QCD matter under external electromagnetic fields

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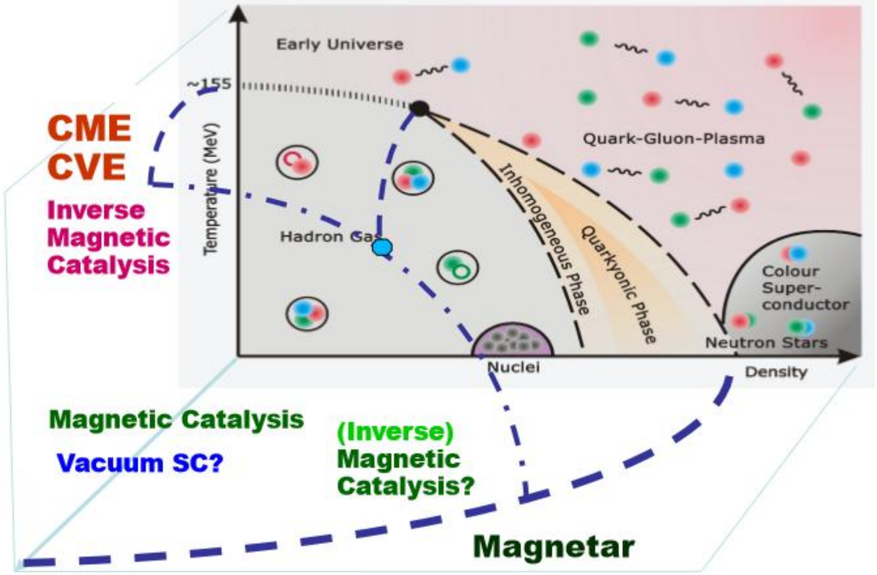
Outline

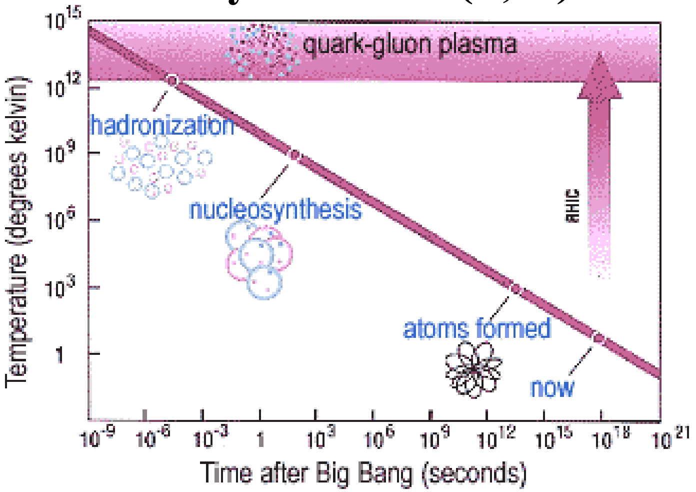
1. background
2. mesons @ eB
3. fluctuations and correlations @ eB
4. summary and outlook

1 Background

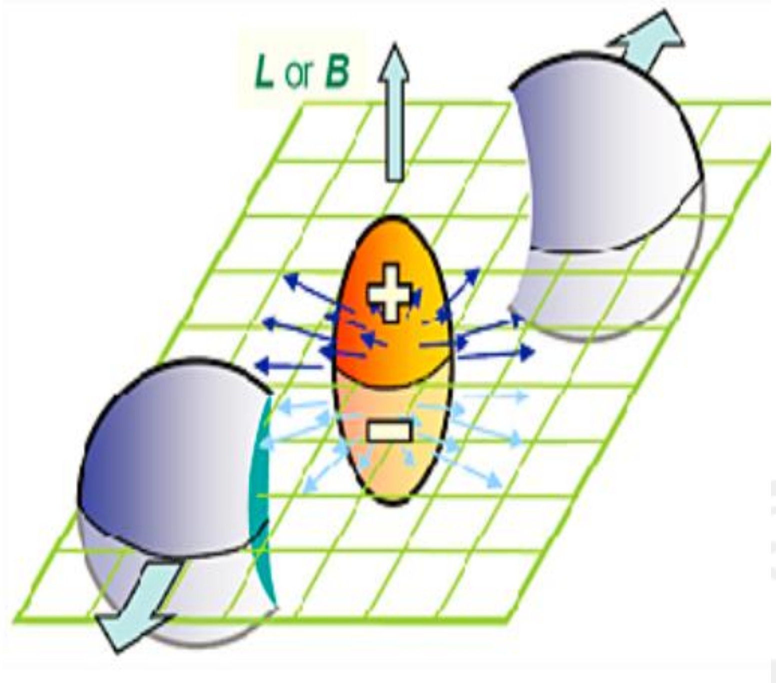
(R,E,) B



Early Universe (T, B)



HIC: LHC, RHIC, FAIR,NICA, …



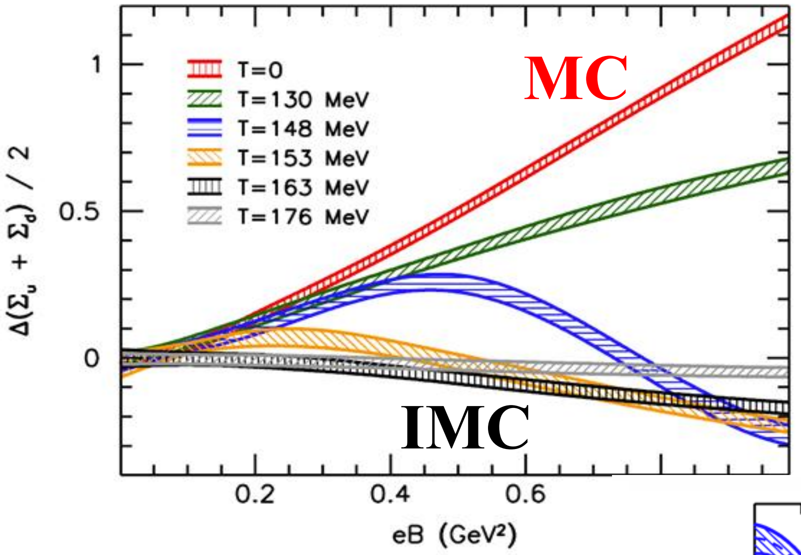
T, density,
E, B, R

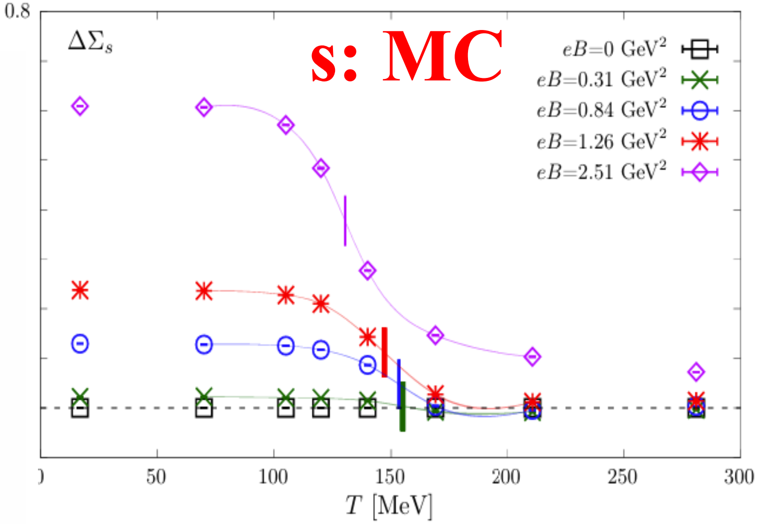
$$\mathrm{B}, E \sim \gamma \frac{Z\alpha_{EM}}{R_A^2} \sim 10^{18\sim 20}\,\mathrm{Gauss}$$

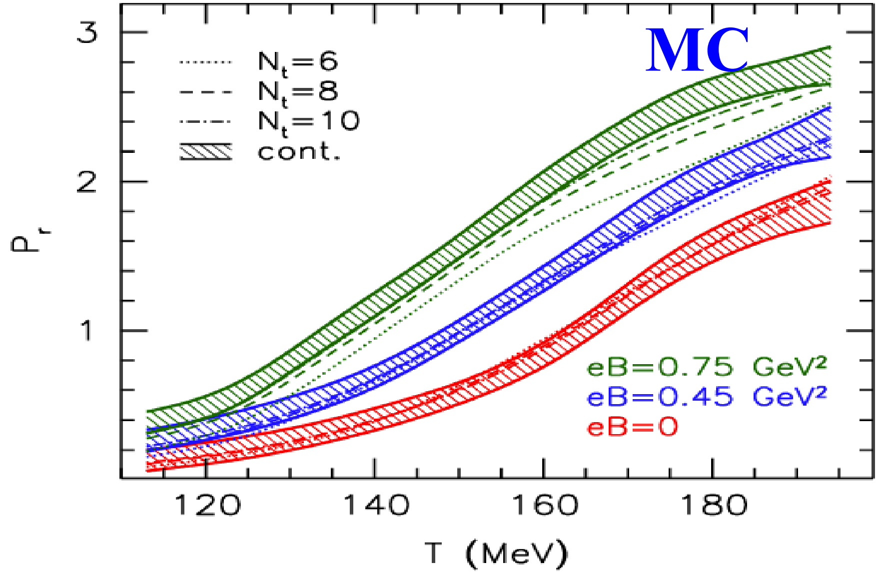
1.1 Inverse Magnetic Catalysis (IMC)

Polyakov loop

chiral condensate

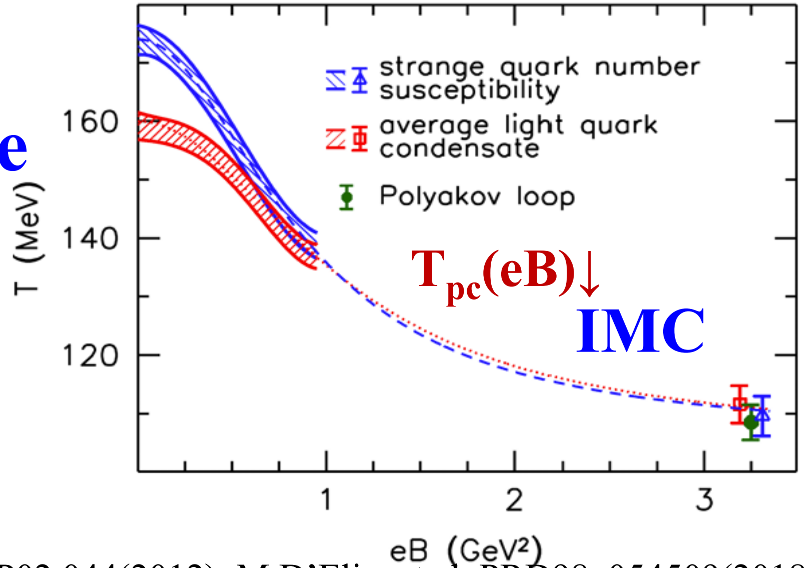


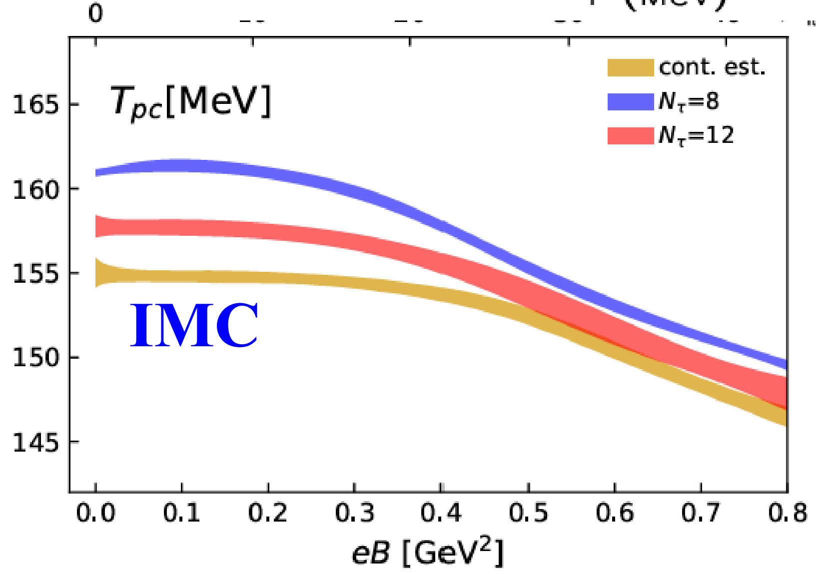




critical temperature

$T_{pc}(eB)\downarrow$





G. Bali, et.al, PRD86, 071502(2012);JHEP02,044(2012); M.D'Elia, et.al, PRD98, 054509(2018);H.T. Ding, et.al, PRD105, 034514(2022); PRD111, 114522 (2025).

1.1 Analytical efforts for IMC ---- Beyond mean field

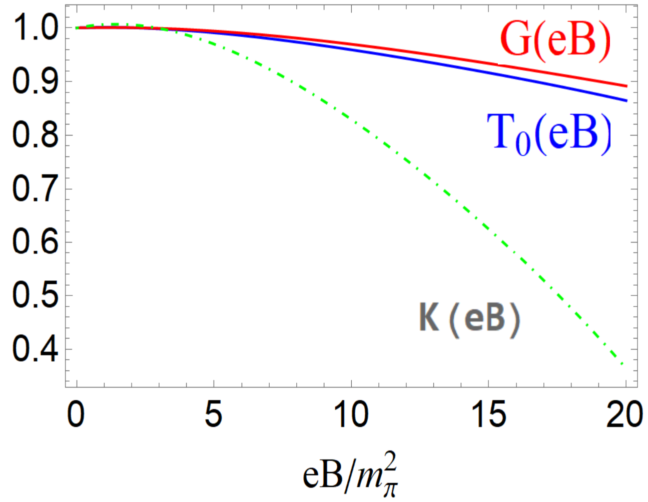
Quarks

(1) magnetic inhibition: feedback from neutral pion to quarks

Fukushima et al., PRL110, 031601 (2013); Mao, PLB758,195(2016); PRD94,036007(2016); PRD97,011501(2018); Mei, Mao, Huang et al., PRD110, 034024(2024).

(2) thermo-magnetic effect: fluctuations @(P)QM

Kamikado et al. , JHEP 03,009(2014); Ayala et al.PRD90,036001(2014); PRD89,116017(2014); PRD92,096011(2015); EPJA 57, 234(2021).



(3) weakening (eB dependence) of coupling between quarks

Farias et al. , PRC 90, 025203 (2014); Ferreira et al. , PRD89,116011(2014); PRD89, 036006(2014); PRD89,016002(2014); PRD89, 056013(2014); Mueller et al. , PRD91,116010(2015); Braun et al. , PLB755, 265(2016); Mao,arXiv:2406.13531.

(4) quark anomalous magnetic moment:

Chaudhuri et al. , PRD99, 116025 (2019); EPJA56, 213 (2020); PRD103, 096021 (2021), Mao et al. , PRD102, 114035 (2020); PRD106, 034018 (2022); Ghosh, PRD103, 116008 (2021); Xu, Chao, Huang, PRD103,076015(2021); Feng, PRD**107**, 076004(2023); Tavares, PRD109, 016011 (2024) .

Gluons

(5) eB-dependent interaction between Polyakov loop and quarks

Fraga, PLB731, 154(2014); Ferreira, PRD89, 016002(2014); Mao, PRD110, 054002(2024).

(6) sphalerons & instantons :chirality imbalance, $U_A(1)$ anomaly

Chao, Huang et al. , PRD88, 054009 (2013); Fukushima PRD81, 114031 (2010); Mao et.al JPS Conf. Proc. 20, 011009 (2017);Zhou, Mao, arXiv:2025.XXXXX.

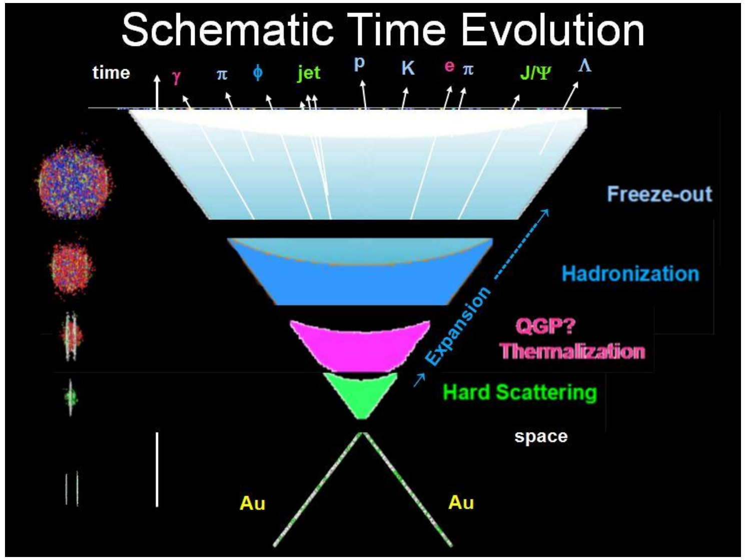
2 What is the influence of eB to QCD matter

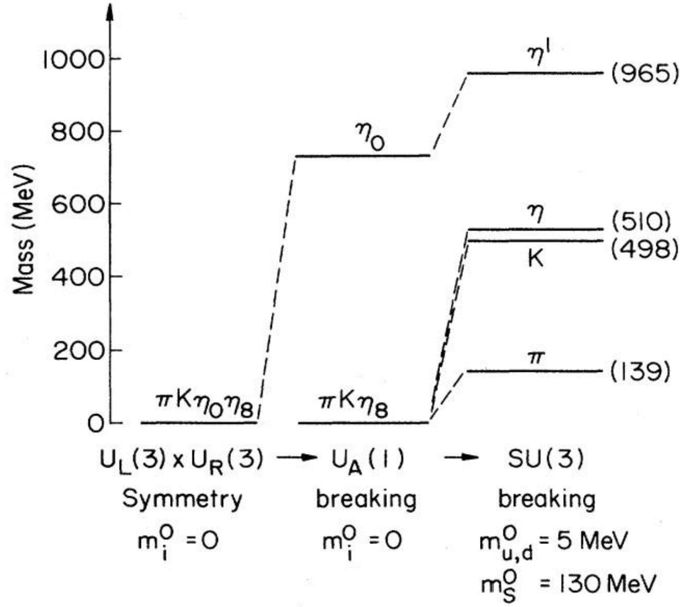
- mass spectra of mesons
- fluctuations and correlations of quark system

