New insight on QCD (de-)confinement
& its implications on thermodynamics

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Based on: YL, Fei Gao (高飞), Yu-xin Liu (刘玉鑫), Jan M. Pawlowski: 2504.05099

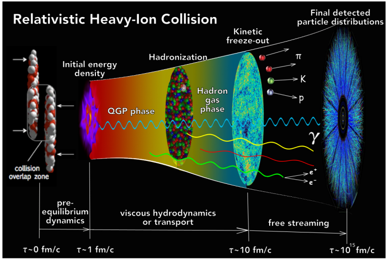
16th Workshop on QCD Phase Transition
and Relativistic Heavy-Ion Physics (QPT 2025)

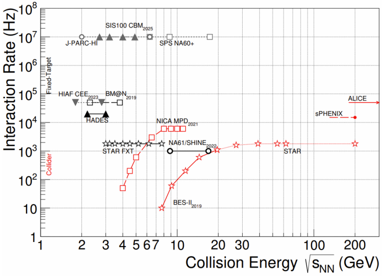
Guilin 26th 11 2025

Experimental research on QCD phase diagram at high baryon density is entering the precision physics era: CBM (FAIR), CEE+ (HIAF), ...

Great demand on theoretical side:

- Phase structure at high μ_B in parallel of experimental landscape - beyond the μ_B range of current best lattice QCD extrapolation.
- Emergent phenomena - “more is different”.
- (Equilibrium) QCD baselines?
  - macroscopically & microscopically.
- How to connect theory with EXP.: thermodynamics & related observables?
- . . .





(Plot: Nucl.Phys.A 982 (2019) 163-169)

QCD chiral phase diagram: the theoretical landscape

- QCD chiral condensate:

Lattice (extrapolation from $\mu_B^2 \leq 0$):
Wuppertal-Budapest Collab., HotQCD Collab, ..

Functional (direct computation at $\mu_B^2 > 0$):
PLB 820: 136584 (2021), PRD 104: 054022 (2021), PRD 101: 054032 (2020)

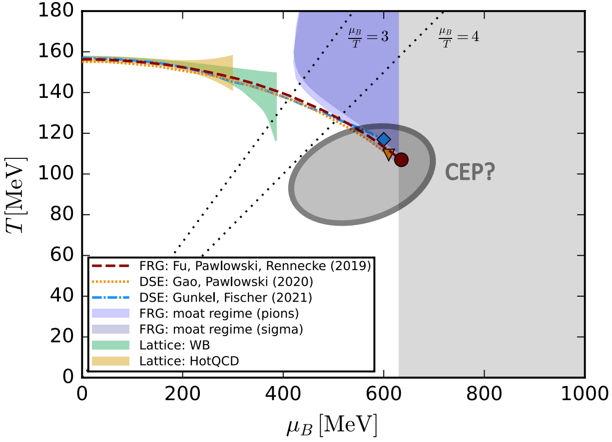
◦ Lee-Yang edge singularities:

Basar, PRC 110: 015203 (2024),
Clarke et al., 2405.10196,
Schmidt, 2504.00629,
Borsanyi et al., 2507.13254, ...

◦ Thermodynamics extrapolations:

Hippert et al., PRD 110: 094006 (2024),
Shah et al. 2410.16206,
Borsanyi et al. 2502.10267, ...

(Plot courtesy: F. Rennecke, Quark Matter 2025)



Continuum functional approaches for QFT:

$$Z[J] = \int \mathcal{D}[\varphi] \exp\left[-S[\varphi] + \int d^D x\, J(x)\varphi(x)\right].$$

$$\left\langle \frac{\delta S[\varphi]}{\delta \varphi(x)} \right\rangle_{J=0} = 0;$$

$$\langle f[\varphi]\rangle_J \equiv \int \mathcal{D}[\varphi]\, f[\varphi]\, e^{-S[\varphi]+\int d^D x\, J(x)\varphi(x)}.$$

(The generating equation of)

Dyson Schwinger equations (DSE):

QFT version of the “least action” principle; manifest field translational invariance.

$$Z_k[J] = \int \mathcal{D}[\varphi]\, e^{-S[\varphi]-\Delta S_k[\varphi]+\int d^D x\, J(x)\varphi(x)},$$

$$\Delta S_k[\varphi] = \frac{1}{2}\int d^D x d^D y\, \varphi(x)\, R_k(x,y)\, \varphi(y);$$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{STr}\left[(\partial_t R_k)\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1}\right].$$

Functional renormalization group (fRG):

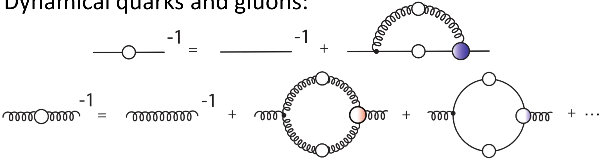
“Coarse-grained” action with the regularization scale k; renormalization group flow and the flow equations.

