# Phase transition and correlations in a system with temperature gradients

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L. Jiang, J-H. Zheng, PRD 104, 016031(2021),
L. Jiang, T. Yang, J-H. Zheng, in preparation

# OUTLINE

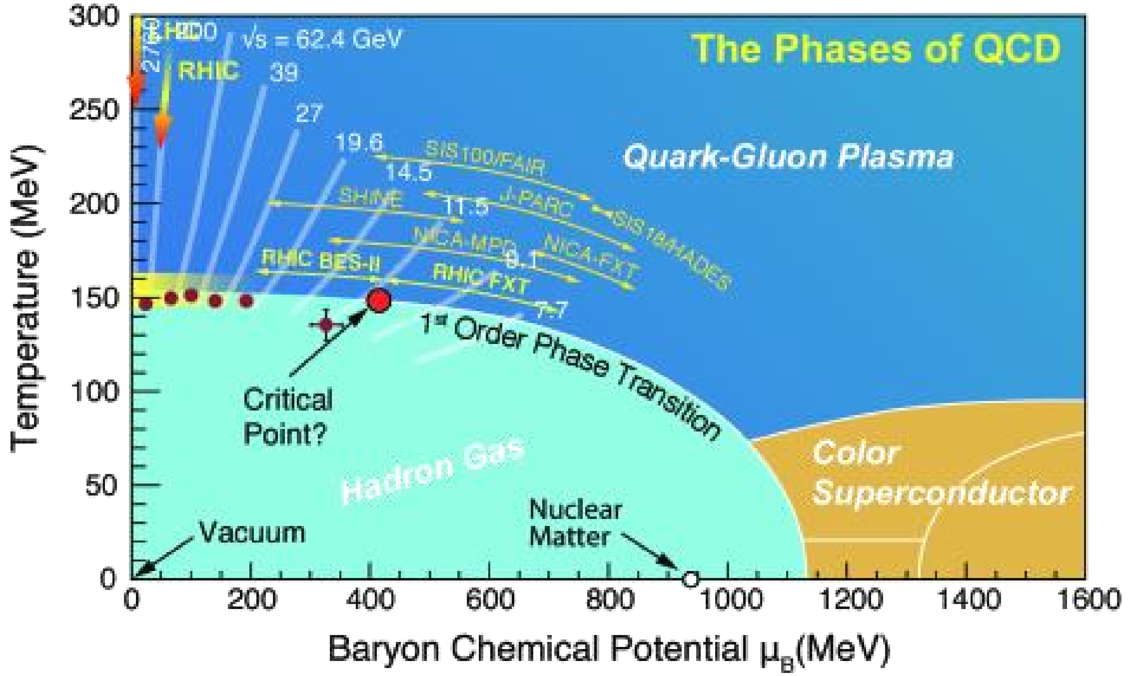
I. Background: QCD Phase Transition and RHIC

II. The partition function and ground state

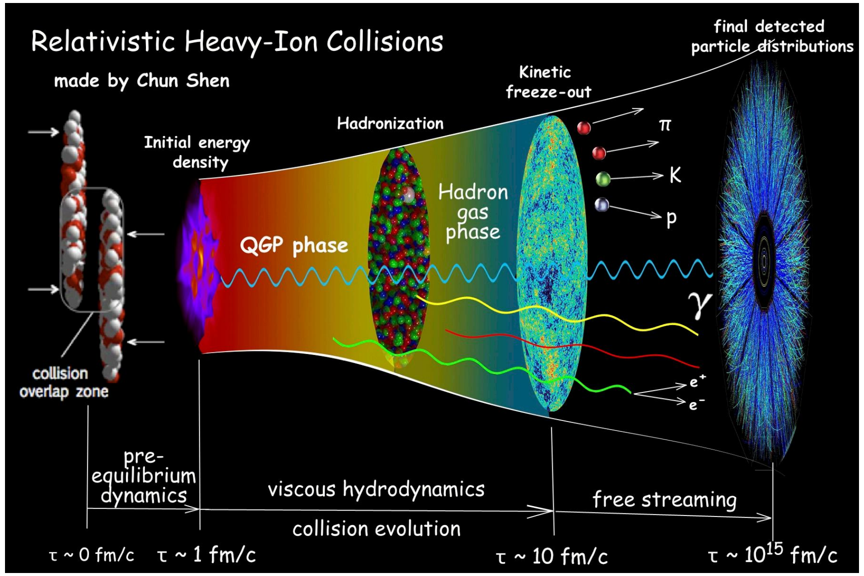
III. The 2-point correlation and high-order fluctuations

IV. Summary

# QCD phase transition and phase diagram




A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, N. Xu, Phys. Rept. 853 (2020)




- **Theoretical analysis (Lattice, nonperturbative QCD, effective theories), CP is predicted.**
- **Experimental facilities:** RHIC (BES), FAIR, NICA, HIAF
- ➢ **Critical Point -- the landmark of the QCD phase diagram.**

**Location of CP? The 1st order phase transition? The signals?**

# Signals for the CP

$g\sigma NN$ coupling

M. Stephanov, PRL 102, 032301(2009)

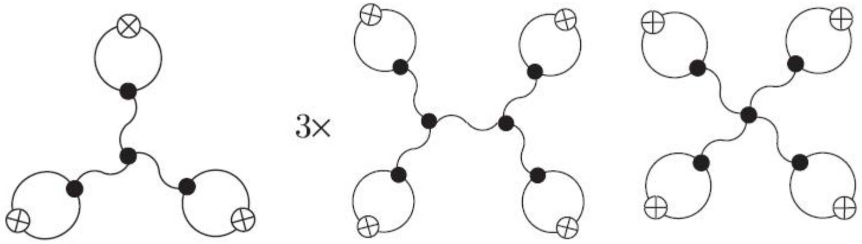
**With this interaction, fluctuating $\sigma$ → fluctuating mass of $N$, particle number correlation becomes**

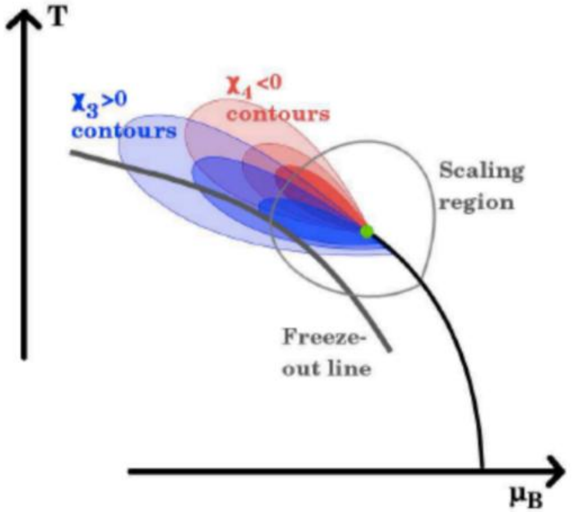
$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k} \xi^2$$

