



QCD phase diagram at high baryon densities

Wei-jie Fu

Dalian University of Technology

The 16th Workshop on QCD Phase Transition and Relativistic Heavy-Ion Physics (QPT2025), Guilin, October 24-28, 2025

Based on:

WF, Xiaofeng Luo, Jan M. Pawłowski, Fabian Rennecke, Shi Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508;

Yang-yang Tan, Yong-rui Chen, WF, Wei-Jia Li, *Nature Commun.* 16 (2025) 2916, arXiv:2403.03503;

WF, Jan M. Pawłowski, Robert D. Pisarski, Fabian Rennecke, Rui Wen, Shi Yin, *PRD* 111 (2025) 094026, arXiv:2412.15949;

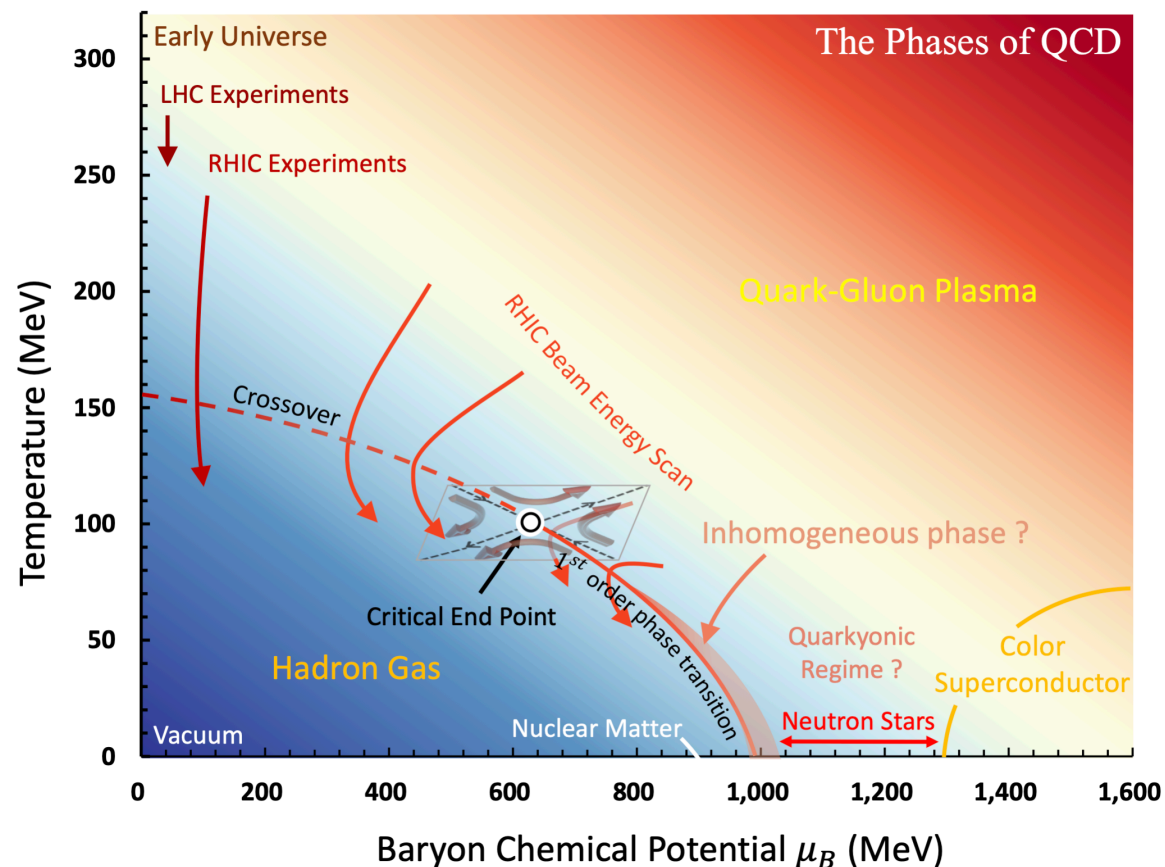
Zi-ning Wang, Li-jun Zhou, Chuang Huang, Rui Wen, Shi Yin, WF, in preparation;

WF, C. Huang, J. M. Pawłowski, F. Rennecke, R. Wen, S. Yin, in preparation;

Yang-yang Tan, Shi Yin, Yong-rui Chen, Chuang Huang, WF, in preparation.

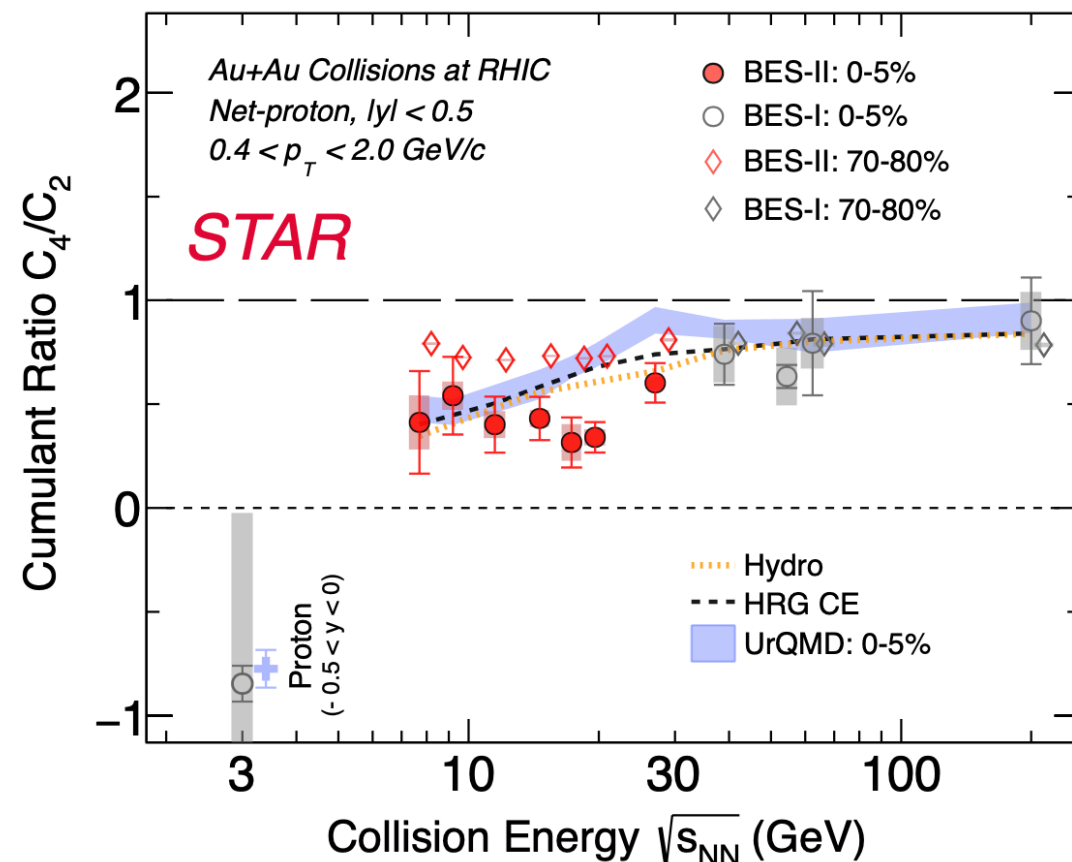
CEP in QCD phase diagram

QCD phase diagram



Non-monotonicity:
M. Stephanov, *PRL* 107 (2011) 052301

Fluctuations measured in BES-II



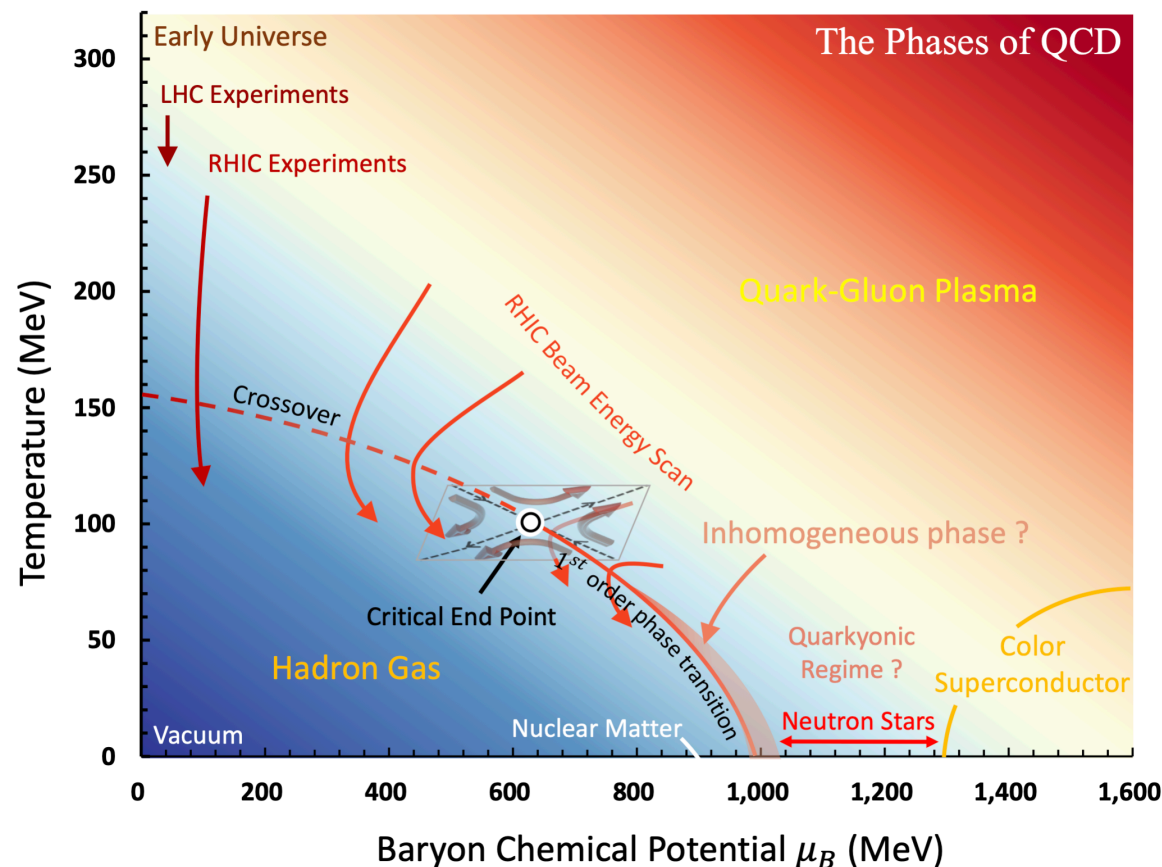
STAR Collaboration, *PRL* 135 (2025) 142301,
arXiv:2504.00817

- Is there a “peak” structure serving as the smoking gun signal for the critical end point in the QCD phase diagram?

See the plenary talk by Yu Zhang on Sat

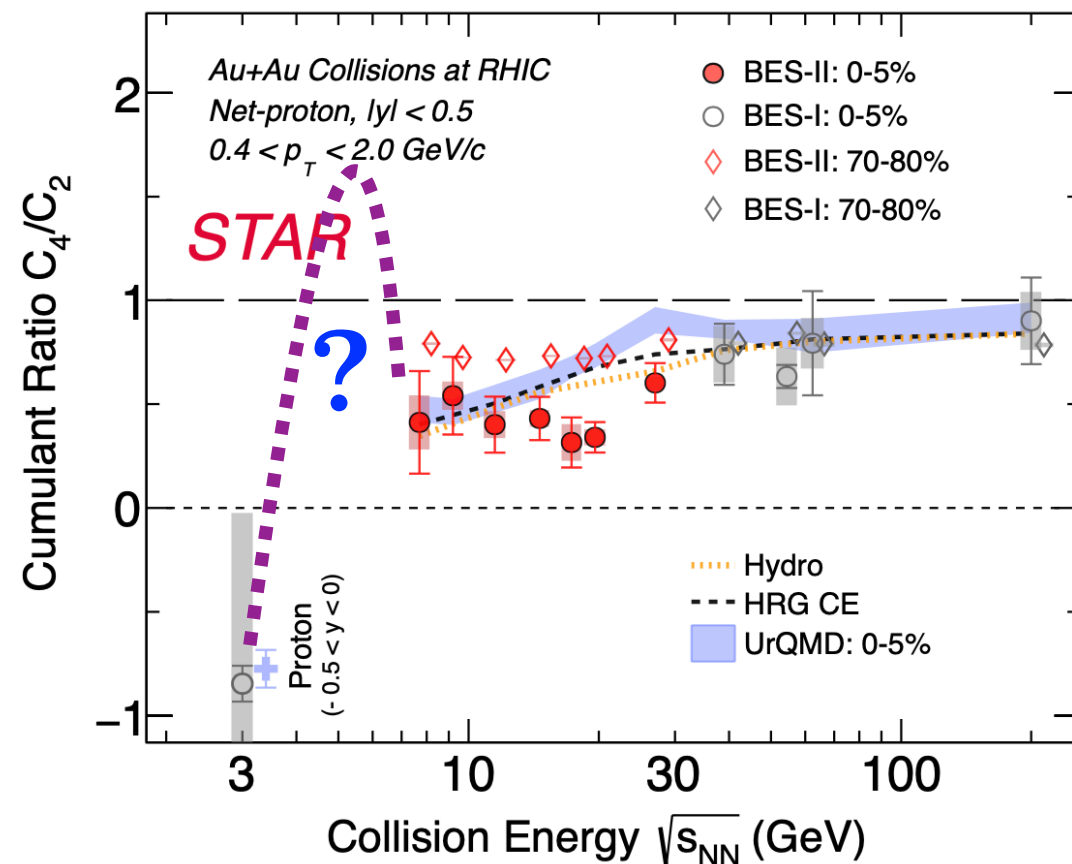
CEP in QCD phase diagram

QCD phase diagram



Non-monotonicity:
M. Stephanov, *PRL* 107 (2011) 052301

Fluctuations measured in BES-II

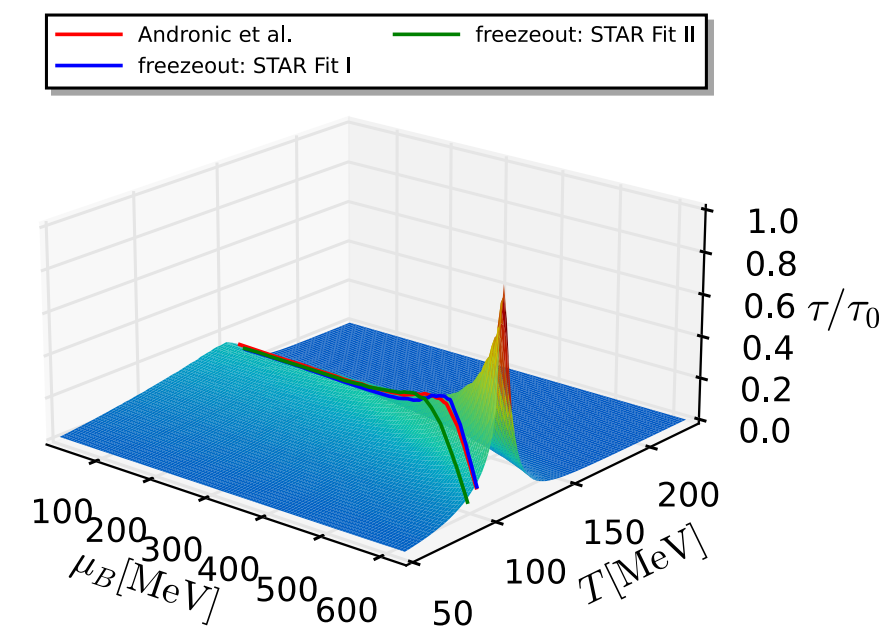
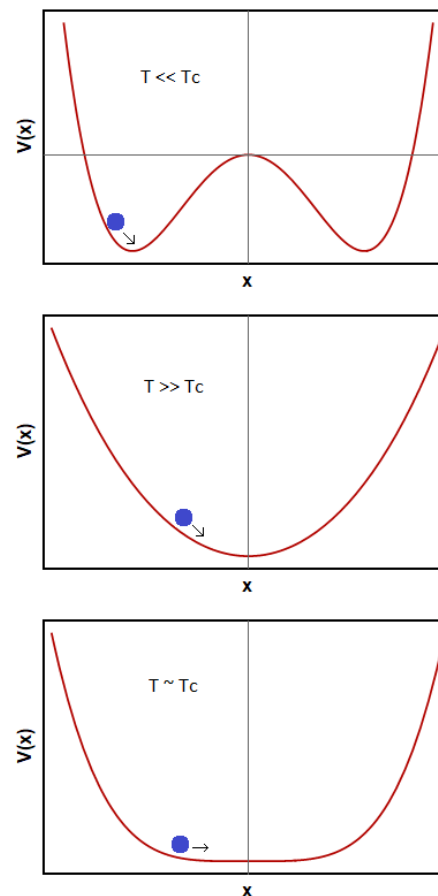
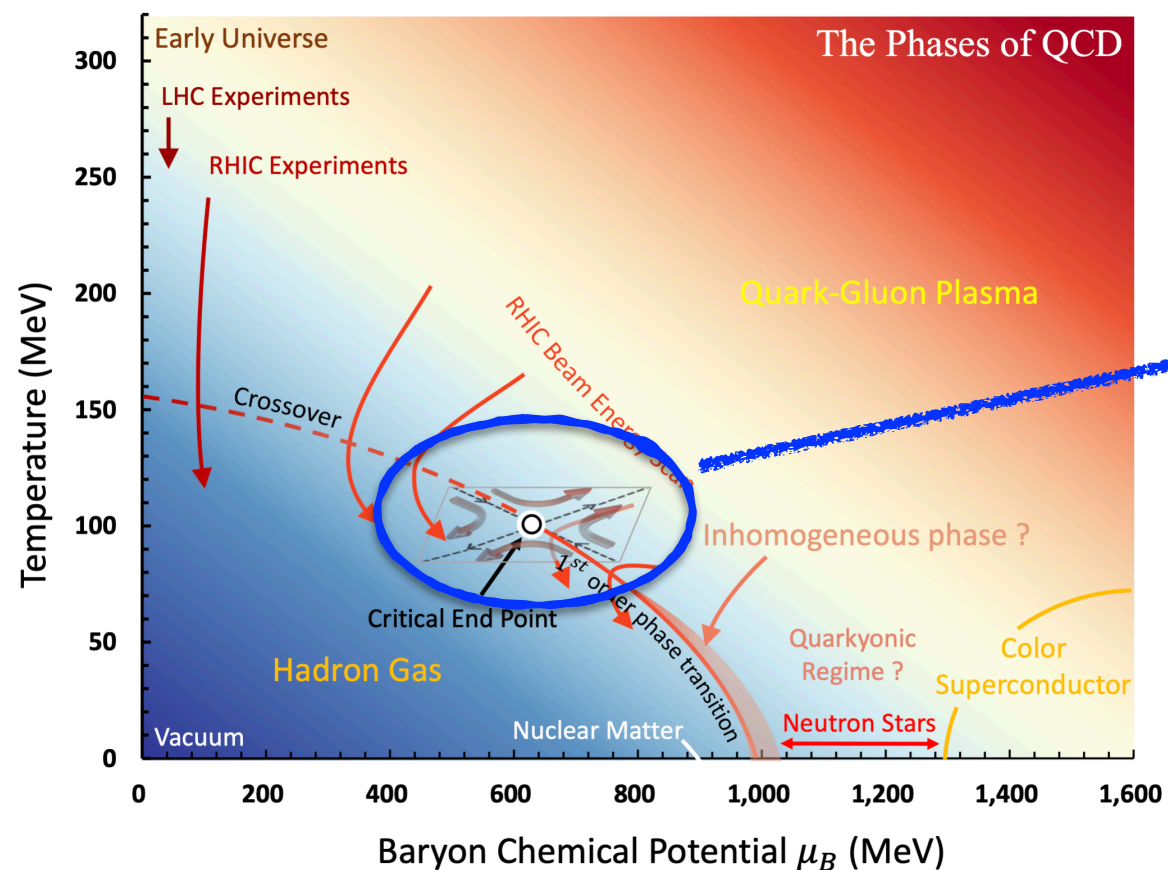


STAR Collaboration, *PRL* 135 (2025) 142301,
arXiv:2504.00817

- Is there a “peak” structure serving as the smoking gun signal for the critical end point in the QCD phase diagram?

See the plenary talk by Yu Zhang on Sat

Critical slowing down near CEP

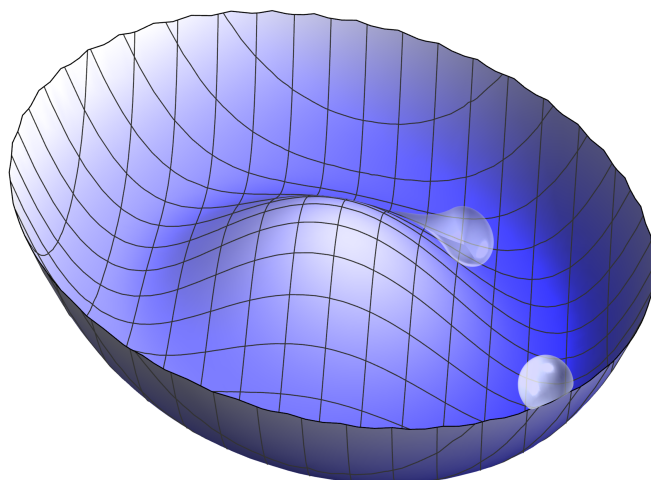


Relaxation time:

$$\tau = \xi^Z f(k\xi)$$

Z : dynamic critical exponent

Goldstone damping



Call for:

- Real-time description of strongly interacting systems.
- Nonperturbative approach of QCD.

See the talk by Shan-jin Wu in parallel I on Mon

Outline

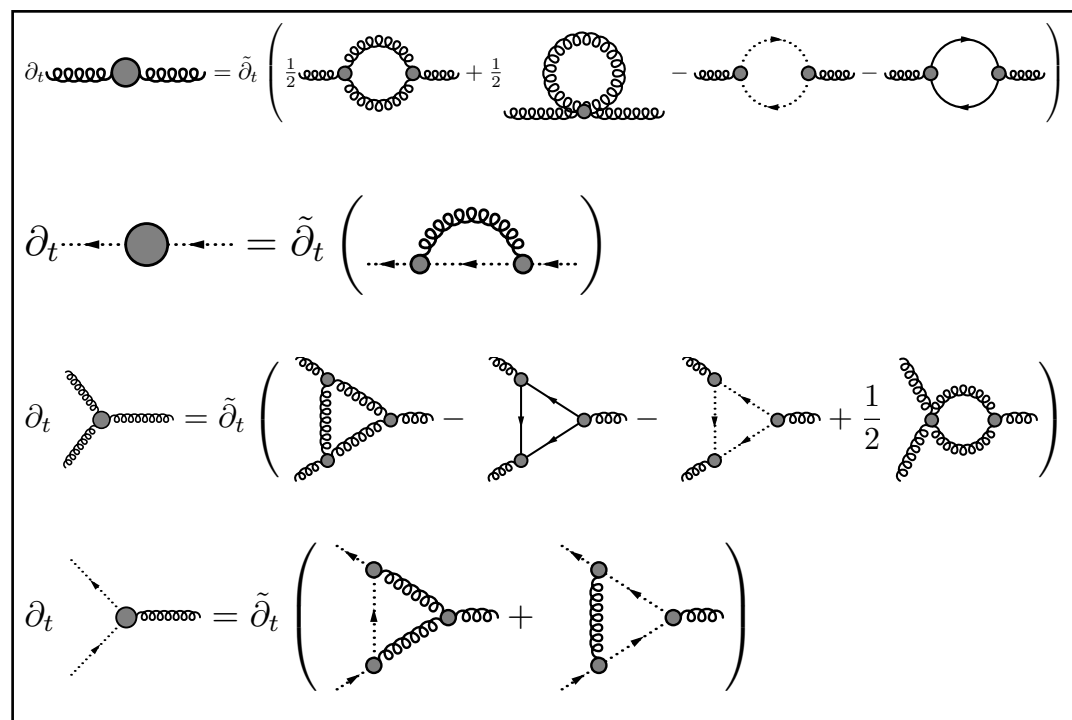
- **Introduction**
- **Recent estimates of the location of CEP**
- **Baryon number fluctuations**
- **Moat regime**
- **Real-time dynamics near phase transitions**
- **Summary and outlook**

QCD within fRG

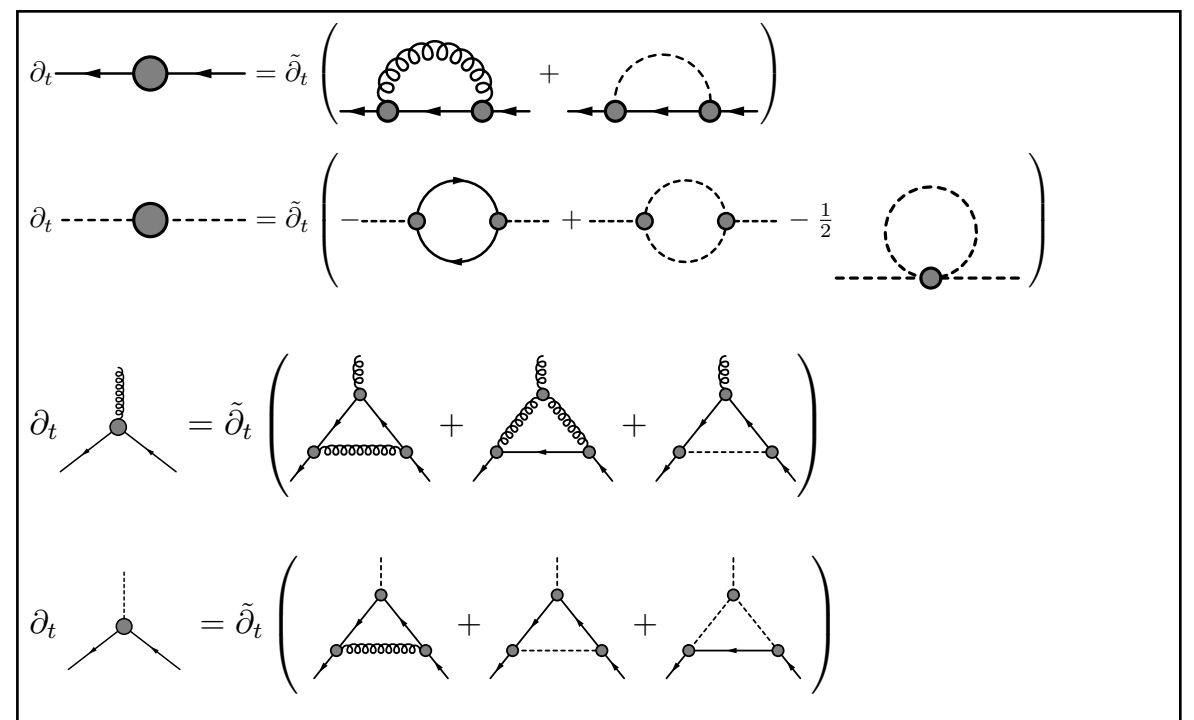
QCD flow equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{ (orange loop) } - \text{ (dotted loop) } - \text{ (solid loop) } + \frac{1}{2} \text{ (blue dashed loop) }$$

