RELATIONS BETWEEN QCD PHASE TRANSITION AND SPIN PHYSICS

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Hao-Lei Chen

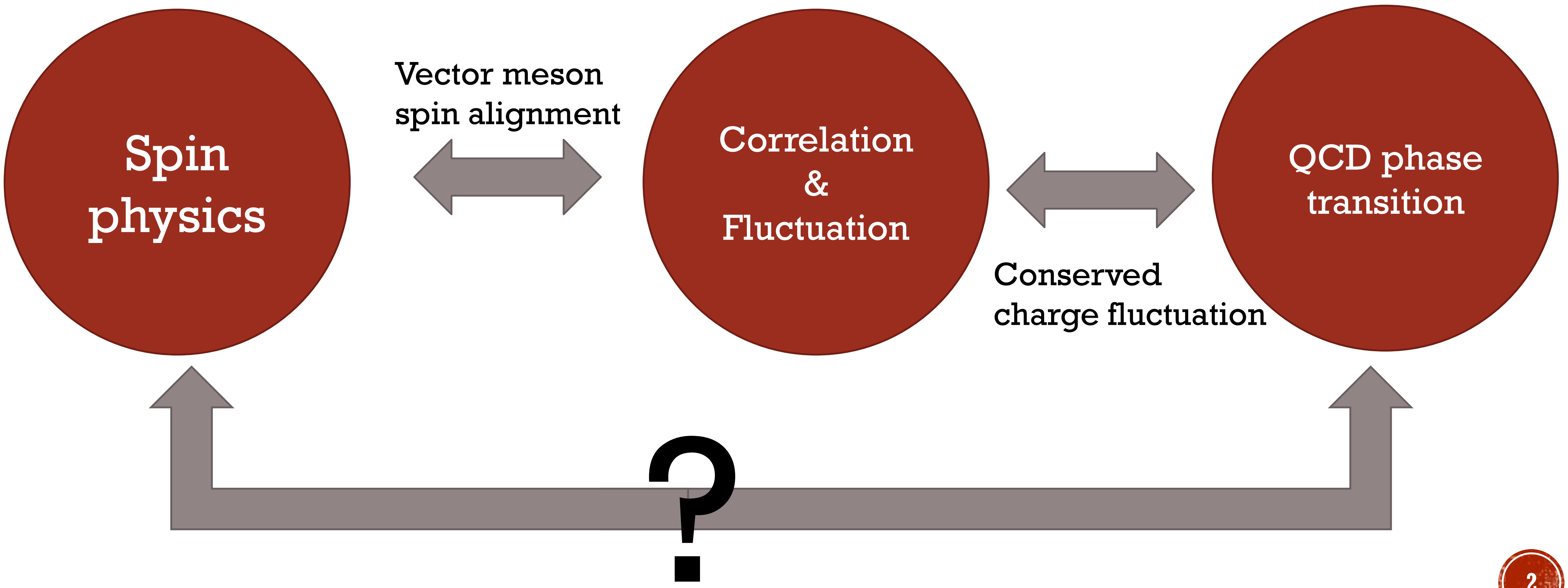
Fudan Univeristy

Based on: HLC, Wei-jie Fu, Xu-Guang Huang, Guo-Liang Ma, Phys. Rev. Lett. 135， 032302 (2025)

HLC, 25xx.xxxx

The 16th Workshop on Quantum Chromodynamics Phase Transition and Relativistic Heavy Ion Physics
@ Guilin, 26th October 2025

OUTLINE

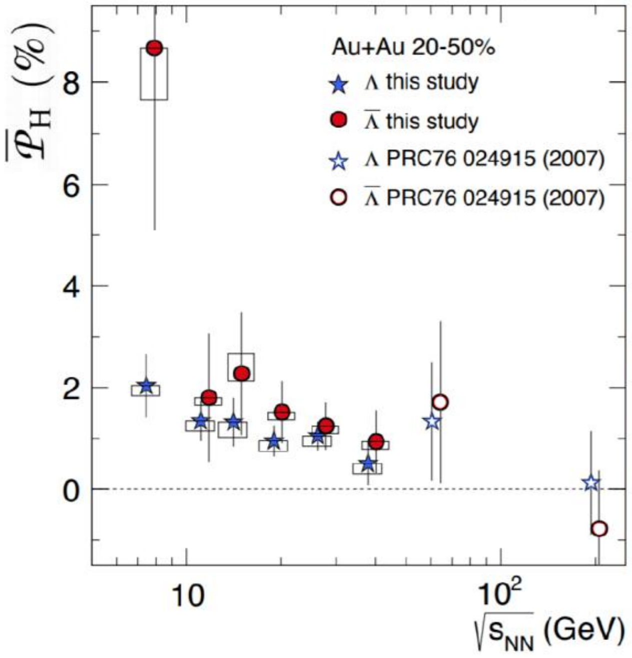


Spin physics
Vector meson spin alignment
Correlation & Fluctuation
Conserved charge fluctuation
QCD phase transition
?

SPIN POLARIZATION AND ALIGNMENT

- Polarization

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

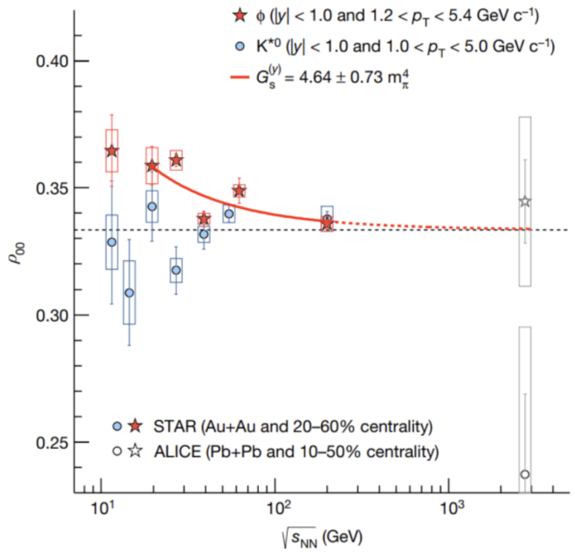


most vortical fluid produced in the laboratory .

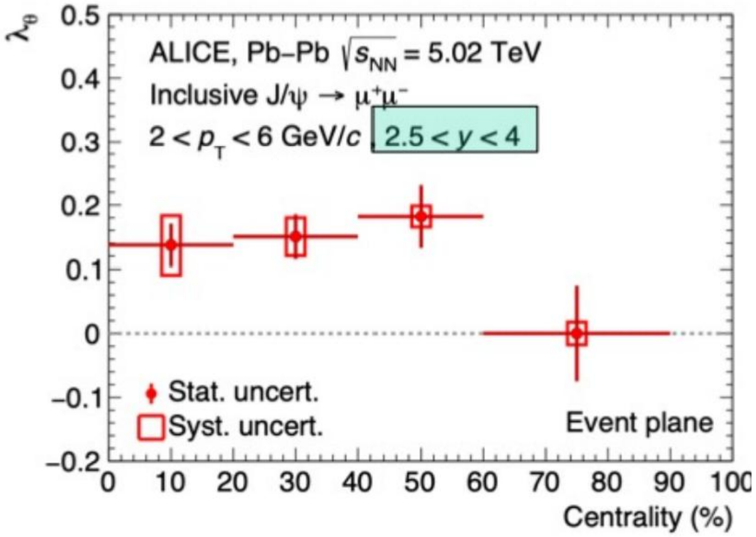
$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

- Alignment

STAR Collaboration, Nature 614 (2023) 7947.