Continuum functional QCD

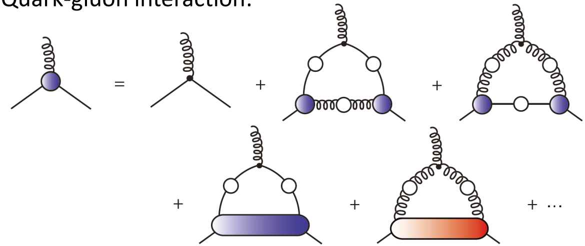
Dyson Schwinger equations [DSE]

(Plots: Eichmann, 0909.0703)

Dynamical quarks and gluons:

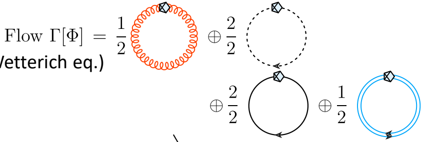


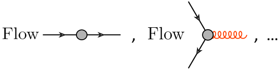
Quark-gluon interaction:



Alkofer & von Smekal, 0007355 Roberts & Schmidt, 0005064
Eichmann at al, 1606.09602 Fischer, 1810.12938 …

functional renormalization group [fRG]





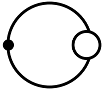
"LEGO principles" 2408.08413

Pawlowski, 0512261 Gies, 0611146
Dupuis at al., 2006.04853 Fu, 2205.00468 …

$$\langle \mathbf{vac.}\,|\bar{\boldsymbol{q}}\boldsymbol{q}|\,\mathbf{vac.}\rangle \quad \text{vs} \quad \sum \langle \text{vac.}\,|\psi\rangle\langle\psi|\,\text{vac.}\rangle$$

$\psi \in$ asymptotic states: $\pi, K, p, n, \ldots$

$$\langle \bar{q}(x)\Gamma q(x)\rangle = -\mathrm{Tr}\,\langle \Gamma q\bar{q}\rangle = -\Gamma$$



"Going medieval" on QCD (de-)confinement

Status before: gluonic background field $\langle A_0 \rangle$, a gauge invariant QCD correlation function, gives a direct access on center symmetry breaking.

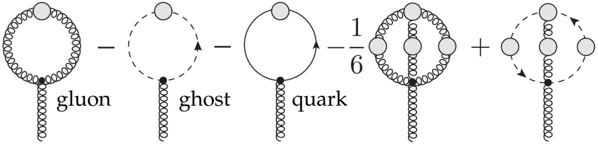
Yang-Mills: fRG & DSE: Fister, Pawlowski, PRD 88: 045010 (2013).

2+1-flavor: fRG: Braun, Gies, Pawlowski, PLB 684: 262 (2010); DSE: Fischer, Fister, Luecker, Pawlowski, PLB 732: 273 (2014).

perturbative: Curci-Ferrari model: Reinosa, Serreau, Tissier, PRD 92: 025021 (2015).

Polyakov loop potential; relatively simple truncation in fQCD, e.g. DSE:

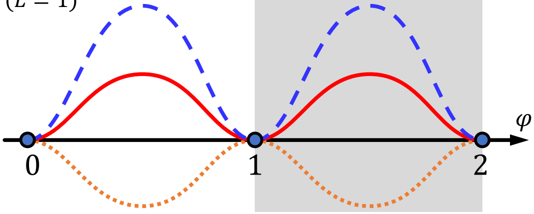
$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2}$$



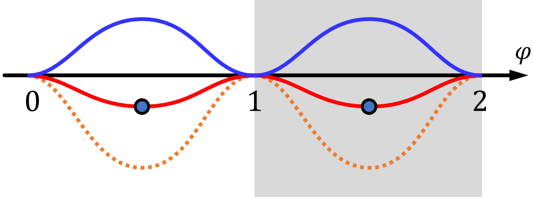
• The diagrammatic representation in functional QCD helps us understand how the chiral dynamics overlaps with the center symmetry breaking / confinement-deconfinement phase structure!