**Stronger signals:** high-order cumulants

$$\omega_3(N_p)_\sigma \approx 6\left(\frac{\tilde{\lambda}_3}{4}\right)\left(\frac{g}{10}\right)^3\left(\frac{\xi}{1\ \mathrm{fm}}\right)^{9/2},$$

$$\omega_4(N_p)_\sigma \approx 46\left(\frac{2\tilde{\lambda}_3^2 - \tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4\left(\frac{\xi}{1\ \mathrm{fm}}\right)^7.$$

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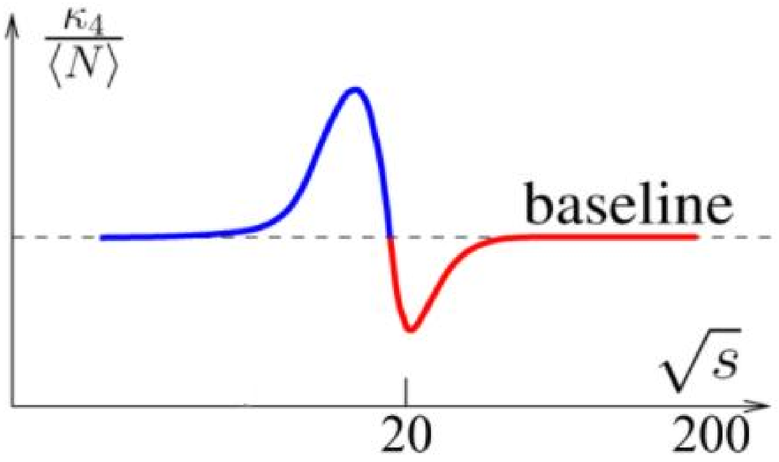
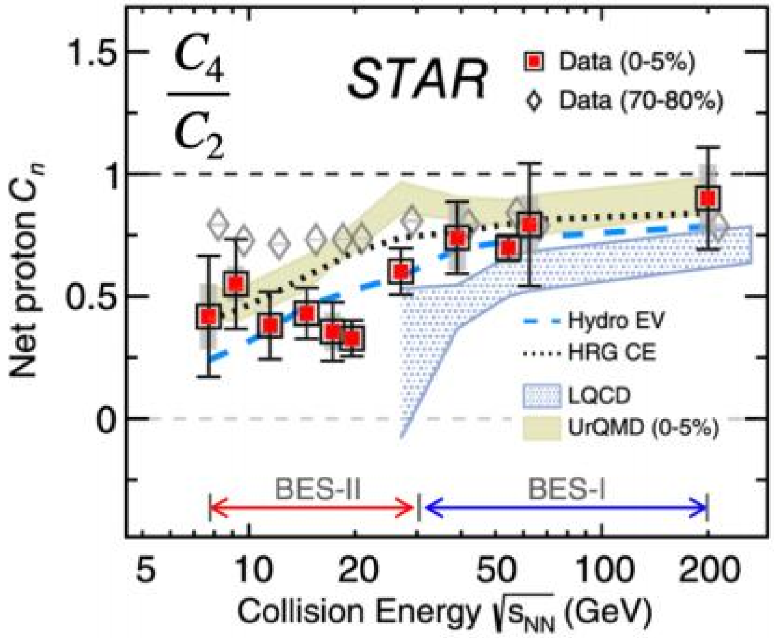
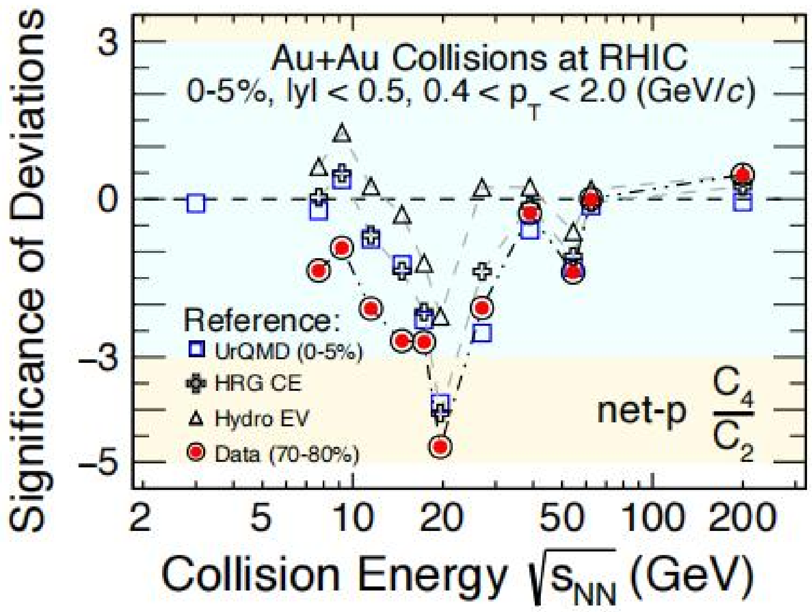


# STAR BES data




*S. Bass, et al: PPNP 41 255 (1998)*
*P. B. Munzinger, et al: NPA 1008 122141 (2021)*
*V. Vovchenkov, et al: PRC 105 014904 (2022)*
*HotQCD: PRD 101 074502 (2020)*
***STAR: PRL135 142301 (2025)***

$$\text{Significance} = \frac{\text{Data} - \text{Ref.}}{\sigma_{total}}$$

Data: STAR 0-5% Result

See Yu Zhang’s talk

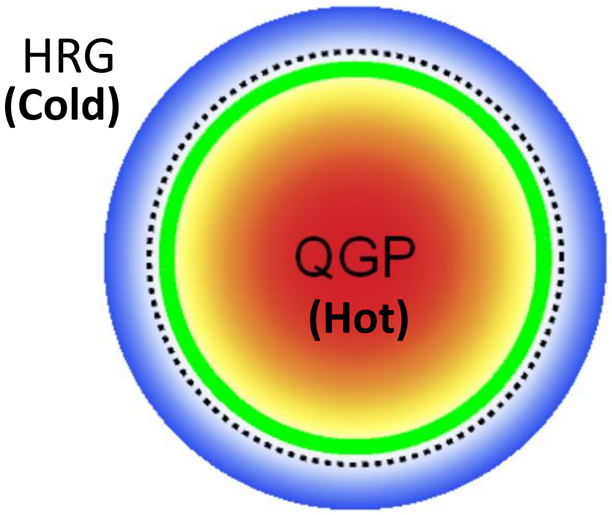
**Indications from experimental data:**

- Deviations from statistical baselines.
- Nonmonotonic at $\sqrt{s_{NN}} \sim 20\ GeV$.
  Could not be explained by UrQMD, HRG, and equi/non-equi critical models.