ALICE, PRL 131 042303 (2023)



Polarization only relates to single quark

While spin alignment relates to quark pair:

Liang, Wang, PRL (2005)

$$\rho_{00}^V = \frac{1-\langle P_q P_{\bar{q}} \rangle}{3+\langle P_q P_{\bar{q}} \rangle} \approx \frac{1}{3} - \frac{4}{9}\langle P_q P_{\bar{q}} \rangle$$

Correlation and fluctuation of quark spin are important

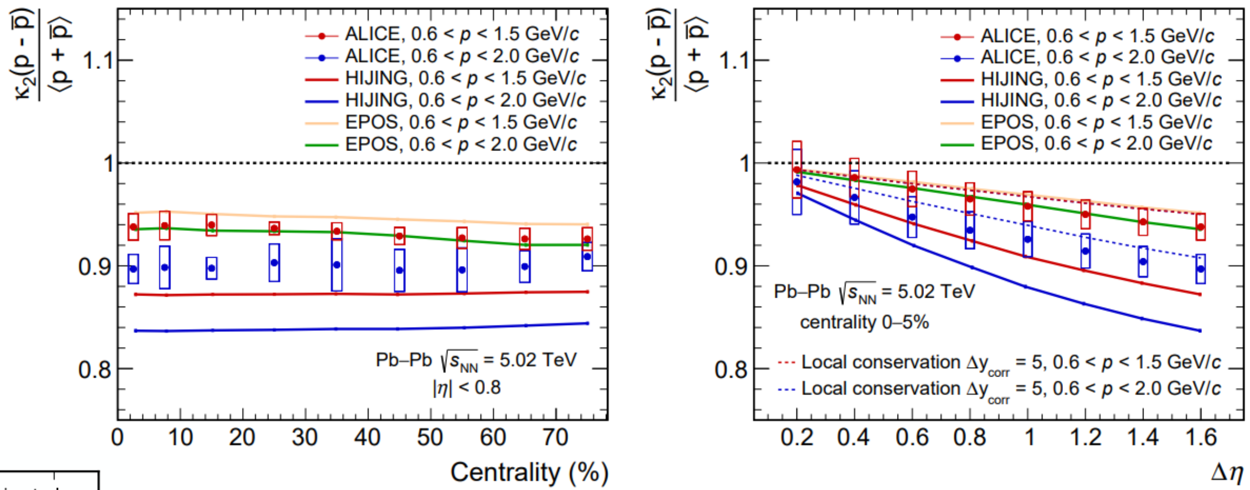
CORRELATION AND FLUCTUATION

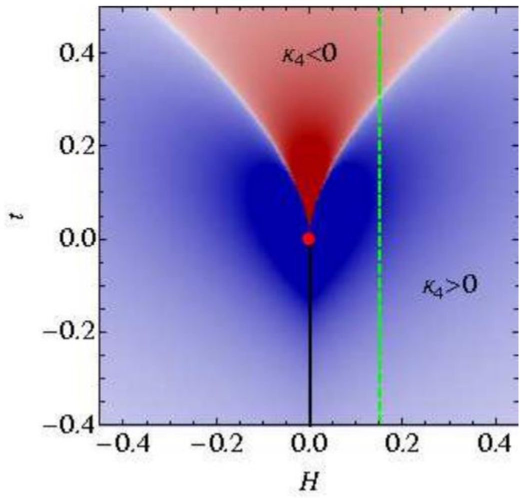
- Baryon number fluctuation

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}$$

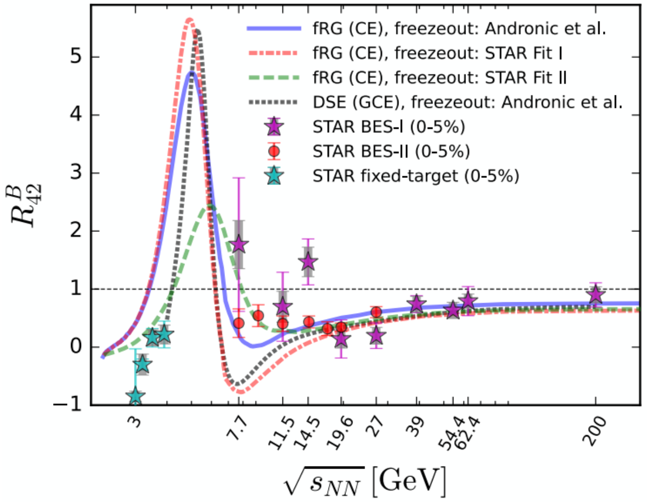
- Quantifying the nature of the phase transition
- At large density: critical endpoint (CEP)

ALICE, PLB 844 (2023) 137545



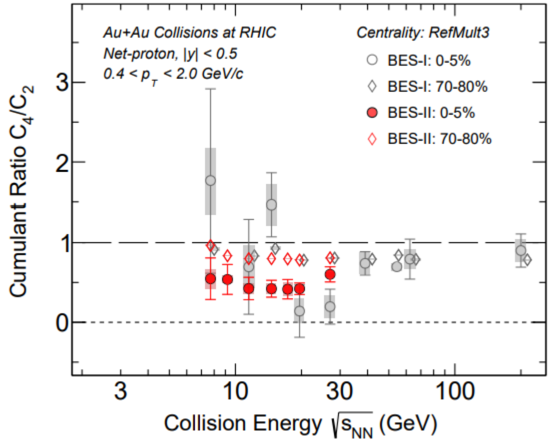


M. Stephanov, PRL 107 (2011) 052301



From W. Fu's HENPIC seminar

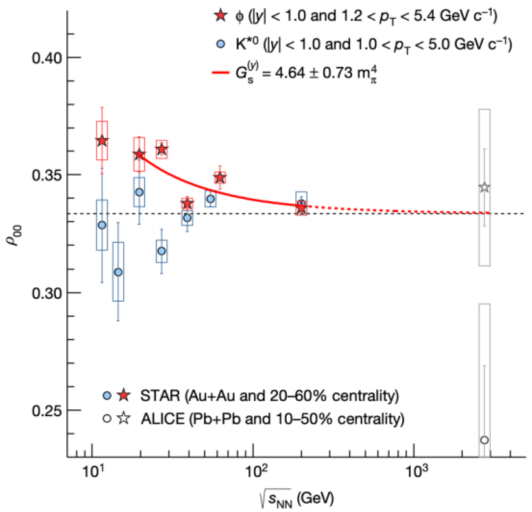
STAR Collaboration, arXiv:2504.00817

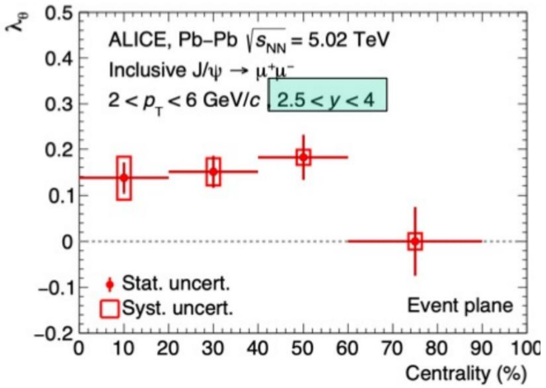


MOTIVATION

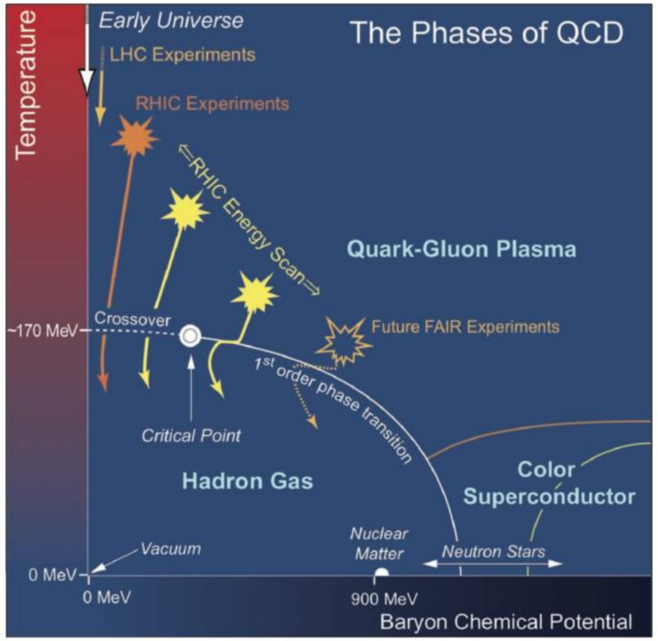
- Although these two topics are almost studied separately in different contexts

Spin physics





QCD Phase transition

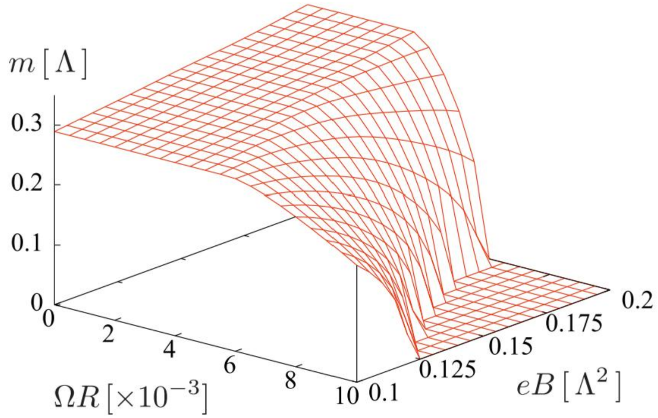


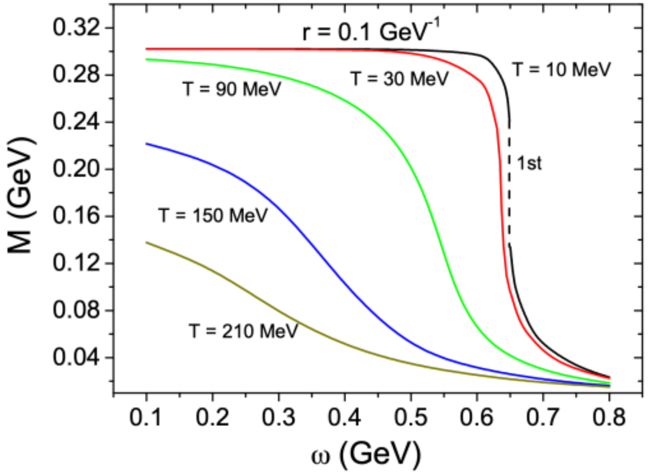
Question: does spin also fluctuates? similar as baryon number?