A_0 & confining dynamics in $SU(N_c = 2)$

- YM, perturbative: (Weiss potential, high T)
  $(L = 1)$



- YM, non-perturbative:
  $(L = 0)$



gluon (pert.)
gluon (with mass gap)
ghost
quark
full ● physical point

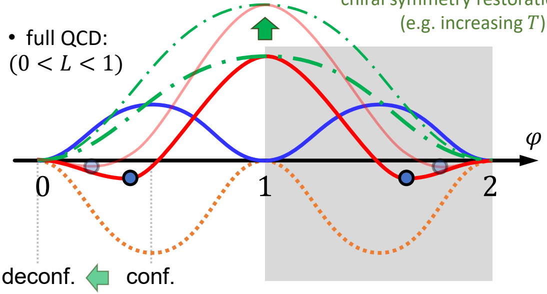
see e.g.:

Fister & Pawlowski
PRD 88: 045010 (2013),

Fukushima & Skokov
PPNP 96 (2017) 154–199

$gA_0 = 2\pi T \varphi \tau_3; \;\; L = \cos \pi\varphi$. (Polyakov loop)

- full QCD:
  $(0 < L < 1)$



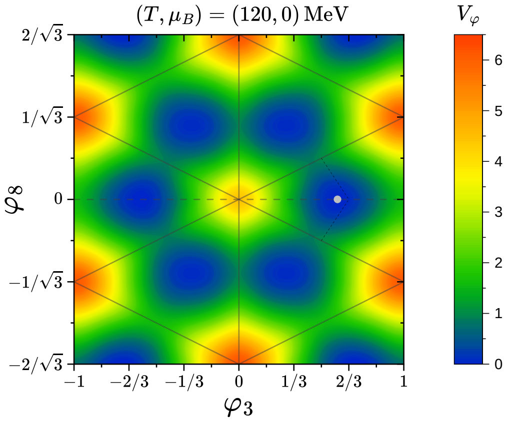
Now: New insights on QCD thermodynamic functions in the view of A_0:

- Complete color SU(3) structure (Cartan sub-algebra) of A_0 at finite μ_B:

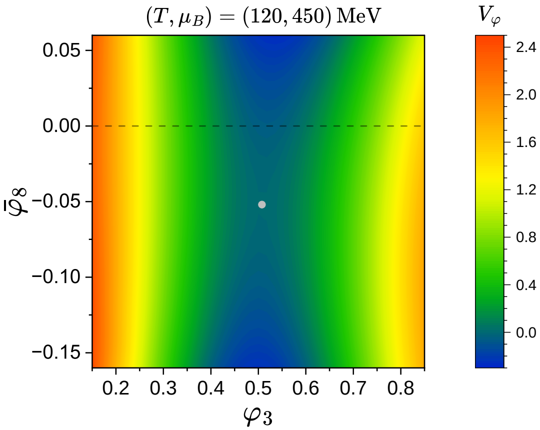
$$A_0 = \frac{2\pi T}{g}\varphi^c; \qquad \text{eigenvalues}: \quad \varphi^c = \begin{cases} \varphi_3/2 \pm \varphi_8/2\sqrt{3},\ \varphi_8/\sqrt{3}, & \text{fund.} \\ 0,\ \pm\varphi_3,\ \pm(\varphi_3 \pm \varphi_8\sqrt{3})/2, & \text{adj.} \end{cases}$$

Polyakov loop potential in 2+1-flavor QCD, within the field space relevant to physical observables: Reinosa, Serreau, Tissier, PRD (2015), Ihssen and Pawlowski, SciPost Phys. (2023)

$\boxed{\varphi_8 \in \mathbb{R} \quad \text{at} \quad \mu_B = 0}$



$\boxed{\bar{\varphi}_8 = -\mathrm{i}\varphi_8 \in \mathbb{R} \quad \text{at} \quad \mu_B \in \mathbb{R}}$



