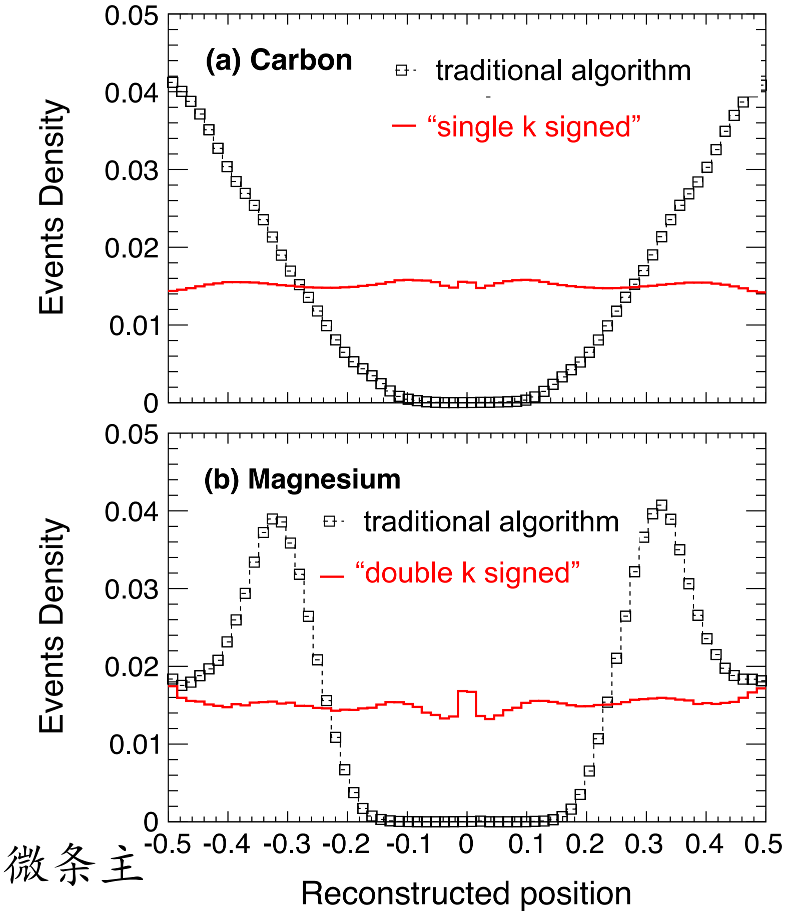


AMS的粒子重建算法

开发了“Single k signed” 数学线性模型，对具有单个微条非线性效应的位置进行重建：

$$X_L = \eta'_L = \hat{s}\,\frac{A_2^m - \frac{c}{1-2c}[A_1^m + (s-1)(1-2|x|)A_1^m - A_2^m]}{A_1^m + (s-1)(1-2|x|)A_1^m + A_2^m}$$

其中，c 是用于修正条形线之间电容耦合和电荷共享的参数；s 是用于修正读出电子学（饱和效应）的非线性增益行为的参数；x 是粒子真实的位置，经过迭代后可以用重建的位置 X_L 替代。



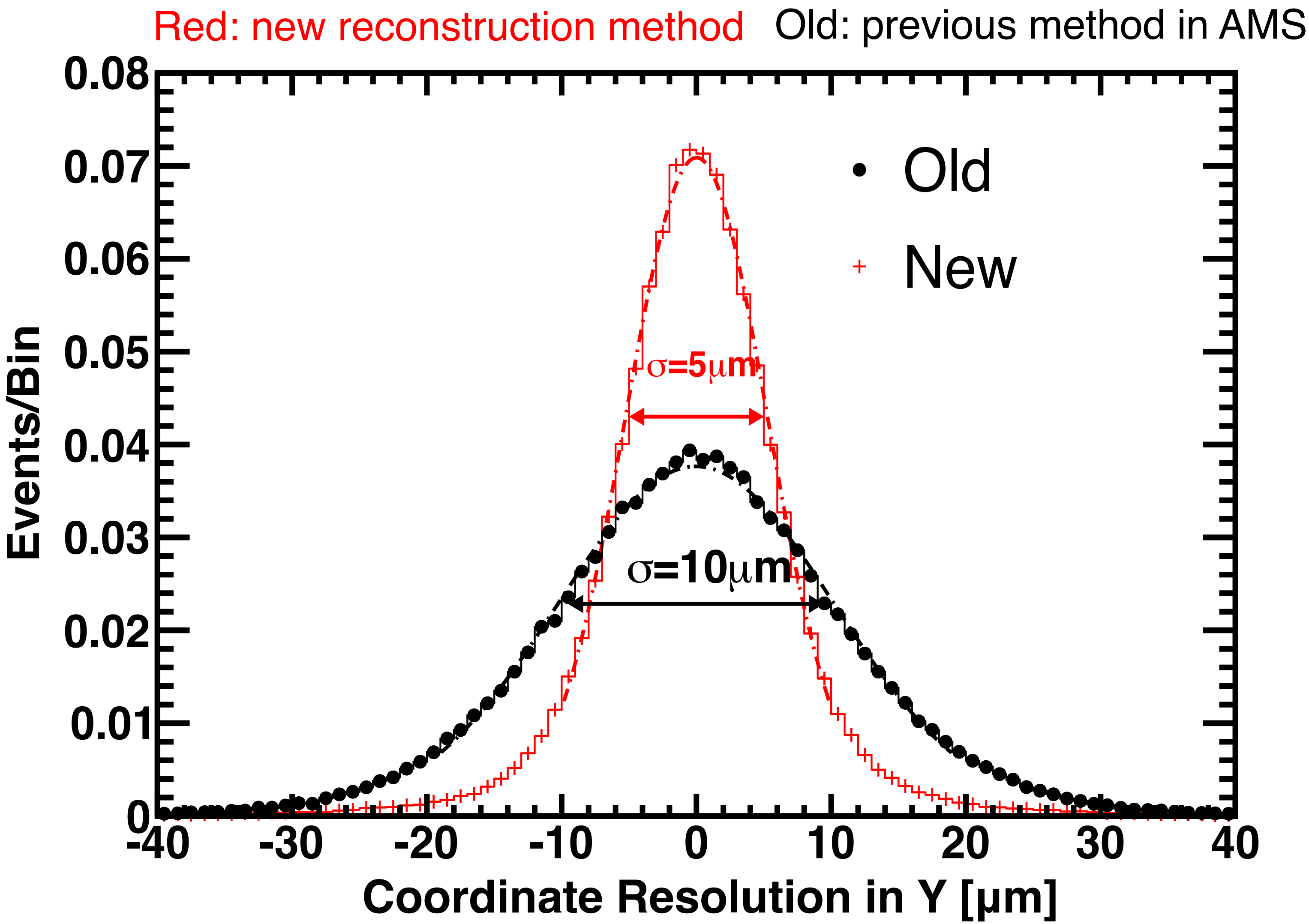
“Double k signed”对具有两个微条非线性效应的位置进行重建:

$$X_P = \eta'_P = \hat{s}\,\frac{A_2^m - \frac{c}{1-2c}[A_1^m + (s-1)(1-2|x|)(1-\frac{|x|}{x_0})A_1^m - A_2^m]}{A_1^m + (s-1)(1-2|x|)(1-\frac{|x|}{x_0})A_1^m + A_2^m}$$

其中，x_0 是额外的参数，用于描述种子微条和第二微条主导饱和的过渡位置。

方法详见我之前发表的文章“Nucl. Instrum. Methods Phys. Res. A 869, 29 (2017)”.

AMS 硅径迹探测器的位置分辨(Carbon Nuclei)

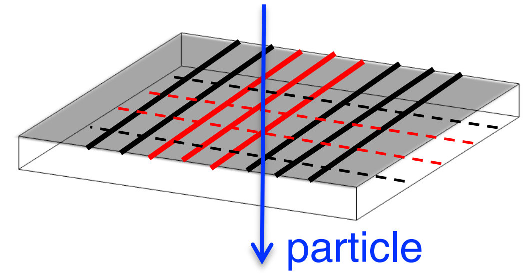


通过这种新方法，碳核的位置分辨率达到5μm的精度。对于更高电荷的原子核，分辨改进效果更加显著。这项工作将AMS的测量扩展到高刚性区域，并提高了所有电荷原子核的刚度分辨率。

AMS的径迹寻找算法

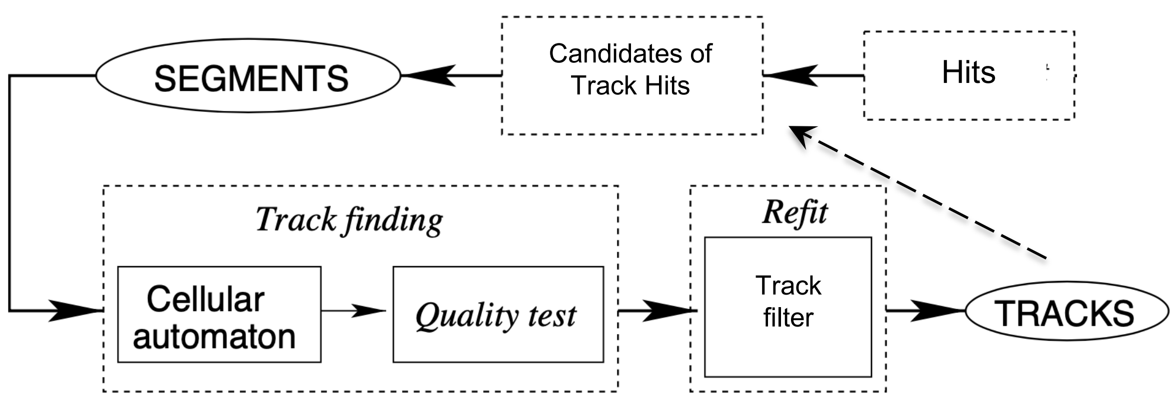
来自同一单元的X方向和Y方向的簇被组合成3D命中。为了减少不相关的组合，每对簇在一个击中都需要有信号幅度的匹配。

AMS径迹探测器拥有大约20万读出道。在一个宇宙线事件中，由于粒子与探测器物质相互作用产生的Δ电子，所产生的击中数数可以达上万个。为了高效地重建所有粒子的径迹，开发了一种复杂的AMS径迹寻找算法。



A 3D Hit in a sensor

首先，通过幅度相关性识别一组来自不同径迹的相关“种子”命中点。然后，将这些种子命中点收集到一个“池”中，以选择可能属于同一径迹的其他候选命中点。通过这种方式，主粒子的命中点和低电荷二次粒子的击中得到一定地区分。接着，来自每两层的选定命中点相互连接，形成用于元胞自动机径迹寻找的预阶段。

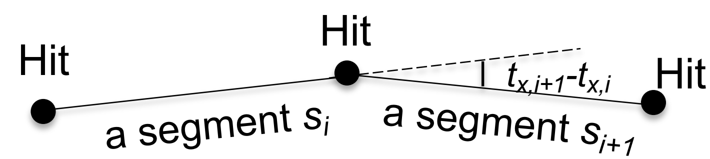


Cellular Automation算法

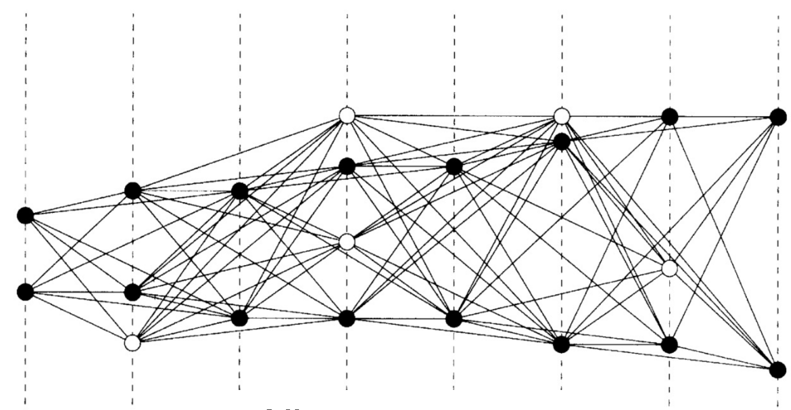
元胞自动机（Cellular automata）由Stanislaw Ulam和John von Neumann于1940年代发现，并在1970年代随着Conway的“生命游戏”（Life）而获得更多关注。在高能物理（HEP）中，径迹寻找算法面临的主要挑战之一是从庞大的击中组合中高效重建出径迹。一个有效的解决方案是采用元胞自动机方法。

元胞自动机是动态系统，它在离散的、通常是二维的空间中演化，由单元格组成。在径迹重建中，短径迹段作为元胞单元使用。一个径迹段s是连接每两层的两个击中点的直线。相邻轨迹段之间的匹配角度由以下公式给出：

$$\varphi(s_i, s_{i+1}) = \sqrt{(t_{x,i+1} - t_{x,i})^2 + (t_{y,i+1} - t_{y,i})^2}.$$



其中，s_i 和 s_{i+1} 是两个相邻的径迹段；t是径迹段的投影角度；φ(si, si+1) 是相邻径迹段之间的角度。



All segments

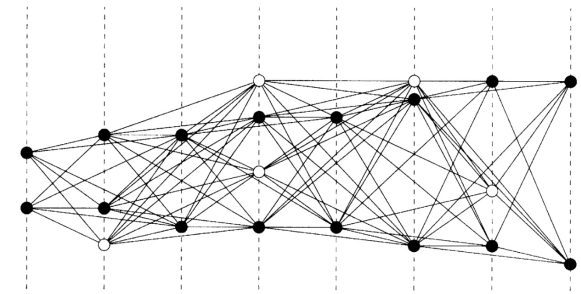
为减少在径迹段pipe up无不相关的排列组合数目，每个相邻径迹段的匹配角度要求满足$\varphi(s_i, s_{i+1}) \leq \phi$.

当给定完整的径迹段集，寻找一条径迹可以数学描述为一个优化问题，目的是找到一个径迹段序列 $U(s_1, \cdots, s_N)$：

1. 通过最大化径迹段数 N 来最大化径迹长度；
2. 通过最小化相邻径迹段之间匹配角度的和来最大化径迹的平滑度。

最优化的问题用以下数学公式描述：

$$J(U) = N + w(s_1 \ldots, s_N) - \gamma \sum_{i=1}^{N-1} \frac{\varphi(s_{i+1}, s_i)}{z_{i+1,r} - z_{i,l}} \rightarrow \max_U$$



其中，$w(s_1, \cdots, s_N)$ 是径迹质量权重，除径迹长度外；γ 是控制径迹段平滑度和长度之间权衡的系数；$z_{i+1,r} - z_{i,l}$ 是相邻径迹段的长度。

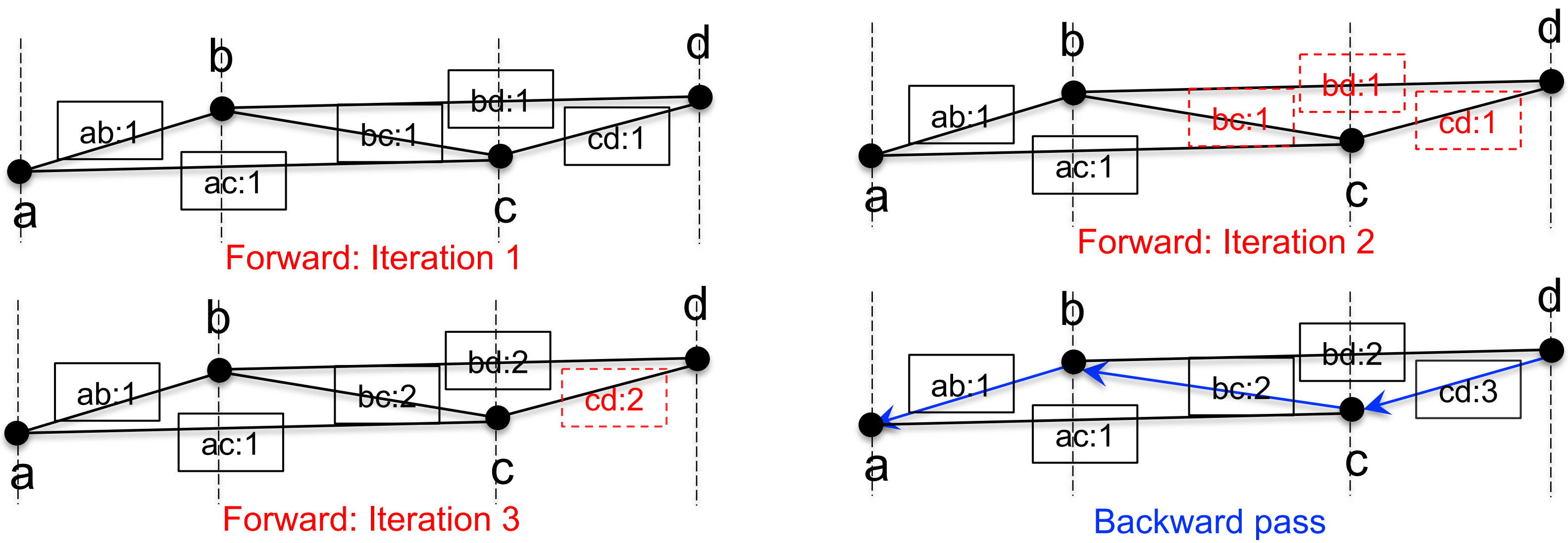
元胞自动机的演化过程分为以下两个阶段：

- <u>前向演化</u>：当自动机迭代更新所有相邻的、具有相同单元格状态时，进行前向演化。
- <u>反向传递</u>：当自动机从具有最高状态的单元格开始收集最优序列时，进行反向传递。

