Extension of Bargmann-Michel-Telegdi equation and Spin correlation

in Martin-Siggia-Rose (MSR) approach of effective field theory (EFT)

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Outline

- Introduction to spin polarization in relativistic heavy ion collisions
- Extension of Bargmann-Michel-Telegdi equation from spin hydrodynamics
- Spin correlations from modified BMT equations
- Summary and discussion

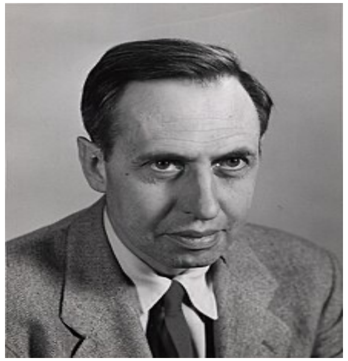
Introduction

Discovery of spin in physics

This year marks the 100th anniversary of the discovery of spin and the 20th anniversary of the proposal for spin polarization in relativistic heavy-ion collisions.

Wolfgang Pauli

Ralph Kronig



George Uhlenbeck

Samuel Goudsmit

Llewellyn H. Thomas

Wolfgang Pauli

1924 — two valued -ness not describable classically

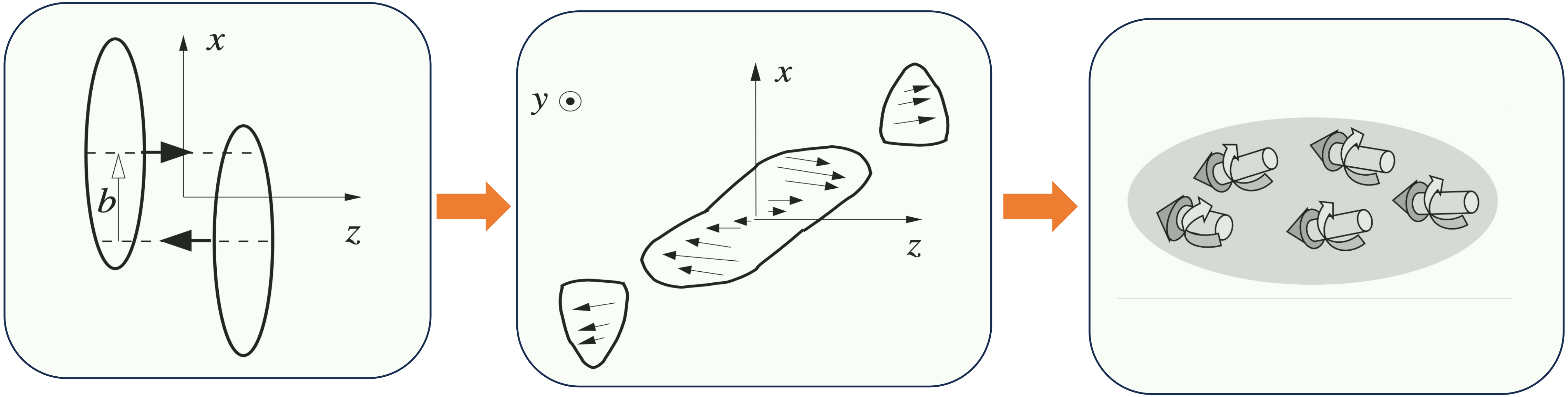
1925 — self-rotation of the electron

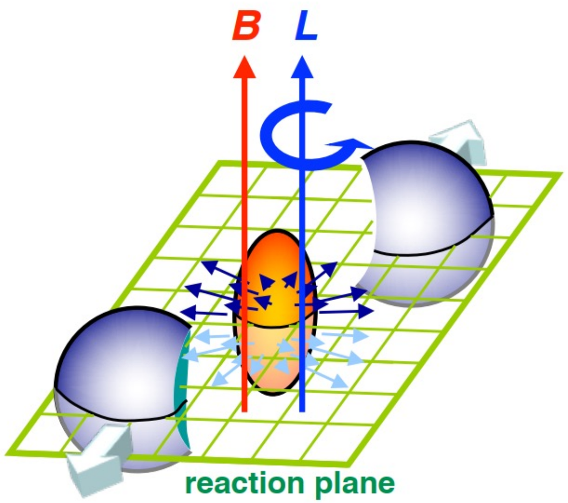
1925 — Published the first paper on spin

1926 — Thomas procession

1927 — spin in quantum mechanics

Early Pioneer work on spin polarization in heavy ion collisions





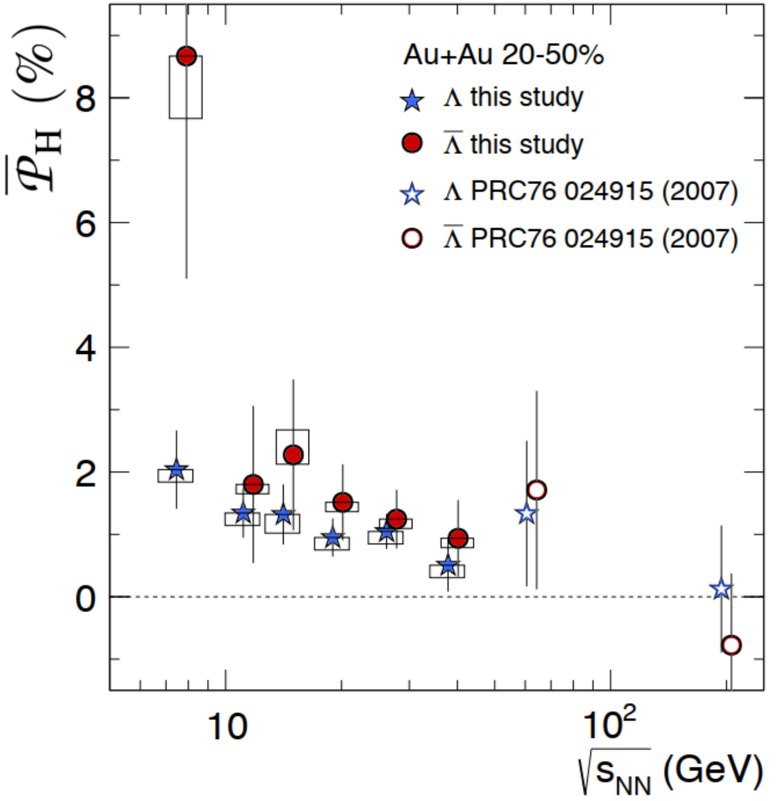
- Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.

- Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.

  Liang, Wang, PRL (2005); PLB (2005);
  Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\overline{\Lambda}$ hyperons

STAR, Nature 548, 62 (2017)

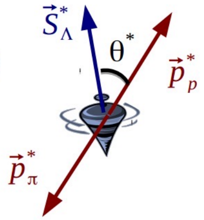


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1+\alpha\mathbf{P}_{\mathbf{\Lambda}}\cdot\mathbf{p}_{\mathbf{p}}^*)$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$\Lambda \to p + \pi^+$
(BR: 63.9%, c τ ~7.9 cm)

- **Estimation given by**

Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)

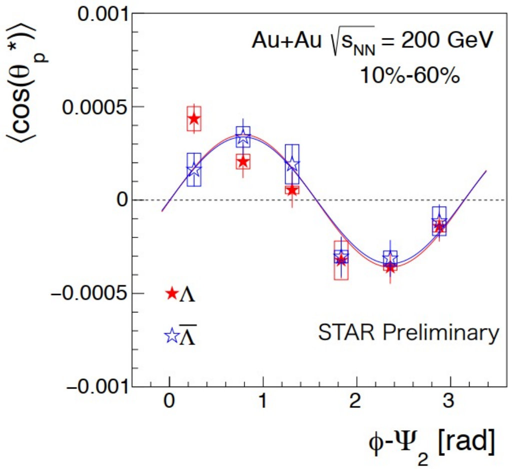
$$\mathbf{P}_\Lambda \simeq \frac{\omega}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}$$
$$\mathbf{P}_{\overline{\Lambda}} \simeq \frac{\omega}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}$$

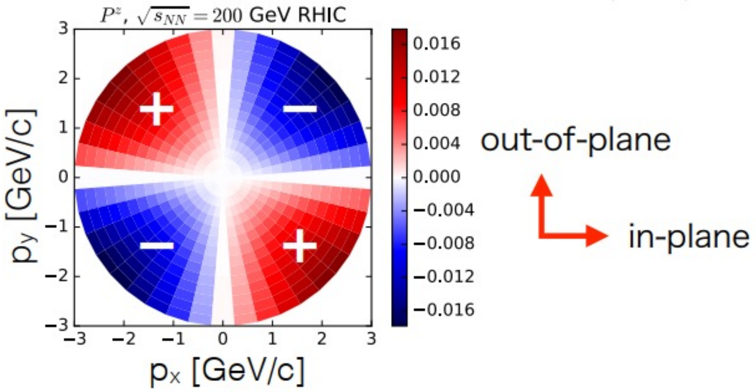
- **ω = (9 ± 1)x10^{21}/s, greater than previously observed in any system.**
- **QGP is most vortical fluid so far.**
- **Global polarization can be well described by thermal vorticity.**

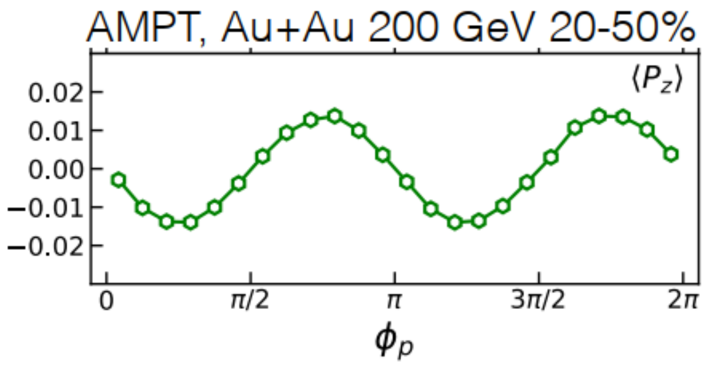
Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

