Study of the Quark-Meson Model with Vector Mesons within fRG

Jing Wu

Dalian University of Technology

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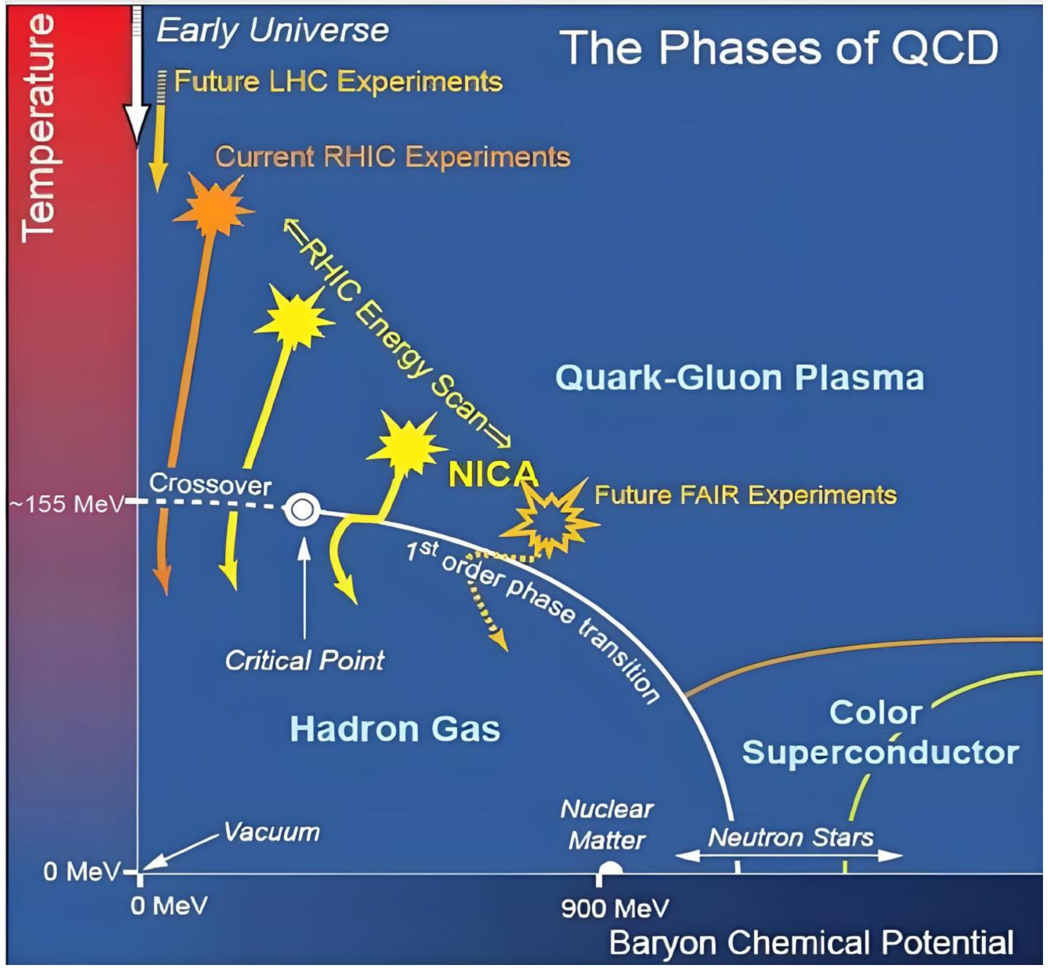
Based on