2.1 强相互作用相变与介子紧密相关

- 朗道相变理论： 连续相变对应于对称性自发破缺/恢复。
- Goldstone定理：连续对称性自发破缺，若不存在长程力，则一定存在零质量的Goldstone玻色子。

※ 利用介子性质研究QCD相变





2.1 Mass spectra of meson @ eB

(P)NJL model beyond mean field

$\mathbf{B} = (0, 0, B)$

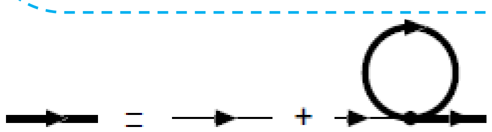
2-flavor $\boxed{\pi}$

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu - m_0\right)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - \mathcal{U}(\Phi,\bar{\Phi}),$$

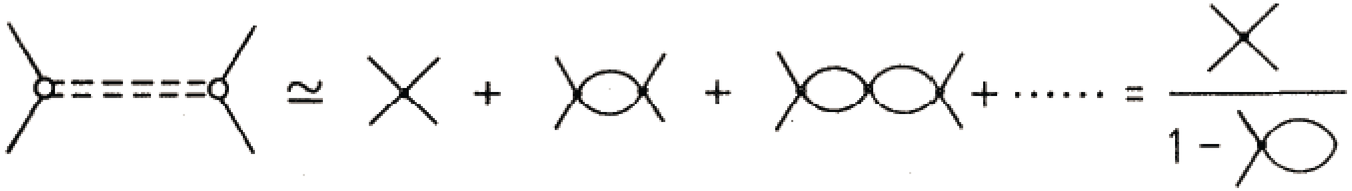
3-flavor $\boxed{K, \pi, \eta, \eta'}$

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - \hat{m}_0\right)\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi,\bar{\Phi}),\\
\mathcal{L}_4 &= G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2\right],\\
\mathcal{L}_6 &= -K\left[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi\right],\\
\mathcal{U}(\Phi,\bar{\Phi}) &= T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3+\Phi^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].
\end{aligned}$$

(1)Quarks： mean field



(2)**Mesons**: RPA resummation (quantum fluctuation)



S.J. Mao and Y.X. Wange, PRD96, 034004(2017); L.Y. Li and S.J. Mao, PRD108, 054001(2023); J.Mei, T.Xia, S.J. Mao, PRD107,074018(2023); arXiv: 2025.XXXXX

quark propagator under external magnetic field
---- Ritus propagator

$$\boxed{\mathbf{B} = (0, 0, B)}$$

$$\underline{S_f(x, y)} = i\sum_{n=0}^{\infty}\int \frac{d\tilde{p}}{(2\pi)^3} e^{-i\tilde{p}\cdot(x-y)} P_n(x_1, p_2) D_f(\bar{p}) P_n(y_1, p_2),$$

$$P_n(z, q) = \frac{1}{2}\left[g_n^{s_f}(z, q) + I_n g_{n-1}^{s_f}(z, q)\right] + \frac{i s_f}{2}\left[g_n^{s_f}(z, q) - I_n g_{n-1}^{s_f}(z, q)\right]\gamma^1\gamma^2,$$

$$\underline{D_f^{-1}(\bar{p})} = \gamma\cdot\bar{p} - m, \quad \bar{p} = (p_0, 0, -s_f\sqrt{2|Q_f B|n}, p_3)$$

Landau level

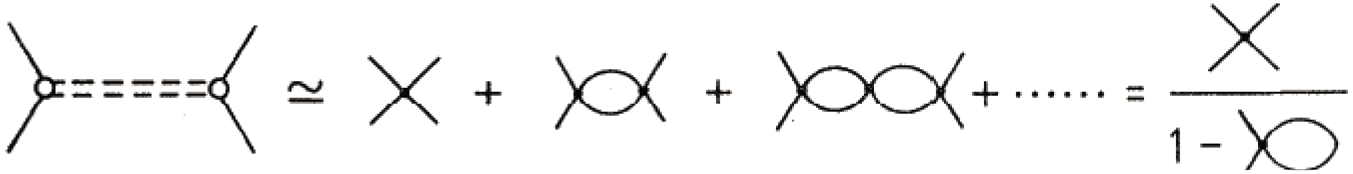
$$\tilde{p} = (p_0, 0, p_2, p_3) \qquad I_n = 1 - \delta_{n0}$$

$$g_n^{s_f}(z, q) = \phi_n(z - s_f q/|Q_f B|)$$

$$\phi_n(z) = \left(2^n n!\sqrt{\pi}|Q_f B|^{-1/2}\right)^{-1/2} \times e^{-z^2|Q_f B|/2} H_n\left(z/|Q_f B|^{-1/2}\right)$$

refs: Ann. Phys. 69, 555 (1972); Nucl. Phys. B747, 266 (2006);Ann. Phys. 295, 33 (2002) .

Mesons: RPA resummation



meson propagator:

$$\mathcal{D}_M(x,z) = 2G\delta(x-z) + \int d^4y\, 2G\Pi_M(x,y)\mathcal{D}_M(y,z),$$

$$\Pi_M(x,y) = i\mathrm{Tr}[\Gamma_M^* S(x,y)\Gamma_M S(y,x)]$$

with the neutral meson vertex

$$\Gamma_M = \begin{cases} i\gamma_5\lambda_0, & M=\eta_0 \\ i\gamma_5\lambda_3, & M=\pi_0 \\ i\gamma_5\lambda_8, & M=\eta_8 \\ i\gamma_5(\lambda_6 \pm i\lambda_7)/\sqrt{2}, & M=K_0,\bar{K}_0 \end{cases},$$

spectral function

$$\rho_M(\omega,\mathbf{0}) = -2\mathrm{Im}\mathcal{D}_{\pi_0}(\omega+i\epsilon,\mathbf{0}) = -2\mathrm{Im}\left(\frac{2G}{1-2G\Pi_{\pi_0}(\omega+i\epsilon,\mathbf{0})}\right).$$

and charged meson vertex

$$\Gamma_M = \begin{cases} i\gamma_5(\lambda_1 \pm i\lambda_2)/\sqrt{2}, M=\pi^{\pm} \\ i\gamma_5(\lambda_4 \pm i\lambda_5)/\sqrt{2}, M=K^{\pm} \end{cases}.$$

pole mass:

$$1 - G\Pi_M(m_M,\mathbf{0}) = 0,$$

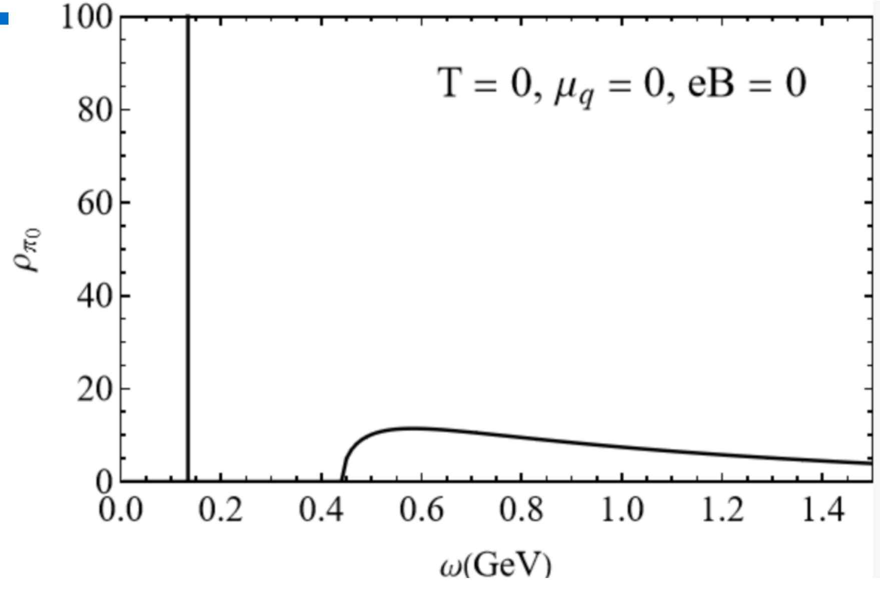
- **neutral mesons** K_0, π_0, η, η'

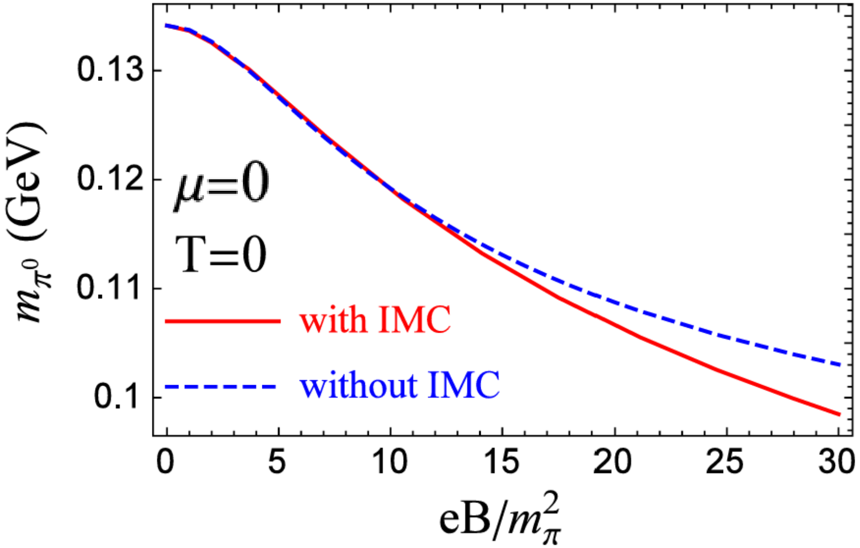
Spectral function of π_0 @ eB

$T=0 \quad \mu=0$

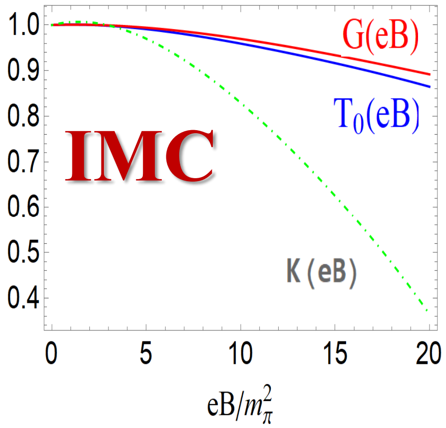
Zhou, **Mao**, arXiv:2025.XXXXX;Li, **Mao**, PRD 108, 054001 (2023)

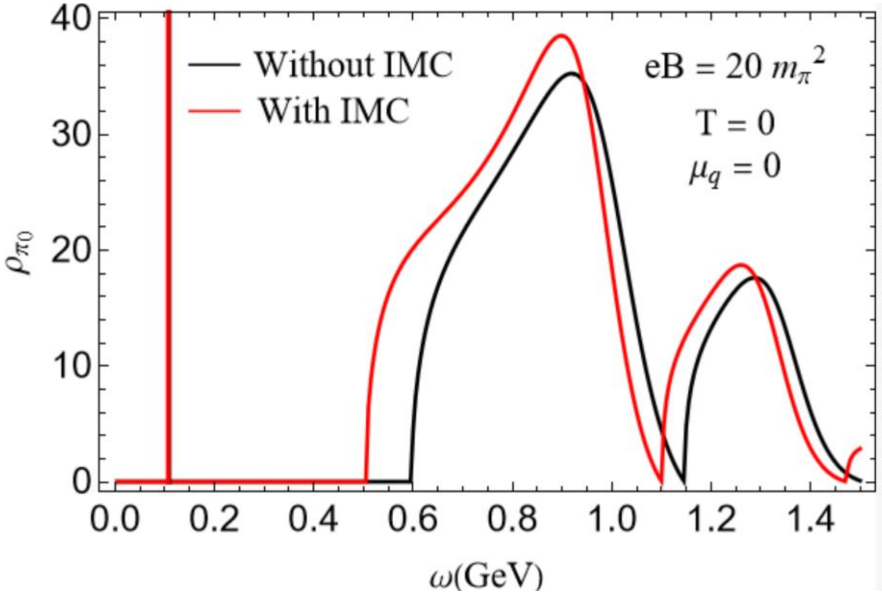
eB=0

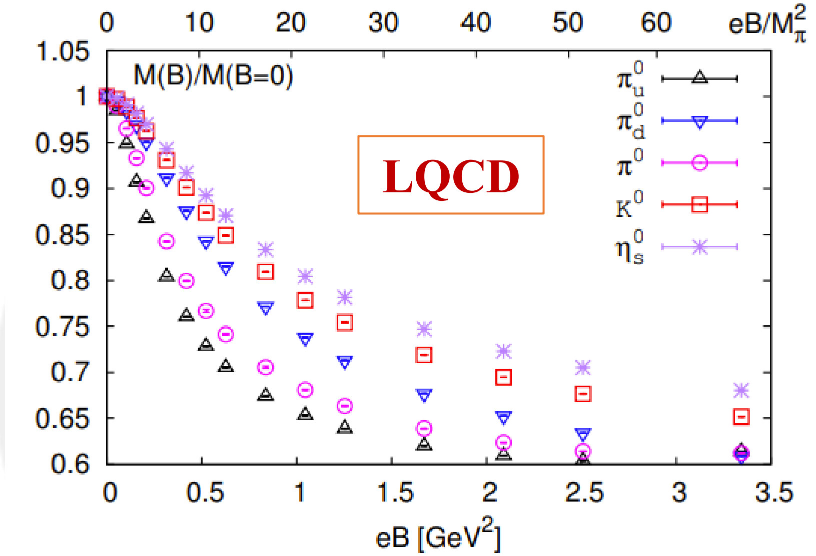




finite eB (Landau levels)



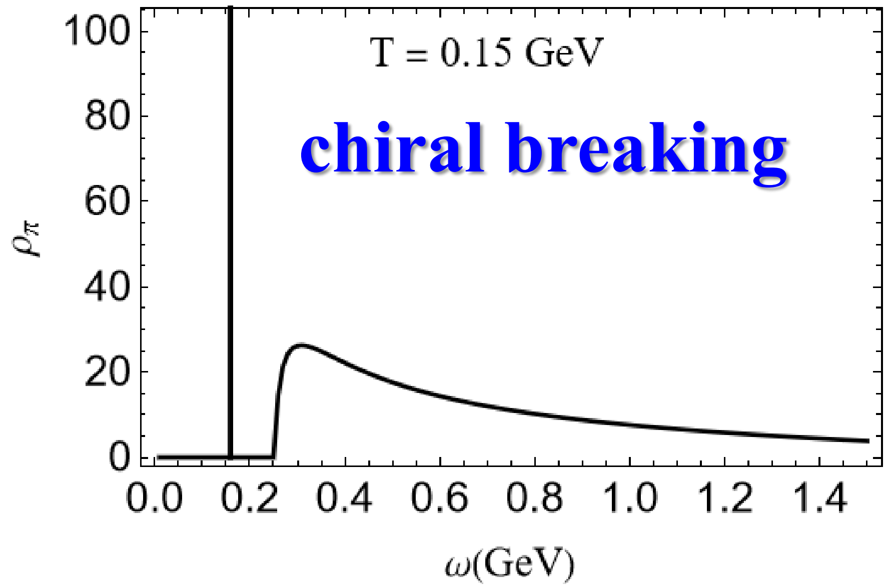


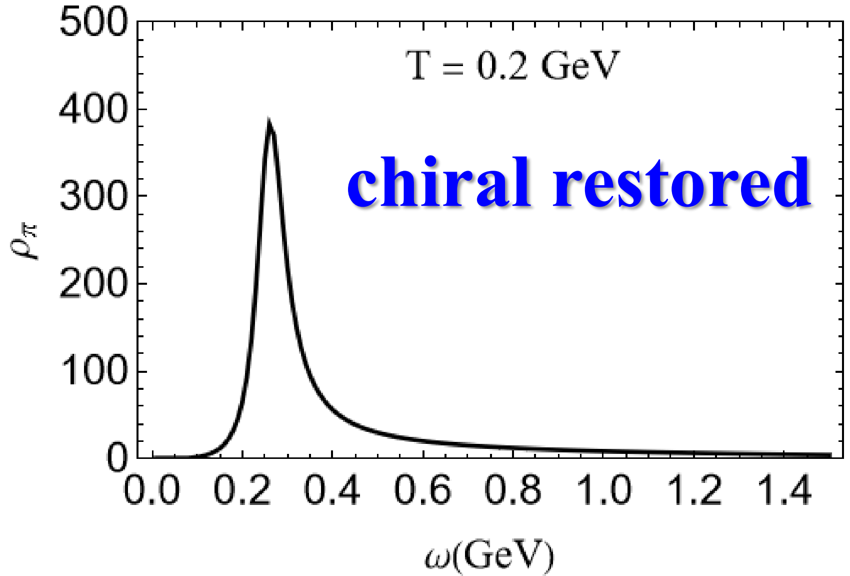


Spectral function of π_0 @ eB $\boxed{\mu=0}$

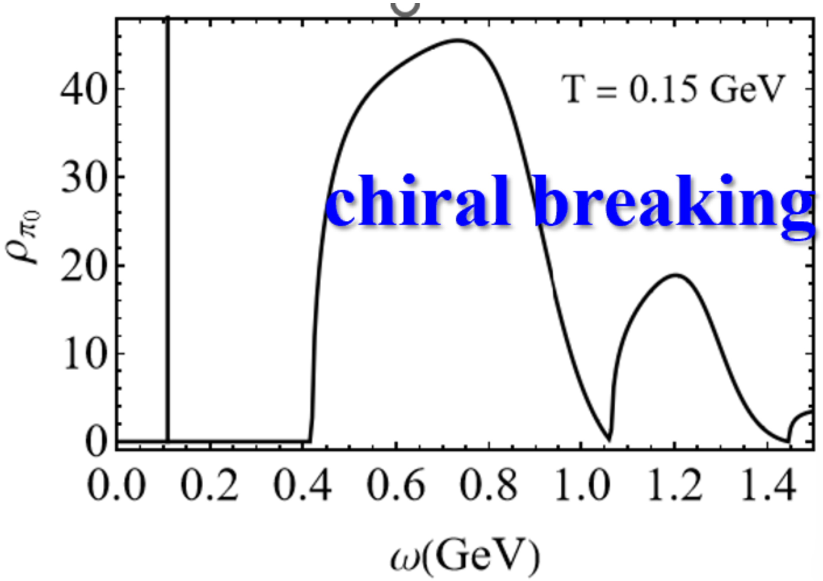
Zhou, **Mao**, arXiv:2025.XXXXX;
Wang, **Mao**, PRD 96, 034004 (2017)