- Which means spin physics and QCD phase structure are related
- To answer this question, we need knowledge about QCD phase diagram under vorticity/rotation

QCD PHASE DIAGRAM UNDER ROTATION

HLC, K. Fukushima, X-G. Huang, K. Mameda, Phys. Rev. D 93. 104052 (2016)





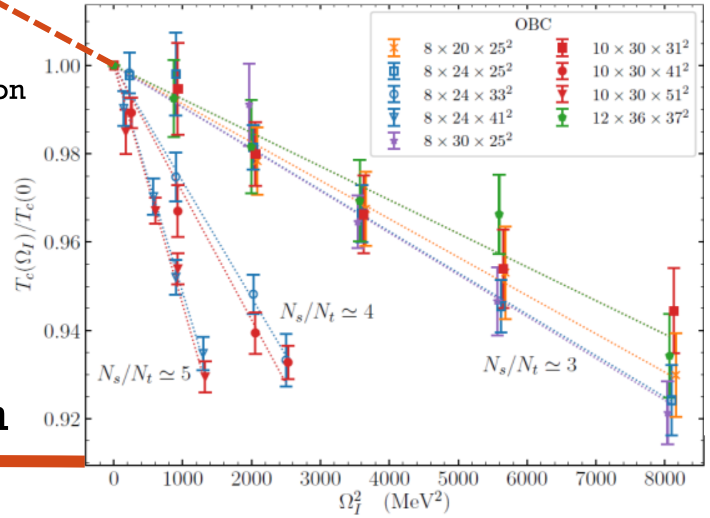
Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016)

- First studied by NJL model
- Rotation restores chiral symmetry
- However, lattice studies give opposite results!

analytical continuation

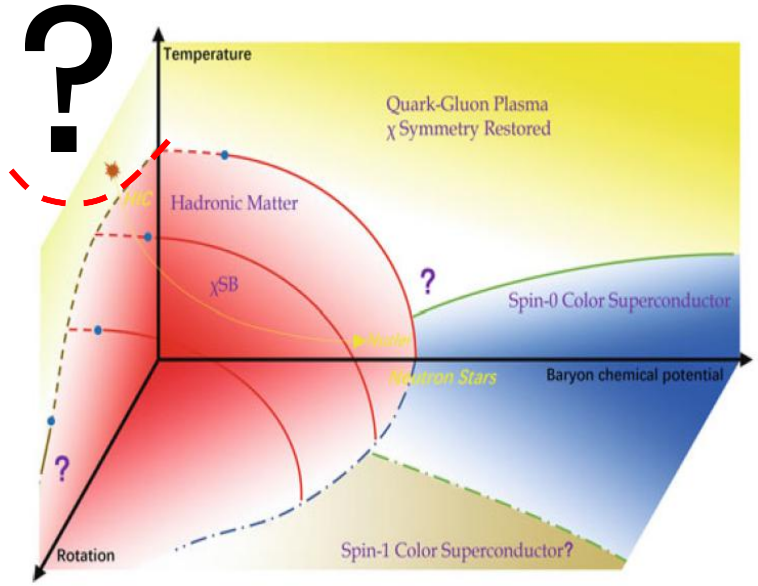
$\Omega_I^2 \leftrightarrow -\Omega^2$

Real rotation



Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.

Unsolved discrepancy



QUALITATIVE STUDY: NJL MODEL

Rotating spacetime metric

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- We are now ready to answer our question: Does spin fluctuates near CEP?
- By NJL model in rotating frame

$$\mathcal{L}_{NJL} = \bar{\psi} i \gamma^\mu \nabla_\mu \psi - m_0 \bar{\psi}\psi + \mu_B \bar{\psi}\gamma^0\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma^5\vec{\tau}\psi)^2]$$

- Hamiltonian for fermion

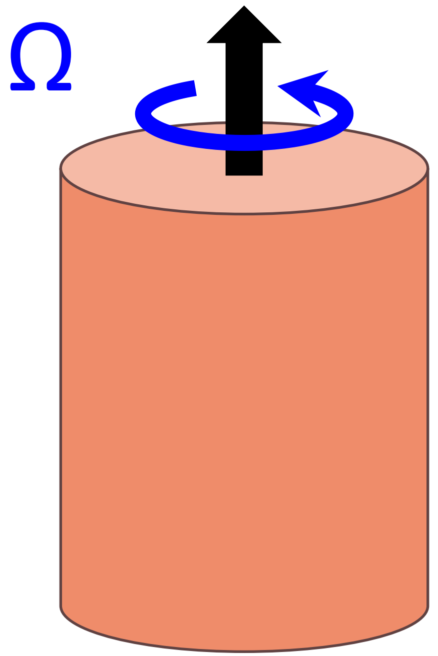
$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}.$$

L. Landau and E. Lifshitz, Statistical Physics, Part 1

- Similar as finite density

$$E \rightarrow E - \mu$$

Rotation behaves as an effective chemical potential $\Rightarrow$ Restore chiral symmetry



QUALITATIVE STUDY: NJL MODEL

- We further introduce spin chemical potentials which only couple to quark or antiquark spin

$$
\begin{aligned}
&V_{\text{eff}}(\boxed{\Omega_q^s, \Omega_{\bar{q}}^s}, \Omega, \mu_q, \mu_{\bar{q}}, \mu; r) \\
&= \frac{[m(r)-m_0]^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^3} J_l^2(p_t r) \Big[T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\Omega l-\mu_q)/T}) \\
&+ T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\Omega l-\mu_q)/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega_{\bar{q}}^s/2+\Omega l-\mu_{\bar{q}})/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega_{\bar{q}}^s/2+\Omega l-\mu_{\bar{q}})/T}) \Big]
\end{aligned}
$$

- Then by taking derivative, we can get correlation of quark/antiquark spin and particle number

$$
\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \Omega_q^s \partial \Omega_{\bar{q}}^s}\Big|_{\Omega_q^s=\Omega_{\bar{q}}^s=\Omega} \qquad \langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \mu_q \partial \mu_{\bar{q}}}\Big|_{\mu_q=\mu_{\bar{q}}=0}
$$

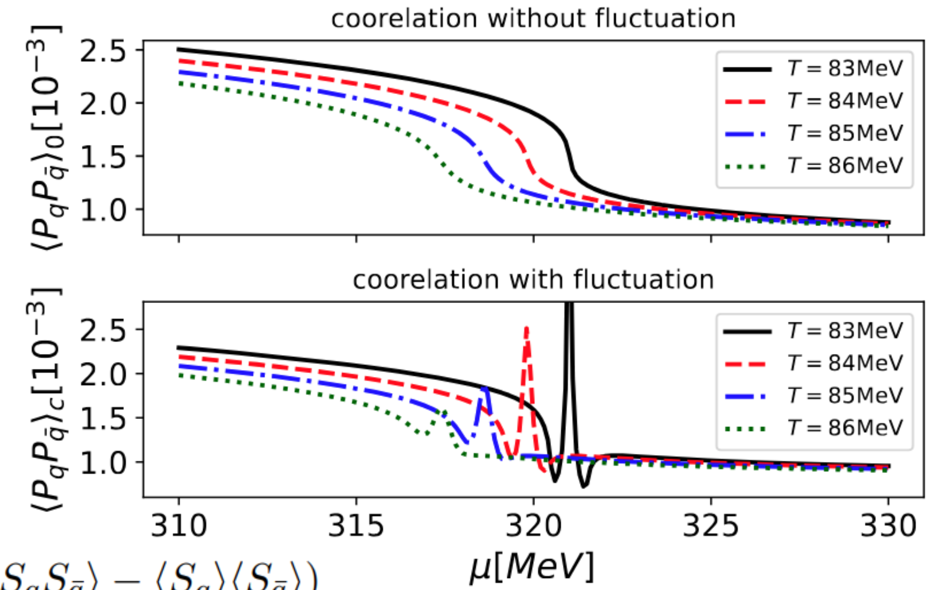
  - Then we can define the spin correlation of quark-antiquark as

