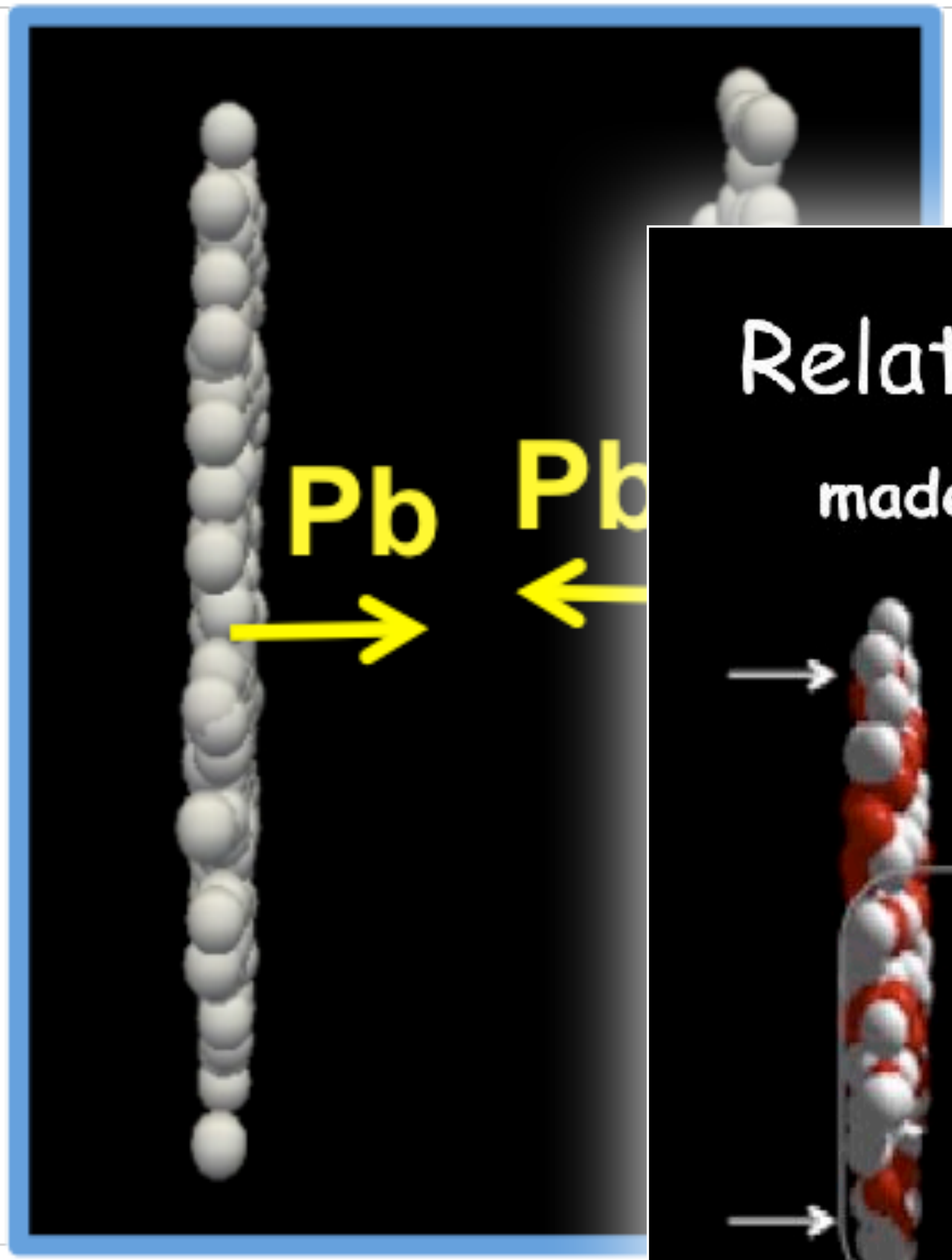
2025.10.25, QPT, Guilin

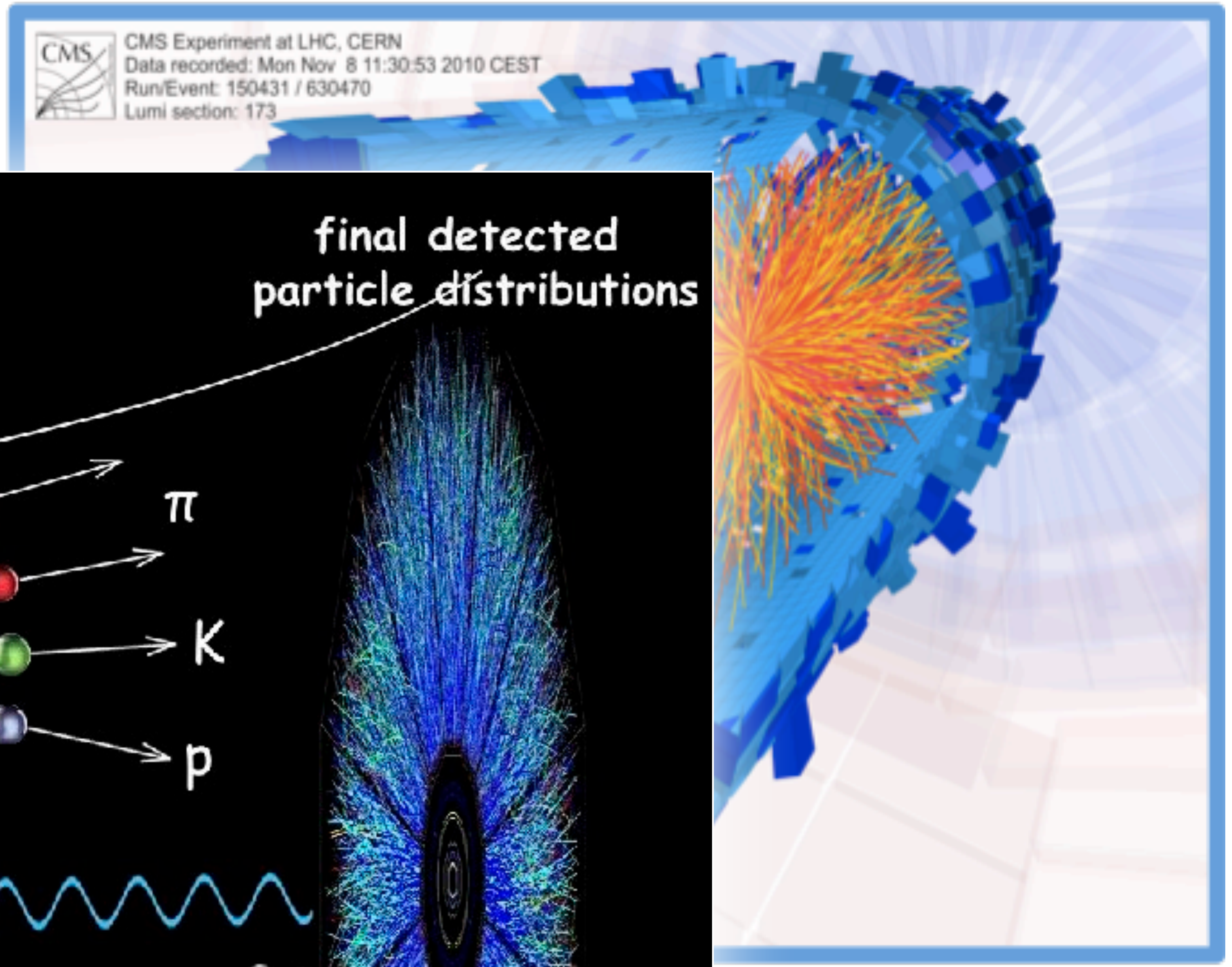
Hydrodynamization of QGP from classical and quantum perspectives

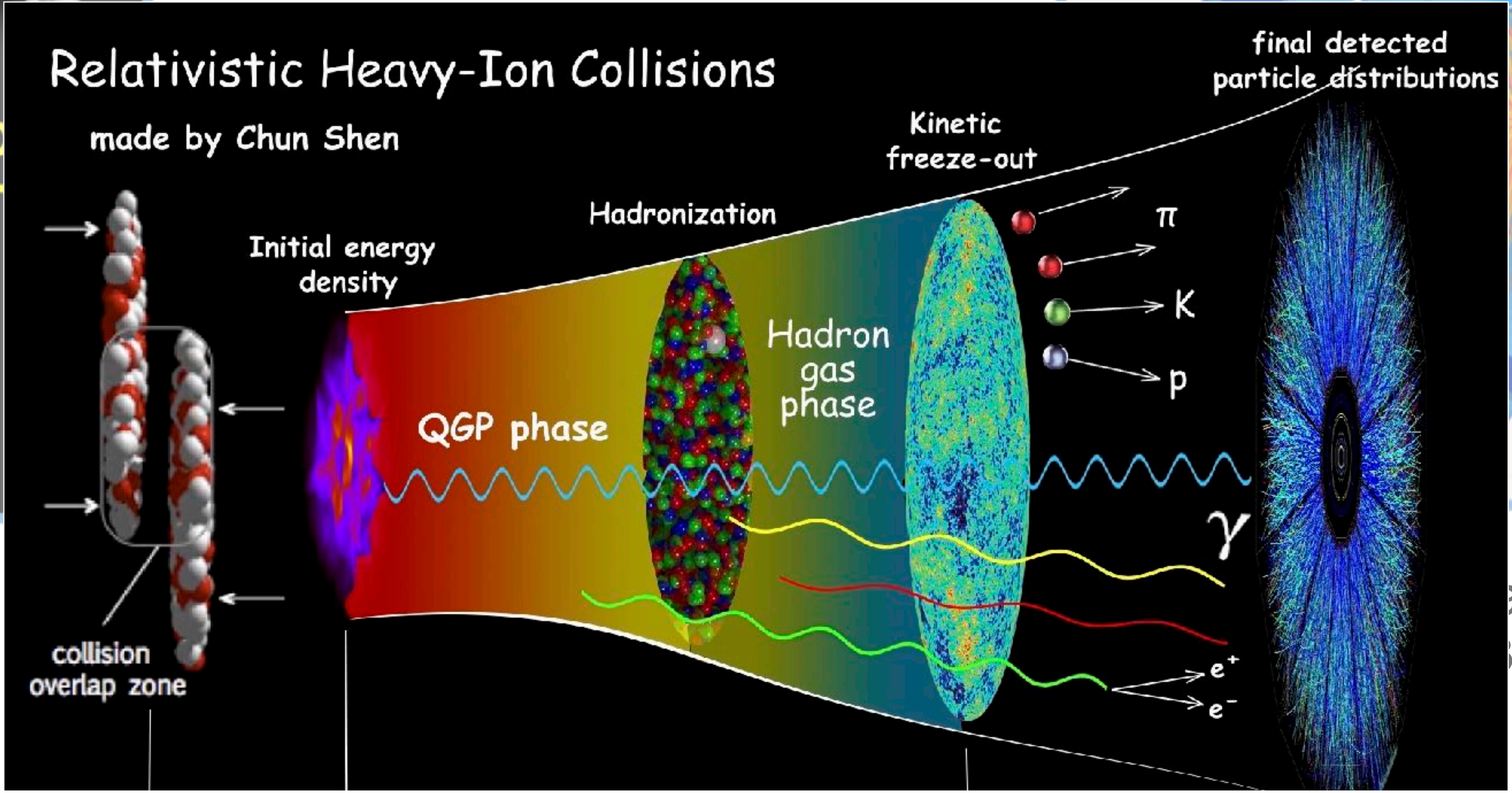
Shuzhe Shi (施舒哲), Tsinghua University

Pre-reaction

Detection







articles

–4

Hydrodynamic attractors

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly

(boost invariant, transverse homogeneous)

$$\tau \partial_\tau \varepsilon = -\frac{4}{3}\varepsilon + \pi_\eta^\eta$$

$$\partial_\tau \pi_\eta^\eta + \frac{4\pi_\eta^\eta}{3\tau} = -\frac{\pi_\eta^\eta - 4\eta/3\tau}{\tau_\pi} + \frac{c_2\,(\pi_\eta^\eta)^2}{\tau_\pi \varepsilon}$$