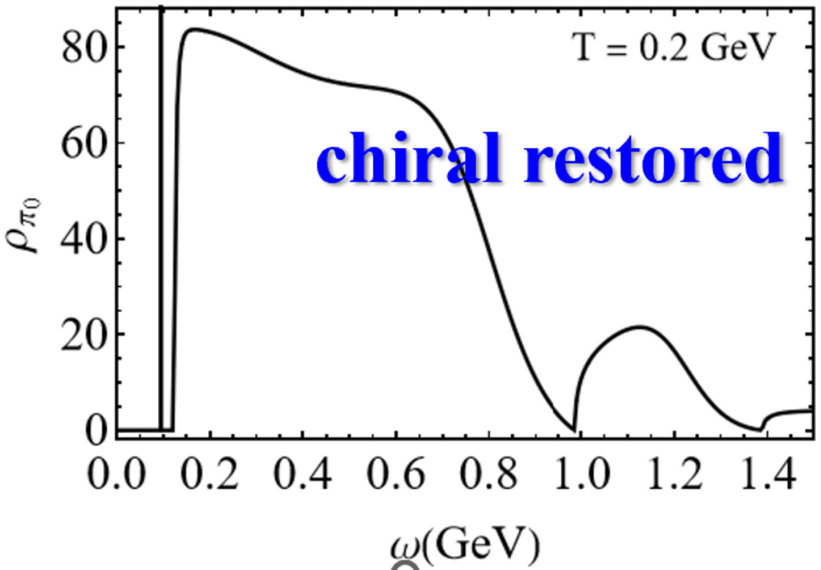
eB=0



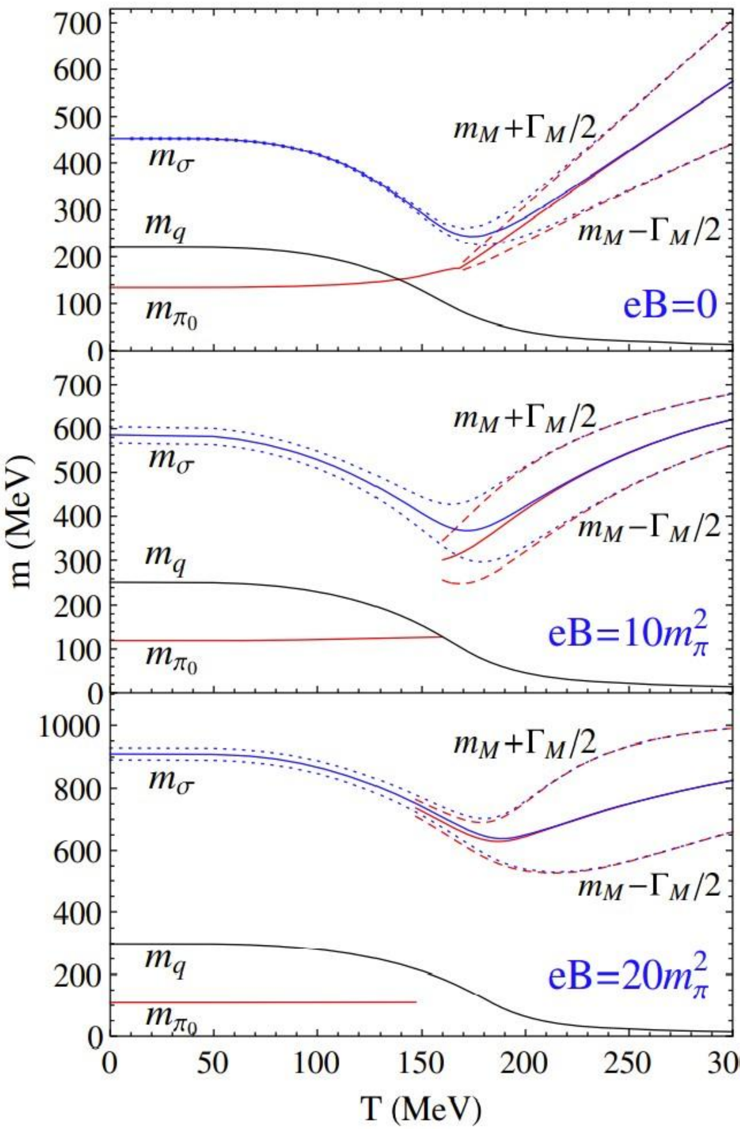


finite eB (Landau levels)

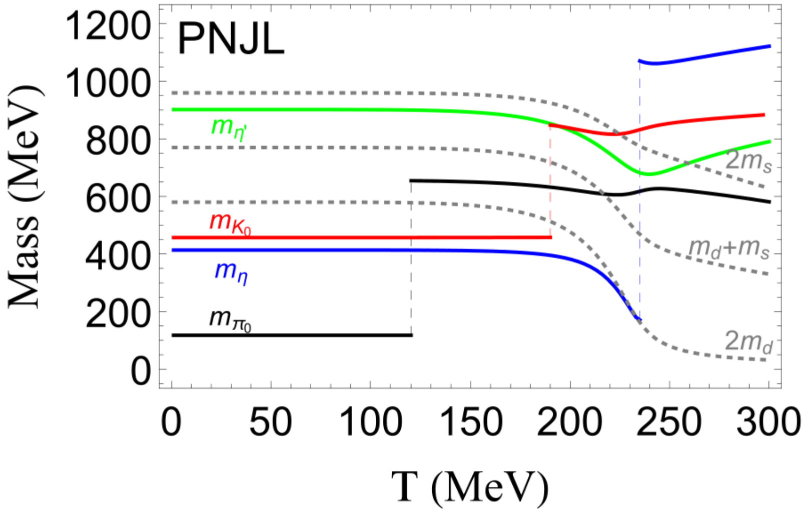


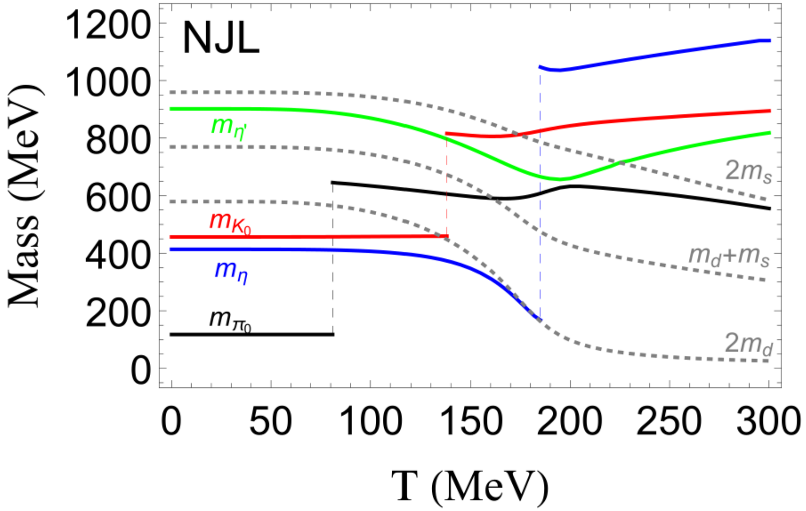


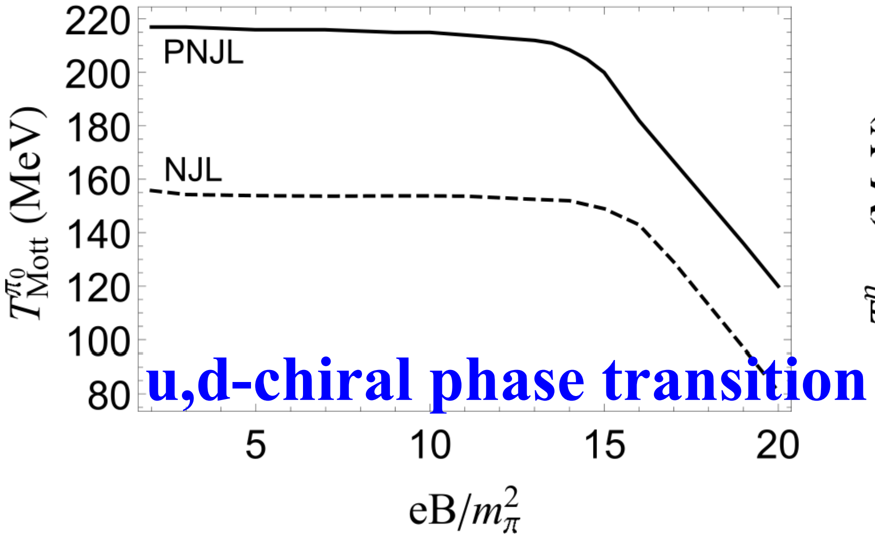
磁场引起质量跳变@Mott 相变

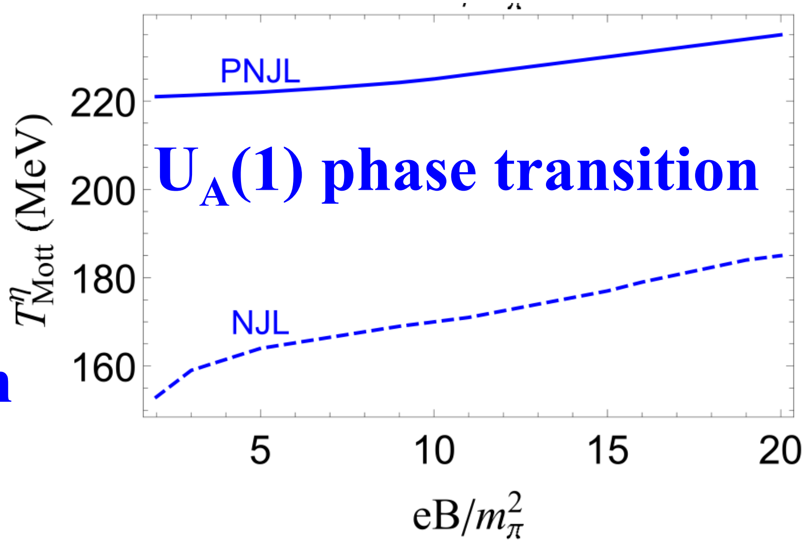


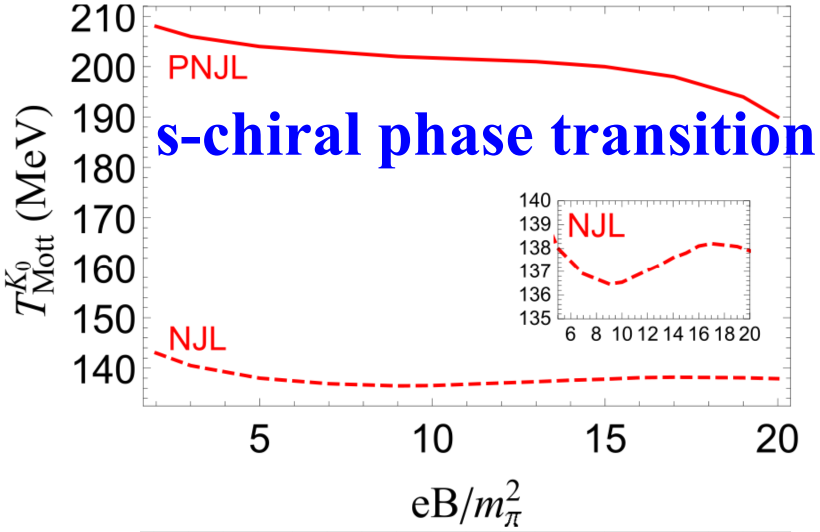
2.1 K_0, π_0, η, η' mesons @ eB $\mu=0$





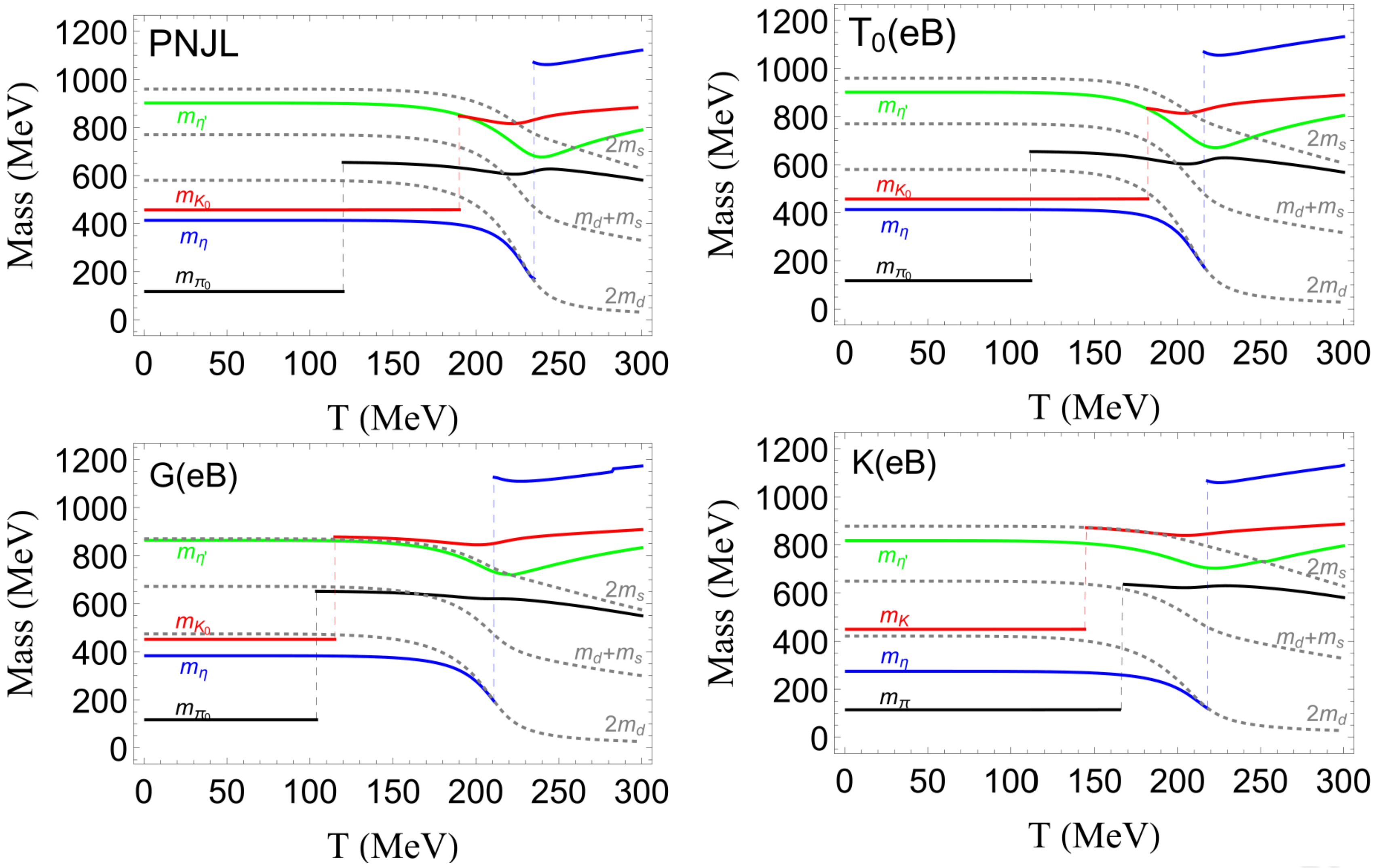


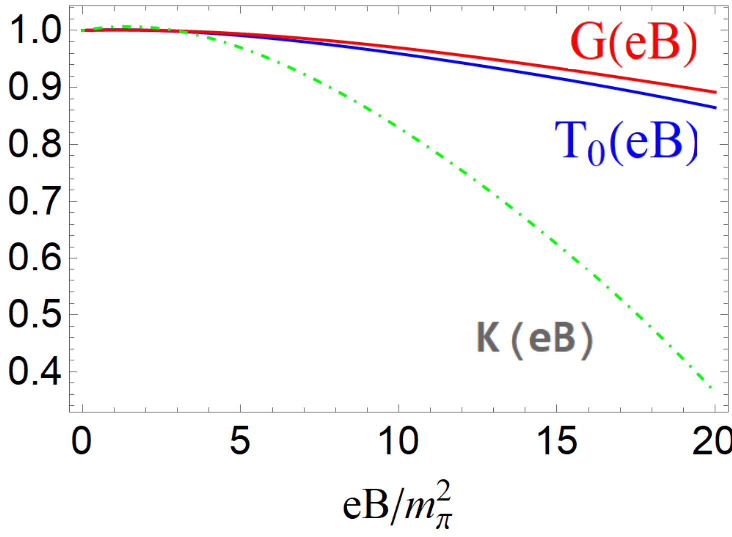




Mei,Xia,Mao, PRD 107, 074018 (2023), Li, Mao, arXiv:2025.XXXXX

2.1 K_0, π_0, η, η' with IMC μ=0





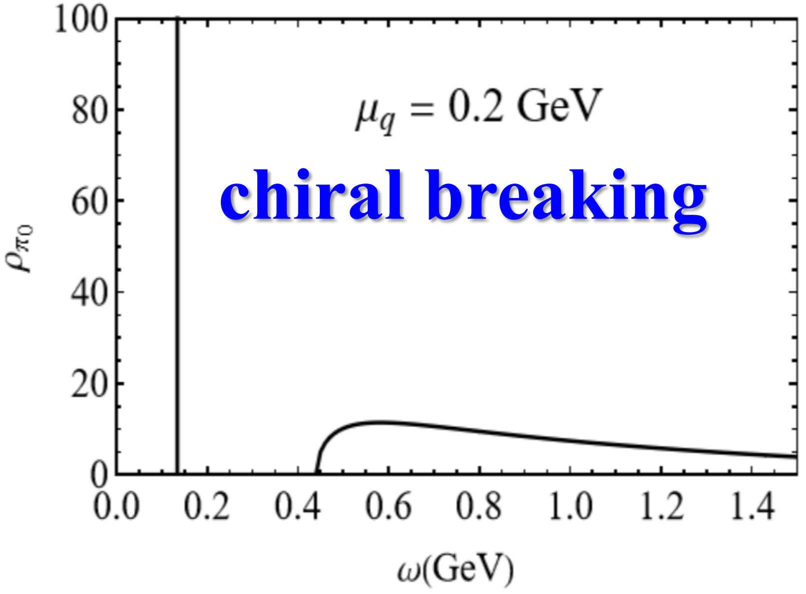
➢ **IMC leads to no qualitative difference**