# Theoretical progress

➢ **Dynamical effects, the finite-size effects.**

B. Berdnikov and K. Rajagopal, PRD 61, 105017 (2000).
M. Stephanov, PRL 102, 032301 (2009) and 107, 052301 (2011).
S. Mukherjee, R. Venugopalan, and Y. Yin, PRC 92, 034912 (2015).
L. Jiang, P. Li, and H. Song, PRC 94, 024918 (2016).
L. Jiang, S. Wu, and H. Song, NPA 967, 441(2017).
S. Wu, Z. Wu, and H. Song, PRC 99, 064902 (2019).
M. Stephanov and Y. Yin, PRD 98, 036006 (2018).
L. Du, U. Heinz, K. Rajagopal, and Y. Yin PRC 102, 054911 (2020).
L. Jiang, H. Stoecker, and J.-H. Zheng, EPJC 83, 117(2023).....



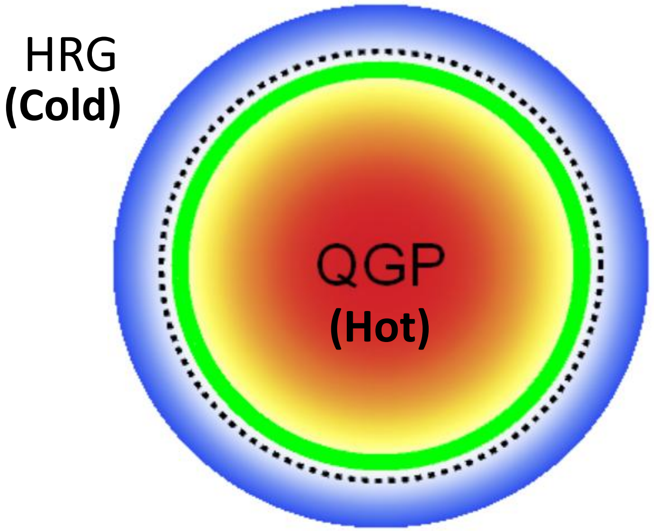

**A slice of the fireball**

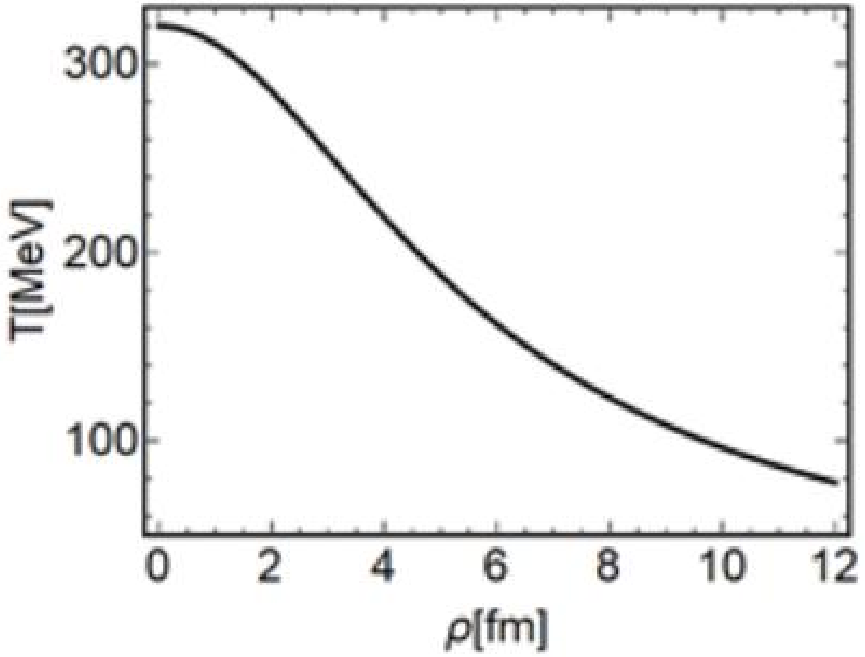
➢ **System with temperature gradients:**

- **Phase transition occurs in a narrow 2D shell.**
- **The spatial temperature gradients on the PT, fluctuations and correlation in the PT region?**

**Jiang, Zheng, PRD 104, 016031(2021) (1D case)**

# Assumptions and Temperature profile




**Assumptions**:

- **Markov process**
- **Fast relaxation**
- **Local equilibrium**
- **Boost invariant (2D Disk)**
- **Constant chemical potential**

**Temperature profile from Gubser flow**

$$T(\tau, r) = \frac{C}{\tau} \frac{(2q\tau)^{2/3}}{\left[1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2\right]^{1/3}}$$

with *C = 2.8, q = 1/4.3 fm and* $\tau$*=1fm.*

**S. Gubser, PRD 82, 085027 (2010)**

# Partition function and e.o.m

**Partition function (local equilibrium)**

$$\mathcal{Z} = \prod_{\boldsymbol{r}} \mathcal{Z}_{\Delta \boldsymbol{r}} = \mathrm{Tr}\exp\left\{-\int d^3\boldsymbol{r}\frac{\mathcal{H}(\hat{\pi},\hat{\sigma})}{T(\boldsymbol{r})}\right\},$$

$$\boxed{\mathcal{H}(\pi,\sigma) = \pi^2/2 + (\nabla\sigma)^2/2 + \mathcal{V}(\sigma).}$$

**Path integral form:** $\mathcal{Z} = \int_{\text{periodic}} \mathcal{D}\sigma e^{S[\sigma]}.$

**The action**

$$S[\sigma] = -\int_0^1 d\tau \int d^3\boldsymbol{r}\left[\frac{T(\partial_\tau\sigma)^2}{2} + \frac{(\nabla\sigma)^2}{2T} + \frac{\mathcal{V}(\sigma,T,\mu)}{T}\right].$$

**Maximum probability gives the e.o.m for the order parameter field:** $\left.\frac{\delta S}{\delta\sigma}\right|_{\sigma=\sigma_c} = 0.$