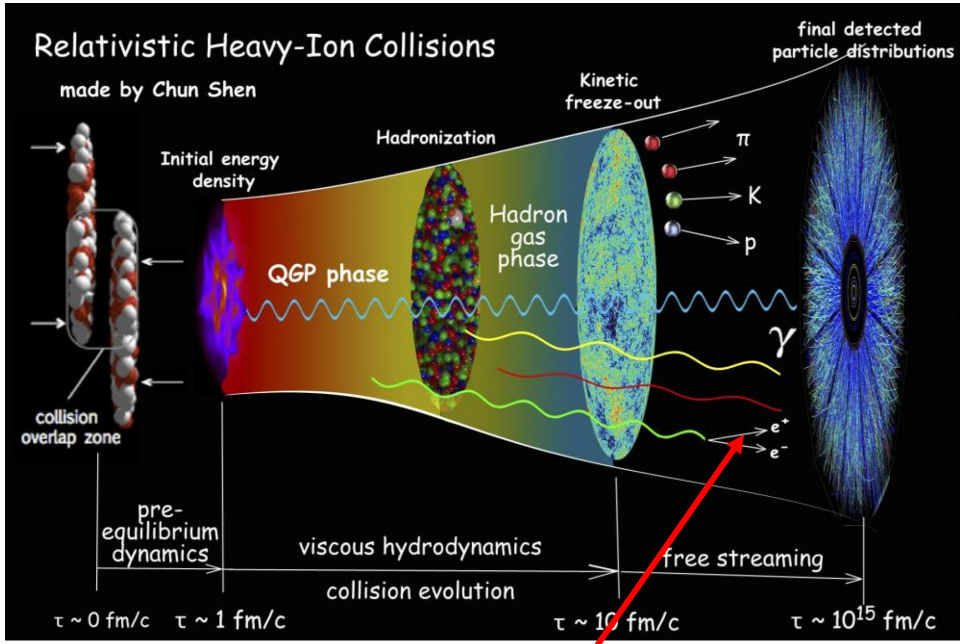
Jing Wu, Wei-jie Fu, in preparation.

Outline

- Introduction
- Construction of the Effective Action
- Numerical result
  - Meson mass
  - The spectral function of ρ meson
- Summary & Outlook