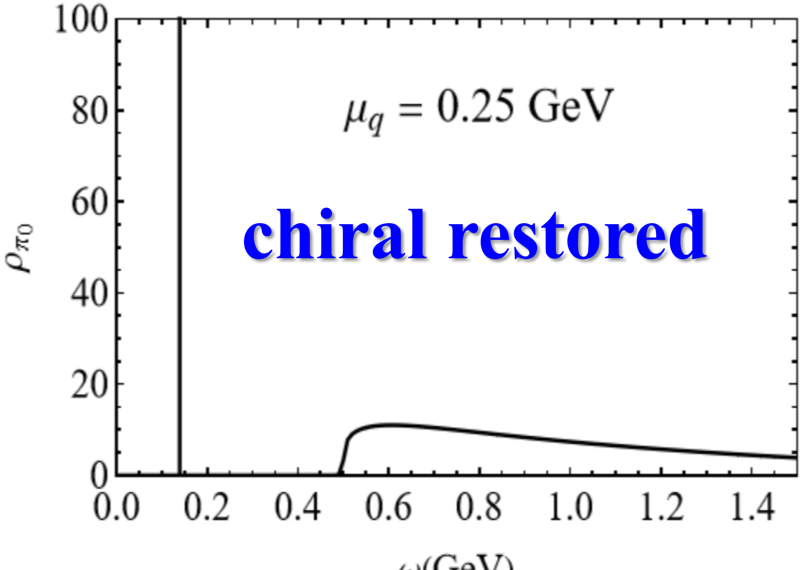
Spectral function of π_0 @ eB

T=0

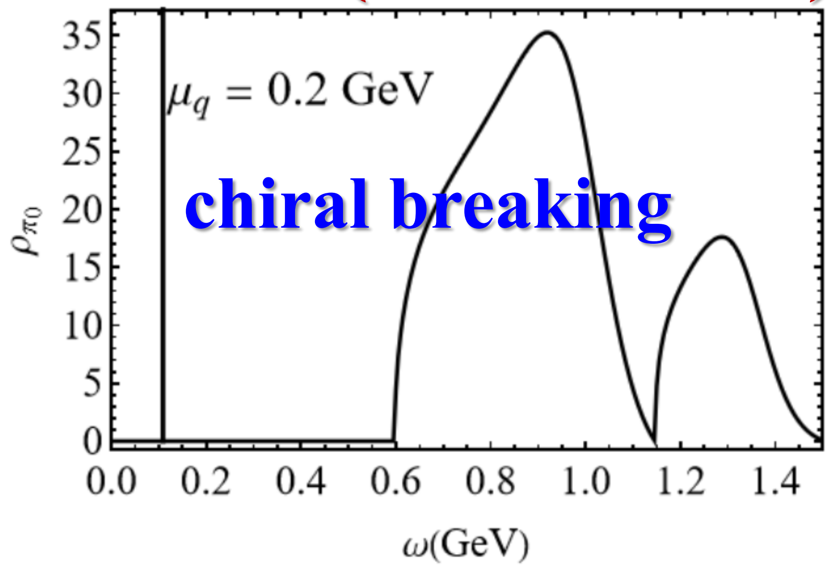
Zhou, **Mao**, arXiv:2025.XXXXX;
Wang, **Mao**, PRD 96, 034004 (2017)

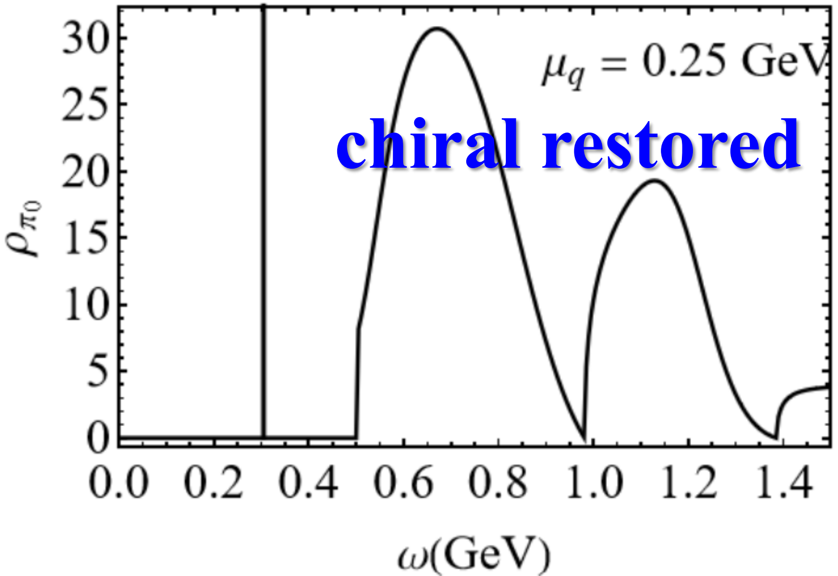
eB=0



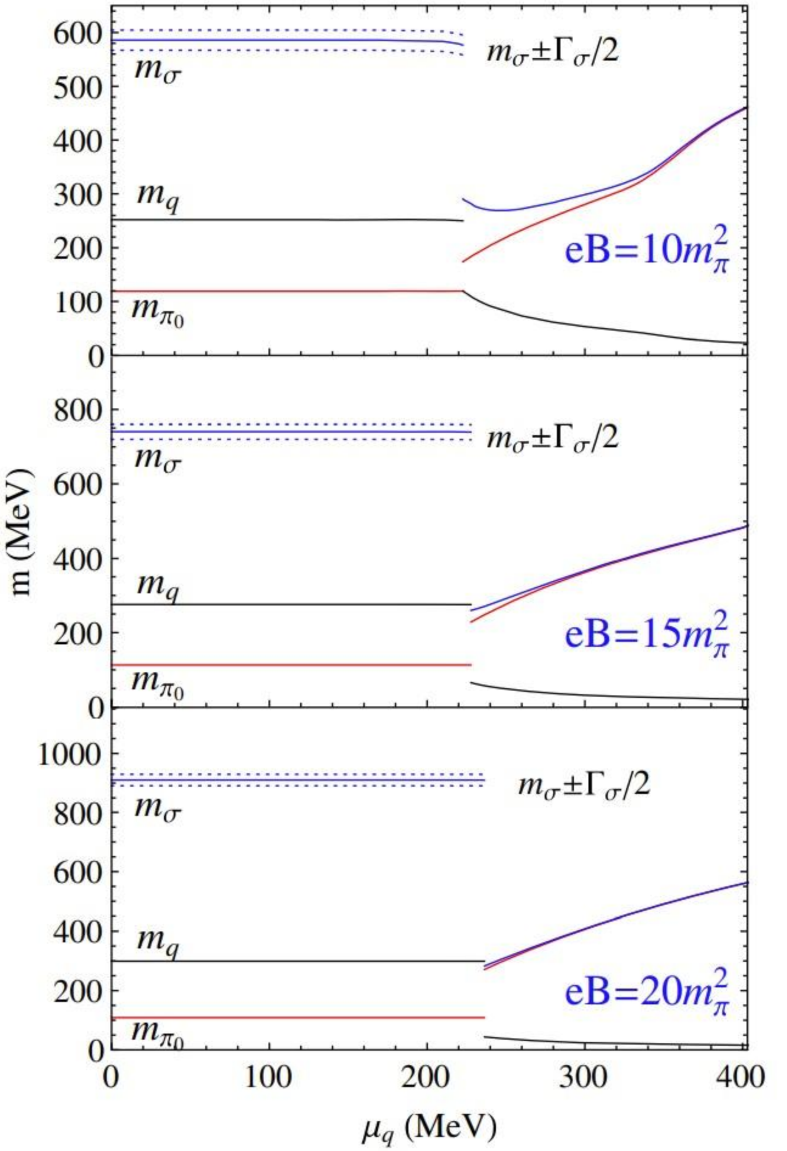


finite eB (Landau levels)



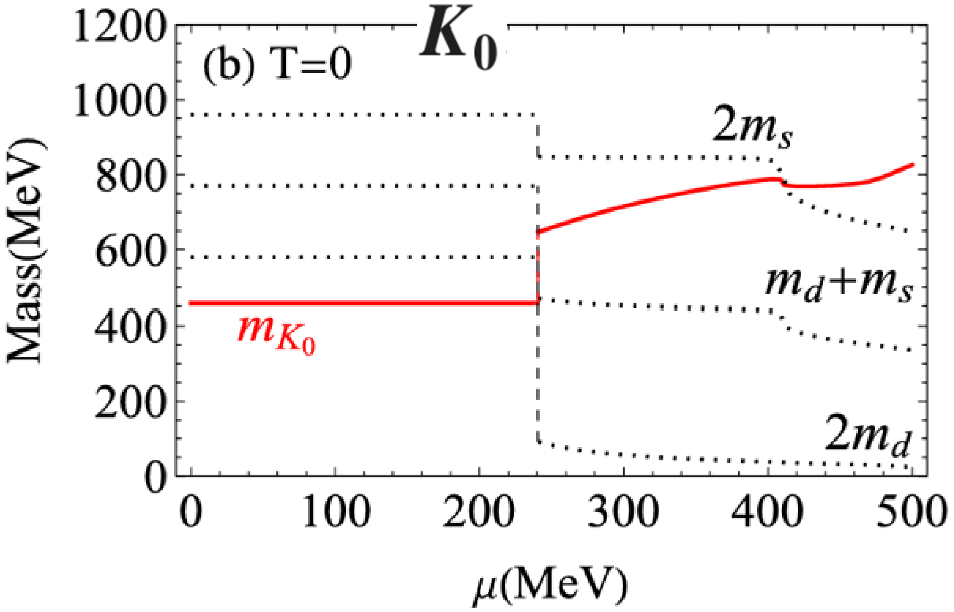


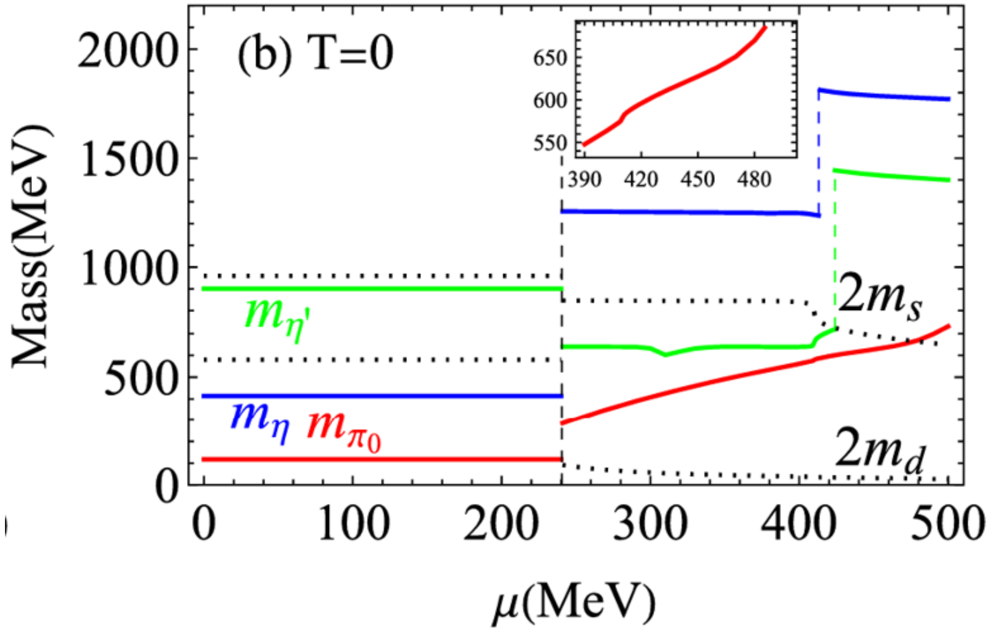
一级相变引起质量跳变



K_0, π_0, η, η' mesons @ T, μ , eB

Mei,Xia, Mao, PRD 107, 074018 (2023)



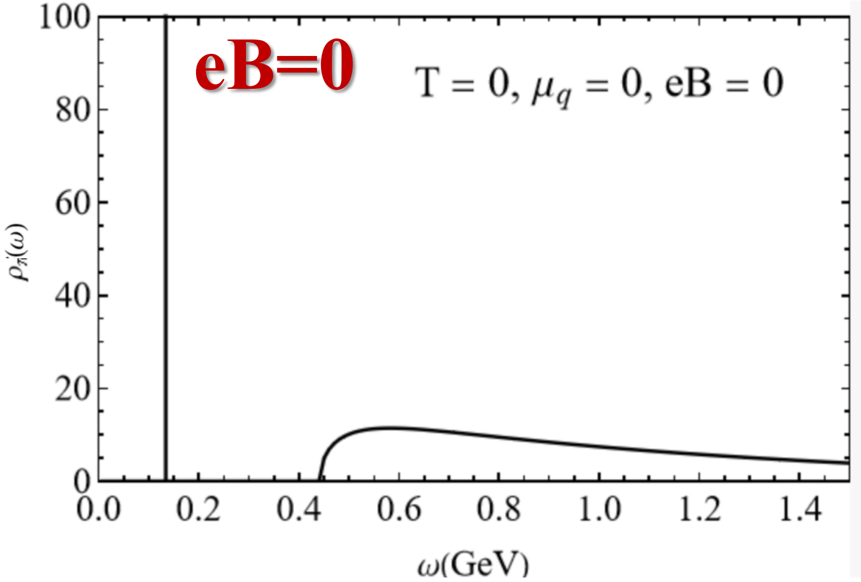


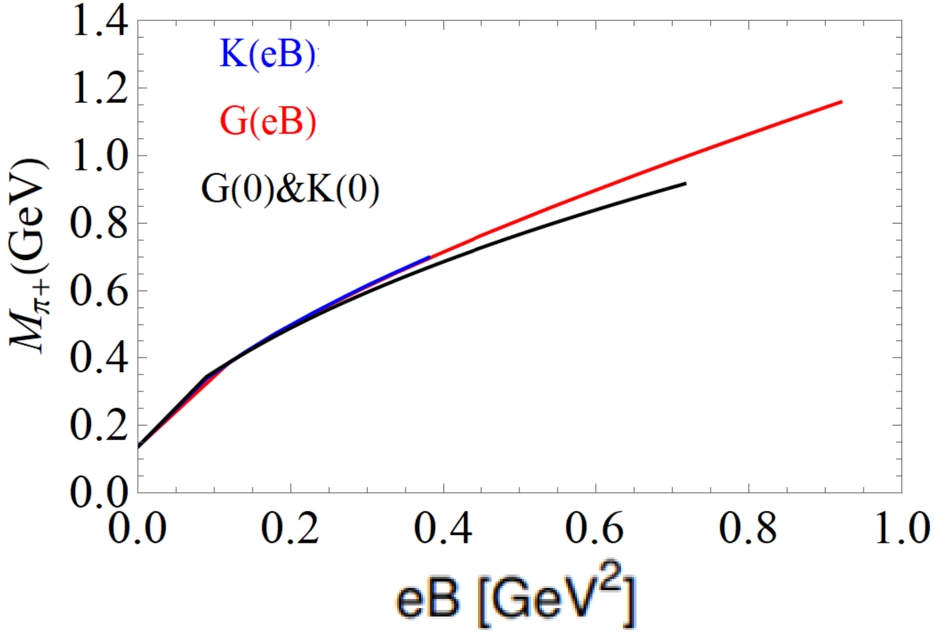
- **charged mesons** π^+, K^+

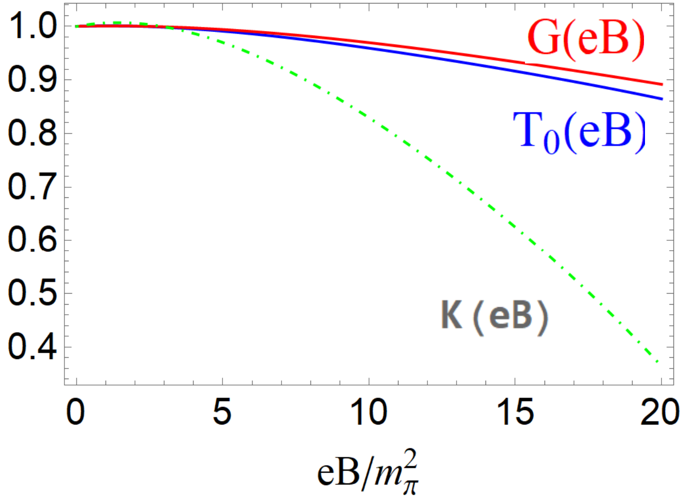
Spectral function of π_+ @ eB

$\mu=0$

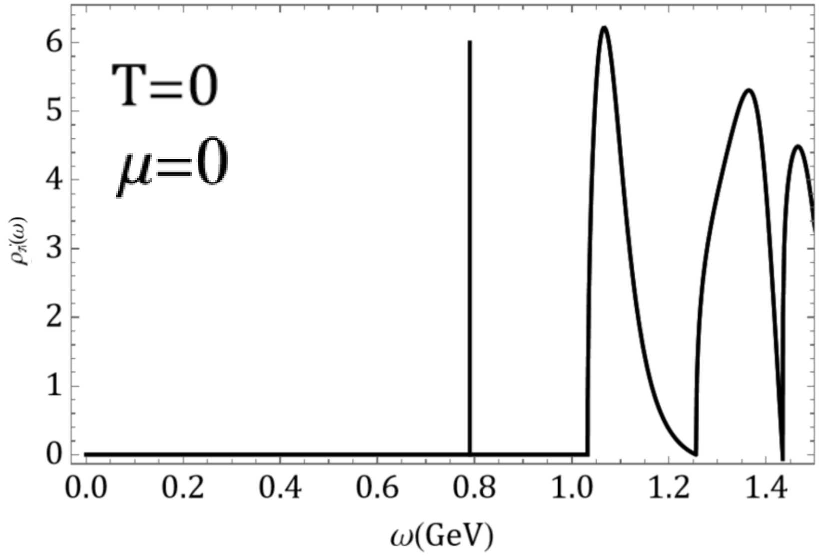
$|Q_u| \neq |Q_d|$

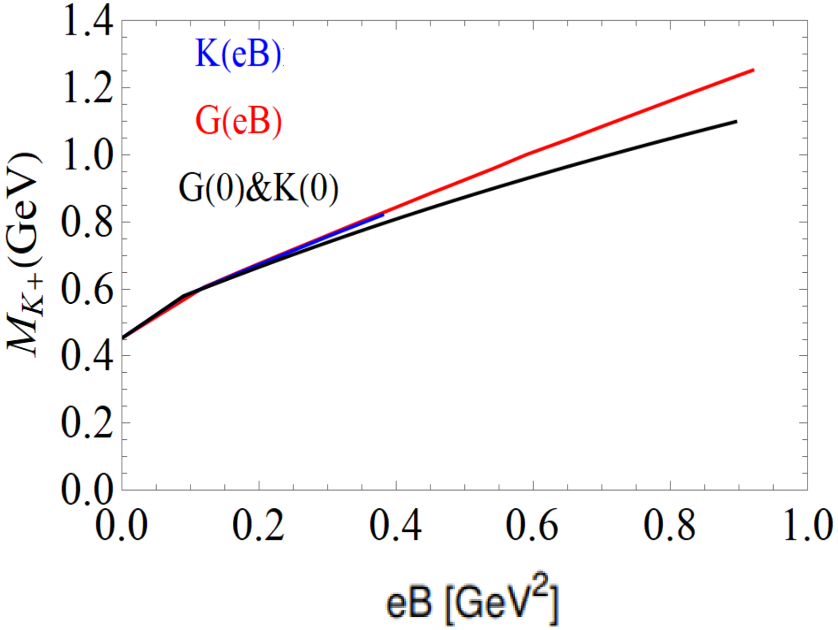


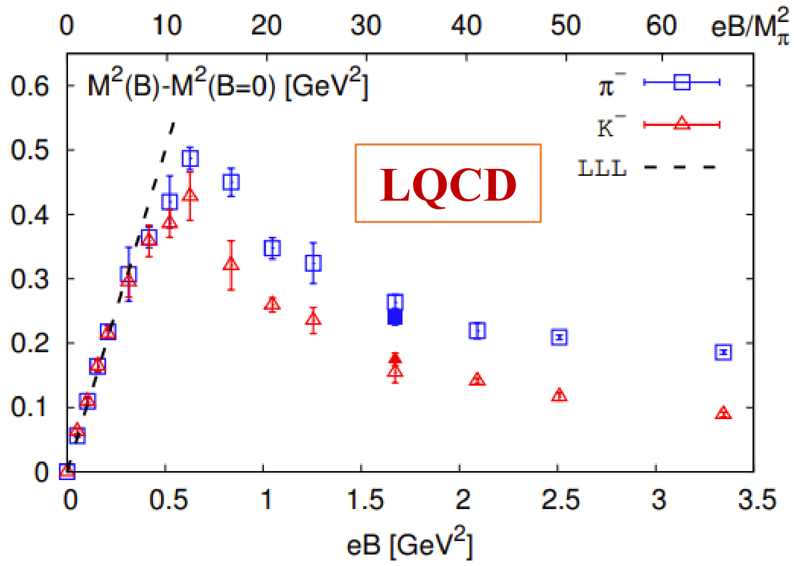




finite eB (Landau levels)