元胞自动机的演化过程分为以下两个阶段：：

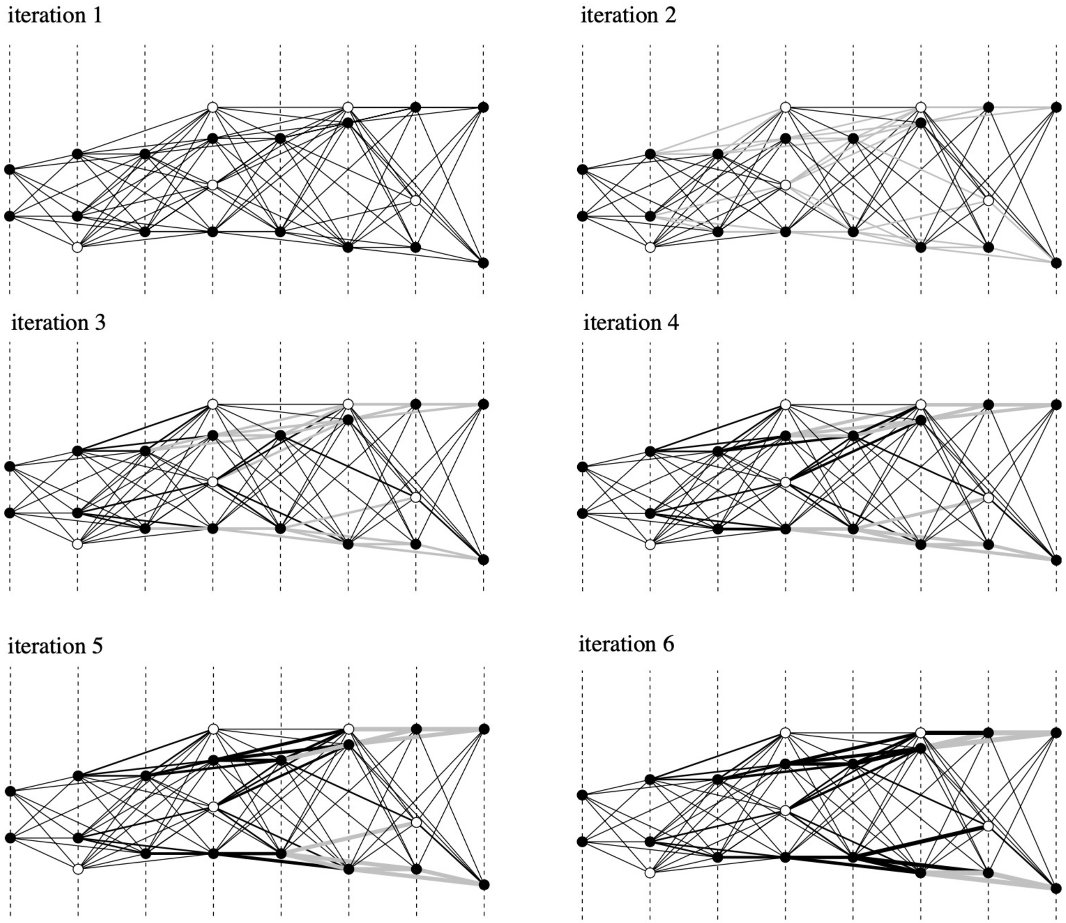
- <u>前向演化</u>:

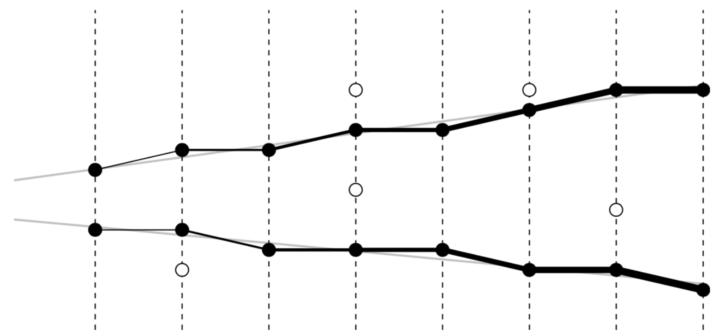
自动机处理每个单元格，并寻找其左侧邻居。如果有左邻且其状态与该单元格的状态相同，则该单元格的状态将增加1。当自动机遍历所有单元格并完成一次循环时，它们之前的状态将被更新。这个过程会反复迭代，直到没有左邻单元格与当前单元格具有相同的状态为止。



- <u>反向传递</u>:

从具有最高状态的单元格（集）开始，自动机寻找其左侧邻居，其状态比当前单元格低一个单位，依此类推。当分配给候选径迹的径迹段状态为1时，候选追踪停止。基线算法将所有分配给径迹的径迹段标记为已使用。自动机通过从另一个未使用的、具有最高状态的单元格开始，继续组装下一个径迹候选。



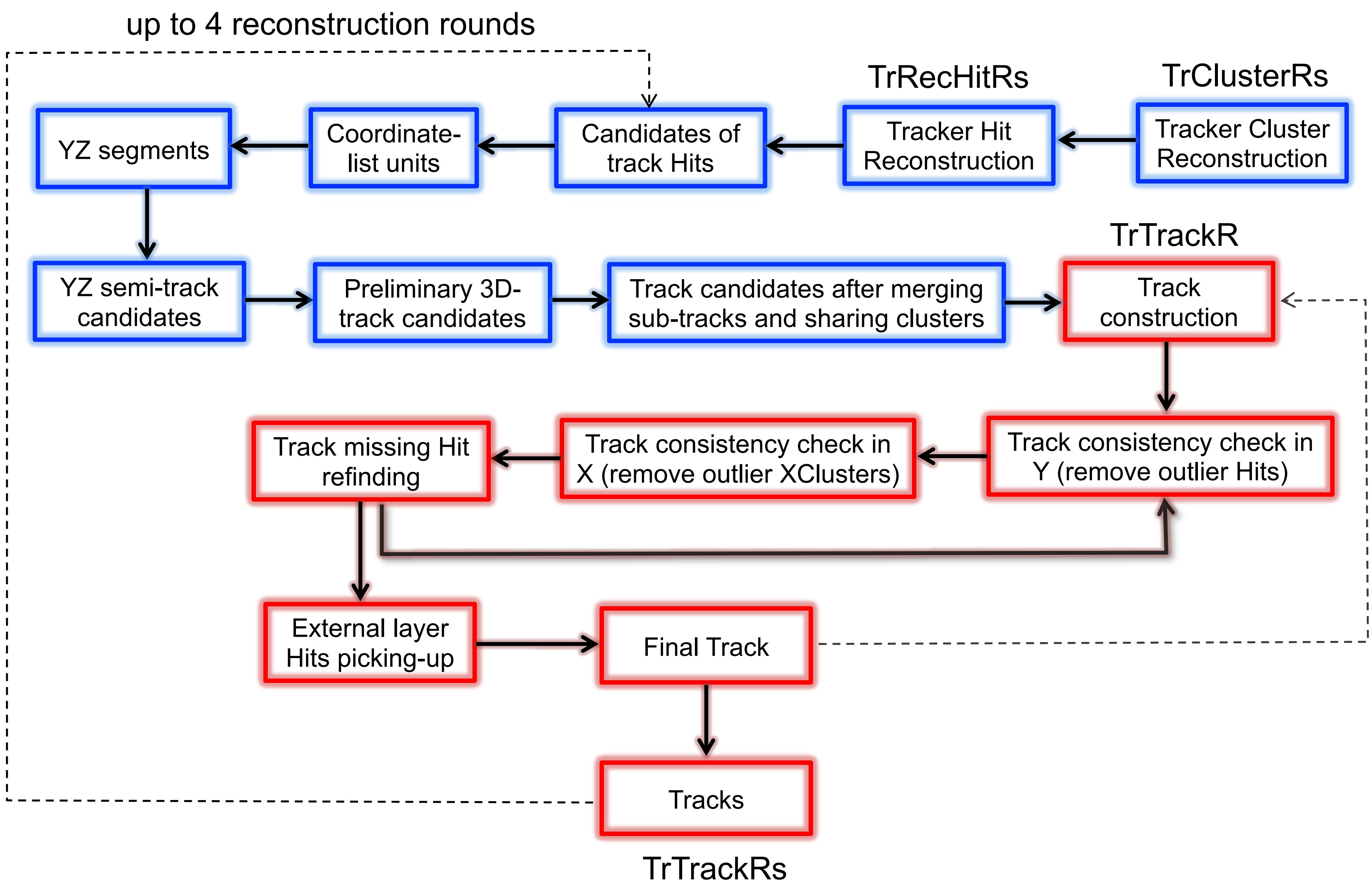


反向传递

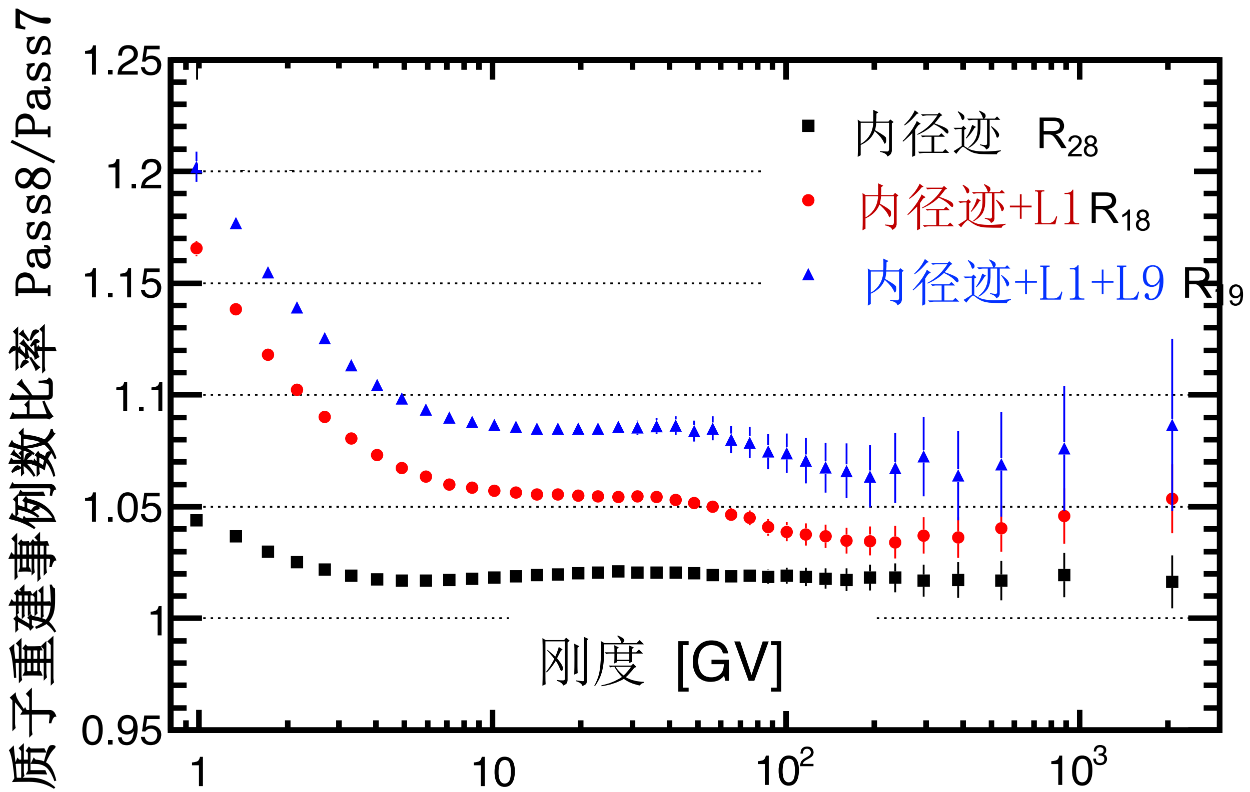
元胞自动机的前向演化与迭代过程：

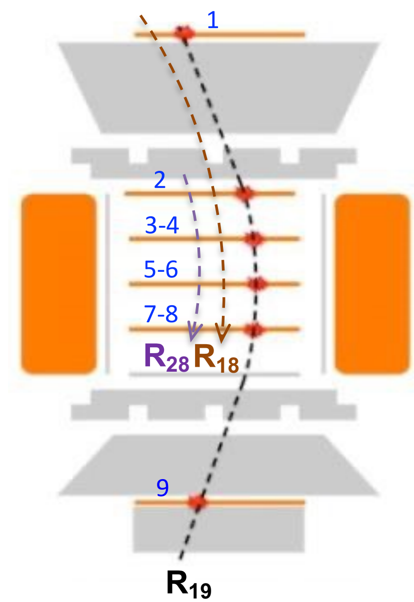
径迹段的粗细代表其当前状态，即状态为1的径迹段用单线表示；状态为2的径迹段则用宽度为两倍的线表示，以此类推。灰色线表示在一次迭代过程中状态发生变化的径迹段。开放圆圈表示噪声击中点。

开发的AMS完整径迹寻找算法（Pass8）示意图



开发的最新AMS径迹重建Pass8与之前Pass7的对比





AMS最新一轮重建Pass8与前一轮重建Pass7的事例数比率。AMS径迹探测器的结构很特殊性：内径迹（L2-L8）的物质量小，总物质量约0.05X_0（辐射长度）；内径迹到第1层（L1）和第9层（L9）外径迹的物质量大，分别约为0.3X_0和0.2X_0。内径迹效率的提升得益于径迹寻找算法的优化，而外径迹效率的提升主要归功于对多重散射效应的特殊处理。

径迹拟合

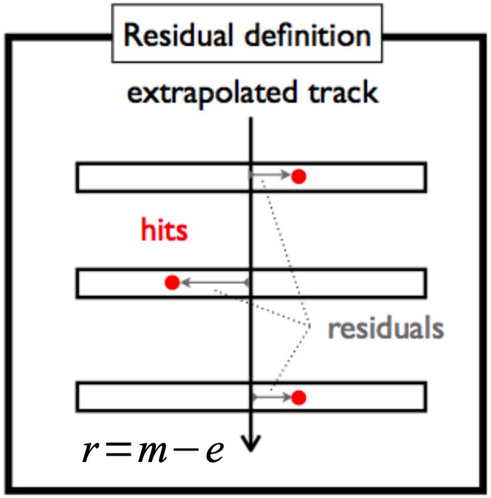
A track in a magnetic field is characterized by 5 parameters at a given plane:

x, y, dx/dz, dy/dz, and 1/R

where 1/R is inverse rigidity. The task of the track fitting is to find the optimal estimate of track parameters for each set of measurements. The Least Squares (X^2) Method best meets the requirements of track fitting.

In the simplest track model, the track parameters (q_l) are determined by minimization of the track residual X^2:

$$\chi^2 = \sum_{k=1}^{n_{meas}} \left(\frac{r_k}{\sigma_k}\right)^2 \longrightarrow \quad 0 = \frac{\partial \chi^2}{\partial q_l} = 2\sum_{k=1}^{n_{meas}} \frac{\partial r_k}{\partial q_l}\frac{r_k}{\sigma_k^2}$$



Rewrite the X^2 using the matrix algebra:

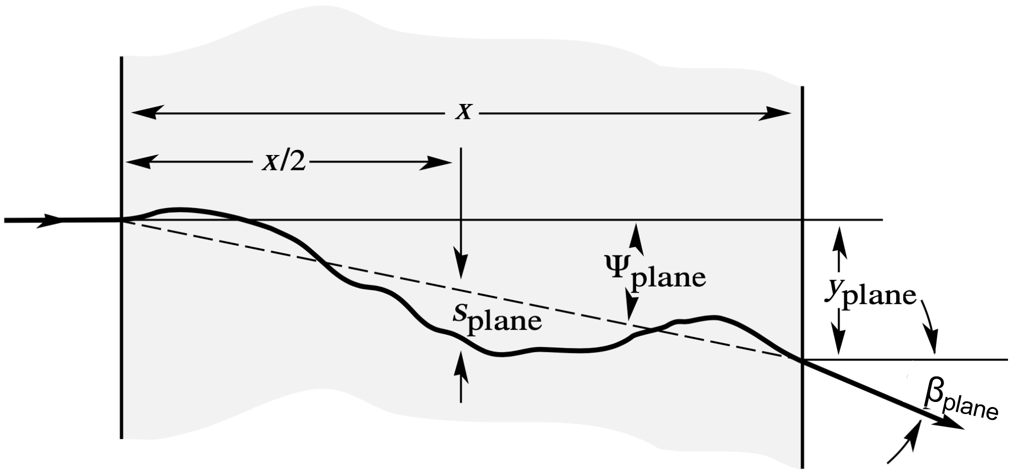
$$r = \begin{pmatrix} r_1 \\ \vdots \\ r_{n_{meas}} \end{pmatrix} \quad V = \begin{pmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{n_{meas}}^2 \end{pmatrix} \longrightarrow \quad \chi^2 = r^T V^{-1} r$$

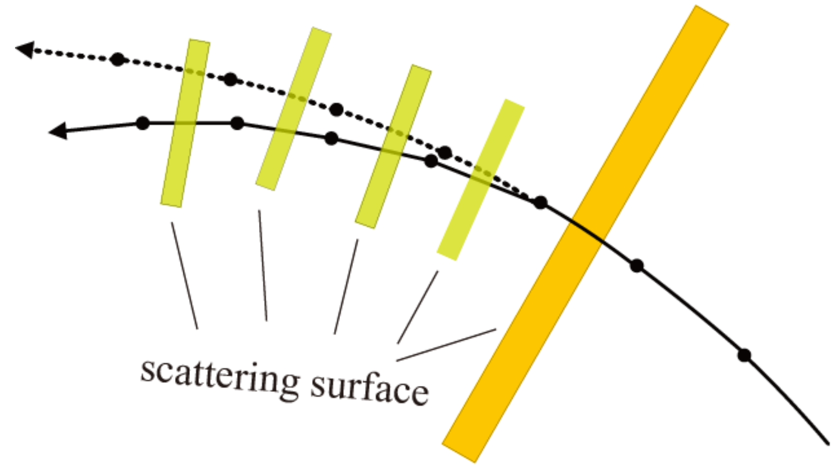
Apply X^2 minimization with respect to the track parameters (q):

$$0 = \frac{\partial \chi^2}{\partial q} = 2\left(\frac{\partial r}{\partial q}\right)^T V^{-1} r \quad \text{where } q = \begin{pmatrix} q_1 \\ \vdots \\ q_{n_{par}} \end{pmatrix} = \begin{pmatrix} x \\ y \\ dx/dz \\ dy/dz \\ 1/R \end{pmatrix} \quad \frac{\partial r}{\partial q} = \begin{pmatrix} \partial r_1/\partial q_1 & \cdots & \partial r_1/\partial q_{n_{par}} \\ \vdots & \ddots & \vdots \\ \partial r_{n_{meas}}/\partial q_1 & \cdots & \partial r_{n_{meas}}/\partial q_{n_{par}} \end{pmatrix}$$

Multiple Scattering

In reality, the track fitting procedure is more complicate. When the incoming particle intersect with detector materials, the track trajectory or its parameters will continuously change.