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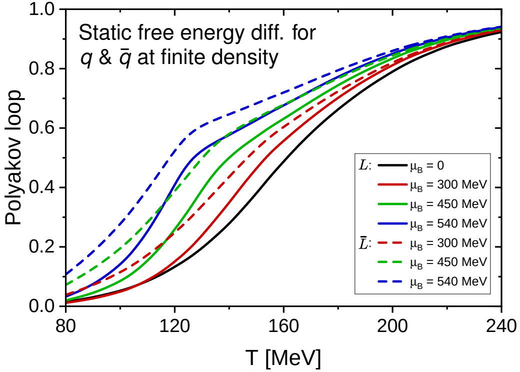
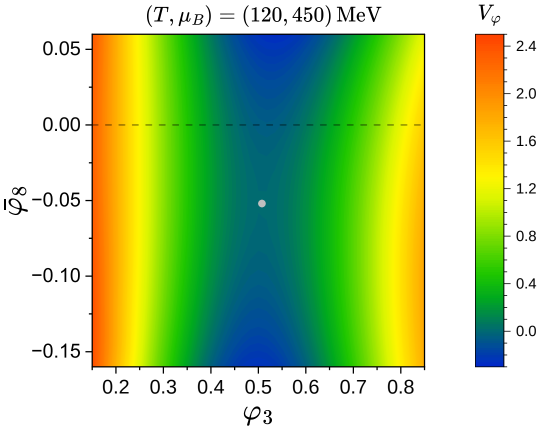
$$\boxed{\begin{aligned} L(\langle A_0\rangle) &= \left(e^{2\pi\bar\varphi_8/\sqrt{3}} + 2\,e^{-\pi\bar\varphi_8/\sqrt{3}}\cos\pi\bar\varphi_3\right)/3\,, \\ \bar L(\langle A_0\rangle) &= \left(e^{-2\pi\bar\varphi_8/\sqrt{3}} + 2\,e^{\pi\bar\varphi_8/\sqrt{3}}\cos\pi\bar\varphi_3\right)/3. \end{aligned}}$$

$$\boxed{\bar\varphi_8 = -\mathrm{i}\varphi_8 \in \mathbb{R} \quad \text{at} \quad \mu_B \in \mathbb{R}}$$





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difference between the static free energies of q and $\bar{q}$ captured.

- Matter sector - an off-shell representation is complete: e.g. net density
  $n_q = \langle \bar{q}\gamma_0 q \rangle = -\mathrm{Tr}\,[\gamma_0 G_{qq}]$; YL et al., PRD (2024), Isserstadt et al., PRD (2019), ...

both confining and chiral dynamics are included self-consistently:

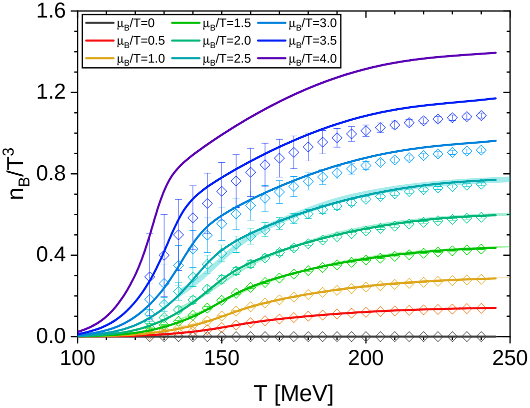
$$G_{qq}^{-1}(p_q) = \mathrm{i}\gamma_0 p_{q,0}\, Z_q^E(p_q) + \mathrm{i}\boldsymbol{\gamma}\cdot\boldsymbol{p}\, Z_q^M(p_q) + Z_q^E(p_q)\boxed{M_q(p_q)}\,,$$

$$p_{q,\mu} = (\omega_{q,n} + \boxed{2\pi T\varphi^c} - \mathrm{i}\mu_q\,,\ \boldsymbol{p}).$$

QCD equation of state: “apples to apples” in continuum & discrete approaches:

fQCD: $n_q(T,\mu_B) = \langle \bar{q}\gamma_0 q \rangle = -\mathrm{Tr}\,[\gamma_0 G_{qq}]$;

lQCD: $n_B(T,\mu_B) = \sum_{n=0}^{N} \frac{P_{2n+2}(T^{(\prime)},0)}{(2n+1)!}\mu_B^{2n+1}$;



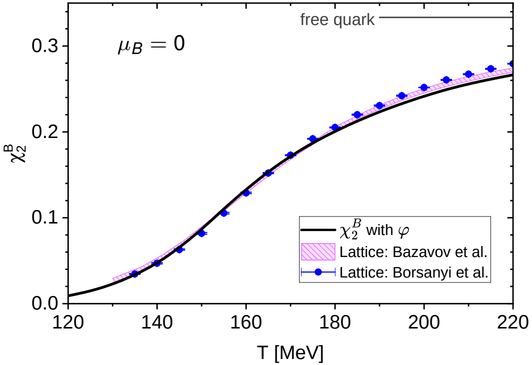
- Conserved charge relation: $n_B = \frac{1}{3}(n_u + n_d + n_s)$.
- Testing ground for the convergence of lattice extrapolations: Wen, Yin, Fu, PRD 110: 016008 (2024).
- Going beyond $\mu_B/T \gtrsim 3$ - no QCD benchmarks there yet. Merits to cover the future experimental landscape.