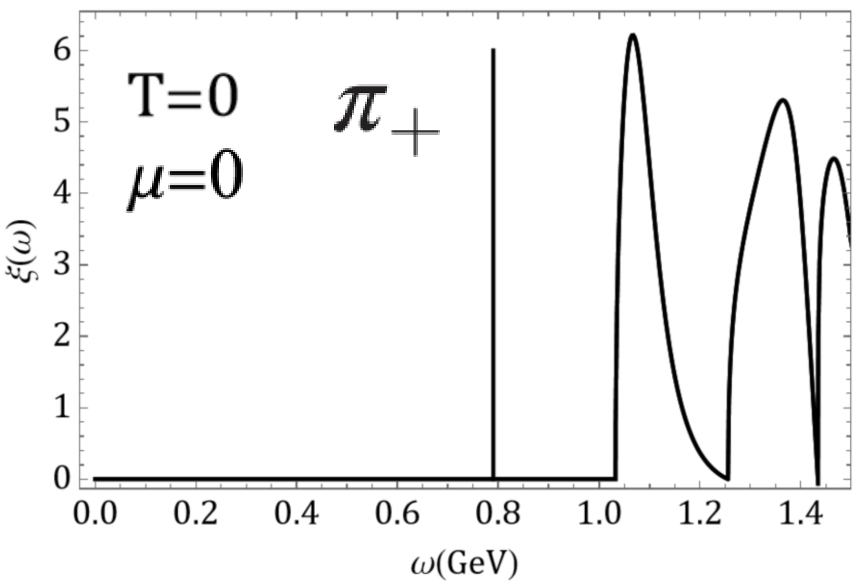


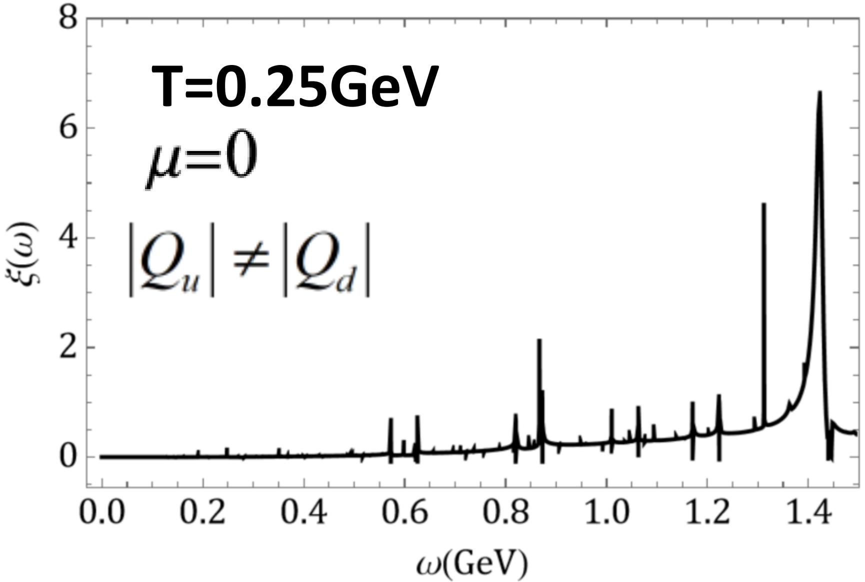




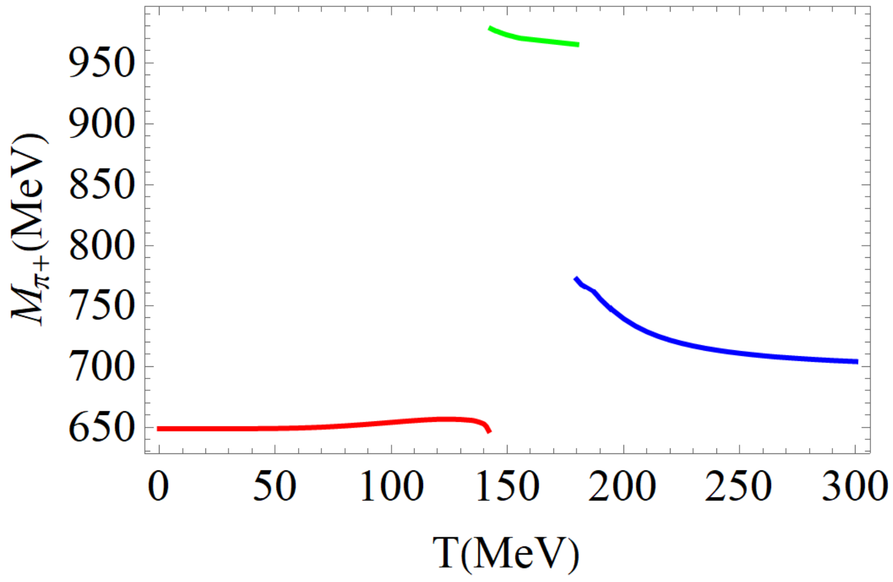
charged pion and kaon@ eB μ=0

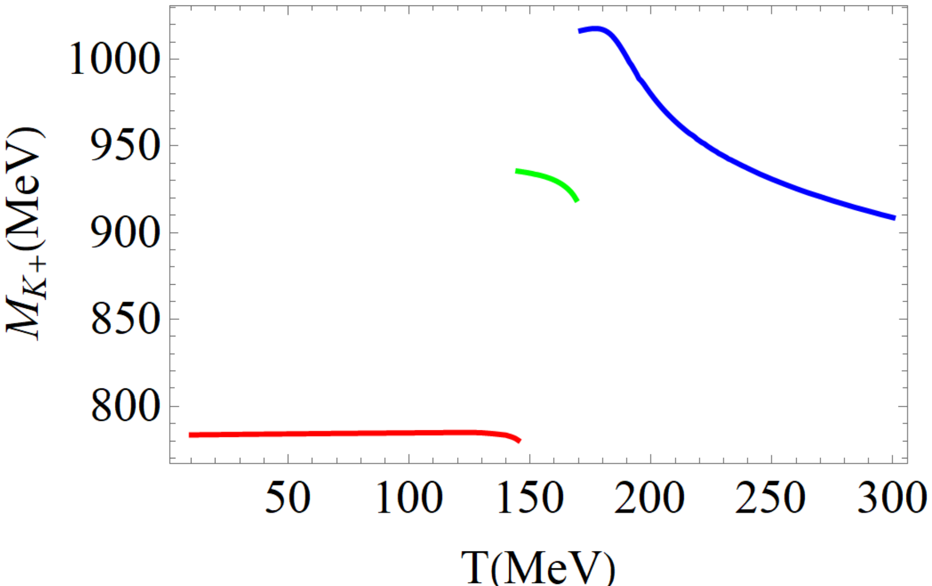
spectral function





Landau levels





What is the influence of eB to QCD matter

- mass spectra of mesons
- fluctuations and correlations of quark system

fluctuations and correlations with/without IMC

Fluctuations & correlations:

$$\chi_{i,j,k}^{B,Q,S} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\bigg|_{\mu_X=0}$$

chiral crossover

PNJL Lagrangian：

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - \hat{m}_0\right)\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi,\bar{\Phi}), \\
\mathcal{L}_4 &= G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2\right], \\
\mathcal{L}_6 &= -K\left[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi\right], \\
\mathcal{U}(\Phi,\bar{\Phi}) &= T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3+\Phi^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].
\end{aligned}$$

Thermodynamical potential:

$$\begin{aligned}
\Omega &= 2G(\sigma_u^2+\sigma_d^2+\sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s + \mathcal{U}(\Phi,\bar{\Phi}) + \Omega_q, \\
\Omega_q &= -\sum_{f=u,d,s}\frac{|Q_fB|}{2\pi}\sum_l \alpha_l\int\frac{dp_z}{2\pi}\Big[3E_f \\
&+ T\ln\left(1+3\Phi e^{-\beta E_f^+} + 3\bar{\Phi}e^{-2\beta E_f^+} + e^{-3\beta E_f^+}\right) \\
&+ T\ln\left(1+3\bar{\Phi}e^{-\beta E_f^-} + 3\Phi e^{-2\beta E_f^-} + e^{-3\beta E_f^-}\right)\Big],
\end{aligned}$$

$$E_f = \sqrt{p_z^2 + 2l|Q_fB| + m_f^2}$$

2.2 introducing IMC: G(eB), T_0(eB)

Lagrangian density：

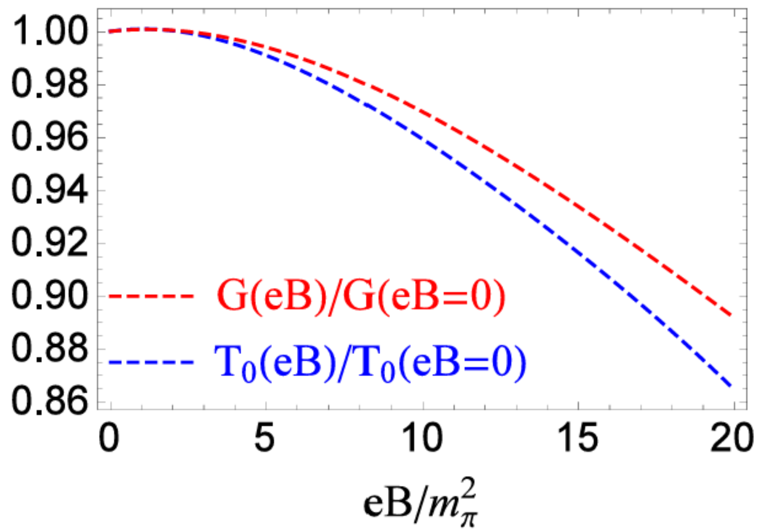
$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - \hat{m}_0\right)\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi,\bar{\Phi}), \quad (1)$$

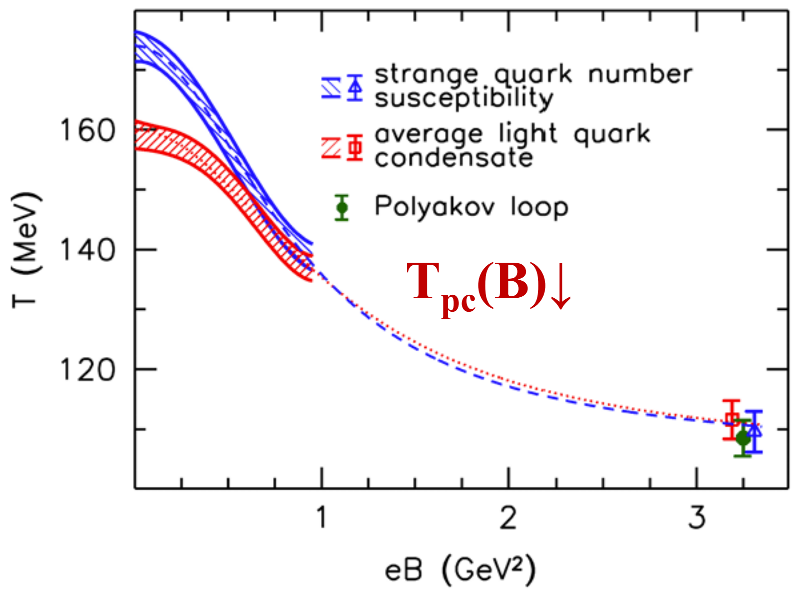
$$\mathcal{L}_4 = G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2\right],$$

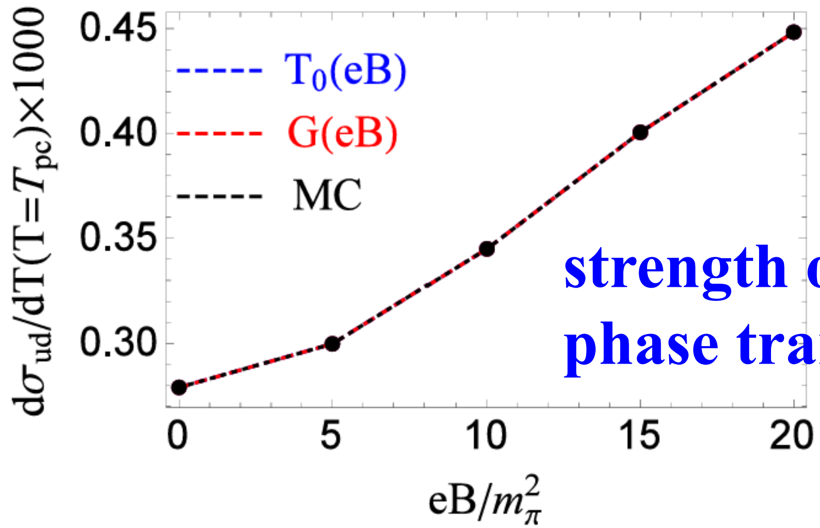
$$\mathcal{L}_6 = -K\left[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi\right],$$

$$\mathcal{U}(\Phi,\bar{\Phi}) = T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3+\Phi^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].$$

$$t' = T_0/T$$



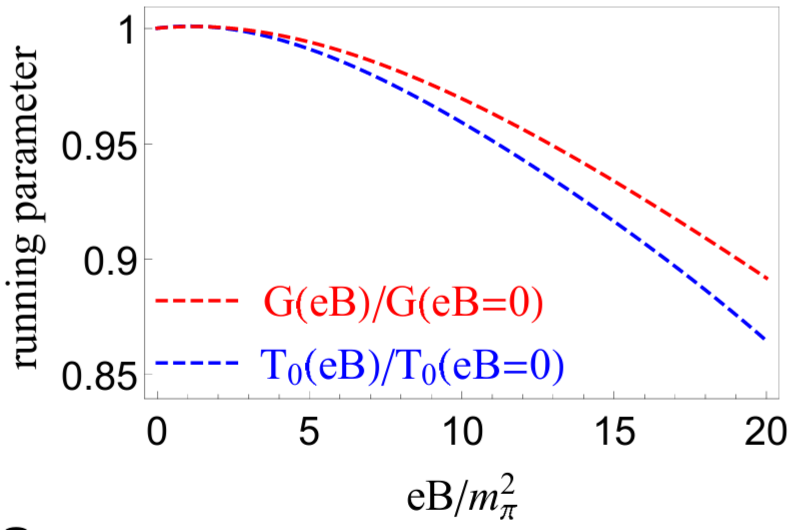


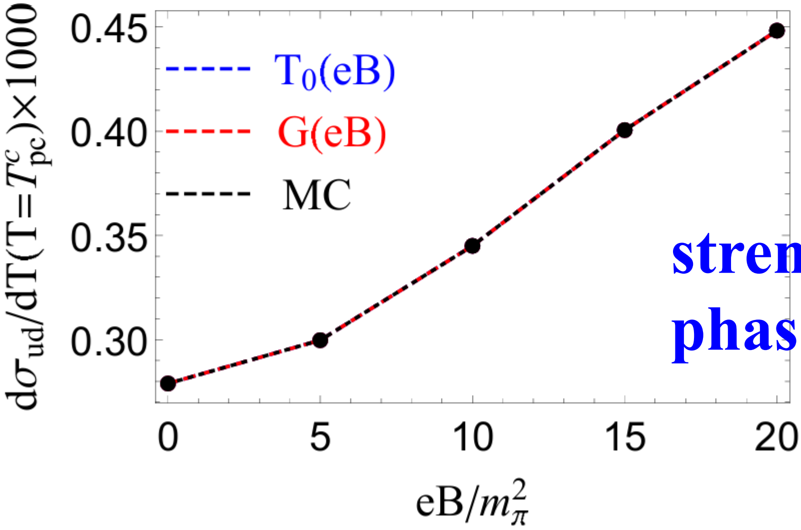


strength of phase transition

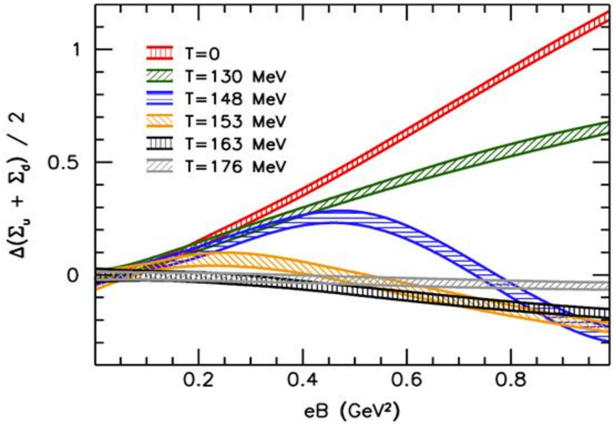
Mao,Yang, PRD112, 014026 (2025);CPC49, 063106(2025) .