Heller, Spalinski, Phys.Rev.Lett. 115 (2015) 072501

Hydrodynamic attractors
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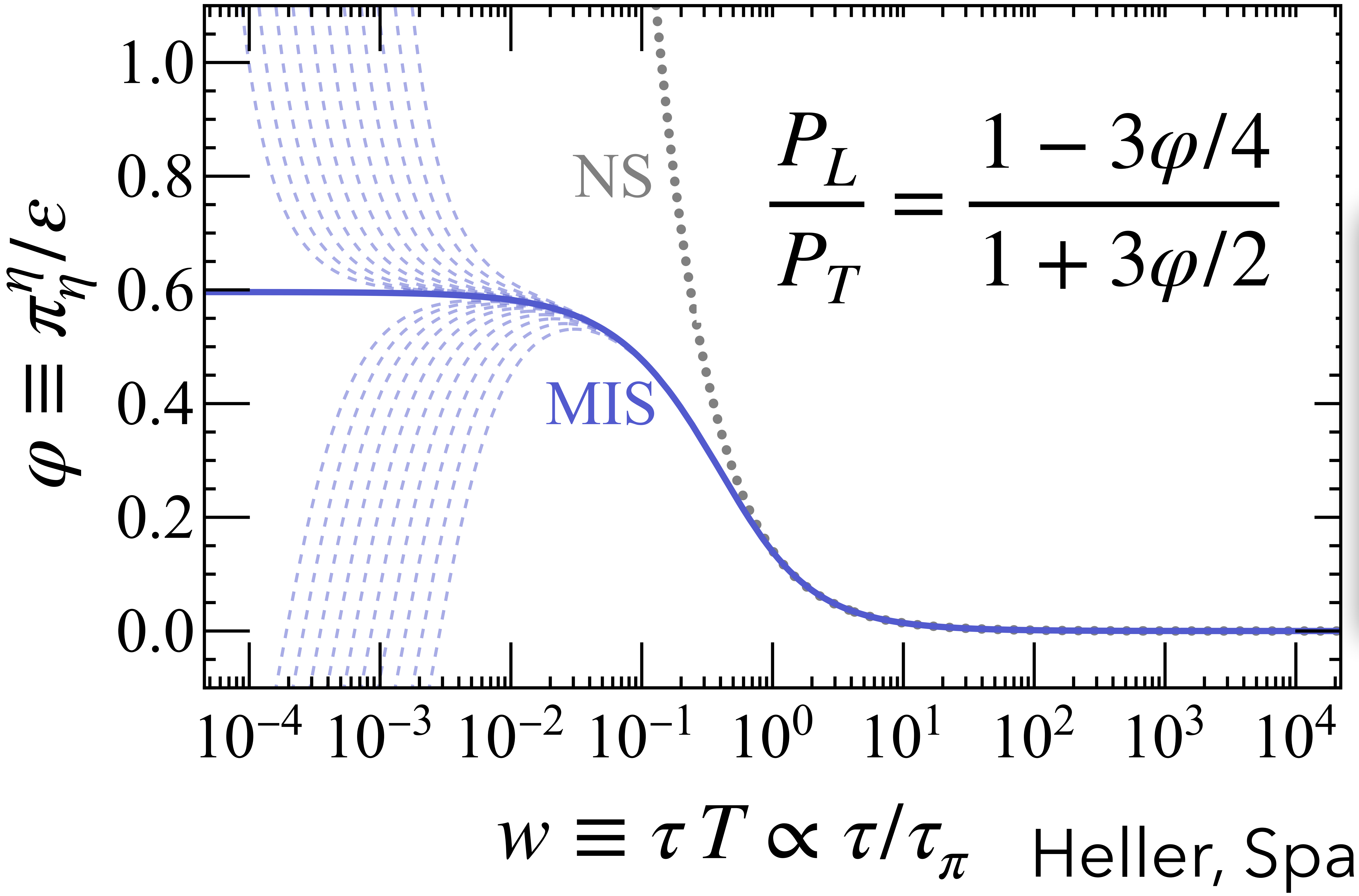

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$$\tau\partial_\tau\varepsilon = -\frac{4}{3}\varepsilon + \pi_\eta^\eta$$

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Heller, Spalinski, Phys.Rev.Lett. 115 (2015) 072501

Hydrodynamic attractors

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attractors of more coupled modes in hydrodynamics

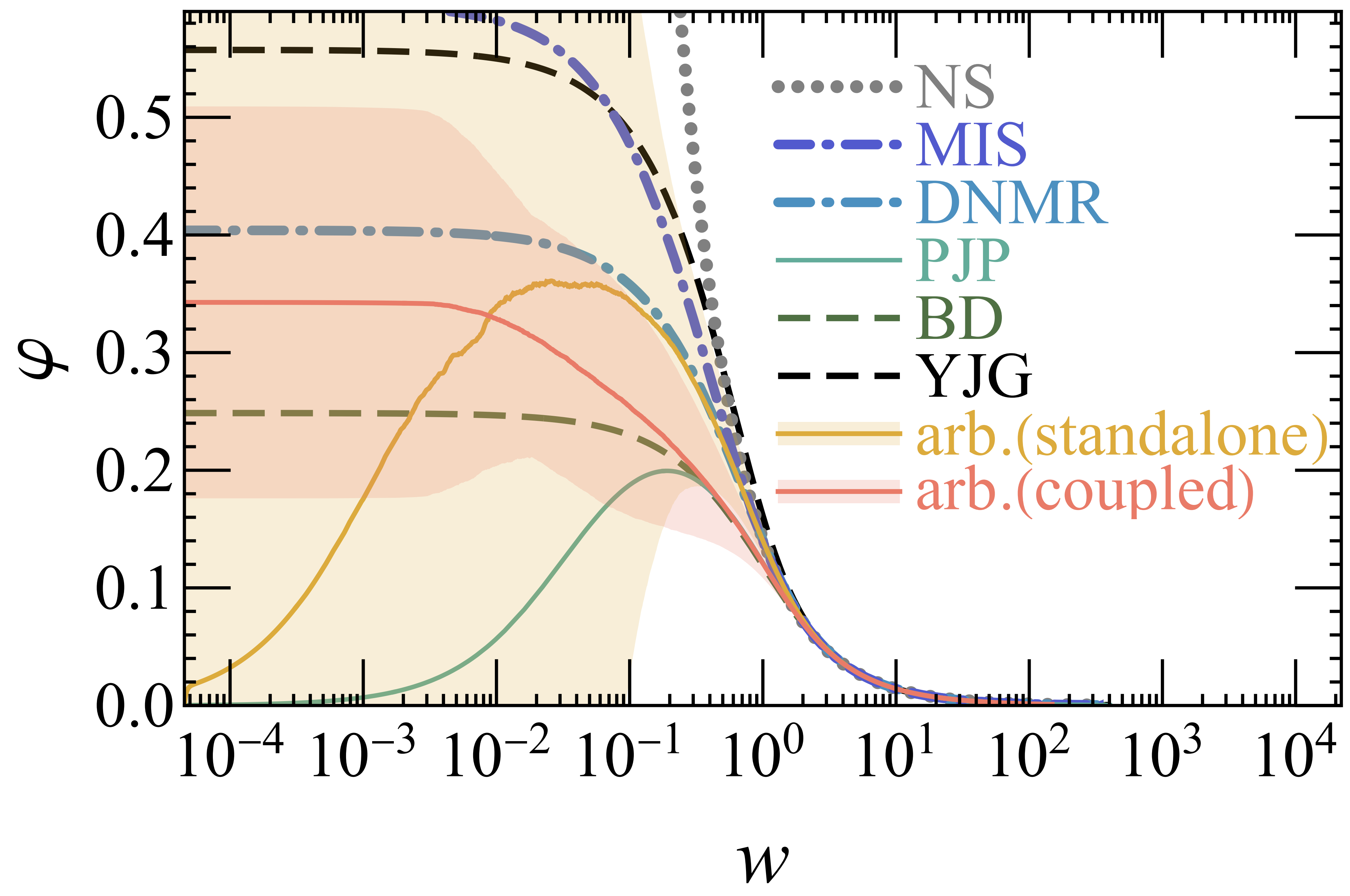
J.-P. Blaizot and Li Yan (严力),
Phys. Lett. B 820, 136478 (2021), *Phys. Rev. C* 104, 055201 (2021)