**in 2D-disk case:**

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\sigma}{\partial\rho}\right) + \frac{\hat{L}_z^2}{\rho^2}\sigma + \frac{1}{T}\frac{\partial\sigma}{\partial\rho}\frac{\partial T}{\partial\rho} + \underline{\eta_1 + \eta_2\sigma + \eta_4\sigma^3} = 0$$

Ising parameterization

# The ground state $\sigma_c$

**Ising-like potential**

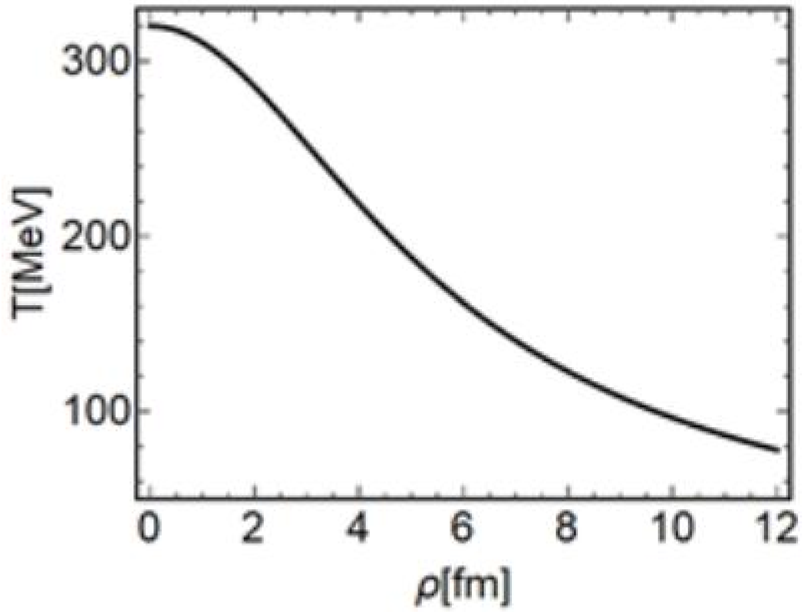
$$\mathcal{V}(\sigma) = \eta_1\sigma + \eta_2\sigma^2/2 + \eta_4\sigma^4/4$$

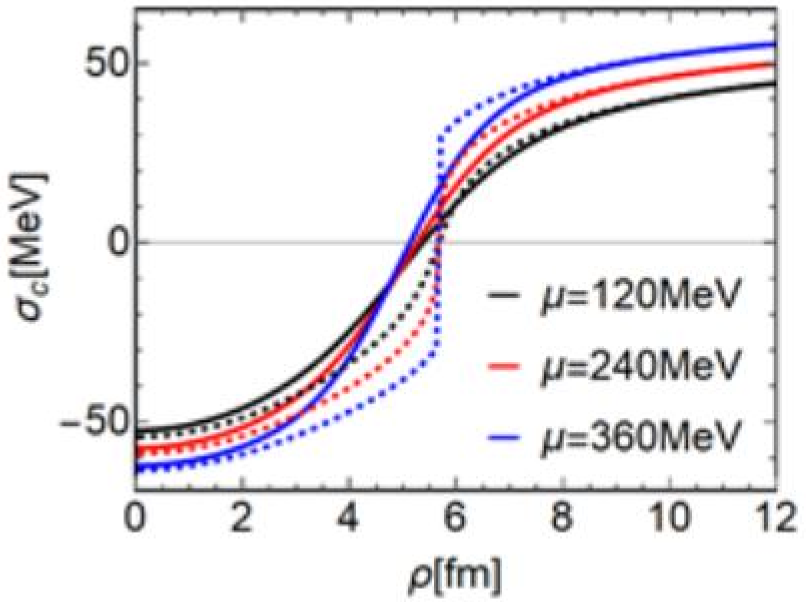
$\eta_1 = 0.5 fm^{-2}\,(T - T_c),$

$\eta_2 = -0.5\, fm^{-1}\,(\mu - \mu_c)$

$\eta_4 = 14.4$

**CP**: $(T_c, \mu_c) = (170, 240)$ **MeV.**




dashed lines are for minimum points of V($\sigma$)

- $\sigma_c(x)$ changes its sign at higher T region.
- The discontinuity of first order phase transition is rounded.

# The fluctuations around $\sigma_c$

**Expanding the action around the ground state**

$$\sigma(\boldsymbol{r}) = \sigma_c(\rho) + \tilde{\sigma}(\boldsymbol{r})$$

**we have**

$$S[\sigma_c(r) + \tilde{\sigma}(\boldsymbol{r})] = S[\sigma_c(r)] + \Delta S[\tilde{\sigma}(\boldsymbol{r})]$$

**where the fluctuating part reads**

$$\Delta S[\tilde{\sigma}(\boldsymbol{r})] = -\int_0^1 d\tau \int d^3\boldsymbol{r} \left(\frac{1}{2}\tilde{\sigma}\hat{O}\tilde{\sigma} + \frac{\lambda_3}{3}\tilde{\sigma}^3 + \frac{\lambda_4}{4}\tilde{\sigma}^4\right),$$

**Gaussian**

$$\hat{O} = \overleftarrow{\partial_\tau} T \overrightarrow{\partial_\tau} + \overleftarrow{\nabla}\frac{1}{T}\cdot\overrightarrow{\nabla} + \frac{(\eta_2 + 3\eta_4\sigma_c^2)}{T},$$

**Non-Gaussian**

$$\lambda_3 = \frac{3\eta_4\sigma_c}{T}, \qquad \lambda_4 = \frac{\eta_4}{T}.$$

# 2-point correlation

**Complete basis (of the Hermitian operator $\hat{\mathcal{O}}$ ) expansion**

$$\tilde{\sigma}(\boldsymbol{r},\tau)=\sum_{\lambda,l=-\infty}^{\infty}\sum_{n}c_{\lambda nl}R_{\lambda nl}(\rho)\,\Theta_l(\theta)\,e^{-i\omega_\lambda\tau},$$

Radial angular: $\frac{e^{il\theta}}{\sqrt{2\pi}}$ Matsubare frequency: $\omega_\lambda=2\pi\lambda$