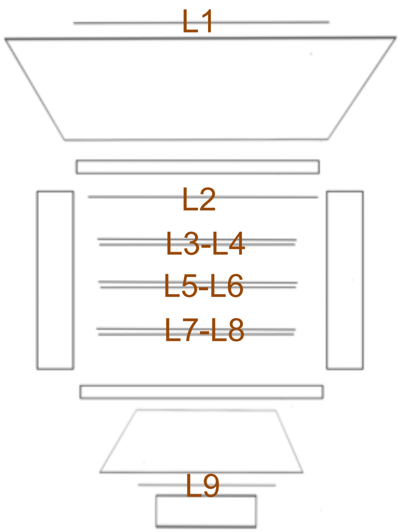
Charge particles traversing materials experience multiple scattering (MS), mainly due to Coulomb interaction with the electrons in the atoms. The multiple scattering successive deflects the particle trajectory, which must be included in the track fitting algorithm.

In the track fitting, the assumption is that the MS angle β follows a Gaussian distribution. It is known that the tails are larger than just Gaussian tails.

$$V(\beta) = \left(\frac{13.6\ \mathrm{MeV}}{\beta c p / z}\right)^2 \frac{x}{X_0}\left[1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right]^2$$

where the mean value of scattering angle in material β is zero; and the variance $V[\beta]=\sigma(\beta)^2$ depends on the amount of traversed material (x/X_0) and the particle rigidity ($R=p/z$).

Track Fitting with Multiple Scattering



Including the MS terms in track fitting model will require precise description of the detector materials:

a) AMS materials between neighboring measurements

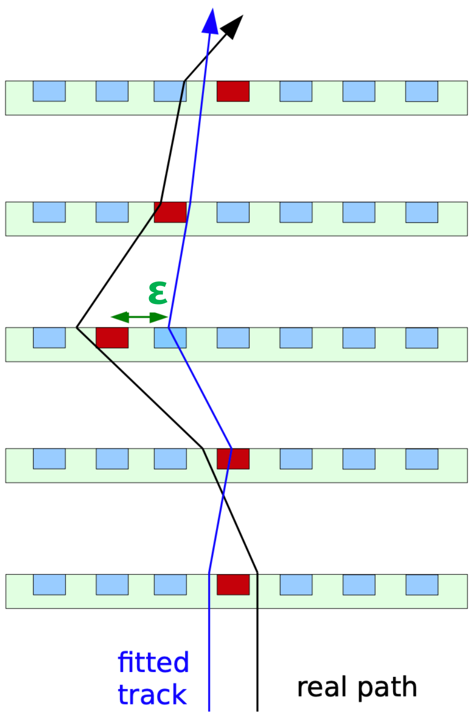
AMS Materials	L1-L2	L2-L3	L3-L4	L4-L5	L5-L6	L6-L7	L7-L8	L8-L9
$x/X_0 \cdot \cos(\theta)$	0.28	0.01	0.01	0.005	0.01	0.005	0.01	0.21

b) the details of the material location

In the track fitting, the scattering angles are treated as the measurement quantity and extra track fitting parameters will be introduced. The track parameters (q_l) are determined by minimization of the χ^2:

$$\chi^2 = \sum_{j=1}^{n_{meas}} \underline{\varepsilon_j^T V_j^{-1} \varepsilon_j} + \sum_{j=2}^{n_{scat}-1} \underline{\beta_j^T W_j^{-1} \beta_j} \quad \text{MS}$$

$$\longrightarrow \quad 0 = \frac{\partial \chi^2}{\partial q_l} = 2 \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_j}{\partial q_l}\right)^T V_j^{-1} \varepsilon_j + 2 \sum_{j=2}^{n_{scat}-1} \left(\frac{\partial \beta_j}{\partial q_l}\right)^T W_j^{-1} \beta_j$$



where **ε** and V is the coordinate residual and its variance on the coordinate measurement, respectively; and β and $W=V[\beta]=\sigma(\beta)^2$ is the scattering angle and its variance, respectively.

Rewrite in matrix form:

$$\chi^2 = \varepsilon^T V^{-1}\varepsilon + \beta^T W^{-1}\beta$$

$$\begin{aligned} 0 = \frac{\partial \chi^2}{\partial q} &= 2\left[(\frac{\partial \varepsilon}{\partial q})^T V^{-1}\varepsilon + (\frac{\partial \beta}{\partial q})^T W^{-1}\beta\right] \\ &= 2\left[(\frac{\partial \varepsilon}{\partial q})^T V^{-1}(\underline{\varepsilon^0 + \frac{\partial \varepsilon}{\partial q}\Delta q}) + (\frac{\partial \beta}{\partial q})^T W^{-1}\underline{\frac{\partial \beta_j}{\partial q}\Delta q}\right] \end{aligned}$$

1st Taylor's expansion 1st Taylor's expansion

The track parameters (Δq) are solved from:

$$\left[(\frac{\partial \varepsilon}{\partial q})^T V^{-1}\frac{\partial \varepsilon}{\partial q} + (\frac{\partial \beta}{\partial q})^T W^{-1}\frac{\partial \beta_j}{\partial q}\right]\Delta q = -(\frac{\partial \varepsilon}{\partial q})^T V^{-1}\varepsilon^0$$

Or simplified as:

$$\Gamma\Delta q = b$$

where Γ is $n_{par} \times n_{par}$ matrix and b is the vector. Their elements are given by:

$$\Gamma_{ll'} = \sum_{j=1}^{n_{meas}} (\frac{\partial \varepsilon_j}{\partial q_l})^T V_j^{-1}\underline{\frac{\partial \varepsilon_j}{\partial q_{l'}}} + \sum_{j=2}^{n_{scat}-1} (\frac{\partial \beta_j}{\partial q_l})^T W_j^{-1}\underline{\frac{\partial \beta_j}{\partial q_{l'}}} \qquad b_l = -\sum_{j=1}^{n_{meas}} (\frac{\partial \varepsilon_j}{\partial q_l})^T V_j^{-1}\varepsilon_j^0$$

Solving above matrix equation requires to invert a $n_{par} \times n_{par}$ matrix.

The advanced track fitting model should be easy to compute the derivatives of the coordinate residual and scattering angle with respect to the track parameters; and the matrix Γ should be invertible.

General Broken Lines (GBL) Track Fitting

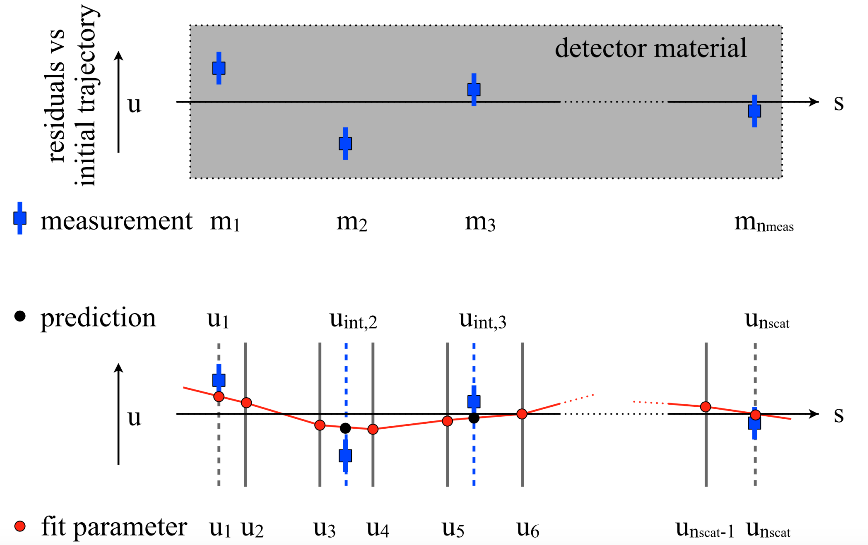
In the GBL model, the materials between two adjacent measured planes is represented by 1 or 2 thin scatterers with zero geometric thickness. Each thin scatterer produces a scattering angle β. For each scatterer, the track position offset u_i is defined as a track parameter. The track parameters $q(\Delta\kappa=\Delta(1/R), u_1, \dots, u_{nscat})$ are determined by:

$$0 = \frac{\partial \chi^2}{\partial q_l} = 2\sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_j}{\partial q_l}\right)^T V_j^{-1} \varepsilon_j + 2\sum_{j=2}^{n_{scat}-1} \left(\frac{\partial \beta_j}{\partial q_l}\right)^T W_j^{-1} \beta_j$$

In this algorithm, the derivatives of the scattering angle with respect to the track parameters can be easily obtained as follows:

$$\boldsymbol{\alpha}_+ = \mathbf{W}_+(\mathbf{u}_+ - \mathbf{J}_+\mathbf{u}_0 - \mathbf{d}_+\Delta\kappa), \quad \mathbf{W}_+ = \mathbf{S}_+^{-1}$$

$$\boldsymbol{\alpha}_- = \mathbf{W}_-(\mathbf{J}_-\mathbf{u}_0 - \mathbf{u}_- + \mathbf{d}_-\Delta\kappa), \quad \mathbf{W}_- = -\mathbf{S}_-^{-1}$$



where α_+ and α_- are the angle after and before u_0 scatterer determined by nearby 3 position offsets (u_-, u_0, u_+) and Δκ. Hence, the direction change $\beta = \alpha_+ - \alpha_-$ is:

$$\boldsymbol{\beta} = \mathbf{W}_+\mathbf{u}_+ - (\mathbf{W}_+\mathbf{J}_+ + \mathbf{W}_-\mathbf{J}_-)\mathbf{u}_0 + \mathbf{W}_-\mathbf{u}_- \quad \mathbf{N} = (\mathbf{W}_+\mathbf{J}_+ + \mathbf{W}_-\mathbf{J}_-)^{-1}$$

When the measurement plane is the location of the scatterer, the derivatives of the coordinate residual with respect to the track parameter is $\partial\varepsilon_j/\partial q_l=1$.

The GBL is an advanced global track fitting approach. The matrix inversion is using root free Cholesky decomposition. The algorithm for AMS was developed in 2020.

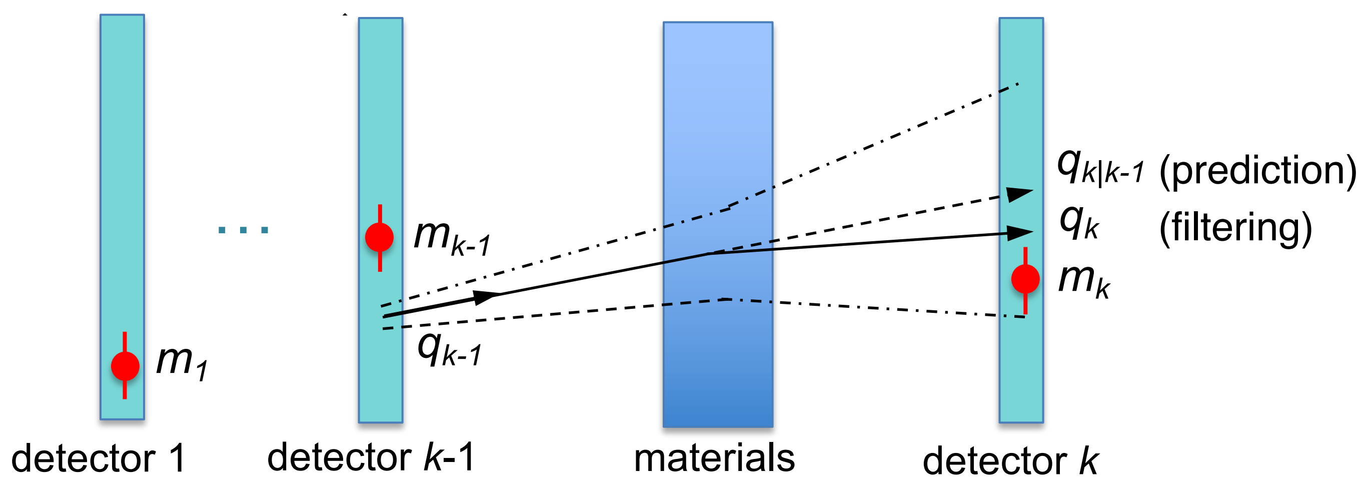
Kalman Track Fitting

The Kalman filter was developed by R.E. Kalman during the 1950's

- To solve differential matrix equations without matrix inversions.
- It is a method of estimating the states of dynamic systems
  - Applied by the NASA in the rocket trajectory control for the Apollo program
  - Military applications: compute plane trajectory by radar tracking.

The Kalman filter is a recursive filter, which evaluates the state of a linear dynamic system using a set of inaccurate measurements with the errors distributed according to the Gauss distribution. Supposing we have k-1 layers of tracker coordinate measurements, the track parameters in $(k\text{-}1)^{th}$ layer were denoted as a state q_{k-1}. Next, when the new layer of k^{th} measurement was added, the track parameters will update to be the new state q_k:

$$\Big\{\{m_1, .., m_{k-1}\}, q_{k-1}\Big\} + m_k \to q_k$$



Prediction: The trajectory of a particle between two adjacent surfaces can be described by a deterministic function plus random disturbances (material effects, etc). The prediction of the state from (k-1)th to the next k layer is:

$$q_{k|k-1} = F_k q_{k-1} + \delta_{k-1}$$

where the mean of the stochastic noise is $<\delta_{k-1}>=0$ and its covariance matrix (error^2) is Q_{k-1}. The total state prediction covariance matrix (error^2) is given by :

$$C_{k|k-1} = F_k C_{k|k-1} F_k^T + Q_{k-1}$$

Filtering: We assume the coordinate measurement can be described by a linear function of the state:

$$m_k = H_k q_k + \varepsilon_k$$

where the mean of the measurement is expected to have no bias $<\varepsilon_k>=0$ and its covariance matrix (measurement error^2) is V_k.

Based on the prediction $q_{k|k-1}$ and k^{th} measurement, the new track state q_k can be derived from:

$$\chi^2 = (q_{k|k-1} - q_k)^T C_{k|k-1}^{-1} (q_{k|k-1} - q_k) + (m_k - H_k q_k)^T V_k^{-1} (m_k - H_k q_k)$$

Minimization X^2 will get the new state:

$$q_k = q_{k|k-1} + \left[C_{k|k-1}^{-1} + (H_k^T V_k^{-1} H_k)\right]^{-1} \left[(H_k^T V_k^{-1})(m_k - H_k q_{k|k-1})\right]$$
$$= q_{k|k-1} + K_k (m_k - H_k q_{k|k-1}) \;= q_{k|k-1} + C_k \left[(H_k^T V_k^{-1})(m_k - H_k q_{k|k-1})\right]$$

where

$$\underline{K_k = \left[C_{k|k-1}^{-1} + (H_k^T V_k^{-1} H_k)\right]^{-1} (H_k^T V_k^{-1}) = C_{k|k-1} H_k^T \left[V_k + H_k C_{k|k-1} H_k^T\right]^{-1}}$$ Filter gain

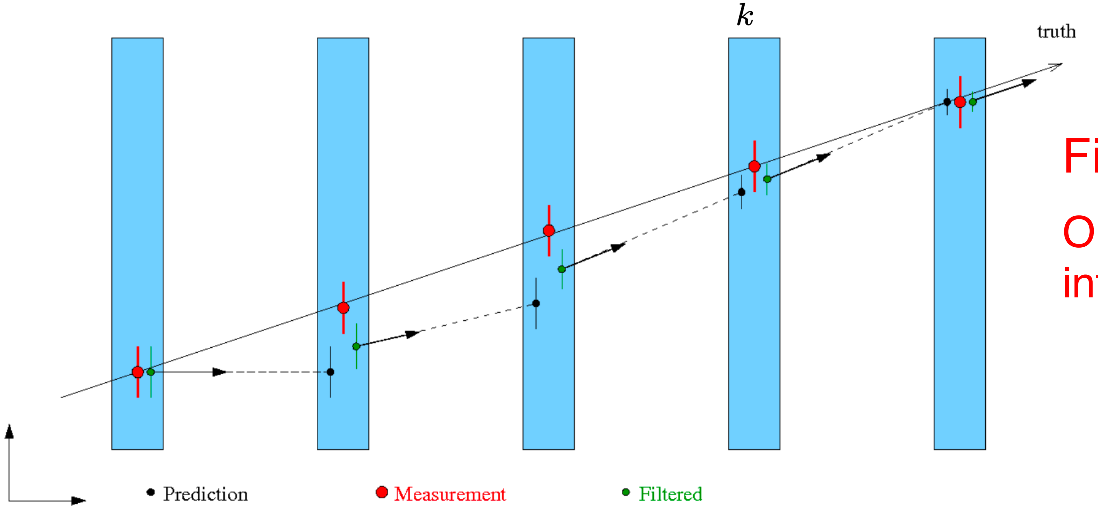
$$\underline{C_k = \left[C_{k|k-1}^{-1} + (H_k^T V_k^{-1} H_k)\right]^{-1} = (I - K_k H_k) C_{k|k-1}}$$ New q_k covariance matrix (error^2)

The coordinate residual is thus:

$$r_k = m_k - H_k q_k$$

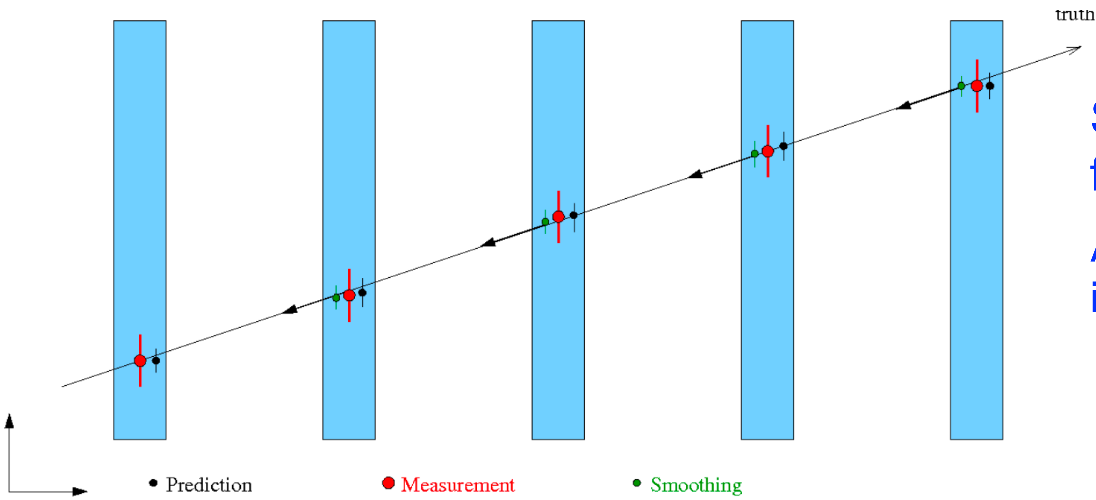
Which allows to compute a X^2 in order to test the goodness of the fit:

$$\chi^2_k = r_k^T V_k^{-1} r_k \qquad \chi^2 = \sum_k \chi^2_k$$



Filtering: from k-1 to k:

Outer points estimates have more information than inner points.