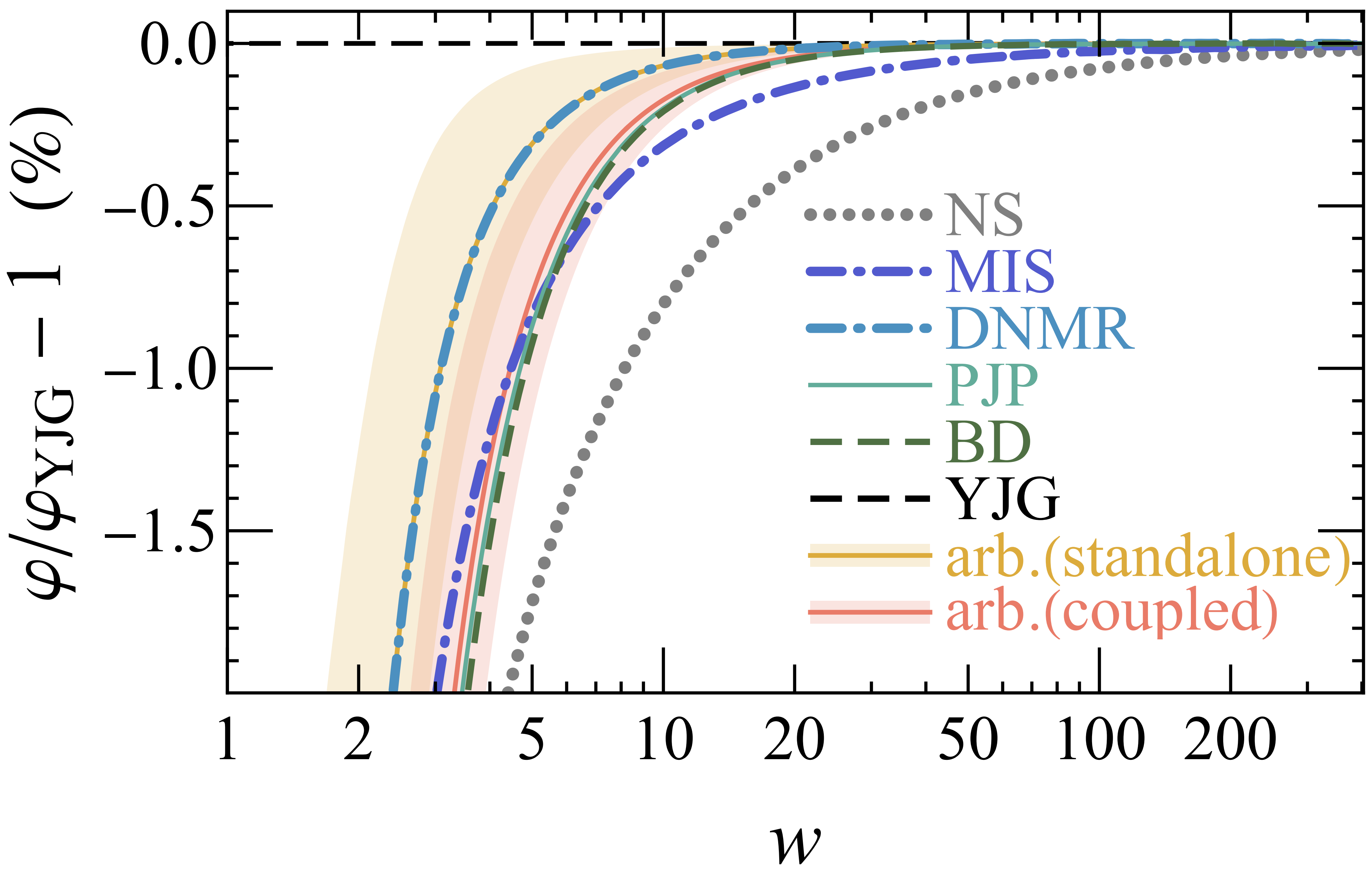
attractors of spin hydrodynamics

Donglin Wang (王栋林), Li Yan (严力) and Shi Pu (浦实),
Phys.Rev.D 111 (2025) 034033

Hydrodynamic attractors

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly





convergence of attractors in 2nd- and high-order hydrodynamics

Shile Chen (陈诗乐) and SS, 2509.08864

Hydrodynamic attractors

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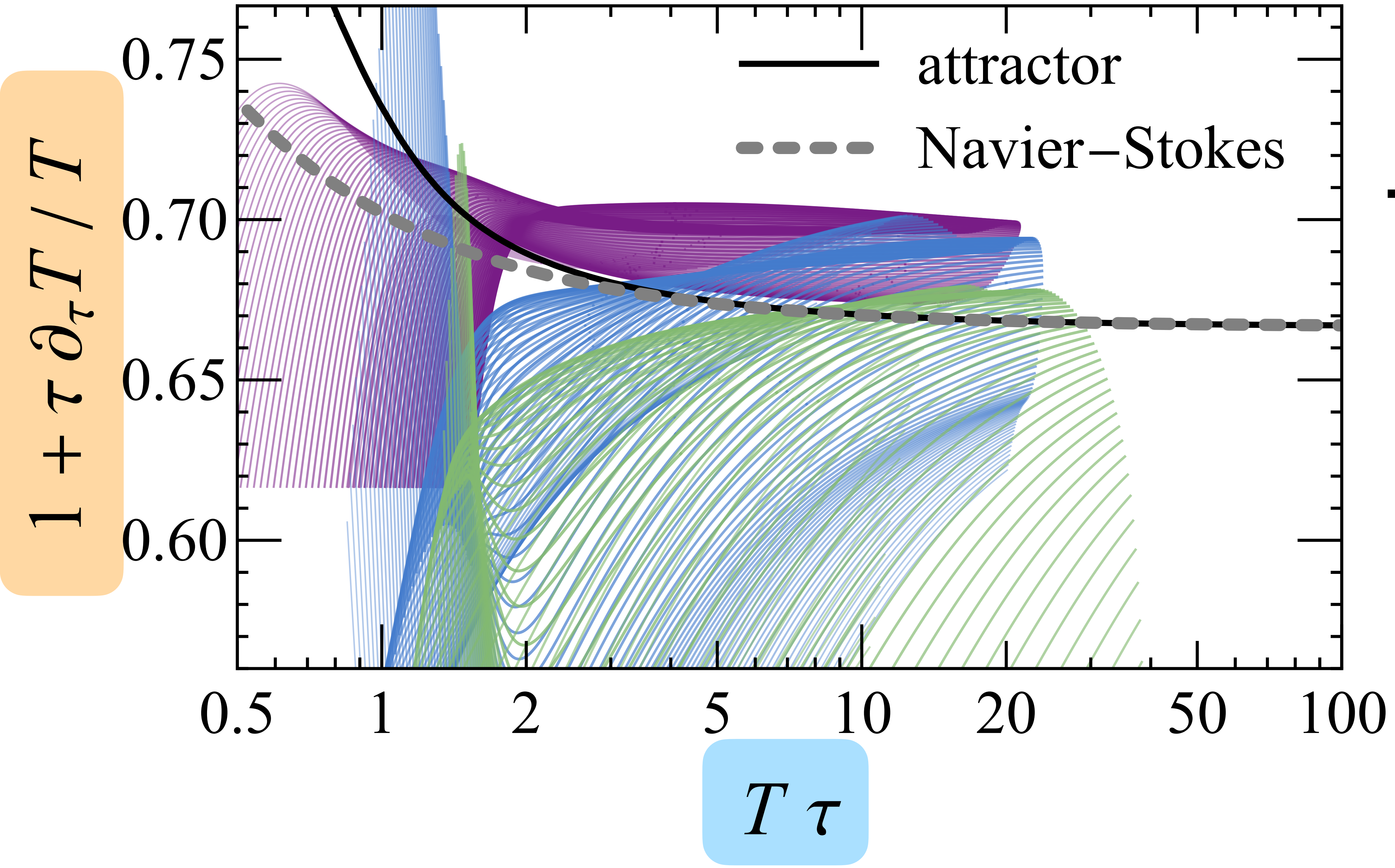
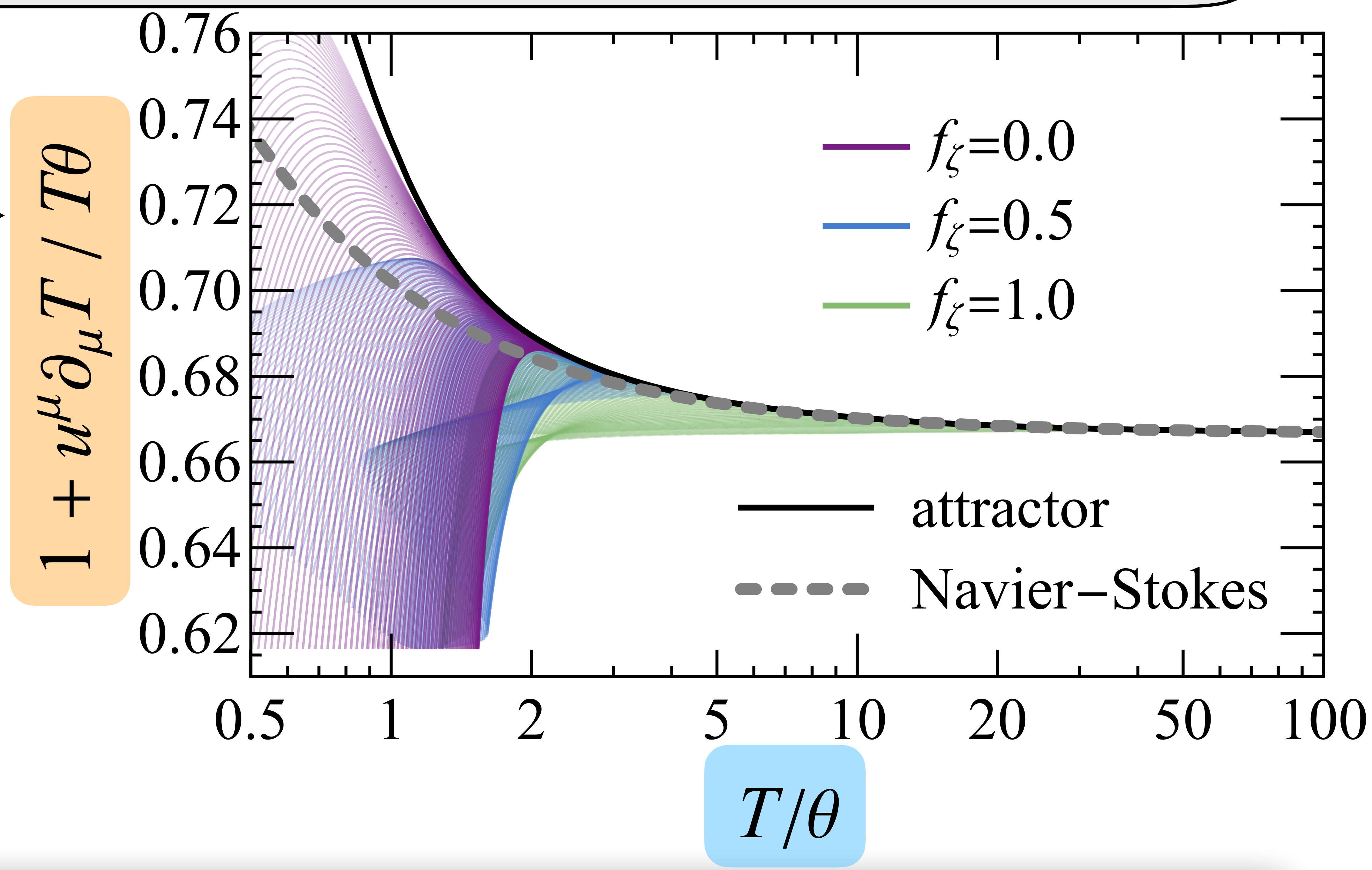


Substitute w/ Lorentz invariant variables



extended to boost-non-invariant systems

Shile Chen (陈诗乐) and SS, PhysRevC.111.L021902

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro

$$p^\mu \partial_\mu f(x,p) = \mathscr{C}[f]\,, \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu f(x,p)$$

usually *linearized* Boltzmann, e.g., Relaxation Time Approximation

see e.g. Strickland, JHEP 12 (2018) 128

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro

$$p^\mu \partial_\mu f(x,p) = \mathscr{C}[f]\,, \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu f(x,p)$$