Local polarization and shear induced polarization

- **Local polarization cannot be explained by thermal vorticity alone.**

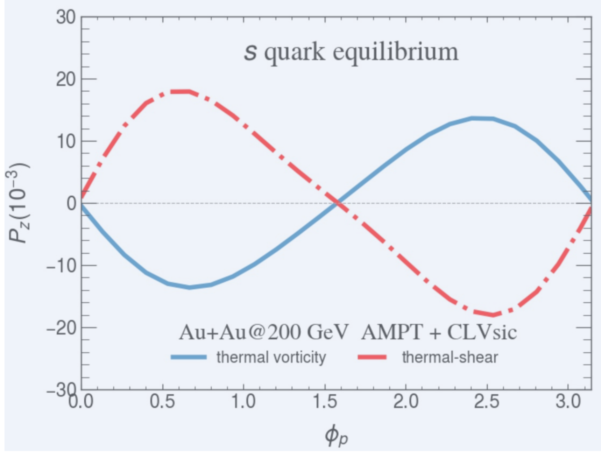




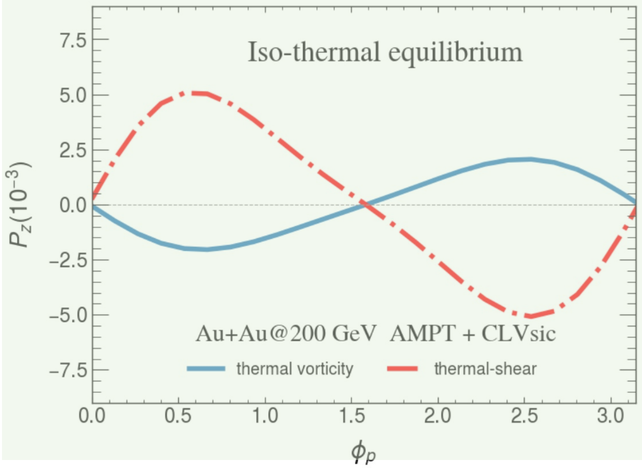


STAR, PRL 123, 132301 (2019) UrQMD : Becattini, Karpenko, PRL (2018) AMPT:Xia, Li, Tang, Wang, PRC (2018)

- **Shear induced polarization plays a crucial role in understanding the data.**



s quark equilibrium:
The spin of Λ hyperons is assumed to be primarily carried by the constituent s quark. One needs to take the s quark's mass instead of mass of Λ in the simulation.
Fu, Liu, Pang, Song, Yin, PRL 2021

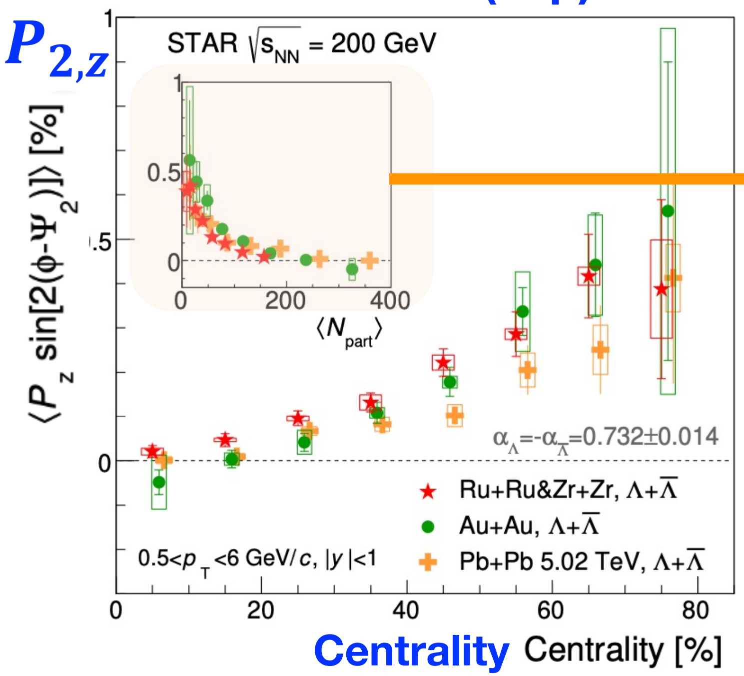


Isothermal equilibrium:
The temperature of the system at the freeze-out hyper-surface is assumed to be constant.
Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

Puzzle: Local polarization in pPb systems

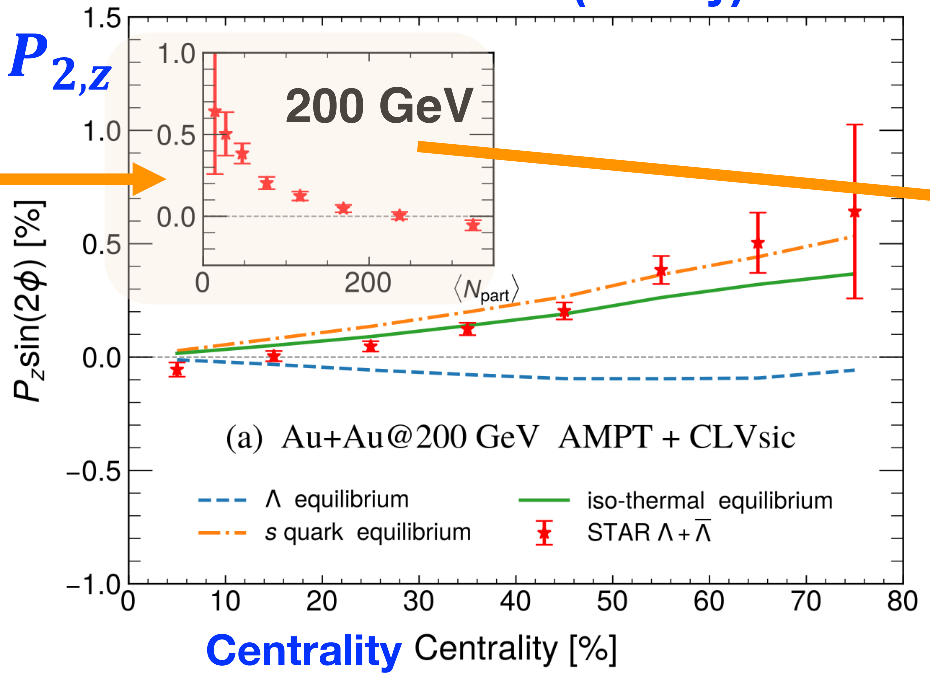
Also see: Cong Yi's talk at 9:25 AM in 10/26/2025 in Parallel II

AA collisons (exp)



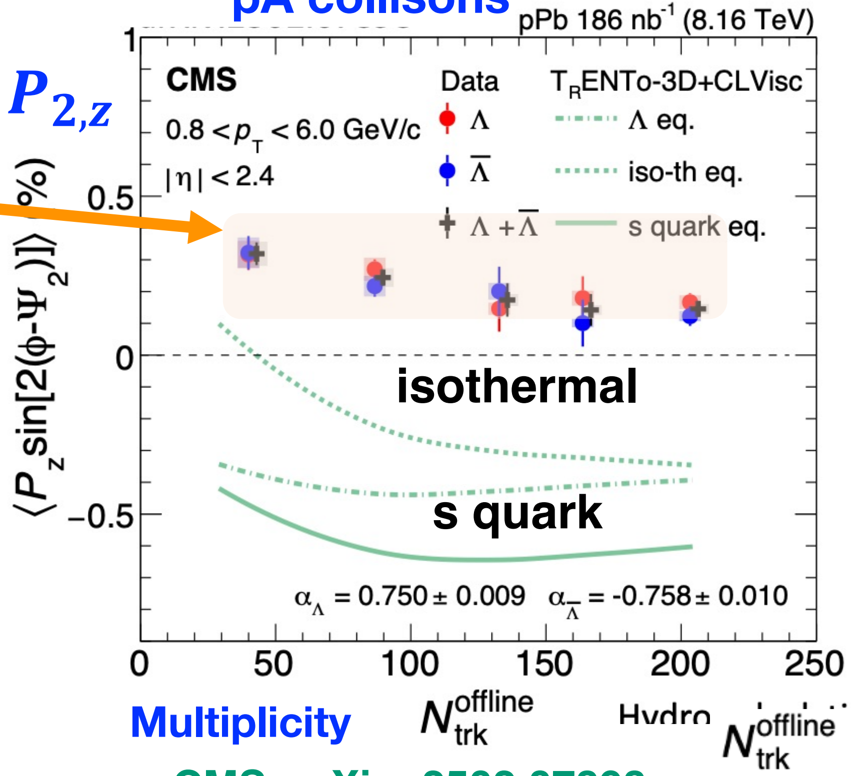
STAR, PRL 123, 132301 (2019)
ALICE, PRL 128, 172005 (2022)

AA collisons (theory)



Yi, et. al., arXiv: 2509.00377

pA collisons



CMS, arXiv: 2502.07898

Prediction: Yi, Wu, Zhu, SP, Qin, PRC (2025)

- P_z in pA systems is closely aligned with that observed in AA systems.
- Hydrodynamic simulations across three scenarios fail to describe the data.

Weak system and collision energy dependence

A key question:

What is the evolution equation for spin?

How is it connected to well-known spin phenomena?