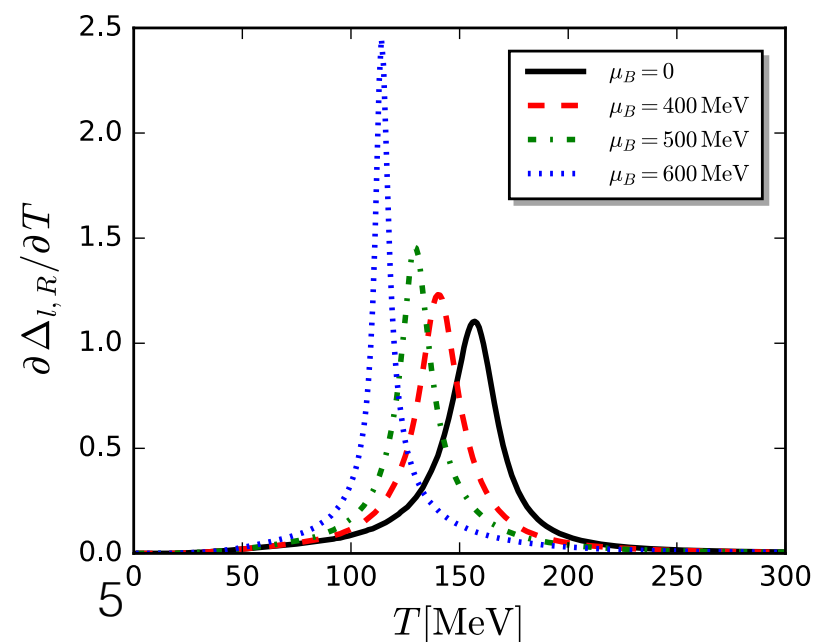
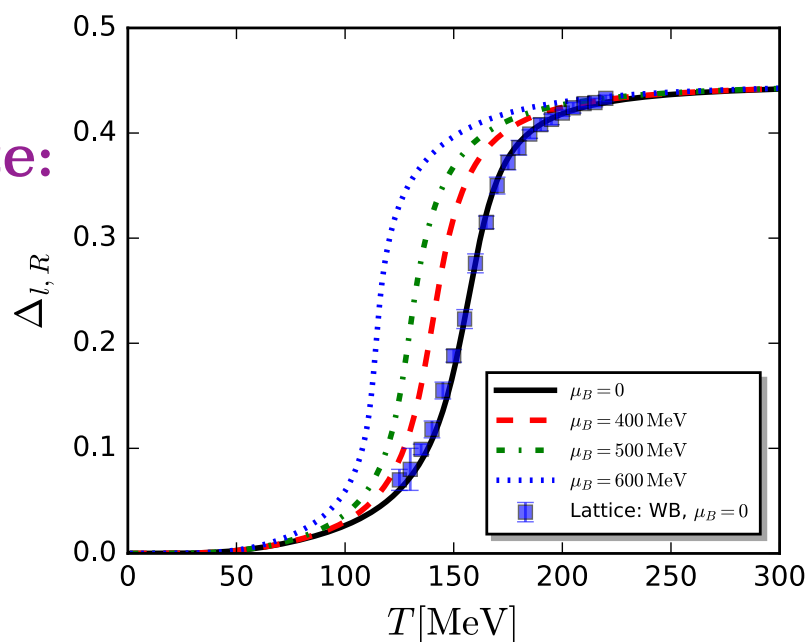
Glue sector:



Matter sector:



quark condensate:

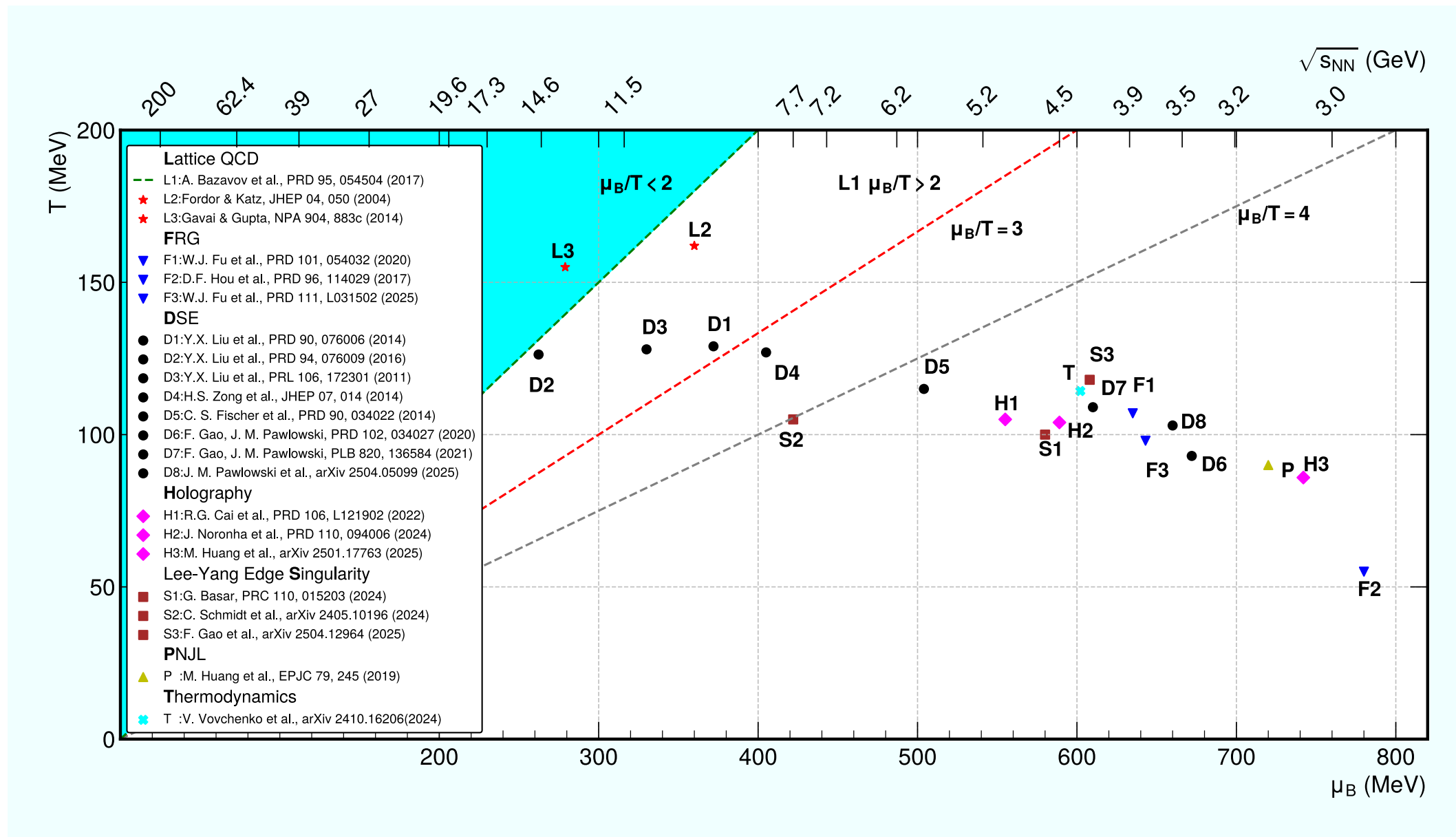


fRG: WF, Pawłowski, Rennecke,
PRD 101 (2020) 054032

Lattice: Borsanyi *et al.* (WB),
JHEP 09 (2010) 073

Quantitative errors analysis in fRG:
Ihssen, Pawłowski, Sattler, Wink,
arXiv:2408.08413

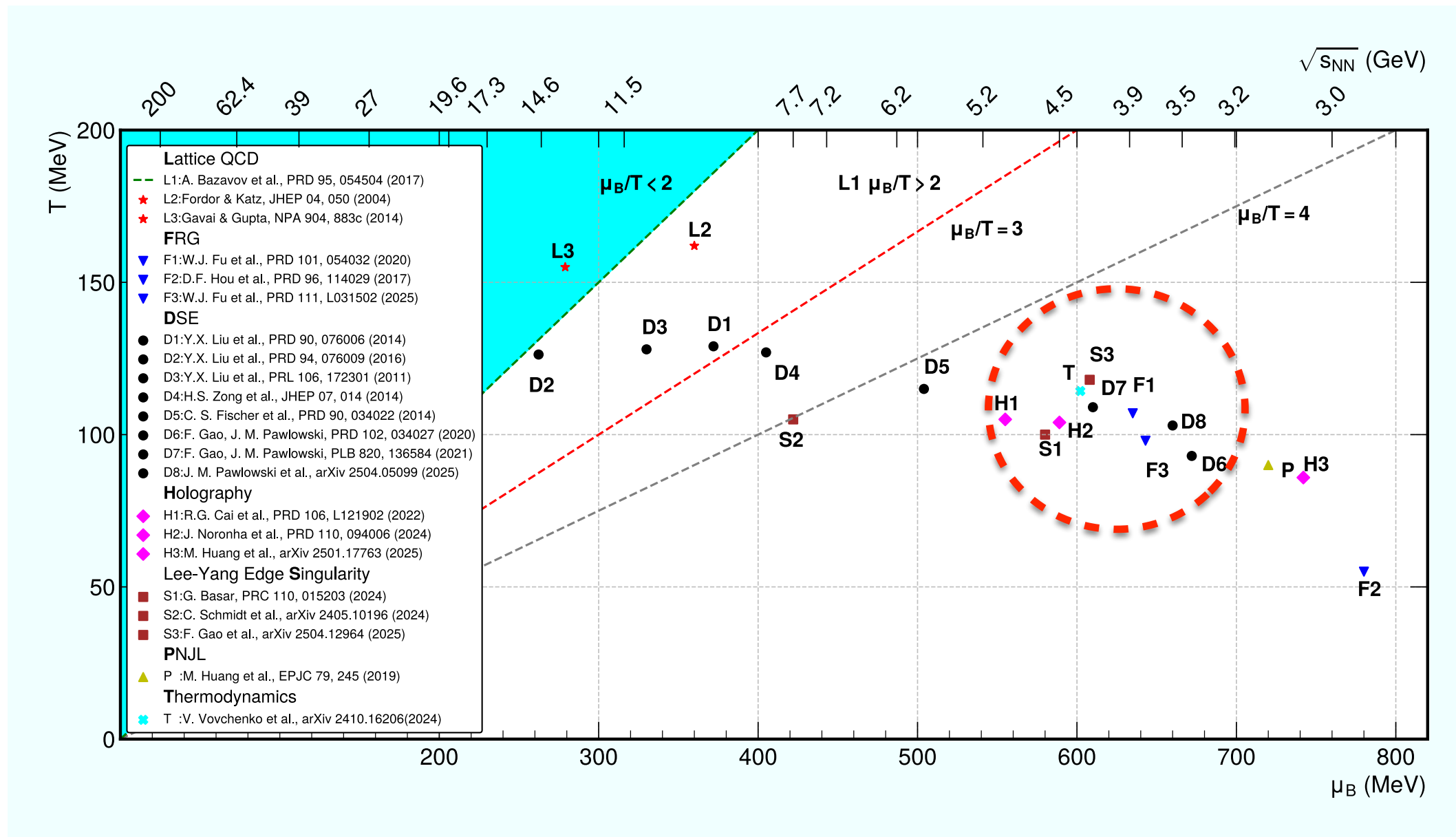
CEP from different theoretical calculations



By courtesy of Xiaofeng Luo

See talks by Heng-Tong Ding, Gao-qing Cao, Hao-Lei Chen, Zhou-run Zhu, Yi Lu, ...

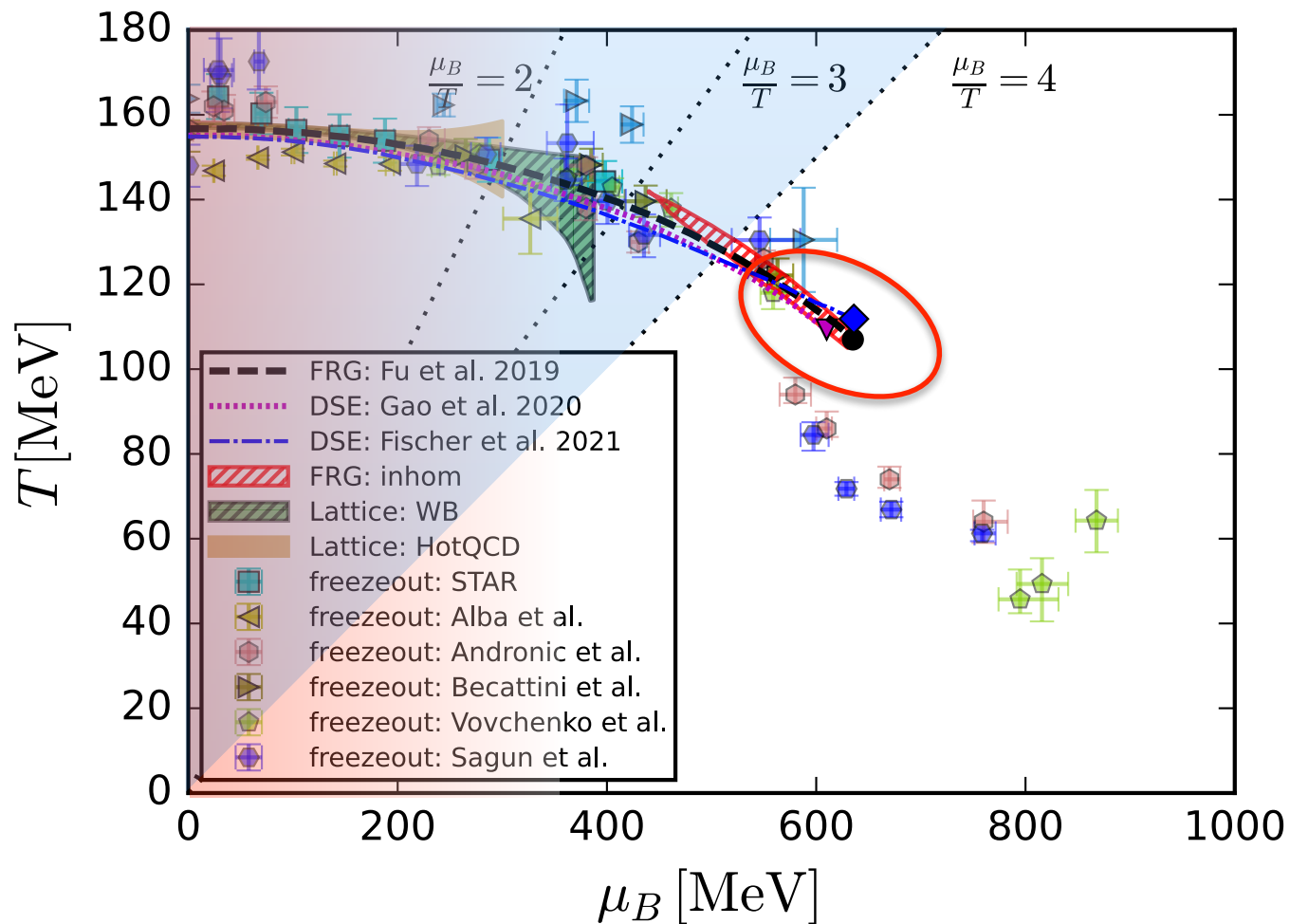
CEP from different theoretical calculations



By courtesy of Xiaofeng Luo

See talks by Heng-Tong Ding, Gao-qing Cao, Hao-Lei Chen, Zhou-run Zhu, Yi Lu, ...

CEP from functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B .

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

Estimates of the location of CEP from first-principles functional QCD:

fRG:

$$\bullet (T, \mu_B)_{\text{CEP}} = (107, 635) \text{ MeV}$$

fRG: WF, Pawłowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

$$\blacktriangledown (T, \mu_B)_{\text{CEP}} = (109, 610) \text{ MeV}$$

DSE : Gao, Pawłowski, *PLB* 820 (2021) 136584

$$\blacklozenge (T, \mu_B)_{\text{CEP}} = (112, 636) \text{ MeV}$$

DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- No CEP observed in $\mu_B/T \lesssim 2 \sim 3$ from lattice QCD. Karsch, *PoS CORFU2018* (2019)163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP: $600 \text{ MeV} \lesssim \mu_{B\text{CEP}} \lesssim 650 \text{ MeV}$.

CEP from other approaches

Recent estimates of the location of CEP:

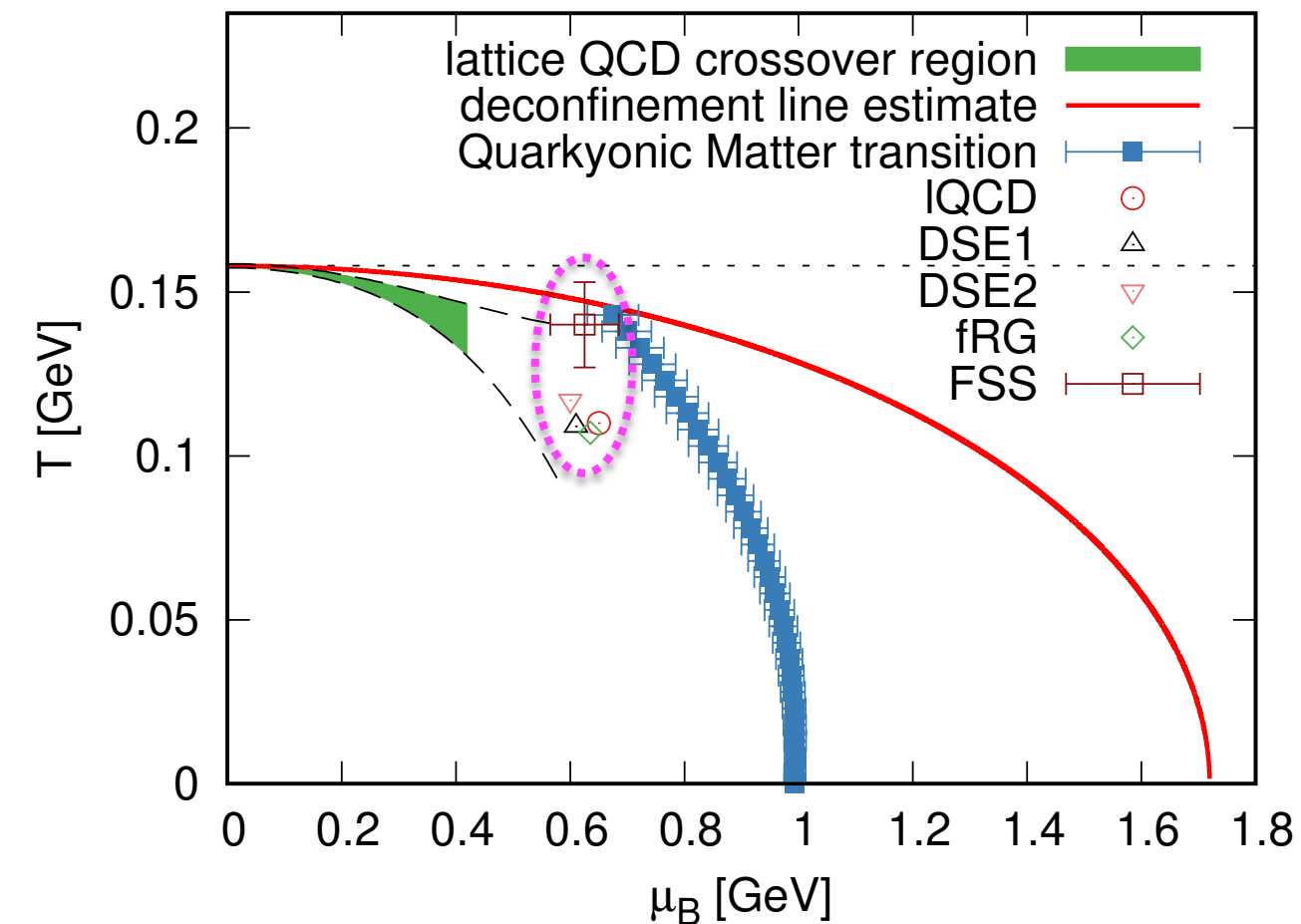


Figure from:

Bluhm, Fujimoto, McLerran, Nahrgang, *PRC* 111 (2025) 044914.

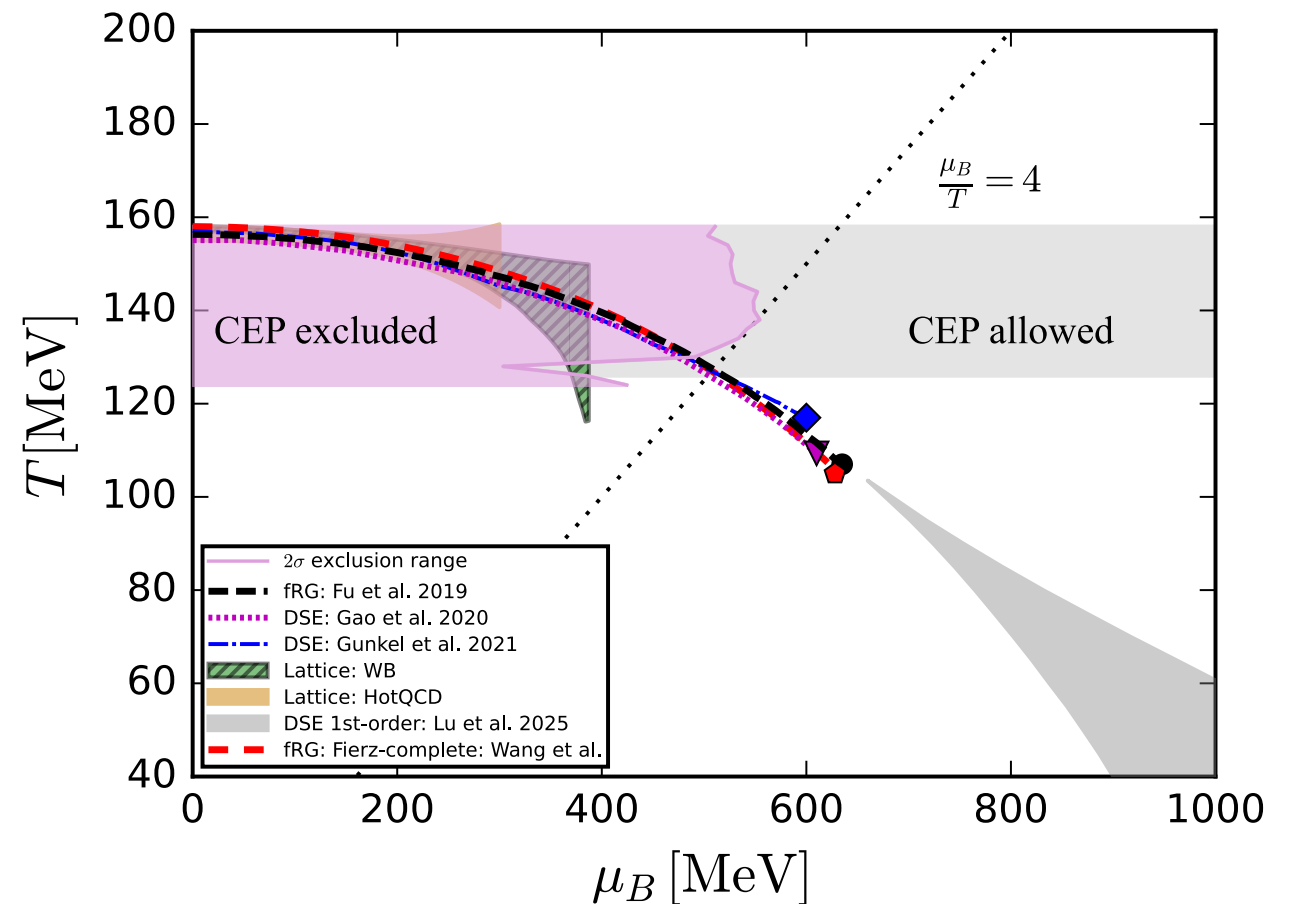
fRG:

WF, Pawłowski, Rennecke, *PRD* 101 (2020), 054032, arXiv:1909.02991.

DSE:

Gao, Pawłowski, *PLB* 820 (2021) 136584, arXiv:2010.13705.
Gunkel, Fischer, *PRD* 104 (2021) 054022, arXiv:2106.08356.
Lu, Gao, Liu, Pawłowski, arXiv:2504.05099.

See the talk by Yi Lu in parallel I on Sun



Lattice extrapolation (Yang-Lee edge singularities):

David A. Clarke *et al.*, arXiv:2405.10196.

S. Borsanyi *et al.*, arXiv:2502.10267.

Finite-size-scaling analysis:

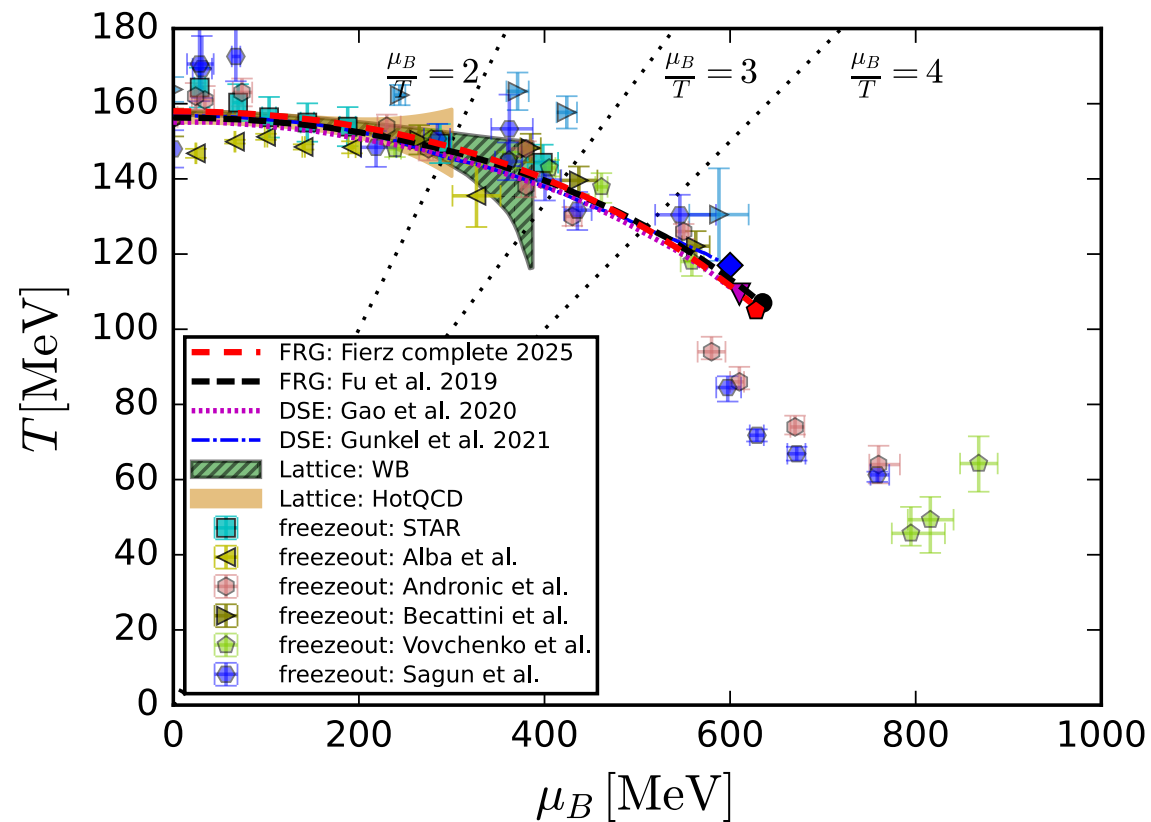
A. Sorensen, P. Sorensen, arXiv:2405.10278.

- Estimates of the location of CEP in the QCD phase diagram have arrived at convergence from different approaches.