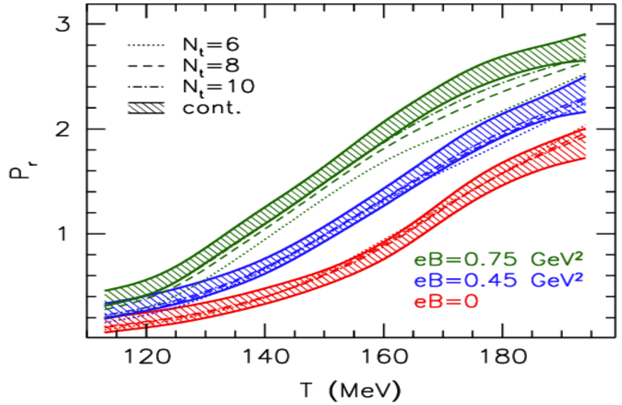
2.2 Comparison with LQCD results

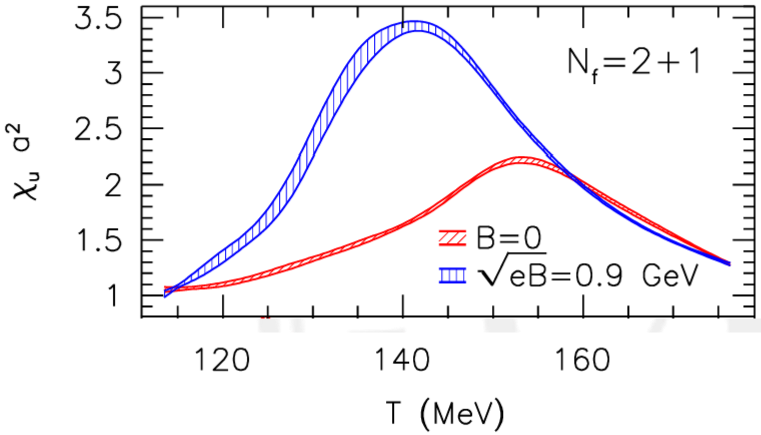




strength of phase transition





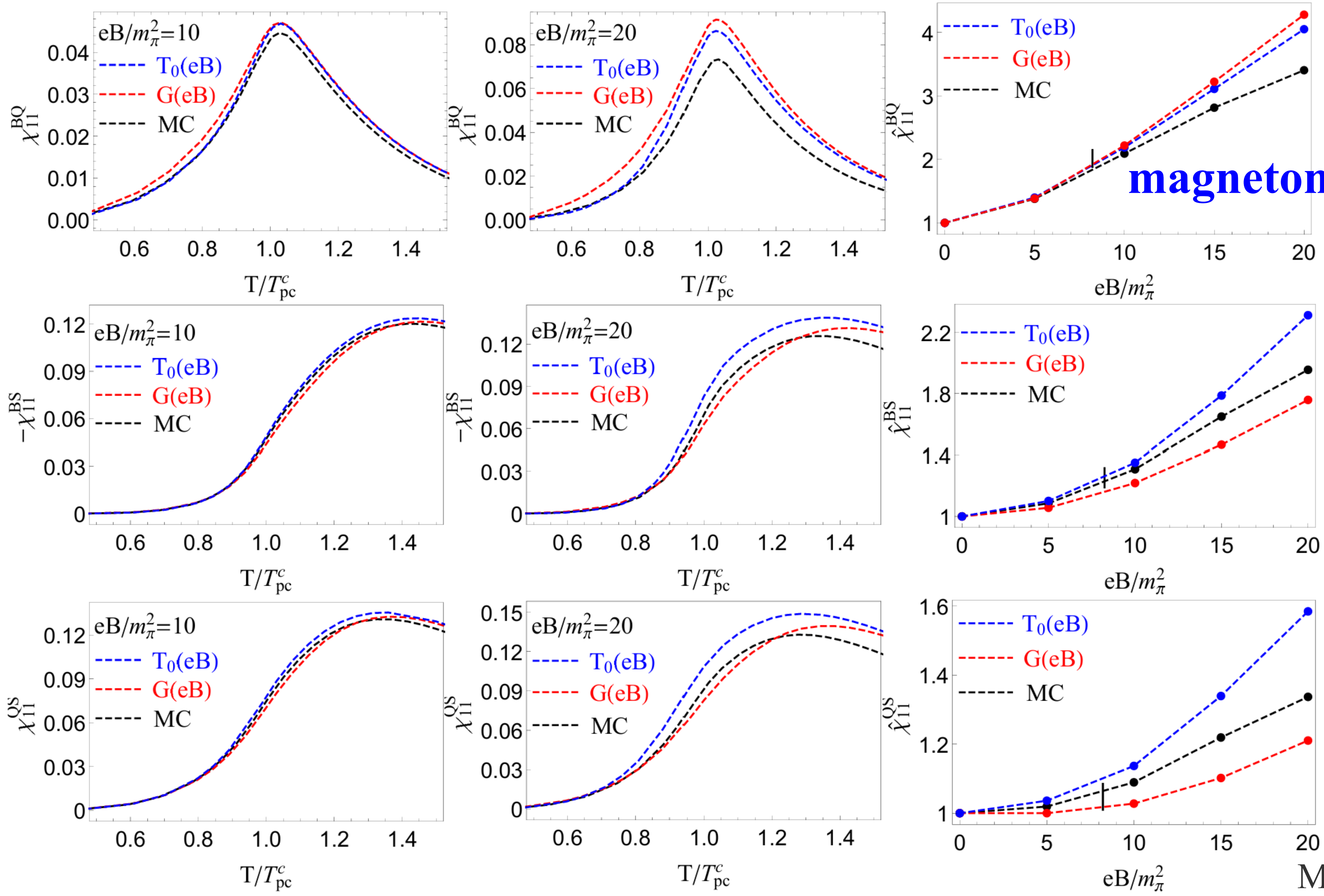


Mao, Yang, PRD112, 014026 (2025).

G. Bali, et.al, PRD86, 071502(2012);JHEP02,044(2012);

2.2 correlations with IMC

$$\chi^{B,Q,S}_{i,j,k} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\bigg|_{\mu_X=0}$$



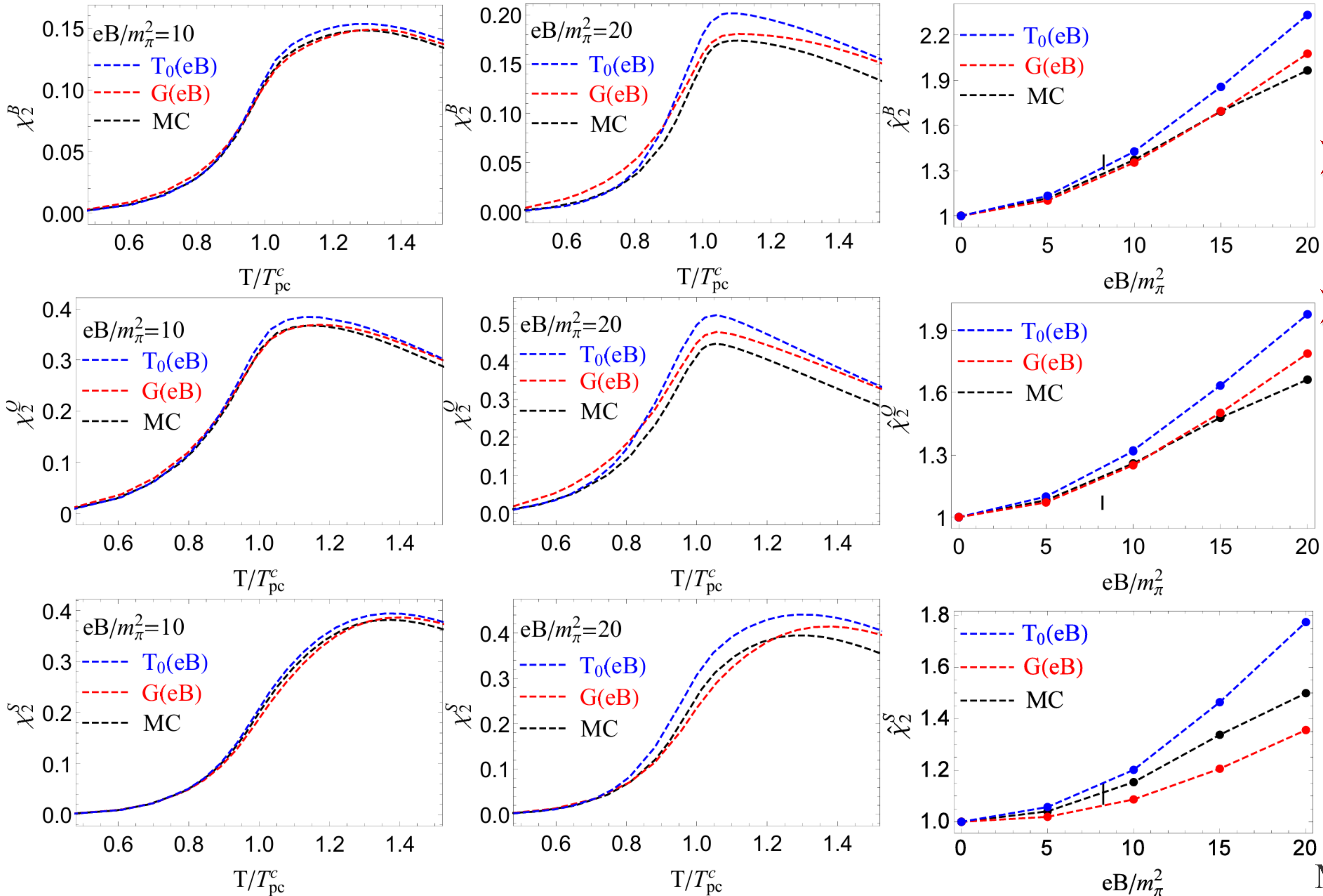
magnetometer of QCD

- **IMC leads to no qualitative difference**
- **consistent with LQCD**

Mao, Yang, PRD112, 014026 (2025)

2.2 fluctuations with IMC

$$\chi^{B,Q,S}_{i,j,k} = -\left.\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\right|_{\mu_X=0}$$

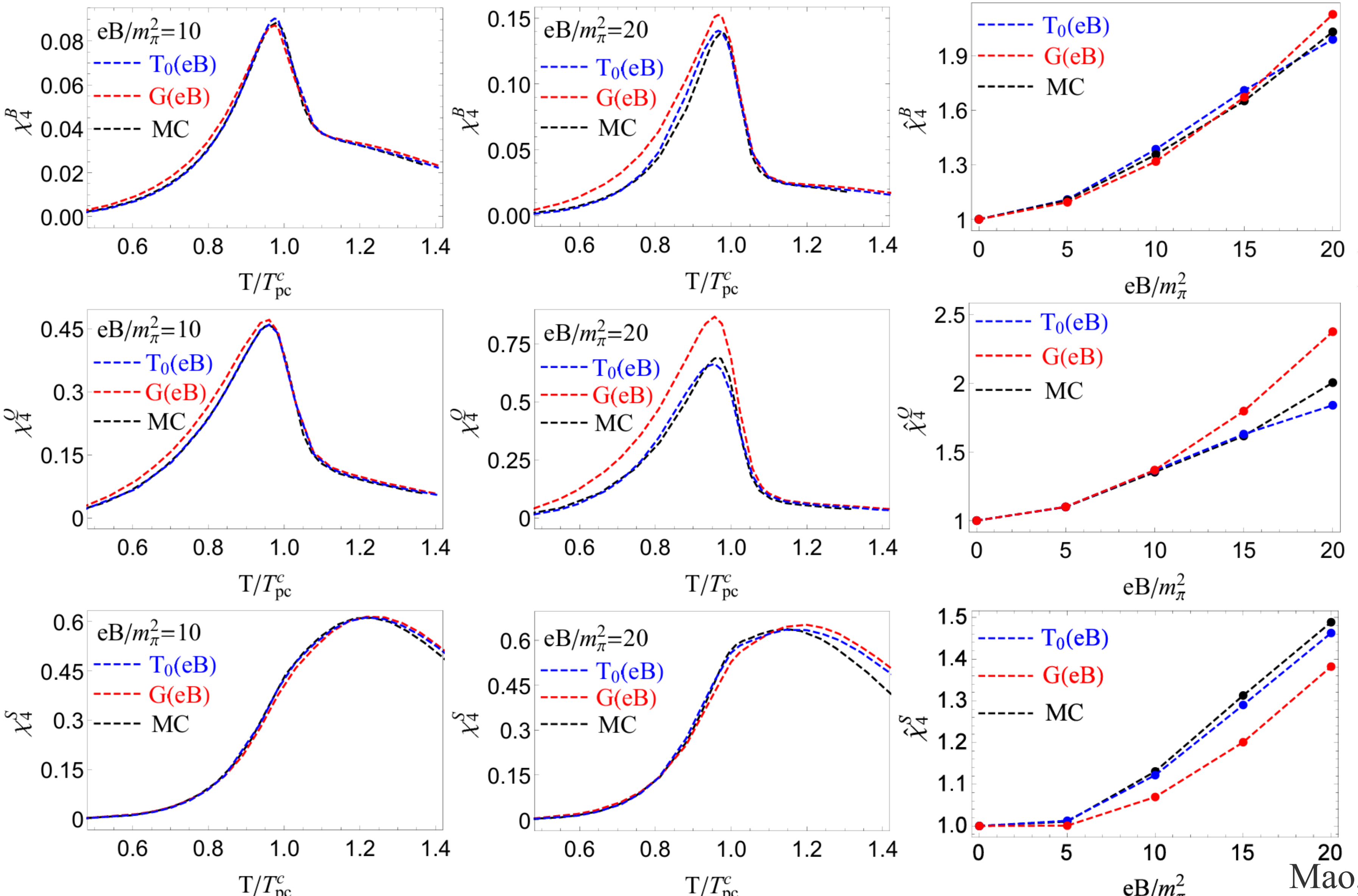


- IMC leads to no qualitative difference
- consistent with LQCD

Mao, Yang, PRD112, 014026 (2025)

2.2 fluctuations with IMC

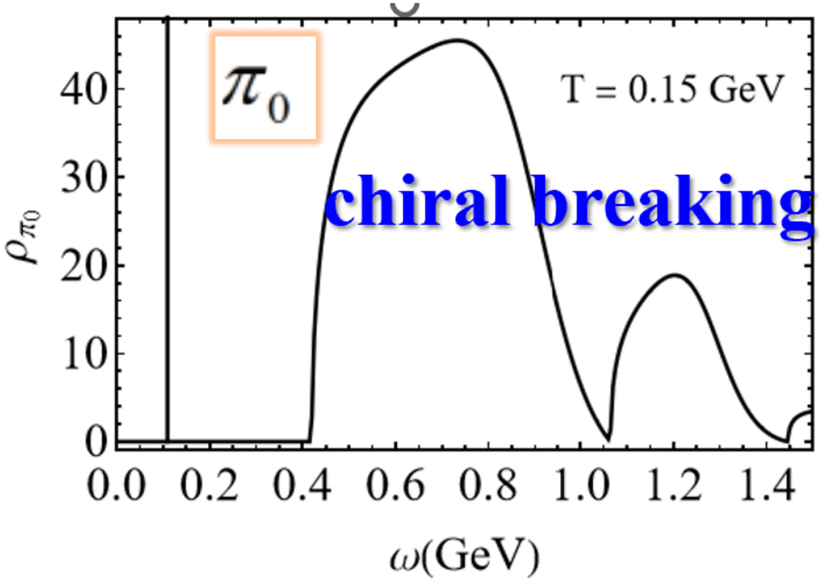
$$\chi_{i,j}^{B,Q,S} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\bigg|_{\mu_X=0}$$

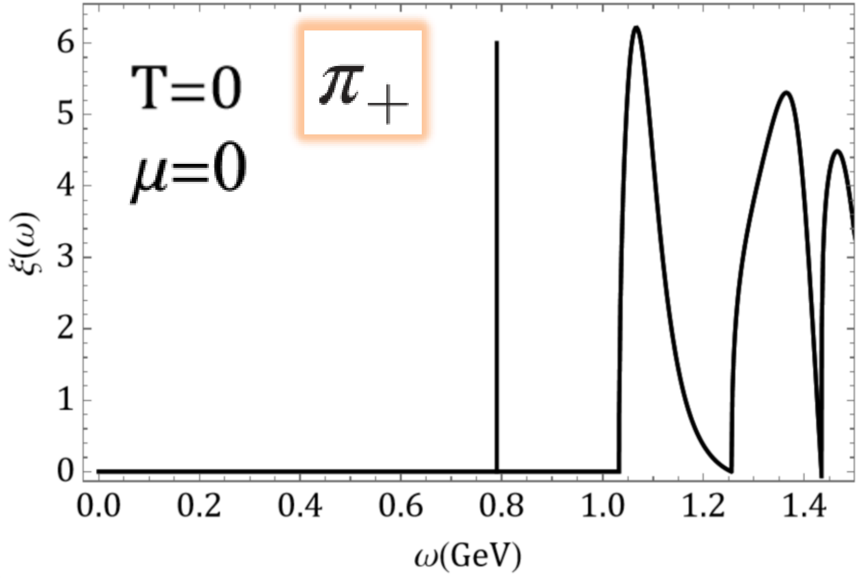


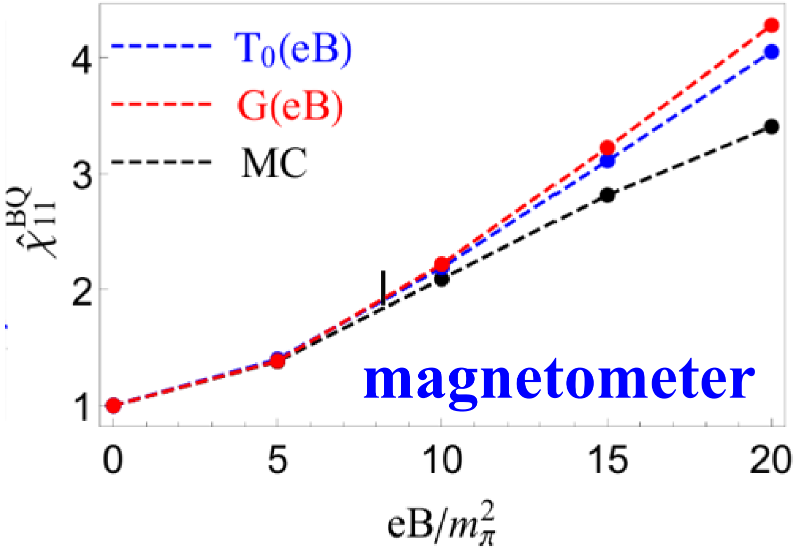
- IMC leads to no qualitative difference

