What neutron stars tell about the properties of strong interaction

Gyuri Wolf

Wigner Research Centre for Physics, Institute for Particle and Nuclear Physics, Theoretical Physics Department

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Collaborators: J. Takátsy, P. Kovács, J. Bielich-Schaffner

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Overview

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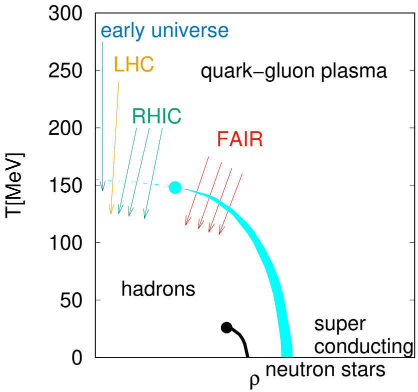
Dense strongly interacting matter

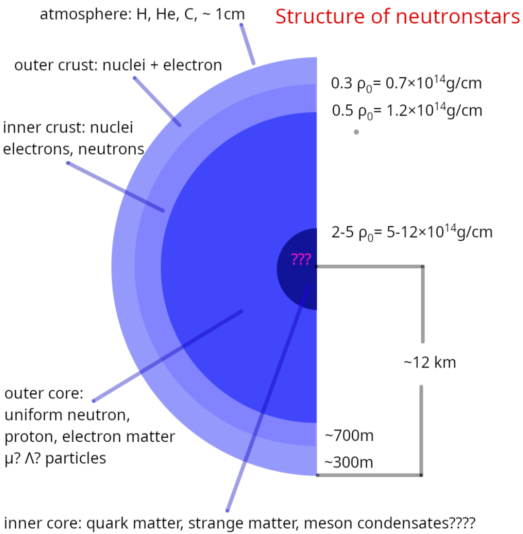


What is the phase diagram and EOS for dense strongly interacting matter?

At $\mu = 0$: lattice and experiments (STAR/PHENIX and ALICE).

For $\mu >> 0$ no precise theory and no heavy ion experiment.

Neutron Stars a challenge and a possibility



Neutron stars are contain cold, dense matter (T≈0, $\rho > 3\rho_0$) not available in terrestial experiments (Laboratory for strong interaction)

What is the structure of neutron stars (what are the constituents), hybrid stars? Superfluids?

YN, YNN interactions are important, three-body repulsion for Λ, Σ (Weise)

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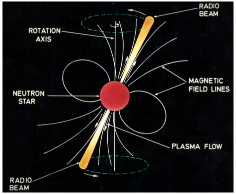
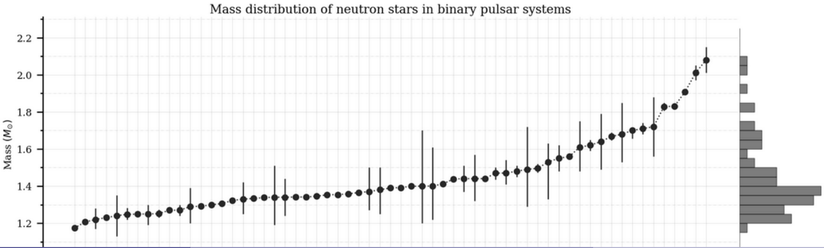
Pulsar mass distribution



lighthouse effect, very precise frequency (1-700 Herz)

Pulsar mass measurements and tests of general relativity



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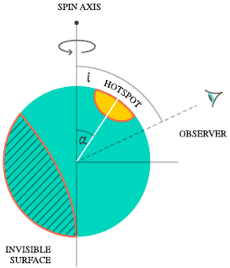
Neutron Star observations

- Discovering heavy neutron stars $M > 2M_\odot$ Demorest, et al., Nature. 467, 1081-1083 (2010). largest mass observed: $2.14 + 0.20 - 0.18 M_\odot$ (2019)
  (Shapiro-delay: pulsar+another star, at almost full covering the second member of the binary delays the radiation of the pulsar)
- Advanced gravitation wave detectors: Advanced Ligo and Virgo (soon Kagra):single neutron stars, multichannel astronomy
  neutron star collision: GW170817 (130 million lightyears)
- Modern telescopes: NICER X-ray telescope: precise (<5%) mass and radius measurements (2020) “for nearby” neutron stars,
  only 2 stars yet, but more to come
  radiation bent by strong gravity: hotspots observation: M, R



Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \tag{1}$$

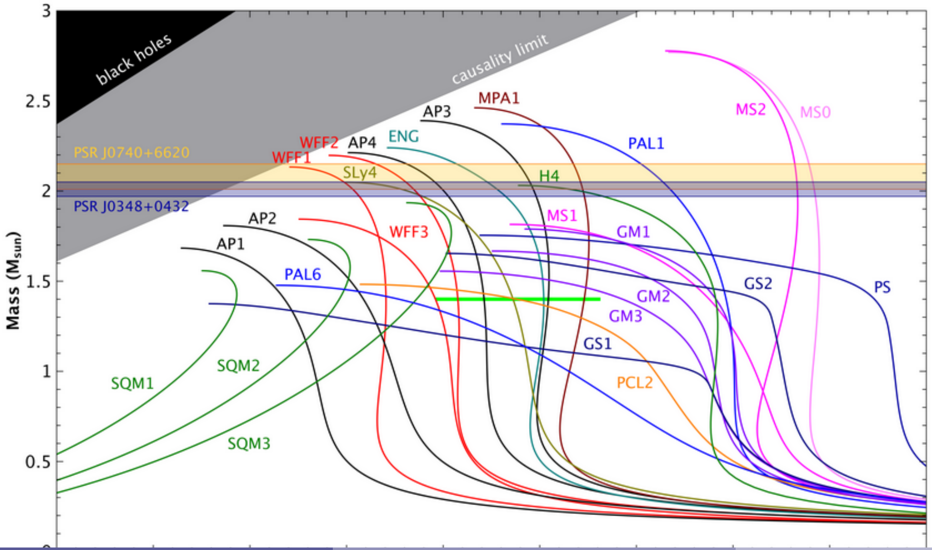
with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

MR curves


black holes
causality limit
PSR J0740+6620
PSR J0348+0432
Mass (M_{sun})
3
2.5
2
1.5
1
0.5
MPA1
AP3
MS2
MS0
WFF2
AP4
ENG
PAL1
WFF1
SLy4
H4
MS1
AP2
WFF3
GM1
AP1
PAL6
GS2
PS
GM2
GM3
GS1
SQM2
SQM1
PCL2
SQM3

Observables for dense strongly interacting matter

1. Nuclear physics
   - $\rho = 0$
     nucleon-nucleon scattering, YN and YNN data from femtoscopy (ALICE), BB interaction and potential (lattice: HALQCD)
     femtoscopy data and HALQCD calculations are consistent
   - $\rho \approx \rho_0$
     masses of nuclei, isobaric analog states, hypernuclei, giant dipole and pigmy resonances, nuclear dipole polarizabilities, neutron skin thickness $\rightarrow$ normal nuclear density: ρ_0, binding energy, compressibility, symmetry energy (1st order in asymmetry expanded in density the 0th and 1st term)
2. Perturbative QCD: $\rho \approx 40\rho_0$
   N^3LO calculation, hard thermal loops: μ =2.6 GeV, p = 3.8 GeV/fm^3.
   T. Gorda, A. Kurkela, et al., Phys. Rev. Lett. 127 (2021) 162003, arXiv:2103.05658.
3. Heavy ion collisions: $\rho : 1 - 8\rho_0$
   not very conclusive; there are many competing effects, like momentum dependent interaction, nonequilibrium, nonzero temperature
4. Neutron stars: $\rho : 1 - 8\rho_0$
   M, R, Λ. Quite strong constraints even with not yet very precise data

Features of our approach

Quark-meson model, very good agreement with lattice at $\mu = 0$.

P. Kovács, Zs. Szép and Gy. Wolf, Phys. Rev. D93 (2016) 114014

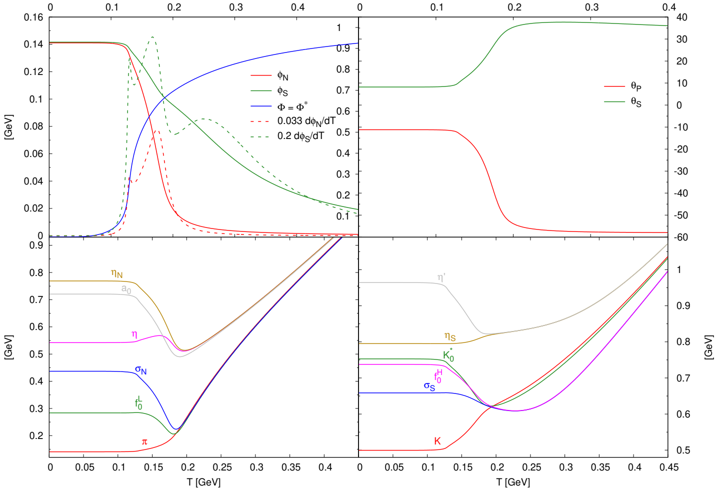
Effective field theory: the same symmetry pattern as in QCD