Extension of the Bargmann-Michel-Telegdi (BMT) equation

S. Fang, Kenji Fukushima, SP, D. L. Wang, arXiv:2506.20698

Spin dynamics in classical electrodynamics

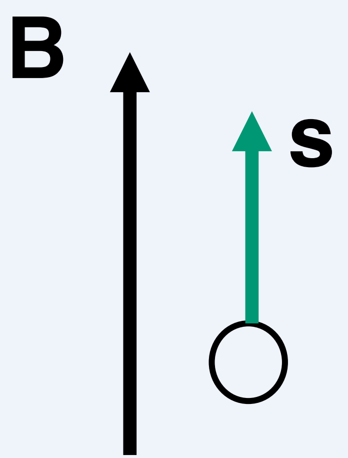
Let us consider a spin-1/2 particle moving in a magnetic field,

$$H = -\gamma \boldsymbol{B} \cdot \boldsymbol{s}$$

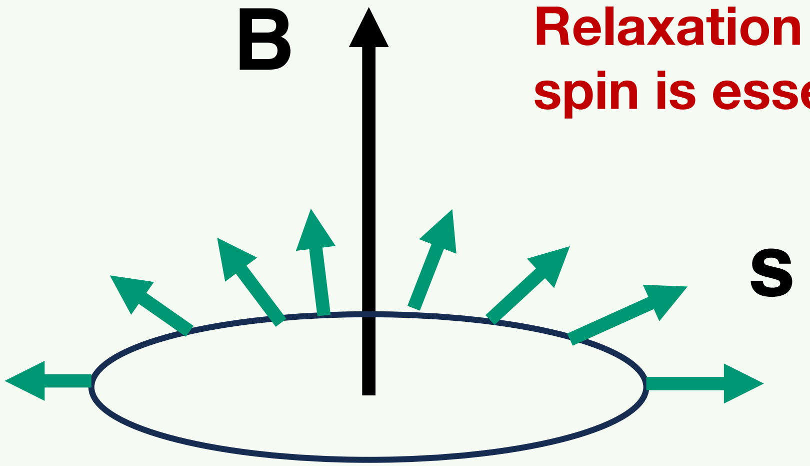
γ:**gyromagnetic ratio**

$$\frac{\partial \boldsymbol{s}}{\partial t} = -\gamma \boldsymbol{B} \times \boldsymbol{s}$$

We expect:

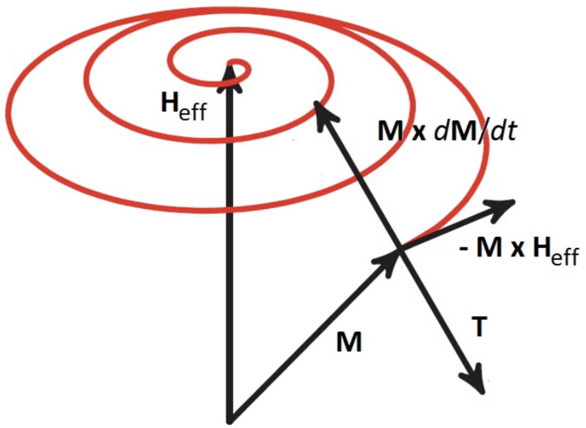


Solution:
The spin will not be parallel to B field with some initial conditions.



Relaxation for spin is essential!

Landau-Lifshitz-Gilbert equation for spin



$$
\begin{aligned}
\frac{\partial \boldsymbol{s}}{\partial t} &= -\gamma \boldsymbol{B} \times \boldsymbol{s} + \frac{\alpha}{s}\gamma[\boldsymbol{s} \times (\boldsymbol{s} \times \boldsymbol{B})] \\
&\simeq -\gamma \boldsymbol{B} \times \boldsymbol{s} + \boxed{\frac{\alpha}{s}\gamma\left(\boldsymbol{s} \times \frac{\partial \boldsymbol{s}}{\partial t}\right)}
\end{aligned}
$$

Effective relaxation for spin

Lesson: relaxation (dissipative) effects are crucial for spin dynamics!

L. Landau and E. Lifshitz, Phys. Z. Sowjetunio, vol. 8, pp. 153-169, 1935
T. Gilbert, Phys Rev, vol. 100, pp. 1243, 1955.
Textbook: L. D. Landau and E. M. Lifshitz, Statistical physics. 2

Relativistic extension for spin

$$s^\mu \xrightarrow{\text{Rest frame}} (0, \boldsymbol{s}) \qquad s \cdot u = 0$$

$$\frac{\partial \boldsymbol{s}}{\partial t} = \boxed{-\gamma \boldsymbol{B} \times \boldsymbol{s}}$$

$$\frac{ds^\mu}{dt} = \boxed{\gamma F^{\mu\nu} s_\nu} + \boxed{\kappa u^\mu},$$

Can be easily derived by contracting this equation with u

Original BMT equation

$$\dot{s}^{\mu} = \boxed{\gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu}} \boxed{- u^{\mu} s^{\nu} \dot{u}_{\nu}}$$

Spin-EM fields coupling

Thomas procession

The key to get the correct relativistic correction for hydrogen

+ possible relaxation (dissipative) effects

$$\dot{a} = da/dt \qquad \Delta^{\mu\rho} = g^{\mu\rho} - u^{\mu} u^{\rho}$$

Textbook: Jackson, classical electrodynamics

Spin evolution in relativistic heavy ion collisions

What is the BMT equation for a relativistic many-body system in the presence of rotation (vorticity)?

Our strategy:

Conservation equations

Energy momentum

$$\partial_\mu \Theta^{\mu\nu} = 0$$

Charge number

$$\partial_\mu j^\mu = 0$$

Total angular momentum

$$\partial_\lambda J^{\lambda\mu\nu} = 0$$

+

Second law of thermodynamics

$$\partial_\mu \mathcal{S}^\mu \geq 0$$

Spin tensor

$$s^\mu \xrightarrow{\text{Rest frame}} (0, \boldsymbol{s})$$

Two different choices for spin tensor operators:

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\gamma^\lambda[\gamma^\mu, \gamma^\nu]\psi$$

Anti-symmetric on $\mu\nu$,
NOT Hermitian
d.o.f for spin tensor is 6.
Commonly used in our field

Hermitian,
Also can be derived by using EoS for feilds

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\{\gamma^\lambda, [\gamma^\mu, \gamma^\nu]\}\psi$$

Total anti-symmetric,
Hermitian
d.o.f for spin tensor is 3.
Commonly used in many other fields
See cosmology textbook by Weinberger

Total anti-symmetry spin tensor

We introduce the total anti-symmetric spin tensor in spin hydrodynamics:

$$\Sigma^{\lambda\mu\nu} = u^{\lambda}\boxed{S^{\mu\nu}} + u^{\mu}\boxed{S^{\nu\lambda}} + u^{\nu}\boxed{S^{\lambda\mu}} + \mathcal{O}(\partial^1)$$

Spin density (tensor) $\boxed{S^{\mu\nu} = -S^{\nu\mu}}$ $\boxed{S^{\mu\nu}u_{\nu} = 0}$

Frenkel-Mathisson-Pirani condition (1926)

Spin density (vector) $s^{\mu} := -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}u_{\nu}S_{\rho\sigma}$

Rest frame $\longrightarrow$ $(0, \boldsymbol{s})$

Entropy production rate

$$
\begin{aligned}
\partial_\mu \mathcal{S}^\mu \quad = \quad & (h^\mu - \mathcal{H}\nu^\mu)(\partial_\mu \beta + \beta \dot{u}_\mu) \\
& + \beta \pi^{\mu\nu} \partial_{<\mu} u_{\nu>} + \phi^{\mu\nu}(2\beta\omega_{\mu\nu} + \partial_{[\mu}\beta u_{\nu]}) \\
& \boxed{+2\beta\omega_{\mu\nu} S^{\lambda\mu}\partial_\lambda u^\nu + q^\mu(\partial_\mu \beta - \beta \dot{u}_\mu)} \\
& + \mathcal{O}(\partial^3)
\end{aligned}
$$

Most of the terms can easily be written as squared terms, but …

It is challenging to ensure that the entropy increases with the total antisymmetric spin tensor!

Hongo, Huang, Kaminski, Stephanov, Yee, JHEP 2022

Cao, Hattori, Hongo, Huang, Taya, PRD 2022

…..

Spin correction

$$\partial_\mu(\mathcal{S}^\mu + \delta\mathcal{S}^\mu) = (h^\mu - \mathcal{H}\nu^\mu + \boxed{h_s^\mu})(\partial_\mu\beta + \beta\dot{u}_\mu) + \beta(\pi^{\mu\nu} + \boxed{\pi_s^{\mu\nu}})\partial_{(\mu}u_{\nu)} + (\phi^{\mu\nu} + \boxed{\phi_s^{\mu\nu}})(2\beta\omega_{\mu\nu} + \partial_{[\mu}\beta u_{\nu]}) + \mathcal{O}(\partial^3)$$

Second law of thermodynamics

New spin corrections

Normal dissipative terms from spin hydro

Heat flow	$h^\mu - \mathcal{H}\nu^\mu$	$+ h_s^\mu$	$=$	$-\sigma\Delta^{\mu\nu}(\partial_\nu\beta + \beta\dot{u}_\nu)$,
Viscous tensor	$\pi^{\mu\nu}$	$+ \pi_s^{\mu\nu}$	$=$	$\zeta\Delta^{\mu\nu}(\partial\cdot u) + \eta\partial^{<\mu}u^{\nu>}$,
Anti-symmetric part of energy Momentum tensor	$\phi^{\mu\nu}$	$+ \phi_s^{\mu\nu}$	$=$	$\gamma_\phi\Delta^{\mu\rho}\Delta^{\nu\sigma}(2\beta\omega_{\rho\sigma} - \Omega_{\rho\sigma})$,

Non-relativistic limit

In non-relativistic limit,

$$\boldsymbol{j} = \boxed{-\frac{1}{2}(\nabla \times \boldsymbol{s})} + \frac{1}{2}(\dot{\boldsymbol{s}} \times \boldsymbol{v}) + (1+\xi)(\boldsymbol{s} \times \dot{\boldsymbol{v}}) \boxed{+\frac{\xi}{T}(\boldsymbol{s} \times \nabla T)} + \frac{\xi}{T}\dot{T}(\boldsymbol{s} \times \boldsymbol{v}) + \mathcal{O}(v^2)$$

inverse spin Hall effect

Sinova, Valenzuela, Wunderlich, Back, Jungwirth, Rev. Mod. Phys. 87, 1213 (2015)

anomalous Hall effect

Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. 82, 1539 (2010).