[SUN] 2:00, *J. Hu*

*Special exact solution to **full non-linear** Boltzmann equation.*

Jin Hu (胡进), JHEP 2025 (2025) 07, 066

see also: Bazow, Denicol, Heinz, Martines, Noronha, Phys. Rev. Lett. 116 (2016) 022301

Thermalization in phase space distribution

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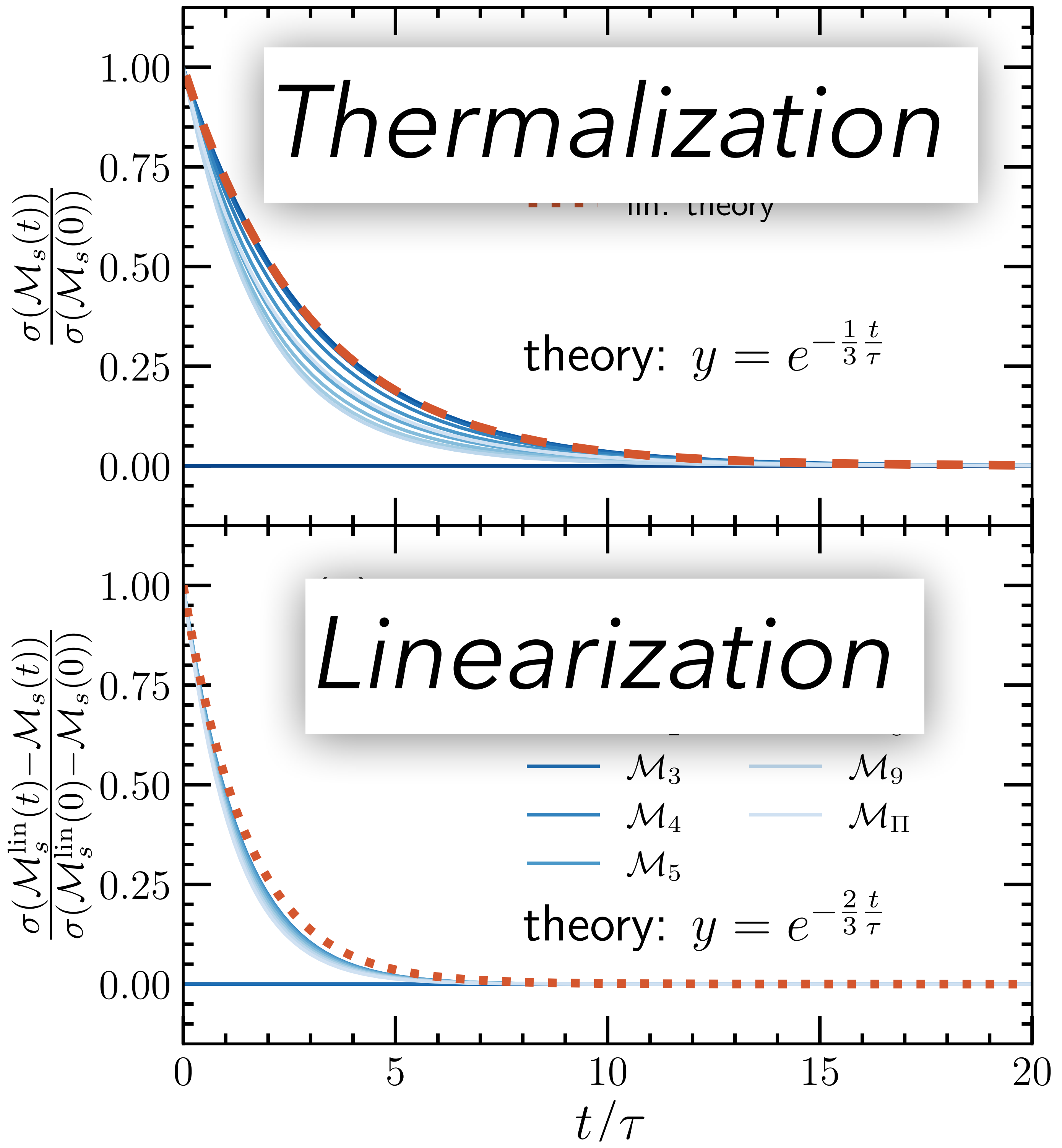
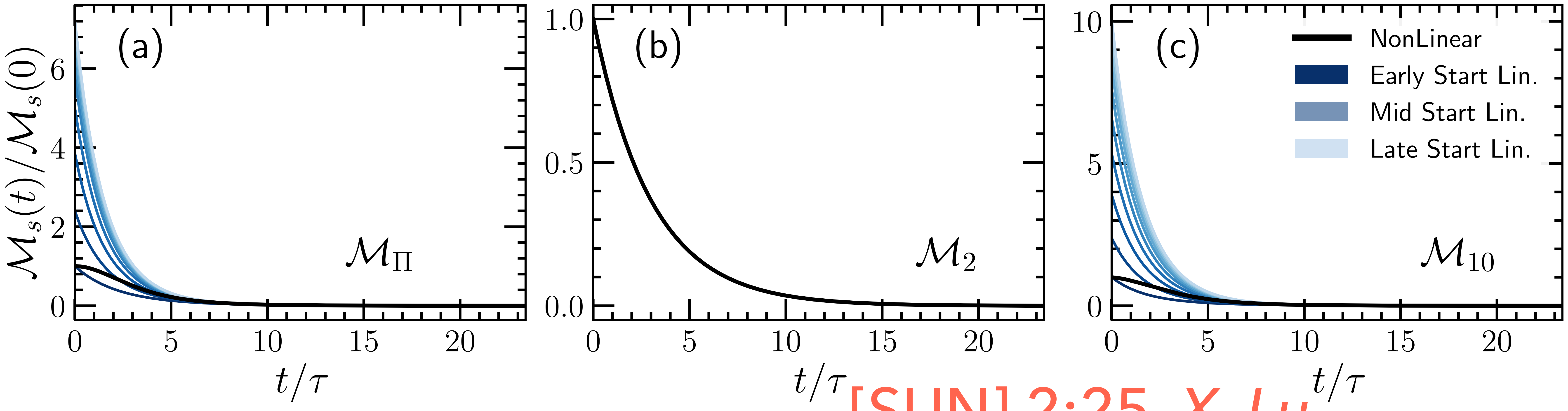




[SUN] 2:25, *X. Lu*

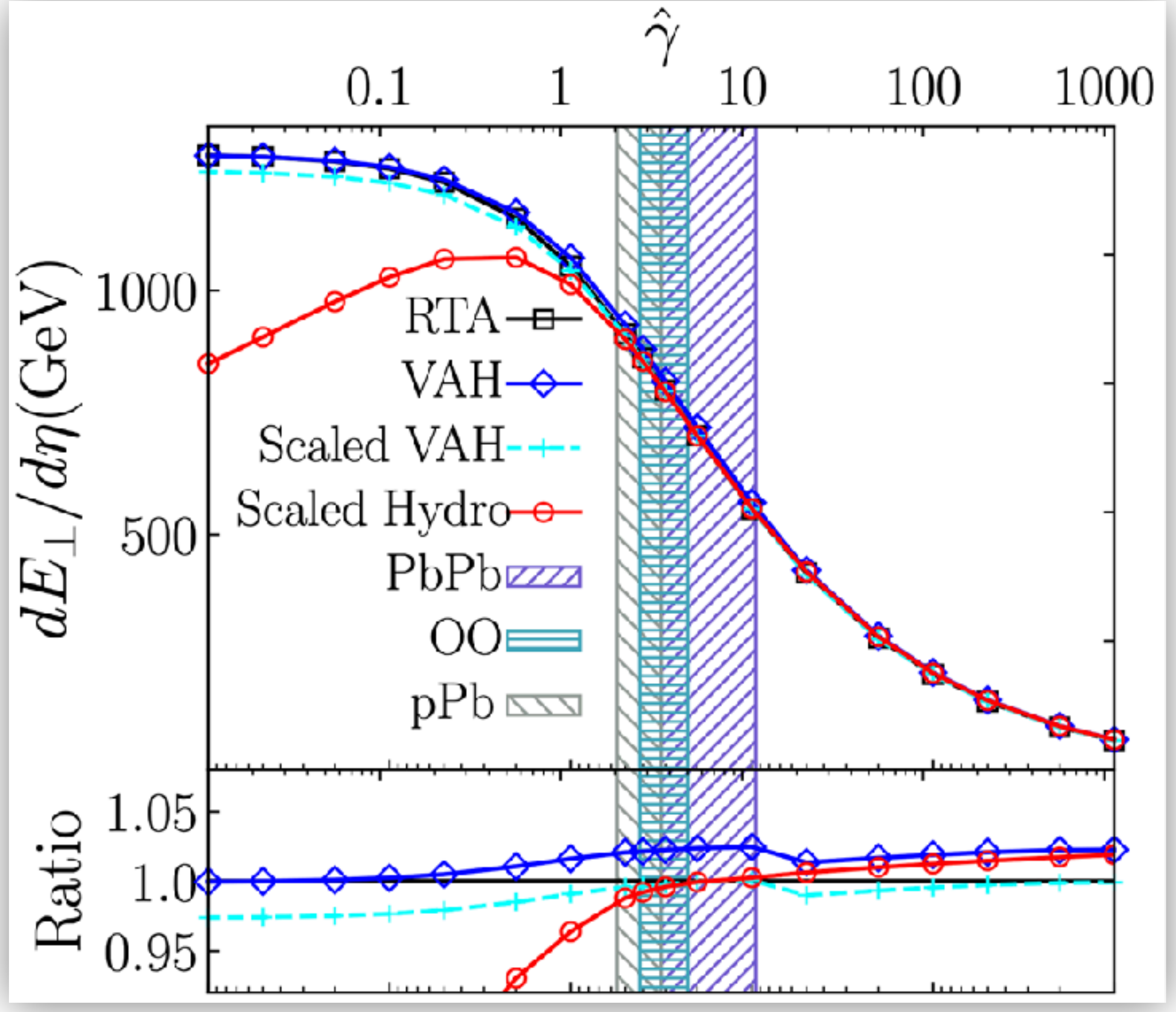
Linearization (evolution well approximated by linearized Boltzmann) before thermalization.

full non-linear Boltzmann: Xingjian Lu (路行简) and SS, 2507.14243+2509.23978

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro

$$p^\mu \partial_\mu f(x,p) = \mathscr{C}[f]\,, \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu f(x,p)$$