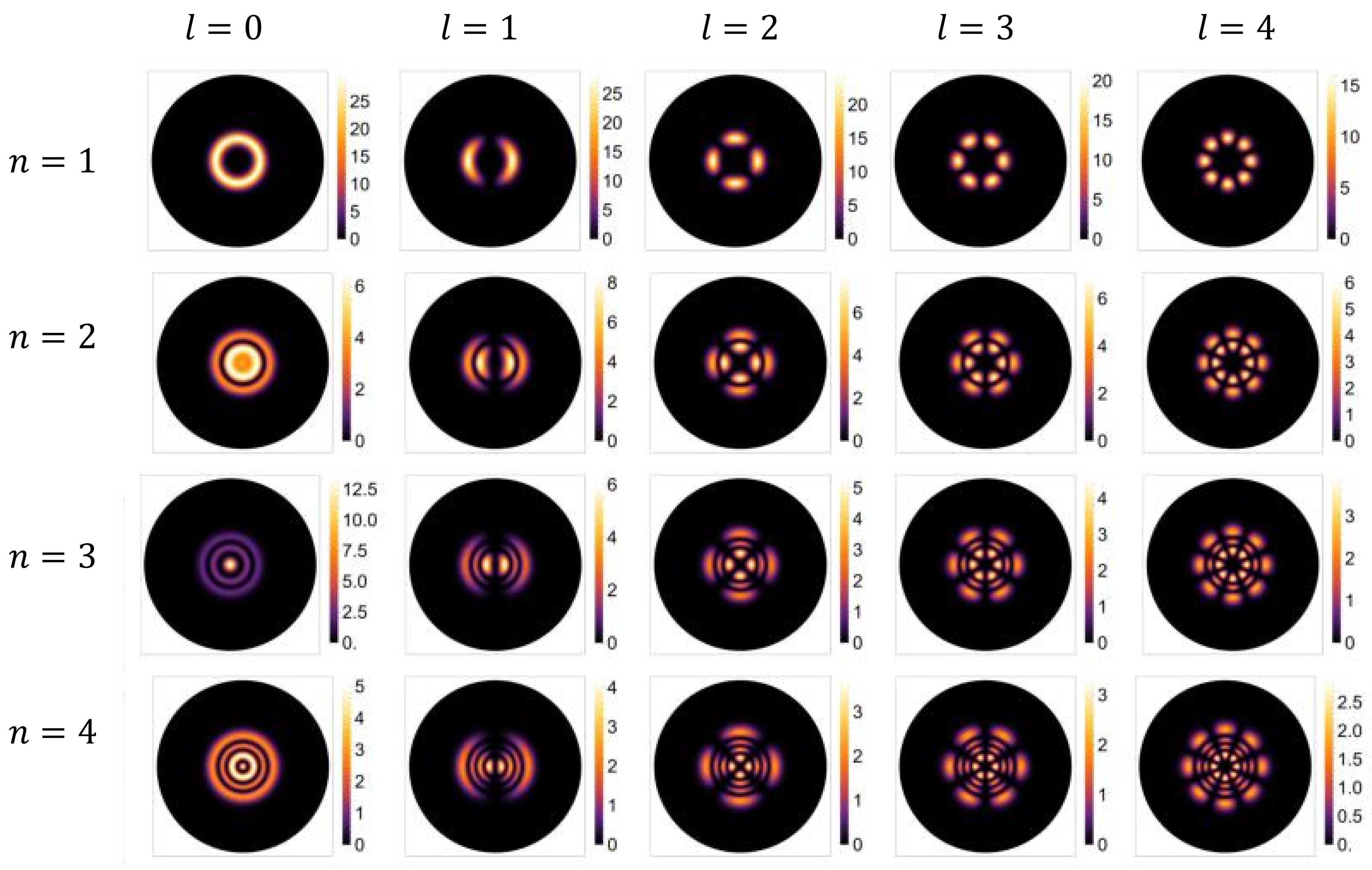
**Gaussian part:**

$$\begin{aligned}\Delta S_0[\tilde{\sigma}] &= -\frac{1}{2}\int_0^1 d\tau\int d^3\boldsymbol{r}\tilde{\sigma}\hat{\mathcal{O}}\tilde{\sigma}\\ &= -\frac{1}{2}d_z\sum_{\lambda nn'l}c^*_{\lambda n'l}c_{\lambda nl}\int d\rho[\sqrt{\rho}R^*_{\lambda n'l}]\hat{\mathcal{O}}_{\lambda l}[\sqrt{\rho}R_{\lambda nl}]\\ &= -\frac{1}{2}d_z\sum_{\lambda nl}c^*_{\lambda nl}\varepsilon_{\lambda nl}c_{\lambda nl},\end{aligned}$$

**Two-point correlation:**

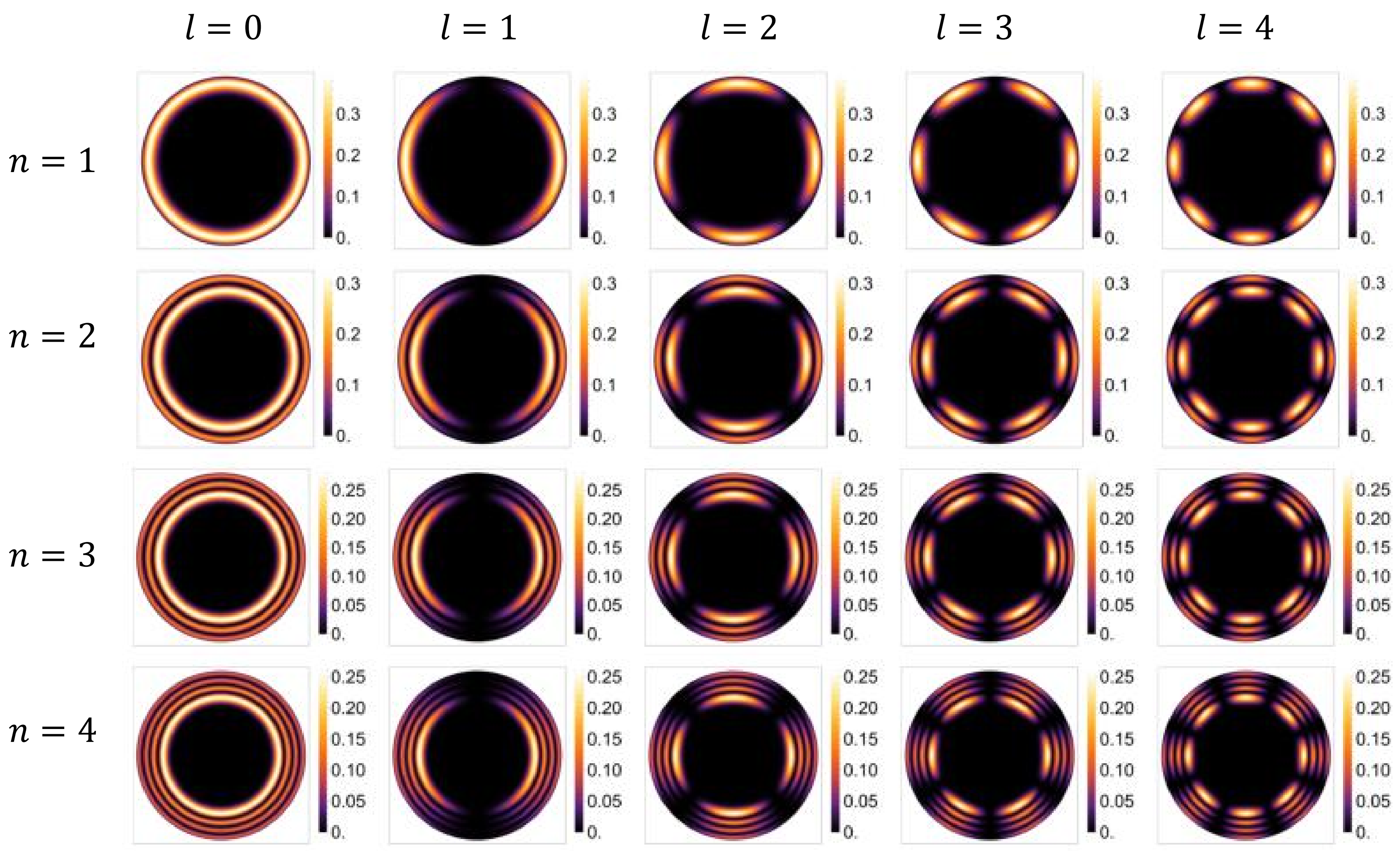
$$\boxed{\langle\tilde{\sigma}(\boldsymbol{r},\tau)\tilde{\sigma}(\boldsymbol{r}',\tau)\rangle=\sum_{\lambda nl}\frac{R_{\lambda nl}(\rho)\,R_{\lambda nl}(\rho')\,\Theta_l(\theta)\,\Theta^*_l(\theta')}{d_z\varepsilon_{\lambda nl}}.}$$