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

With low mass scalars, $m_{f_0^L} = 300$ MeV

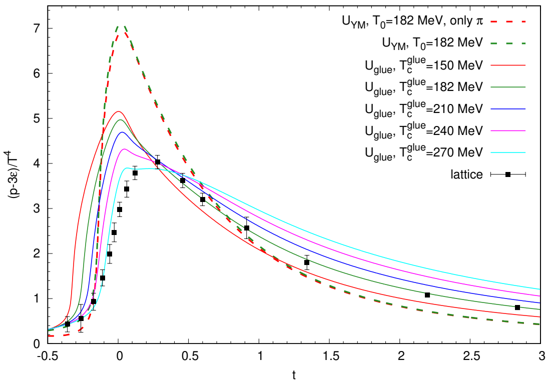


chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

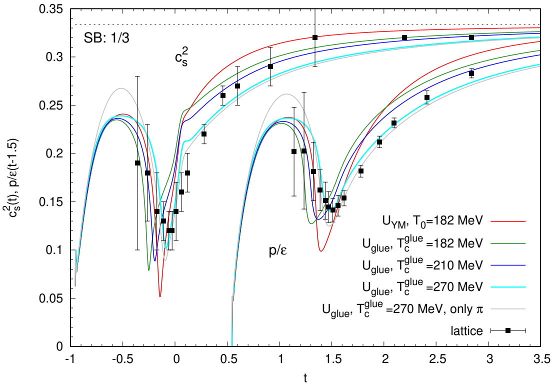
$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate, temperature dependent coupling of the $U(1)_A$ breaking term?

Observables

interaction measure

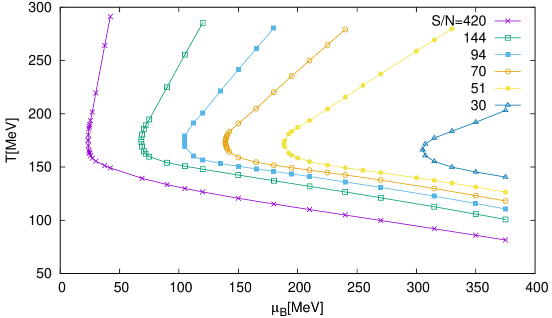


speed of sound

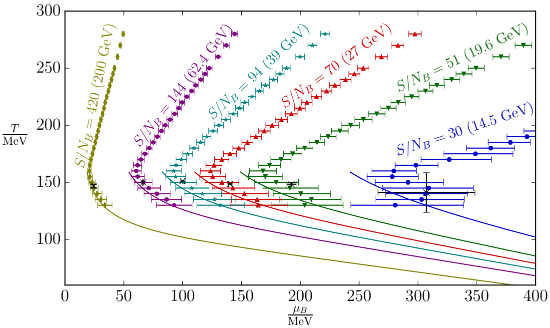


Isentropic trajectories in the $T - \mu_{\rm B}$ plane

our model, where $\mu_B^{\rm CEP} > 850$MeV



lattice analytic cont. arXiv:1607.02493



same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV

$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of μ_B

EOS

1. $\rho \leq 2-4\rho_0$ ordinary nuclear potentials, CEFT, ...
2. $2-4\rho_0 \leq \rho \leq 6-8\rho_0$ quark matter model
3. $6-8\rho_0 \leq \rho$ extrapolation to the pQCD point

hadronic matter - soft: SFHo
(Steiner et al., Astrophys. J. 774 (2013) 17) and Hempel et al., J. Nucl. Phys. A837 (2010) 210)
relativistic mean-field model (nucleons, σ, ω, ρ with quartic copulings), with K=245 MeV, L=47.1 MeV, m^*/m_n=0.76.

hadronic matter - stiff: DD2
(S. Typel, et al., Phys. Rev. C81 (2010) 015803
relativistic mean-field + light clusters, K = 243 MeV, L=58 MeV m^*/m_n=0.63