$$
\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}
$$

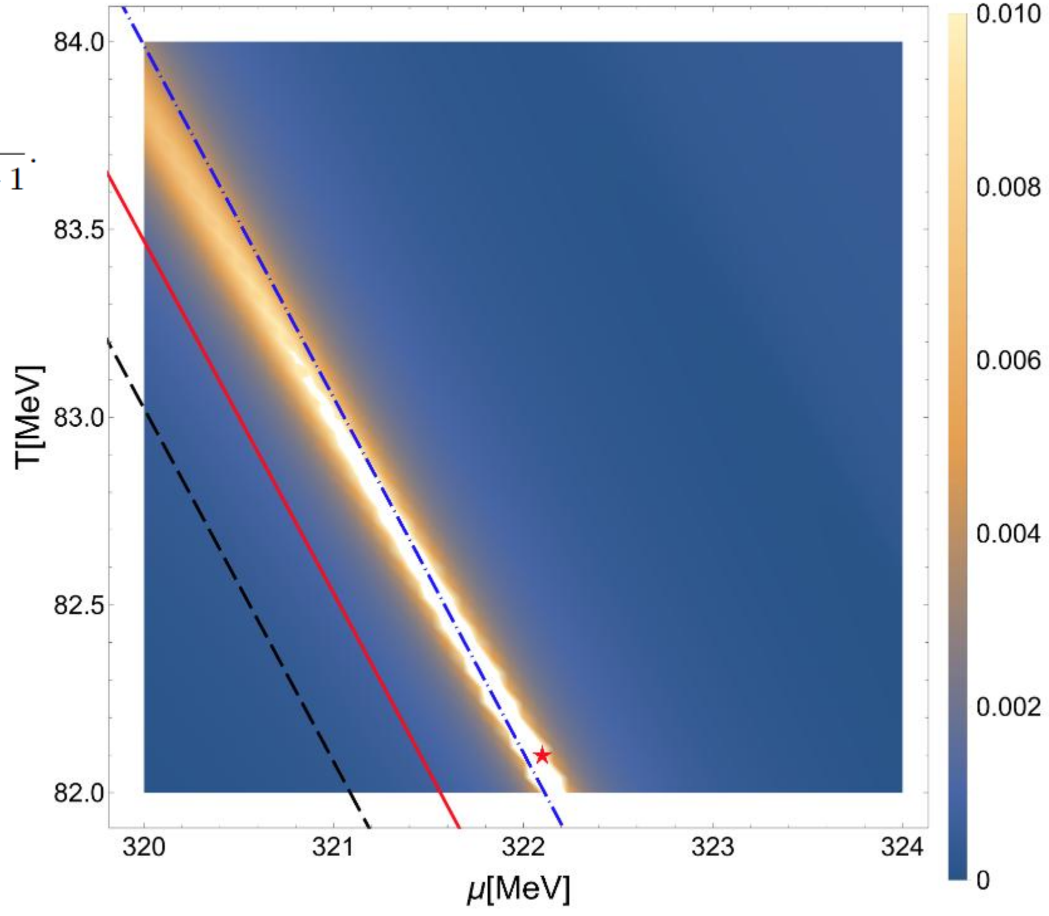
SPIN CORRELATION ENHANCED BY CEP!

- Comparison with the case w/o fluctuation

$$\langle P_q P_{\bar{q}}\rangle_0 = \frac{\int \mathrm{d}^3p (f_q^{\uparrow} - f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} - f_{\bar{q}}^{\downarrow})}{\int \mathrm{d}^3p (f_q^{\uparrow} + f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} + f_{\bar{q}}^{\downarrow})}. \qquad f_q^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p - \mu \mp \frac{\Omega}{2})} + 1}, \qquad f_{\bar{q}}^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p + \mu \mp \frac{\Omega}{2})} + 1}.$$



$$\langle P_q P_{\bar{q}}\rangle_c = \frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\langle N_q N_{\bar{q}}\rangle - \langle N_q\rangle\langle N_{\bar{q}}\rangle}$$

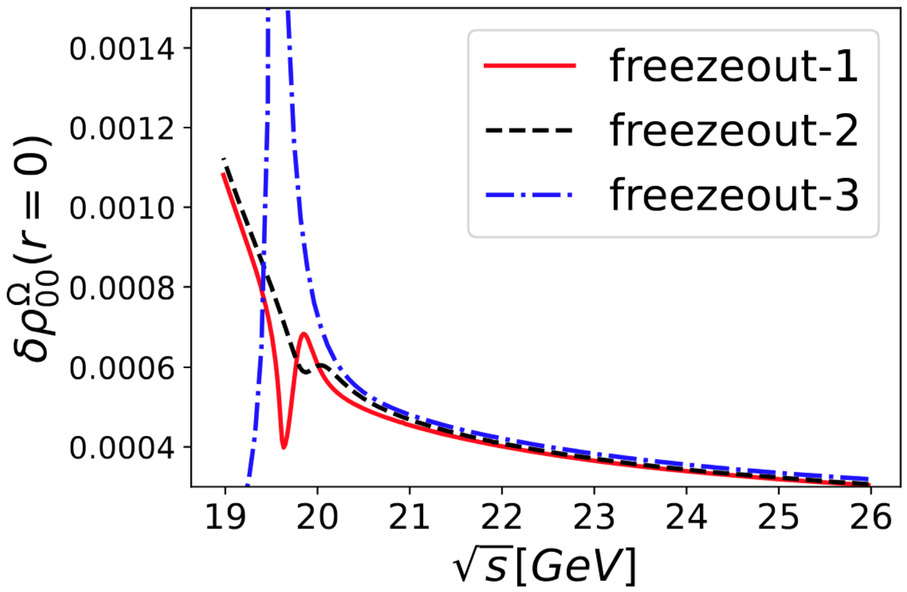


VECTOR MESON SPIN ALIGNMENT

- Along imaginary freezeout lines

$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} \boxed{- \delta\rho_{00}^{\Omega}.}$$

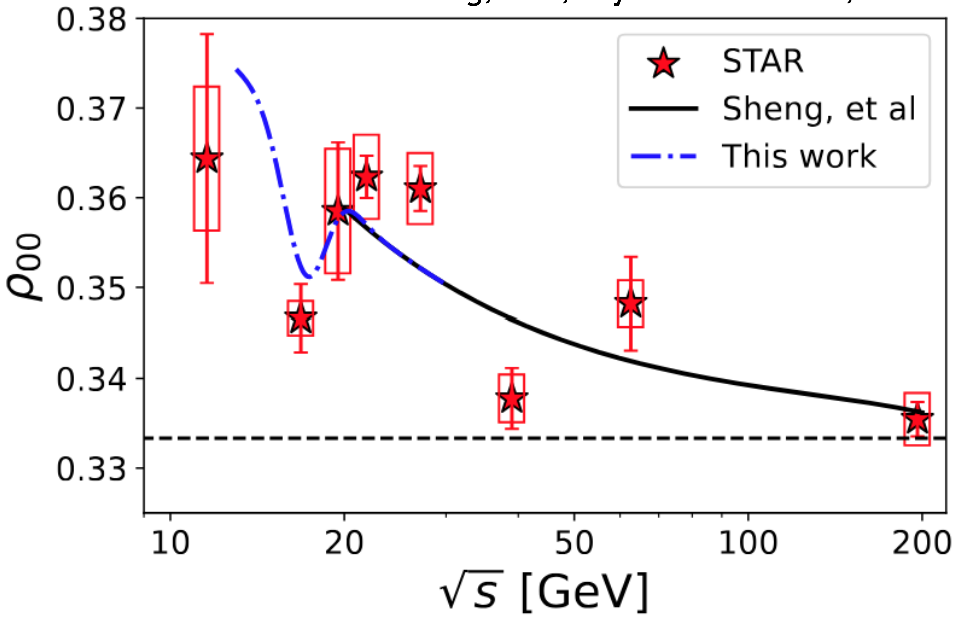
critical fluctuation
as an additional contribution (to
strong force field contribution)



A SCHEMATIC FIGURE

G. Wilks' talk@SQM2024

X.-L. Sheng, et al, Phys. Rev. Lett. 131, 042304 (2023)