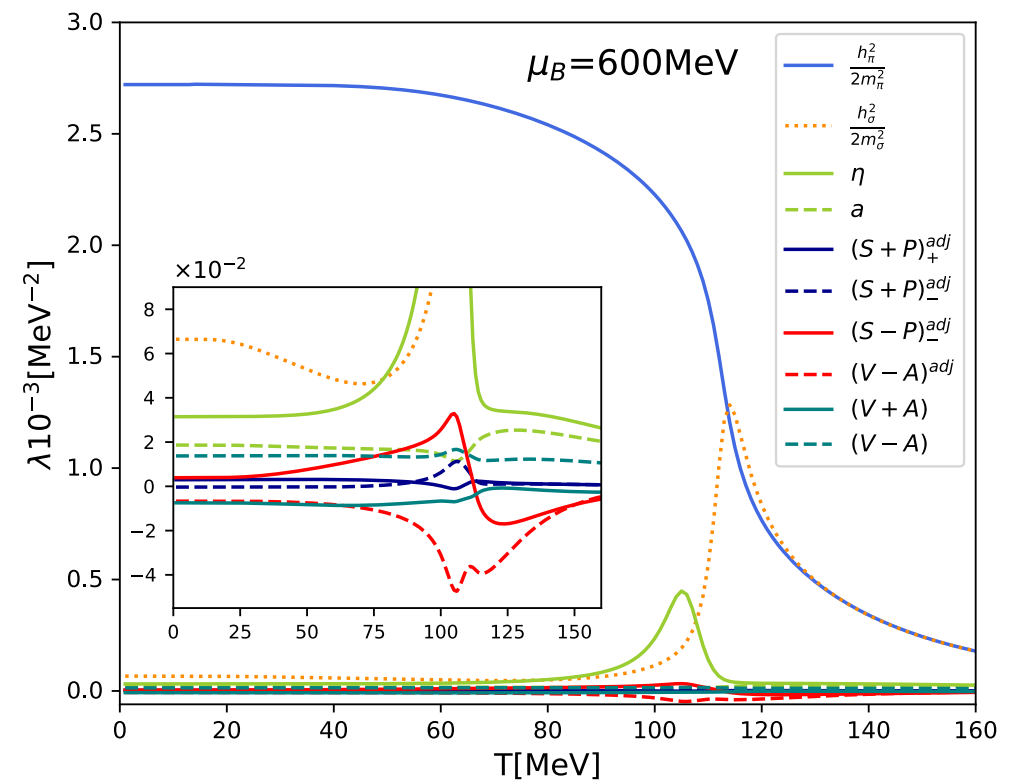
See the plenary talk by Heng-Tong Ding on Sat

CEP and Fierz-complete four-quark basis

QCD phase diagram with Fierz-complete basis:



Four-quark couplings:



Scalar-pseudoscalar channel:

$$(T, \mu_B)_{\text{CEP}} = (107, 635) \text{ MeV} \quad \longrightarrow$$

$$\partial_t \text{ (diagram)} = \tilde{\partial}_t \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} \right)$$

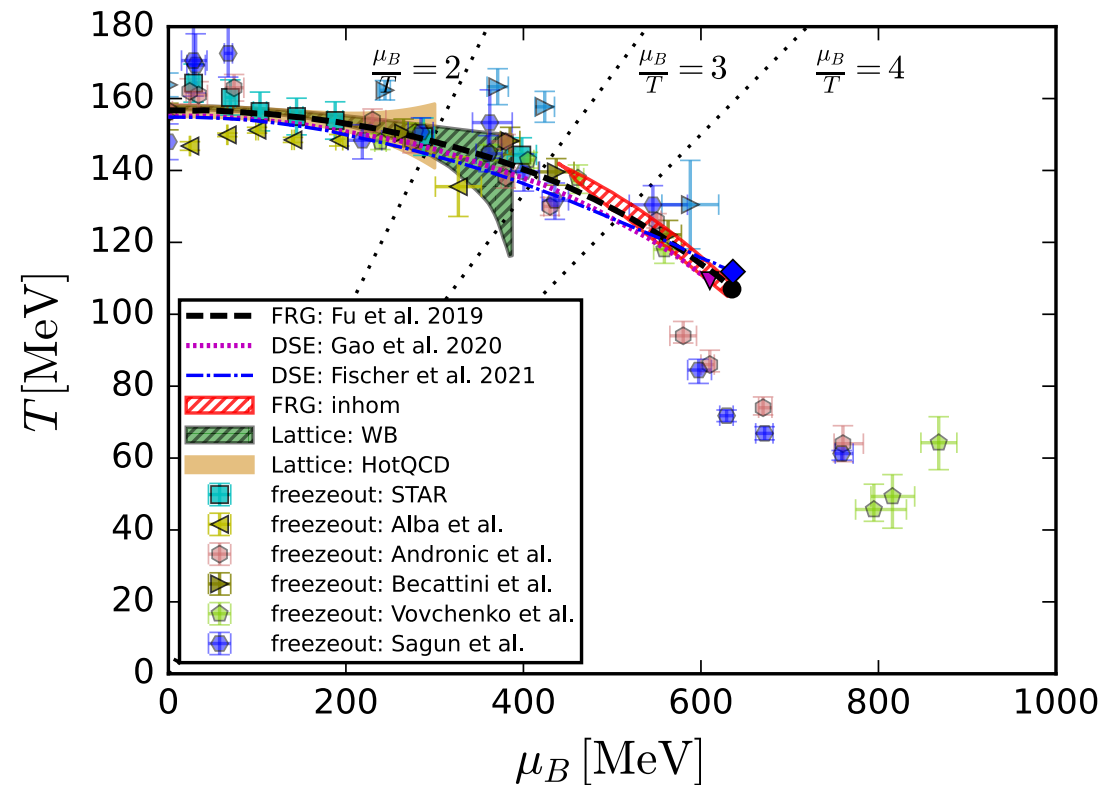
Fierz-complete channels:

$$(T, \mu_B)_{\text{CEP}} = (105, 630) \text{ MeV}$$

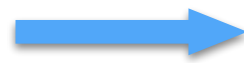
Wang, Zhou, Huang, Wen, Yin, WF, in preparation.

CEP and strangeness neutrality

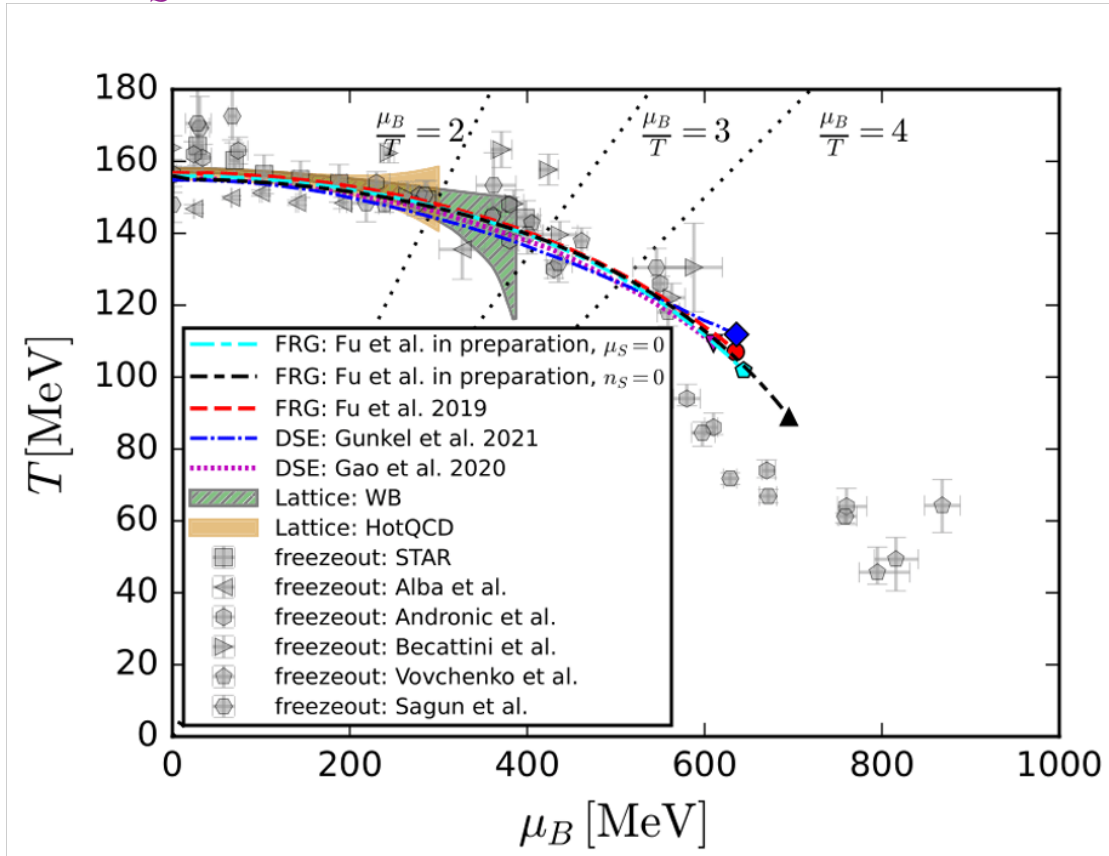
Phase diagram with $\mu_S = 0$



$$(T, \mu_B)_{\text{CEP}} = (107, 635) \text{ MeV}$$



$n_S = 0$ (strangeness neutrality)



$$(T, \mu_B)_{\text{CEP}} = (89, 695) \text{ MeV}$$

Curvature of the phase boundary:

$$\frac{\kappa_2(n_S = 0)}{\kappa_2(\mu_S = 0)} = \frac{0.0130(2)}{0.0145(2)} = 0.90(2)$$

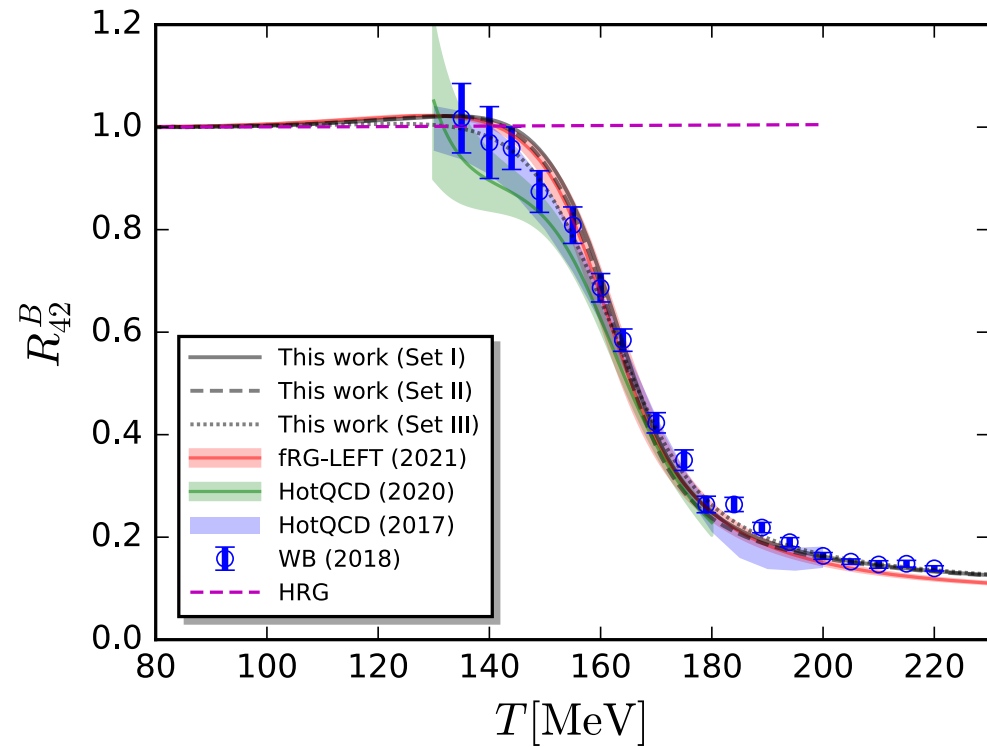
Lattice QCD:

$$\frac{\kappa_2(n_S = 0)}{\kappa_2(\mu_S = 0)} = 0.893(35)$$

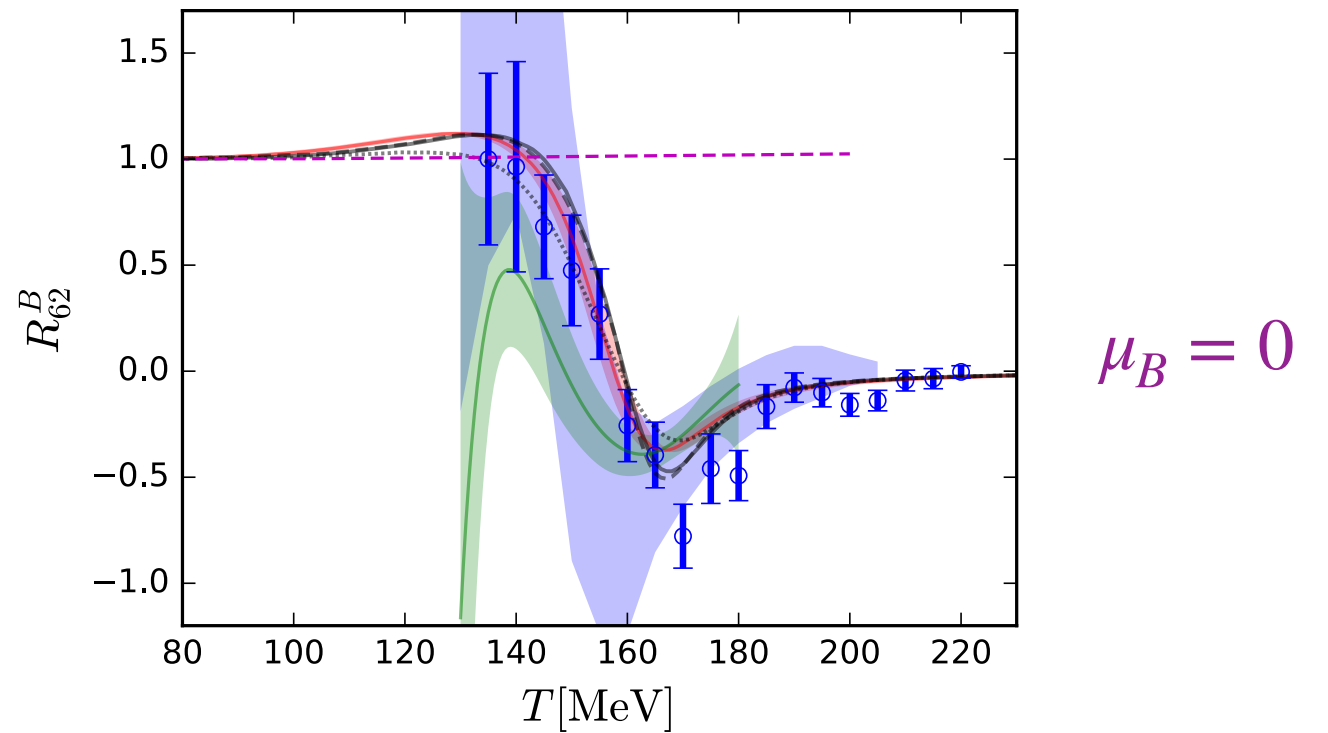
WF, Huang, Pawłowski, Rennecke, Wen, Yin, in preparation.

HotQCD: H.-T. Ding *et al.*, *PRD* 109 (2024), 114516.

Baryon number fluctuations



fRG: WF, Luo, Pawłowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508; WF, Luo, Pawłowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047



HotQCD: A. Bazavov *et al.*, *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, *JHEP* 10 (2018) 205

baryon number fluctuations

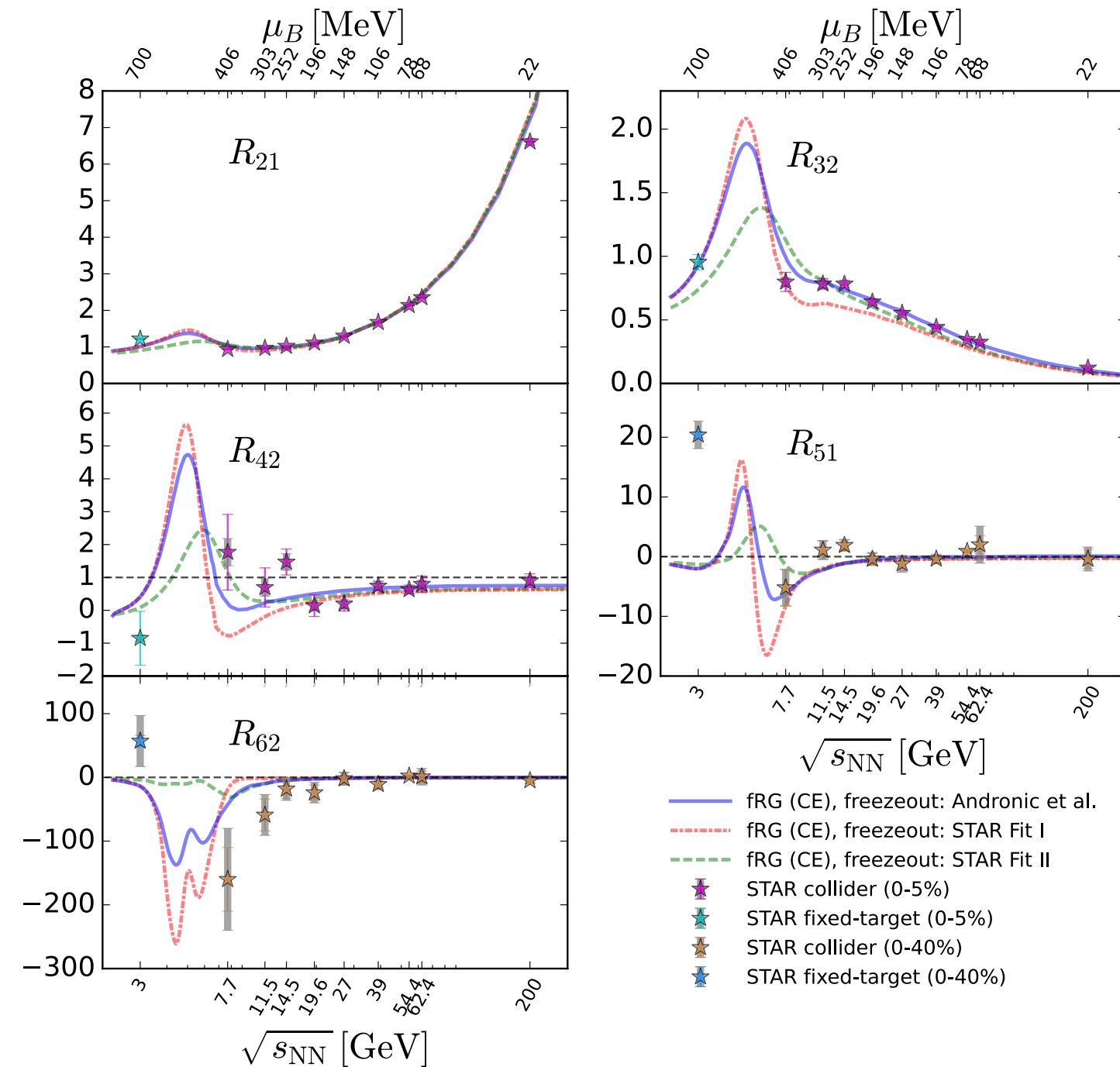
$$\chi_n^B = \frac{\partial^n}{\partial (\mu_B/T)^n} \frac{p}{T^4} \quad R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B, \quad \frac{\sigma^2}{VT^3} = \chi_2^B, \quad S = \frac{\chi_3^B}{\chi_2^B \sigma}, \quad \kappa = \frac{\chi_4^B}{\chi_2^B \sigma^2},$$

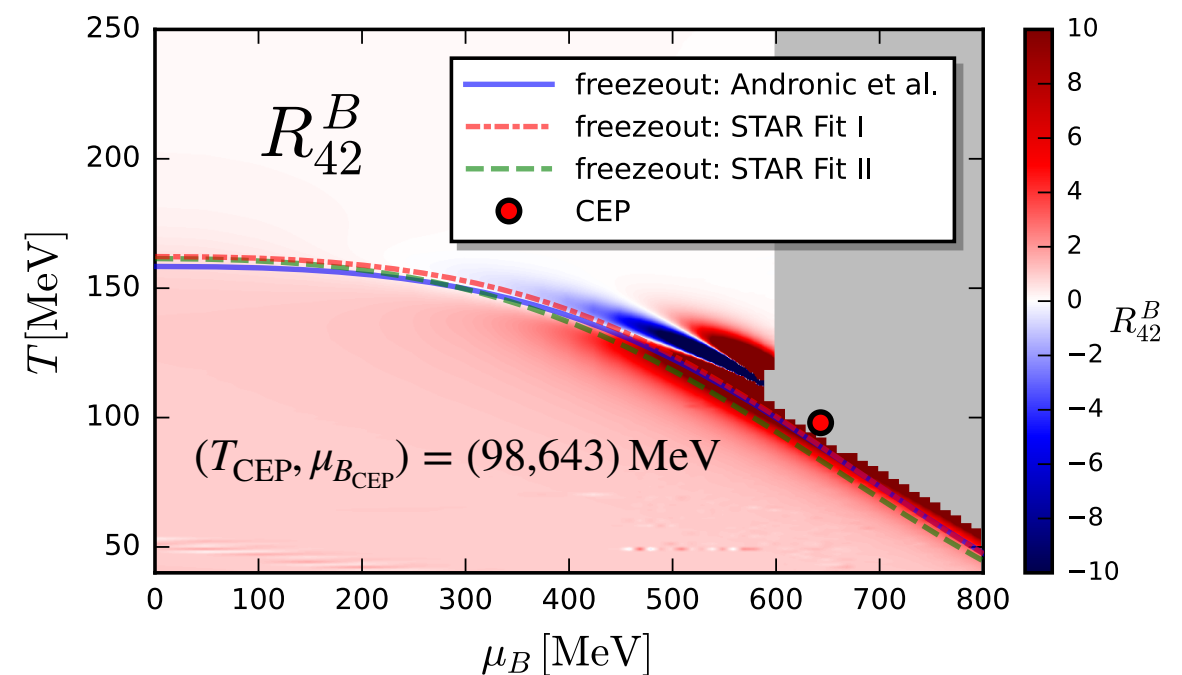
- In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are **convergent** and **consistent**.

Canonical fluctuations at the freeze-out



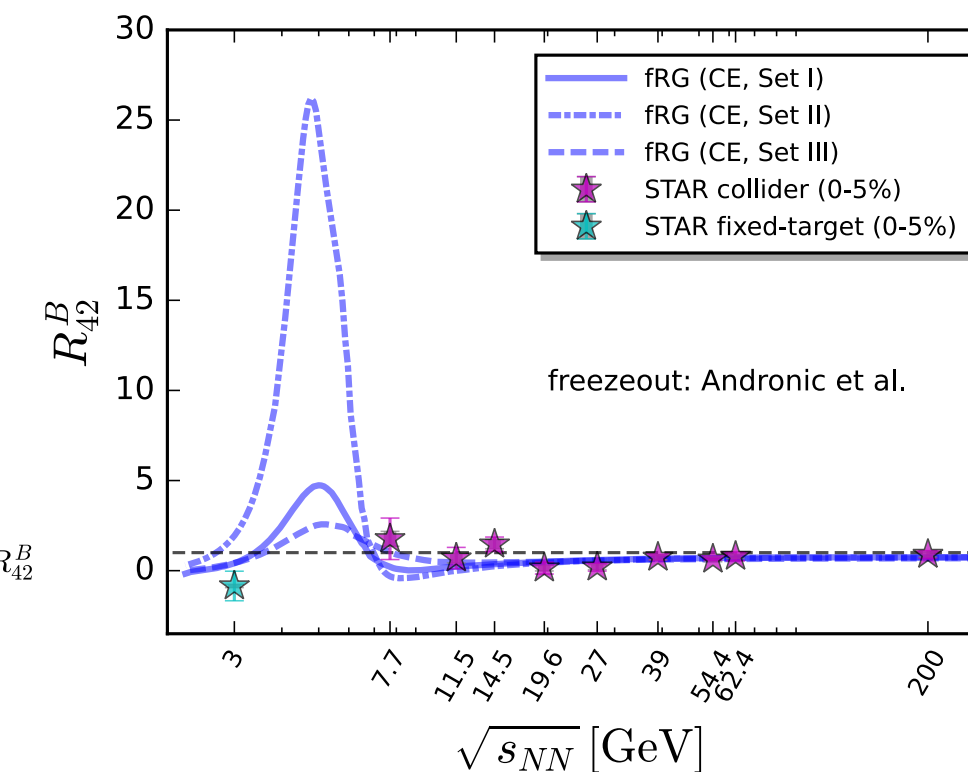
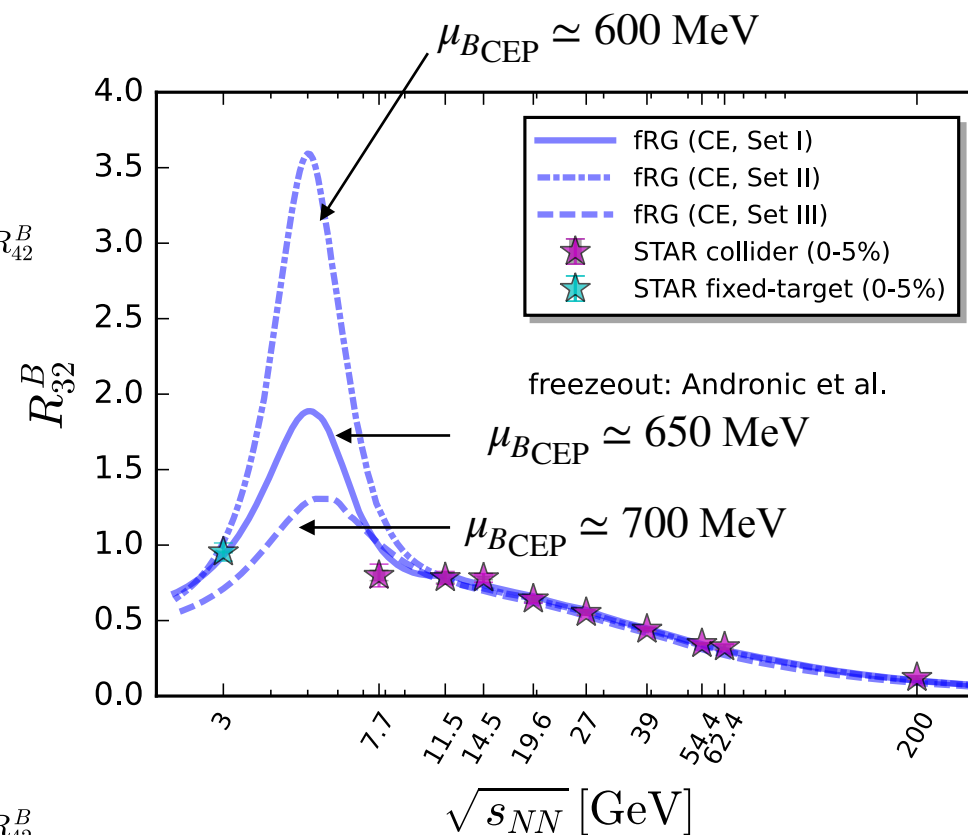
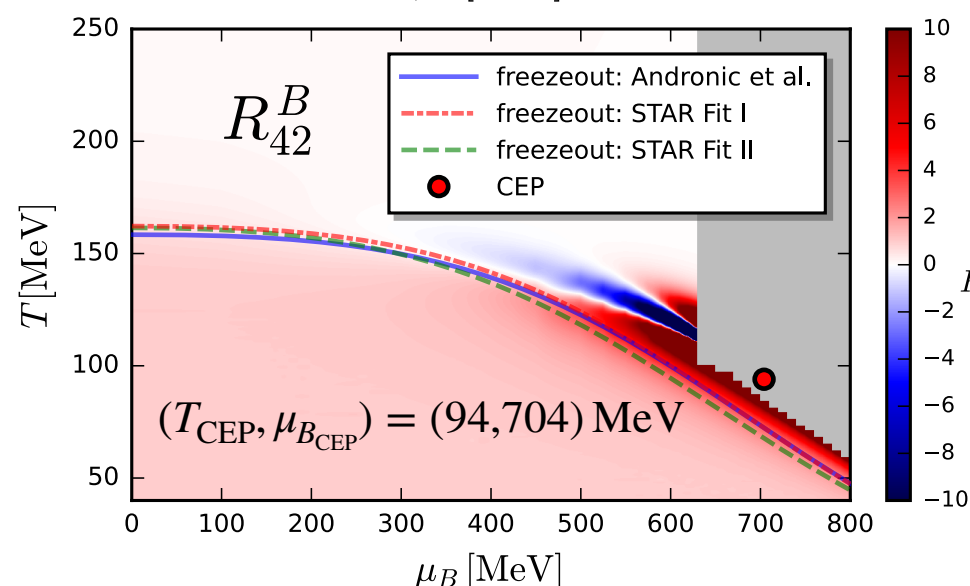
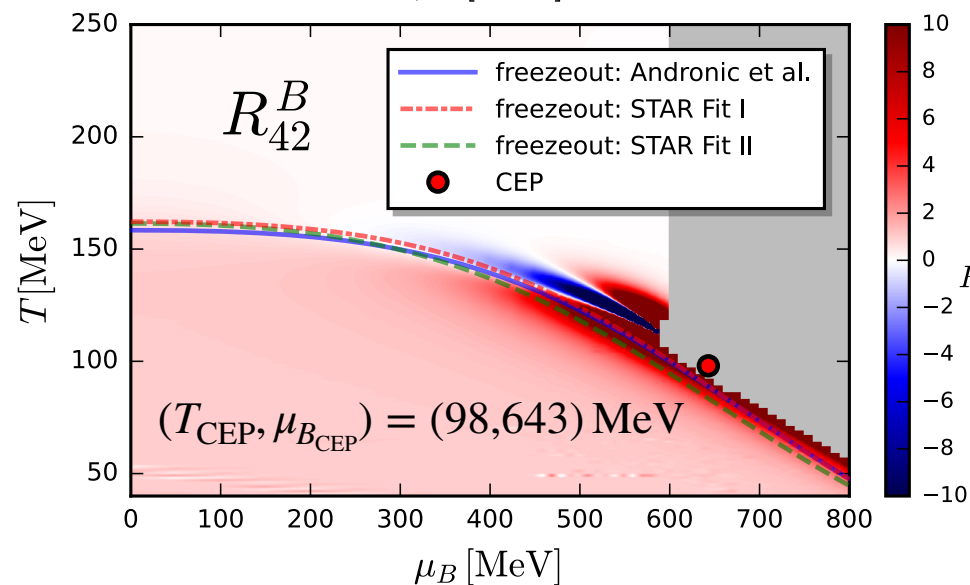
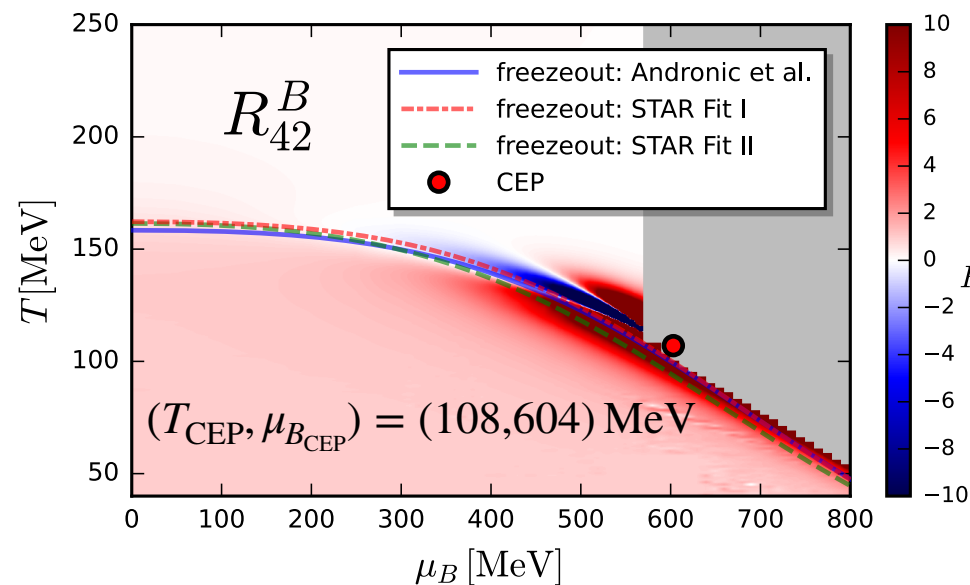
fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508

STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303;
Aboona *et al.* (STAR), *PRL* 130 (2023) 082301



- Peak structure is found in 3 GeV $\lesssim \sqrt{s_{NN}} \lesssim 7.7$ GeV.
- Position of peak in R_{42} is $\mu_{B_{\text{peak}}} = 536, 541$ and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{\text{CEP}}} = 643$ MeV.