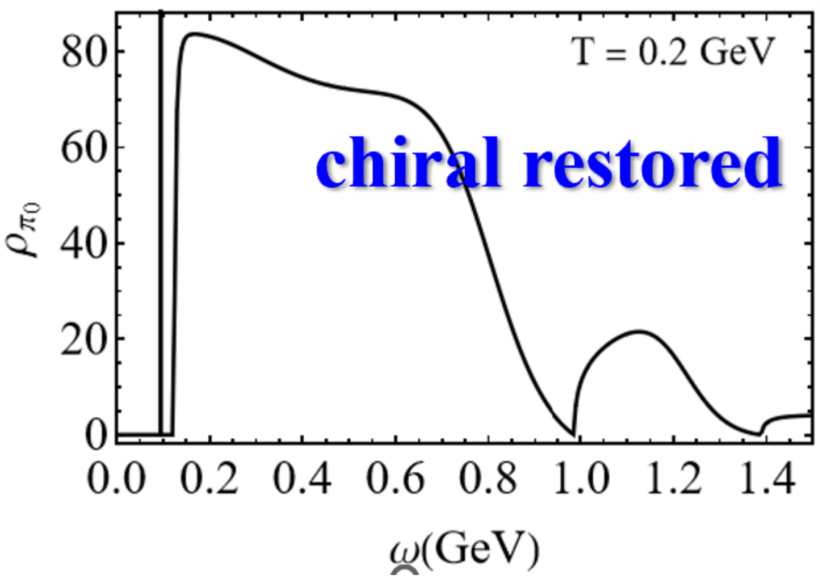
Mao, Yang, PRD112, 014026 (2025)

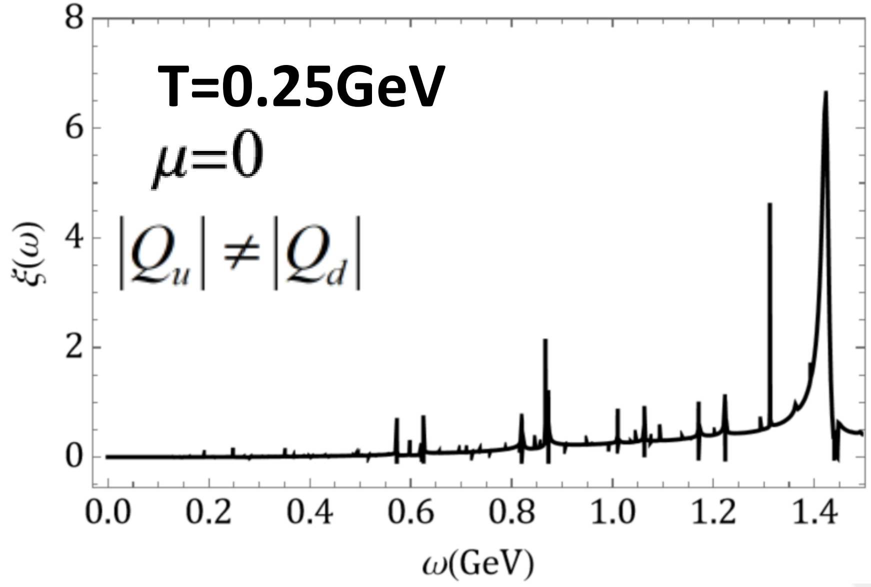
3 summary and outlook

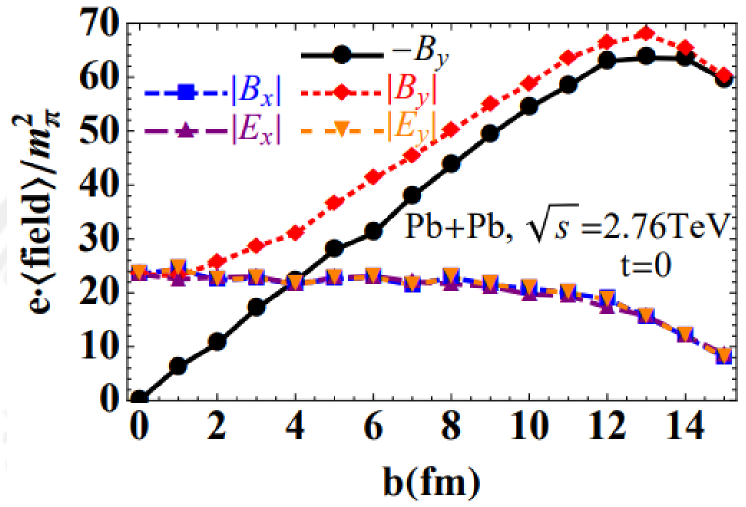




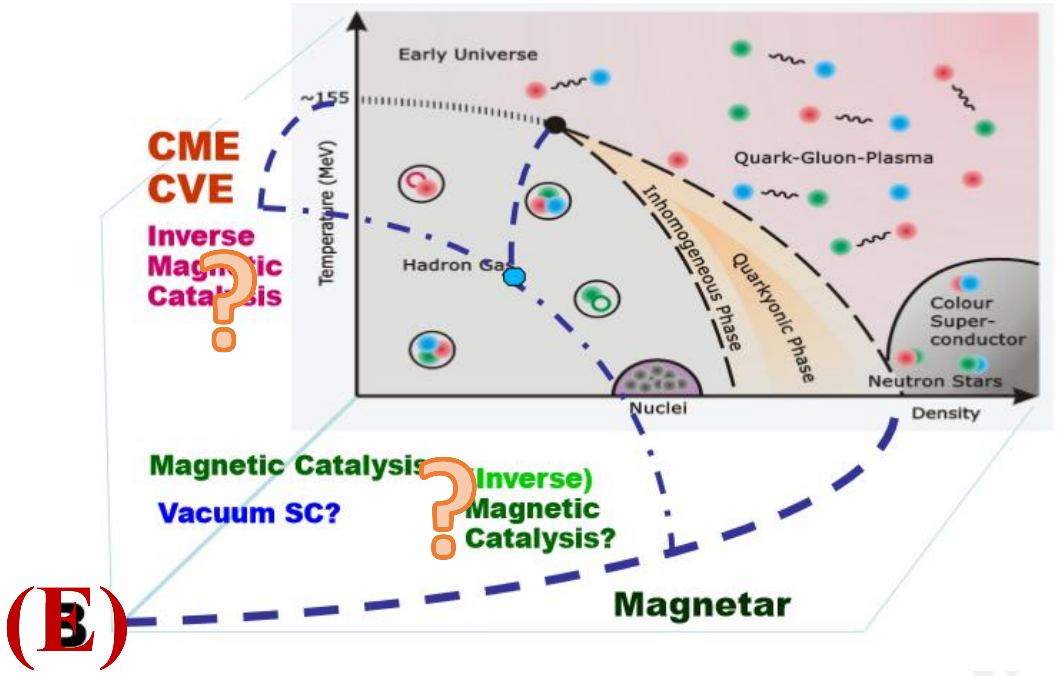




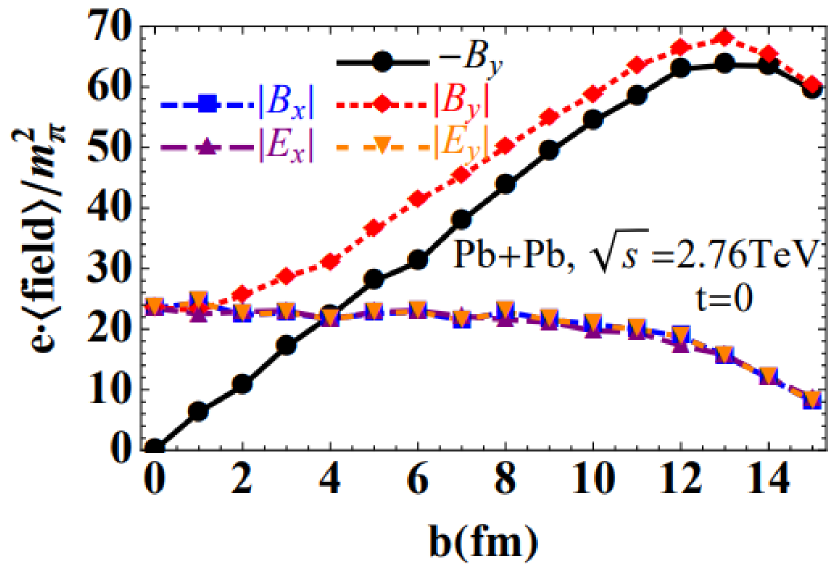




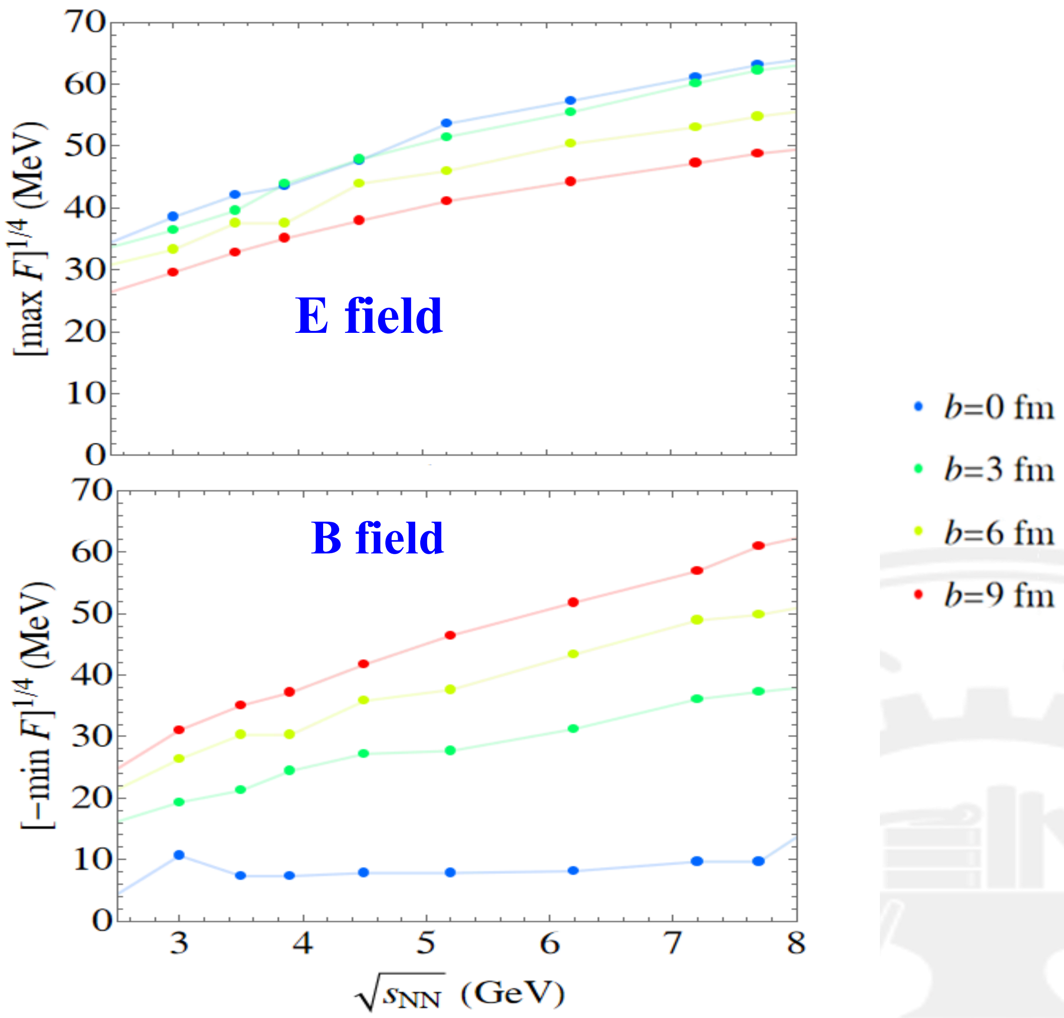
3 Electric Catalysis (EC)



3 E field needs more attention.



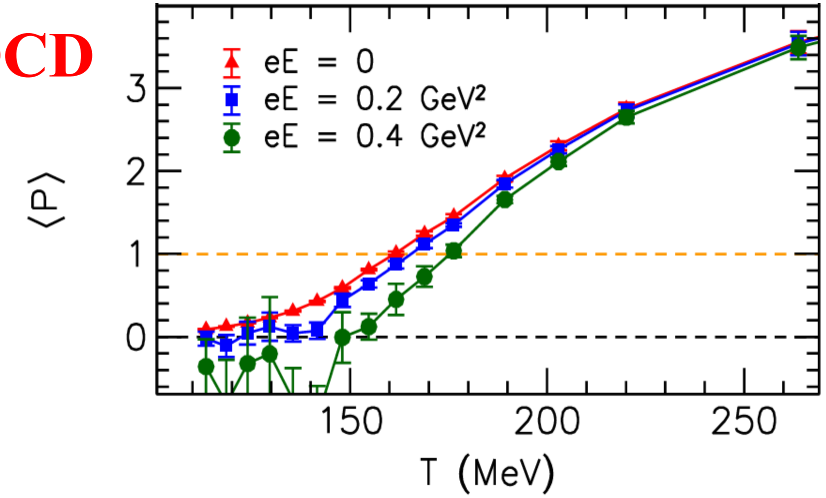
Deng&Huang, PRC85, 044907(2012)

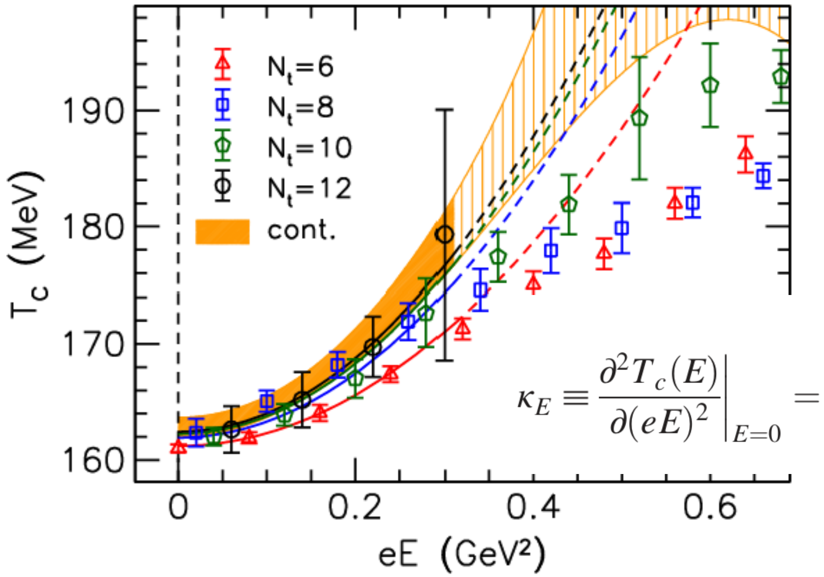


Taya,et.al,2402.17136; 2409.07685

3.1 Electric Catalysis (EC)

LQCD



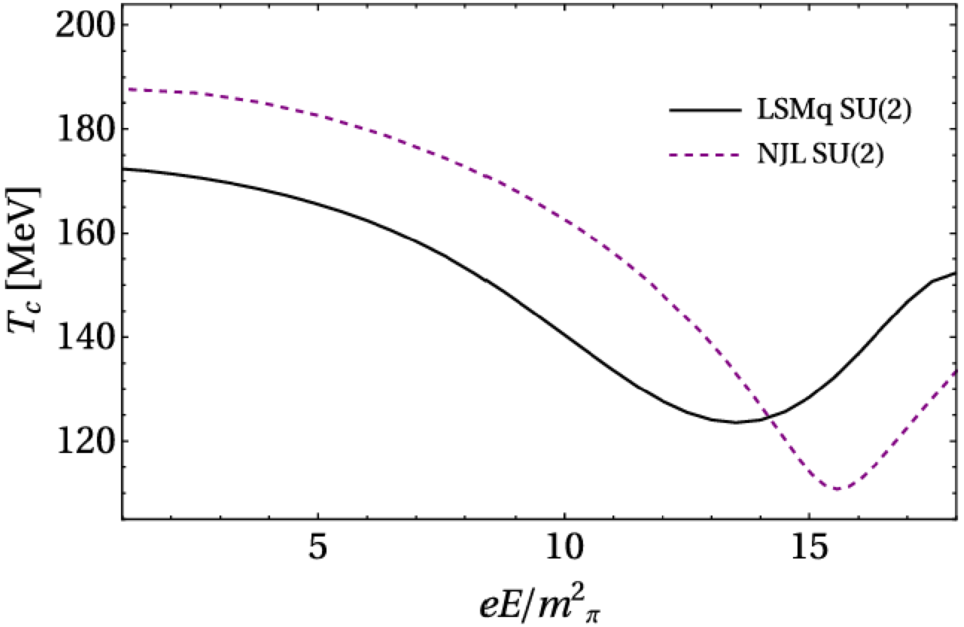


$$\kappa_E \equiv \frac{\partial^2 T_c(E)}{\partial (eE)^2}\bigg|_{E=0} = 0.37(9)\ \mathrm{GeV}^{-3}$$

G.Endroedi and G. Marko, PRD109, 034506(2024).

???