OTHER CORRELATIONS

Kun Xu, Mei Huang, Phys.Rev.D 110 (2024) 9, 094034

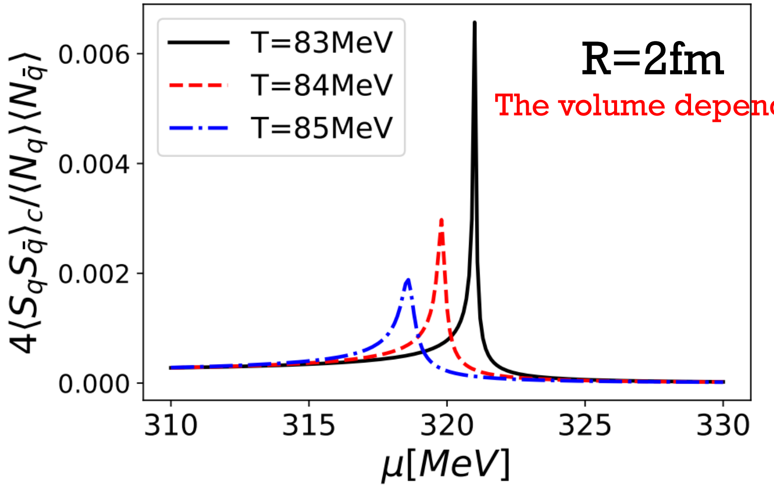
$$\delta\rho_{00}(\phi) \approx -\frac{4}{9}\frac{\langle \delta N_s \delta N_{\bar{s}}\rangle}{N_s N_{\bar{s}}} = -\frac{32}{9c^2}\frac{T}{V}\frac{N_c^2 G_A}{\rho_s^2}L^2$$

- Analogy to baryon number case

angular velocity $\omega \Longleftrightarrow \mu$ chemical potential

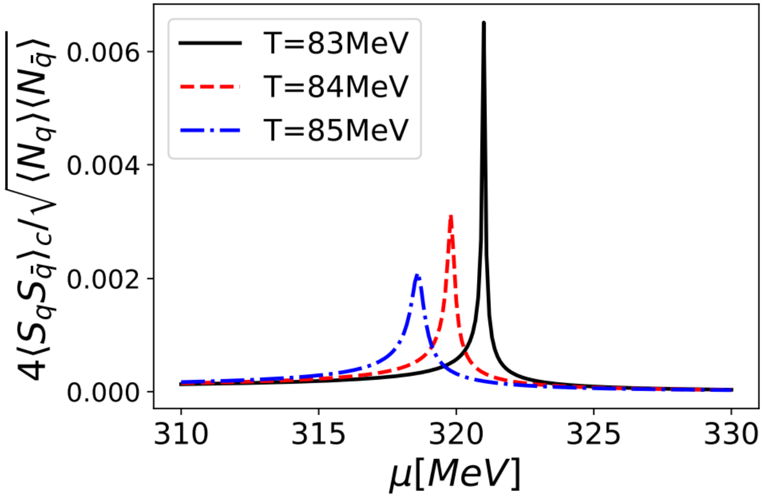
conserved charge : spin $S \Longleftrightarrow N$ quark number

$$\frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\langle N_q\rangle\langle N_{\bar{q}}\rangle}$$

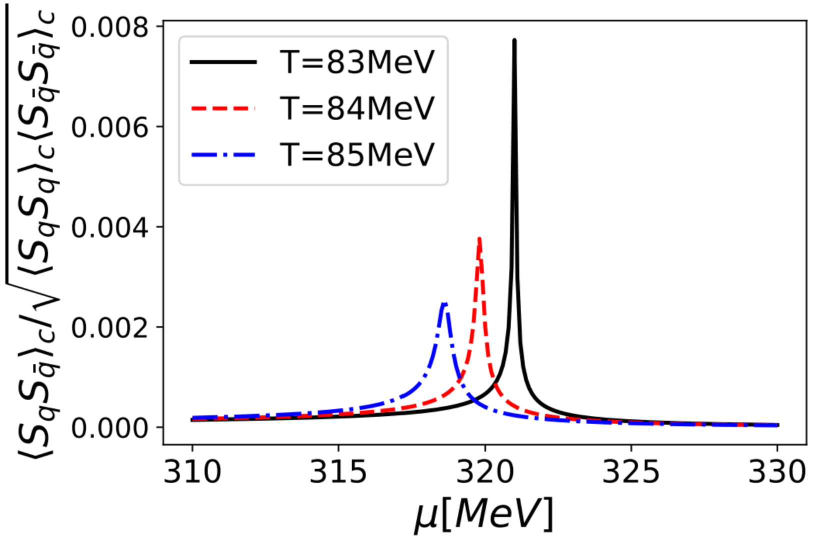


The volume dependence is annoying

$$\frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\sqrt{\langle N_q\rangle\langle N_{\bar{q}}\rangle}}$$



$$\frac{\langle S_q S_{\bar{q}}\rangle_c}{\sqrt{\langle S_q S_q\rangle_c \langle S_{\bar{q}} S_{\bar{q}}\rangle_c}}$$ Pearson correlation coefficient

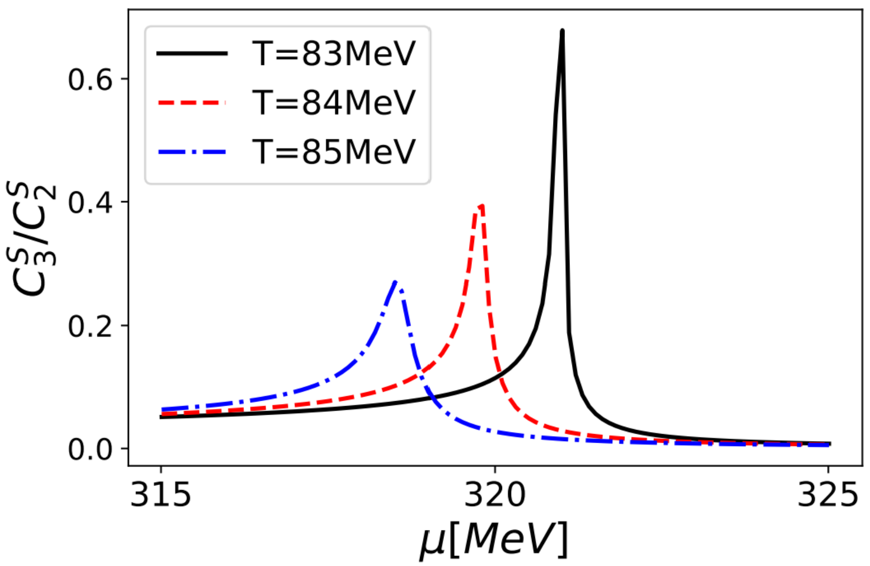


Second order quark-antiquark spin correlations

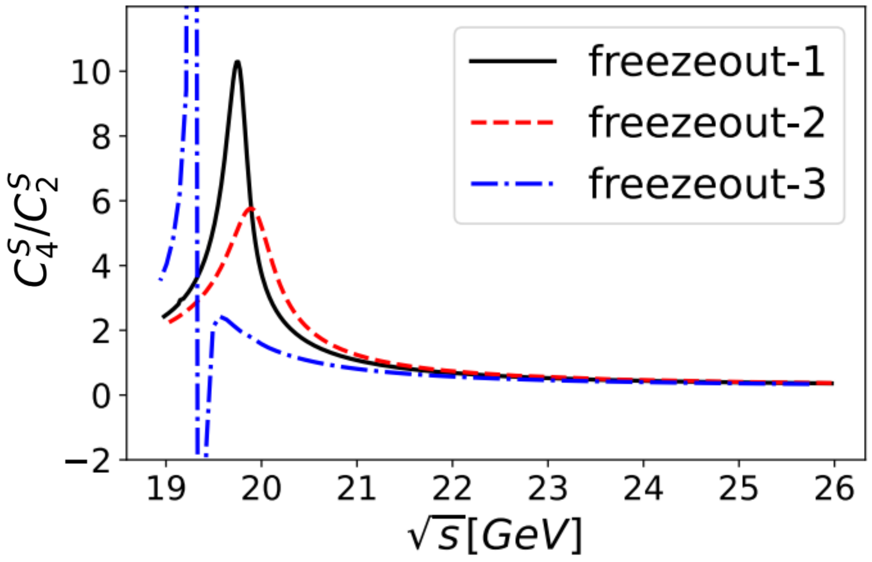
OTHER CORRELATIONS

- Cumulants $C_n^S = VT^{n-1}\frac{\partial^n p}{\partial \omega^n}$
- Higher orders are more sensitive to CEP

Skewness of spin fluctuation:



Kurtosis of spin fluctuation:



GLUON SPIN

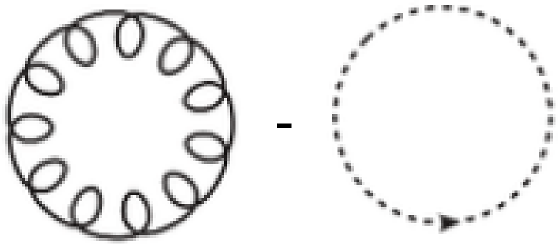
- So far, we only focused on the quark sector
- Gluon has spin one: more sensitive to rotation in principle
- It is interesting to see how rotation affects deconfinement and gluon spin