# Probability density with $\lambda$=0



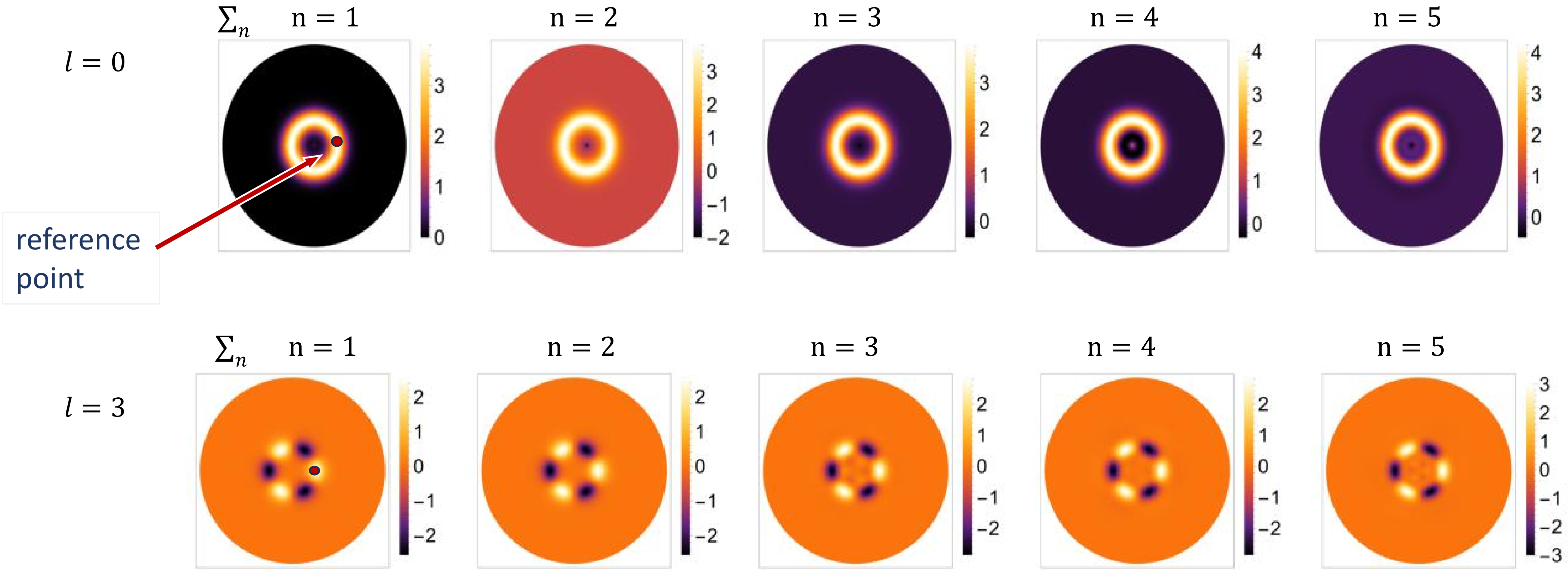

- **Fluctuations excited in the phase transition region.**

# Probability density with $\lambda$=2



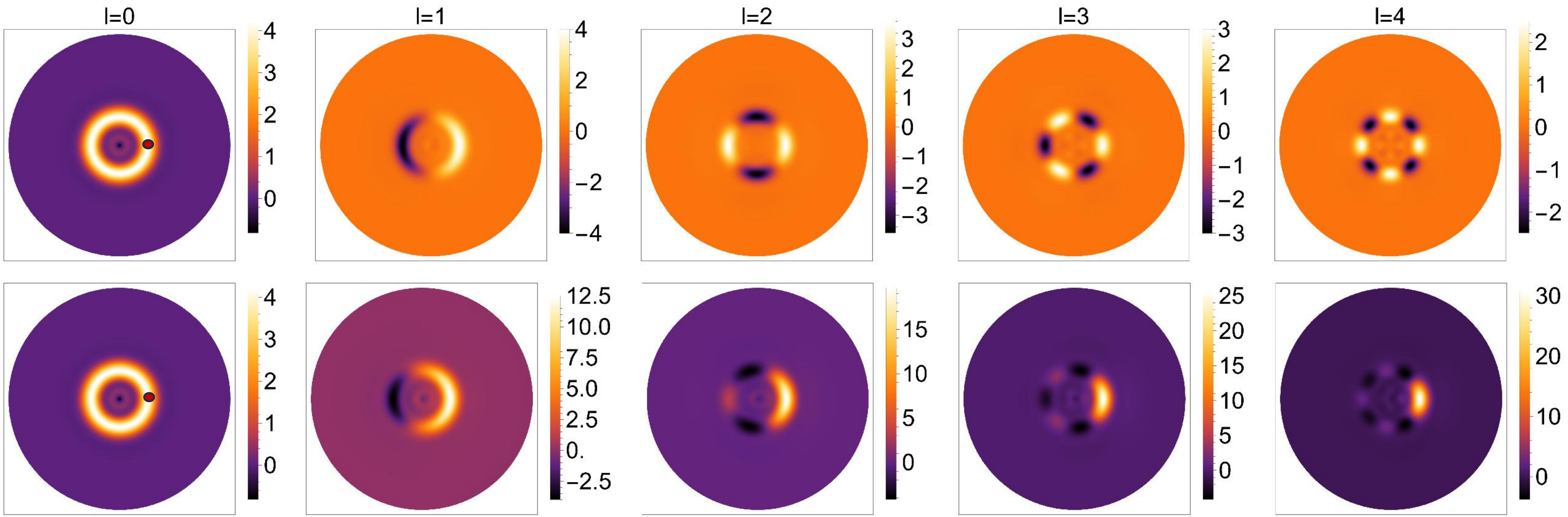

- **High frequency modes contribute to the peripheral fluctuations, depressed in the center.**

# Nonlocal 2-point correlation




- **Correlation between an reference point and the other point arbitrary in the disk.**
- **Strong correlations along the isothermal ring for different $l$-modes.**

# Nonlocal 2-point correlation




$\lambda$ and n components is summed over .

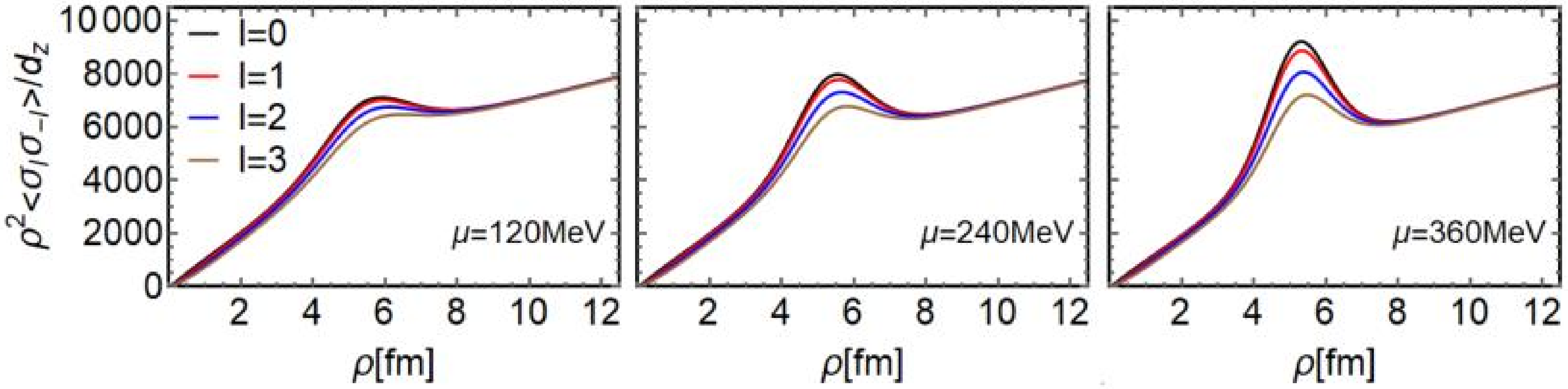
- **Decoherence of the correlation along the isothermal ring as the acummation of $l$-modes .**