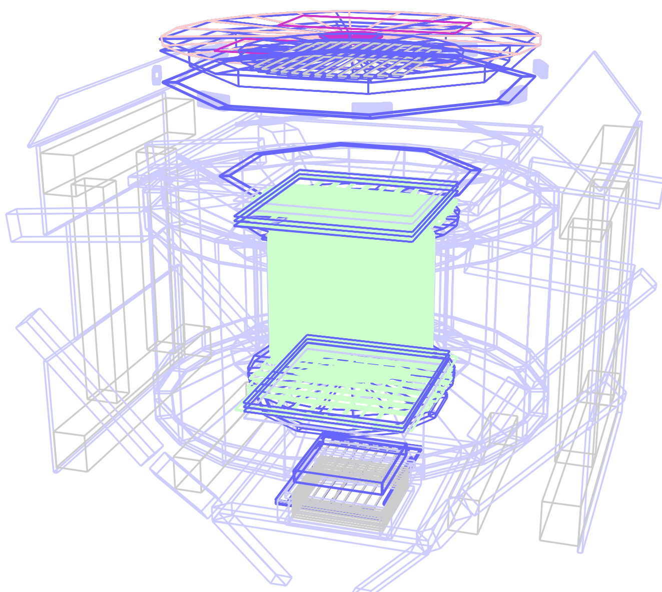


Smoothing: from k to k-1(backward filter):

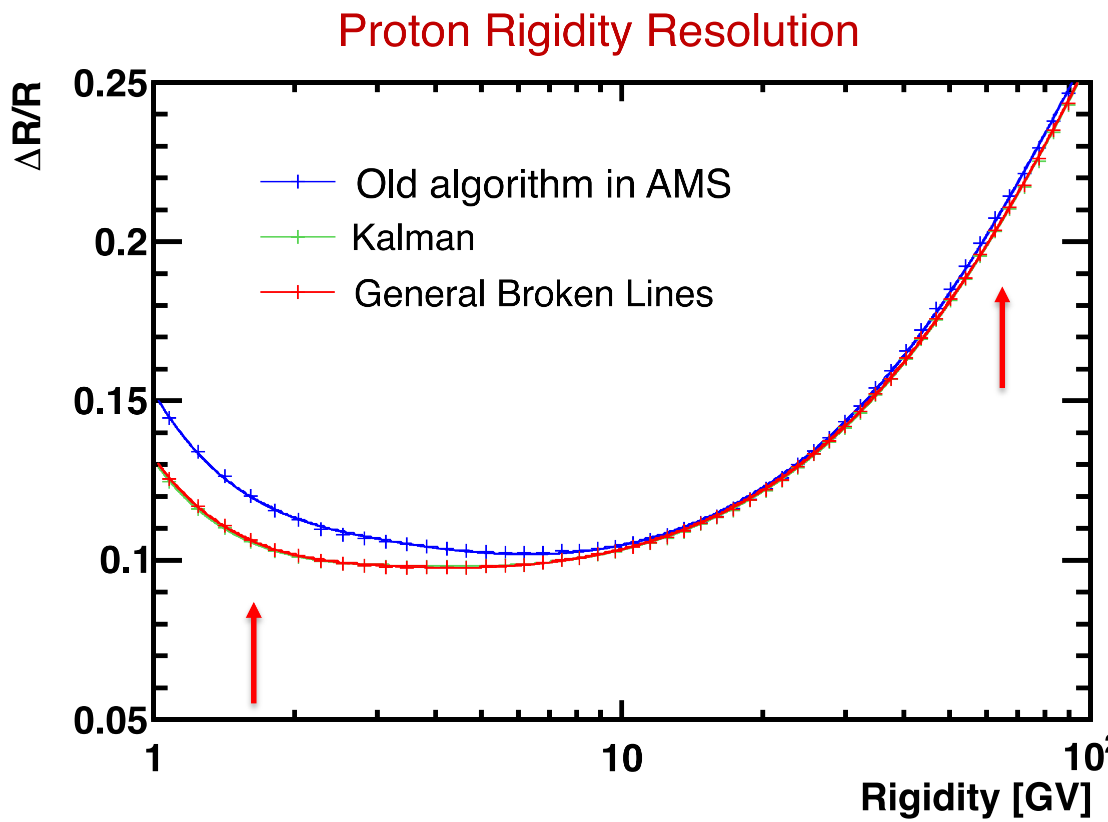
All points estimates have the best information.

Kalman Track Fitting for AMS

The Kalman filter track fitting for AMS was developed in 2018 based on GeFit-package. The precise description of the AMS geometry (materials) and physics processes (multiple scattering, energy loss, and bremsstrahlung for e+/e- only) allows accurate determination of the track parameters for all charge particles.



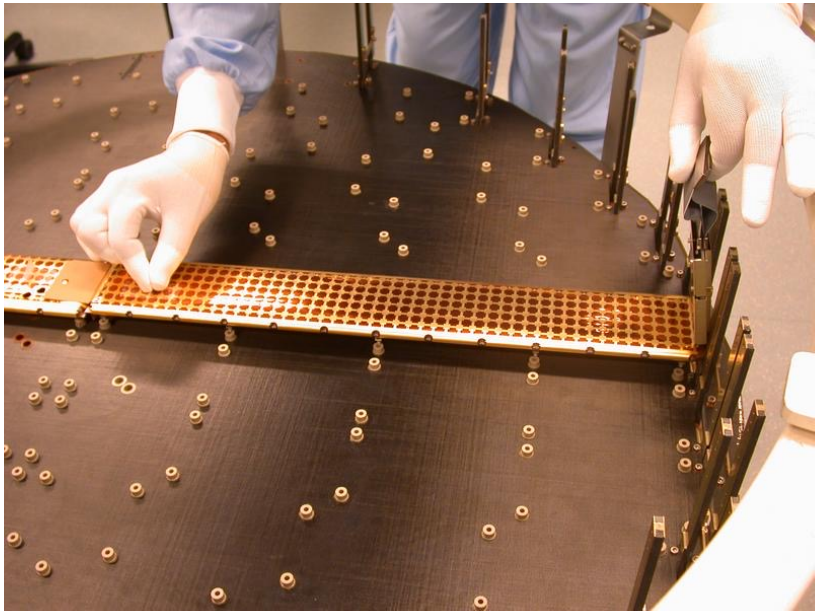
AMS geometry description used in Kalman

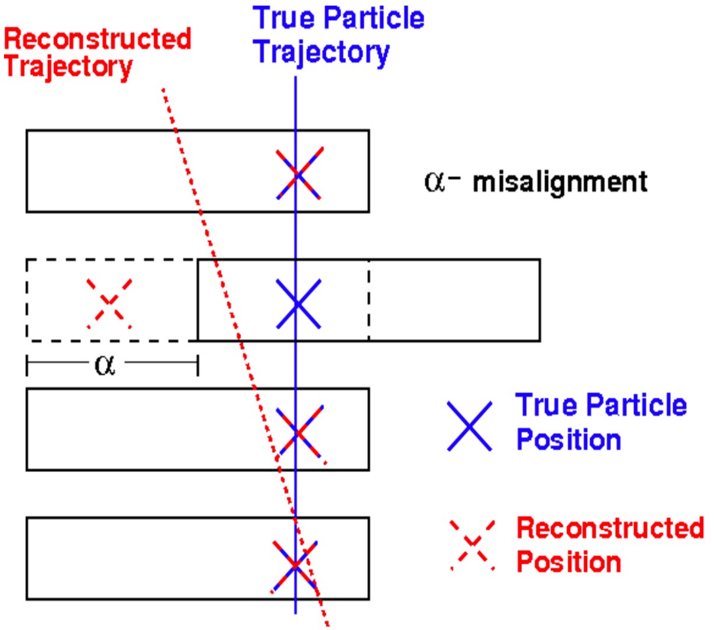


Note that the Kalman track fitting (recursive filter) mathematically is equivalent to the General Broken Lines track fitting (global fitting approach).

Tracker Alignment

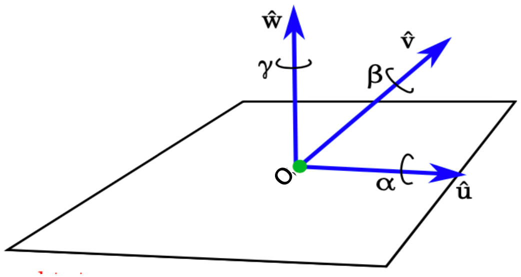
Each detector module have its own initial mechanical mounting precision varying from tens of micron to thousands of micron, which is far away from the detector requirement of a few micron.



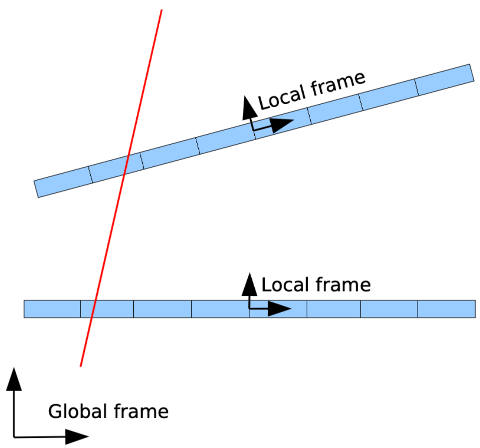


In order to have high resolution and unbiased tracking, the detector elements must be correctly aligned. The track-based alignment was an approach to align the detector modules to ultimate precision. The alignment is the most sophisticated procedure in the detector calibration. For the AMS silicon tracker, there are 2,284 silicon sensors, 192 ladders, and 9 layers of detector modules to be aligned. The total number of the alignment parameters reach up to 15,000.

The Global and Local Coordinate System



Each detector module is positioned with 6 degrees of freedom: 3 translations (x_0, y_0, z_0) and 3 rotations (α_0, β_0, γ_0).



The transformation of the position and track state between global and local coordinate system is heavily used in the alignment:

- The track parameters are necessary to delivered in the experimental global reference frame
- The measurements are usually given in the detector module (sensor, ladder, or layer) so called local reference frame

  a) The covariance matrix usually has a diagonal form

  b) The residual can be easily linearized with respect to the alignment parameters

Without displacement, the transformation from a local position $q_l=(u,v,w)^T$ to the global coordinate position $r_g=(x,y,z)^T$ is given by:

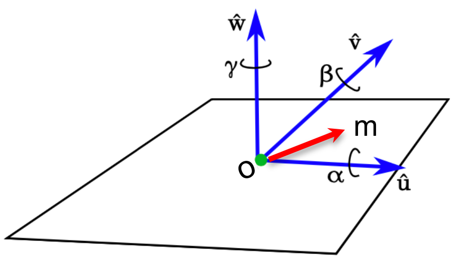
$$r_g = R^T q_l + r_0$$

where the geometric center of the detector module is defined as its original point, which is positioned at $r_0=(x_0, y_0, z_0)^T$ in the global reference frame; $R^T(\alpha_0, \beta_0, \gamma_0)$ is the detector module rotation matrix.

With displacement, the transformation from a local position $q_l=(u,v,w)^T$ to the global coordinate postion $r_g=(x,y,z)^T$ becomes:

$$r_g = R^T \Delta R(q_l + \Delta q) + r_0$$

where the correction to the transformation by an offset $\Delta q=(\Delta u,\Delta v,\Delta w)^T$ and a rotation $\Delta R= \Delta R_\gamma \cdot \Delta R_\beta \cdot \Delta R_\alpha$ has to be determined by the alignment procedure. The rotation matrices are given by:



$$\Delta R_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \quad \Delta R_\beta = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \quad \Delta R_\gamma = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For AMS, sensors are assembled in the ladders; ladders are mounted in the layers; and layers are built into the tracker. The hit coordinate measured in sensor local reference frame can be transformed recursively to the next reference frame :

Sensor ↓ Ladder ↓ Layer ↓ Tracker

$$q_s \longrightarrow \text{Hit coordinate in the sensor}$$

$$r_L = R_s^T \Delta R_s (q_s + \Delta q_s) + r_{0s} \longrightarrow \text{Hit coordinate in the ladder}$$

$$r_P = R_L^T \Delta R_L (r_L + \Delta q_L) + r_{0L} \longrightarrow \text{Hit coordinate In the layer}$$

$$r_g = R_P^T \Delta R_P (r_P + \Delta q_P) + r_{0P} \longrightarrow \text{Hit coordinate in global reference}$$

Finally, the hit position measured in the sensor local coordinate system (q_s) can be transformed into the global coordinate system (r_g):

$$r_g = R^T q_s^c + r_0 = R^T (q_s + \Delta q) + r_0$$

where

$$\begin{aligned}
R^T &= R_P^T R_L^T R_s^T \\
\Delta q &= [(R_s R_L \Delta R_P R_L^T R_s^T)(R_s \Delta R_L R_s^T)\Delta R_s - E]q_s + \Delta q_s + [(R_s R_L \Delta R_P R_L^T R_s^T) R_s \Delta R_L - R_s] r_{0s} \\
&+ R_s \Delta q_L + R_s R_L (\Delta R_P - E) r_{0L} + R_s R_L \Delta q_P \\
r_0 &= R_P^T R_L^T r_{0s} + R_P^T r_{0L} + r_{0P}
\end{aligned}$$

The partial derivatives of the measured residual (**ε**=track projection-hit position) with respect to the alignment parameters are shown (as input for the alignment):

$$\begin{aligned}
\frac{\partial \varepsilon}{\partial u_s} &= P e_1 \longrightarrow \text{sensor offset} \\
\frac{\partial \varepsilon}{\partial u_L} &= P R_s e_1 \longrightarrow \text{ladder offset} \\
\frac{\partial \varepsilon}{\partial u_P} &= P R_s R_L e_1 \longrightarrow \text{layer offset} \\
\frac{\partial \varepsilon}{\partial \alpha_s} &= P \frac{\partial \Delta R_s}{\partial \alpha_s} q_s \longrightarrow \text{sensor rotation} \\
\frac{\partial \varepsilon}{\partial \alpha_L} &= P R_s \frac{\partial \Delta R_L}{\partial \alpha_L} (R_s^T q_s + r_{0s}) \longrightarrow \text{ladder rotation} \\
\frac{\partial \varepsilon}{\partial \alpha_P} &= P R_s R_L \frac{\partial \Delta R_P}{\partial \alpha_P} [R_L^T (R_s^T q_s + r_{0s}) + r_{0L}] \longrightarrow \text{layer rotation}
\end{aligned}$$

where $P = \begin{pmatrix} -1 & 0 & \frac{du_s^p}{dw_s^p} \\ 0 & -1 & \frac{dv_s^p}{dw_s^p} \\ 0 & 0 & 0 \end{pmatrix}$

The Global Tracker Alignment

In the global alignment, the global detector alignment parameters $\Delta p(\Delta p_1, \Delta p_2, \ldots)$ and local track fitting parameters $\Delta q(\Delta q_1, \Delta q_2, \ldots)$ for every track are determined simultaneously though vast χ^2 minimization, taking account of both residual measurements and multiple-scattering effects.

$$\chi^2(q,p) = \sum_{i=1}^{m_{ev}} \left[\overbrace{\sum_{j=1}^{n_{meas}} \varepsilon_{ij}(q_i,p)^T V_{ij}^{-1} \varepsilon_{ij}(q_i,p)}^{\text{residual measurements } (\boldsymbol{\varepsilon})} + \overbrace{\sum_{j=2}^{n_{scat}-1} \beta_{ij}(q_i)^T W_{ij}^{-1} \beta_{ij}(q_i)}^{\text{multiple-scattering } (\beta)} \right]$$

Minimization of the χ^2 leads that the partial derivative with respect to each global parameter Δp_g is zero (the multiple scattering is uncorrelated with the global parameter):

$$0 = \frac{\partial \chi^2}{\partial p_g} = 2\sum_{i=1}^{m_{ev}} \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^T V_{ij}^{-1} \varepsilon_{ij} = 2\sum_{i=1}^{m_{ev}} \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^T V_{ij}^{-1} \underbrace{\left[\varepsilon_{ij}(q_i^0, p^0) + \sum_{l'} \frac{\partial \varepsilon_{ij}}{\partial q_{il'}} \Delta q_{il'} + \sum_{g'} \frac{\partial \varepsilon_{ij}}{\partial p_{g'}} \Delta p_{g'}\right]}_{\text{1st Taylor's expansion}}$$

Above equation can be simplified as:

$$\sum_{i=1}^{m_{ev}} d_g^i = \sum_{g'} \left(\sum_{i=1}^{m_{ev}} C_{gg'}^i\right) \Delta p_{g'} + \sum_{i=1}^{m_{ev}} \sum_{l'} G_{gl'}^i \Delta q_{il'}$$

where
$$C_{gg'}^i = \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^T V_{ij}^{-1} \frac{\partial \varepsilon_{ij}}{\partial p_{g'}} \qquad G_{gl'}^i = \sum_{j=1}^{n_{meas}} \left(\underbrace{\frac{\partial \varepsilon_{ij}}{\partial p_g}}_{\text{global derivative}}\right)^T V_{ij}^{-1} \underbrace{\frac{\partial \varepsilon_{ij}}{\partial q_{il'}}}_{\text{local derivative (from track fitting)}}$$

$$d_g^i = -\sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^T V_{ij}^{-1} \varepsilon_{ij}(q_i^0, p^0)$$

All global derivatives make the matrix equation: $\displaystyle \sum_{i=1}^{m_{ev}} \boldsymbol{d}^i = \left(\sum_{i=1}^{m_{ev}} \boldsymbol{C}^i\right) \boldsymbol{\Delta p} + \sum_{i=1}^{m_{ev}} \boldsymbol{G}^i \boldsymbol{\Delta q}_i$

$$\chi^2(q,p) = \sum_{i=1}^{m_{ev}} \Bigg[\underbrace{\sum_{j=1}^{n_{meas}} \varepsilon_{ij}(q_i,p)^T V_{ij}^{-1} \varepsilon_{ij}(q_i,p)}_{\text{residual measurements } (\boldsymbol{\varepsilon})} + \underbrace{\sum_{j=2}^{n_{scat}-1} \beta_{ij}(q_i)^T W_{ij}^{-1} \beta_{ij}(q_i)}_{\text{multiple-scattering } (\beta)} \Bigg]$$