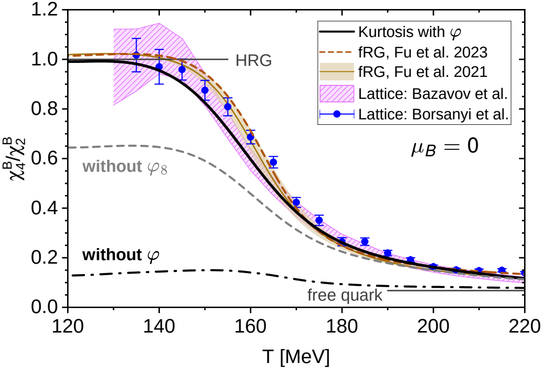
In particular, through **baryon number fluctuations.**

Baryon number “fluctuations” (susceptibility) at vanishing μ_B

Hints on the finite-μ_B signatures of (de-)confinement phase transition.

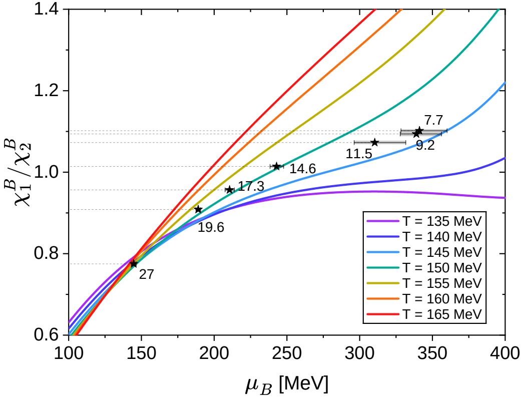
$$\text{cumulants:}\quad \chi_k^B = \frac{\partial^k (P/T^4)}{\partial(\mu_B/T)^k} \propto \partial_{\mu_B}^{(k-1)} n_B = \begin{cases} -\frac{1}{3}\sum_{q=u,d,s} \mathrm{Tr}\left[\gamma_0(\partial_{\mu_B}^{(k-1)} G_{qq})\right], & \text{fQCD;} \\ \sum_{n=0}^{N-k} \frac{P_{2n+k}(T^{(\prime)},0)}{(2n)!}\mu_B^{2n}, & \text{lQCD.} \end{cases}$$

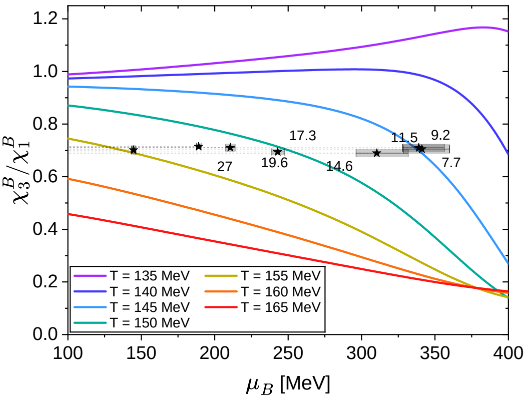




Bridge the gap between theory and high precision experiment - STAR, PRL 135: 142301 (2025).
Systematic extraction of T_f and $\mu_{B,f}$ at different $\sqrt{s_{\mathrm{NN}}}$ with the leading 3 cumulants:

$$\chi_2^B/\chi_1^B\left(T_f,\mu_{B,f}\right)=C_2/C_1\left(\sqrt{s_{\mathrm{NN}}}\right)\quad\text{and}\quad\chi_3^B/\chi_1^B\left(T_f,\mu_{B,f}\right)=C_3/C_1\left(\sqrt{s_{\mathrm{NN}}}\right).$$