[MON] 9:40, *Y. Peng*

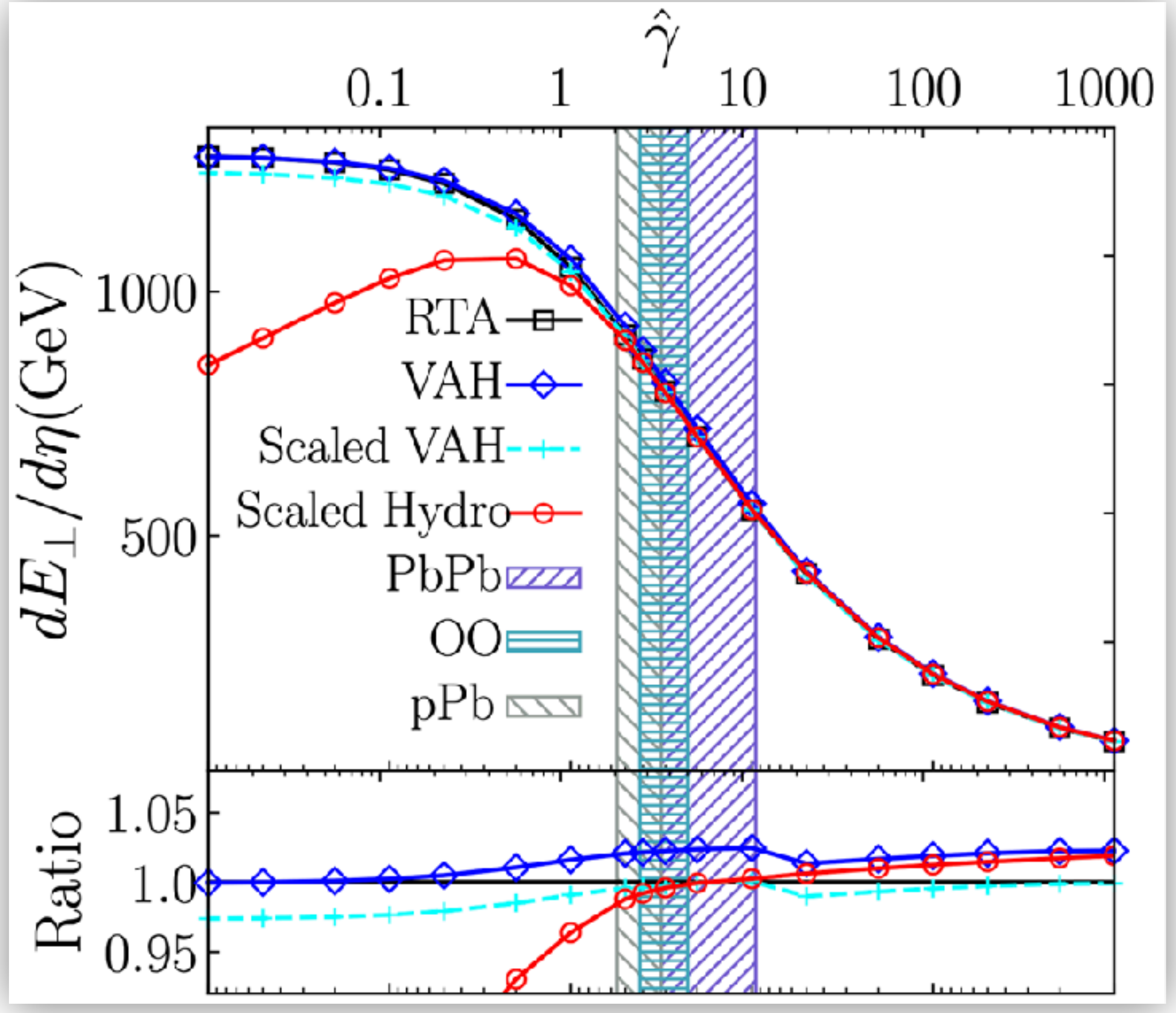
Anisotropic hydrodynamics needed in small systems

Yiyang Peng(彭亦扬), Ambrus, Werthmann, Schlichting, Heinz, Huichao Song(宋慧超), 2509.04431

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro

$$p^\mu \partial_\mu f(x,p) = \mathscr{C}[f]\,, \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu f(x,p)$$



How to probe the anisotropy?

heavy flavor: [SUN] 4:55, *Daria Prokhorova*

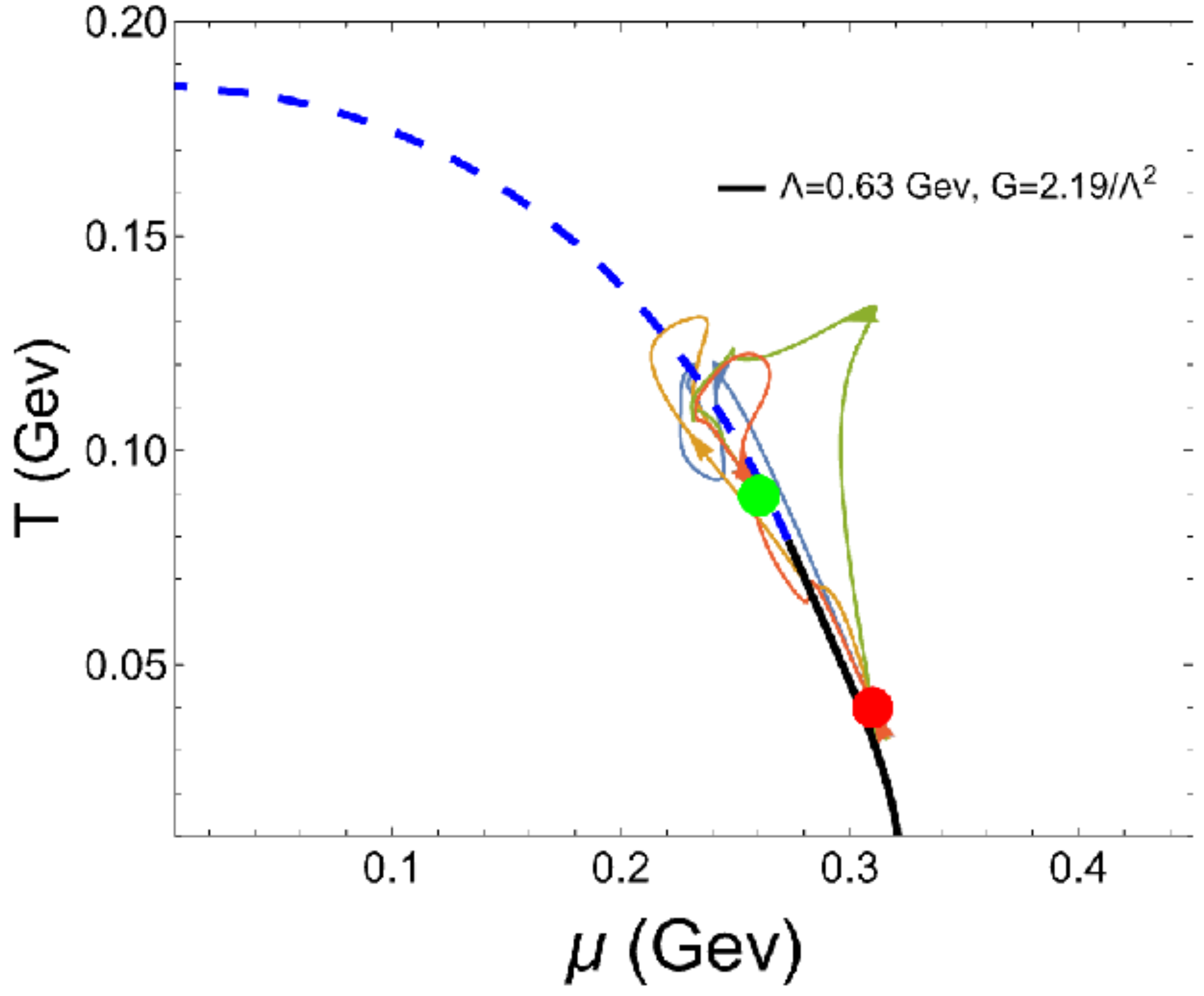
jets? [SUN] 2:45, *Yu Guo*(郭钰)

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro

$$p^\mu \partial_\mu f(x,p) = \mathscr{C}[f]\,, \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu f(x,p)$$

far from the CEP at right-hand side



Effect of a Critical Point?

[MON] 10:50, *Anping Huang* (黄安平)

transport theory in a more formal side

- Boltzmann eq. in RTA:

$$\mathbf{D}f = -\frac{f - f^{(0)}}{\tau}$$

$$\frac{\mathbf{D}}{1+\tau\mathbf{D}} f^{(0)} = -\frac{f - f^{(0)}}{\tau}$$

$$\boxed{\mathbf{D} = \partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla}}$$

$$\boxed{f^{(0)}(x,\boldsymbol{p}) = \exp\left[-\frac{p-\mu(x)}{T}\right]}$$

- Resummed diffusion eq.

$$\boxed{\int_\Omega \frac{\mathbf{D}}{1+\tau\mathbf{D}} n(x) = 0}$$

[NA, Rischke JHEP (2025)]

- All-order diffusion eq.: going beyond Fick's law

$$\Bigg[\underbrace{\left(\tilde{\partial}_t - \frac{1}{3}\tilde{\boldsymbol{\nabla}}^2\right)}_{\text{Fick's law}} + \left(-\tilde{\partial}_t^2 + \tilde{\partial}_t\tilde{\boldsymbol{\nabla}}^2 - \frac{1}{5}\tilde{\boldsymbol{\nabla}}^4\right) + \left(\tilde{\partial}_t^3 - 2\tilde{\partial}_t^2\tilde{\boldsymbol{\nabla}}^2 + \tilde{\partial}_t\tilde{\boldsymbol{\nabla}}^4 - \frac{1}{7}\tilde{\boldsymbol{\nabla}}^6\right) + \dots\Bigg] n(x) = 0$$

- Connection to SK-EFT:

$$\boxed{\mathcal{L}_{\text{EFT}}\left[\varphi_r, \varphi_a\right]_{A=0} = \underbrace{E[\varphi_r]}_{\text{deterministic hydro}}\varphi_a + \varphi_a \underbrace{F[\varphi_r, \partial]}_{\text{fixed by KMS}}\varphi_a + \mathcal{O}(\varphi_a^3)}$$

$$E[\varphi_r] \equiv \partial_\mu J^\mu[\varphi_r] = 0$$

[Crossley, Glorioso, Liu]
[Jensen, Pinzani-Fokeeva, Yarom]
[Heahl, Loganayagam, Rangamani]
[Grozdanov, Polonyi PRD (2015)]

hydrodynamization of two- and higer-point correlators.

proposed new conjecture of late-time behavior.

[SUN] 4:15, *Navid Abbasi*

Quark-Gluon Plasma:

A strongly coupled, quantum, (quasi)many-body system!