Extension of BMT equations – Thomas procession

Original BMT equation:

$$\dot{s}^{\mu} = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu} \boxed{- u^{\mu} s^{\nu} \dot{u}_{\nu}}$$

Thomas procession

+ possible relaxation (dissipative) terms

Extension of BMT equation:

$$\begin{aligned} \dot{s}^{\mu} &= \boxed{-u^{\mu} s^{\nu} \dot{u}_{\nu}} \\ &\quad + (\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} - 2\beta\gamma_{\phi} g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma}) \\ &\quad - s_{\nu} \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right) s^{\mu} (\partial \cdot u)\,, \end{aligned}$$

Thomas procession

Extension of BMT equations – Spin-EM coupling

Original BMT equation:

$$\dot{s}^{\mu} = \boxed{\gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu}} - u^{\mu} s^{\nu} \dot{u}_{\nu}$$

$$\boxed{\begin{aligned} H &= -\gamma \boldsymbol{B} \cdot \boldsymbol{s} \\ \frac{\partial \boldsymbol{s}}{\partial t} &= -\gamma \boldsymbol{B} \times \boldsymbol{s} \end{aligned}}$$

Spin-EM fields coupling

+ possible relaxation (dissipative) terms

Extension of BMT equation:

Spin-vorticial fields coupling

$$\boxed{H_{\omega} = -\boldsymbol{\omega} \cdot \boldsymbol{s}}$$

$$\begin{aligned} \dot{s}^{\mu} = & -u^{\mu} s^{\nu} \dot{u}_{\nu} \\ & + (\underline{\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho}} - 2\beta\gamma_{\phi} g^{\mu\sigma})(2\omega_{\sigma} - \underline{\mathfrak{w}_{\sigma}}) \\ & - s_{\nu} \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right) s^{\mu} (\partial \cdot u), \end{aligned}$$

Extension of BMT equations – Killing condition

Original BMT equation:

$$\dot{s}^{\mu} = \gamma\Delta^{\mu\rho}F_{\rho\nu}s^{\nu} - u^{\mu}s^{\nu}\dot{u}_{\nu}$$

+ possible relaxation (dissipative) terms

Extension of BMT equation:

Thermal vorticity combined with spin chemical potential (Killing condition)

$$\begin{aligned}\dot{s}^{\mu} \quad = \quad & -u^{\mu}s^{\nu}\dot{u}_{\nu} \\ & +(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho} - 2\beta\gamma_{\phi}g^{\mu\sigma})\boxed{(2\omega_{\sigma} - \mathfrak{w}_{\sigma})} \\ & -s_{\nu}\partial^{<\mu}u^{\nu>} - \left(\frac{1}{3} + 2v_{n}^{2}\right)s^{\mu}(\partial\cdot u)\,,\end{aligned}$$

Extension of BMT equations – Dissipative effects

Original BMT equation:

$$\dot{s}^\mu = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^\nu - u^\mu s^\nu \dot{u}_\nu$$

+ possible relaxation (dissipative) terms

Extension of BMT equation:

$$\begin{aligned}\dot{s}^\mu &= -u^\mu s^\nu \dot{u}_\nu \\ &\quad +(\varepsilon^{\mu\nu\rho\sigma} s_\nu u_\rho \boxed{- 2\beta\gamma_\phi g^{\mu\sigma})(2\omega_\sigma - \mathfrak{w}_\sigma)} \\ &\quad \boxed{-s_\nu \partial^{<\mu} u^{\nu>}} \boxed{-\left(\frac{1}{3} + 2v_n^2\right) s^\mu (\partial \cdot u)},\end{aligned}$$

Spin coupled to shear tensor, bulk pressure and other dissipative effects

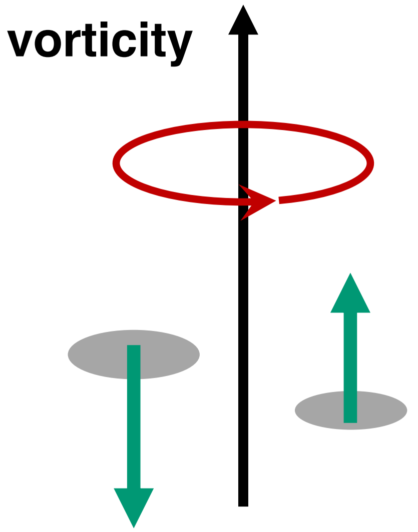
Equilibrium

In global equilibrium,

$$
\begin{aligned}
\dot{s}^{\mu} \quad = \quad & -u^{\mu}s^{\nu}\dot{u}_{\nu} \\
& +(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho}-2\beta\gamma_{\phi}g^{\mu\sigma})\boxed{\cancel{(2\omega_{\sigma}-\mathfrak{w}_{\sigma})}} \\
& -s_{\nu}\boxed{\cancel{\partial^{<\mu}u^{\nu>}}}-\left(\frac{1}{3}+2v_{n}^{2}\right)s^{\mu}\boxed{\cancel{(\partial\cdot u)}},
\end{aligned}
$$

$$\mathfrak{w}^{\mu}s^{\nu}-\mathfrak{w}^{\nu}s^{\mu}=0$$

Spin is parallel to vorticity in equilibrium.



Extension of BMT equation

Original BMT equation for EM fields

$$\dot{s}^{\mu} = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu} - u^{\mu} s^{\nu} \dot{u}_{\nu}$$

+ other dissipative effects

Thomas procession

Spin-vorticial fields coupling

Killing condition

Extension of BMT equation for vorticity

$$\dot{s}^{\mu} = -u^{\mu} s^{\nu} \dot{u}_{\nu} + (\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} - 2\beta\gamma_{\phi} g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma}) - s_{\nu} \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right) s^{\mu} (\partial \cdot u),$$

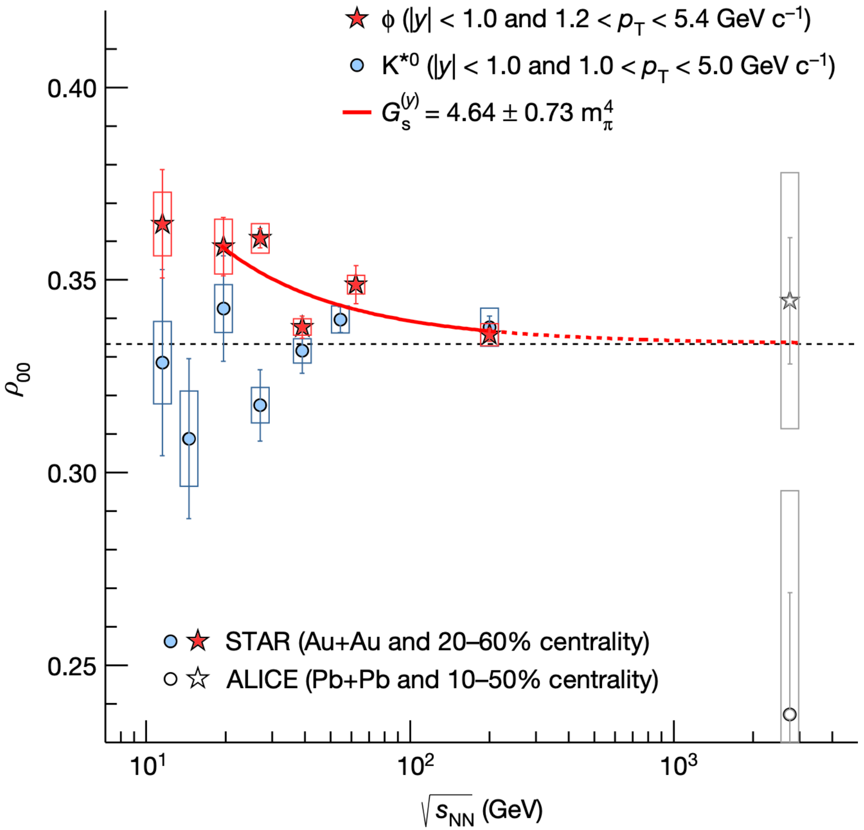
Spin coupled to shear tensor, bulk pressure and other dissipative effects