# Extraction of nonzero $l$-modes

**Introducing**
$$\tilde{\sigma}_l(\rho,\tau) \equiv \int dz \int_0^{2\pi} d\theta \tilde{\sigma}(\boldsymbol{r},\tau)\, e^{-il\theta}$$

**2-point correlation**
$$\langle \tilde{\sigma}_{-l}(\rho,\tau)\, \tilde{\sigma}_{l'}(\rho',\tau')\rangle = \langle \tilde{\sigma}_l^*(\rho,\tau)\, \tilde{\sigma}_{l'}(\rho',\tau')\rangle = 2\pi d_z \delta_{l'l} \sum_{\lambda=-\infty}^{\infty} G_{\lambda l}(\rho,\rho') e^{i\omega_\lambda(\tau-\tau')},$$

**with** $G_{\lambda l}(\rho,\rho') = \sum_n R_{\lambda nl}(\rho)\, R_{\lambda nl}(\rho')/\varepsilon_{\lambda nl}.$

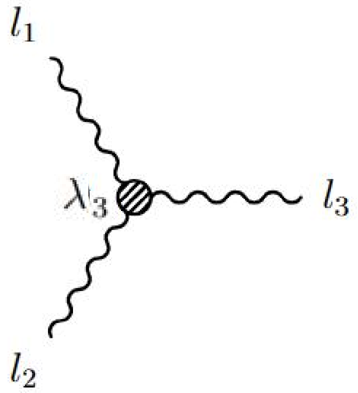


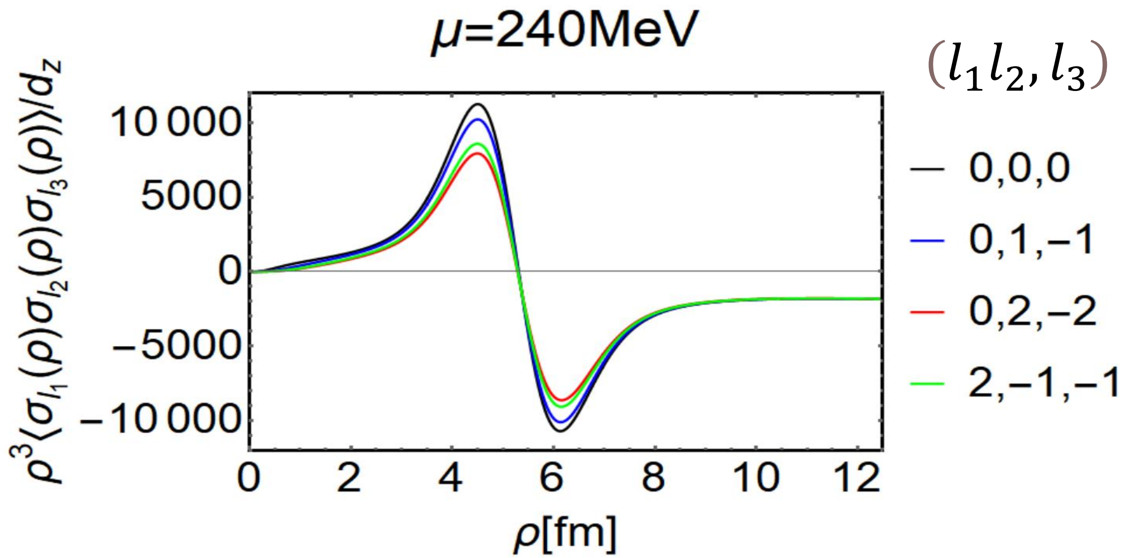

- **Enhancements of zero and nonzero $l$-modes in the phase transition region for different $\mu$.**