Minimization of the χ^2 leads that the partial derivative with respect to each local track parameter in each track fitting Δq_{il} is also zero:

$$0 = \frac{\partial \chi^2}{\partial q_{il}} = 2 \sum_{j=1}^{n_{meas}} \Big(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\Big)^T V_{ij}^{-1} \varepsilon_{ij} + 2 \sum_{j=2}^{n_{scat}-1} \Big(\frac{\partial \beta_{ij}}{\partial q_{il}}\Big)^T W_{ij}^{-1} \beta_{ij}$$

$$= 2 \sum_{j=1}^{n_{meas}} \Big(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\Big)^T V_{ij}^{-1} \underbrace{\Big[\varepsilon_{ij}(q_i^0,p^0) + \sum_{l'} \frac{\partial \varepsilon_{ij}}{\partial q_{il'}} \Delta q_{il'} + \sum_{g'} \frac{\partial \varepsilon_{ij}}{\partial p_{g'}} \Delta p_{g'}\Big]}_{1^{st}\text{ Taylor's expansion}} + 2 \sum_{j=2}^{n_{scat}-1} \Big(\frac{\partial \beta_{ij}}{\partial q_{il}}\Big)^T W_{ij}^{-1} \underbrace{\sum_{l'} \frac{\partial \beta_{ij}}{\partial q_{il'}} \Delta q_{il'}}_{1^{st}\text{ Taylor's expansion}}$$

Above equation can be simplified as:

$$b_l^i = \sum_{g'} G_{lg'}^i \Delta p_{g'} + \sum_{l'} \Gamma_{ll'}^i \Delta q_{il'}$$

where $$G_{lg'}^i = \sum_{j=1}^{n_{meas}} \Big(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\Big)^T V_{ij}^{-1} \frac{\partial \varepsilon_{ij}}{\partial p_{g'}} \qquad \Gamma_{ll'}^i = \sum_{j=1}^{n_{meas}} \Big(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\Big)^T V_{ij}^{-1} \frac{\partial \varepsilon_{ij}}{\partial q_{il'}} + \sum_{j=2}^{n_{scat}-1} \Big(\frac{\partial \beta_{ij}}{\partial q_{il}}\Big)^T W_{ij}^{-1} \frac{\partial \beta_{ij}}{\partial q_{il'}}$$

$$b_l^i = -\sum_{j=1}^{n_{meas}} \Big(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\Big)^T V_{ij}^{-1} \varepsilon_{ij}(q_i^0,p^0)$$

All local derivatives from each track make matrix equation: $\boldsymbol{b^i = G^i \Delta p + \Gamma^i \Delta q_i}$

The global detector alignment parameters Δp and the local track fitting parameters Δq_i should satisfy:

$$\sum_{i=1}^{m_{ev}} d^i = (\sum_{i=1}^{m_{ev}} C^i)\Delta p + \sum_{i=1}^{m_{ev}} G^i \Delta q_i$$

$$b^i = G^i \Delta p + \Gamma^i \Delta q_i$$

All the parameters can be solved simultaneously from following vast matrix inversion:

$$\begin{pmatrix} \sum_i C^i & G^1 & G^2 & \dots & G^j & \dots & G^{m_{ev}} \\ (G^1)^T & \Gamma^1 & 0 & \dots & 0 & \dots & 0 \\ (G^2)^T & 0 & \Gamma^2 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (G^j)^T & 0 & 0 & \dots & \Gamma^j & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (G^{m_{ev}})^T & 0 & 0 & \dots & 0 & \dots & \Gamma^{m_{ev}} \end{pmatrix} \cdot \begin{pmatrix} \Delta p \\ \Delta q_1 \\ \Delta q_2 \\ \vdots \\ \Delta q_j \\ \vdots \\ \Delta q_{m_{ev}} \end{pmatrix} = \begin{pmatrix} \sum_i d^i \\ b^1 \\ b^2 \\ \vdots \\ b^j \\ \vdots \\ b^{m_{ev}} \end{pmatrix}$$

The solution requires the inversion of the matrix with the dimension of $n+m_{ev}v$.

n: the number of the global parameters (up to 15 000 for the AMS tracker)

m_{ev}: the number of tracks in the alignment

v: the averaged number of the local track fitting parameters for each track

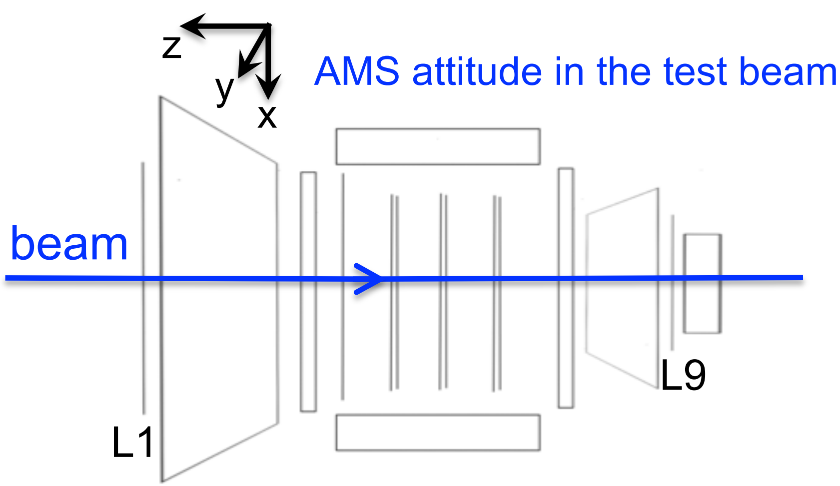
Huge matrix, impossible to invert? Mathematically can be solved by PEDE's approach!

Alignment based on 400 GeV/c proton Test Beam (CREN)

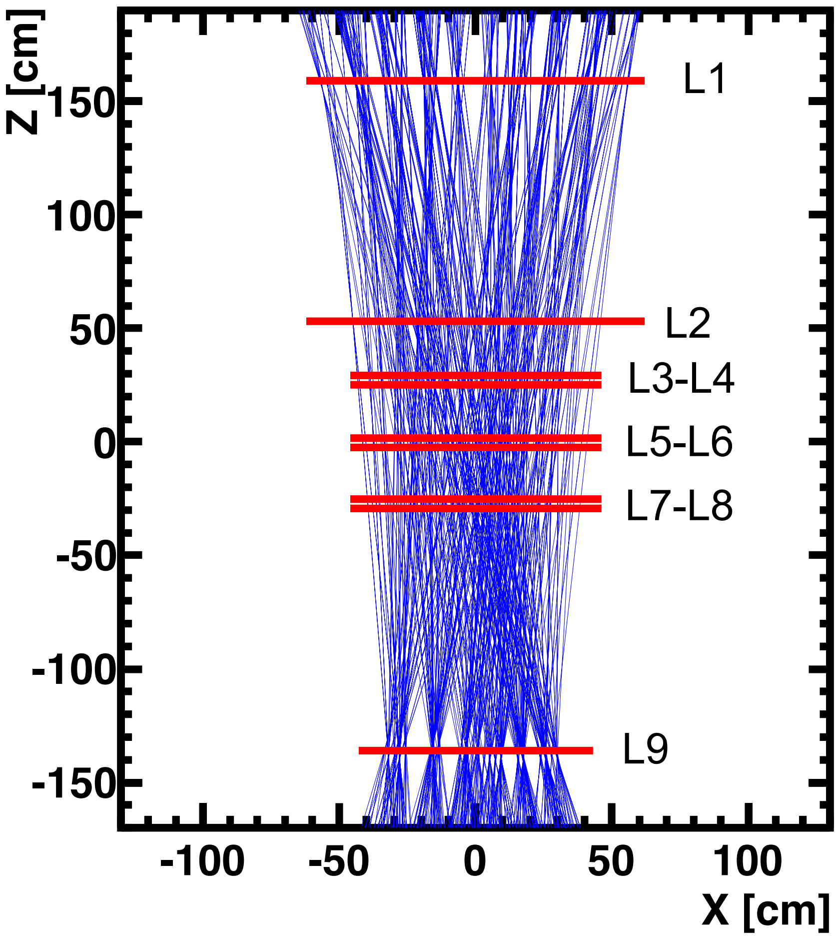
TB data used in current alignment   TB data for misalignment study

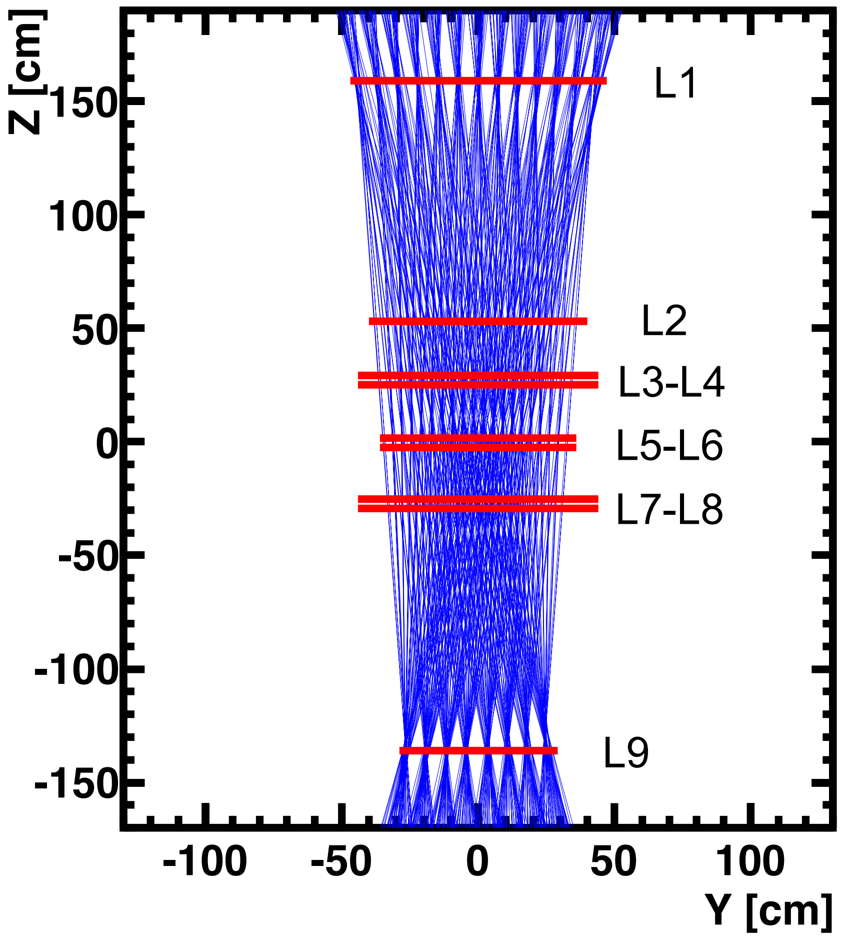
Particle	Momentum	C1/C2 Pr (Bar)	Min Events Per Pos	Spills Per Pos	Time (hrs) Total	Positions	Rate (p/sp) Expected	Comment
Protons	400 GeV	2/2	10^6	75	1	Center, 5°	20k	Initial Setup
Protons	400 GeV	2/2	10^4	3	5	TRACKER60	20k	Inner Tracker Alignment
Protons	400 GeV	2/2	10^4	7	1	TRACKER10	20k	Laser Correlation
Protons	400 GeV	2/2	10^4	3	30	TRACKER416	20k	Layers 1/9 Alignment
Protons	400 GeV	2/2	10^4	3	6	TRACKER80	20k	Layers 2/9 Alignment
Protons	400 GeV	2/2	10^4	3	24	TRACKER280	20k	Layers 1/8 Alignment
				67				

Particle	Momentum	C1/C2 Pr (Bar)	Min Events Per Pos	Spills Per Pos	Time (hrs) Total	Positions	Rate (p/sp) Expected	Comment
Protons	400 GeV	2/2	10^4	3	5	TRACKER60	20k	Inner Tracker Alignment
Protons	400 GeV	2/2	10^4	3	5	TRACKERMISS	20k	Inner Tracker Alignment