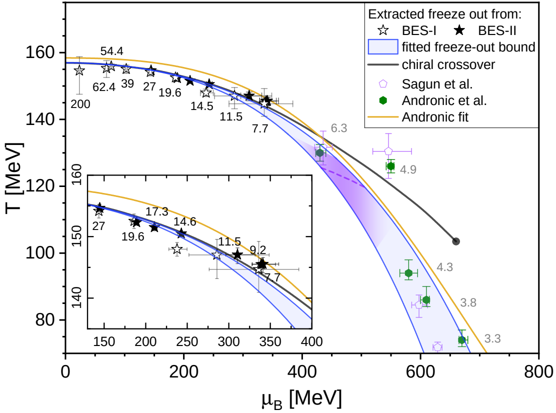


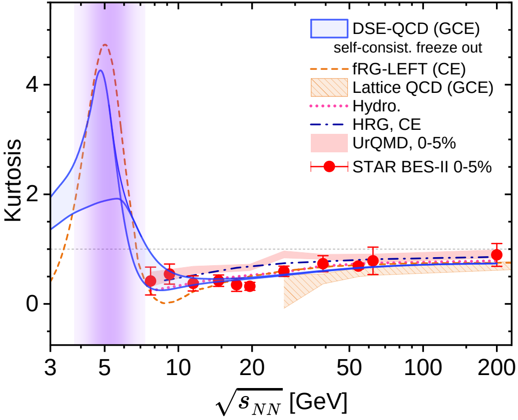
Fischer, Gao, Liu, YL, Pawlowski, in prep.

see also: Bazavov et al. PRL 109, 192302 (2012),
Borsanyi at al. PRL 111, 062005 (2013)

Baryon number fluctuations along freeze out line

- "Yet another" baseline on the baryon number kurtosis (equilibrium QCD).
- Peak of $\chi_4^B/\chi_2^B \to$ region in the phase diagram for experimental insights.





Fischer, Gao, Liu, YL, Pawlowski, in prep.

Summary

New insights on several aspects in QCD thermodynamics:

- QCD (de-)confinement and center symmetry (breaking) at finite T & μ_B.
- confining + chiral dynamics: unified approach toward QCD thermodynamics.
- Freeze out determination: accessing equilibrium baselines in BES experiment.

Access wide open on:

- Strangeness physics, e.g. B-S correlations - $A_0(T,\mu_B) \to A_0(T,\mu_B,\mu_S)$.
- Incorporating with non-equilibrium evolution study?
- Chiral and confinement phase structure beyond the onset of CEP?
- $\dots$

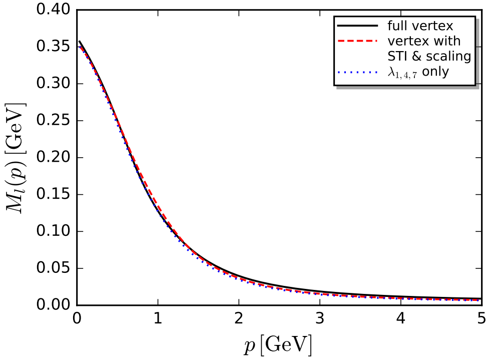
Thanks for your attention!

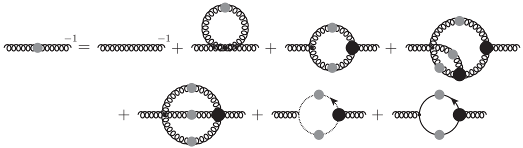
fqcd-collaboration.github.io

Back-up

Minimal truncation of Dyson-Schwinger equations

- **Minimal correlation functions**: phase-space localisation of strong interaction → high-order proper vertices are greatly suppressed - quick convergence with a non-perturb. expansion; practically sufficient up to quark-gluon vertex.
  Gao, Papavassiliou & Pawlowski, PRD 2021
- **Minimal "fluctuations"**: subtracting the in-medium truncation scheme dependence at the level of functional equations → "regularisation-free" at finite temperature and chemical potential. Gao & Pawlowski, PLB 2020, YL, Gao, Liu & Pawlowski, PRD 2024





Gluon prop. at $\boldsymbol{v} = (T, \mu)$ & $\boldsymbol{v}_0 = (0, 0)$:

$$G^{-1}_{A,\mu\nu,\boldsymbol{v}}(k) = G^{-1}_{A,\mu\nu,\boldsymbol{v}_0}(k) + \Delta\Pi^{\boldsymbol{v},\boldsymbol{v}_0}_{A;\mu\nu}(k)\,,$$

$$\Delta\Pi^{\boldsymbol{v},\boldsymbol{v}_0}_{A;\mu\nu}(k) = \Delta\Pi^{\mathrm{gauge};\boldsymbol{v},\boldsymbol{v}_0}_{A;\mu\nu}(k) + \Delta\Pi^{\mathrm{quark};\boldsymbol{v},\boldsymbol{v}_0}_{A;\mu\nu}(k).$$