Quark matter: Quark-meson model - chiral $U(3)xU(3) \rightarrow SU(2)xU(1)$ model
degrees of freedom: 4 meson nonets, constituent quarks, Polyakov loops
condensates: 2 scalar (N,S), Polyakov loops (T>0), vector mesons ($\mu > 0$)
P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Concatenation

It seems that a strong first order phase transition is ruled out by astrophysical constraints: J.-E. Christian and J. Schaffner-Bielich, Phys. Rev. D 103, 063042 (2021), The allowed $p(\varepsilon)$ functions are in a rather narrow band, there can be no big jump

Hadron-quark crossover with polynomial interpolation ($\rho = \rho_B$):

$$\begin{aligned}
\varepsilon(\rho_B) &= \varepsilon_{hadronic}(\rho_B) \qquad \rho_B < \rho_{BL}, \\
\varepsilon(\rho_B) &= \sum_{k=0}^{5} C_k \rho_B^k \qquad \rho_{BL} \le \rho_B \le \rho_{BU} \\
\varepsilon(\rho_B) &= \varepsilon_{qm}(\rho_B) \qquad \rho_{BU} < \rho_B.
\end{aligned}$$

C_k is determined by the requirement that the energy density, ε and its first two derivatives with respect to ρ_B, pressure and sound velocity is continuous at the boundaries.
2 parameters: $\Gamma = 0.5 * (\rho_{BU} - \rho_{BL})$ and $\overline{\rho_B} = 0.5 * (\rho_{BL} + \rho_{BU})$

Bayesian inference

Unsetted parameters: m_σ, g_v, $\overline{\rho_B} \equiv 0.5(\rho_{BL} + \rho_{BU})$, $\Gamma \equiv 0.5(\rho_{BU} - \rho_{BL})$

290 MeV $\leq m_\sigma \leq$ 700 MeV, $0 \leq g_v \leq 10$

$2\rho_0 \leq \overline{\rho_B} \leq 5\rho_0$

$\rho_0 \leq \Gamma \leq 4\rho_0$ with the constraint: $\rho_{BL} = \overline{\rho_B} - \Gamma > \rho_0$

We created $\sim$ 18000 EOSs to be used in the Bayesian analysis

We believe that our model is quite generic and with running the parameters we cover the possible EOSs.

Bayes theorem:

θ is a parameter set, $p(\theta)$ is the prior probability for θ, $p(data|\theta)$ is the probability that for given θ, the data is measured. Then

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

$p(data)$ is a normalization constant. We assume $p(\theta)$ is uniform in the allowed hypersurface. For independent observations:

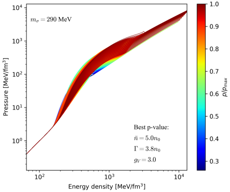
$$p(data|\theta) = p(M_{max}|\theta)p(NICER|\theta)p(\overline{\Lambda}|\theta)$$

J. Takátsy, P. Kovács, Gy. Wolf, J. Bielich-Schaffner: Phys. Rev. D105 (2022) 103014, Phys. Rev. D108 (2023)

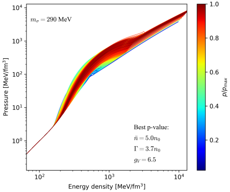
Data

- $2M_\odot$: PSR J0348+0432 with a mass $2.01 \pm 0.04 M_\odot$,
  PSR J1614-2230 with a mass $1.908 \pm 0.016 M_\odot$
- perturbative QCD EOS should converge to it keeping $c_s < 1$,
  $\mu_{QCD} = 2.6 GeV, n_{QCD} = 6.471/fm^3, p_{QCD} = 3823 MeV/fm^3$
- NICER: (M,R) values for PSR J0030+0451, PSR J0740+6620 (Miller)
- tidal deformability GW170817: : $70 < \Lambda(1.4 M_\odot) < 720$ Abbot (2019)
- Hess J1731-347 neutron star: mass= $0.77 \pm 0.19 M_\odot$, $R = 10.4 \pm 0.8 km$
- massgap neutron star: $2.59 \pm 0.09 M_\odot$

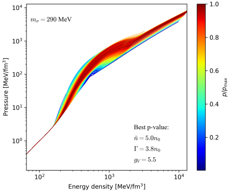
Bayesian analysis: EOS



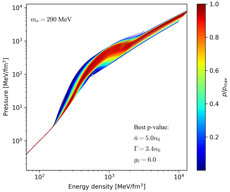
prior (M_{max} + p_{QCD})