Dependence on the location of CEP



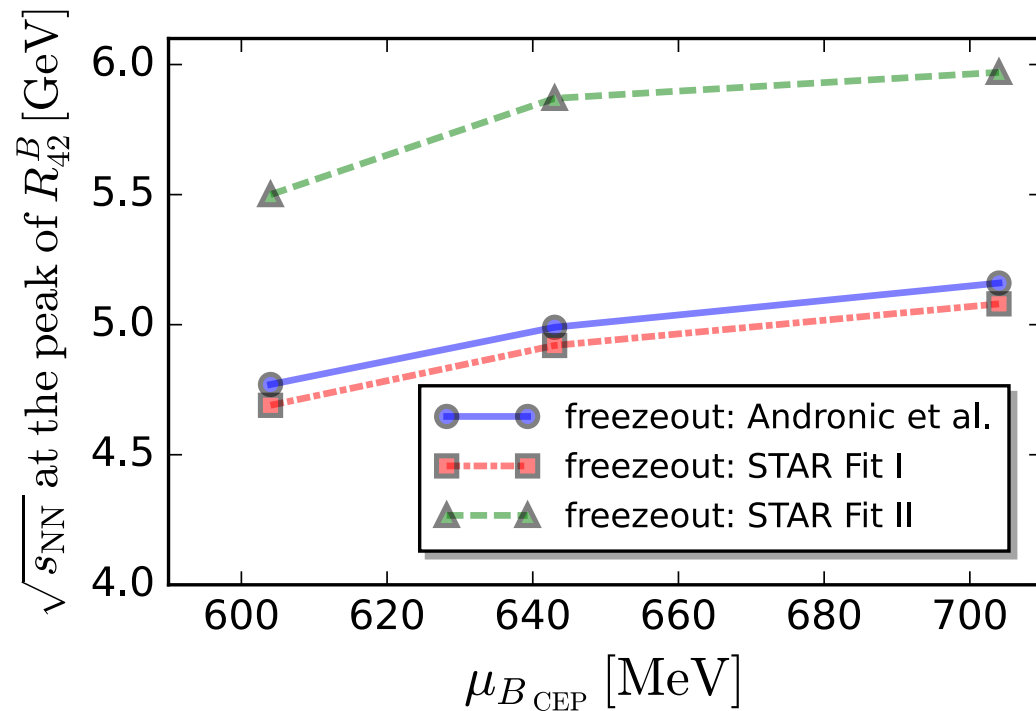
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301.

fRG: WF, Luo, Pawłowski, Rennecke, Yin, *PRD* 111 (2025) L031502.

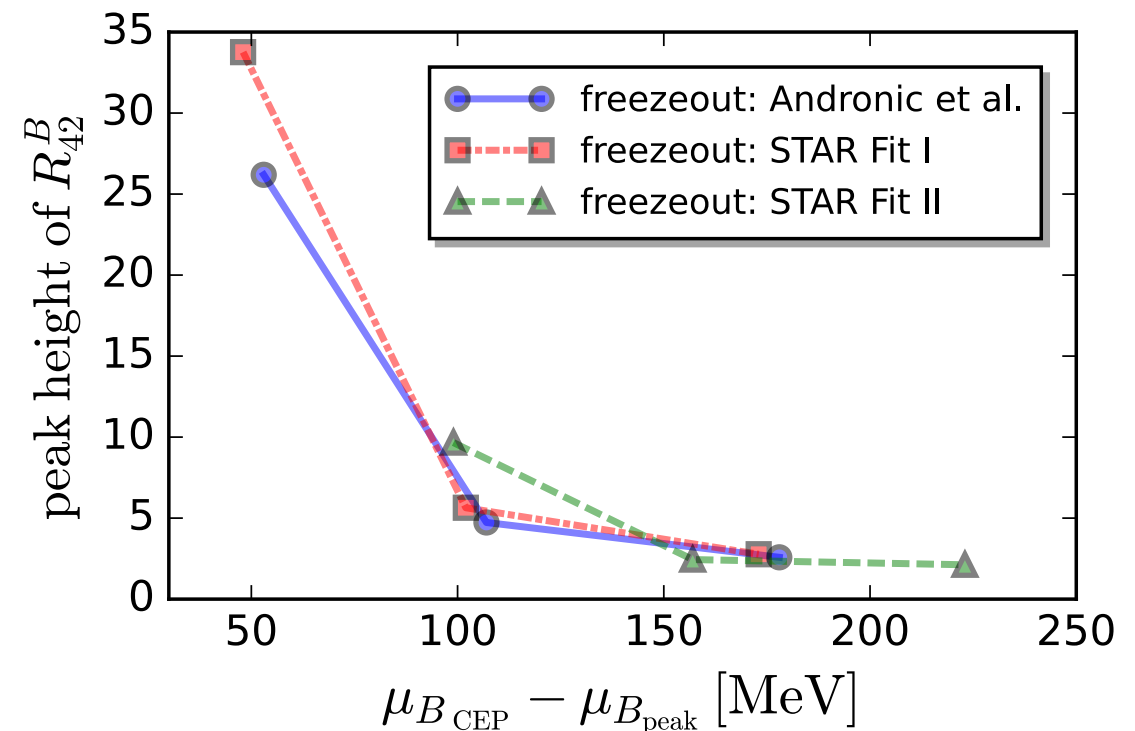
- **Position** of the peak is **insensitive** to the location of CEP.
- **Height** of peak **decreases** as CEP moves towards larger μ_B .

Ripples of the QCD critical point

Position of peak:



Height of peak:



IRG: WF, Luo, Pawłowski, Rennecke, Yin, *PRD* 111 (2025) L031502.

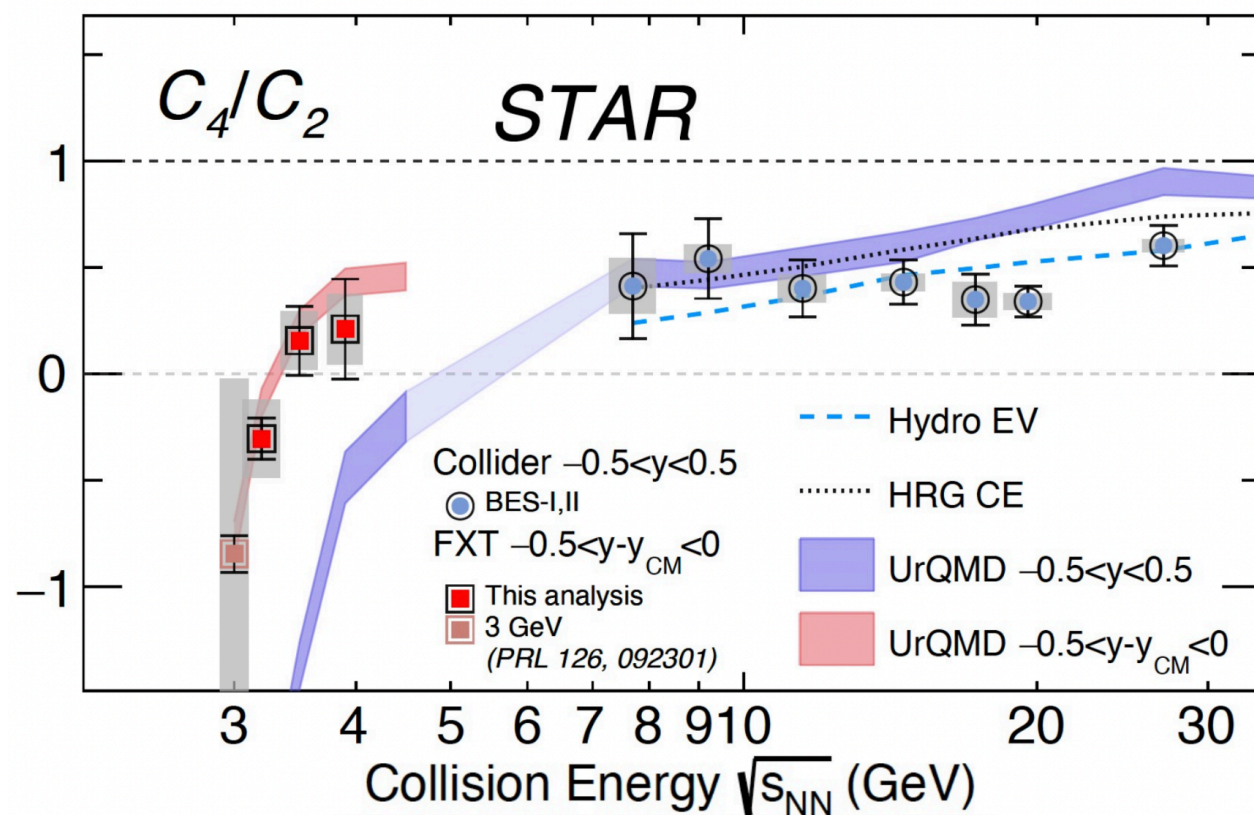


- Note that the ripples of CEP are far away from the critical region characterized by the universal scaling properties, e.g., the critical slowing down.
- But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.

C₄/C₂: Comparison to STAR data

FXT energies at 3.2, 3.5, 3.9 GeV:

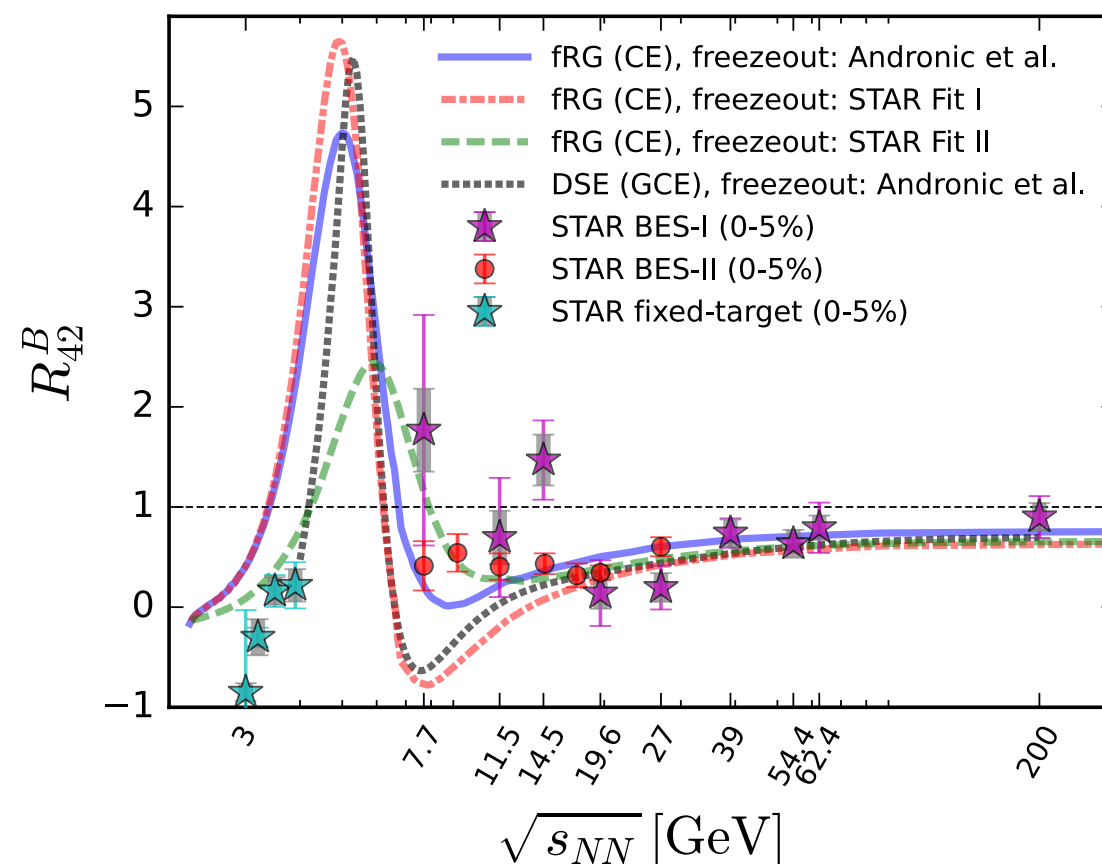
0-5% Au+Au Collisions at RHIC



STAR: Z. Sweger, Quark Matter 2025

STAR: arXiv:2504.00817

Net baryon (proton) number kurtosis:



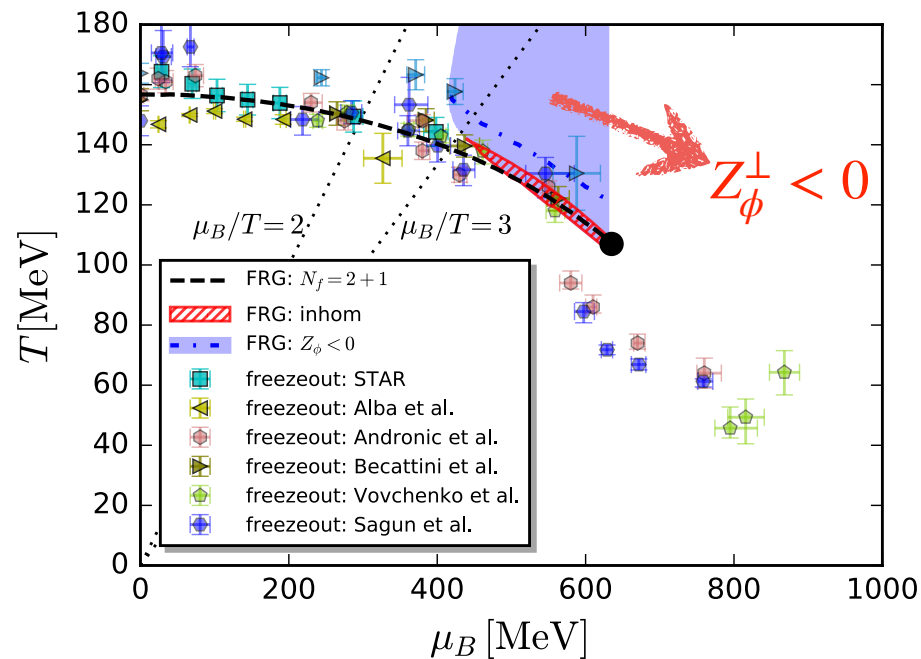
fRG: WF, Luo, Pawłowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508

DSE: Lu, Gao, Liu, Pawłowski, arXiv: 2504.05099

- Theoretical prediction with critical fluctuations (fRG and DSE) is consistent with STAR data.
- A peak structure is predicted in the energy regime of fixed-target experiments, i.e. $3 \text{ GeV} \lesssim \sqrt{s_{NN}} \lesssim 7.7 \text{ GeV}$. Experimental search of this peak is very important.

Moat regime in QCD phase diagram

Moat (护城河) regime:



Indicated in QCD calculations:

WF, Pawłowski, Rennecke, *PRD* 101 (2020) 054032.



See the talk by Gao-qing Cao in parallel I on Sun

Mesonic two-point correlation function:

$$\Gamma_{\phi\phi}^{(2)}(p) = [Z_\phi^\parallel(p_0, \mathbf{p}) p_0^2 + Z_\phi^\perp(p_0, \mathbf{p}) \mathbf{p}^2] + m_\phi^2$$

with

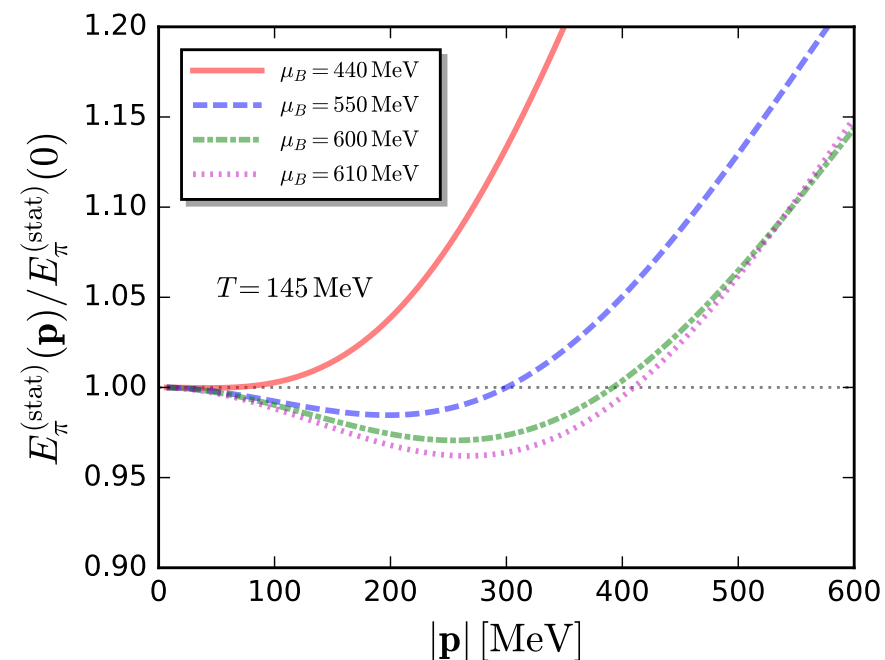
$$\Gamma_{\phi\phi,k}^{(2)} = \left. \frac{\delta^2 \Gamma_k[\Phi]}{\delta\phi\delta\phi} \right|_{\Phi=\Phi_{\text{EoM}}}$$

Phenomenology:

Pisarski, Rennecke, *PRL* 127 (2021) 152302;

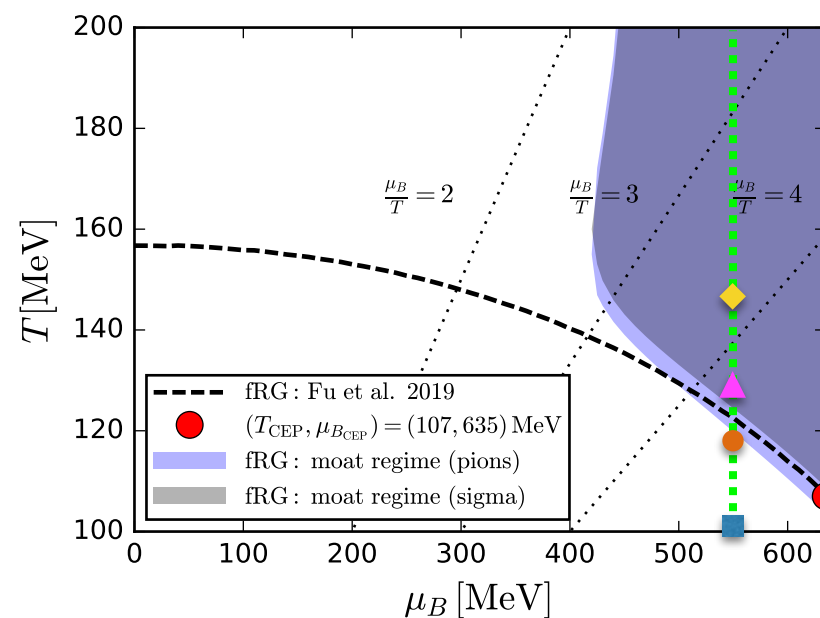
Rennecke, Pisarski, Rischke, *PRD* 107 (2023) 116011.

Dispersion relation:



WF, Pawłowski, Pisarski, Rennecke, Wen, Yin, *PRD* 111 (2025) 094026.

Spectral functions in moat regime

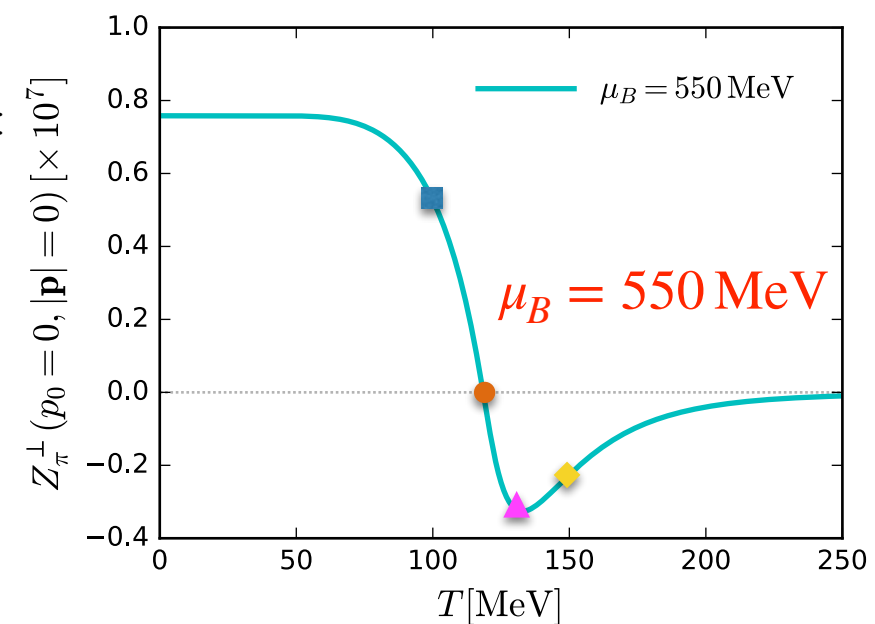


Analytic continuation on the flow equation:

$$G_\phi^R(\omega, \vec{p}) = \lim_{\epsilon \rightarrow 0^+} G_\phi(-i(\omega + i\epsilon), \vec{p})$$

Spectral functions of mesons:

$$\rho_\phi(\omega, \vec{p}) = \frac{1}{\pi} \text{Im} G_\phi^R(\omega, \vec{p})$$

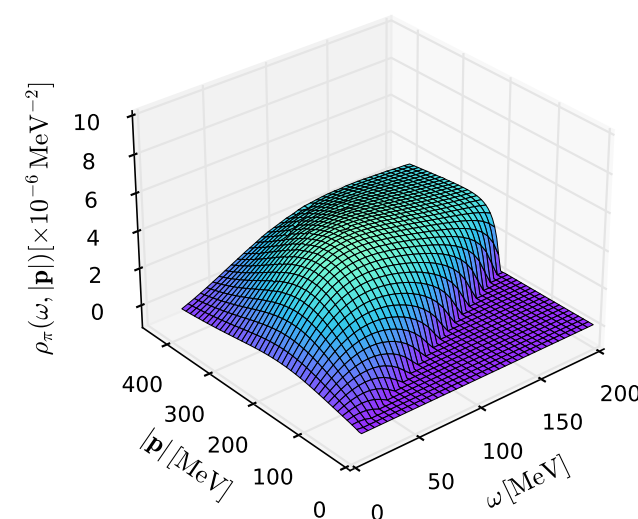
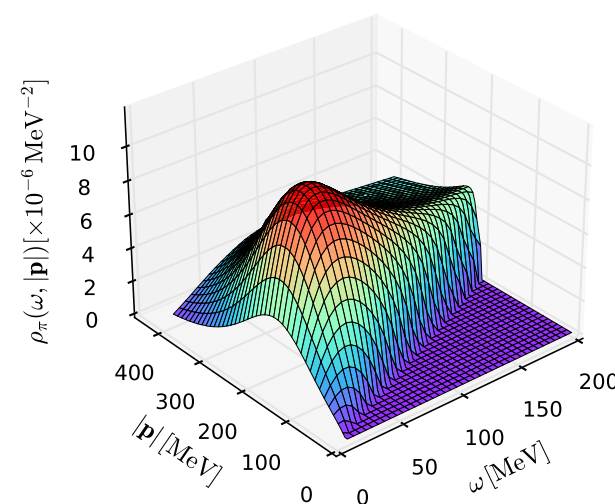
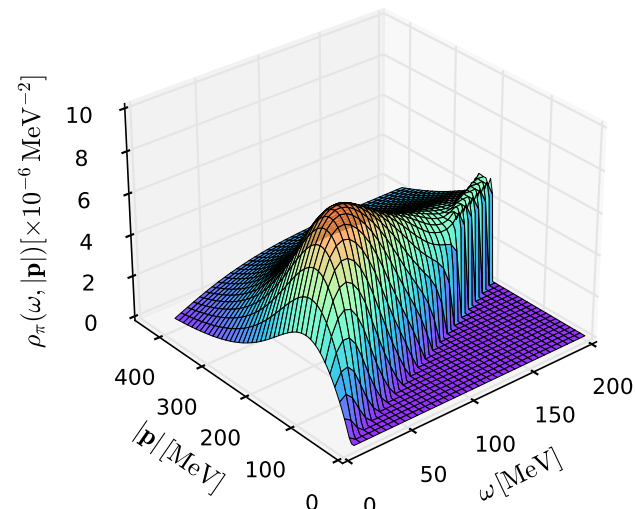
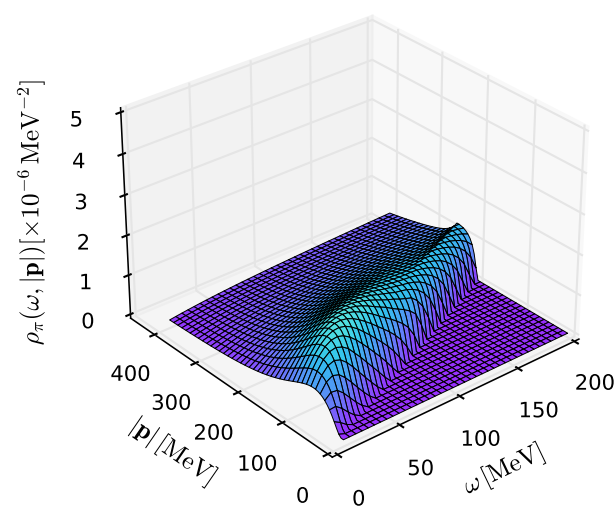


■ $T = 100$ MeV

● $T = 120$ MeV

▲ $T = 130$ MeV

◆ $T = 150$ MeV

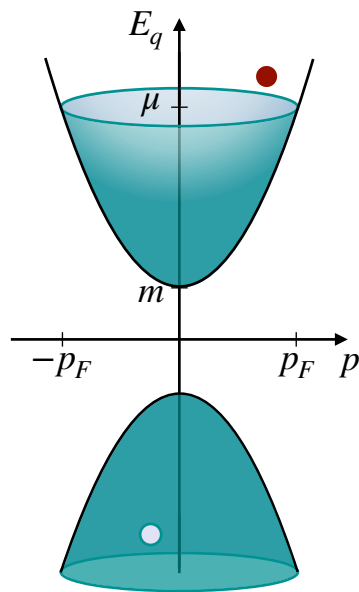


WF, Pawłowski, Pisarski, Rennecke, Wen, Yin, *PRD* 111 (2025) 094026.