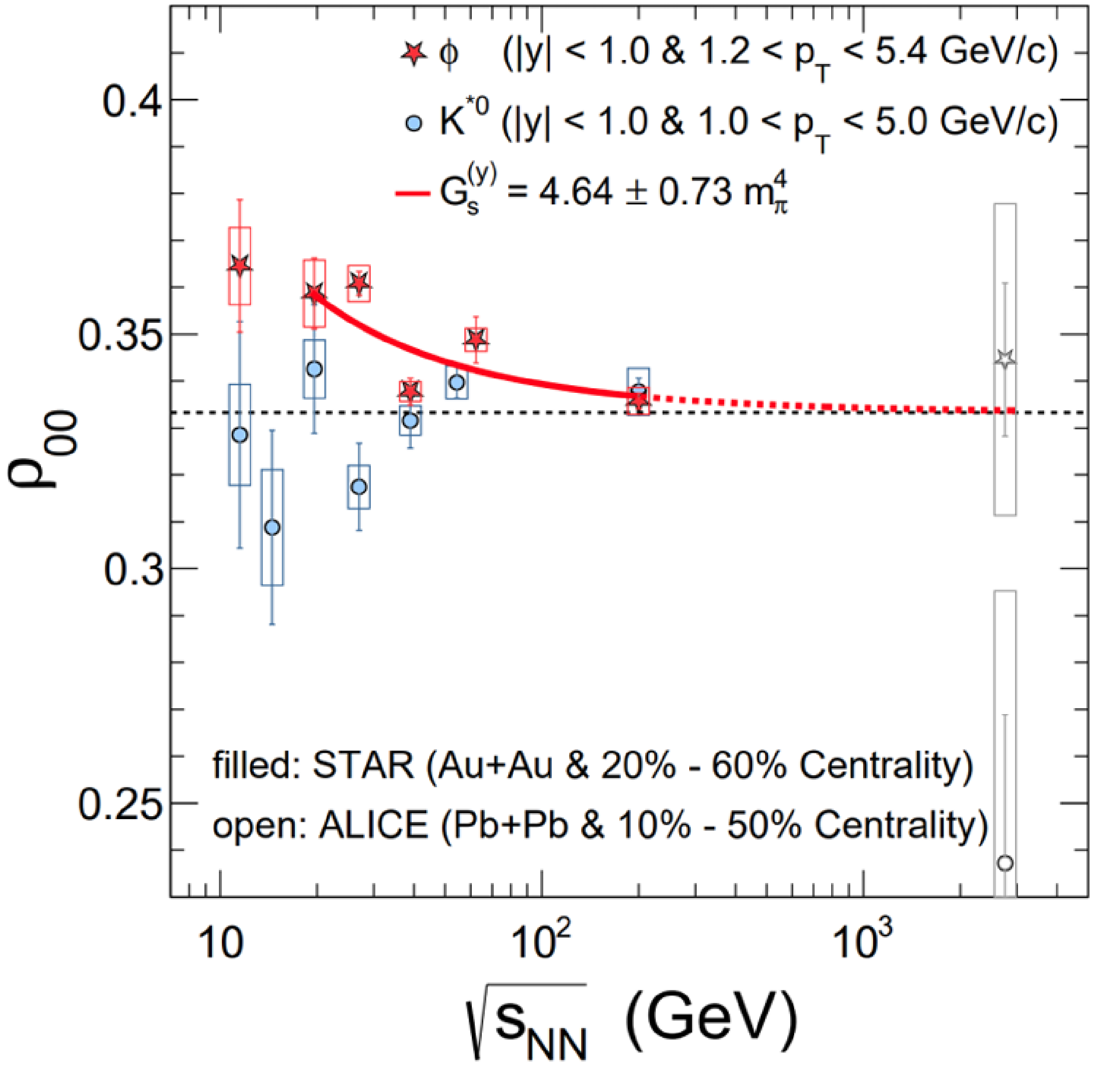
Vector Mesons as Key to Dilepton Probes





dileptons

The spin alignment of vector mesons



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Why a New Framework? Beyond Previous Models

Existing FRG studies with vector mesons

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Scalar(pseudoscalar) $\phi = (\vec{\pi}, \sigma)^T$

Vector(axial-vector) $V_\mu = \vec{\rho}_\mu \cdot \vec{T} + \vec{a}_{1\mu} \cdot \vec{T}^5$

$$(T_i)_{jk} = \begin{pmatrix} -i\epsilon_{ijk} & \vec{0} \\ \vec{0}^T & 0 \end{pmatrix} \quad T_i^5 = \begin{pmatrix} 0_{3\times 3} & -i\vec{e}_i \\ i\vec{e}_i^T & 0 \end{pmatrix}$$

- **Incomplete full meson (scalar & vector).**
- **Non–trivial extension to the (2+1) flavor case.**

Our Goal: To develop a unified and extensible Quark-Meson model within FRG that consistently incorporates **all** scalar, pseudoscalar, vector, and axial-vector mesons.

The Building Blocks: Fields and Symmetries

- Quark Fields: ψ (coupled to mesons via Yukawa interaction).

- Meson Fields:
  - Scalar & Pseudoscalar: $\Phi = T^a(\sigma_a + i\pi_a)$ (where T^a are generators of $SU(N_f)$)
  - Vector & Axial-Vector: Introduced via chiral left/right fields: $\mathrm{L}_\mu = T^b(v_{b,\mu} + a_{b,\mu})$ $\quad$ $\mathrm{R}_\mu = T^b(v_{b,\mu} - a_{b,\mu})$

- Symmetry: The action is constructed to be invariant under $SU(N_f)_L \otimes SU(N_f)_R$ chiral transformations.

Construction of effective action

$$\Gamma_k = \int_0^\beta d^4x \Big[\bar{\psi}(\partial^\mu \gamma_\mu - \mu\gamma_0 + h_s\Phi_5 + i h_v \gamma_\mu V_5^\mu)\psi + \tilde{U}_k[\Phi] + \frac{1}{4}\mathrm{Tr}\left[D_\mu \Phi D^\mu \Phi^\dagger\right]$$
$$+ \frac{1}{16}\mathrm{Tr}\left[L_{\mu\nu}^2 + R_{\mu\nu}^2\right] + \frac{1}{8} m_{k,V}^2 \mathrm{Tr}\left[L_\mu^2 + R_\mu^2\right] + \frac{1}{4}\Delta m_{k,V}^2 (\mathrm{Det}[L_\mu] + \mathrm{Det}[R_\mu]) \Big]$$