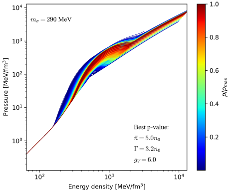
prior + NICER



prior + Nicer + GW

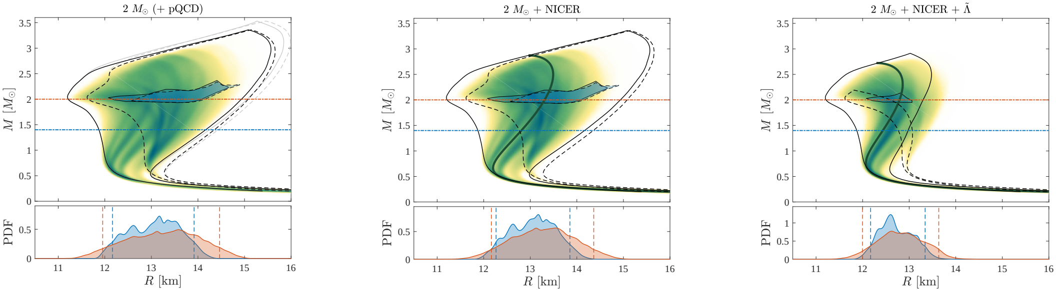


prior + Nicer + GW + HESS



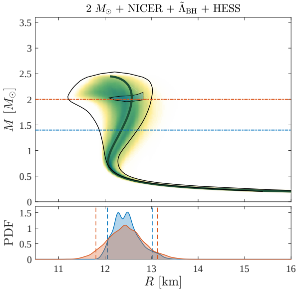
prior + Nicer + GW + HESS + GAP

Bayesian analysis

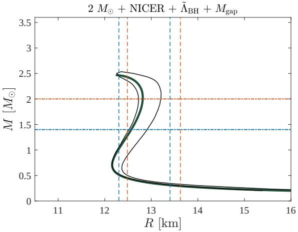


prior (M_{max} + p_{QCD}) prior + NICER prior + NICER + GW

M(R) curve probabilities

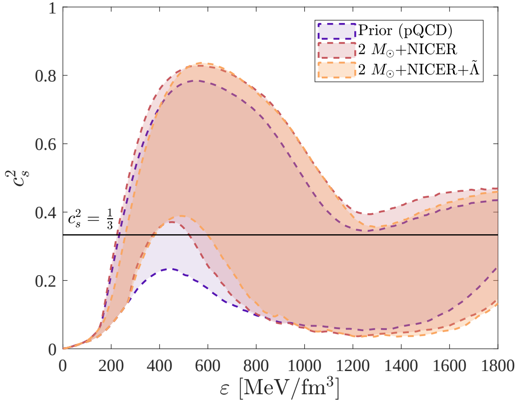


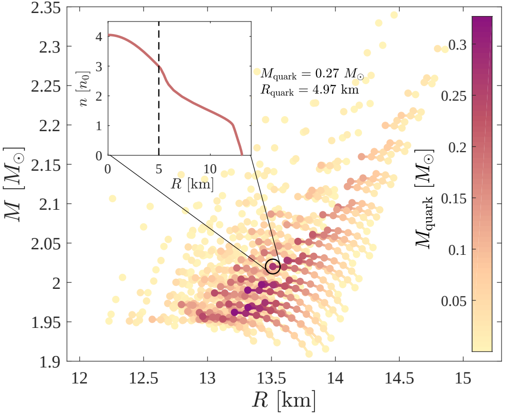
prior + NICER + GW + HESS



prior + NICER + GW + Mgap

Sound velocity and Quark content





Summary and Conclusions

- Our model can reproduce the lattice calculations at $\mu = 0$
- With our model we can fulfill the present astronomical constraints
- The central density do not go above $6\rho_0$.
- The radius of the neutron stars are 12.8 ± 0.8 km.
- strangeness in hadronic phase should be included
- hadronic and quark phase ought to be handled with the same model to drop ad-hoc parameters

Modelling the strongly interacting matter

Plan is to have an interaction with the right global symmetry pattern describing the hadronic (with binding) and the quark phase as well.