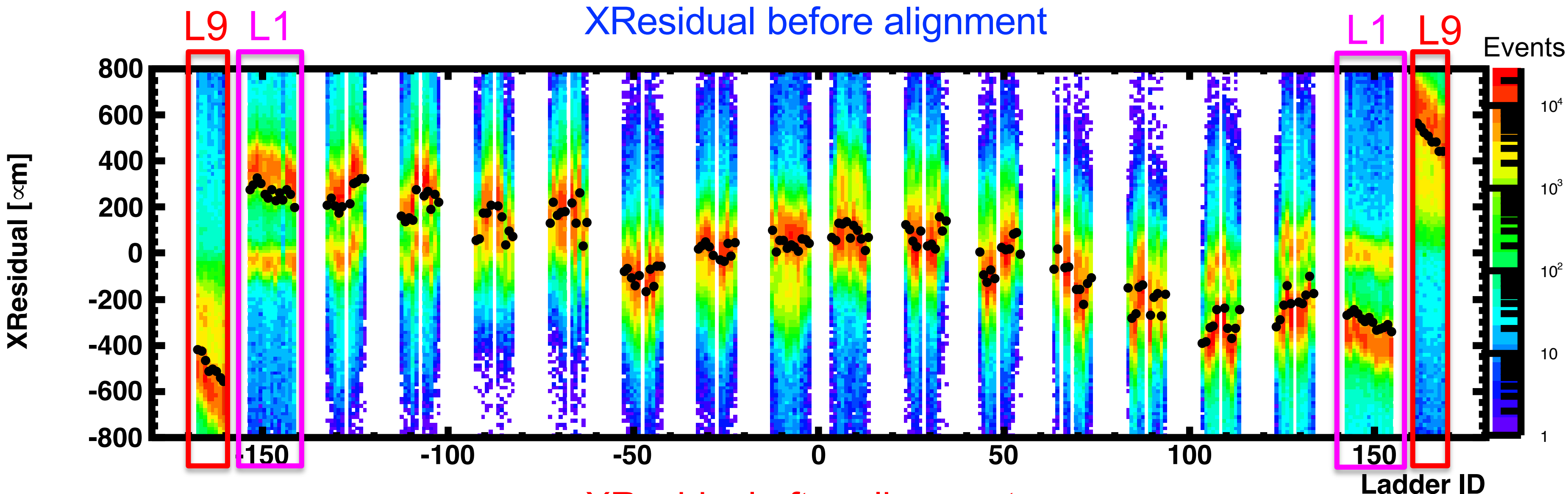


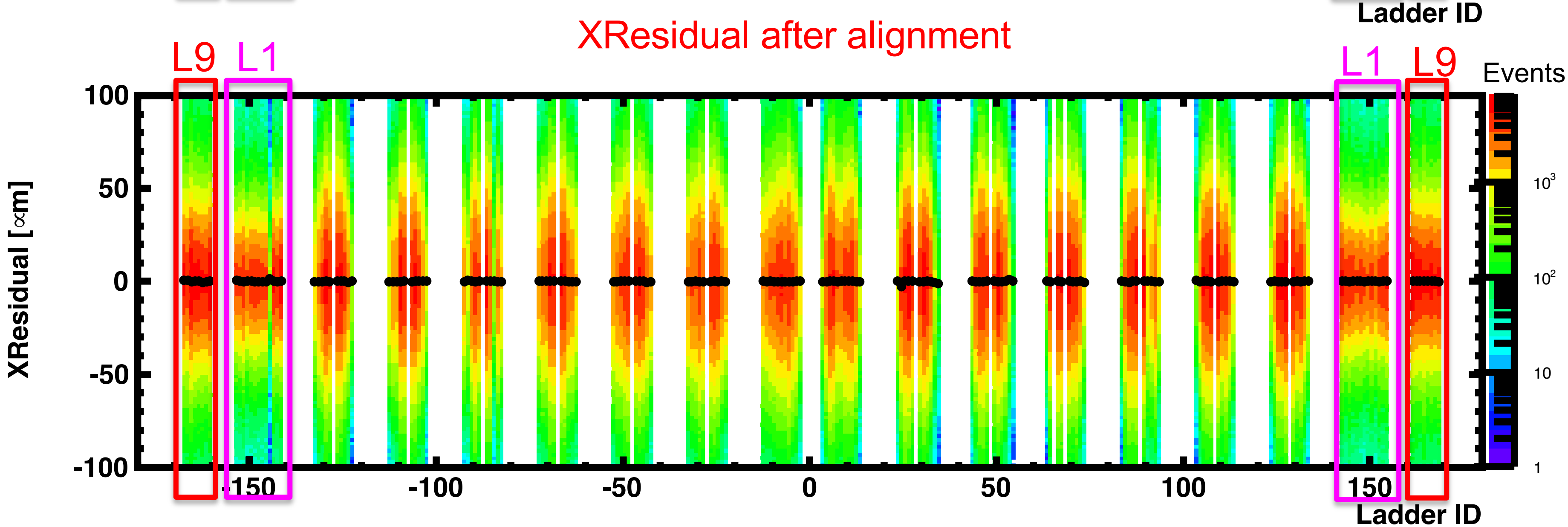
400 GeV/c proton Test Beam at CERN-SPS





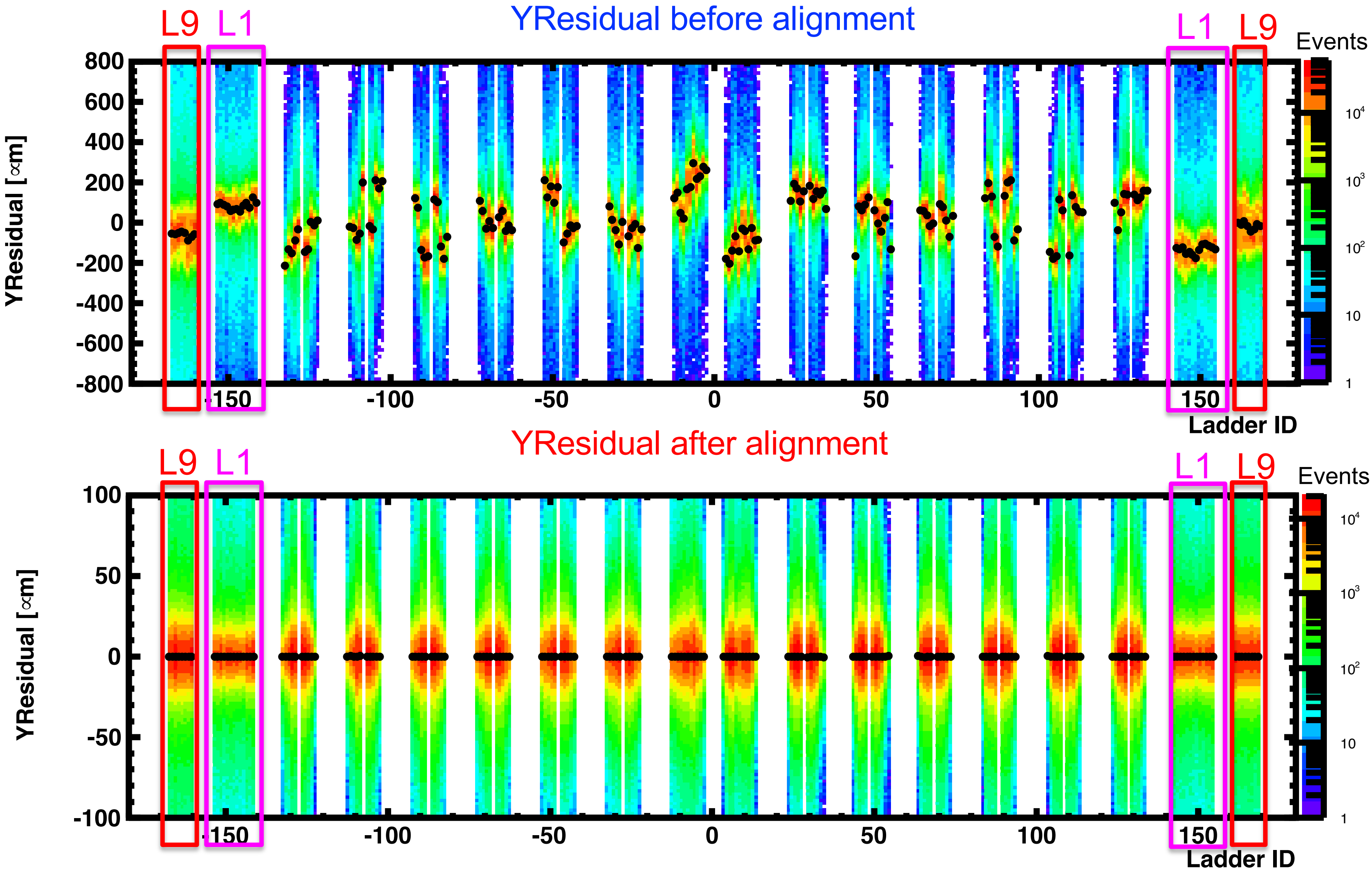
Tracker XResiduals for Each Ladder Before and After Alignment





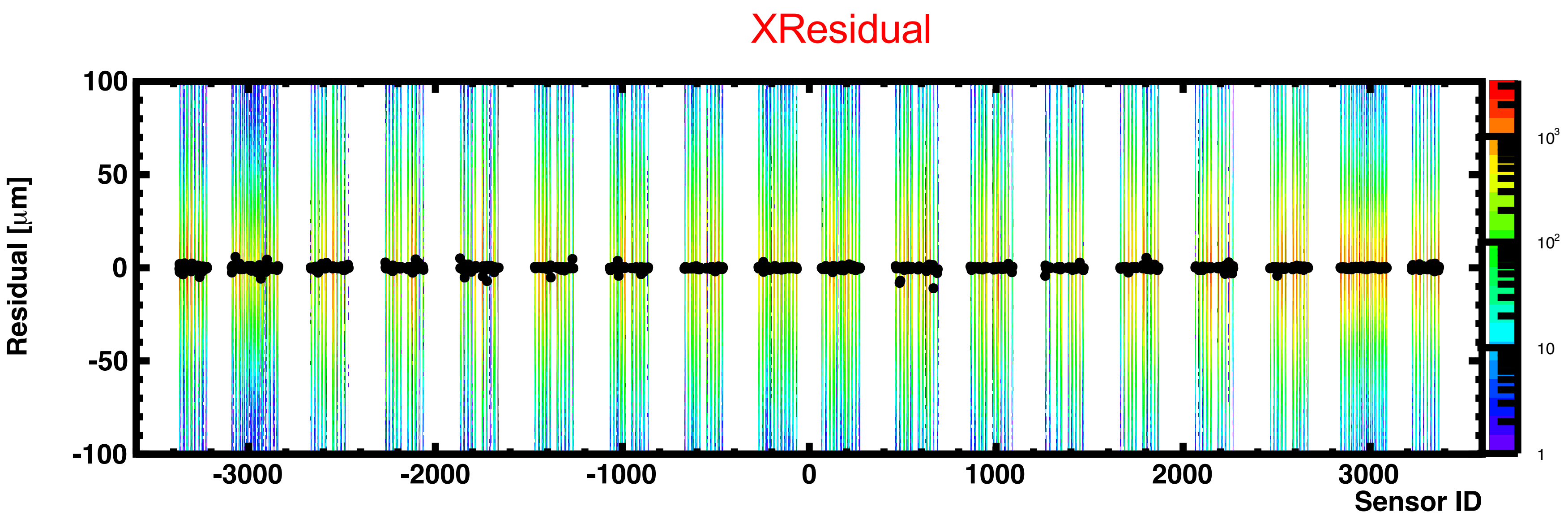
There is no bias in each of ladder after alignment.

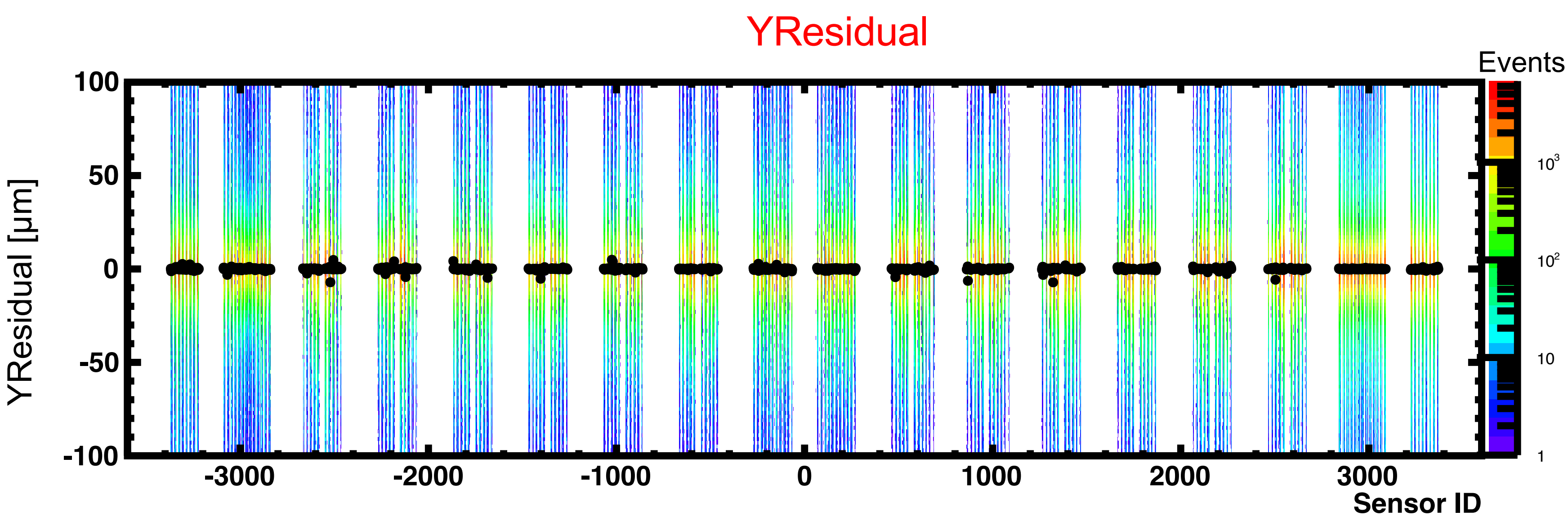
Tracker YResiduals for Each Ladder Before and After Alignment



There is no bias in each of ladder after alignment.

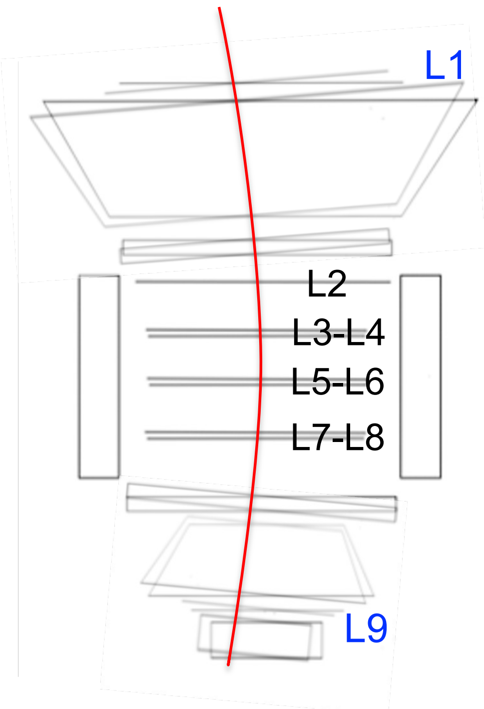
Tracker Residuals for Each Sensor After Alignment





There is no bias in each of sensor after alignment.

Dynamic Alignment of the External Layers in Flight

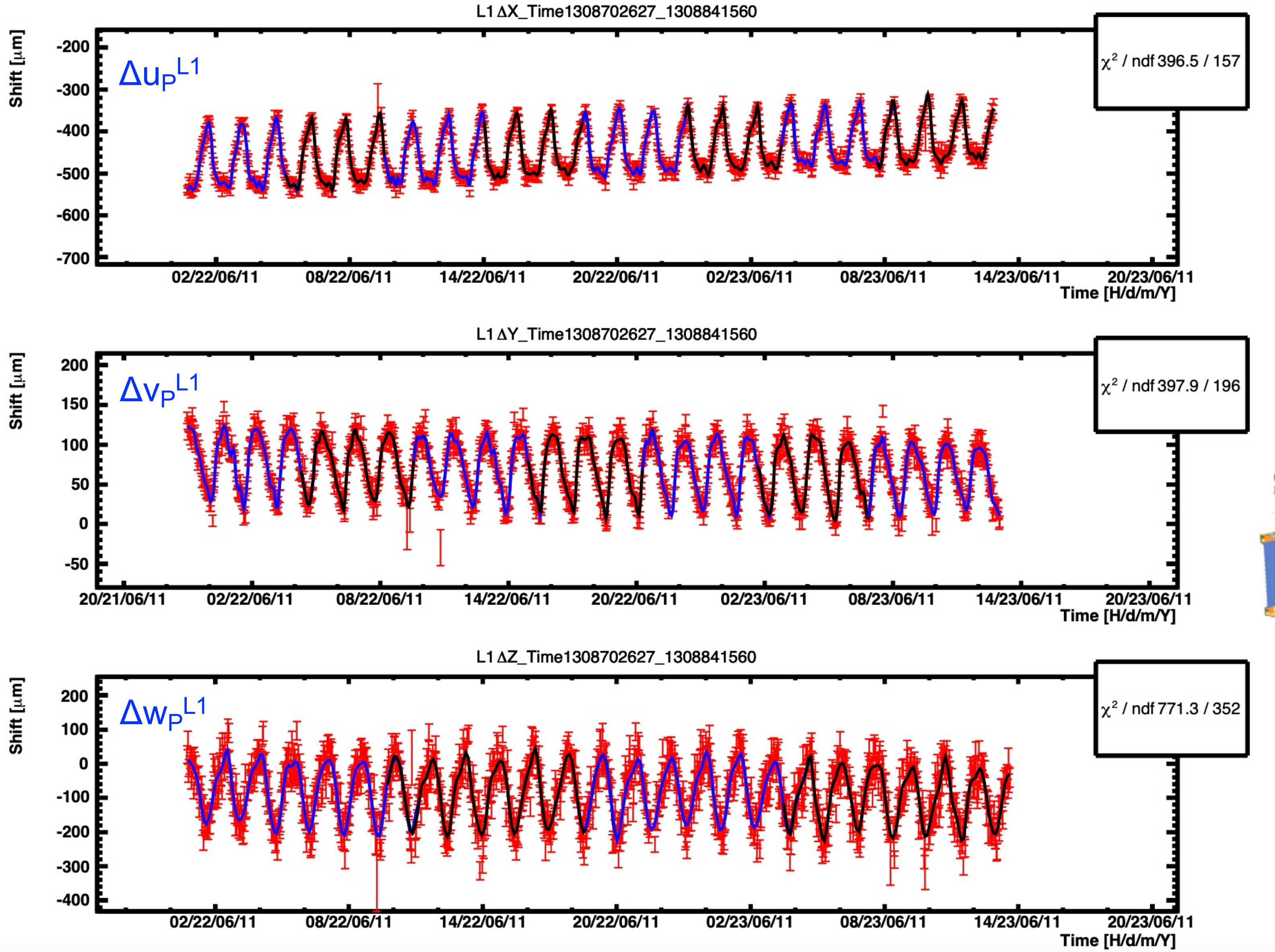


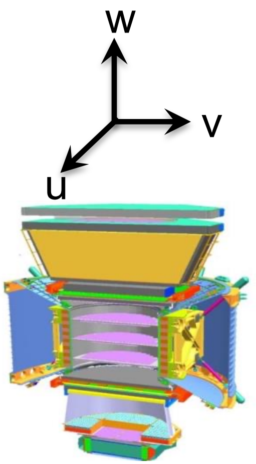
1: The mechanical and thermal deformation coefficients of the external layers (L1 and L9) supporting structures are different from that of the inner tracker (L2-L8).

2: When AMS in flight, the two external layers are continually moving over time with respect to the inner tracker. The movement reaches 200 µm/half-obit (~45 minutes) and 1000 µm/month.

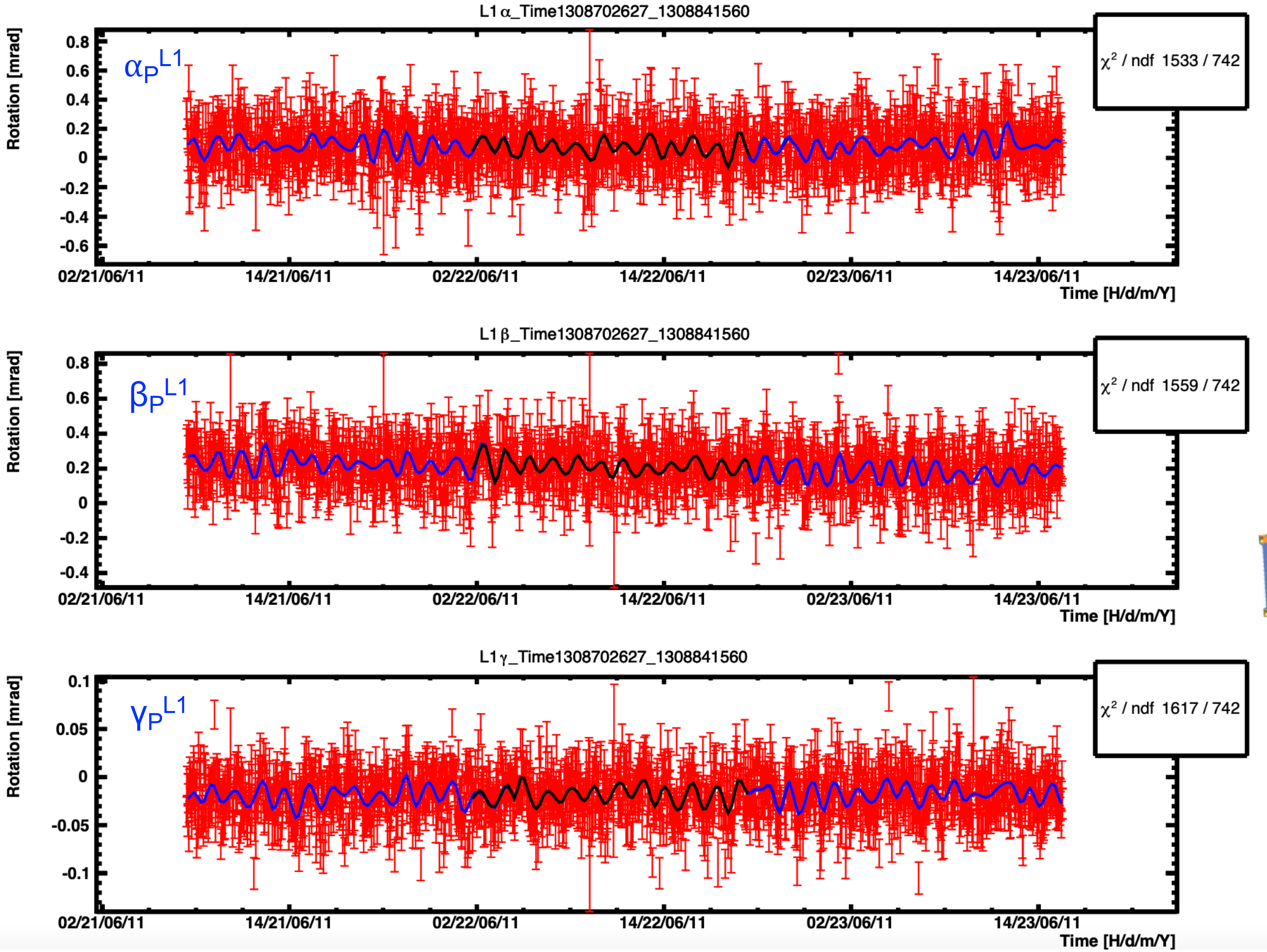
3: To correct the position, the external layers are dynamically aligned by using cosmic ray events every 90 sec.