Shi Chen, Kenji Fukushima, Yusuke Shimada, Phys.Rev.Lett. 129 (2022) 24, 242002
Victor V. Braguta, Maxim N. Chernodub, Ilya E. Kudrov, Artem A. Roenko, Dmitrii A. Sychev, Phys.Rev.D 110 (2024) 1, 014511
Yin Jiang, Phys.Lett.B 853 (2024) 138655
Guojun Huang, Shile Chen, Yin Jiang, Jiaxing Zhao, Pengfei Zhuang, Phys.Lett.B 862 (2025) 139274
Kenji Fukushima, Yusuke Shimada, Phys.Lett.B 868 (2025) 139716
Sheng Wang, Jun-Xia Chen, Defu Hou, Hai-Cang Ren, arXiv: 2505.15487

POLYAKOV LOOP CONDENSATE

S. Chen, K. Fukushima, and Y. Shimada
Phys.Rev.Lett. 129 (2022) 24, 242002

- Pure Yang-Mills theory under imaginary rotation
- Weiss potential (one loop potential for Polyakov loop)

$V_{eff} =$ 

$$V(\phi;\tilde{\Omega}_\mathrm{I})|_{\tilde{r}=0} = \frac{\pi^2 T^4}{3}\sum_{\boldsymbol{\alpha}}\sum_{s=\pm 1} B_4\left(\left(\frac{\boldsymbol{\phi}\cdot\boldsymbol{\alpha} + s\tilde{\Omega}_\mathrm{I}}{2\pi}\right)_{\mathrm{mod}\ 1}\right).$$

- Simple replacement

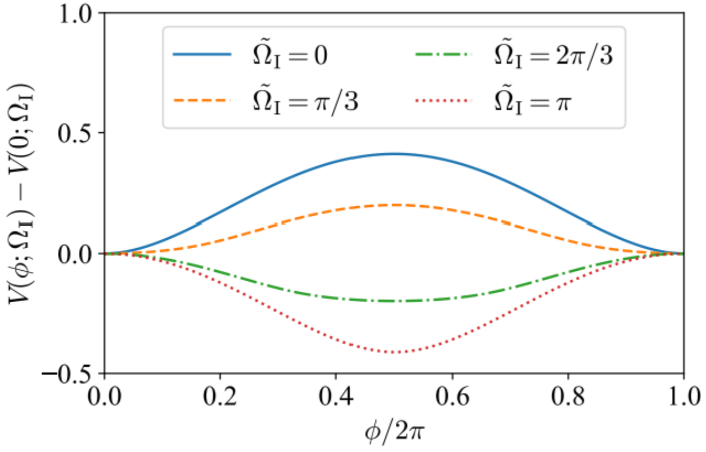
$$\phi\cdot\alpha \to \phi\cdot\alpha \pm \beta\Omega_I$$

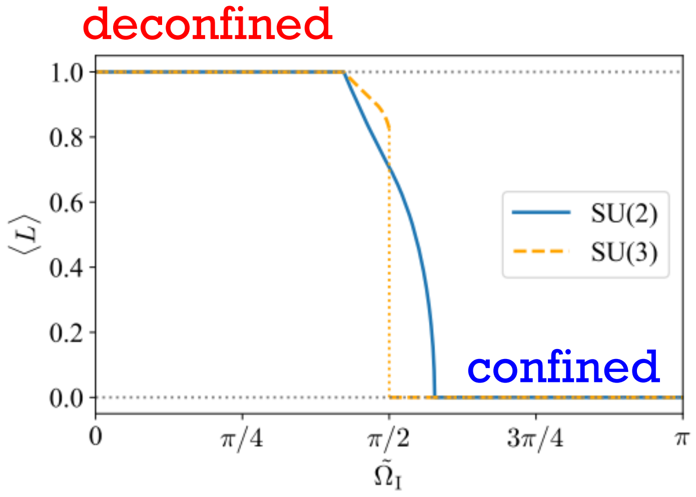
- Imaginary rotation prefer confinement

analytical continuation

Real rotation prefer deconfinement

opposite to lattice





CHROMOMAGNETIC CONDENSATE

- Another well known gluon condensate is chromomagnetic condensate

G.K. Savvidy, Phys. Lett. B, 71:133, 1977

- Although the constant chromomagnetic configuration is not stable

N.K Nielsen and P. Olesen., Nucl. Phys. B, 144(2-3):376–396, 1978.

- Asymptotic freedom can be understood in term of vacuum polarizability

$$E_{\mathrm{vac,QCD}} = -\tfrac{1}{2} V H^2 \frac{(33 - 2N_F) g^2}{48\pi^2} \log \frac{\Lambda^2}{|gH|}$$

QCD Beta function

$$\beta(g) = -\frac{(33 - 2N_F) g^3}{48\pi^2}.$$

Asymptotic freedom as a spin effect

N. K. Nielsen[a)]
Fysisk Institut, Odense University, Odense, Denmark
(Received 25 August 1980; accepted 26 November 1980)

It is shown how both the qualitative and the quantitative features of the asymptotic freedom of quantum chromodynamics can be understood in a rather intuitive way. The starting point is the spin of the gluon, which because of the gluon self-coupling makes the vacuum behave like a paramagnetic substance. Combining this result with Lorentz invariance, we conclude that the vacuum exhibits dielectric antiscreening and hence asymptotic freedom. The calculational techniques are with some minor modifications those of the Landau theory on the diamagnetic properties of a free-electron gas.

- It is interesting to see how rotation affects polarized gluon field(chromomagnetic condensate)

SU(2) YANG-MILLS THEORY UNDER ROTATION

- We include both Polyakov loop and chromomagnetic field by background field

$$\bar{A}_\mu^3 = (\phi, \frac{1}{2}Hy, -\frac{1}{2}Hx, 0)$$

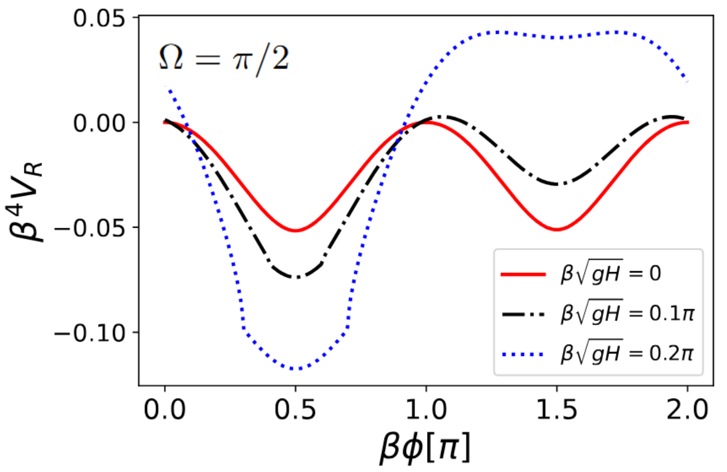
- As an extension of Kenji's work, we consider imaginary rotation and r=0

$$V(r=0) = \frac{1}{2}H^2 + \frac{gH}{2\pi\beta}\sum_{n=-\infty}^{\infty}\sum_{\lambda=0}^{\infty}\sum_{s=\pm 1}\int\frac{\mathrm{d}k_z}{2\pi}\ln[(\omega_n + s\Omega_I + g\phi)^2 + gH(2\lambda+1+2s) + k_z^2]$$

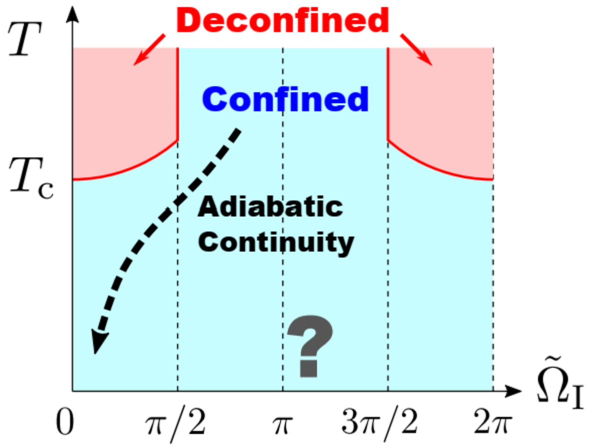
Tachyonic mode when $\lambda = 0, \quad s = -1$