Application of Extended BMT equation

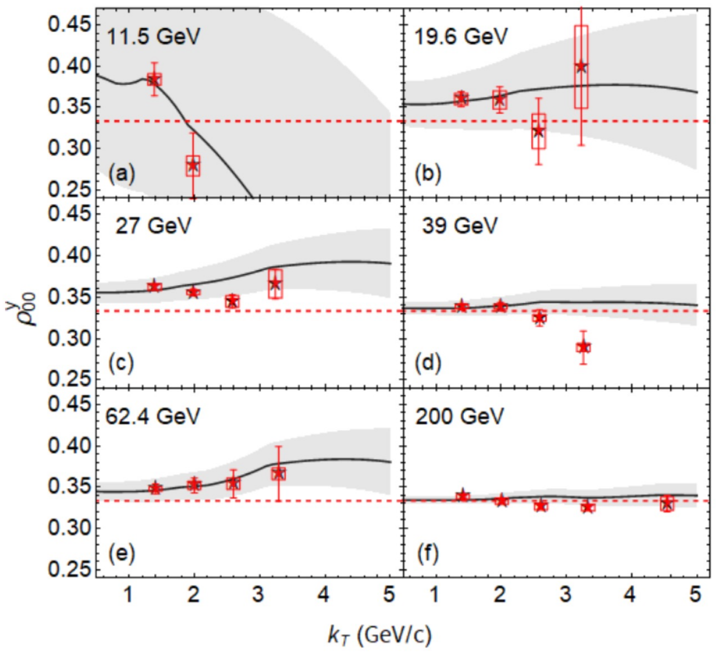
Spin correlation and fluctuations
in Martin-Siggia-Rose (MSR) approach
of effective field theory (EFT)

In collaboration with
Cong Yi, Dong-Lin Wang, Navid Abassi, et al, in preparation

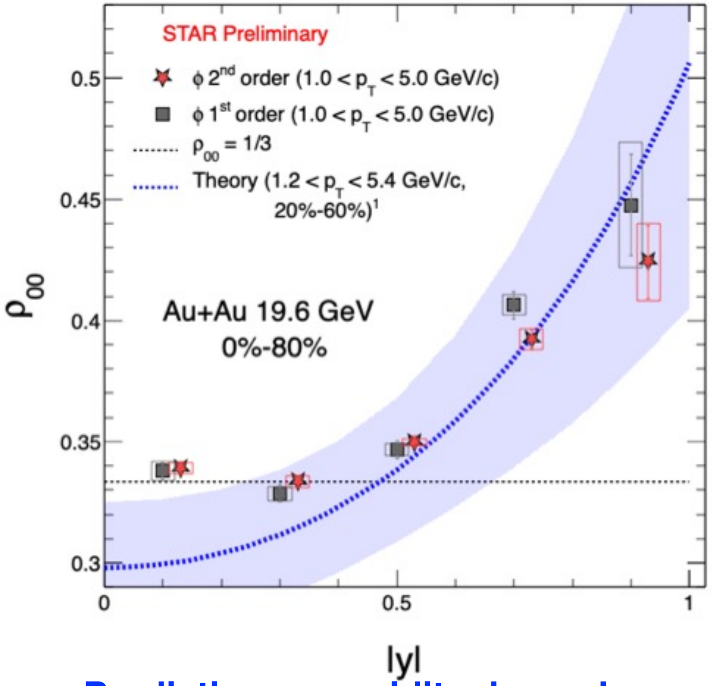
Spin alignment of ϕ mesons



STAR, Nature 614, 244 (2023)

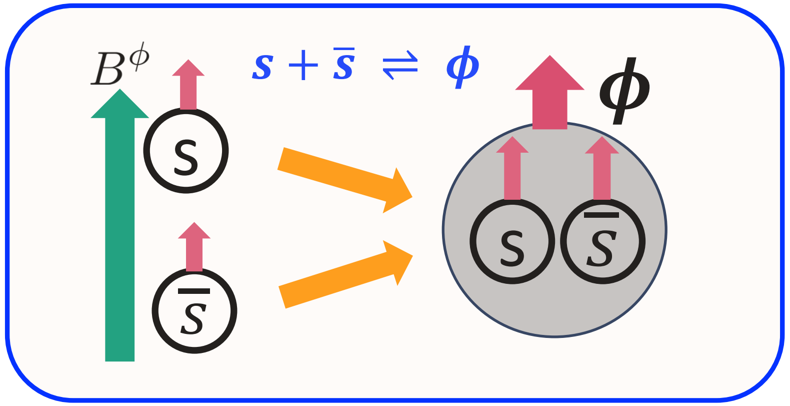


Sheng, Oliva, Liang, Wang, Wang, PRL (2023)



Prediction on rapidity dependence in Sheng, Pu, Wang, PRC 2023 agrees with data

- STAR has observed a huge spin alignment of ϕ meson and it cannot be explained by vorticity effects.



- One successful phonemical model: vector meson strong force field
  Sheng et al., PRD (2020); PRD (2020)

Spin correlation of hyperons and spin alignment

- **Lesson:**
  Large spin alignment of ϕ suggests strong spin correlation of s and $\bar{s}$

$$\rho_{00}^{\phi} - \frac{1}{3} \sim \langle P_s P_{\bar{s}} \rangle \neq \langle P_s \rangle \langle P_{\bar{s}} \rangle. \quad \langle \rho_{00}^{\phi} \rangle \sim \frac{1 - \bar{c}_{zz;\phi}^{(s\bar{s})} - \langle P_s \rangle^2}{3 + \bar{c}_{zz;\phi}^{(s\bar{s})} + \langle P_s \rangle^2},$$

- Since $P_{\Lambda/\bar{\Lambda}} \sim P_{s/\bar{s}}$,

$$\langle c_{zz}^{\Lambda\bar{\Lambda}} \rangle \sim \bar{c}_{zz;\Lambda\bar{\Lambda}}^{(s\bar{s})} + \langle P_s \rangle^2,$$

one can further estimate the spin correlation of Λ and $\bar{\Lambda}$,

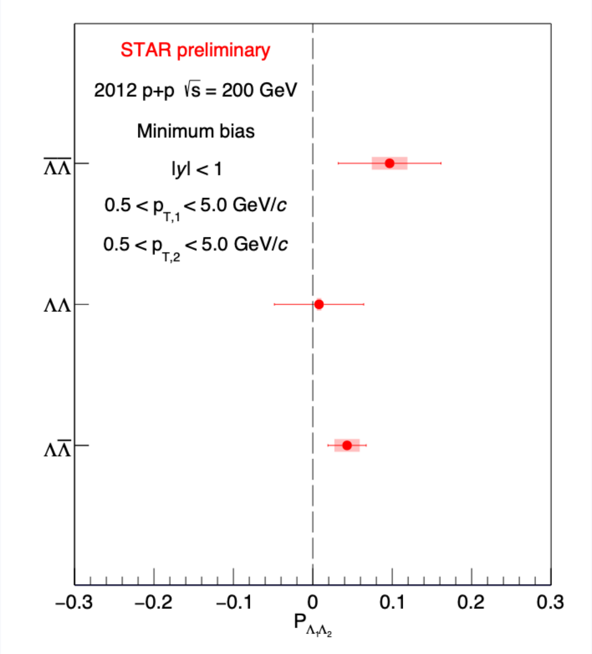
Definition of spin correlation for hyperons

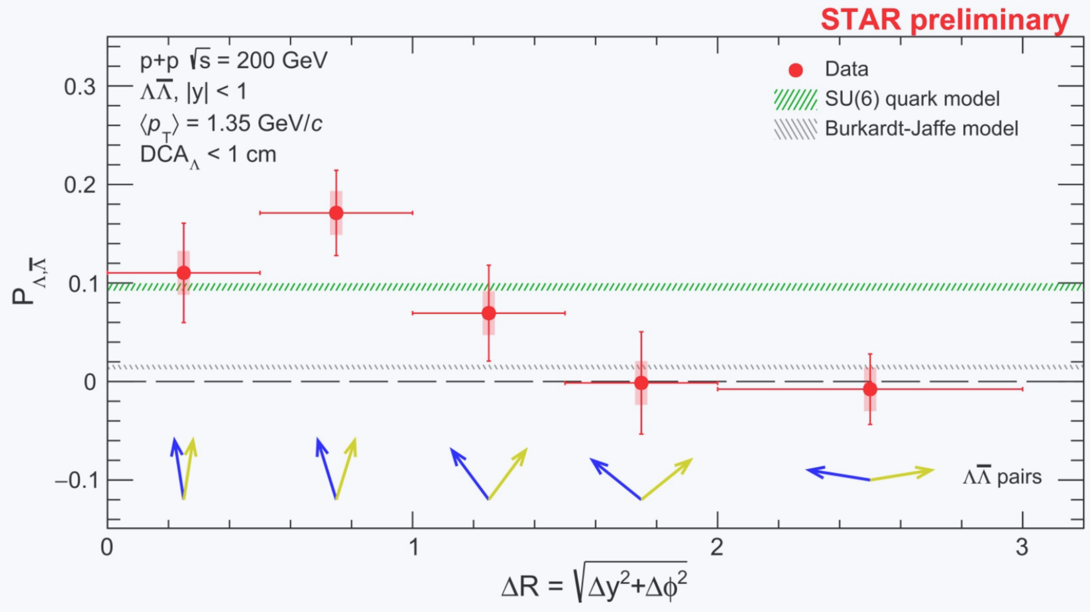
$$c_{nn}^{H_1\bar{H}_2} = \frac{f_{++}^{H_1\bar{H}_2} + f_{--}^{H_1\bar{H}_2} - f_{+-}^{H_1\bar{H}_2} - f_{-+}^{H_1\bar{H}_2}}{f_{++}^{H_1\bar{H}_2} + f_{--}^{H_1\bar{H}_2} + f_{+-}^{H_1\bar{H}_2} + f_{-+}^{H_1\bar{H}_2}},$$

Lv, Yu, Liang, Wang, Wang PRD (2024)
Sheng, Rischke, Wang, Wu, arXiv: 2508.03496

Experimental developments on spin correlation

- Spin correlation in STAR pp collisions, talk given by Jan Vanek in Quark Matter 2025





The mechanism for spin correlation in pp collisions is different from the one in AA collisions.

Spin correlations arises from gluon splitting? chiral condensation?