1. Starting point is an SU(3) linear sigma model with (pseudo)scalar and (axial)vector nonets. We obtained a very good description for the meson masses and decay widths.
   D. Parganlija, P. Kovács, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011
2. We added te isospin breaking
   P. Kovács, Gy. Wolf, N. Weickgenannt and D.H. Rischke, Phys. Rev. D (2024)
3. We added the baryon octet and decuplet, baryon masses are from spontanous breaking of chiral symmetry
   P. Kovács, Á. Lukács, J. Váróczy, Gy. Wolf, M. Zétényi, Phys. Rev. D89 (2014) 054004
4. Nonzero temperature, chemical potential: We added Polyakov-loops, quarks: Quark-meson model, very good agreement with lattice at $\mu = 0$.
   P. Kovács, Zs. Szép and Gy. Wolf, Phys. Rev. D93 (2016) 114014
5. Future: hadronic phase is not yet well described.

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980$ or $1450), K_0^\star(800$ or $1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: \quad -g_1\phi_N a^\mu_{1N}\partial_\mu\pi_N, \\
\pi - a^\mu_1 &: \quad -g_1\phi_N(a^{\mu+}_1\partial_\mu\pi^- + a^{\mu 0}_1\partial_\mu\pi^0) + \text{h.c.}, \\
\pi_S - a^\mu_{1S} &: \quad -\sqrt{2}g_1\phi_S a^\mu_{1S}\partial_\mu\pi_S, \\
K_S - K^\star_\mu &: \quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star 0}_\mu\partial^\mu K^0_S + K^{\star -}_\mu\partial^\mu K^+_S) + \text{h.c.}, \\
K - K^\mu_1 &: \quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K^{\mu 0}_1\partial_\mu\bar{K}^0 + K^{\mu +}_1\partial_\mu K^-) + \text{h.c.}.
\end{aligned}$$

Diagonalization→ Wave function renormalization

Thermodynamical Observables

We include mesonic thermal contribution to p for (π, K, f_0^l)

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

Inclusion of the vector meson- quark interaction

$$\mathcal{L}_{\mathrm{Vq}} = -g_V\sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu \Psi$$
$$V_0^\mu = \frac{1}{\sqrt{6}}\mathrm{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8) \tag{2}$$

vector fields: like Walecka model, nonzero expectation values are built up at nonzero chemical potential. For simplicity

$$< v_0^\mu >= v_0\delta^{0\mu}, \quad < v_8^\mu >= 0$$

Modification of the grand canonical potential:

$$\Omega(T = 0, \mu_q, g_v) = \Omega(T = 0, \tilde{\mu}_q, gv = 0) - \frac{1}{2}m_v^2 v_0^2,$$

with $\tilde{\mu}_Q = \mu_q - g_v v_0$

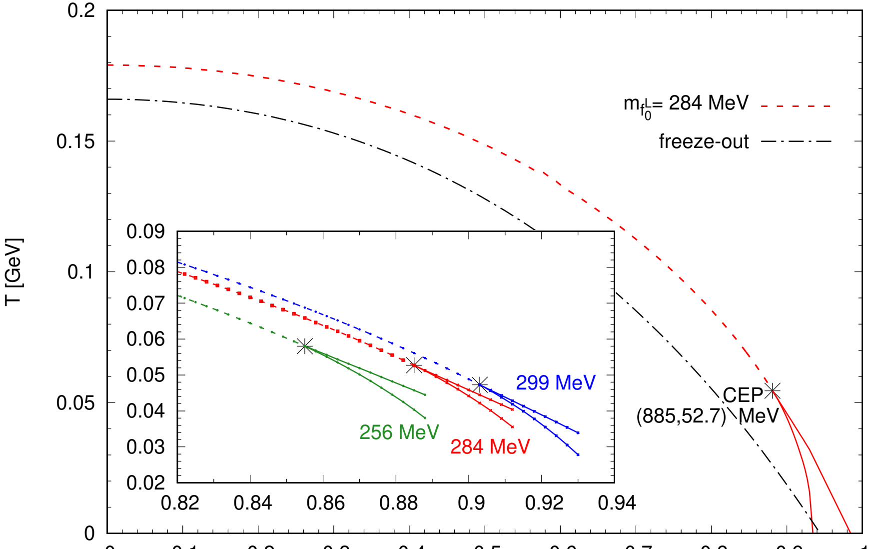
T/μ_B dependence of the condensates

By deriving the grand canonical potential for Polyakov loops (Ω) according to Φ and $\bar{\Phi}$ and equate to zero (extremum eqn.)
Ω: grand canonical potential