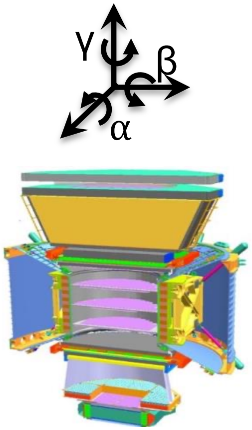
The Movement of the Tracker L1 In a Day





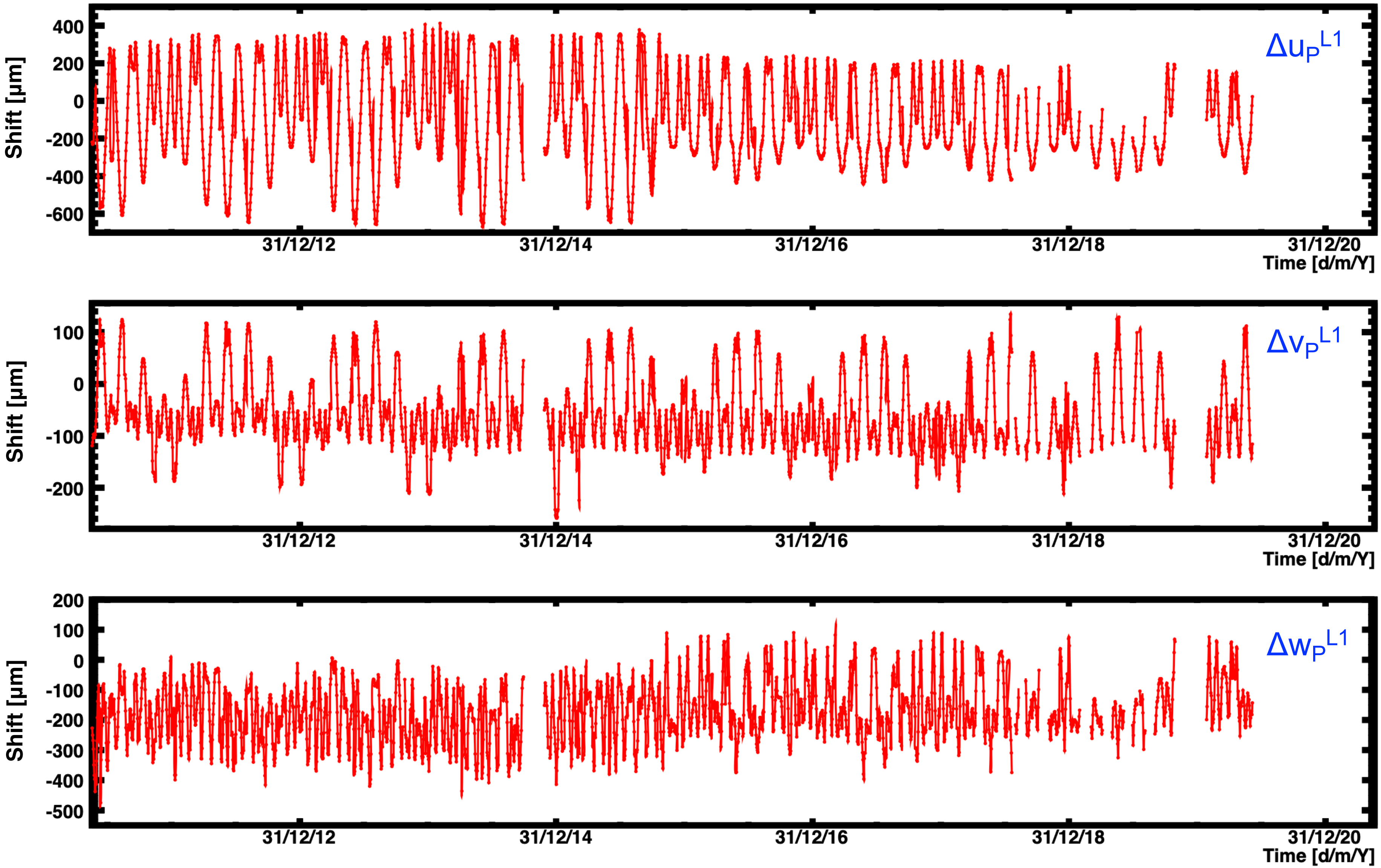
The orbital period of ~1.5 hours can be clearly seen in the alignment parameters:
The L1 movement in shift Δu_P^{L1}, Δv_P^{L1} and Δw_P^{L1}: 100-200 μm/half obit



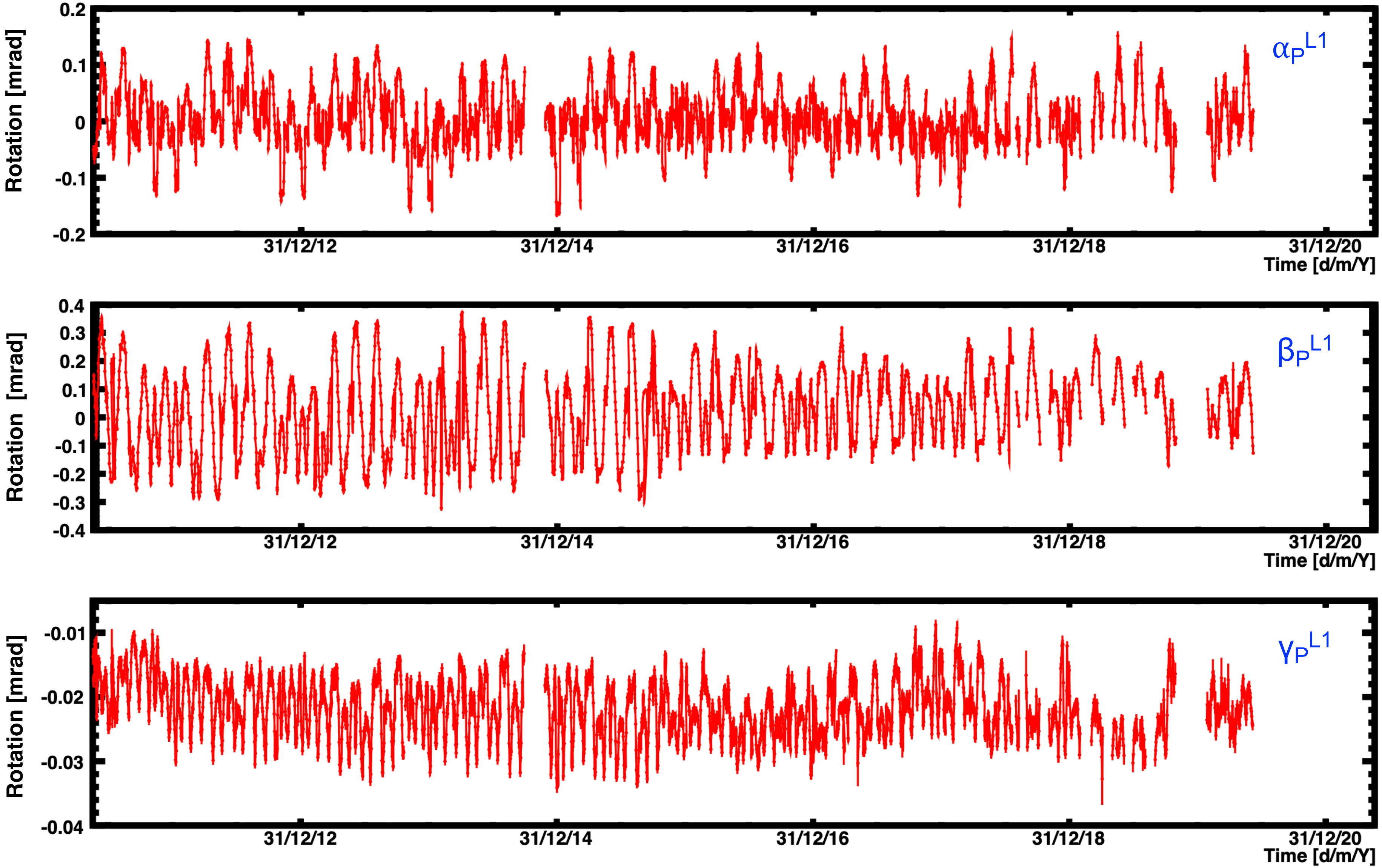


The orbital period of ~1.5 hours can be clearly seen in the alignment parameters:
The L1 movement in rotation α_P^{L1}, β_P^{L1}, and γ_P^{L1}: 0.02-0.2 mrad/half obit

The Daily Averaged Movement of the Tracker L1 (2011-2020)

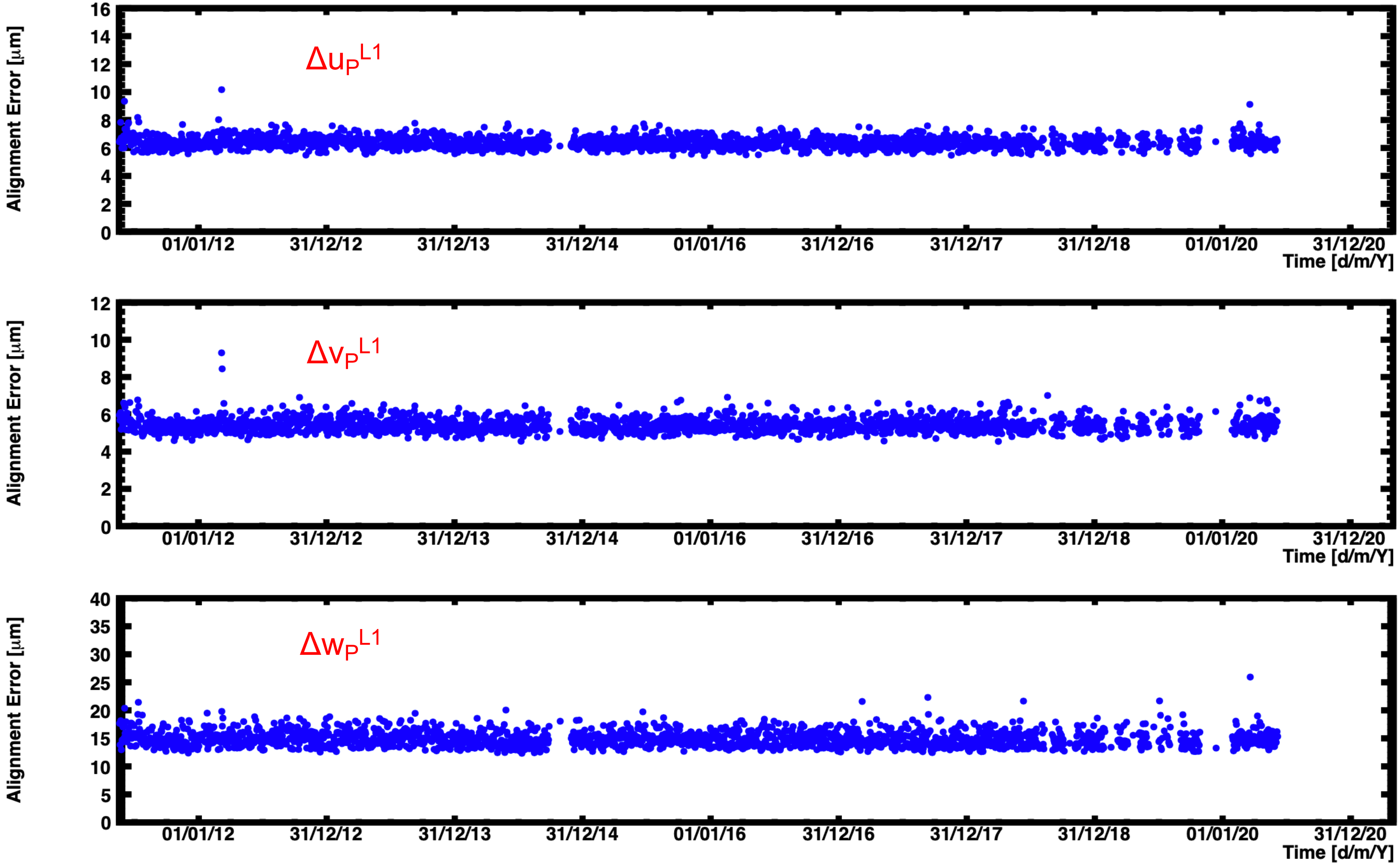


In addition to the orbital movement, external layers also have long-term periodic movement.
The L1 movement in shift Δu_P^{L1}, Δv_P^{L1} and Δw_P^{L1}: 200-1000 µm/month



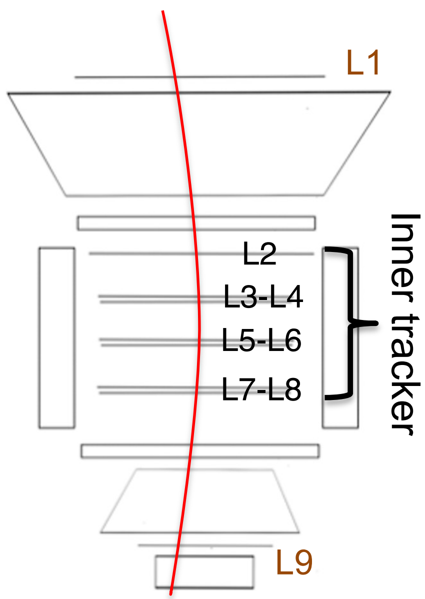
In addition to the orbital movement, external layers also have long-term periodic movement.
The L1 movement in rotation α_P^{L1}, β_P^{L1}, and γ_P^{L1}: 0.02-0.5 mrad/month

The Alignment Precision VS Time (2011-2020)

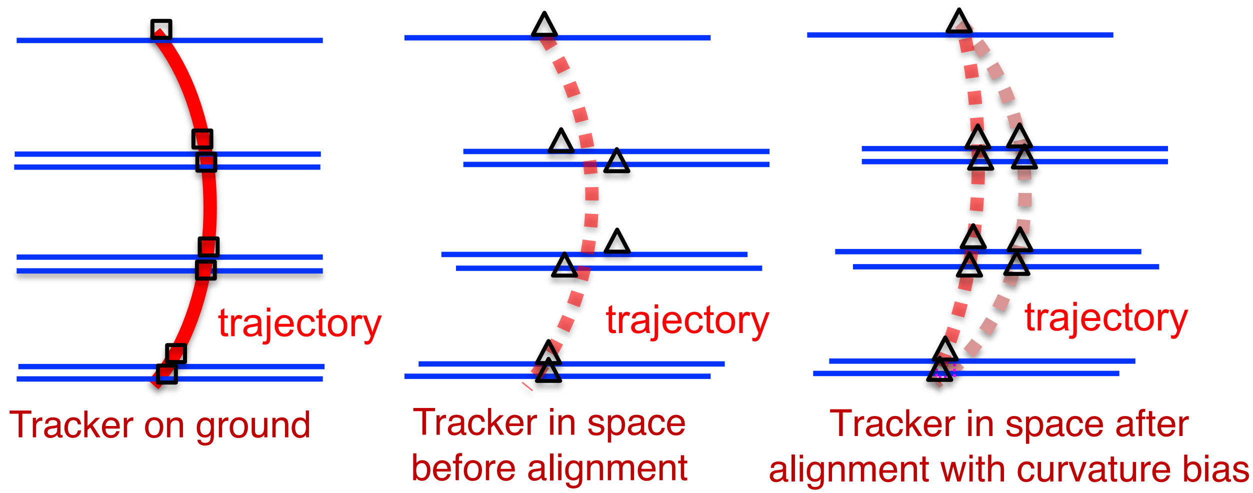


Static Alignment of the Tracker in Space

Before launch, all the AMS tracker modules have been aligned based on the primary 400 GeV/c proton beam. However, the strong accelerations and vibrations during launch, followed by the rapid outgassing of the support structure in vacuum, permanently changed the positions of ladders and sensors up to tens of microns compared to their positions on the ground. Therefore, the entire tracker has to be re-aligned with cosmic ray events to correct the resulting displacement. The unprecedented challenge of this alignment is the unknown curvatures (inverse rigidities) of the incoming particles in the presence of the magnetic field.



A track alignment approach similar to the alignment on ground is not sufficient for this alignment as the curvatures of tracks can be significantly biased due to a lack of constrains. Therefore, we developed a new alignment approach to precisely align the spectrometer (2,000 tracker modules) in space.