Chiral transformation

$\psi_L \to \hat{L}\psi_L \quad \psi_R \to \hat{R}\psi_R$

$\Phi \to \hat{L}\Phi\hat{R}^\dagger \quad \mathrm{L}_\mu \to \hat{L}\mathrm{L}_\mu\hat{L}^\dagger \quad \mathrm{R}_\mu \to \hat{R}\mathrm{R}_\mu\hat{R}^\dagger$

$\Phi_5 = T^a(\sigma_a + i\gamma_5\pi_a) \quad \mathrm{V}_5^\mu = T^b(v_b^\mu + \gamma_5 a_b^\mu)$

$\mathrm{L}_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu \quad \mathrm{R}_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu$

$D_\mu\Phi = \partial_\mu\Phi + \mathrm{ig}(\Phi R_\mu - \mathrm{L}_\mu\Phi)$

- Chiral symmetry
- The covariant derivative D_μ naturally introduces couplings between the vector (axial-vector) and scalar (pseudoscalar) fields.

Two flavor case

$$\Phi = (\sigma + i\eta)I + (\vec{a}_0 + i\vec{\pi})\cdot\vec{\tau}$$
$$L_\mu = (\omega_\mu + f_{1N,\mu})I + (\vec{\rho}_\mu + \vec{a}_{1\mu})\cdot\vec{\tau}$$
$$R_\mu = (\omega_\mu - f_{1N,\mu})I + (\vec{\rho}_\mu - \vec{a}_{1\mu})\cdot\vec{\tau}$$

includes all mesons

Explicit breaking chiral symmetry

$$\widetilde{U}_k(\Phi) = U_k(\rho_1,\rho_2) + \frac{d_k}{8}\mathrm{Tr}\left[\left(\Phi^\dagger\Phi - \frac{1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\right)^2\right] - c\sigma$$

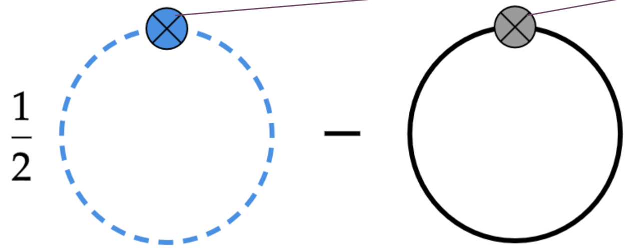
$$\rho_1 = \frac{1}{2}(\sigma^2 + \vec{\pi}^2) \qquad \rho_2 = \frac{1}{2}(\eta^2 + \vec{a}_0^2)$$

$$m_\eta^2 = \frac{\partial U_k}{\partial \rho_2}$$
$$m_{a_0}^2 = \frac{\partial U_k}{\partial \rho_2} + d_k\sigma^2$$

Flow Equation

3D regulator:

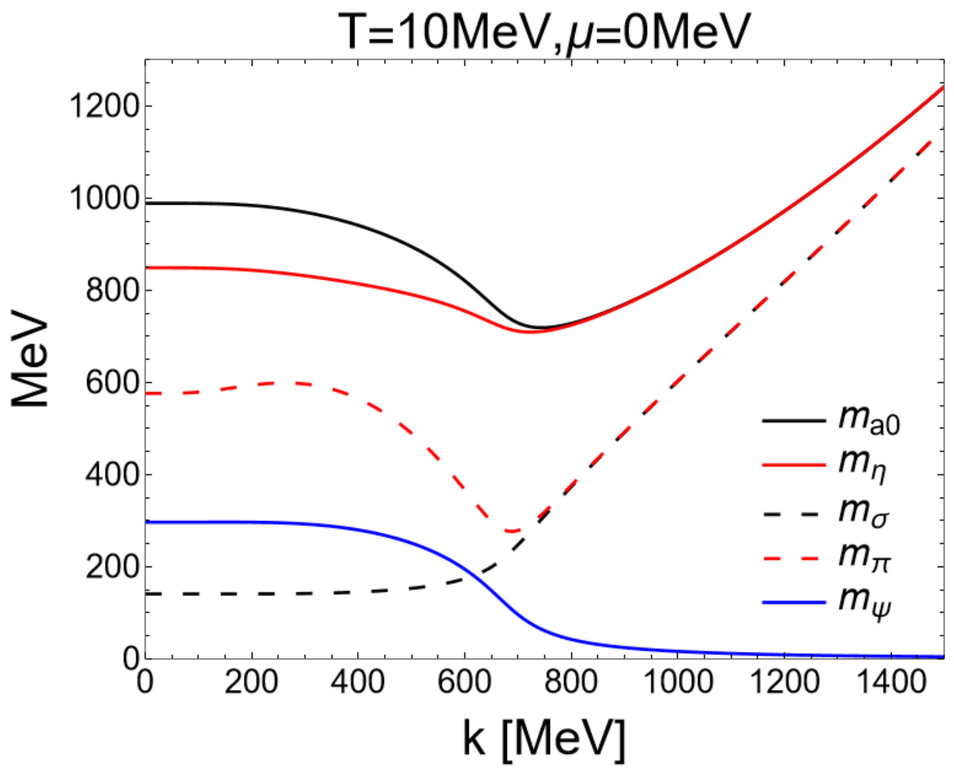
$$\partial_t \widetilde{U}_k(\Phi) =$$

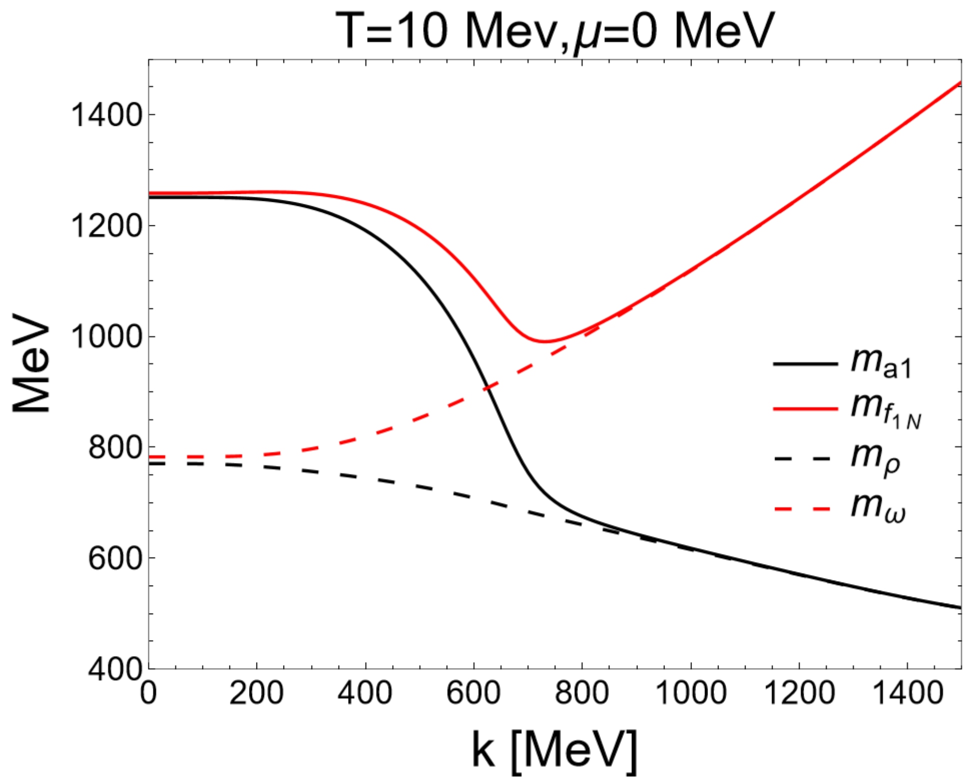


$$R_k^\phi(\vec{q}) = (k^2 - \vec{q}^2)\theta(k^2 - \vec{q}^2)$$
$$R_k^\psi(\vec{q}) = \not{\vec{q}}\left(\sqrt{\frac{k^2}{\vec{q}^2}} - 1\right)\theta(k^2 - \vec{q}^2)$$