Representation of the Polyakov loop potential $V(A_0)$ and its DSE

- A_0 eigenvalues (ν) and the mode potential representation:

$$V(A_0) = \sum_{\nu} V_{\text{mode}}(\nu) = \sum_{\nu_{\text{adj.}}} \left[V^{\text{gluon}}_{\text{mode}}(\nu_{\text{adj.}}) + V^{\text{ghost}}_{\text{mode}}(\nu_{\text{adj.}}) \right] + \sum_{\nu_{\text{fund.}}} V^{\text{quark}}_{\text{mode}}(\nu_{\text{fund.}}).$$

Each mode potential is given by a standard functional loop diagram calculation (w.o. A_0), with a shift on the Matsubara frequencies $\omega_n \to \omega_n + 2\pi T\nu$. Convenience in DSE:

$$\frac{\delta V(A_0)}{\delta A_0} = 0 \quad \to \quad \frac{\partial V(\varphi)}{\partial \varphi_i} = \sum_{\nu} \frac{\partial V_{\text{mode}}(\nu)}{\partial \nu} \frac{\partial \nu}{\partial \varphi_i} = 0, \quad i = 3, 8.$$

- Specifically, the full ghost 1-loop diagram:

$$\frac{\partial V^{\text{ghost}}}{\partial \nu} = -\frac{1}{T^3 \mathcal{V}_3} \operatorname{Tr} \frac{\partial S^{(2)}_{cc}}{\partial \nu} G_{cc} = -\frac{1}{T} \sum_{n \in \mathbb{Z}} \int \frac{d^3\boldsymbol{p}}{(2\pi)} \frac{n+\nu}{Z_c(x^{(\nu)})\, x^{(\nu)}}\,, \quad x^{(\nu)} = (2\pi T)^2 (n+\nu)^2 + \boldsymbol{p}^2\,.$$

with the ghost dressing function Z_c, and similar for gluon and quark.

A_0 back-coupling in the chiral dynamics?

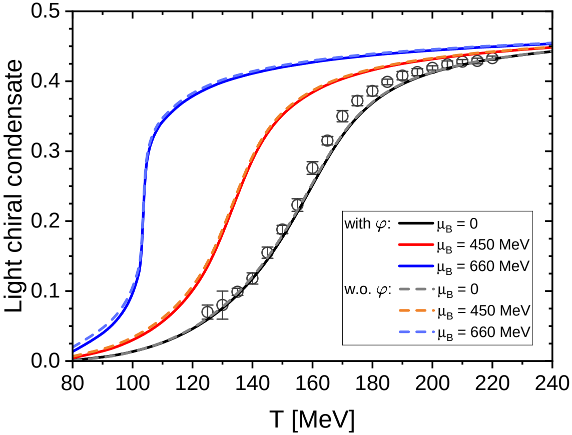
• A_0 back-coupling is seemingly small in the chiral observable:

comparsion of chiral condensate with/without A_0 background indicates that

$$M_q(A_0) \approx M_q(A_0 = 0).$$

for the quark dynamical mass (function).

• Further evidence by solving the A_0-dependent quark gap equation in combination of A_0 DSE: Wan et al. 2504.12964.

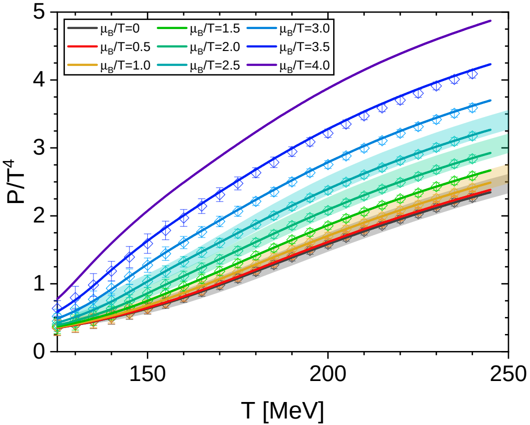


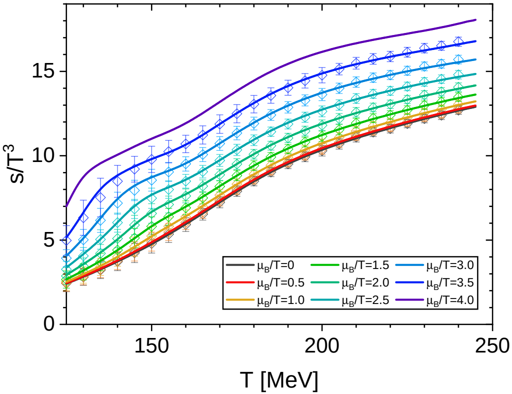
Thermodynamics and QCD equation of state