Ideally, full quantum simulation of QCD in 3+1D

Practical Example: strong coupling QED in 1+1D

– confinement, chiral condensate

Schwinger Hamiltonian

1+1D Schwinger model

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \, .$$

$|0\rangle$:empty

$|1\rangle$:occupied

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$$

field operators represented by matrices

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm} \, , \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0 \, .$$

$$H = \frac{1}{4a} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n + \frac{a\, g^2}{2} \sum_{n=1}^{N-1} L_n^2 \, .$$

Jordan-Wigner representation

Schwinger Hamiltonian

1+1D Schwinger model

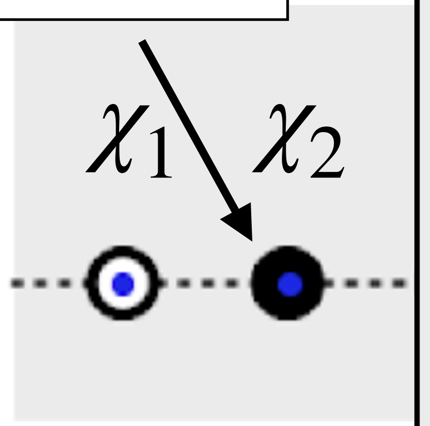
E: electric field

ic potential

nion field

$|0\rangle$:empty

$|1\rangle$:occupied



[TUE]9:15 X. Guo - quantum computation

Eigenstates of H: vacuum and (quasi)-particles

Time evolution:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -iH|\psi(t)\rangle$$

$$O(t) = \langle\psi(t)|\hat{O}|\psi(t)\rangle$$

$_{a,b}\delta(x-y)$

d operators

resented by

rices

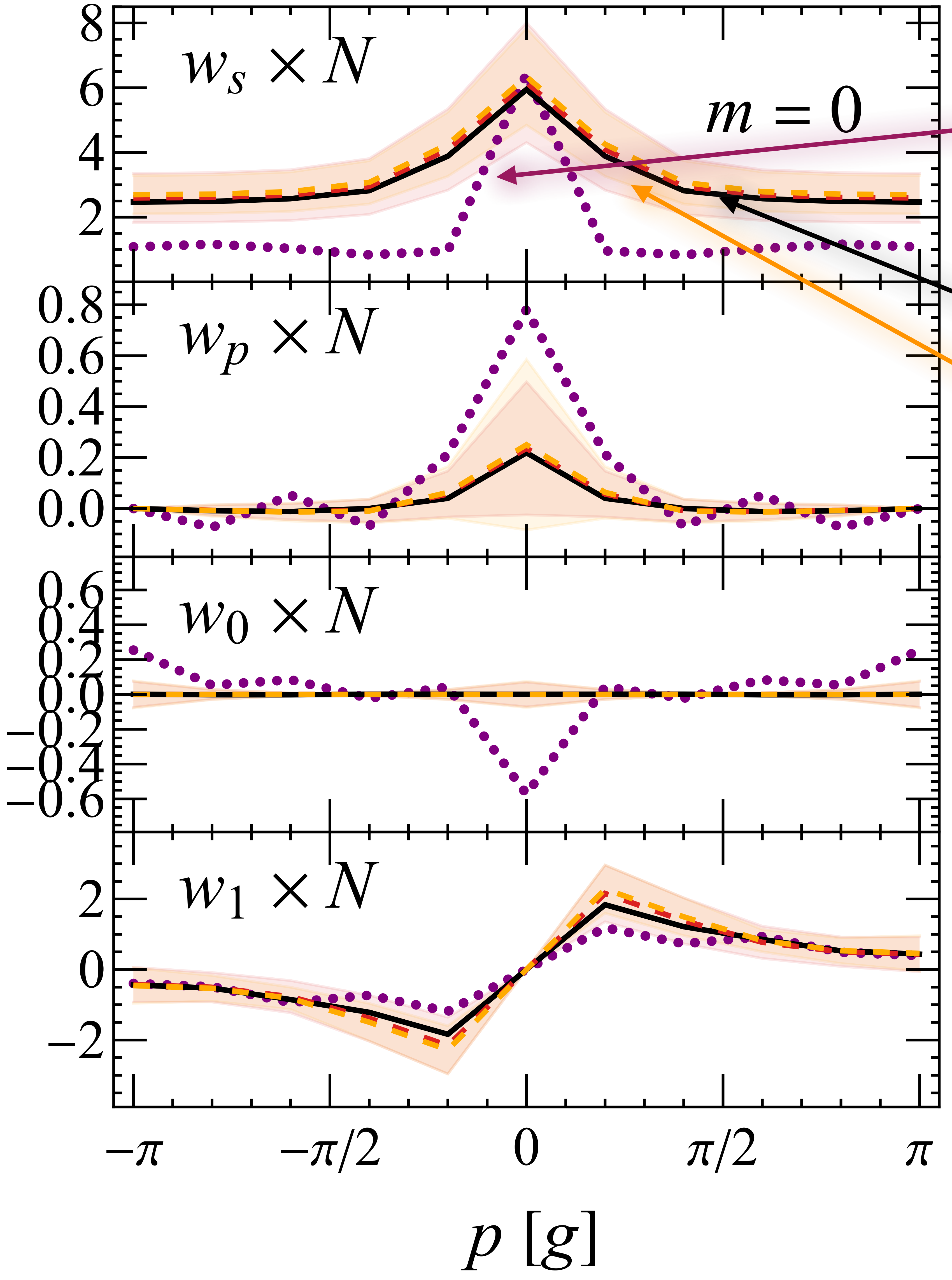
$\{\chi_n, \chi_m\} = 0$.

dan-Wigner

resentation

$n{=}1$ $n{=}1$ $n{=}1$

thermalization of quantum distribution function: Wigner function

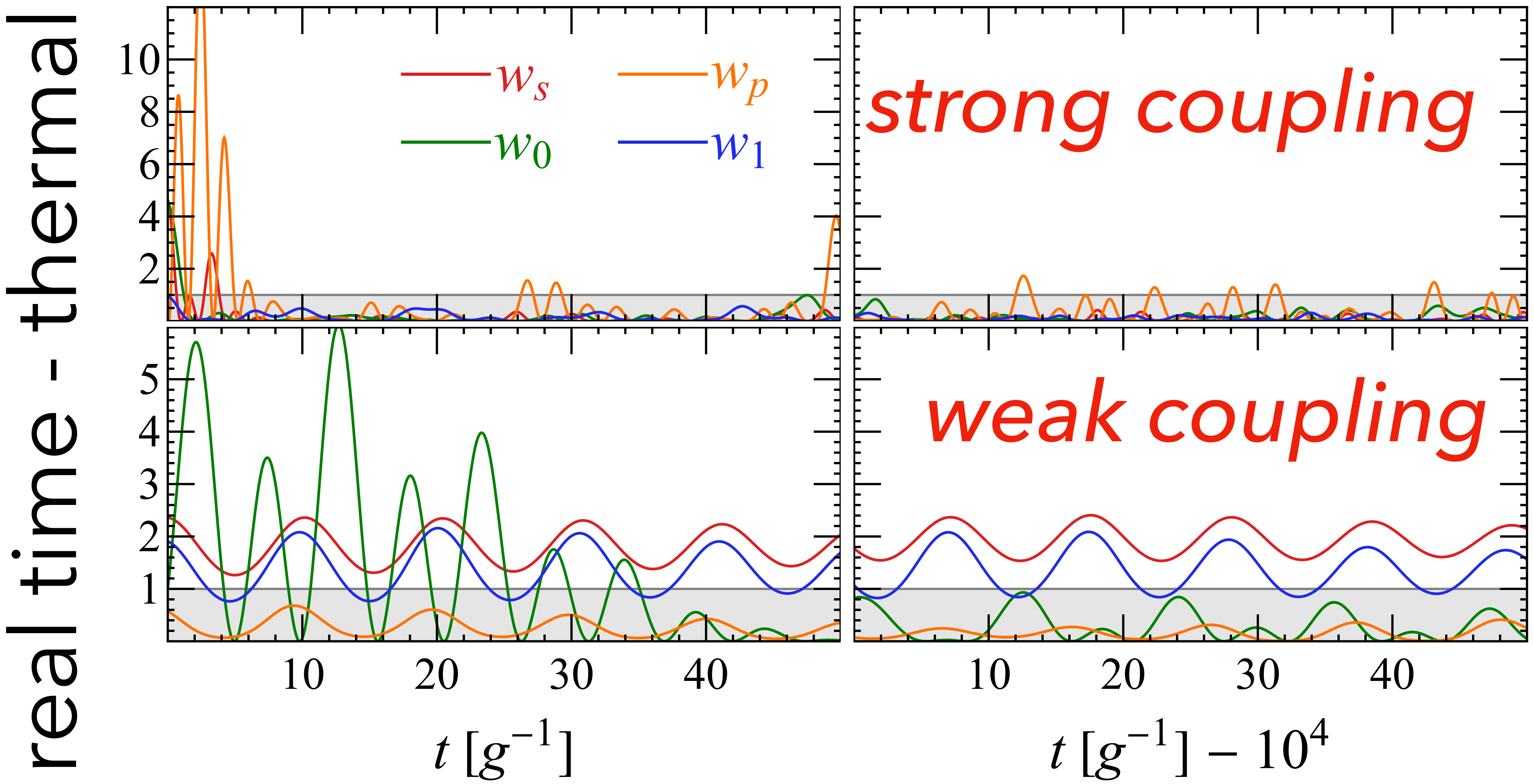


initial condition

$$\hat{W}_{\alpha\beta}(t,z,p) = \int \bar{\psi}_\alpha(z_+)U(z_+,z_-)\psi_\beta(z_-)\, e^{i\frac{py}{\hbar}}\, \mathrm{d}y$$