- The effective potential

$$V_R = \frac{11g^2H^2}{48\pi^2}\ln(\frac{gH}{\mu_0^2}) - \frac{(gH)^{\frac{3}{2}}}{\pi^2\beta}\sum_{n=1}^{\infty}\frac{1}{n}[K_1(n\beta\sqrt{gH}) - \frac{\pi}{2}Y_1(n\beta\sqrt{gH})]\cos n(\tilde{\phi} - \tilde{\Omega}_I)$$
$$- 2\frac{(gH)^{\frac{3}{2}}}{\pi^2\beta}\sum_{n=1}^{\infty}\sum_{\lambda=0}^{\infty}\frac{1}{n}\sqrt{2\lambda+3}K_1(n\beta\sqrt{gH(2\lambda+3)})\cos n\tilde{\phi}\cos n\tilde{\Omega}_I$$
$$V_I = -\frac{(gH)^2}{8\pi} - \frac{(gH)^{\frac{3}{2}}}{2\pi^2\beta}\sum_{n=1}\frac{1}{n}J_1(n\beta\sqrt{gH})\cos n(\tilde{\phi} - \tilde{\Omega}_I)$$



Shi Chen, Kenji Fukushima, Yusuke Shimada,
Phys.Rev.Lett. 129 (2022) 24, 242002



GLUON CONDENSATE

- Minimize the real part of effective potential at high temperature

$$\frac{\partial V_R}{\partial (gH)} = \frac{\partial V_R}{\partial (g\phi)} = 0$$

- For small imaginary rotation, chromomagnetic condensate increases

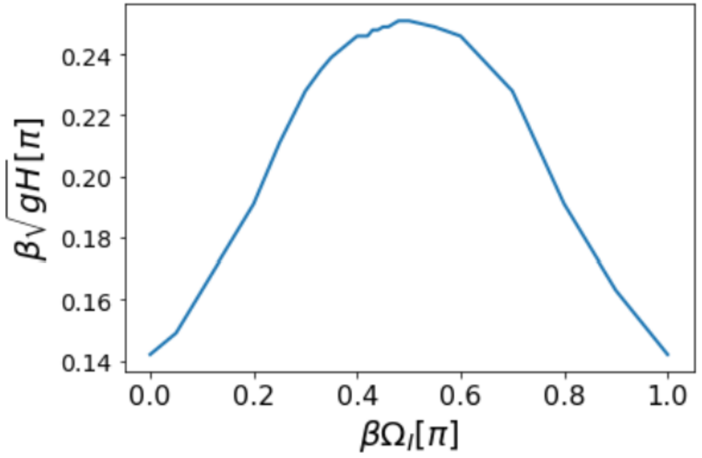


FIG. 3. Chromomagnetic condensate as a function of imaginary angular velocity at $T = 10\mu_0$.

deconfined

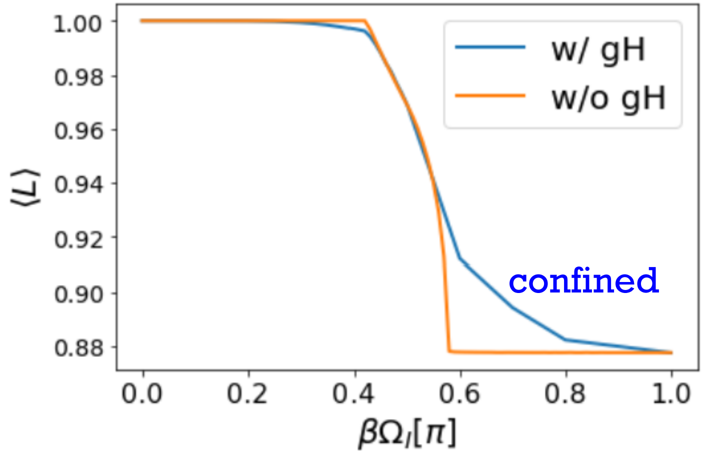


FIG. 6. Ployakov loop at $T = 10\mu_0$.

EFFECTIVE COUPLING CONSTANT

- For small imaginary rotation, we can expand the effective potential (where $b = \beta\sqrt{gH}$ and $\tilde{\Omega}_I = \beta\Omega$)

$$V_R = [-\frac{11}{24\pi^2\beta^4}(\ln\frac{\beta\mu_0}{4\pi} - \gamma) + \frac{7+4C_1}{32\pi^2\beta^4} - \frac{11}{96\pi^4\beta^4}\zeta(3)\tilde{\Omega}_I^2]b^4 - \frac{C_2}{2\pi\beta^4}b^3 - [\frac{11}{24\pi\beta^4} - \frac{C_3}{2\pi\beta^4}]\tilde{\Omega}_I^2 b$$

- We can extract effective coupling constant from VR

$$g^2_{eff}(T,\tilde{\Omega}_I) = \frac{1}{-\frac{11}{12\pi^2}(\ln\frac{\beta\mu_0}{4\pi} - \gamma) + \frac{7+4C_1}{16\pi^2} - \frac{11}{48\pi^4}\zeta(3)\tilde{\Omega}_I^2}$$

- The coupling increase with imaginary rotation: Tc increases with imaginary rotation
- Analytic continuation: Tc decreases with real rotation

In consistent with previous model studies

SUMMARY

- Critical spin fluctuations near CEP can lead to non-monotonic behavior of spin alignment & Hyperon-anti-Hyperon correlation
- In the picture of vacuum polarizability, our results agree with previous model studies
- Connection between spin and phase transition is a very interesting direction
- More realistic and detailed studies in future

THANK YOU FOR ATTENTION！

BACK UP

QUALITATIVE STUDY: NJL MODEL

- General thermodynamic potential under rotation

$$V_{eff}(r) = \frac{(m-m_0)^2}{4G} - N_c N_f \sum_l \int_0^{\Lambda} \frac{p_t \mathrm{d}p_t \mathrm{d}p_z}{(2\pi)^2} [\varepsilon_p + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p-\mu-\Omega j)}) + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p+\mu+\Omega j)})](J_l^2(p_t r) + J_{l+1}^2(p_t r)).$$

- Away from the center, contribution from orbital angular momentum is dominant
- Since we are interested in spin, we first focus on the physics near the center (r=0)

$$\begin{aligned} V_{eff}^0(\Omega,\mu) = & \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p \\ & + N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^2} [\overset{q,\uparrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})} + \overset{q,\downarrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})} \\ & + \underset{\bar{q},\uparrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})} + \underset{\bar{q},\downarrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})}]. \end{aligned}$$

- We can get information about average spin from this expression

QUALITATIVE STUDY: NJL MODEL

- Thermodynamic contribution (without critical fluctuation)

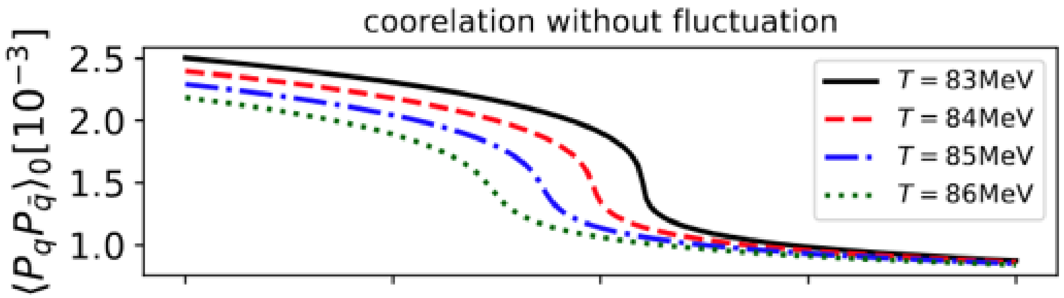
$$
\begin{aligned}
V^0_{eff}(\Omega,\mu) =& \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3p}{(2\pi)^2} 2\varepsilon_p \\
&+ N_c N_f \int_0^\infty \frac{\mathrm{d}^3p}{(2\pi)^2} [\underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})}_{q,\uparrow} + \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})}_{q,\downarrow} \\
&+ \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})}_{\bar{q},\uparrow} + \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})}_{\bar{q},\downarrow}].
\end{aligned}
$$

$$\langle S \rangle = -\frac{V}{T}\frac{\mathrm{d}V^0_{eff}}{\mathrm{d}(\frac{\Omega}{T})} = -\frac{V}{T}\frac{\partial V^0_{eff}}{\partial(\frac{\Omega}{T})} - \frac{V}{T}\boxed{\frac{\partial V^0_{eff}}{\partial m}}\frac{\partial m}{\partial(\frac{\Omega}{T})}.$$

0 (Gap Eq)

$$\langle P_q P_{\bar{q}} \rangle_0 = \frac{\int \mathrm{d}^3p (f_q^\uparrow - f_q^\downarrow)(f_{\bar{q}}^\uparrow - f_{\bar{q}}^\downarrow)}{\int \mathrm{d}^3p (f_q^\uparrow + f_q^\downarrow)(f_{\bar{q}}^\uparrow + f_{\bar{q}}^\downarrow)}.$$