$$
\begin{aligned}
\frac{\partial\Omega}{\partial\Phi} &= \left.\frac{\partial\Omega}{\partial\bar{\Phi}}\right|_{\varphi_N=\phi_N,\varphi_S=\phi_S} = 0 \\
\frac{\partial\Omega}{\partial\phi_N} &= \left.\frac{\partial\Omega}{\partial\phi_S}\right|_{\Phi,\bar{\Phi}} = 0, \qquad \text{(after the SSB)} \\
\frac{\partial\Omega}{\partial v_0} &= 0 \qquad \text{(only contribute at } \mu > 0\text{)}
\end{aligned}
$$

five order parameters:
$(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations
ϕ_N, ϕ_S: expectation values for the scalar field
$\Phi, \bar{\Phi}$: expectation values for the Polyakov loops

$T - \mu_B$ phase diagram



Lagrangian (2/1)

$$\begin{aligned}\mathcal{L}_{\mathrm{Tot}} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ &\quad - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\ &\quad + c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ &\quad + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\ &\quad + \bar{\Psi} i\partial\!\!\!/\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi + g_V\bar{\Psi}\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi \\ &\quad + \text{Polyakov loops}\end{aligned}$$

P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar\Psi = (\bar u, \bar d, \bar s)$$

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv <\sigma_{N/S}>$$

Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\exp}}{\delta Q_i} \right]^2 ,$$

where $(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N)$ calculated from the model, while $Q_i^{\exp}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x}, \tau)\right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement

low T: confined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle = 0$

high T: deconfined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

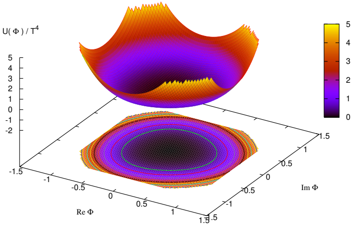
Effects of the gauge fields:

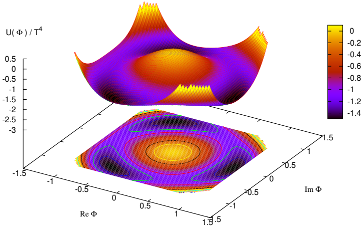
- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential
  $\longrightarrow$ modified quark distribution function.
- Polyakov potential: $\mathcal{U}\left(\Phi, \bar{\Phi}\right)$ models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

Polyakov loop potential

“Color confinement”
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum

“Color deconfinement”
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected





from H. Hansen et al., PRD75, 065004 (2007)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1+3\left(\bar\Phi + \Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1+3\left(\Phi + \bar\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}$$

$\Phi, \bar\Phi \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\Phi, \bar\Phi \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

Features of our approach

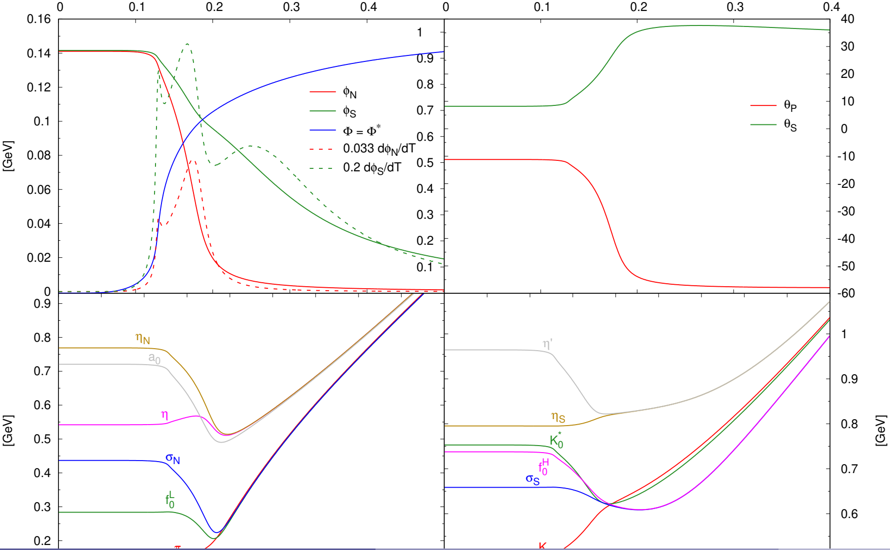
Effective field theory: the same symmetry pattern as in QCD

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

With low mass scalars, $m_{f_0^L} = 300$ MeV



Thank you for your attention!