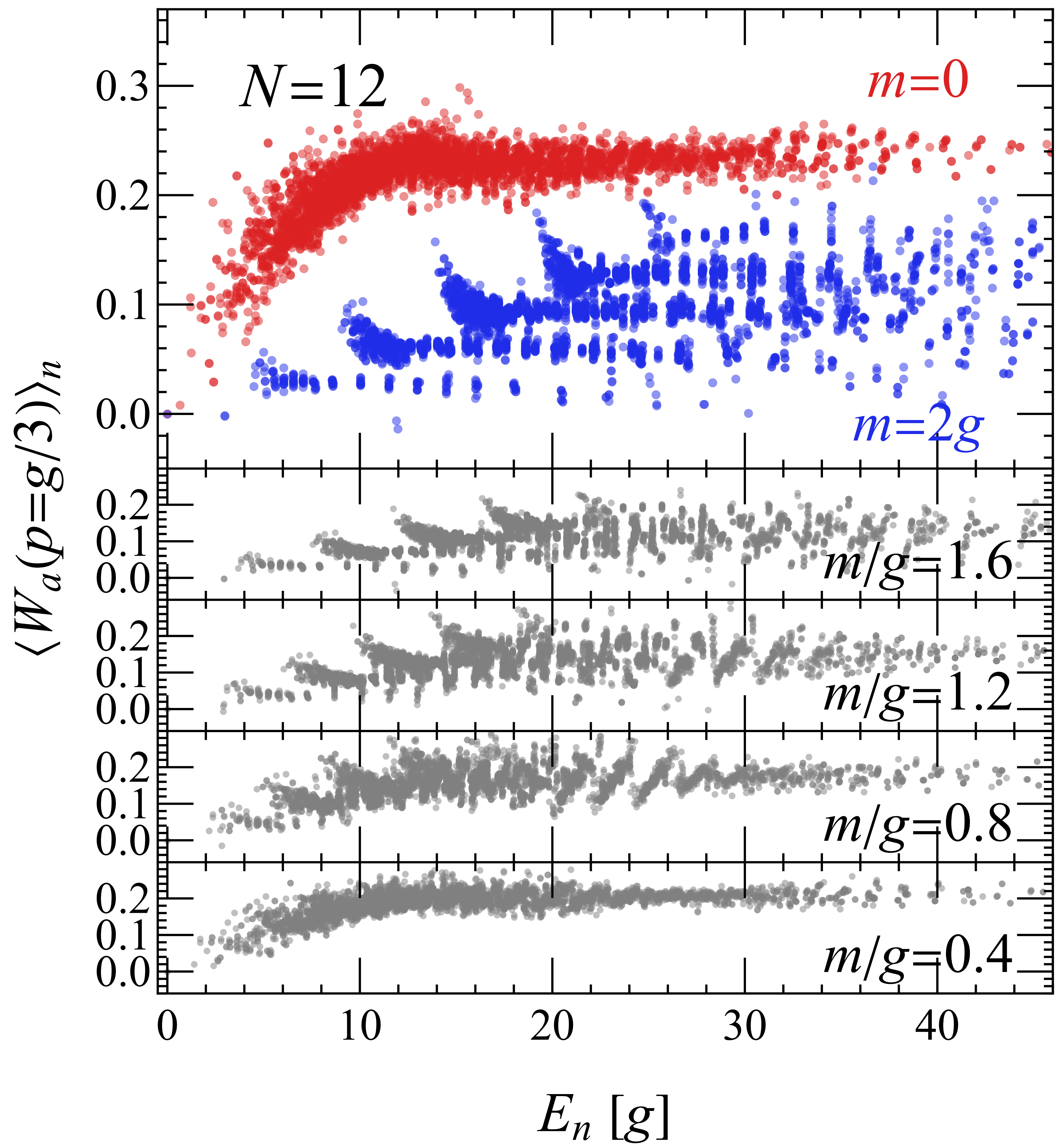
long-time average

MCE/CE average



Shile Chen, SS, and Li Yan, 2412.00662

thermalization of quantum distribution function: Wigner function



eigenstate thermalization hypothesis

$$\langle n|\hat{O}|n\rangle \approx f_O(E_n)$$

$$\sum_n p_n \langle n|\hat{O}|n\rangle \approx f_O(\sum_n p_n E_n)$$

general pure state: $|c_n|^2$

thermal: $e^{-\beta E_n}/Z$

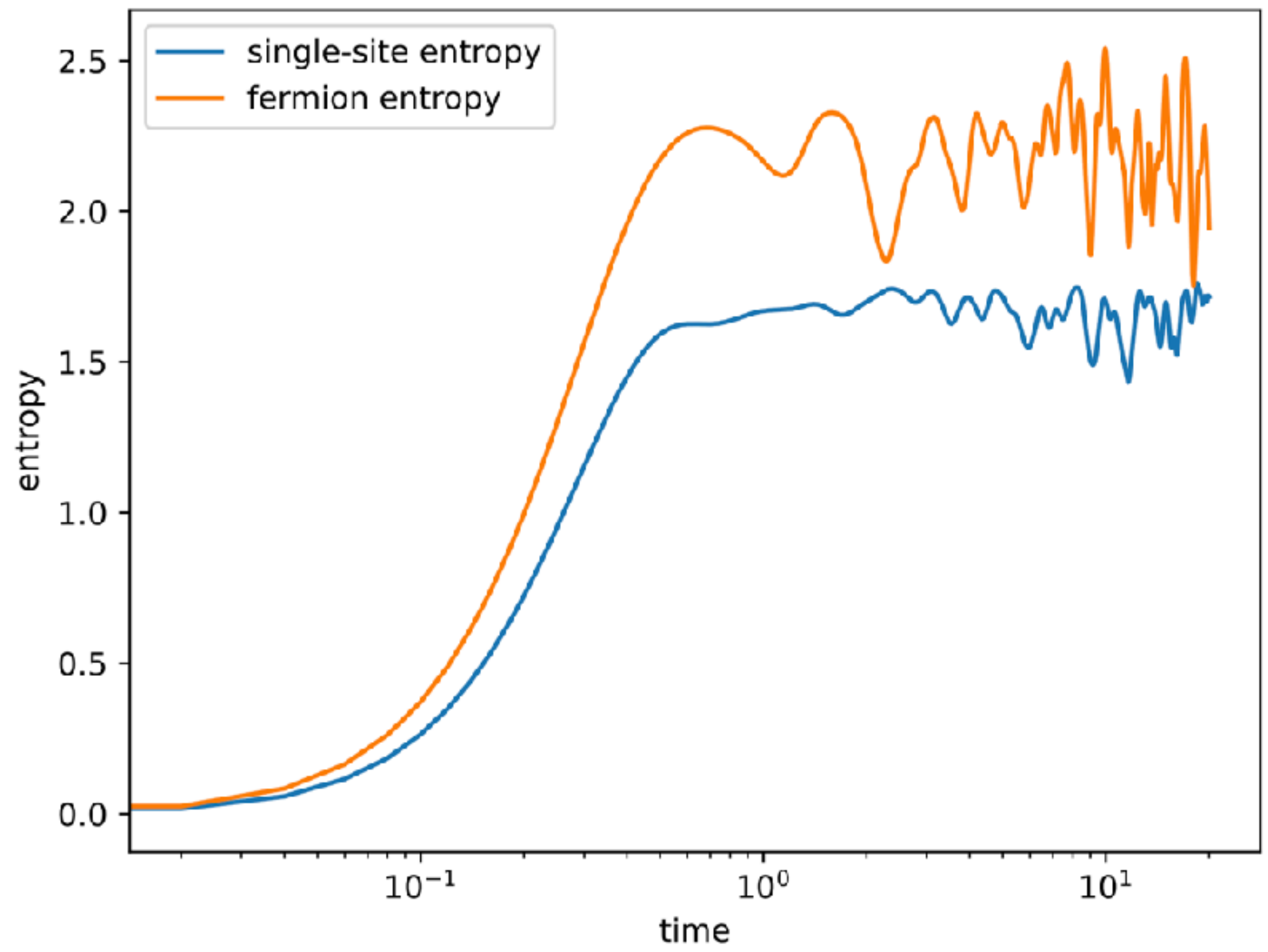
$$\langle O\rangle_{PS} \approx \langle O\rangle_{th}$$

if $\langle E\rangle_{PS} = \langle E\rangle_{th}$

Quantum Thermalization ⇆ ETH

Shile Chen, SS, and Li Yan, 2412.00662

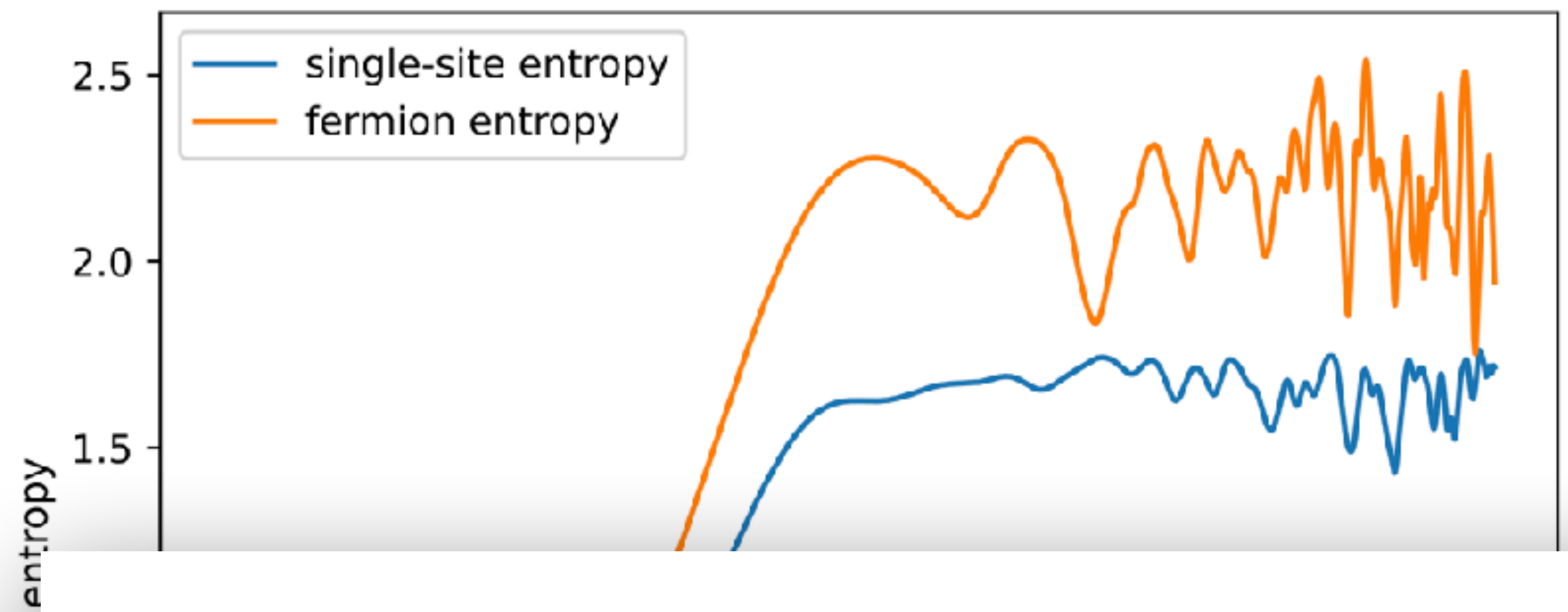
testing thermalization in different quantum systems



entropy production in a quantum simulation of QC_2D

Phys.Rev.D 112 (2025) 3, 034511
Zhen-Xuan Yang, Hidefumi Matsuda, Xu-Guang Huang, Kouji Kashiwa

testing thermalization in different quantum systems



entropy production in a quantum simulation of QC_2D

Eigenstate thermalization in a SU(2) gauge theory:

Xiaojun Yao(姚晓骏), *Phys.Rev.D* 108 (2023) L031504

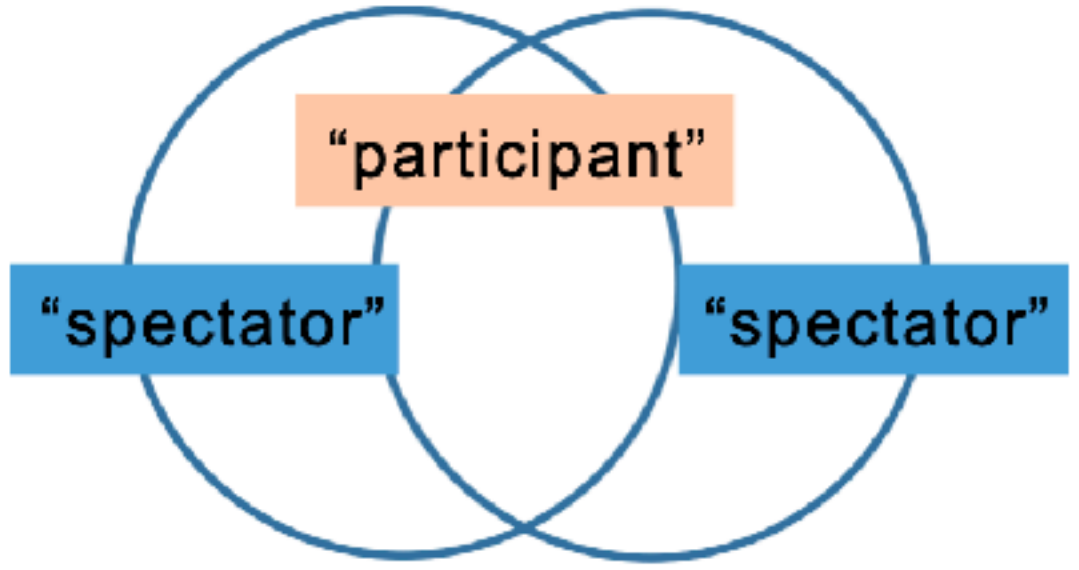
Ebner, Muller, Schafer, Seidl, Yao, *Phys.Rev.D* 109 (2024) 014504