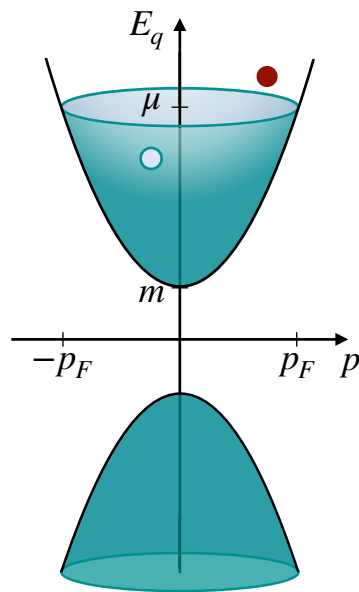
See the talk by Jing Wu in parallel I on Mon

Landau damping and moat regime

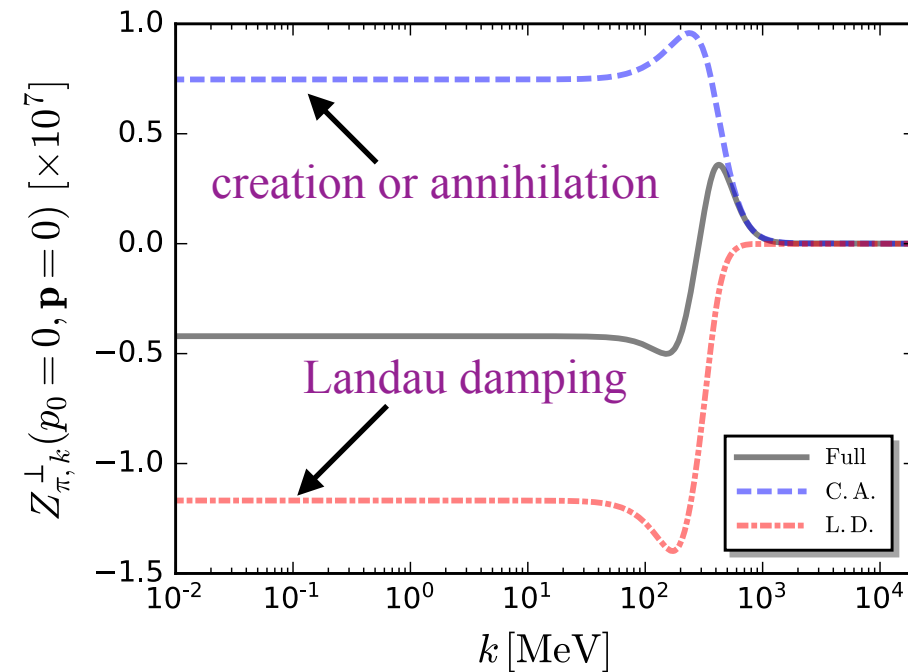
Creation-annihilation process:



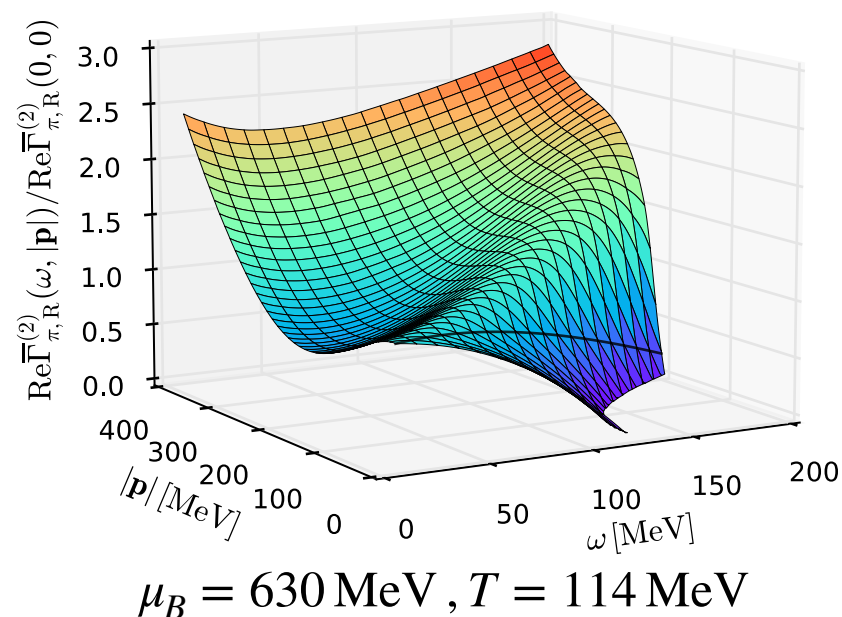
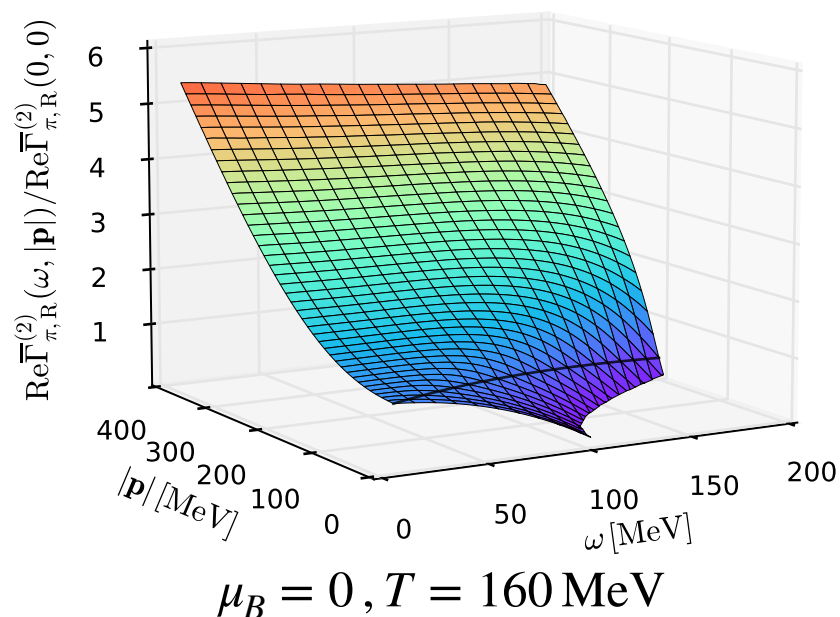
Landau damping (particle-hole) process:



Contributions from two processes:



Real part of $\Gamma_{\phi\phi,R}^{(2)}(p_0, \mathbf{p})$:



- Moat regime is found to be resulted from **Landau damping** of quarks in thermal bath in the regime of large baryon chemical potential.

WF, Pawłowski, Pisarski, Rennecke, Wen, Yin, *PRD* 111 (2025) 094026.

A relaxation critical $O(N)$ model

- The effective action on the Schwinger-Keldysh contour reads Hohenberg and Halperin, *Rev. Mod. Phys.* 49 (1977) 435.

Model A

$$\Gamma[\phi_c, \phi_q] = \int d^4x \left(Z_a^{(t)} \phi_{a,q} \partial_t \phi_{a,c} - Z_a^{(i)} \phi_{a,q} \partial_i^2 \phi_{a,c} + V'(\rho_c) \phi_{a,q} \phi_{a,c} - 2 Z_a^{(t)} T \phi_{a,q}^2 - \sqrt{2} c \sigma_q \right)$$

$\Gamma = 1/Z_a^{(t)}$: relaxation rate

$V'(\rho_c)$: potential $\rho_c \equiv \phi_c^2/4$

Gaussian white noise with coefficient determined by fluctuation-dissipation theorem

$Z_a^{(i)}$: wave function

c : explicit breaking

- Retarded propagator

Pseudo-Goldstone:

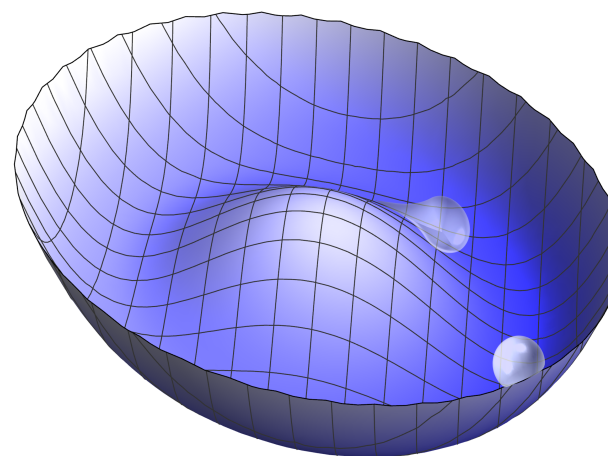
Mass of pseudo-Goldstone

$$G_{ab}^R = \left(\frac{\delta^2 \Gamma[\phi_c, \phi_q]}{\delta \phi_{a,q} \delta \phi_{b,c}} \right)^{-1}$$

$$m_\varphi^2 = \frac{V'(\rho_0)}{Z_\varphi^{(i)}} = \frac{c}{\sigma_0 Z_\varphi^{(i)}}$$

Retarded propagator of Goldstone

$$G_{\varphi\varphi}^R(\omega, q) = \frac{1}{-i Z_\varphi^{(t)} \omega + Z_\varphi^{(i)} (q^2 + m_\varphi^2)}$$



Gell-Mann--Oakes--Renner (GMOR) relation

Emergence of a novel universal damping in the critical region

In the critical region, the two wave function renormalizations read

$$Z_\varphi^{(i)} = t^{-\nu\eta} f^{(i)}(z), \quad Z_\varphi^{(t)} = t^{-\nu\eta_t} f^{(t)}(z)$$

Here $f^{(i)}(z), f^{(t)}(z)$: scaling functions; $z \equiv t c^{-1/(\beta\delta)}$: scaling variable; $t \equiv (T_c - T)/T_c$: reduced temperature. The static and dynamic anomalous dimensions are

$$\eta = -\frac{\partial_\tau Z_\varphi^{(i)}}{Z_\varphi^{(i)}}, \quad \eta_t = -\frac{\partial_\tau Z_\varphi^{(t)}}{Z_\varphi^{(t)}}$$

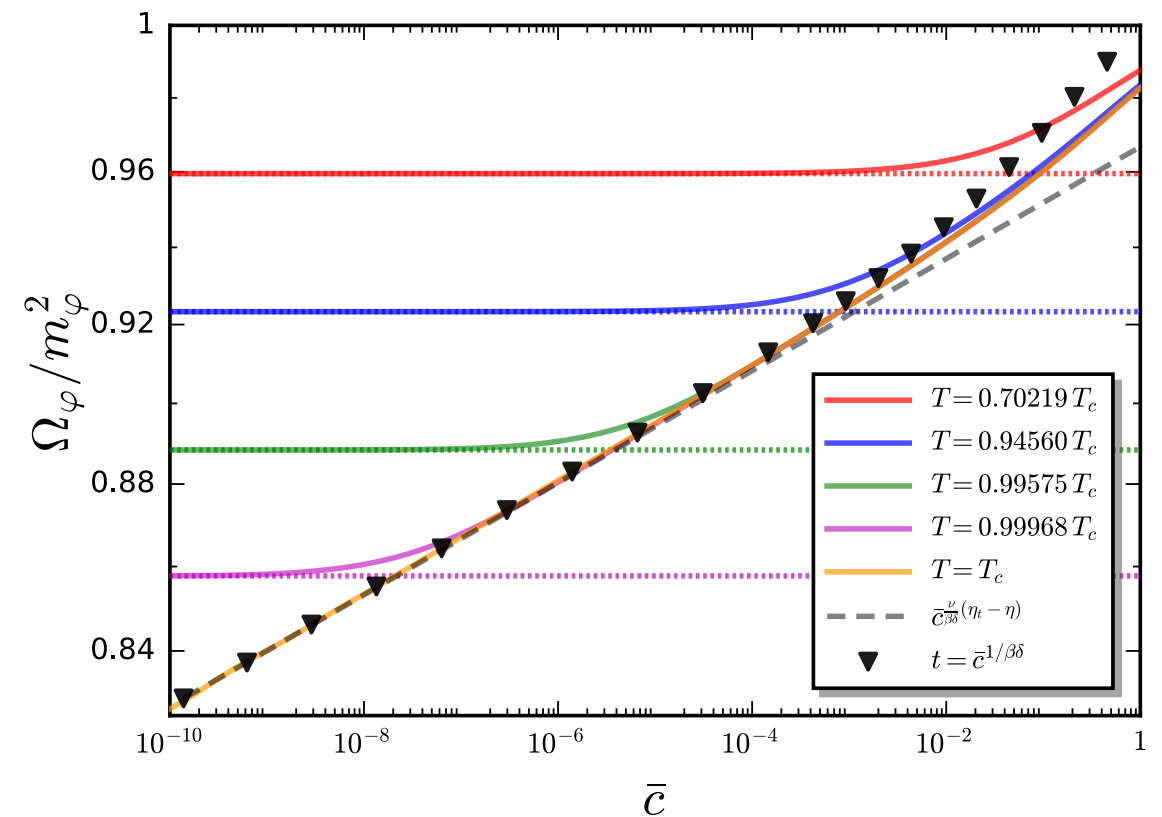
RG time $\tau = \ln(k/\Lambda)$

- In the case of $c \rightarrow 0$

$$\frac{Z_\varphi^{(i)}}{Z_\varphi^{(t)}} \propto t^{\nu(\eta_t - \eta)}$$

- In the other case of $t \rightarrow 0$

$$\frac{Z_\varphi^{(i)}}{Z_\varphi^{(t)}} \propto c^{\frac{\nu}{\beta\delta}(\eta_t - \eta)} \propto m_\varphi^{(\eta_t - \eta)} \quad \text{with} \quad m_\varphi^2 \propto c^{\frac{2\nu}{\beta\delta}}$$



Tan, Chen, WF, Li, *Nature Commun.* 16 (2025) 2916, arXiv: 2403.03503

From the fixed-point equation we determine in the O(4) symmetry

$$\eta \approx 0.0374, \quad \eta_t \approx 0.0546$$

Thus

$$\Delta_\eta \equiv \eta_t - \eta \approx 0.0172$$

Estimate of size of the dynamic critical region:

$$m_{\pi 0} \lesssim 0.1 \sim 1 \text{ MeV}$$

Relaxation dynamics of the critical mode

- Langevin dynamics of the critical mode:

$$Z_{\phi}^{(t)} \partial_t \sigma - Z_{\phi}^{(i)} \partial_i^2 \sigma + U'(\sigma) = \xi$$

with the correlation of the Gaussian white noise

$$\langle \xi(t, \mathbf{x}) \xi(t', \mathbf{x}') \rangle = 2 Z_{\phi}^{(t)} T \delta(t - t') \delta(\mathbf{x} - \mathbf{x}')$$

- Inputs from first-principles functional QCD: [WF, Pawłowski, Rennecke, *PRD* 101 \(2020\) 054032](#)

Effective potential:

$$U'(\sigma) = \left. \frac{\delta \Gamma[\Phi]}{\delta \sigma} \right|_{\substack{\sigma(x) = \sigma \\ \tilde{\Phi} = \tilde{\Phi}_{\text{EoM}}}}$$

Spatial wave function:

$$Z_{\phi}^{(i)} = \left. \frac{\partial \Gamma_{\sigma\sigma}^{(2)}(p_0, \mathbf{p})}{\partial \mathbf{p}^2} \right|_{\substack{p_0 = 0 \\ \mathbf{p} = 0}}$$

Temporal wave function:

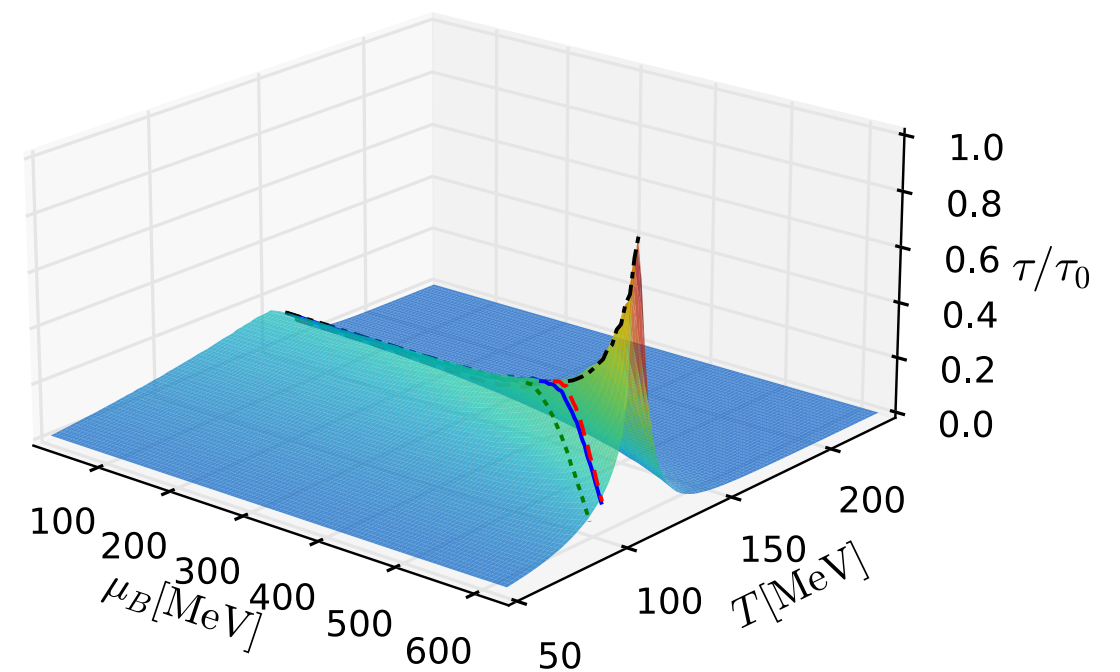
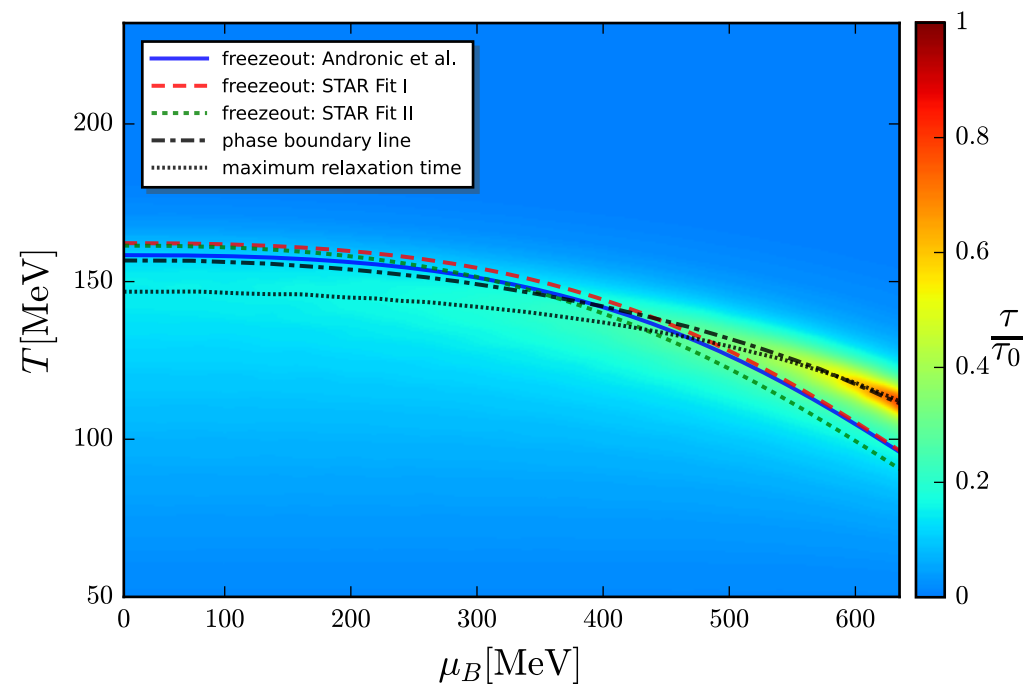
$$Z_{\phi}^{(t)} = \lim_{|\mathbf{p}| \rightarrow 0} \lim_{\omega \rightarrow 0} \frac{\partial}{\partial \omega} \text{Im} \Gamma_{\sigma\sigma, \text{R}}^{(2)}(\omega, \mathbf{p})$$

with

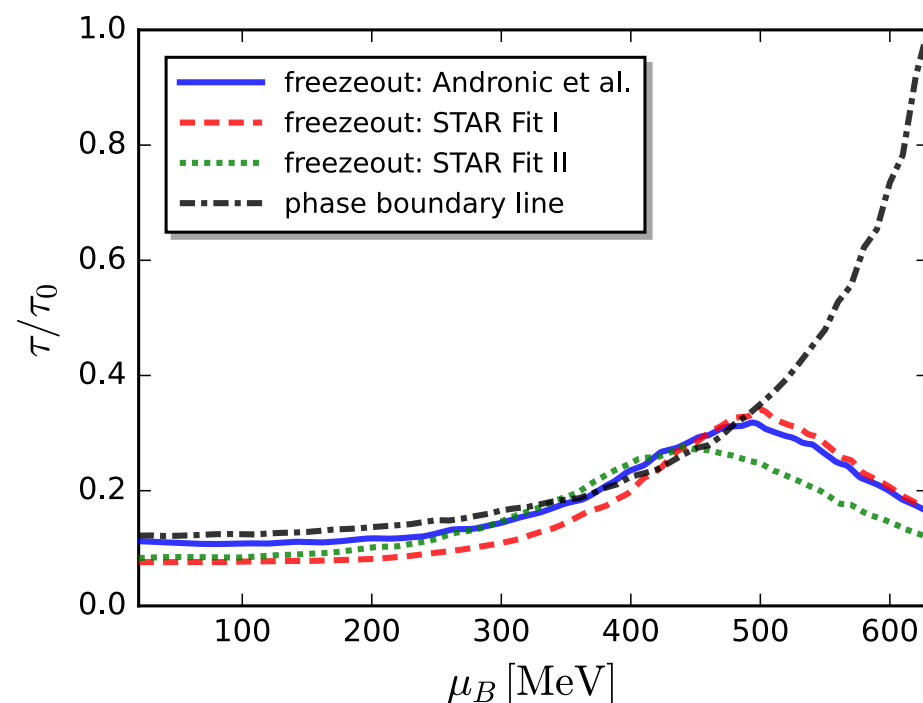
$$\Gamma_{\sigma\sigma, \text{R}}^{(2)}(\omega, \mathbf{p}) = \lim_{\epsilon \rightarrow 0^+} \Gamma_{\sigma\sigma}^{(2)}(p_0 = -i(\omega + i\epsilon), \mathbf{p})$$

Relaxation time in QCD phase diagram

Relaxation time:



Relaxation time at the freezeout :



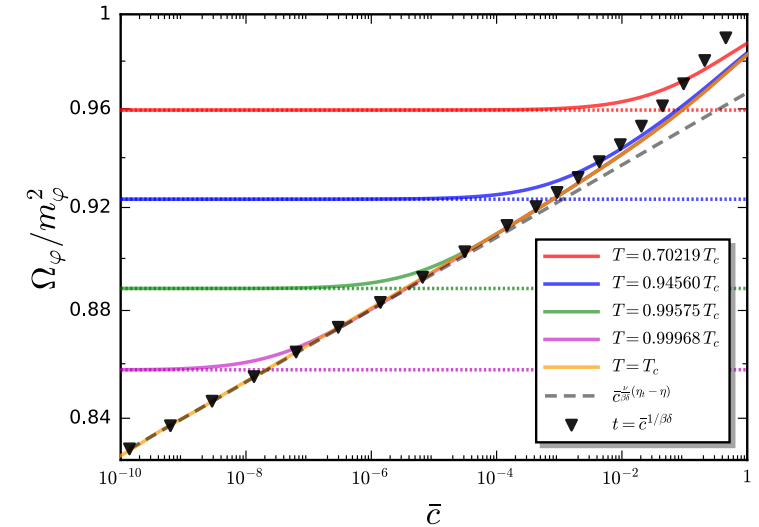
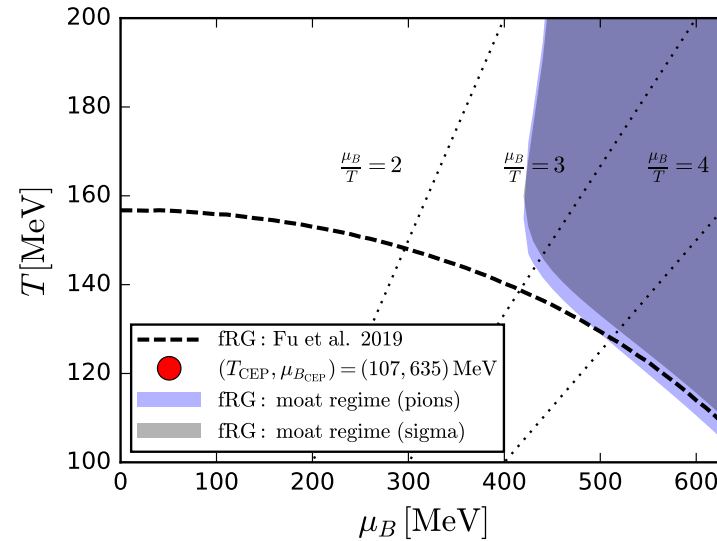
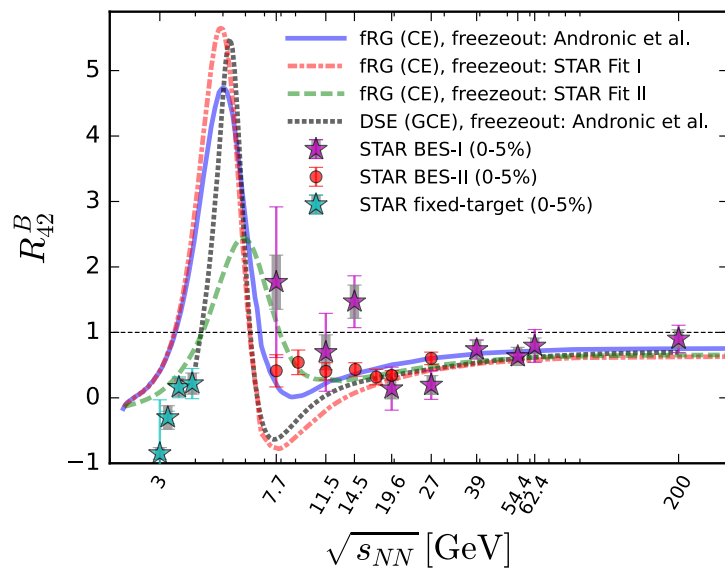
Tan, Yin, Chen, Huang, WF, in preparation

See also:

M. Bluhm *et al.*, *NPA* 982 (2019) 871

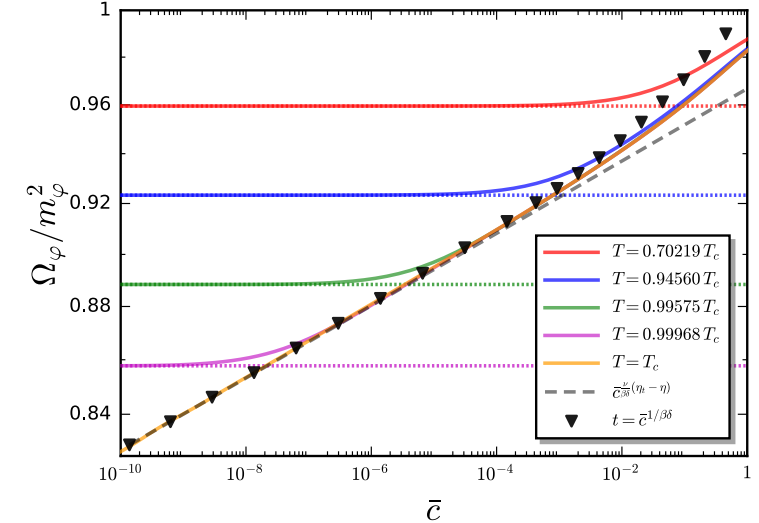
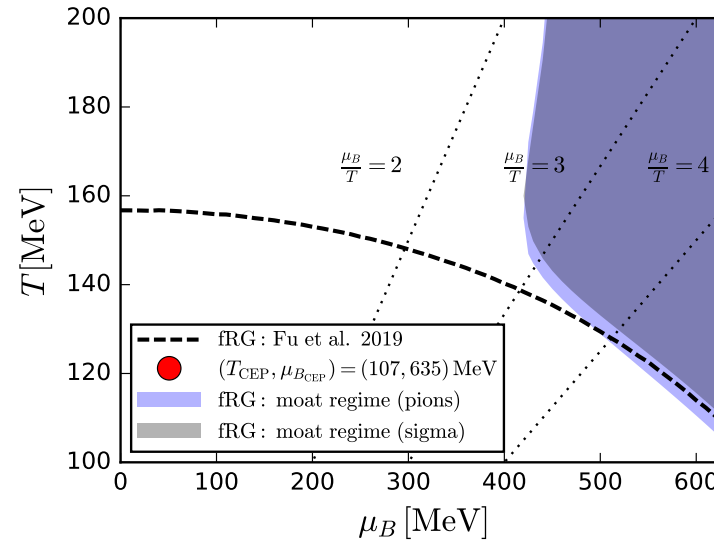
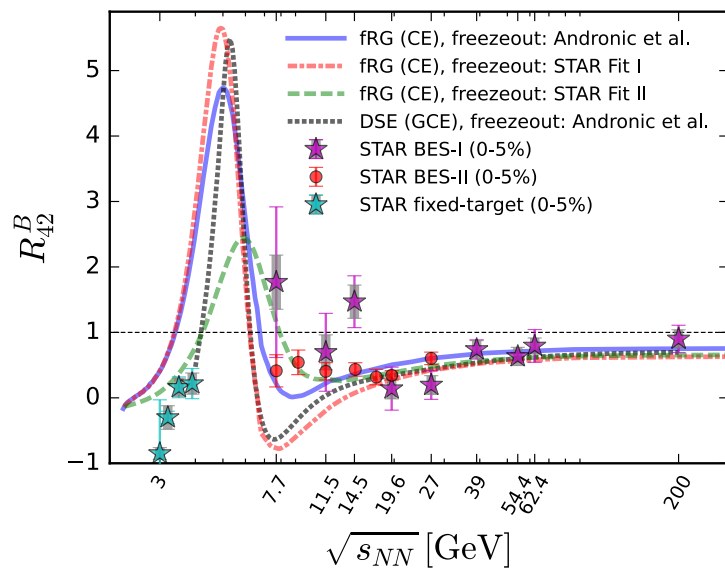
- Relaxation time **drops quickly** once the system is away from the critical regime.