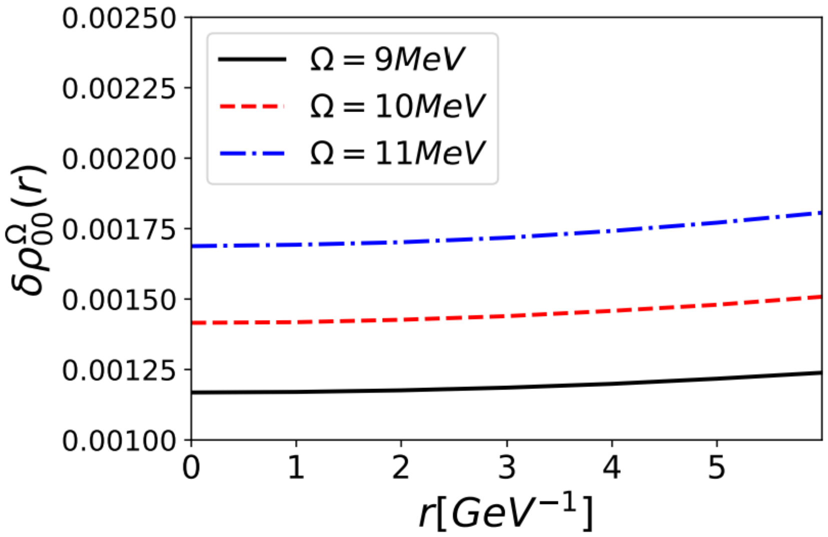
$$f_q^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p-\mu\mp\frac{\Omega}{2})}+1}, \quad f_{\bar{q}}^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p+\mu\mp\frac{\Omega}{2})}+1}.$$



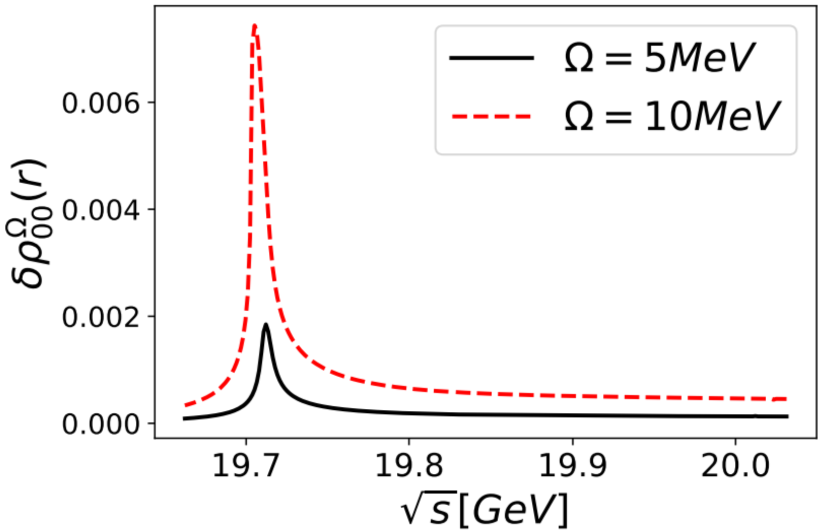
However, what we want is the correlation between quark and antiquark

ROTATION DEPENDENCE

Far from CEP



Freezeout-2 at different rotation

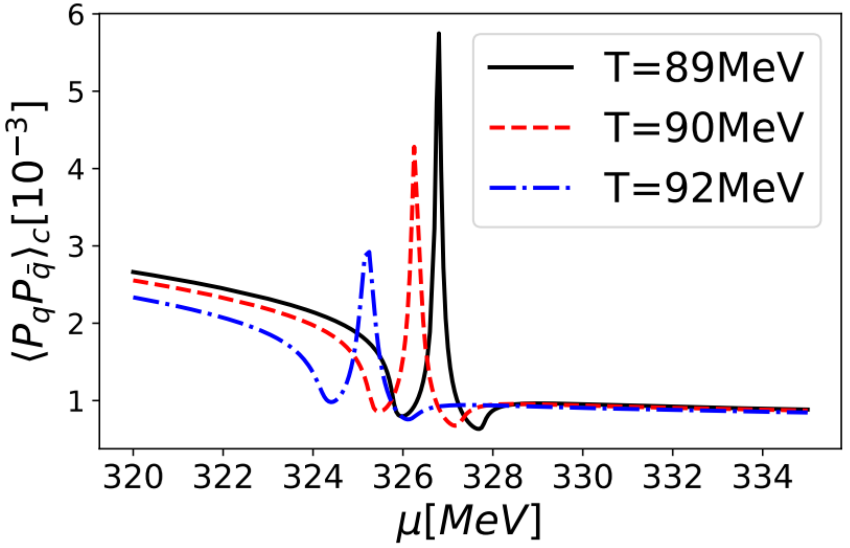


PNJL MODEL

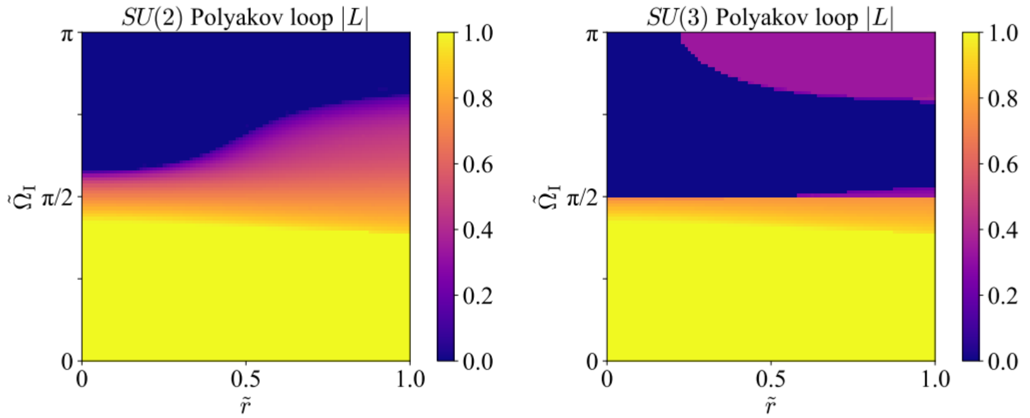
Λ [MeV]	m_0 [MeV]	$G_{\rm PNJL}\Lambda^2$	N_f	a_0	a_1	a_3	b_3	T_0[MeV]
651	5.5	2.135	2	3.51	−2.47	15.2	−1.75	210

$$
\begin{aligned}
&V_{\rm PNJL}(\Omega_q^s, \Omega_{\bar q}^s, \mu_q, \mu_{\bar q}; r=0) \\
=&\frac{[m-m_0]^2}{4G_{\rm PNJL}} - 2N_f\int_0^\Lambda \frac{{\rm d}^3p}{(2\pi)^3} 3\varepsilon_p \\
&- N_f\int_0^\infty \frac{{\rm d}^3p}{(2\pi)^3}\Big[T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T} + 3\bar\Phi{\rm e}^{-2(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T} + {\rm e}^{-3(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T}) \\
&+ T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T} + 3\bar\Phi{\rm e}^{-2(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T} + {\rm e}^{-3(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T}) \\
&+ T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p+\mu-\Omega_{\bar q}^s/2-\mu_{\bar q})/T} + 3\bar\Phi{\rm e}^{-2(\varepsilon_p+\mu-\Omega_{\bar q}^s/2-\mu_{\bar q})/T} + {\rm e}^{-3(\varepsilon_p+\mu-\Omega_{\bar q}^s/2-\mu_{\bar q})/T}) \\
&+ T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p+\mu+\Omega_{\bar q}^s/2-\mu_{\bar q})/T} + 3\bar\Phi{\rm e}^{-2(\varepsilon_p+\mu+\Omega_{\bar q}^s/2-\mu_{\bar q})/T} + {\rm e}^{-3(\varepsilon_p+\mu+\Omega_{\bar q}^s/2-\mu_{\bar q})/T})\Big] \\
&+ T^4\left\{-\frac{1}{2}\left[a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2\right]\bar\Phi\Phi + b_3(\frac{T_0}{T})^3\ln\left[1-6\bar\Phi\Phi+4(\bar\Phi^3+\Phi^3)-3(\bar\Phi\Phi)^2\right]\right\},
\end{aligned}
$$

- Quantitatively agree with NJL model results



FUTURE DIRECTION



- Including Quark degree of freedom
- Inhomogeneity Shi Chen, et al., Phys.Lett.B 859 (2024) 139107
- Two-loop contribution
- Higgs mechanism

might stabilize the system

- Whether this polarized gluon field relates to some observables?
- For example, proton spin