AMS Global Composite Curvature Alignment

To align the detector with particles of unknown rigidity in the presence of a magnetic field, a new alignment mathematical physics model has been developed:

$$\chi^2(\mathbf{q},\mathbf{p}) = \sum_{i=1}^{N_{track}} \left[\sum_{j=1}^{n_{meas}} \overbrace{\varepsilon_{ij}(\mathbf{q}_i,\mathbf{p})^{\mathsf{T}} \mathbf{V}_{ij}^{-1} \varepsilon_{ij}(\mathbf{q}_i,\mathbf{p})}^{\text{residual measurements } (\boldsymbol{\varepsilon})} + \sum_{j=2}^{n_{scat}-1} \overbrace{\beta_{ij}(\mathbf{q}_i)^{\mathsf{T}} \mathbf{W}_{ij}^{-1} \beta_{ij}(\mathbf{q}_i)}^{\text{multiple-scattering } (\boldsymbol{\beta})} + \underbrace{\frac{\rho_i^2(\mathbf{p})}{Z_i}}_{\text{curvature bias } (\boldsymbol{\rho})} \right]$$

With the mathematic linearization of **ε** (residual), **β** (scattering angle), and **ρ** (curvature), all the global detector alignment parameters, Δ**p** (10^4 parameters), and all the local track parameters, Δ**q** (10^{10} parameters), are simultaneously solved through one vast matrix inversion:

$$\begin{pmatrix} \sum_i \mathbf{C}^i & \mathbf{G}^1 & \dots & \mathbf{G}^j & \dots & \mathbf{G}^N \\ (\mathbf{G}^1)^{\mathsf{T}} & \Gamma^1 & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (\mathbf{G}^j)^{\mathsf{T}} & \mathbf{0} & \dots & \Gamma^j & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (\mathbf{G}^N)^{\mathsf{T}} & \mathbf{0} & \dots & \mathbf{0} & \dots & \Gamma^N \end{pmatrix} \begin{pmatrix} \Delta\mathbf{p} \\ \Delta\mathbf{q}_1 \\ \vdots \\ \Delta\mathbf{q}_j \\ \vdots \\ \Delta\mathbf{q}_N \end{pmatrix} = \begin{pmatrix} \sum_i \mathbf{d}^i \\ \mathbf{b}^1 \\ \vdots \\ \mathbf{b}^j \\ \vdots \\ \mathbf{b}^N \end{pmatrix}$$

$$C'^{i}_{gg'} = \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^{\mathsf{T}} \mathbf{V}_{ij}^{-1} \frac{\partial \varepsilon_{ij}}{\partial p_{g'}} + \frac{\partial \rho_i}{\partial p_g} Z_i^{-1} \frac{\partial \rho_i}{\partial p_{g'}}$$

$$G^i_{gl'} = \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^{\mathsf{T}} \mathbf{V}_{ij}^{-1} \frac{\partial \varepsilon_{ij}}{\partial q_{il'}}$$

$$\Gamma^i_{ll'} = \sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\right)^{\mathsf{T}} \mathbf{V}_{ij}^{-1} \frac{\partial \varepsilon_{ij}}{\partial q_{il'}} + \sum_{j=2}^{n_{scat}-1} \left(\frac{\partial \beta_{ij}}{\partial q_{il}}\right)^{\mathsf{T}} \mathbf{W}_{ij}^{-1} \frac{\partial \beta_{ij}}{\partial q_{il'}}$$

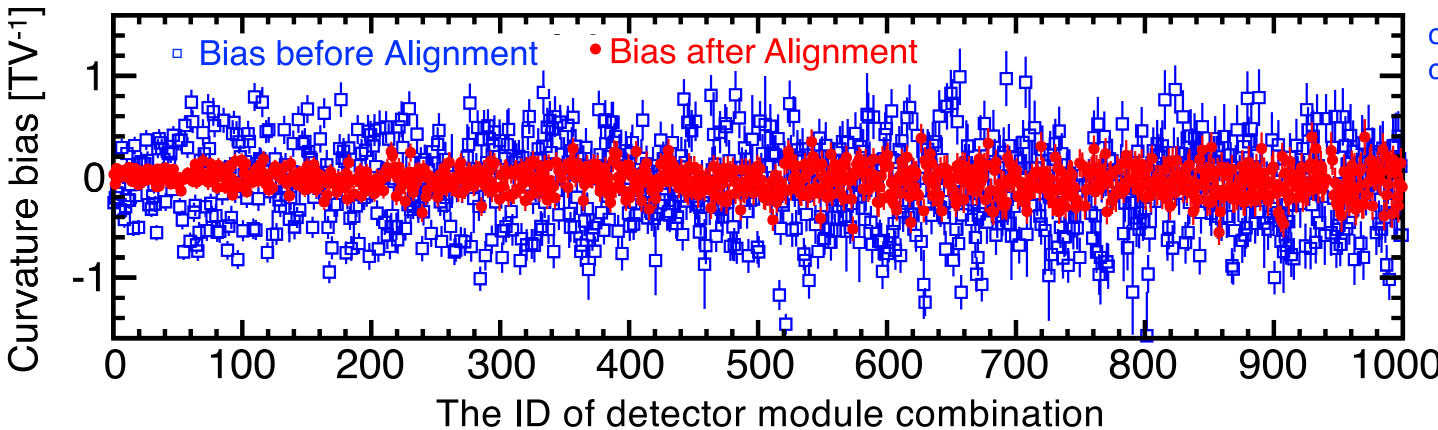
$$d'^{i}_{g} = -\sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial p_g}\right)^{\mathsf{T}} \mathbf{V}_{ij}^{-1} \varepsilon_{ij}(\mathbf{q}_i^0,\mathbf{p}^0) - \frac{\partial \rho_i}{\partial p_g} Z_i^{-1} \rho_i(\mathbf{p}^0)$$

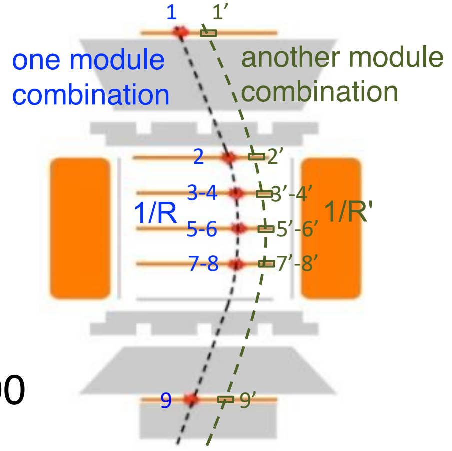
$$b^i_l = -\sum_{j=1}^{n_{meas}} \left(\frac{\partial \varepsilon_{ij}}{\partial q_{il}}\right)^{\mathsf{T}} \mathbf{V}_{ij}^{-1} \varepsilon_{ij}(\mathbf{q}_i^0,\mathbf{p}^0)$$

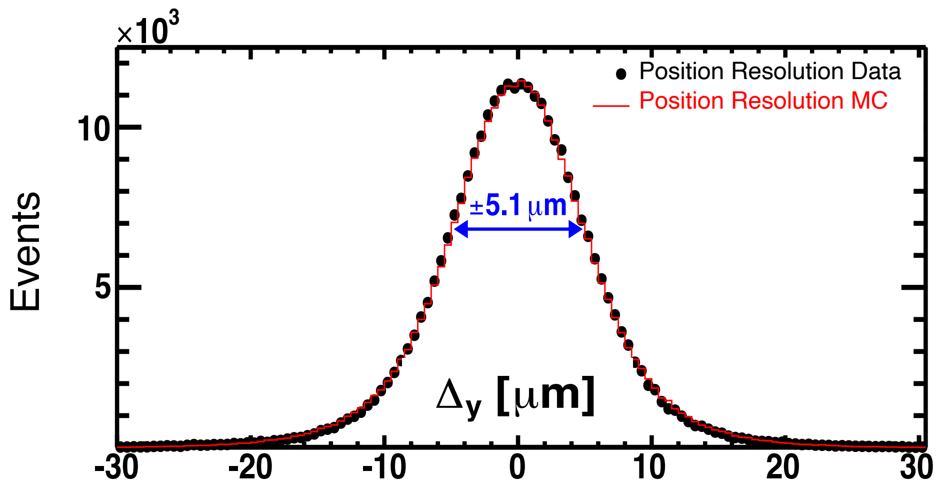
To achieve micron-level alignment accuracy for each sensor, 1.6 billion selected cosmic-ray events were used in this alignment. The track information from all these events was integrated into one matrix for the solution of all detector alignment parameters.

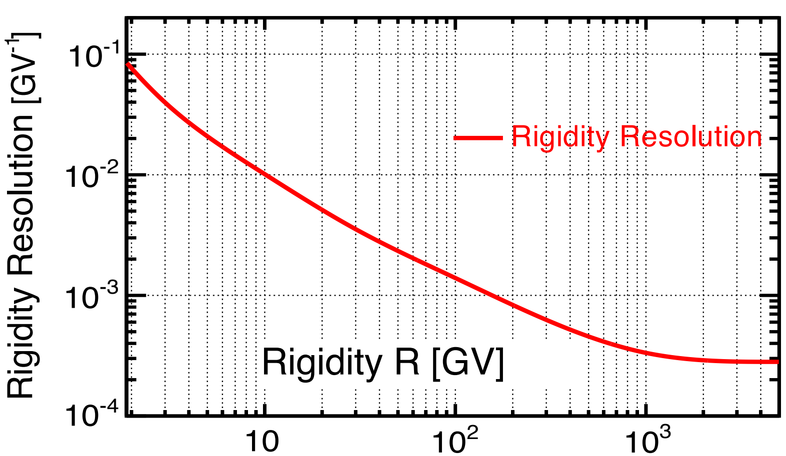
AMS Alignment Precision in Space

Utilizing the isotropy property of cosmic ray flux, a relative curvature bias (inverse rigidity: 1/R) was estimated for various detector module combinations. The average curvature bias after alignment was reduced to 0.11 TV^{-1}, corresponding to a position alignment accuracy of ~2 microns. With alignment, the position measurement resolution for carbon nuclei reaches 5.1 microns. Correspondingly, the maximal detectable rigidities, R^M, with $R^M \sigma(1/R^M) \equiv 1$, are R^M= 3.6 TV.



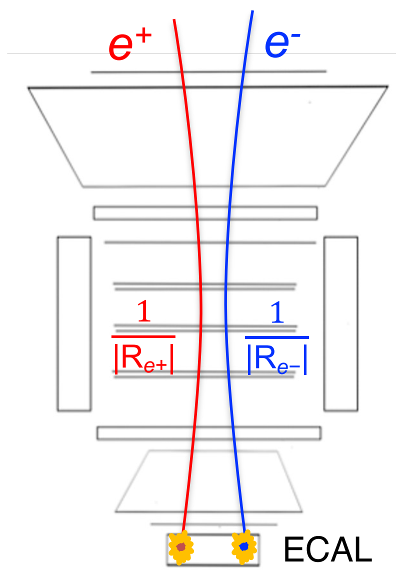


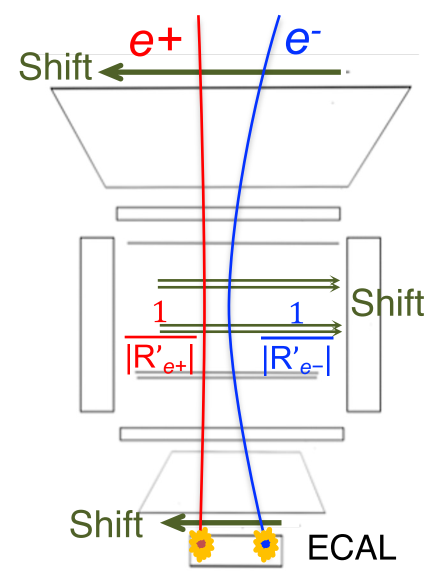




Determination of the Total AMS Absolute Rigidity Scale in Space

After the previous alignment, the tracker has a good rigidity resolution. However, the entire tracker can have an overall shift in the total absolute rigidity scale, resulting in a coherent shift in the positions of the tracker layers. To accurately determine the total absolute rigidity scale in space, a method using cosmic-ray positron (e^+) and electron (e^-) events to calibrate the detector was developed. This method was based on the principle that e^+ and e^- at the same energy exhibit opposite curvature (1/R) in the magnetic field.

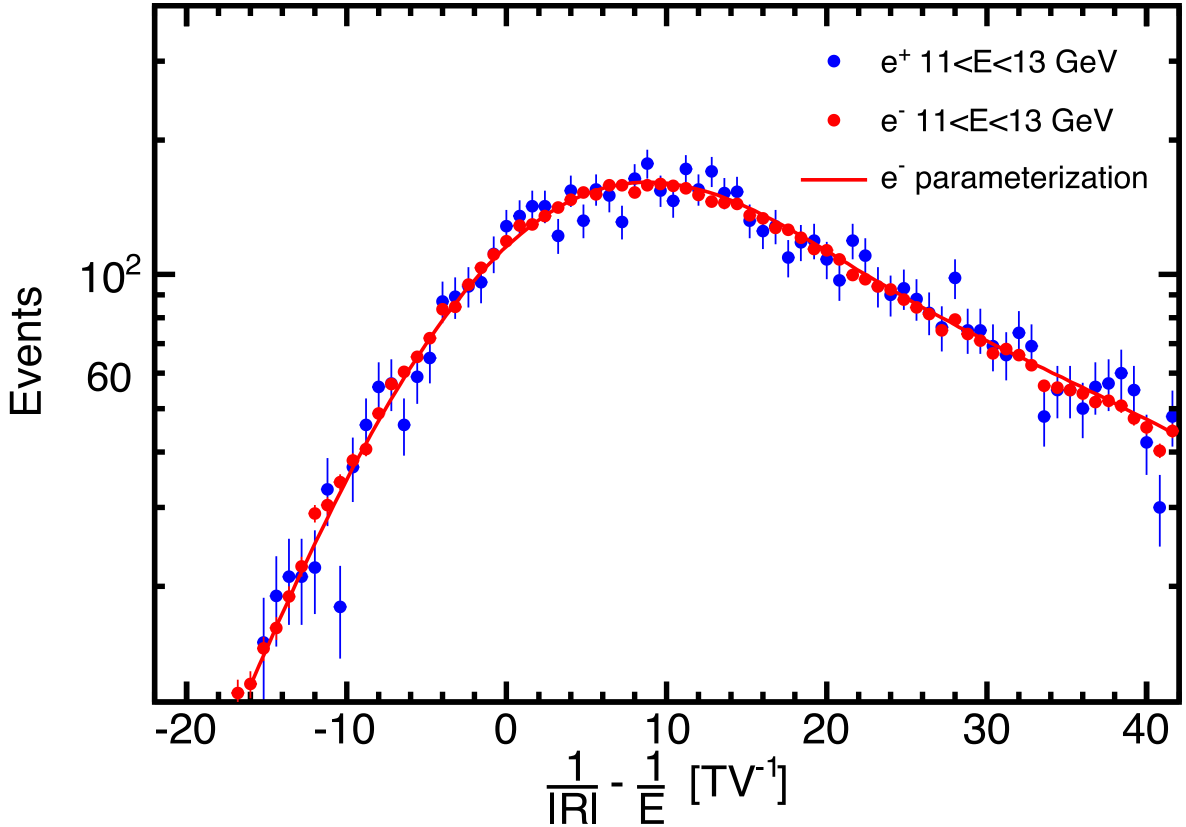




The measured curvatures are opposite for e+ and e- with the same energy, in the absence of the tracker shift.

When a tracker shift occurs, the measured curvatures for e+ and e- are biased in different directions.

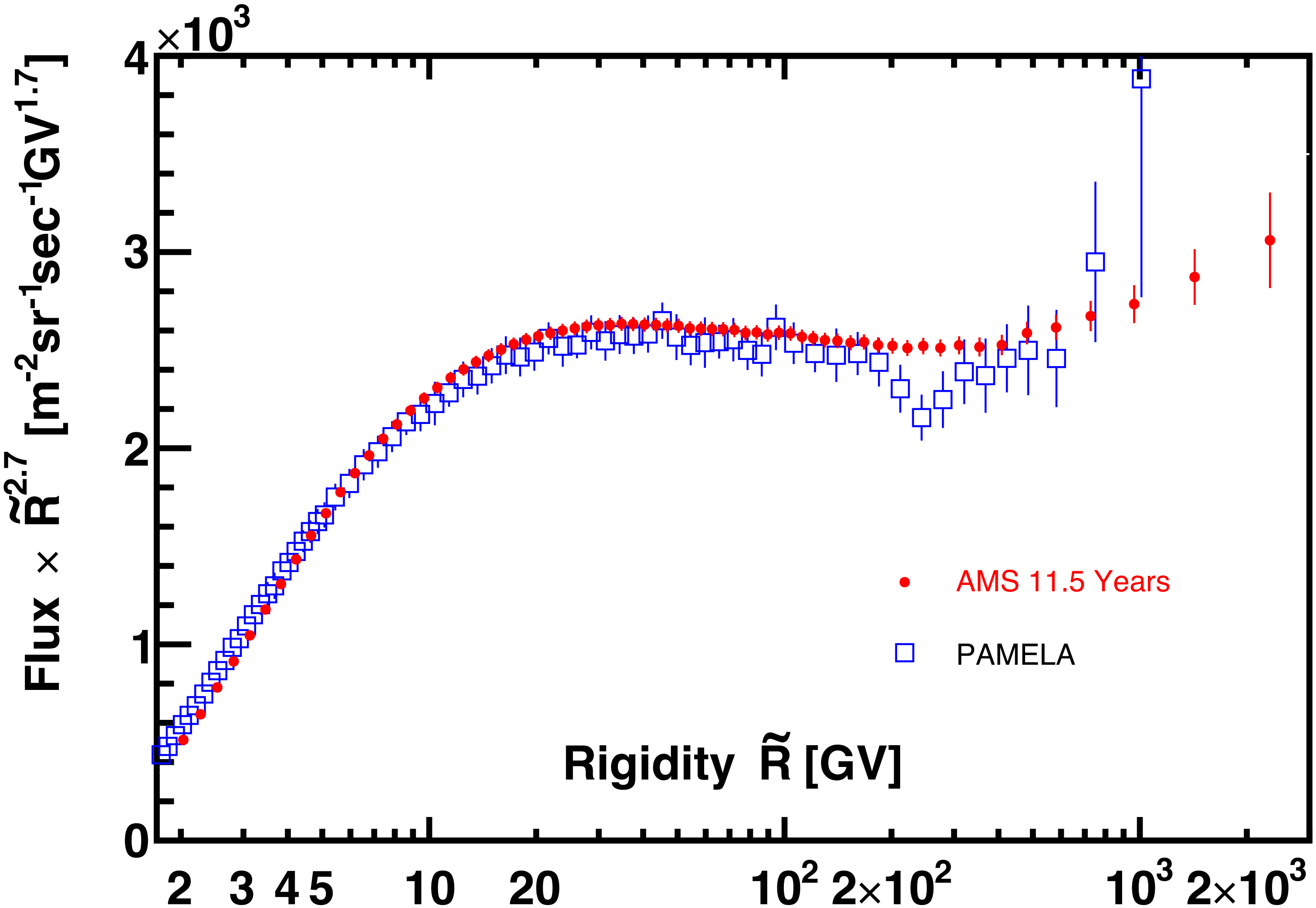
To make full usage of all statistics of e+ and e- events in cosmic rays from different energies, an advanced un-binned likelihood method has been developed. The detailed description about this method can be found in "Nucl. Instrum. Methods Phys. Res. A 869, 10 (2017)". With this method, the rigidity scale of the AMS tracker was established with an accuracy of $\pm 1/30\ \mathrm{TV}^{-1}$, which corresponds to the determination of the coherent displacement of the inner tracker layers with a precision of better than 0.2 µm. This work allows AMS to measure the cosmic ray fluxes with an accuracy of a few percent at the highest energy.



1/IRI-1/E distributions for e+ and e- after calibration

AMS Cosmic Ray Flux Measurement (Helium)

With precise detector alignment/calibration, AMS measures cosmic ray fluxes with an accuracy of a few percent up to TV rigidity region.



AMS Alignment Paper

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Regular Article - Experimental Physics

Alignment of the Alpha Magnetic Spectrometer (AMS) in space

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Abstract The Alpha Magnetic Spectrometer (AMS) is a precision particle physics detector operating at an altitude of ∼ 410 km aboard the International Space Station. The AMS silicon tracker, together with the permanent magnet, measures the rigidity (momentum/charge) of cosmic rays in the range from ∼ 0.5 GV to several TV. In order to have accurate rigidity measurements, the positions of more than 2000 tracker modules have to be determined at the micron level by an alignment procedure. The tracker was first aligned using the 400 GeV/c proton test beam at CERN and then re-aligned using cosmic-ray events after being launched into space. A unique method to align the permanent magnetic spectrometer for a space experiment is presented. The developed underlying mathematical algorithm is discussed in detail.

Contents

1 Introduction

The Alpha Magnetic Spectrometer (AMS), operating aboard