Summary and outlook



- ★ A prominent peak structure is predicted in baryon number fluctuations in the collision energy range of $3 \text{ GeV} \lesssim \sqrt{s_{NN}} \lesssim 7.7 \text{ GeV}$, which need to be confirmed in experiments in the near future.
- ★ Moat regime is found to be resulted from Landau damping of quarks in thermal bath in the regime of large baryon chemical potential.
- ★ A novel universal damping in the critical region is found.

Summary and outlook



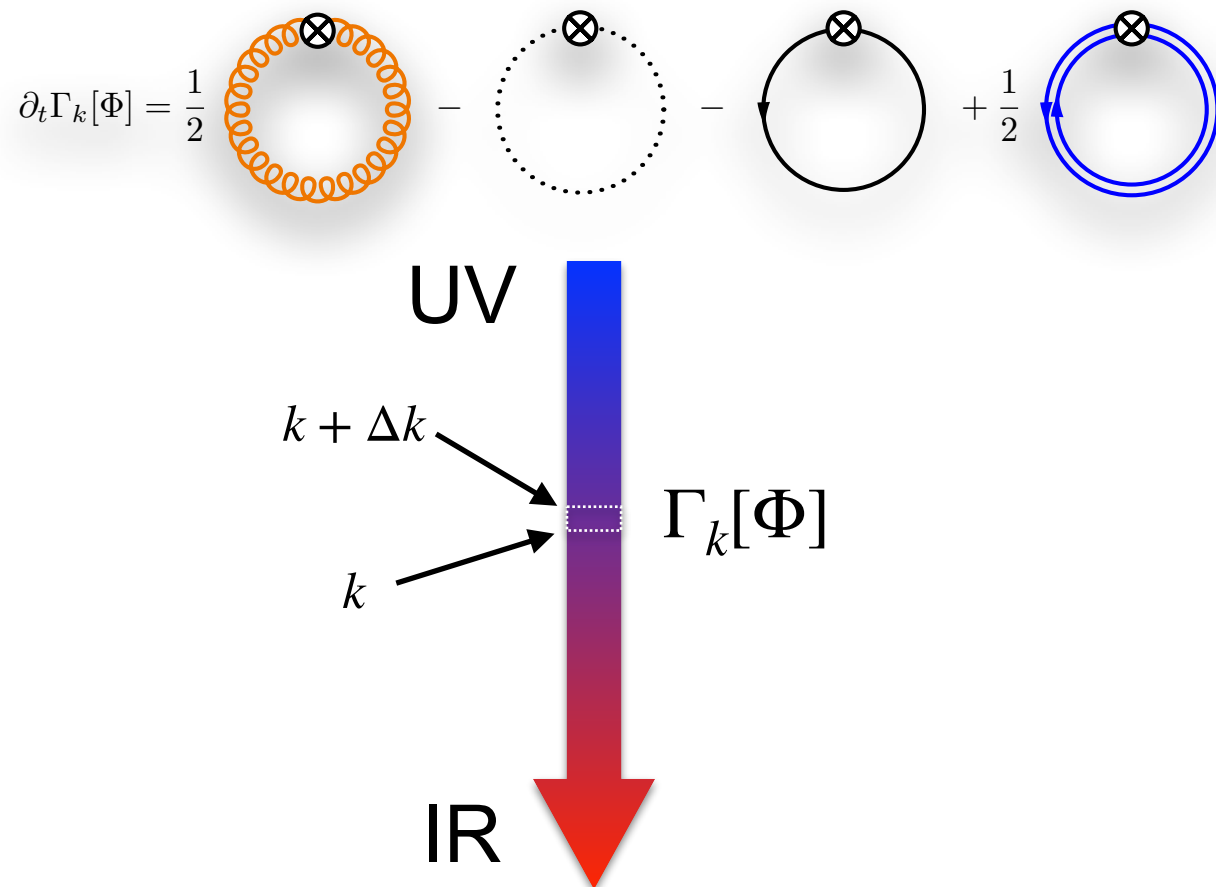
- ★ A prominent peak structure is predicted in baryon number fluctuations in the collision energy range of $3 \text{ GeV} \lesssim \sqrt{s_{NN}} \lesssim 7.7 \text{ GeV}$, which need to be confirmed in experiments in the near future.
- ★ Moat regime is found to be resulted from Landau damping of quarks in thermal bath in the regime of large baryon chemical potential.
- ★ A novel universal damping in the critical region is found.

Thank you very much for your attentions!

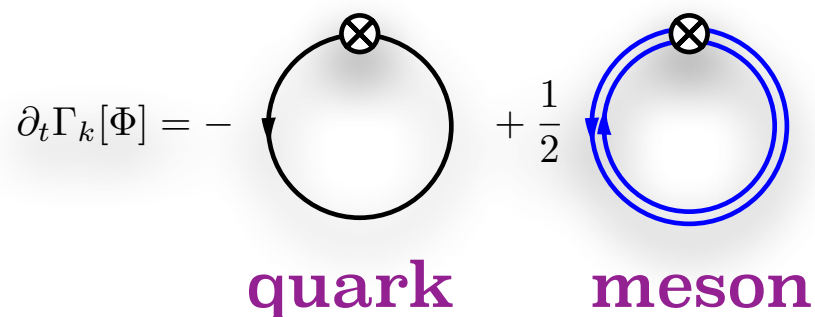
Backup

QCD-assisted LEFT

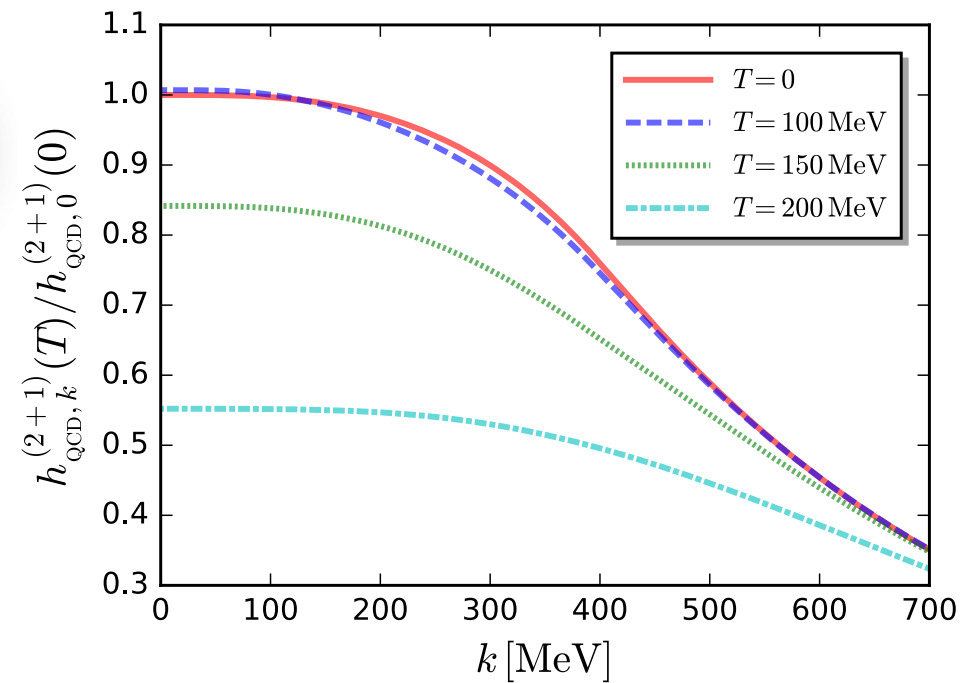
QCD flow equation:



LEFT flow equation:

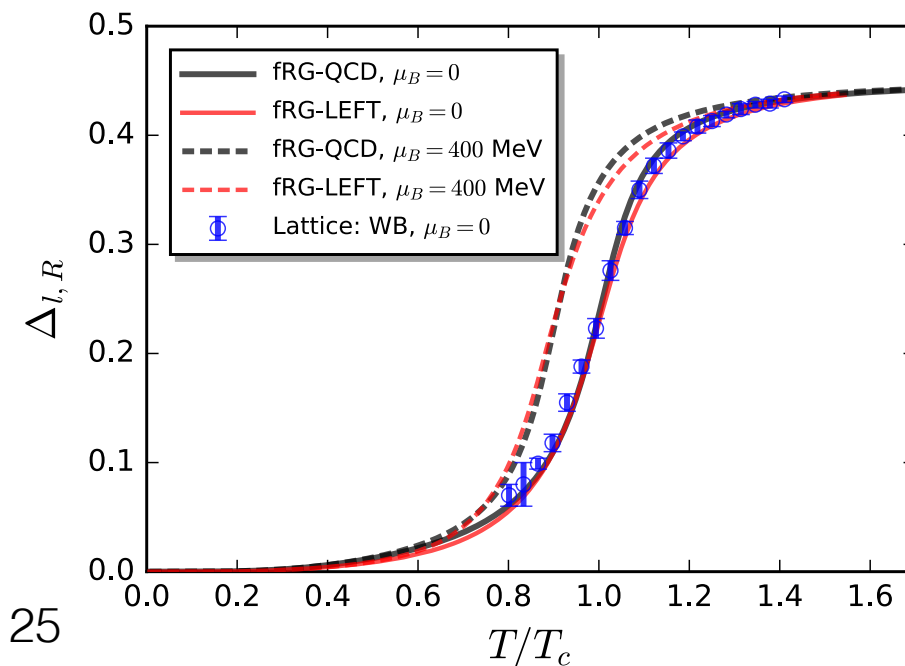


- Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT



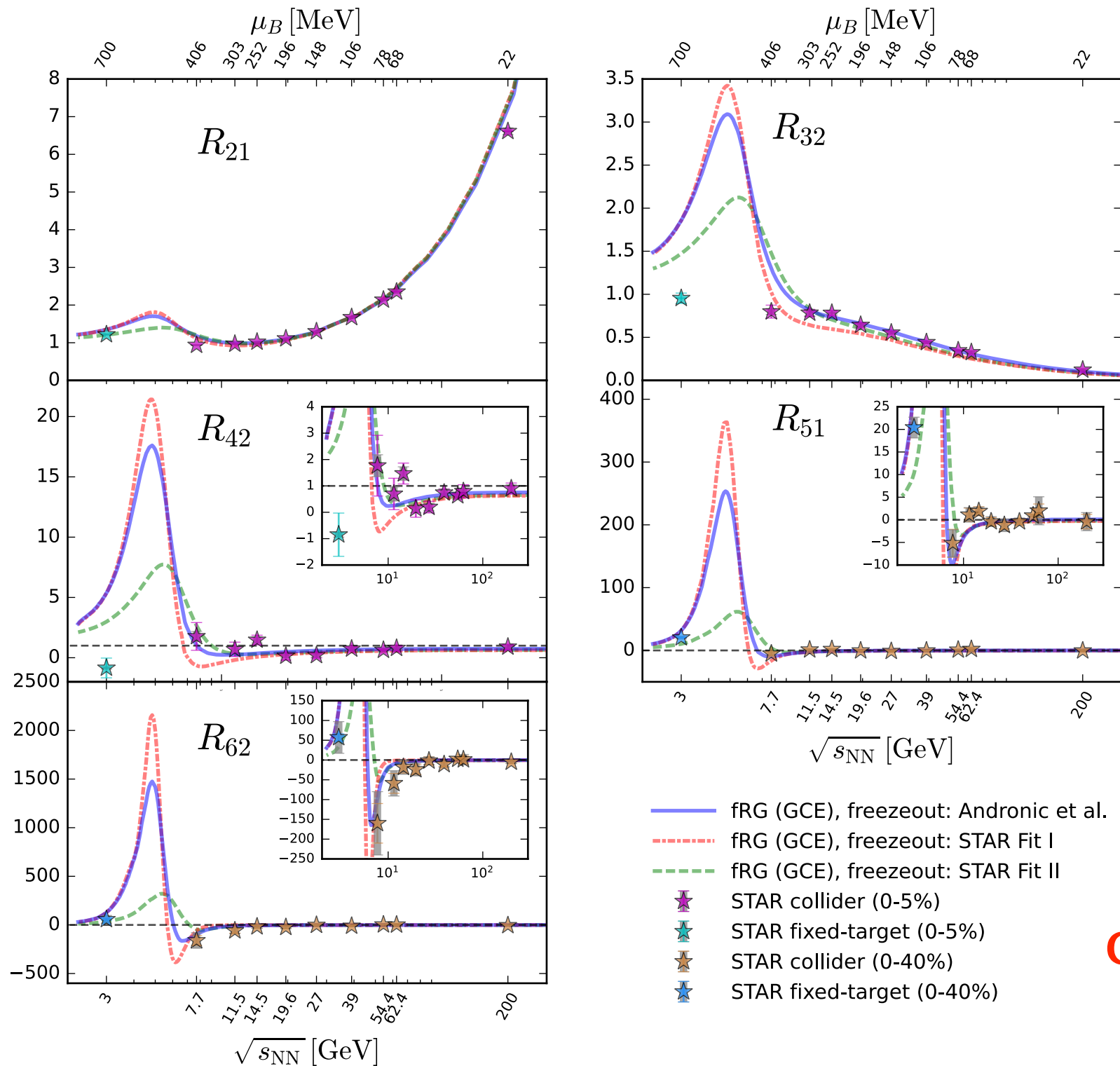
WF, Pawłowski, Rennecke, *PRD* 101 (2020) 054032

- Chiral condensates in QCD and QCD-assisted LEFT in agreement



WF, Luo, Pawłowski, Rennecke, Yin, arXiv: 2308.15508

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
 Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303;
 Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

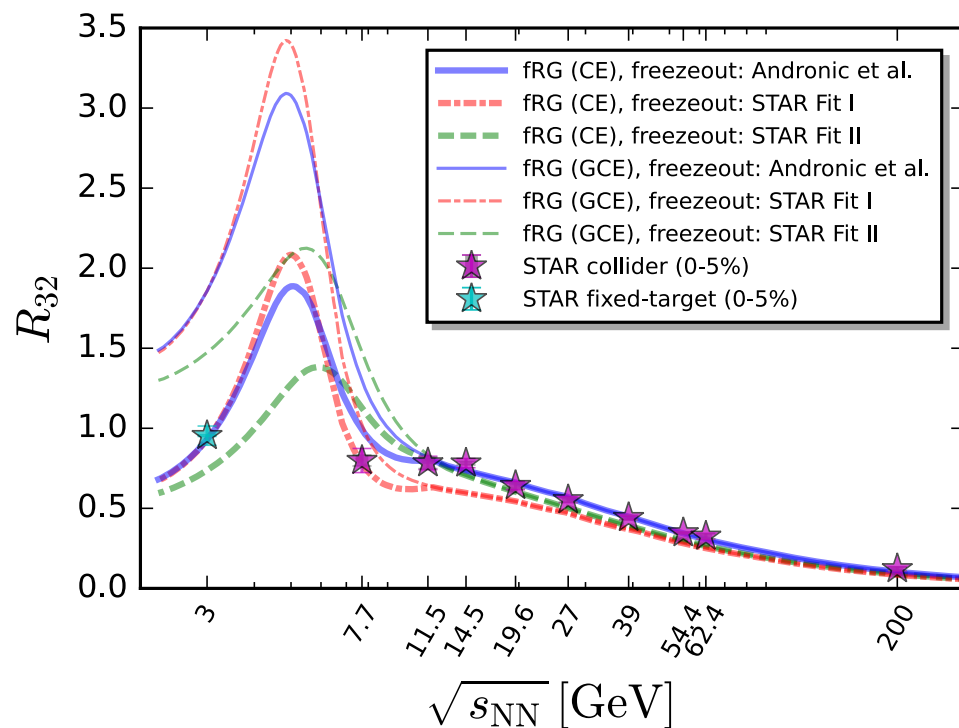
fRG: WF, Luo, Pawłowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508

- Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643)$ MeV.
- Peak structure is found in 3 GeV $\lesssim \sqrt{s_{NN}} \lesssim 7.7$ GeV.
- Agreement between the theory and experiment is worsening with $\sqrt{s_{NN}} \lesssim 11.5$ GeV.
- Effects of global baryon number conservation in the regime of low collision energy should be taken into account.

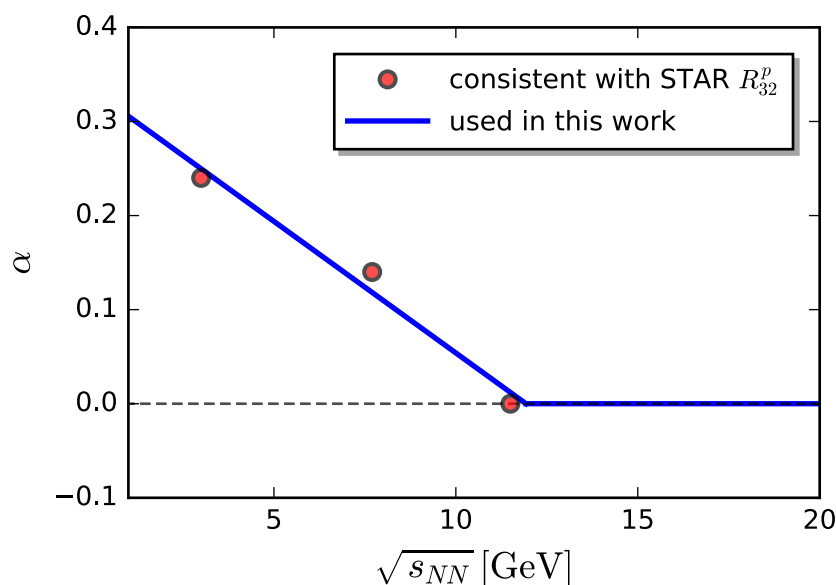
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{NN}} \lesssim 11.5$ GeV.
- We choose the simplest linear dependence



$$\alpha(\bar{s}) = a \left(1 - \sqrt{\bar{s}} \right) \theta(1 - \bar{s})$$

$$a = 0.33, \quad \sqrt{\bar{s}} = \frac{\sqrt{s_{NN}}}{11.9 \text{ GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1 : the subensemble volume measured in the acceptance window, V : the volume of the whole system.

- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

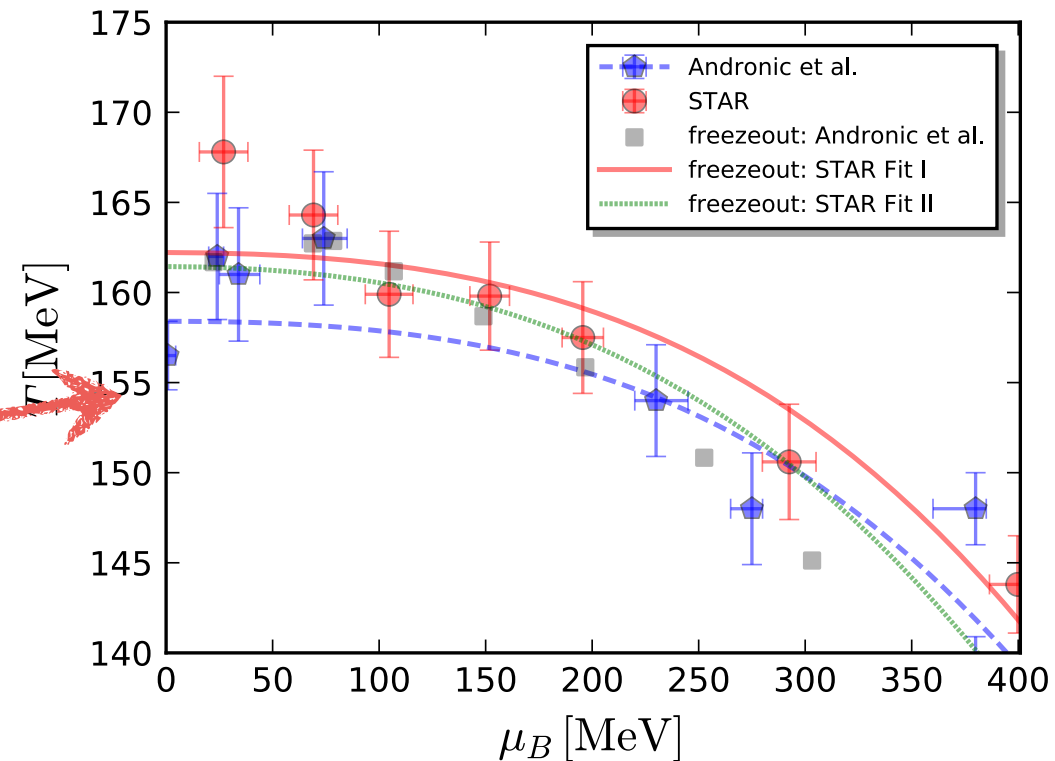
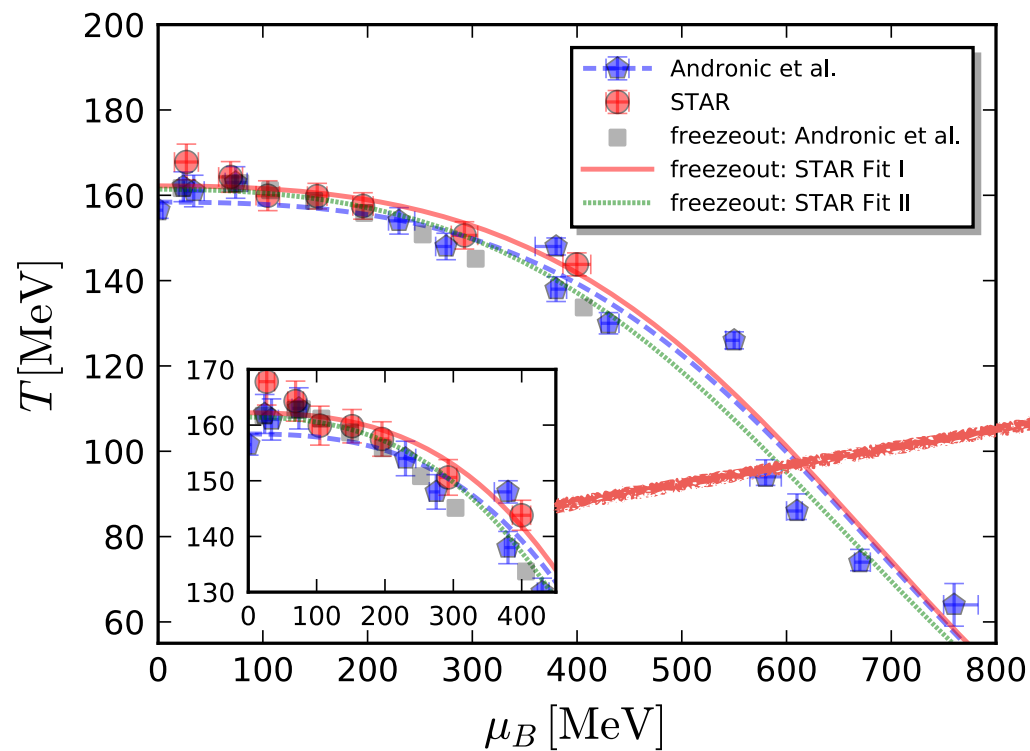
$$\bar{R}_{21}^B = \beta R_{21}^B, \quad \bar{R}_{32}^B = (1 - 2\alpha) R_{32}^B,$$

$$\bar{R}_{42}^B = (1 - 3\alpha\beta) R_{42}^B - 3\alpha\beta (R_{32}^B)^2$$

$$\beta = 1 - \alpha$$

SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, *PLB* 811 (2020) 135868

Determination of the freeze-out curve



three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

3. freeze-out: STAR Fit II

neglecting first two at low μ_B and the last one

$$\mu_{B\text{CF}} = \frac{a}{1 + 0.288\sqrt{s_{\text{NN}}}},$$

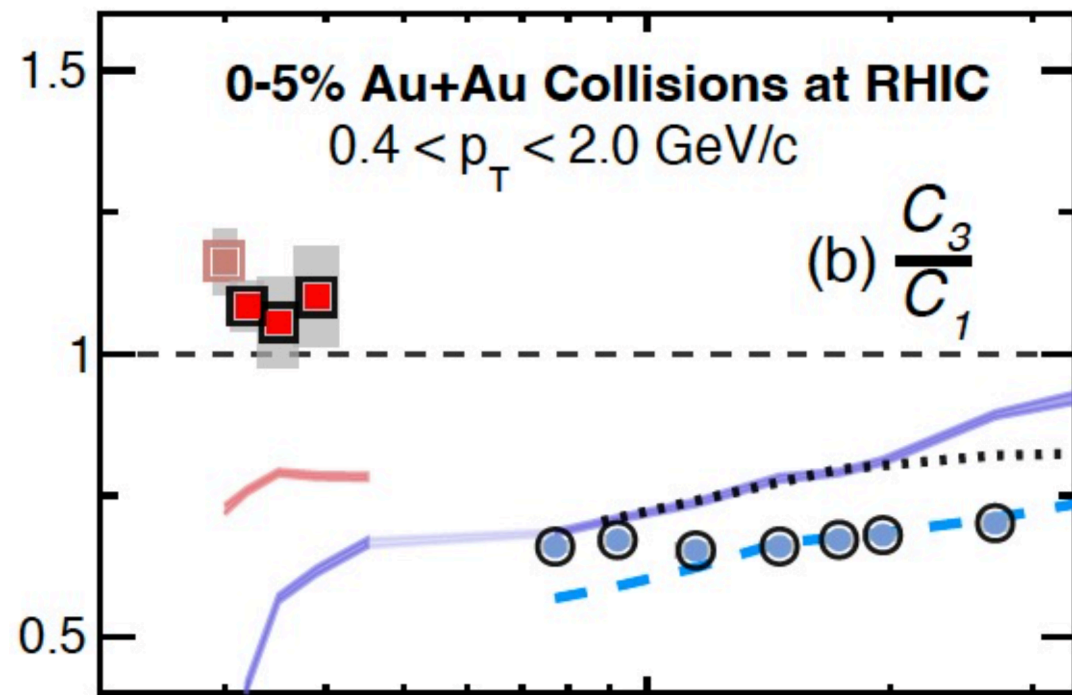
$$T_{\text{CF}} = \frac{T_{\text{CF}}^{(0)}}{1 + \exp(2.60 - \ln(\sqrt{s_{\text{NN}}})/0.45)}$$

all data points

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

C₃/C₁: Comparison to STAR data

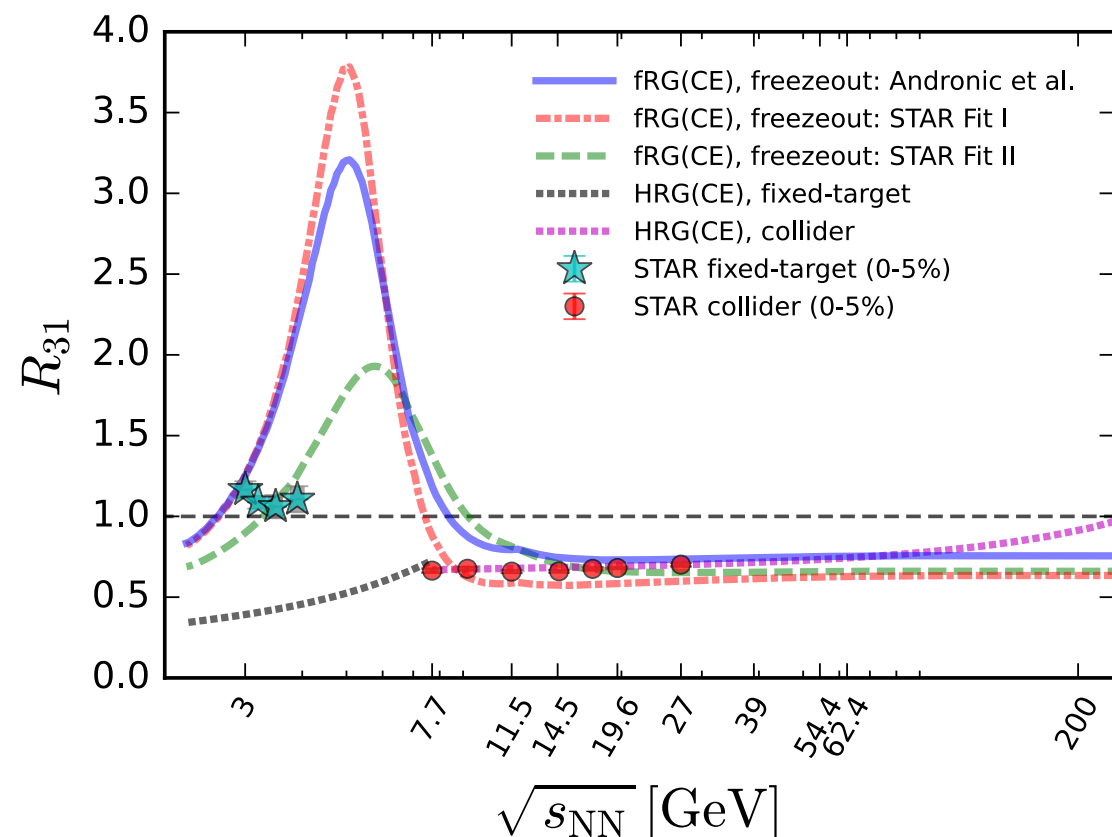
STAR data and UrQMD (baseline):