- Results of spin correlation in AA collisions

Definition of spin correlation and fluctuations in EFT

Spin density: $s^i(x) = \boxed{s_0^i(x)} + \boxed{\delta s^i(x)}$ $\langle \delta s^i(x) \rangle = 0$

Spin density in "equilibrium" (average value)

Spin density "fluctuations"

$$\left\langle\!\left\langle s^i(x_1) s^j(x_2) \right\rangle\!\right\rangle_V = s_0^i(x_1) s_0^j(x_2) + \left\langle\!\left\langle \delta s^i(x_1) \delta s^j(x_2) \right\rangle\!\right\rangle_V$$

Equilibrium "Fluctuation"

$\langle \quad \rangle$ ensemble average ≈ event average

$\langle \quad \rangle_V$ space average through the freeze-out hypersurface.

Stragety in our work

Step 1:
Write down the evolution equation for spin density

Step 2:
Derive the effective action for spin density

Step 3:
Compute the two point correlation function

MSR approach or Schwinger-Keldysh formulism for EFT

Functional differentials in quantum field theory

Review on EFT: P. Kovtun, J. Phys. A: Math. Theor. 45 (2012) 473001

Some evolution equation for spin

Original BMT equation for EM fields

$$\dot{s}^{\mu} = \boxed{\gamma\Delta^{\mu\rho}F_{\rho\nu}s^{\nu}} \boxed{-u^{\mu}s^{\nu}\dot{u}_{\nu}} \boxed{\text{+ other dissipative effects}}$$

Extension of BMT equation for vorticity

Thomas procession

Spin-vorticial fields coupling

Killing condition

$$\begin{aligned}\dot{s}^{\mu} = & \boxed{-u^{\mu}s^{\nu}\dot{u}_{\nu}} \\ & +(\boxed{\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho}} - \boxed{2\beta\gamma_{\phi}g^{\mu\sigma}})(2\omega_{\sigma} - \boxed{\mathfrak{w}_{\sigma}}) \\ & \boxed{-s_{\nu}\partial^{<\mu}u^{\nu>}} \boxed{-\left(\frac{1}{3}+2v_{n}^{2}\right)s^{\mu}(\partial\cdot u)},\end{aligned}$$

Spin coupled to shear tensor, bulk pressure and other dissipative effects

Fang, Fukushima, SP, Wang, arXiv:2506.20698

Step 1: Spin evolution equation based on BMT equation

$$\partial_t \vec{s} - \boxed{D_s \nabla^2 \vec{s}} - \boxed{\gamma \vec{s} \times \vec{B}} + \boxed{\lambda \vec{s} \times (\vec{s} \times \vec{B})} = \boxed{\vec{\xi}}$$

Spin diffusion:

Without it, spin will never be relaxed

Spin-B coupling

B: effective fields coupled to spin
vorticity, (color) magnetic fields,
Other effective forces

Landau-Lifshitz term

Spin relaxation, but it is high order in EFT

Noise term

Auxiliary fields

Static solutions: $\vec{s}_0 \propto \vec{B}, (\vec{\omega}, \vec{B}^{\phi})$

Spin correlation function in “equilibrium”

Example: considering the spin polarized induced by vorticity or vector meson strong force

Static solutions: $\vec{s}_0 \propto \vec{B}, (\vec{\omega}, \vec{B}^{\phi})$

$$\left\langle\!\left\langle s^i(x_1)s^j(x_2)\right\rangle\!\right\rangle_{\mathrm{V}} = \boxed{s_0^i(x_1)s_0^j(x_2)} + \left\langle\!\left\langle \delta s^i(x_1)\delta s^j(x_2)\right\rangle\!\right\rangle_{\mathrm{V}}$$

Equilibrium

“Fluctuation”

$$\frac{1}{N_{\mathrm{events}}}\sum_{\mathrm{events}}\int d\Sigma\cdot p[a(p)\omega^i(x_1)\omega^j(x_2) + b(p)\delta^{ij}B^{\phi}(x_1)^2]$$

vorticity

Pang, Petersen, Wang, Wang, PRL (2016) $\sim 10^{-4}$

vector meson strong force

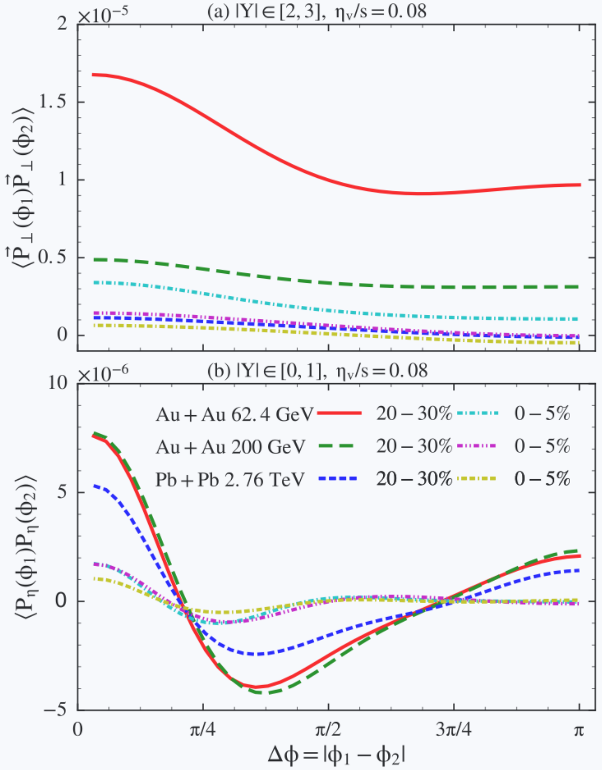
Lv, Yu, Liang, Wang, Wang PRD (2024) $\sim 10^{-1} - 10^{-3}$

Sheng, Wu, Rischke, Wang, arXiv: 2508.03496 $\sim 10^{-4}$

Theoretical developments on spin correlation: strong force dominates

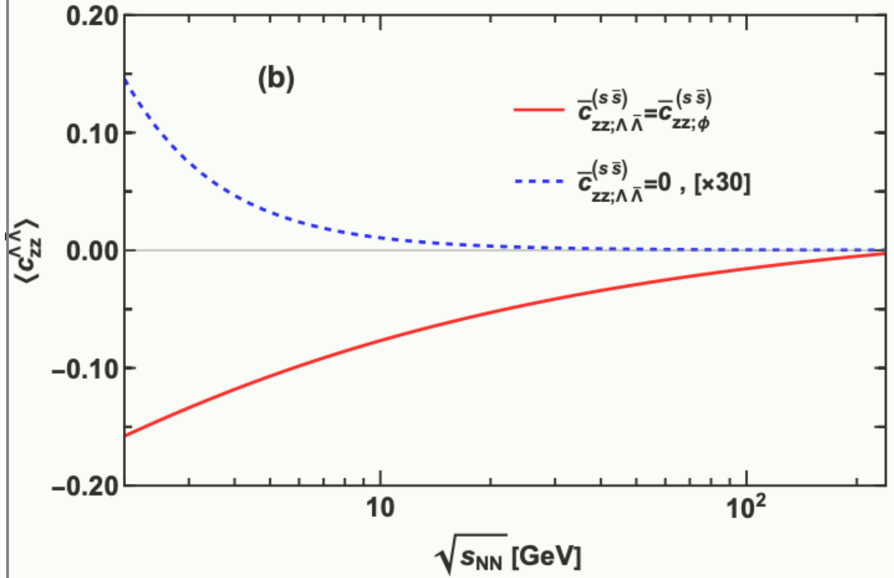
Short range correlation $\sim 10^{-4}$

Early pioneer work: estimated by vorticity effects:
Pang, Petersen, Wang, Wang, PRL (2016)



Short range correlation $\sim 10^{-1} - 10^{-3}$

Estimated by the spin alignment of ϕ meson
Lv, Yu, Liang, Wang, Wang PRD (2024)

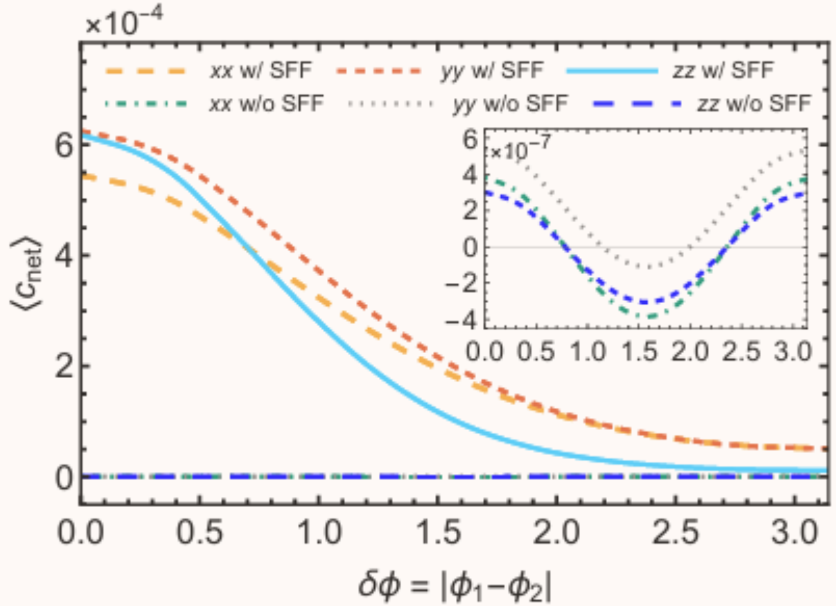


$$\langle \rho_{00}^{\phi}\rangle \sim \frac{1-\bar{c}^{(s\bar{s})}_{zz;\phi}-\langle P_s\rangle^2}{3+\bar{c}^{(s\bar{s})}_{zz;\phi}+\langle P_s\rangle^2},$$

$$\langle c_{zz}^{\Lambda\bar{\Lambda}}\rangle \sim \bar{c}^{(s\bar{s})}_{zz;\Lambda\bar{\Lambda}}+\langle P_s\rangle^2,$$