Analytical results in mean field

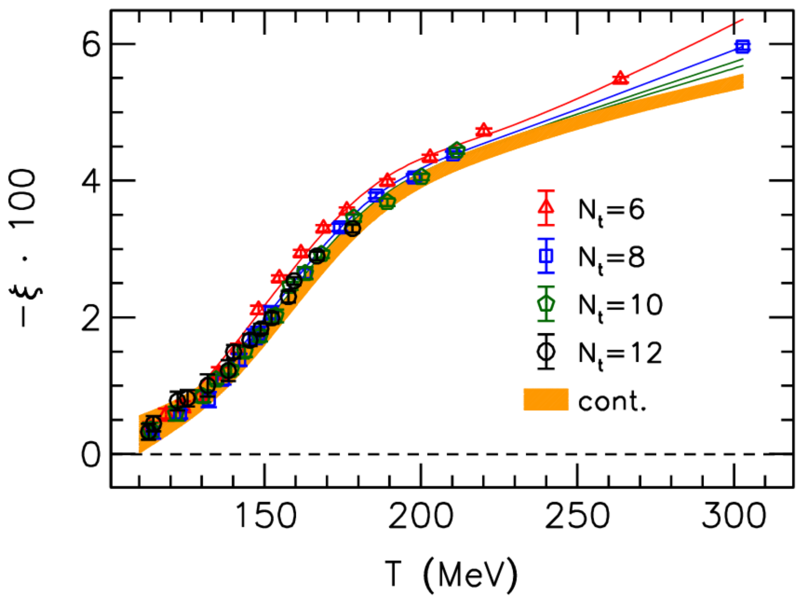


G.Q. Cao, X.G. Huang, PRD93, 016007(2016);
W.R. Tavares, et.al, PRD108, 016017(2023).

3.2 Electric susceptibility

LQCD

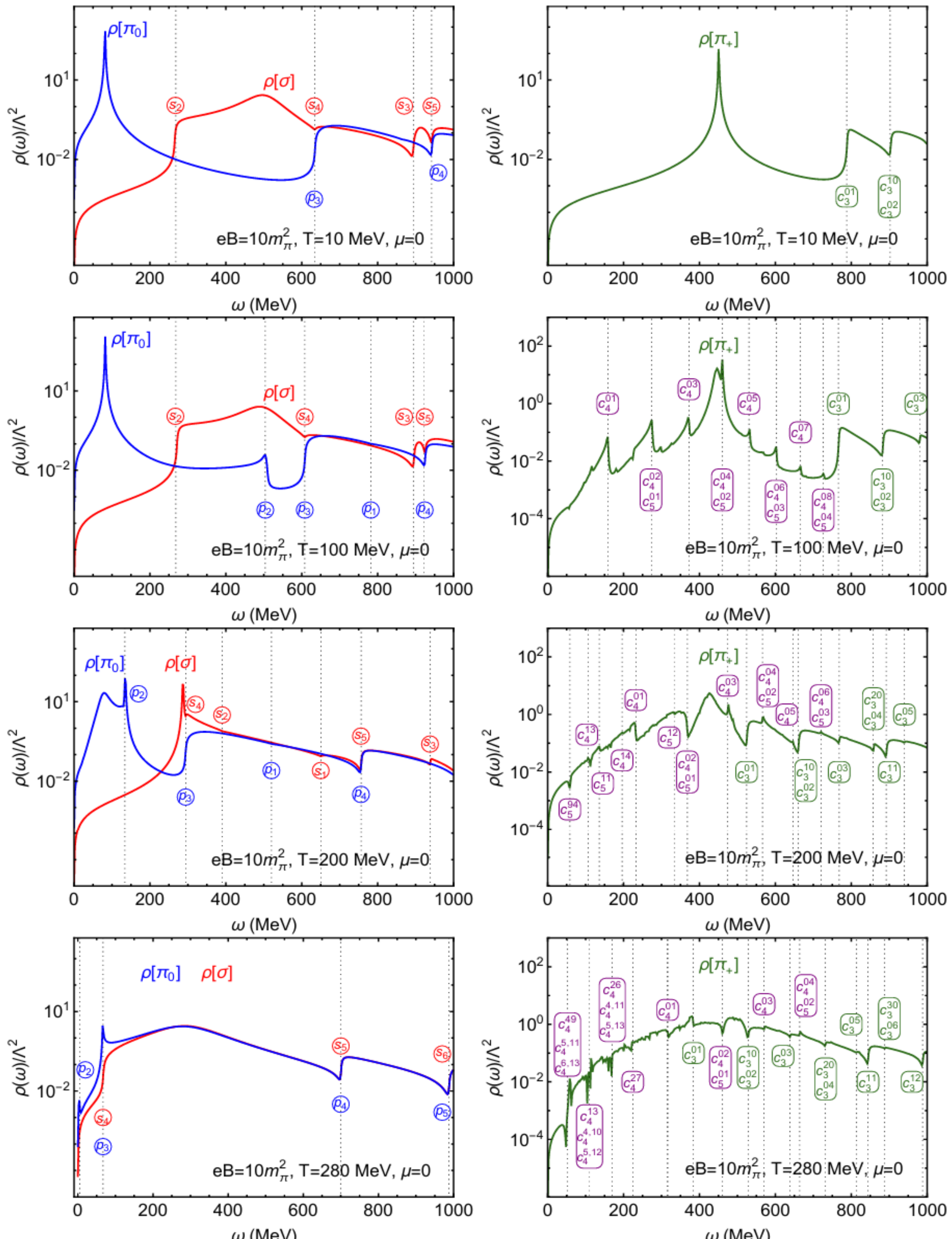
electric susceptibility



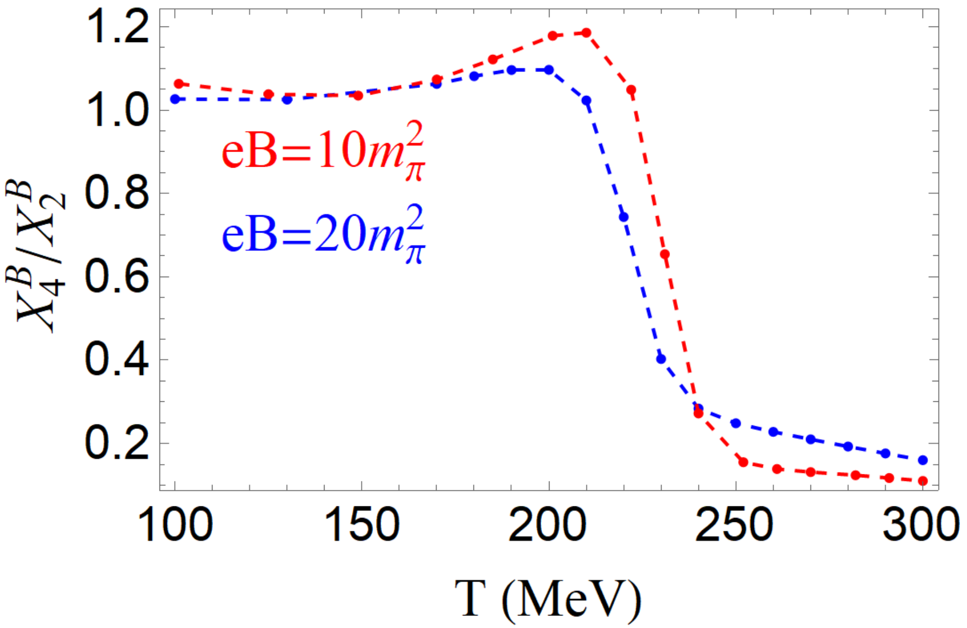
G.Endroedi, G. Marko, PRD109, 034506(2024)

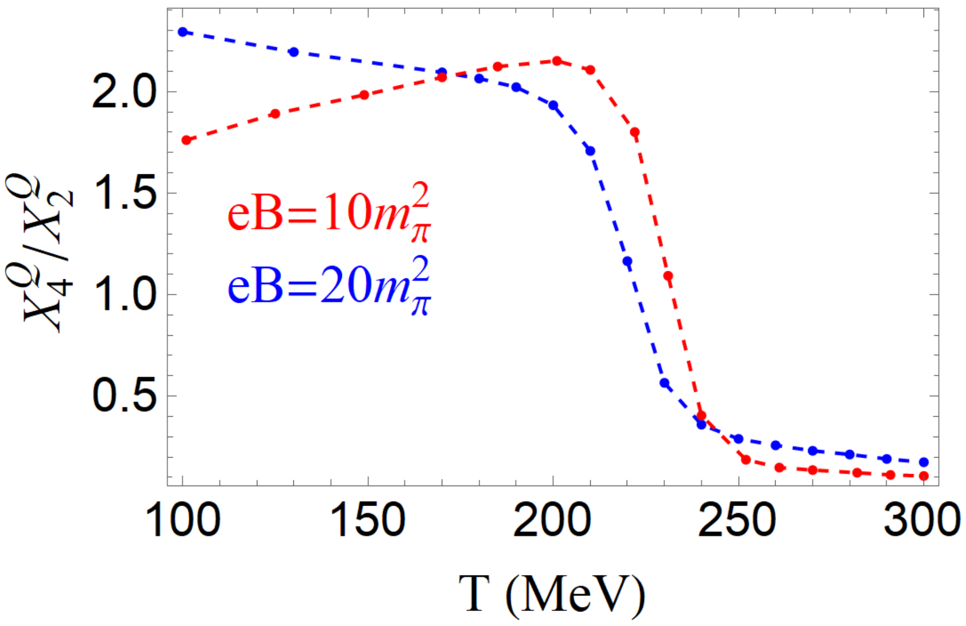
Schwinger mechanism:
quark pair production???

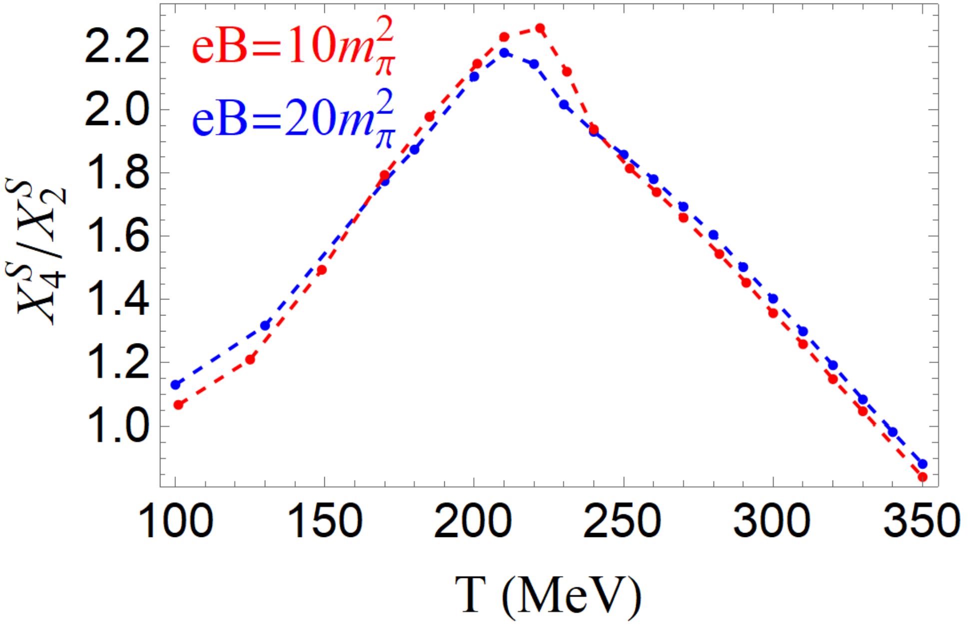
Electric field effect on QCD matter is under progress.

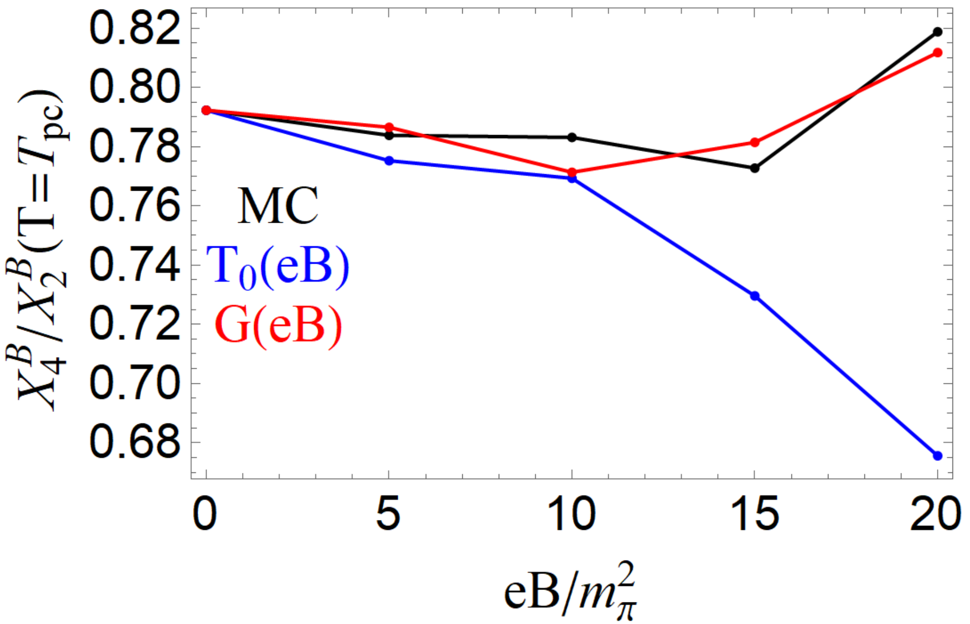


QM model with FRG,
from Ziyue Wang

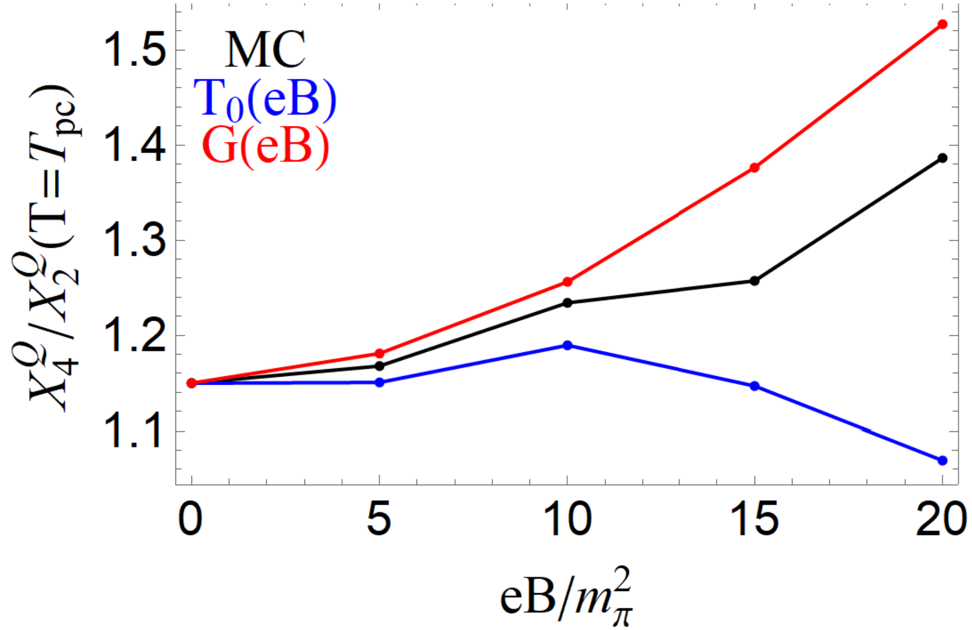

X_4^B/X_2^B
$eB=10m_\pi^2$
$eB=20m_\pi^2$
T (MeV)


X_4^Q/X_2^Q
$eB=10m_\pi^2$
$eB=20m_\pi^2$
T (MeV)

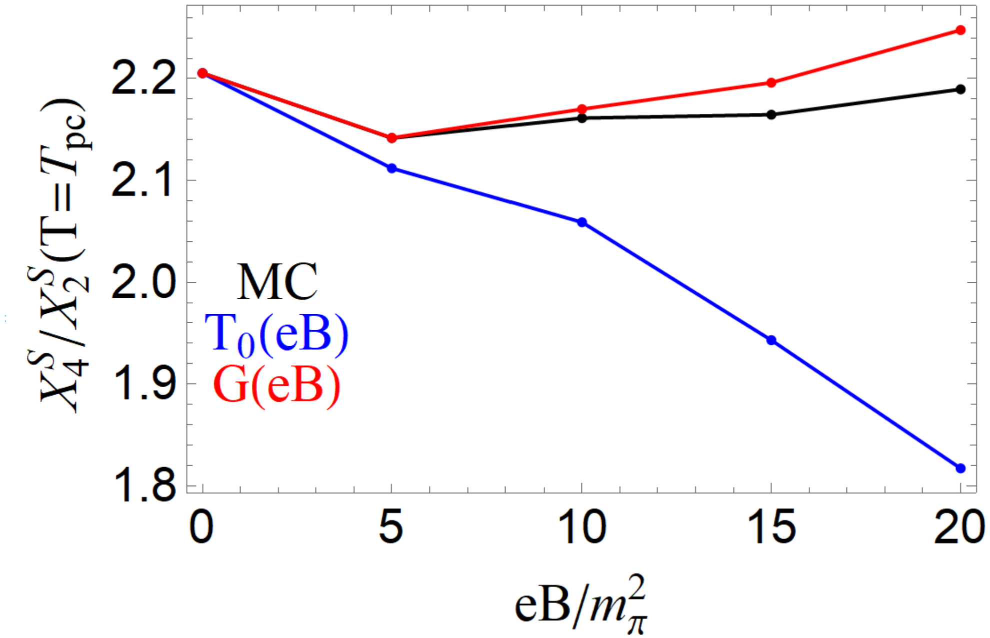

X_4^S/X_2^S
$eB=10m_\pi^2$
$eB=20m_\pi^2$
T (MeV)



$X_4^B/X_2^B(\mathrm{T}=T_{\mathrm{pc}})$
0.82
0.80
0.78
0.76
0.74
0.72
0.70
0.68
MC
$\mathrm{T}_0(\mathrm{eB})$
G(eB)
0
5
10
15
20
eB/m_π^2



$X_4^Q/X_2^Q(\mathrm{T}=T_{\mathrm{pc}})$
1.5
1.4
1.3
1.2
1.1
MC
$\mathrm{T}_0(\mathrm{eB})$
G(eB)
0
5
10
15
20
eB/m_π^2



$X_4^S/X_2^S(\mathrm{T}=T_{\mathrm{pc}})$
2.2
2.1
2.0
1.9
1.8
MC
$\mathrm{T}_0(\mathrm{eB})$
G(eB)
0
5
10
15
20
eB/m_π^2

K_0, π_0, η, η' mesons @ T, μ, eB

Mei, Xia, Mao, PRD 107, 074018 (2023)