STAR: Z. Sweger, Quark Matter 2025

STAR: arXiv:2504.00817

fRG (critical) and HRG (baseline):

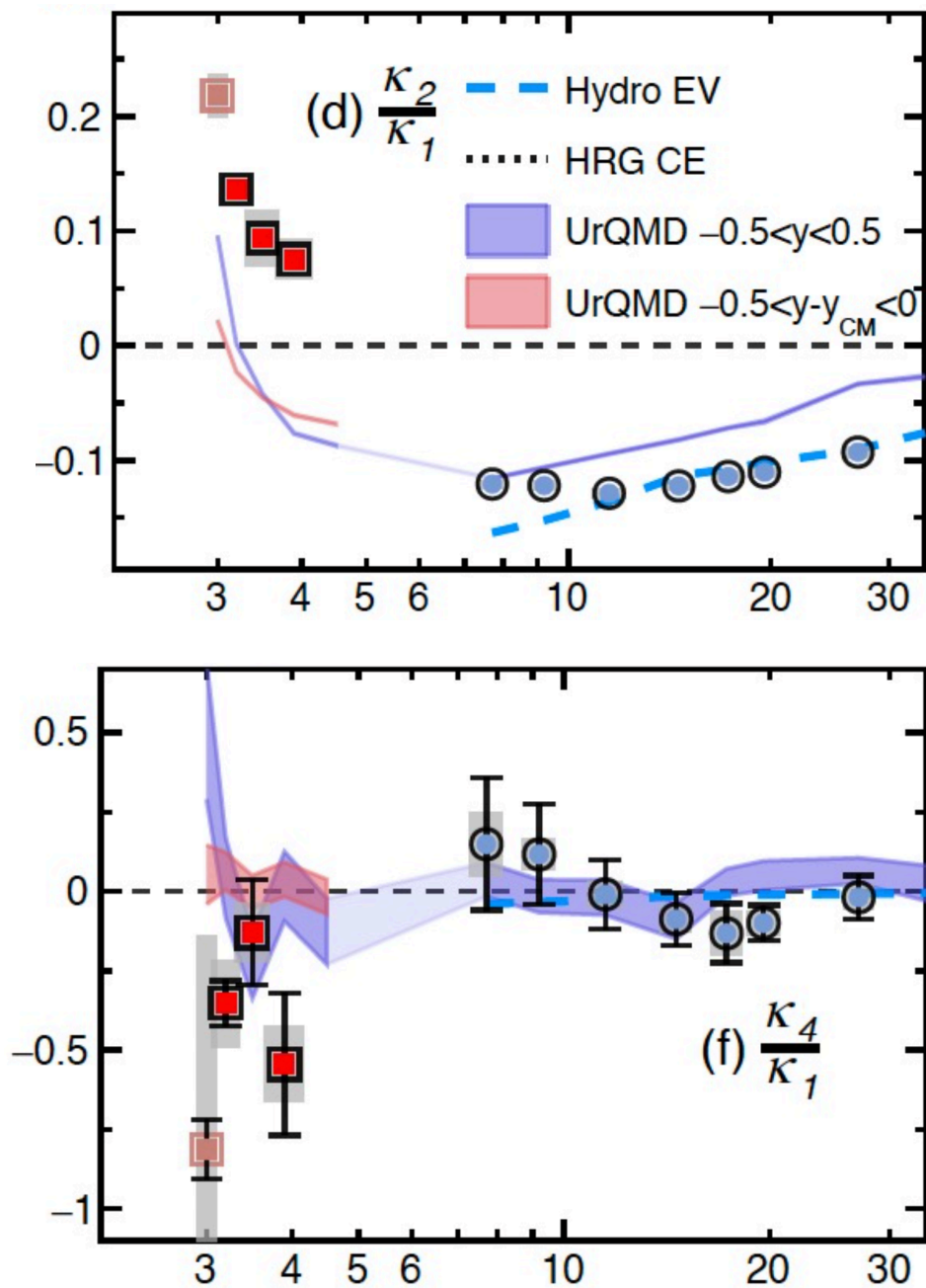


fRG: WF, Luo, Pawłowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508; Zhao, Yin, WF, in preparation

- Significant deviations from both the non-critical baseline results in UrQMD and HRG.
- fRG results are in accordance with data.

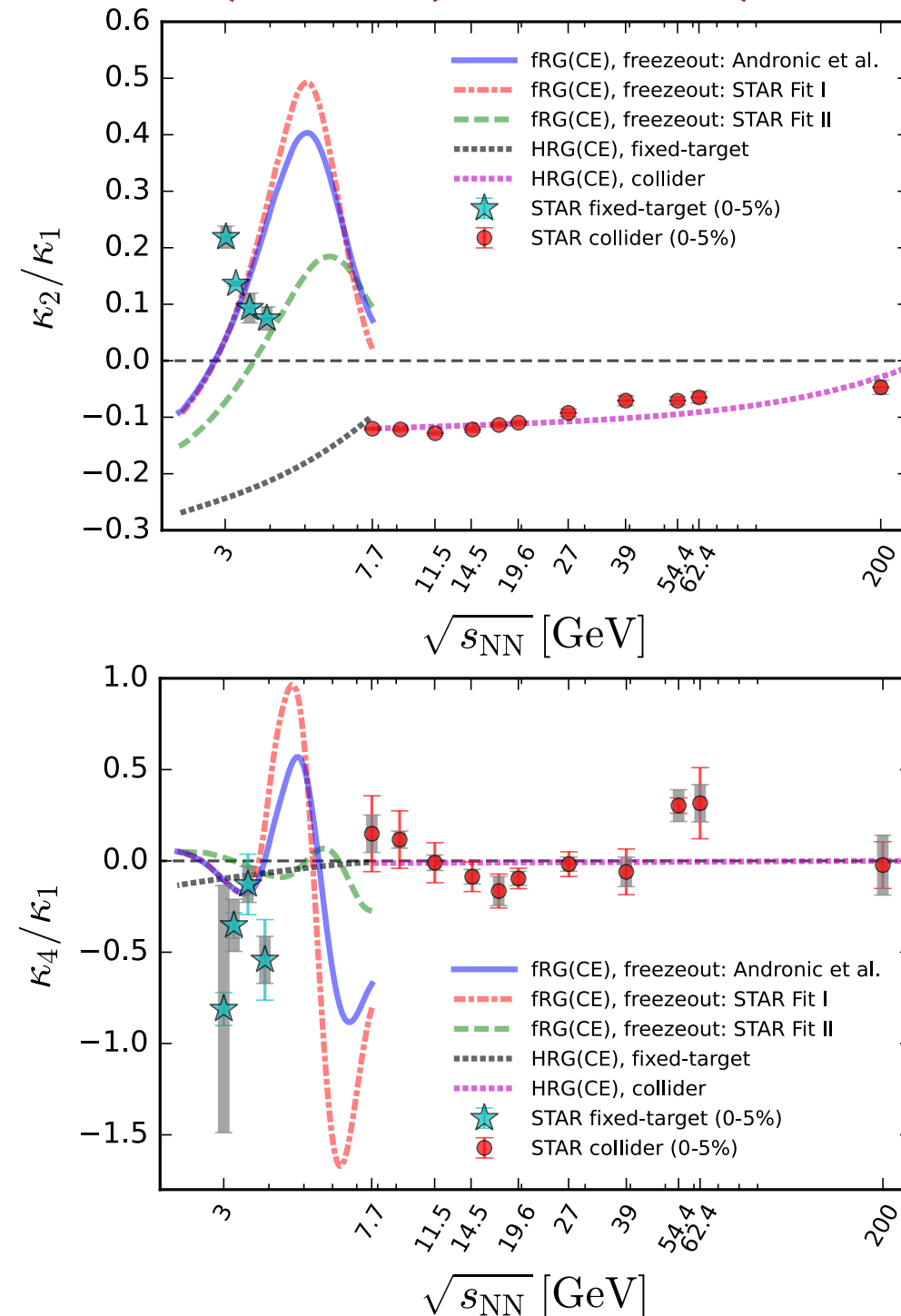
$\kappa_2/\kappa_1, \kappa_4/\kappa_1$: factorial cumulants of proton

STAR data and UrQMD (baseline):



STAR: Z. Sweger, Quark Matter 2025

fRG (critical) and HRG (baseline):

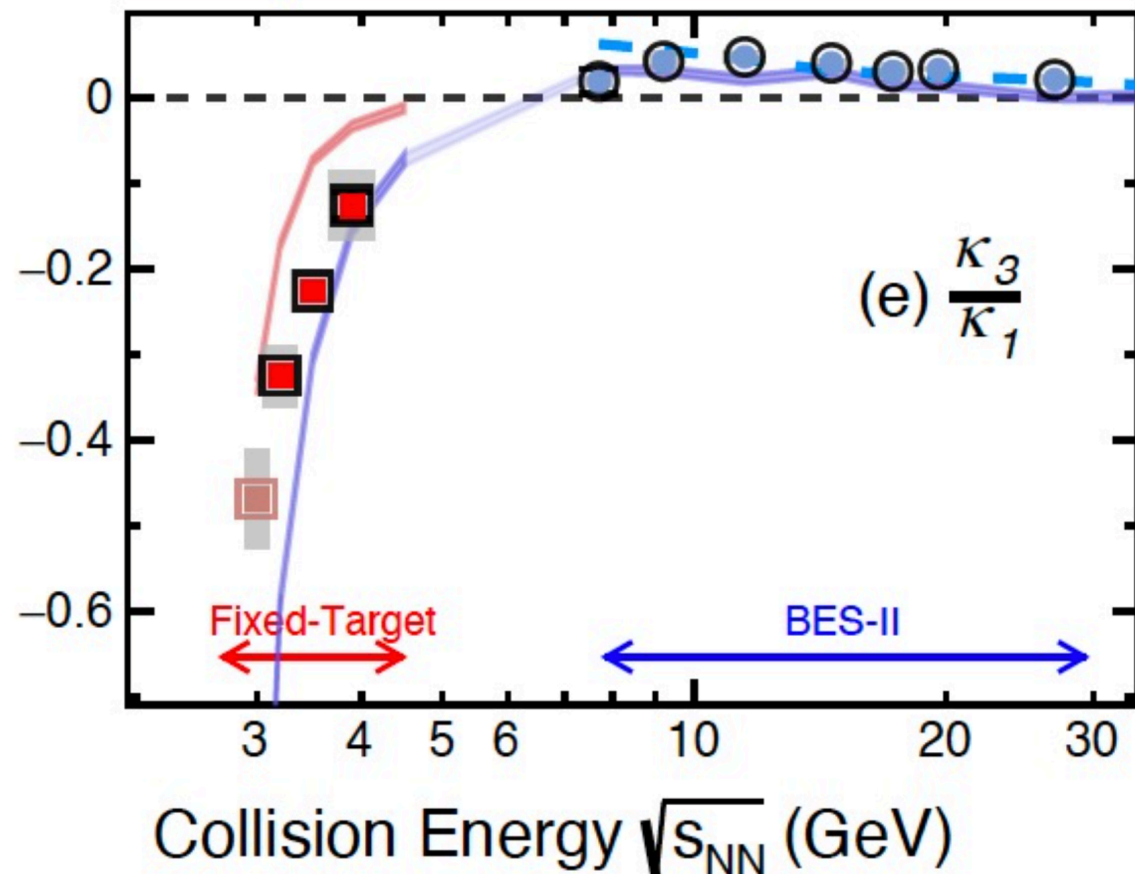


fRG and
HRG: Zhao,
Yin, WF, in
preparation

- For κ_2/κ_1 , the trend at fixed target is different for UrQMD and HRG.

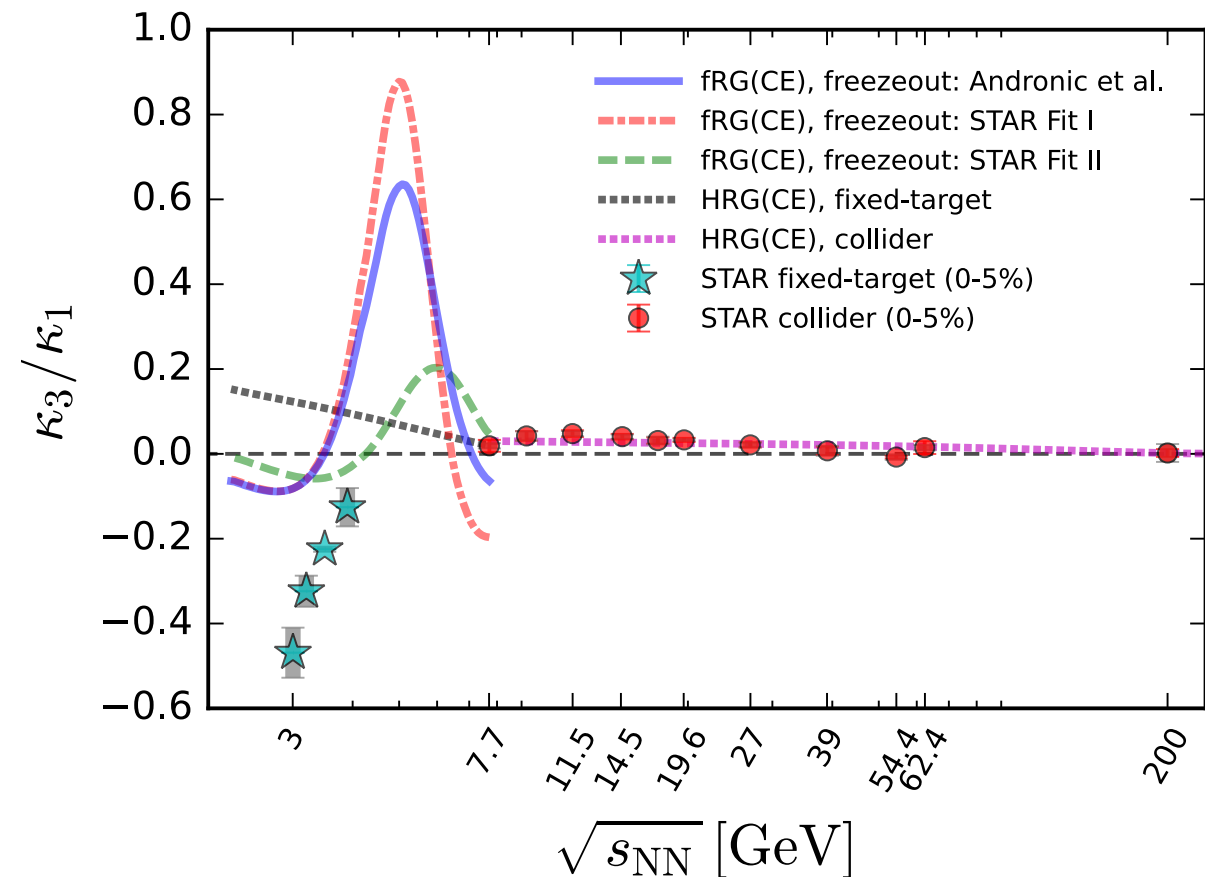
κ_3/κ_1 : factorial cumulants of proton

STAR data and UrQMD (baseline):



STAR: Z. Sweger, Quark Matter 2025

fRG (critical) and HRG (baseline):

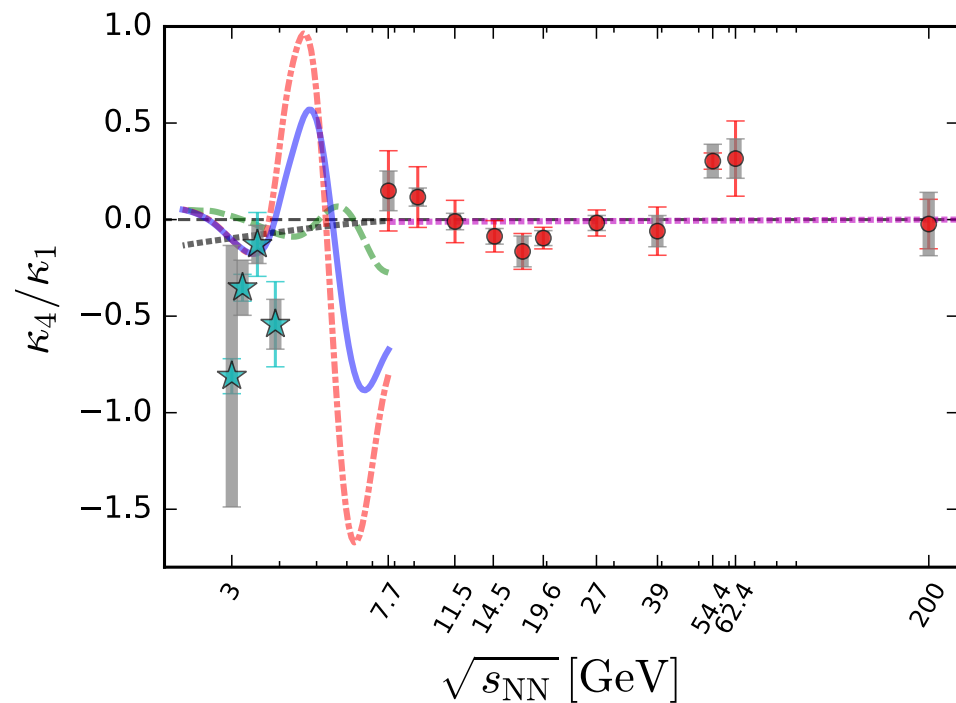
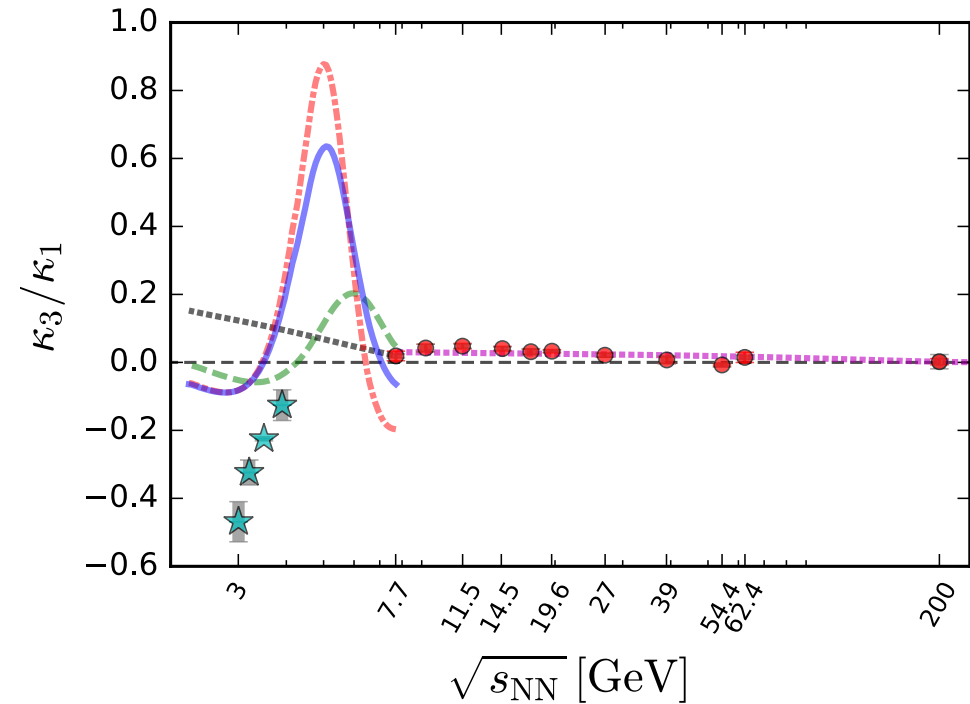
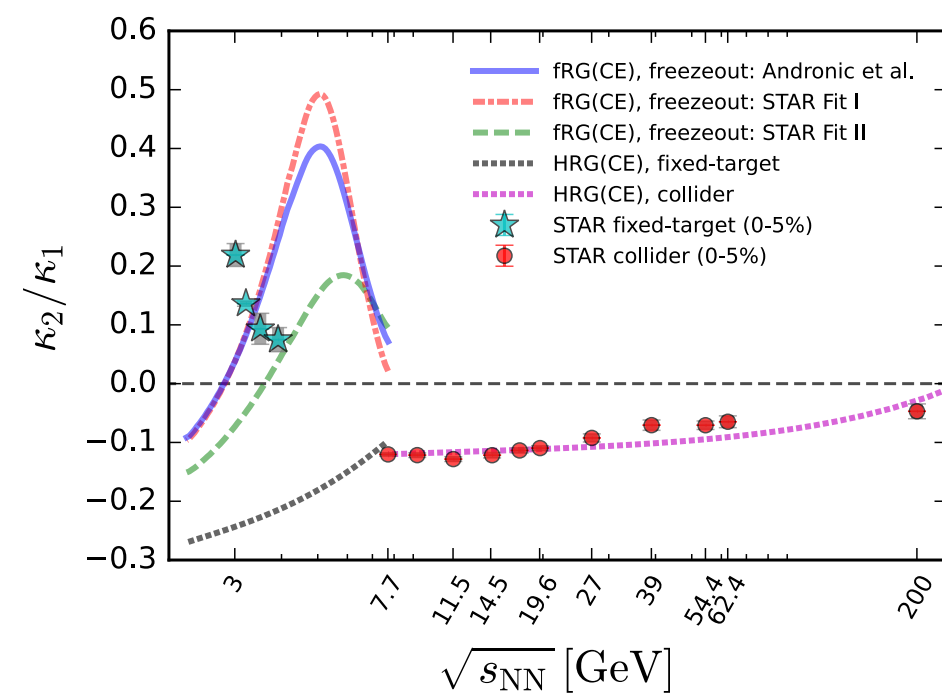


fRG and HRG: Zhao, Yin, WF, in preparation

- The discrepancy between the UrQMD and HRG at the FXT energy might imply non-equilibrium effect becomes sizable at low collision energy?

Factorial cumulants of proton (baryon)

Comparison with non-critical HRG and critical fRG:



STAR: Quark Matter 2025 fRG and HRG: Zhao, Yin,
STAR: arXiv:2504.00817 WF, in preparation

$$\begin{aligned}\kappa_1 &= C_1 \\ \kappa_2 &= -C_1 + C_2 \\ \kappa_3 &= 2C_1 - 3C_2 + C_3 \\ \kappa_4 &= -6C_1 + 11C_2 - 6C_3 + C_4\end{aligned}$$

- In comparison to the non-critical HRG, fRG results with critical fluctuations seem to be better consistent with the data.