Long range correlation $\sim 10^{-4}$

Prediction from spin alignment of of ϕ meson (improved), strong field fluctuations dominate
Sheng, Wu, Rischke, Wang, 2508.03496



$$c_{\rm net}^{ab} \equiv \frac{1}{2}(c^{ab}_{\Lambda\Lambda}+c^{ab}_{\bar{\Lambda}\bar{\Lambda}})-c^{ab}_{\Lambda\bar{\Lambda}}$$

Spin correlation function

Example:
considering the spin polarized induced by vorticity or vector meson strong force

$$\left\langle\left\langle s^i(x_1)s^j(x_2)\right\rangle\right\rangle_V = s_0^i(x_1)s_0^j(x_2) + \left\langle\left\langle\delta s^i(x_1)\delta s^j(x_2)\right\rangle\right\rangle_V$$

Equilibrium

Short range correlation $\sim 10^{-1} - 10^{-3}$

Long range correlation is incapable to MSR approach

"Fluctuation":

Possible sources arising fluctuations:

- **Thermal enviorments**
- **Induced by vorticity**
- **Induced by vector meson strong force**
- **Diffusion of spin**
- **Other effective forces**

Step 2: Derive the effective action in MSR approach

By using Martin-Siggia-Rose (MSR) approach or Schwinger-Keldysh formulism for EFT, for any operator O(s), we derive,

P. Kovtun, J. Phys. A: Math. Theor. 45 (2012) 473001

$$\langle \mathcal{O}[\vec{s}]\rangle = \int \mathcal{D}\vec{s}\mathcal{D}\vec{r}\mathcal{O}[\vec{s}]\boxed{J[\vec{s}]}e^{i\boxed{S_{\mathrm{EFT}}[\vec{s},\vec{r}]}}$$

effective action

Auxiliary fields

source in path integral

$$S_{\mathrm{EFT}}[\vec{s},\vec{r}] = \int d^dx\left[\vec{r}\cdot\left(-\partial_t + D_s\nabla^2 - \gamma\vec{B}\times\right)\vec{s} + \frac{i}{2}\vec{r}\cdot\mathrm{F}\cdot\vec{r}\right]$$
$$-\lambda\int d^dx\vec{r}\cdot[\vec{s}\times(\vec{s}\times\vec{B})].$$

Step 3: Spin correlation function

Assuming that the effective field B is along y direction,

$$\delta s^i(p)\delta s^i(p) = \frac{2\boxed{D_s}T\chi|\vec{p}|^2(B_y^2\gamma^2 + D_s^2|\vec{p}|^4 + \omega^2)}{(\omega^2 - B_y^2\gamma^2)^2 + 2D_s^2|\vec{p}|^4(B_y^2\gamma^2 + \omega^2) + D_s^4|\vec{p}|^8}, \qquad \boldsymbol{i = x, z}$$

$$\delta s^y(p)\delta s^y(p) = \frac{2\boxed{D_s}T\chi|\vec{p}|^2}{D_s^2|\vec{p}|^4 + \omega^2},$$

$$\delta s^z(p)\delta s^x(p) = \frac{4i\boxed{D_s}T\chi\gamma B_y\omega|\vec{p}|^2}{(\omega^2 - B_y^2\gamma^2)^2 + 2D_s^2|\vec{p}|^4(B_y^2\gamma^2 + \omega^2) + D_s^4|\vec{p}|^8},$$

Remarks:

- **Spin correlation can provide us additional insights into spin diffusion D_s**
- **This approach differs from others in the following ways**
  - **Fluctuation in y direction is independent on B^y**
  - **$\delta s^z \delta s^x$ is nonzero even beyond the contributions from vorticity, vector meson strong force**

Summary

Summary

Thomas procession

Spin-vorticial fields coupling

Extension of BMT eq.

$$\dot{s}^{\mu} = \boxed{-u^{\mu}s^{\nu}\dot{u}_{\nu}} + \left(\boxed{\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho}} - \boxed{2\beta\gamma_{\phi}g^{\mu\sigma}}\right)\left(2\omega_{\sigma} - \boxed{\mathfrak{w}_{\sigma}}\right) \boxed{-s_{\nu}\partial^{<\mu}u^{\nu>}} \boxed{-\left(\frac{1}{3}+2v_n^2\right)s^{\mu}(\partial\cdot u)},$$

Killing condition

Spin coupled to shear tensor, bulk pressure and other dissipative effects

Spin correlation in MSR approach of EFT theory

$$\left\langle\!\left\langle s^i(x_1)s^j(x_2)\right\rangle\!\right\rangle_{\mathrm{V}} = s_0^i(x_1)s_0^j(x_2) + \left\langle\!\left\langle \delta s^i(x_1)\delta s^j(x_2)\right\rangle\!\right\rangle_{\mathrm{V}}$$

Thank you for your time!

Any suggestions and comments are welcome!

One More Thing…

热烈欢迎各位专家亲临
第十七届QCD相变与相对论
重离子物理研讨会！

二十年后，重聚科大，
经典回归，敬请期待！

Puzzle: Is shear induced polarization dissipative?

Classical kinetic theory:

H theorem: $\frac{dH}{dt} \geq 0$

Entropy production rate:

$$\partial_\mu s^\mu \geq 0$$

Well-established

Quantum kinetic theory:

H theorem: ?

Entropy production rate: ?

Unknown

$$O(x) - \boxed{\langle \hat{O}(x) \rangle_{\mathrm{GE}}}$$

$$= \lim_{k\to 0} \mathrm{Im} \frac{i}{\beta(x) k_0} \int_{t_0}^{t} \mathrm{d}^4 x' e^{-ik\cdot(x'-x)}$$

$$\boxed{\left\langle -[\hat{O}(x), \hat{T}^{\mu\nu}(x')]\partial_\mu \beta_\nu(x) + [\hat{O}(x), \hat{j}^\mu(x')]\partial_\mu \alpha(x) \right\rangle_{\mathrm{GE}}}$$

$$+\mathcal{O}(\partial^2).$$

non-dissipative

dissipative

Also see:
Jia-Rong Wang's talk at 3:10 PM, 09/23/2025, in Section: Spin in HIC

- We derive the shear induced polarization in Zubarev's approaches，

$$\delta \mathcal{A}^{<,\mu}_{\mathrm{LE},\xi}(q,X) = -q^\beta \boxed{\xi_{\alpha\beta}(X)} 2\pi\delta(q^2 - m^2) \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma}{2|q_0|} f^{(0)}_q (1 - f^{(0)}_q)$$

thermal shear tensor

$$\delta \mathcal{A}^{<,\mu}_{\mathrm{LE},\alpha}(q,X) = \boxed{(\partial_\nu \alpha)} 2\pi\delta\left(q^2 - m^2\right) \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma}{2|q_0|} f^{(0)}_p \left(1 - f^{(0)}_p\right)$$

chemical potential gradient

- Shear induced polarization comes from off-equilibrium corrections, the polarization induced by the shear tensor and chemical potential gradient could be dissipative.

Jia-Rong Wang, Shuo Fang, Di-Lun Yang, SP, arXiv:2507.15238

Anomalous Hall effect in heavy ion collisions

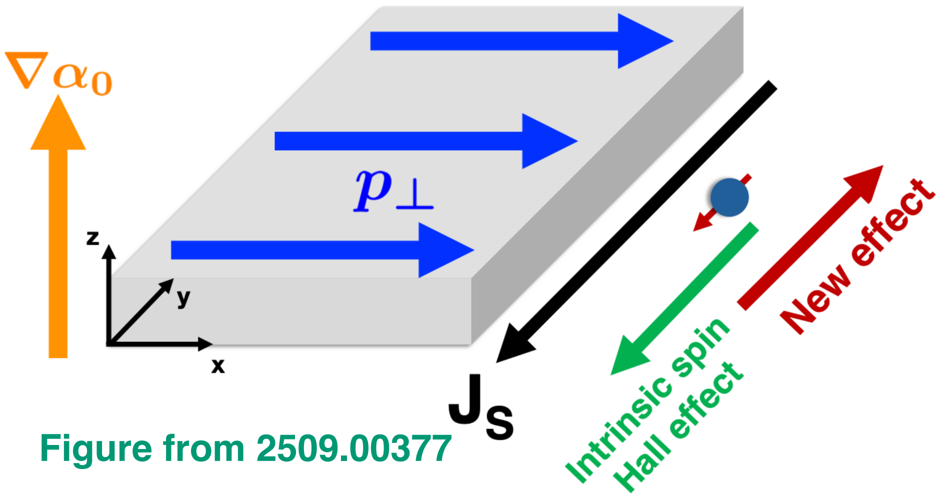
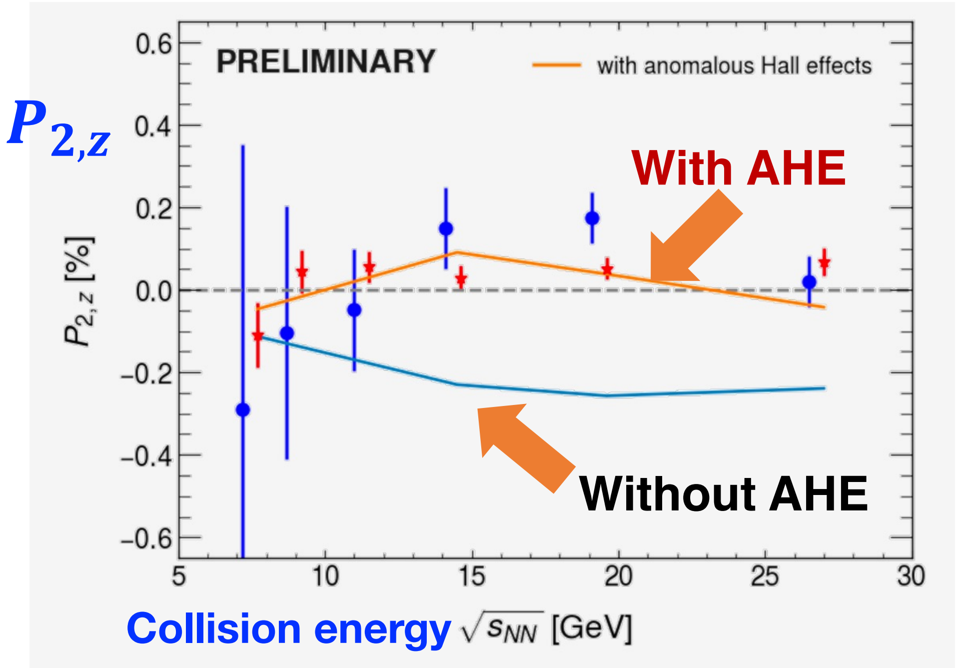