# Non-Gaussian fluctuations

**3-point correlation**

$$\langle \tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\,\tilde{\sigma}_{l_3}(\rho,0)\rangle = -12\pi d_z \eta_4 \delta_{l_1+l_2+l_3,0} \times \sum_{\lambda_1,\lambda_2=-\infty}^{\infty} \int d\rho' \frac{\rho' \sigma_c(\rho')}{T(\rho')} G_{\lambda_1 l_1}(\rho,\rho') G_{\lambda_2 l_2}(\rho,\rho') G_{\lambda_1+\lambda_2, l_3}(\rho,\rho'),$$



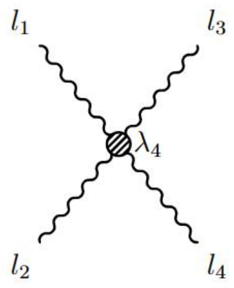



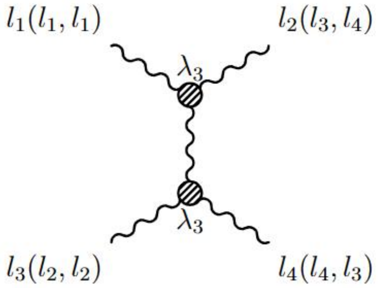

- **Nonzero $l$-modes exhibit typical structure and comparable fluctuations as the zero mode.**

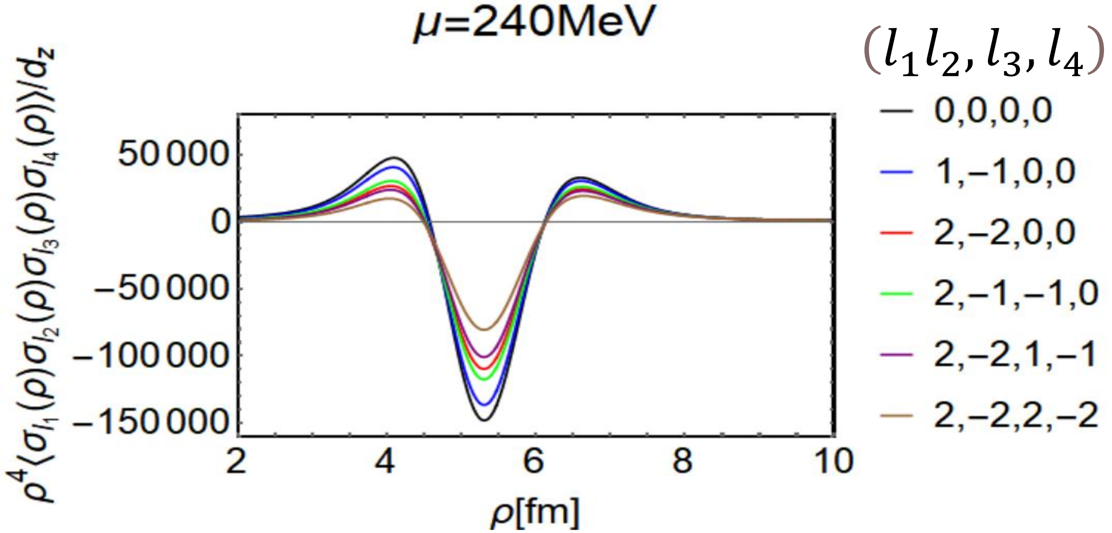
# Non-Gaussian fluctuations

**4-point correlation**

$$
\begin{aligned}
&\langle\tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\,\tilde{\sigma}_{l_3}(\rho,0)\,\tilde{\sigma}_{l_4}(\rho,0)\rangle \\
=\ &-12\pi d_z\eta_4\delta_{l_1+l_2+l_3+l_4,0} \\
&\times\sum_{\lambda_1,\lambda_2,\lambda_3}\int d\rho'\frac{\rho'}{T(\rho')}G_{\lambda_1 l_1}(\rho,\rho')G_{\lambda_2 l_2}(\rho,\rho')G_{\lambda_3 l_3}(\rho,\rho')G_{\lambda_1+\lambda_2+\lambda_3,l_4}(\rho,\rho') \\
&+72\pi d_z\eta_4^2\delta_{l_1+l_2+l_3+l_4,0} \\
&\times\sum_{\lambda_1,\lambda_2,\lambda_3}\int d\rho_1\int d\rho_2\frac{\rho_1\sigma_c(\rho_1)}{T(\rho_1)}\frac{\rho_2\sigma_c(\rho_2)}{T(\rho_2)} \\
&\times\Big[G_{\lambda_1 l_1}(\rho,\rho_1)G_{\lambda_2 l_2}(\rho,\rho_1)G_{\lambda_3 l_3}(\rho,\rho_2)G_{\lambda_1+\lambda_2+\lambda_3,l_4}(\rho,\rho_2)G_{\lambda_1+\lambda_2,l_1+l_2}(\rho_1,\rho_2) \\
&+G_{\lambda_1 l_1}(\rho,\rho_1)G_{\lambda_2 l_3}(\rho,\rho_1)G_{\lambda_3 l_2}(\rho,\rho_2)G_{\lambda_1+\lambda_2+\lambda_3,l_4}(\rho,\rho_2)G_{\lambda_1+\lambda_2,l_1+l_3}(\rho_1,\rho_2) \\
&+G_{\lambda_1 l_1}(\rho,\rho_1)G_{\lambda_2 l_4}(\rho,\rho_1)G_{\lambda_3 l_2}(\rho,\rho_2)G_{\lambda_1+\lambda_2+\lambda_3,l_3}(\rho,\rho_2)G_{\lambda_1+\lambda_2,l_1+l_4}(\rho_1,\rho_2)\Big]
\end{aligned}
$$



# Summary

We studied the ground state, the fluctuations and nonlocal correlations in a system with spatial temperature gradients:

— Lift of the PT temperature, and rounding of the discontinuities in FOPT.

— The fluctuations are enhanced in the PT region.

— Strong anisotropic correlations due to the spatial T distribution.

— Correlations of nonzero angular momentum modes comparable to the zero mode.

**Thanks for your attention!**