Pressure, entropy density, energy density etc.

fQCD: $P(T,\mu_B) = P(T,0) + \int_0^{\mu_B} n_B(T,\mu)\, d\mu;$ $\qquad s(T,\mu_B) = s(T,0) + \int_0^{\mu_B} \frac{\partial n_B(T,\mu)}{\partial T}\, d\mu.$

lQCD: $P(T,\mu_B) = \sum_{n=0}^{N} \frac{P_{2n}(T^{(\prime)},0)}{(2n)!}\mu_B^{2n};$ $\qquad s(T,\mu_B) = \sum_{n=0}^{N} \frac{\partial_T P_{2n}(T^{(\prime)},0)}{(2n)!}\mu_B^{2n}.$

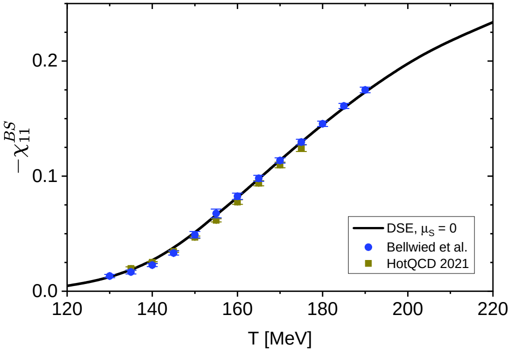


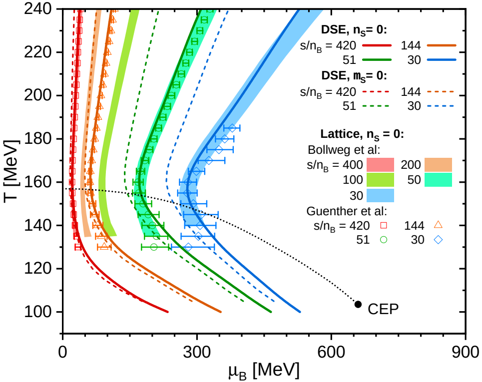


Access wide open for extended studies

Strangeness and baryon-strange correlations: $(\mu_u, \mu_d, \mu_s) = (\frac{\mu_B}{3}, \frac{\mu_B}{3}, \frac{\mu_B}{3} - \mu_S)$.

Resolution on $A_0(T, \mu_B, \mu_S, ...)$ may serve as a unified approach for probing QCD thermodynamics directly: YL, Gao, Liu, Pawlowski, in prep.





Strange hadron production in heavy-ion collision: $\text{EoS}(T, \mu_B, \mu_S)$ + pheno.

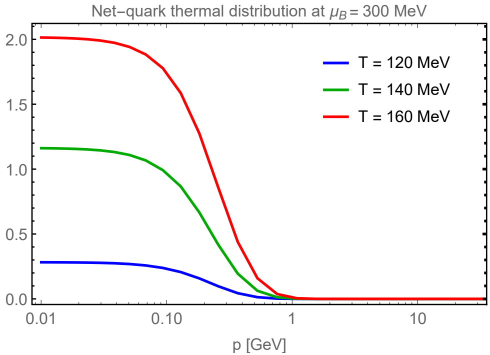
Access wide open for extended studies

Useful by-products at hand: net-quark thermal distribution given by a partial sum:

$$\langle \bar{q}\gamma_0 q\rangle = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} f_{\text{net-}q}(\boldsymbol{p}),$$

$$f_{\text{net-}q}(\boldsymbol{p}) = -T\sum_{\omega_p} \mathrm{tr}[\gamma_0 G_q(\omega_p,\boldsymbol{p})].$$

Pheno. matching: quasi-particles that reflect the QCD equilibrium baseline.



ψ: asymptotic states (experiment)

$$\boxed{\langle\psi_{\mathrm{i}}|q(x_1)\cdots\bar{q}(x_n)A_\mu(y_1)\cdots A_\nu(y_m)|\psi_{\mathrm{o}}\rangle \to f_{\psi_{\mathrm{i}}} * f_q * \cdots f_{\bar{q}} * \cdots f_{\text{gluon}} * \cdots * f_{\psi_{\mathrm{o}}}}$$

$q, \bar{q}, A_\mu$: first-principles QCD

And further $\delta f_{\text{non-eq.}}$: transport model