Figure from 2509.00377



- **Novel spin transport from interactions but does not depend on coupling constant:**

➢ **Gauge theory in Hard-Thermal-Loop approximation**
Fang, SP, PRD (2024)

$$\delta\mathcal{S}^{\mu}_{(\mathrm{I}),\mathrm{shear}} = -\frac{\hbar^2}{4N}\int_\Sigma \frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 \boxed{g_2(E_{\mathbf{p}})}\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \sigma_{\nu\alpha}p^\alpha,$$

$$\delta\mathcal{S}^{\mu}_{(\mathrm{I}),\mathrm{chem}} = -\frac{\hbar^2}{4N}\int_\Sigma \frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 \boxed{g_1(E_{\mathbf{p}})}\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \nabla_\nu\left(\frac{\mu}{T}\right).$$

g1 and g2 but do NOT depend on coupling constant explicitly.

➢ **Relaxation time approaches:**
Wang, Fang, Yang, SP, arXiv:2507.15238

Also see:
Jia-Rong Wang's talk at 3:10 PM, 09/23/2025, in Section: Spin in HIC

$$\delta S_R^{<,\mu} = -\frac{4\pi\hbar}{u\cdot p}\boxed{\frac{\tau_R}{\tau_R'}}\delta\left(p^2\right) S^{\mu\nu}_{(u)} f_p^{(0)}\left(1-f_p^{(0)}\right) c_3\beta p^\sigma \partial_{\langle\nu}u_{\sigma\rangle}$$

- **Our results align consistently with findings in condensed matter physics.** Valet, Raimondi, PRB Lett. (2024)

Puzzle: Is shear induced polarization dissipative?

Step 1: Construct entropy current at $O(\hbar)$

Conserved current

$$j_R^\mu = 2\int \frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left(p^\mu + \hbar S_{(u)}^{\mu\nu}\mathcal{D}_\nu\right)\boxed{f_p^{(u)}} \Rightarrow s_R^\mu = 2\int \frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left(p^\mu + \hbar S_{(u)}^{\mu\nu}\mathcal{D}_\nu\right)\boxed{\mathcal{H}\left[f_p^{(u)}\right]}$$

Entropy current

Step 2: Test physical condition: Is entropy production rate positive-define?

$$\partial_\mu s^\mu \sim \frac{2\hbar}{\tau_R}\int \frac{d^4p}{(2\pi)^3}\frac{\bar{\epsilon}_{(u)}}{p\cdot u}\delta\left(p^2\right)c_1\frac{\left(\delta f_p\right)^2}{f_p^{eq}\left(1-f_p^{eq}\right)} \quad c_1 \geq 0 \Rightarrow \partial_\mu s^\mu \geq 0$$

Step 3: Compute the entropy production rate related to the shear-induced polarization

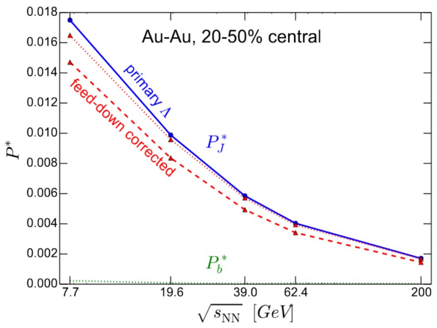
$$\partial_\mu s^\mu = \frac{4}{15}\tau_R\beta^2\boxed{\partial_{\langle\mu}u_{\nu\rangle}\partial^{\langle\mu}u^{\nu\rangle}}\int \frac{d^4p}{(2\pi)^3}\delta\left(p^2\right)\left(p\cdot u\right)^3 f_p^{(0)}\left(1-f_p^{(0)}\right) \geq 0$$

Dilemma:

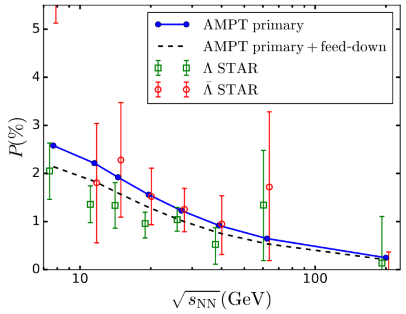
- **The shear-induced polarization and anomalous Hall effect do not contribute to entropy production, suggesting they are non-dissipative.**
- **Total entropy increases due to the classical shear viscous tensor. Dissipative?**

Jia-Rong Wang, Shuo Fang, Di-Lun Yang, SP, arXiv:2507.15238

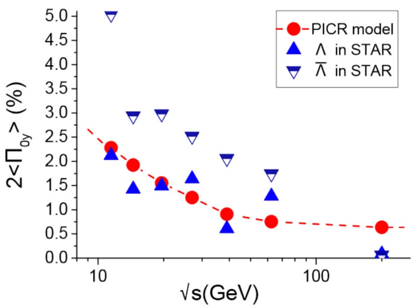
Phenomenological models for global polarization



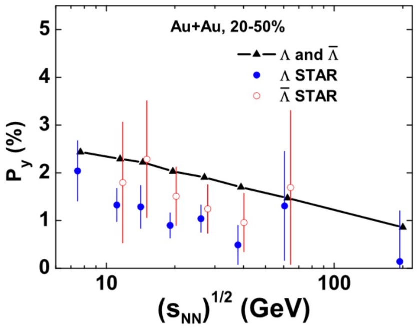
Karpenko, Becattini, EPJC(2017)



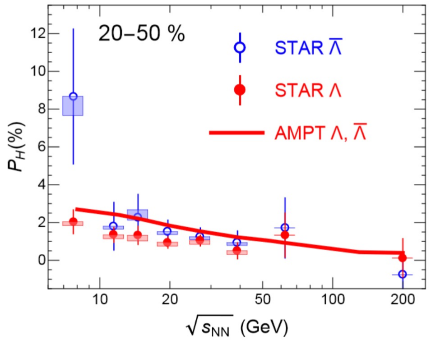
Li, Pang, Wang, Xia PRC(2017)



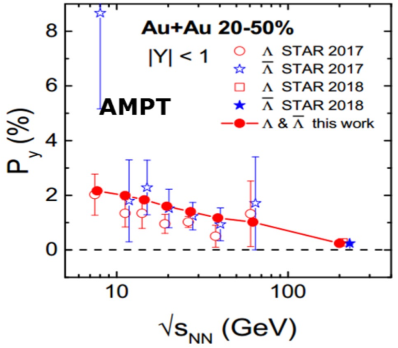
Xie, Wang, Csernai, PRC(2017)



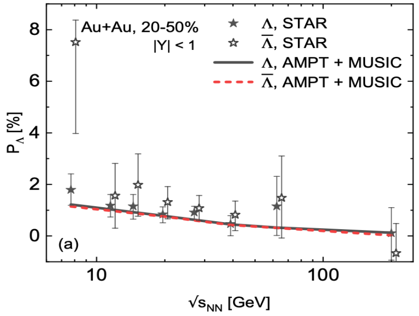
Sun, Ko, PRC(2017)



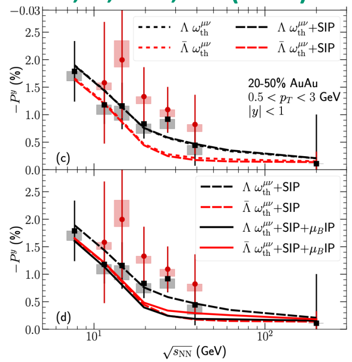
Shi, Li, Liao, PLB(2018)



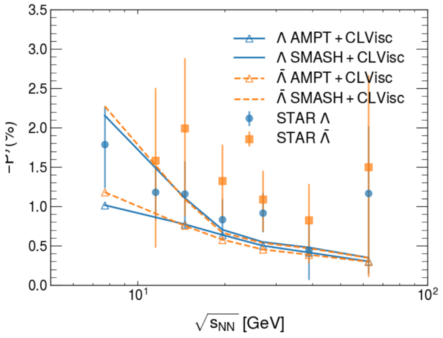
Wei, Deng, Huang, PRC(2019)



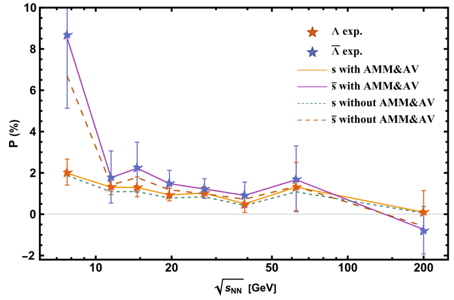
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)



Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)



Xu, Lin, Huang, Huang, PRDL (2022)