Eigenstate thermalization in 1+1D SU(2) gauge theory with Fermion:

Das, Ebner, Kadam, Raychowdhury, Schafer, Yao, 2509.18269

connecting quantum and classical entropies

Quantum thermalization in inclusive scatterings



- "Spectators" are traced away → reduced density matrix

$$|\psi\rangle_p = \sum_n \frac{1}{\sqrt{\mathcal{D}}}|n, x, Q^2\rangle\,, \quad \mathcal{D}\text{: dimension of Fock space}$$

Renormalization scale

$$\rho_p = |\psi\rangle_{pp}\langle\psi|$$

$$\downarrow$$

$$S_{En} = -\mathrm{tr}[\rho_p \log \rho_p]$$

$$\downarrow$$

$$\log \mathcal{D} \approx \log(xG(x, Q^2))$$

$$Q \gg \Lambda_{QCD}\,, \quad x \ll 1$$

$\ln\frac{1}{x}$++ $\downarrow$ $\downarrow$ t++

time scale evolution

$$N_{mn} = N(E)\delta_{mn} + e^{-S} r_{mn}$$

$$\downarrow \text{ ETH}$$

$$xG(x, Q^2) \equiv \langle\psi|\hat{N}|\psi\rangle = \frac{\mathcal{D}}{2}$$

no interference among states of different n_{parton}

participants are the quantum subsystem of interest

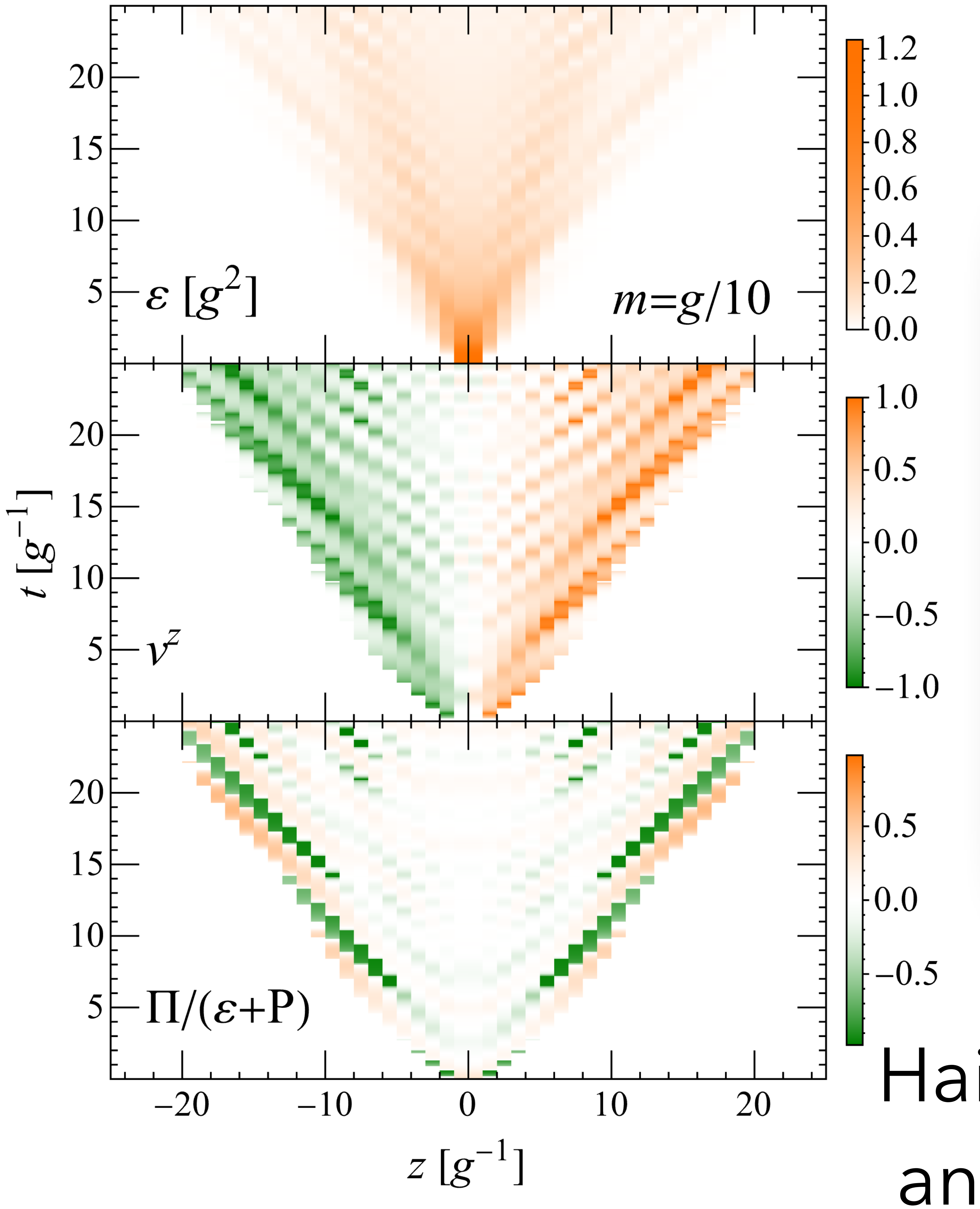
[SUN] 5:35, *C. Liang*

Chenxi Liang (梁晨曦), Li Yan, Xiaojian Du

emergence of Bjorken flow

$$|\Psi(t=0)\rangle = e^{i\omega\int_{-w/2}^{+w/2}\bar{\psi}\psi(z)\mathrm{d}z}|\mathrm{vac}\rangle$$

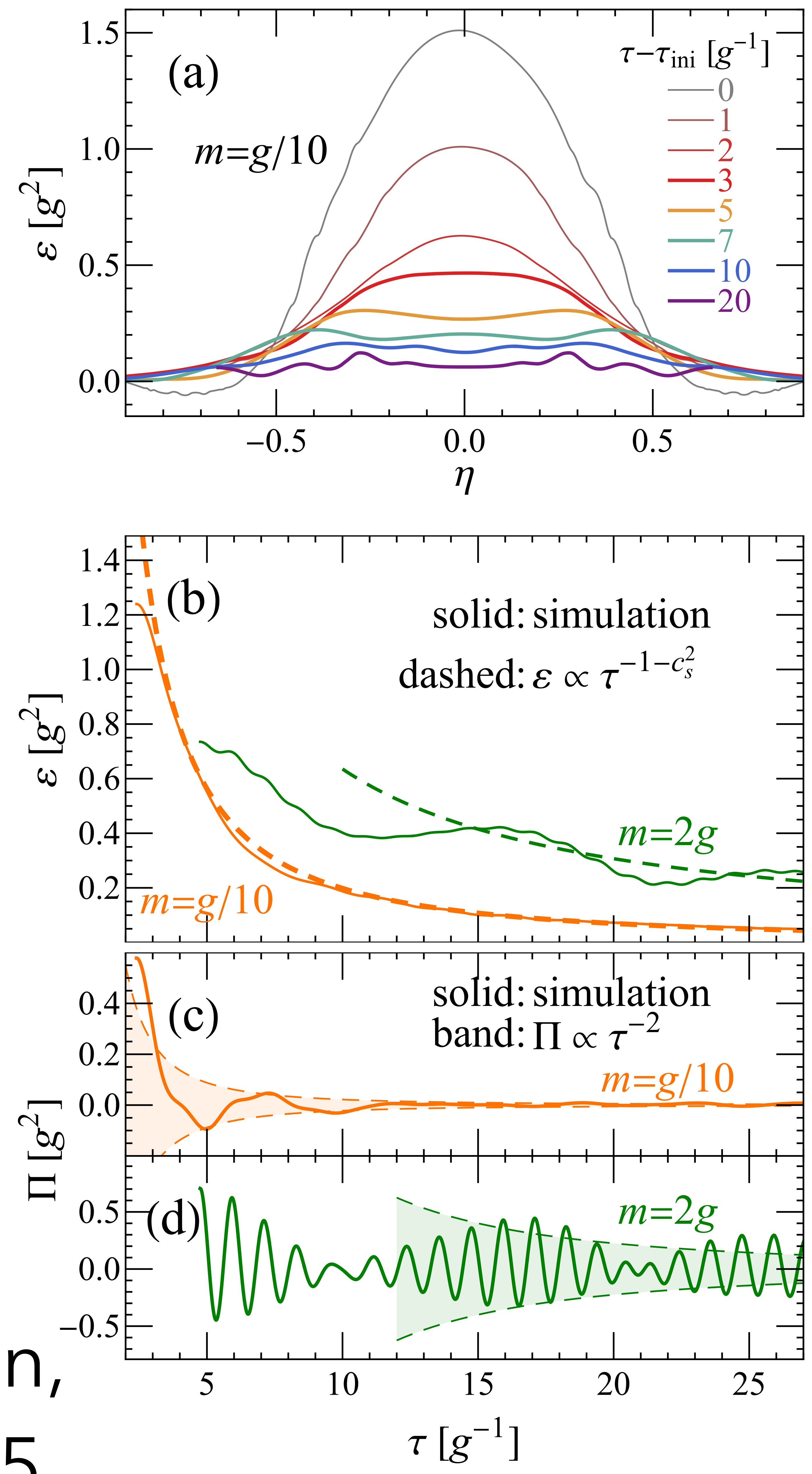
vacuum + [excitation @ center]



$$\langle\Psi(t)|\hat{T}^{\mu\nu}(x)|\Psi(t)\rangle$$
$$= (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu}$$

emergence of Bjorken flow

- *spread within light cone*
- $v \approx z/t$
- *rapidity structure*
- $\varepsilon \propto \tau^{-1-c_s^2}$
- *decreasing of* Π



[SUN] 5:15, *H. Shao*

Haiyang Shao (邵海洋), Shile Chen, and SS, 2509.10835+2509.10855

summary

- hydrodynamics attractors – common behavior of evolution
- transport theory – thermalization and hydrodynamization at microscopic
- quantum simulation of microscopic QFTs