Fierz-complete basis of four-quark interactions

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $SU_A(N_f)$ and $U_A(1)$

$$\begin{aligned}\mathcal{T}_{ijlm}^{(V-A)} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} \gamma_\mu T^0 q)^2 - (\bar{q} i \gamma_\mu \gamma_5 T^0 q)^2, \\ \mathcal{T}_{ijlm}^{(V+A)} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} \gamma_\mu T^0 q)^2 + (\bar{q} i \gamma_\mu \gamma_5 T^0 q)^2, \\ \mathcal{T}_{ijlm}^{(S-P)+} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 q)^2 - (\bar{q} \gamma_5 T^0 q)^2 \\ &\quad + (\bar{q} T^a q)^2 - (\bar{q} \gamma_5 T^a q)^2, \\ \mathcal{T}_{ijlm}^{(V-A)\text{adj}} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} \gamma_\mu T^0 t^a q)^2 - (\bar{q} i \gamma_\mu \gamma_5 T^0 t^a q)^2,\end{aligned}$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $U_A(1)$, breaking $SU_A(N_f)$

$$\begin{aligned}\mathcal{T}_{ijlm}^{(S-P)-} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 q)^2 - (\bar{q} \gamma_5 T^0 q)^2 \\ &\quad - (\bar{q} T^a q)^2 + (\bar{q} \gamma_5 T^a q)^2, \\ \mathcal{T}_{ijlm}^{(S-P)\text{adj}} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 t^a q)^2 - (\bar{q} \gamma_5 T^0 t^a q)^2 \\ &\quad - (\bar{q} T^a t^b q)^2 + (\bar{q} \gamma_5 T^a t^b q)^2.\end{aligned}$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $SU_A(N_f)$, breaking $U_A(1)$

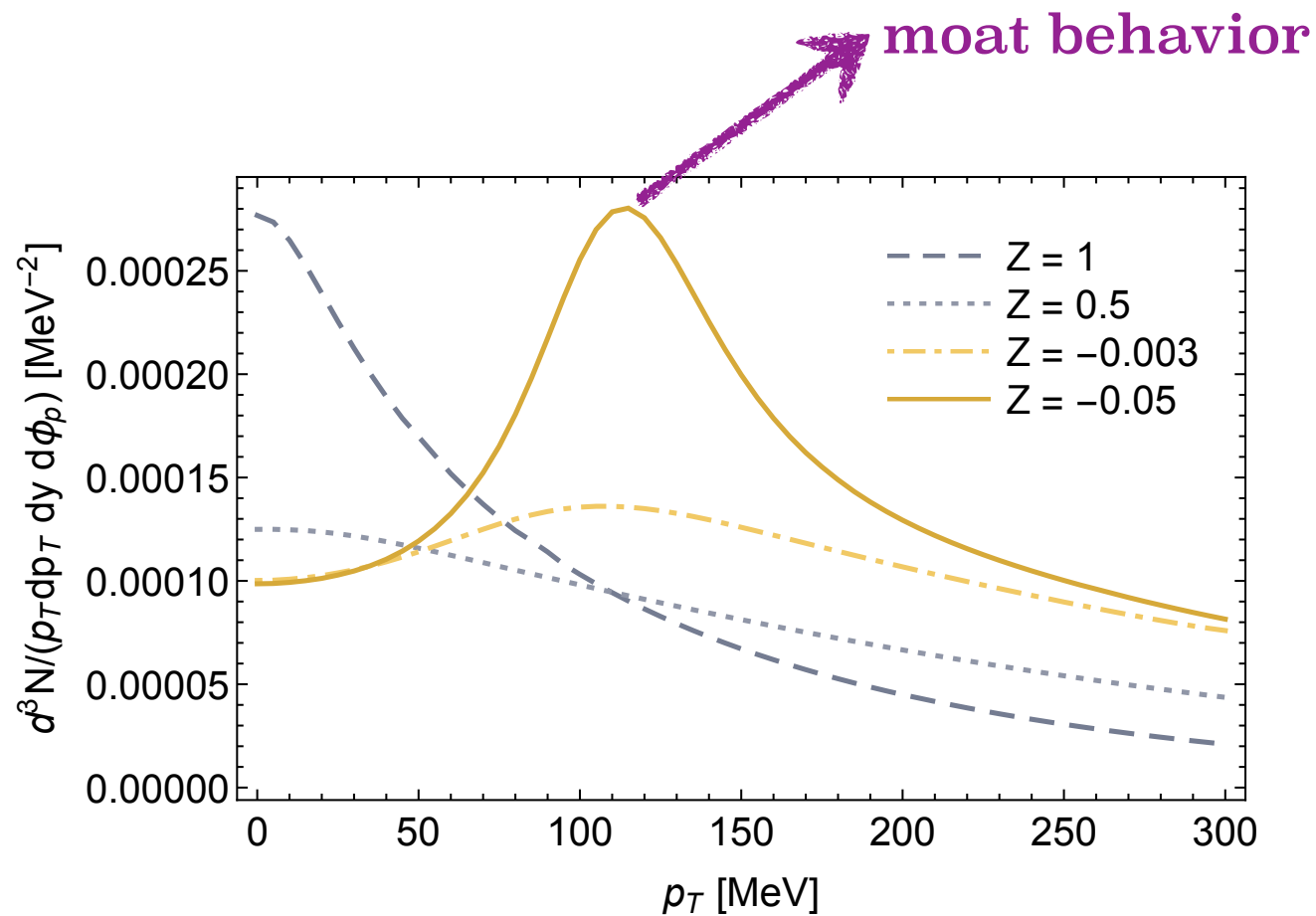
$$\begin{aligned}\mathcal{T}_{ijlm}^{(S+P)-} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 q)^2 + (\bar{q} \gamma_5 T^0 q)^2 \\ &\quad - (\bar{q} T^a q)^2 - (\bar{q} \gamma_5 T^a q)^2, \\ \mathcal{T}_{ijlm}^{(S+P)\text{adj}} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 t^a q)^2 + (\bar{q} \gamma_5 T^0 t^a q)^2 \\ &\quad - (\bar{q} T^a t^b q)^2 - (\bar{q} \gamma_5 T^a t^b q)^2.\end{aligned}$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, breaking $SU_A(N_f)$, $U_A(1)$

$$\begin{aligned}\mathcal{T}_{ijlm}^{(S+P)+} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 q)^2 + (\bar{q} \gamma_5 T^0 q)^2 \\ &\quad + (\bar{q} T^a q)^2 + (\bar{q} \gamma_5 T^a q)^2, \\ \mathcal{T}_{ijlm}^{(S+P)\text{adj}} \bar{q}_i q_l \bar{q}_j q_m &= (\bar{q} T^0 t^a q)^2 + (\bar{q} \gamma_5 T^0 t^a q)^2 \\ &\quad + (\bar{q} T^a t^b q)^2 + (\bar{q} \gamma_5 T^a t^b q)^2.\end{aligned}$$

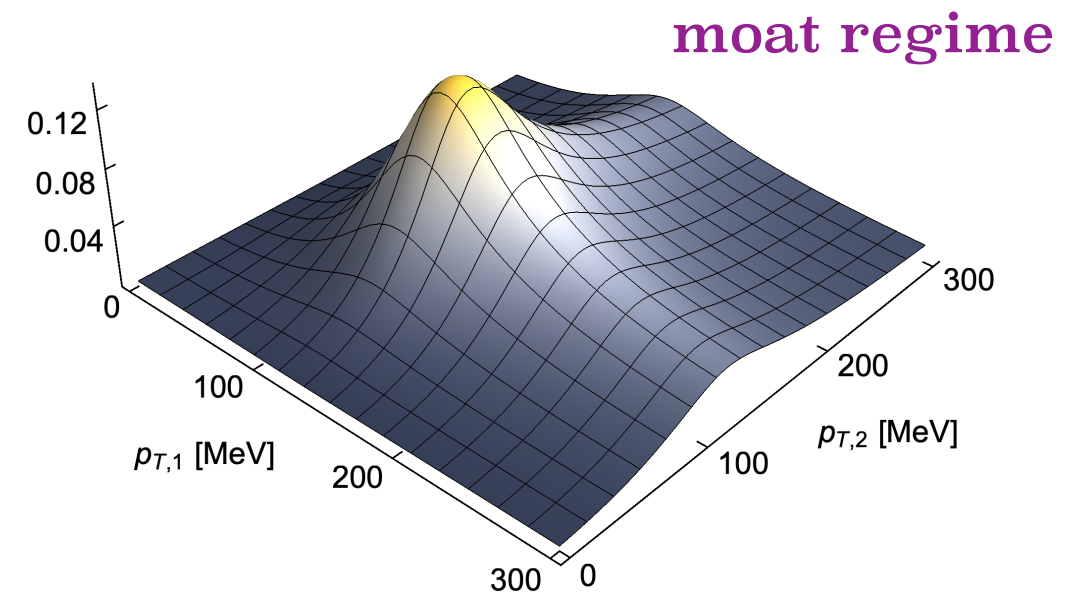
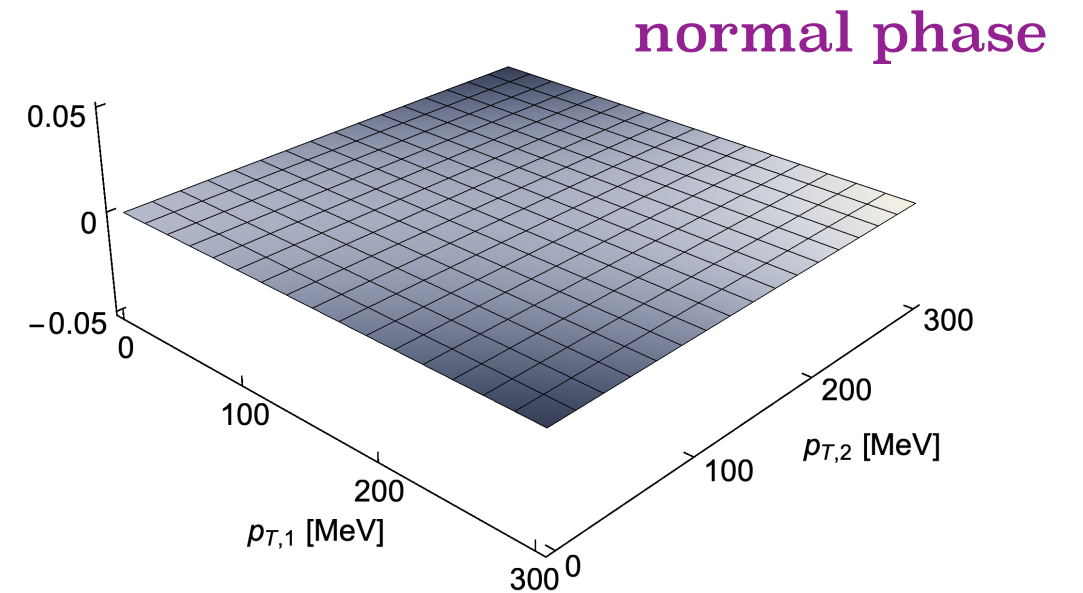
Phenomenology of moat regime in heavy-ion collisions

- Transverse momentum spectrum of one particle:



Pisarski, Rennecke, *PRL* 127 (2021) 152302;
 Rennecke, Pisarski, *PoS CPOD2021* (2022);
 Rennecke, Pisarski, Rischke, *PRD* 107 (2023) 116011

- Two-particle correlation:



$$\Delta n_{12} = \left\langle \left(\frac{d^3N}{d\mathbf{p}^3} \right)^2 \right\rangle_c / \left\langle \frac{d^3N}{d\mathbf{p}^3} \right\rangle^2$$

Schwinger-Keldysh path integral

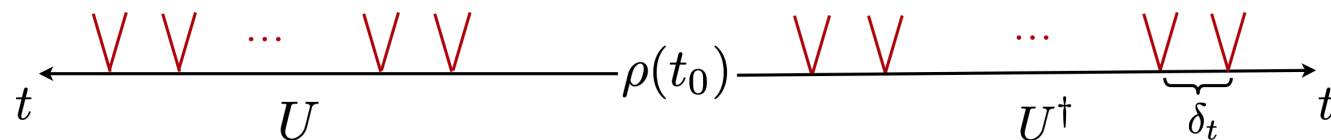
- Schrödinger equation:



$$U(t, t_0) = e^{-iH(t-t_0)}$$

$$i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle \longrightarrow |\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle,$$

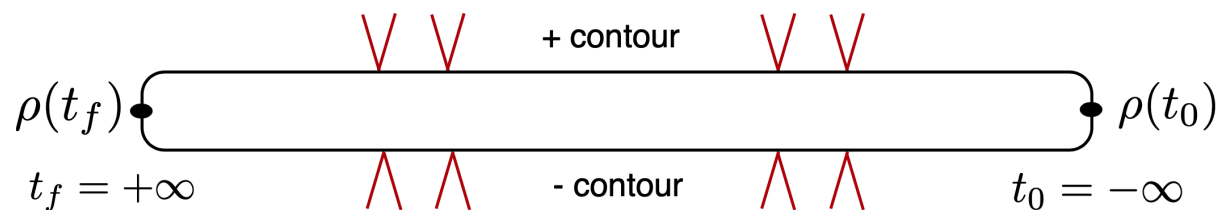
- von Neumann equation:



$$\partial_t\rho(t) = -i[H, \rho(t)] \longrightarrow \rho(t) = U(t, t_0)\rho(t_0)U^\dagger(t, t_0),$$

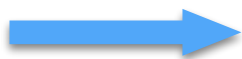
- Keldysh partition function:

$$Z = \text{tr} \rho(t),$$



- two-point closed time-path Green's function:

$$\begin{aligned} G(x, y) &\equiv -i\text{tr}\{T_p(\phi(x)\phi^\dagger(y)\rho)\} \\ &\equiv -i\langle T_p(\phi(x)\phi^\dagger(y)) \rangle, \end{aligned}$$



$$\begin{aligned} G(x, y) &= \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix} \\ &\equiv \begin{pmatrix} G_F & G_+ \\ G_- & G_{\tilde{F}} \end{pmatrix}, \end{aligned}$$

$$G_F(x, y) \equiv -i\langle T(\phi(x)\phi^\dagger(y)) \rangle,$$

$$G_+(x, y) \equiv -i\langle \phi^\dagger(y)\phi(x) \rangle,$$

$$G_-(x, y) \equiv -i\langle \phi(x)\phi^\dagger(y) \rangle,$$

$$G_{\tilde{F}}(x, y) \equiv -i\langle \tilde{T}(\phi(x)\phi^\dagger(y)) \rangle,$$

Schwinger, J. Math. Phys. 2, 407 (1961);
Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964);
Chou, Su, Hao, Yu, Phys. Rept. 118, 1 (1985).

FRG in Keldysh path integral

- Implement the formalism of fRG in the two time branches:

$$Z_k[J_c, J_q] = \int (\mathcal{D}\varphi_c \mathcal{D}\varphi_q) \exp \left\{ i \left(S[\varphi] + \Delta S_k[\varphi] + (J_q^i \varphi_{i,c} + J_c^i \varphi_{i,q}) \right) \right\},$$

with

$$\begin{aligned} \Delta S_k[\varphi] &= \frac{1}{2} (\varphi_{i,c}, \varphi_{i,q}) \begin{pmatrix} 0 & R_k^{ij} \\ (R_k^{ij})^* & 0 \end{pmatrix} \begin{pmatrix} \varphi_{j,c} \\ \varphi_{j,q} \end{pmatrix} \\ &= \frac{1}{2} \left(\varphi_{i,c} R_k^{ij} \varphi_{j,q} + \varphi_{i,q} (R_k^{ij})^* \varphi_{j,c} \right), \end{aligned}$$

Keldysh rotation:

$$\begin{cases} \varphi_{i,+} = \frac{1}{\sqrt{2}} (\varphi_{i,c} + \varphi_{i,q}), \\ \varphi_{i,-} = \frac{1}{\sqrt{2}} (\varphi_{i,c} - \varphi_{i,q}), \end{cases}$$

- Then we derive the flow equation in the closed time path:

$$\partial_\tau \Gamma_k[\Phi] = \frac{i}{2} \text{STr} \left[(\partial_\tau R_k^*) G_k \right], \quad R_k^{ab} \equiv \begin{pmatrix} 0 & R_k^{ij} \\ (R_k^{ij})^* & 0 \end{pmatrix},$$

$$iG(x, y) = \begin{pmatrix} iG^K(x, y) & iG^R(x, y) \\ iG^A(x, y) & 0 \end{pmatrix},$$

$$\begin{aligned} iG^R(x, y) &= \theta(x^0 - y^0) \langle [\phi(x), \phi^*(y)] \rangle, \\ iG^A(x, y) &= \theta(y^0 - x^0) \langle [\phi^*(y), \phi(x)] \rangle, \\ iG^K(x, y) &= \langle \{ \phi(x), \phi^*(y) \} \rangle, \end{aligned}$$

Universal damping or not?

From the pole of the retarded propagator of Goldstone

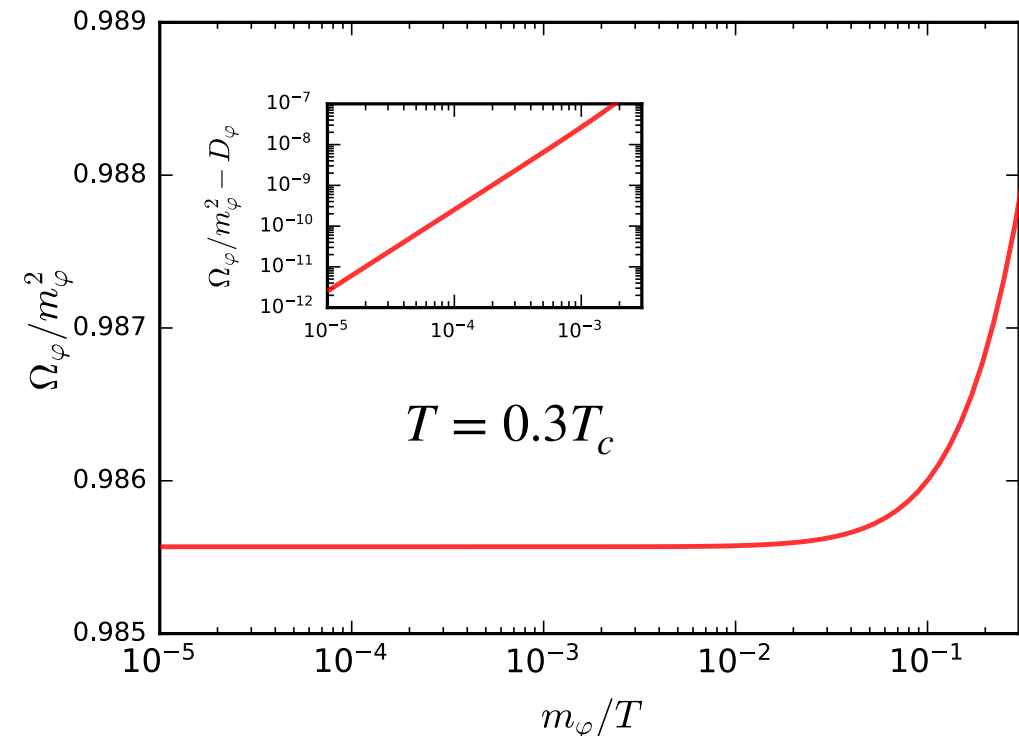
$$G_{\varphi\varphi}^R(\omega, q) = \frac{1}{-iZ_{\varphi}^{(t)}\omega + Z_{\varphi}^{(i)}(q^2 + m_{\varphi}^2)}$$

One obtains the dispersion relation of a damped mode

$$\omega(q) = -i \frac{Z_{\varphi}^{(i)}}{Z_{\varphi}^{(t)}} (m_{\varphi}^2 + q^2)$$

The relaxation rate at zero momentum reads

$$\Omega_{\varphi} \equiv -\text{Im } \omega(q=0) = \frac{Z_{\varphi}^{(i)}}{Z_{\varphi}^{(t)}} m_{\varphi}^2$$



Tan, Chen, WF, Li, *Nature Commun.* 16 (2025) 2916, arXiv: 2403.03503

This seemingly appears as a **universal** relation that was also observed in Holographics, Hydrodynamics, and EFT

Holographics:

Amoretti, Areán, Goutéraux, Musso, *PRL* 123 (2019) 211602;
Amoretti, Areán, Goutéraux, Musso, *JHEP* 10 (2019) 068;
Ammon *et al.*, *JHEP* 03 (2022) 015;
Cao, Baggioli, Liu, Li, *JHEP* 12 (2022) 113

Hydrodynamics:

Delacrétaz, Goutéraux, Ziogas, *PRL* 128 (2022) 141601

EFT:

Baggioli, *Phys. Rev. Res.* 2 (2020) 022022;
Baggioli, Landry, *SciPost Phys.* 9 (2020) 062

● If $T \ll T_c$

$$\frac{\Omega_{\varphi}}{m_{\varphi}^2} \simeq D_{\varphi}(T) + \mathcal{O}\left(\frac{m_{\varphi}^2}{T^2}\right) \quad \text{with} \quad D_{\varphi}(T) \equiv \frac{Z_{\varphi}^{(i)}(T, c=0)}{Z_{\varphi}^{(t)}(T, c=0)}$$

Large N limit

In the large N limit, the static and dynamic anomalous dimensions can be solved analytically

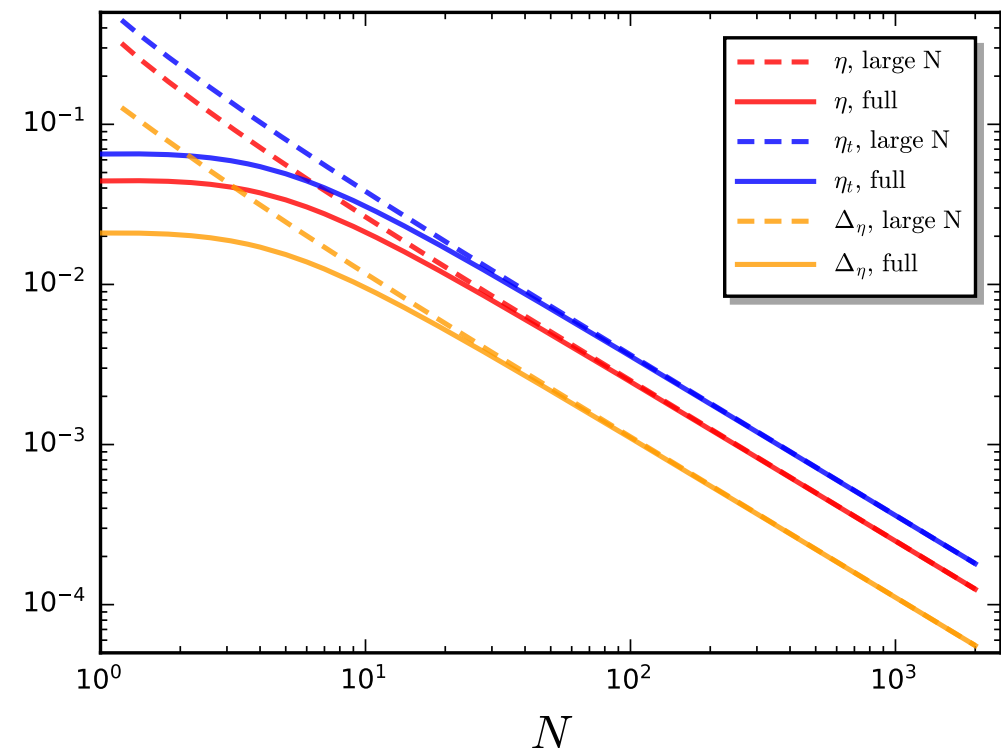
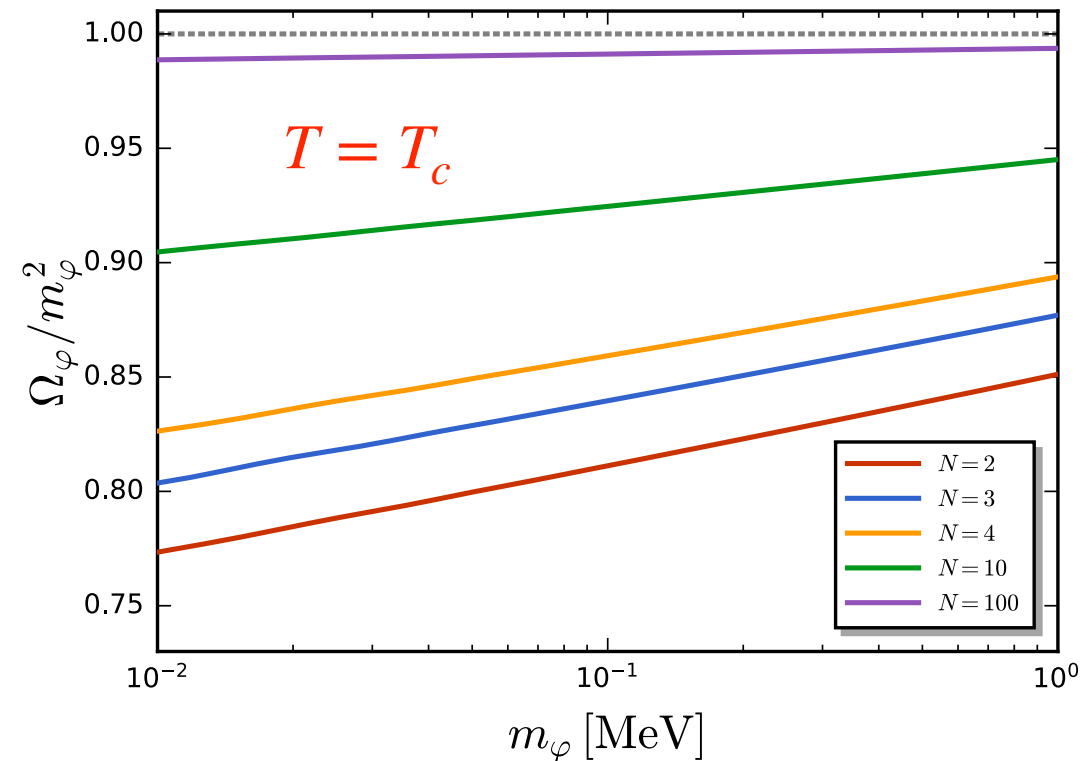
$$\eta = \frac{5}{N-1} \frac{(1+\eta)(1-2\eta)^2}{(5-\eta)(2-\eta)^2}$$

and

$$\eta_t = \frac{1}{9(N-1)} \frac{(1-2\eta)^2(13+15\eta-2\eta^3)}{(2-\eta)^2}$$

Tan, Chen, WF, Li, *Nature Commun.* 16
(2025) 2916, arXiv: 2403.03503

- In the limit $N \rightarrow \infty$, the novel universal damping disappears.
- One should not expect that the anomalous scaling regime can be observed in classical holographic models.



Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathcal{D}\hat{\Phi}) \exp \left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2} \text{STr} \left[(\partial_t R_k) G_k \right] - \frac{1}{2} \Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

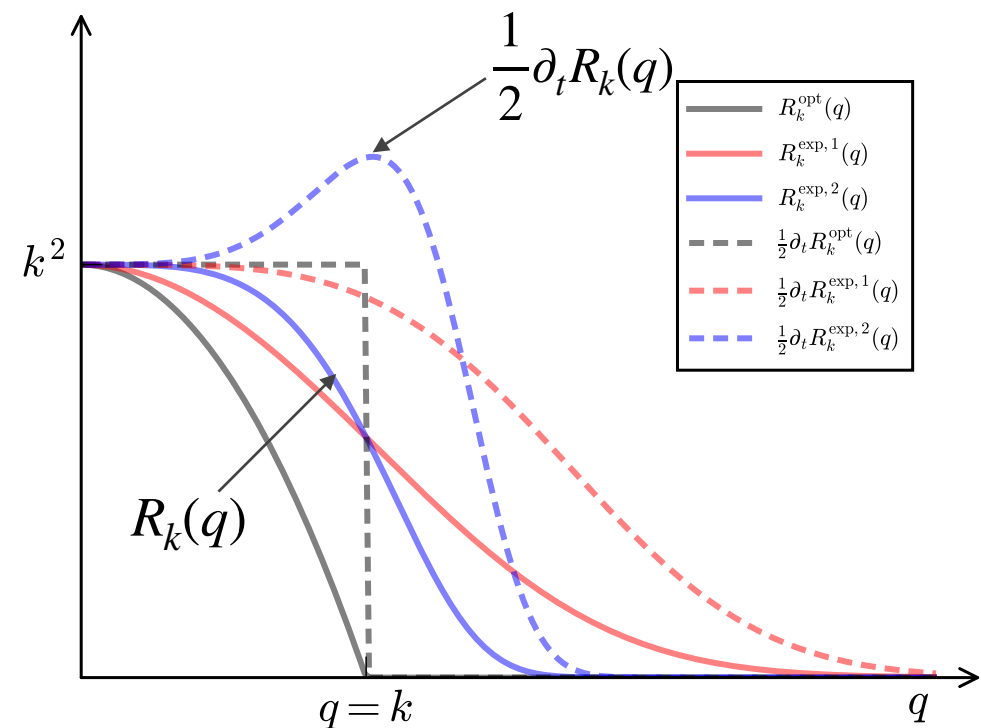
$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

flow of the effective action:

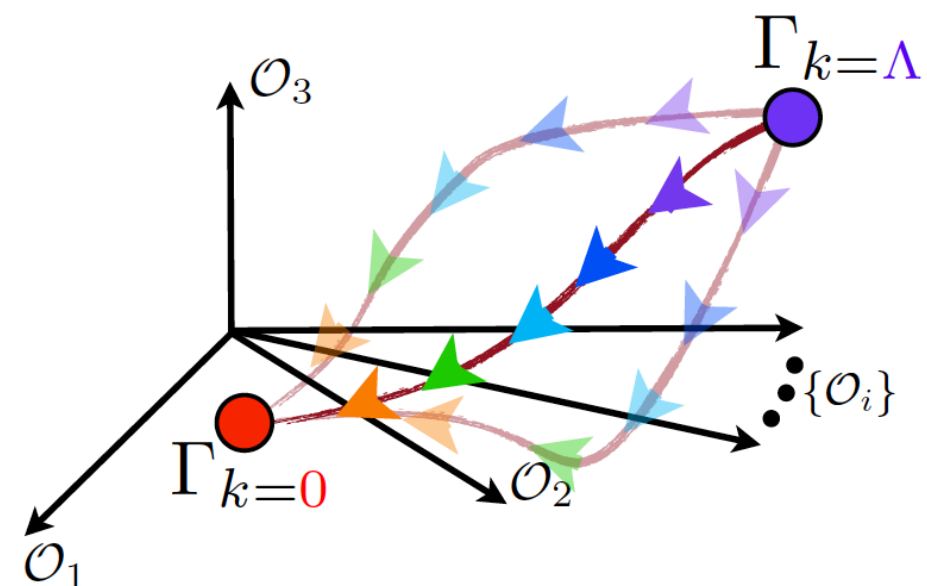
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr} \left[(\partial_t R_k) G_k \right] = \frac{1}{2} \text{ (loop diagram) }$$

Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90



$$G_{k,ab} = \gamma^c_a \left(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi] \right)^{-1}_{cb},$$



Review: WF, *CTP* 74 (2022) 097304,
arXiv: 2205.00468 [hep-ph]