$t = \ln\frac{k}{\Lambda}$ is the FRG time

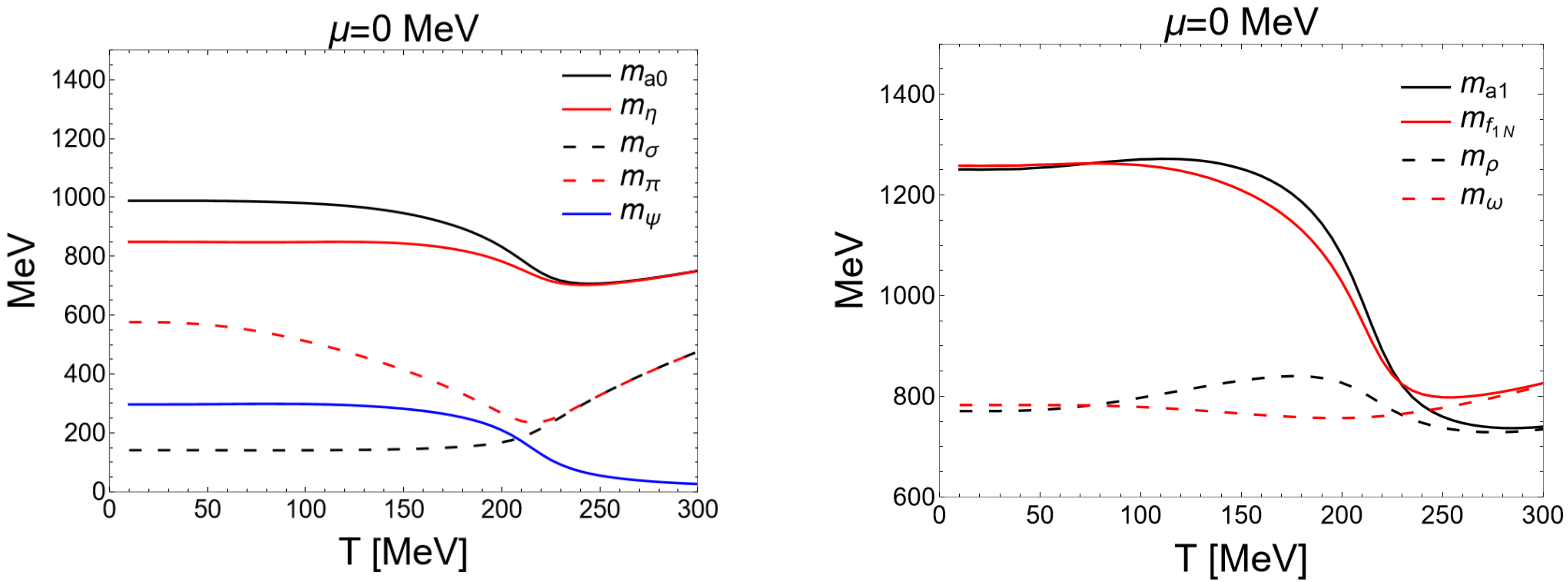
Scale-dependent Masses





- At the UV cutoff $\Lambda = 1500$ MeV, the masses of the chiral partners(ρ and a_1, σ and π etc) are degenerate. The effective mass of quark approaches zero.
- As the FRG scale is lowered, the masses of chiral partners split, and the effective mass of quark increases.

Temperature-dependent Masses



- Analogous to its dependence on k, chiral symmetry is restored at high temperature, where the masses of chiral partner mesons become degenerate and the effective quark mass approaches zero.

The spectral function of ρ meson

Spectral function $$\rho(\omega,\vec{p}) = 2ImD_R(\omega,\vec{p}) = -2\frac{Im\Gamma_R^{(2)}(\omega,\vec{p})}{(Im\Gamma_R^{(2)}(\omega,\vec{p}))^2 + (Re\Gamma_R^{(2)}(\omega,\vec{p}))^2}$$

Analytic continuation $$\Gamma_R^{(2)}(\omega,\vec{p}) = \lim_{\epsilon\to 0^+}\Gamma^{(2)}(p_0 = -i(\omega+i\epsilon),\vec{p})$$

Magnetic Component: $$\Gamma^{(2),\perp}_{\rho_\mu\rho_\nu}(p) = \frac{1}{2}\Gamma^{(2)}_{\rho_\mu\rho_\nu}(p)\Pi^{T,\perp}_{\mu\nu}(p)$$

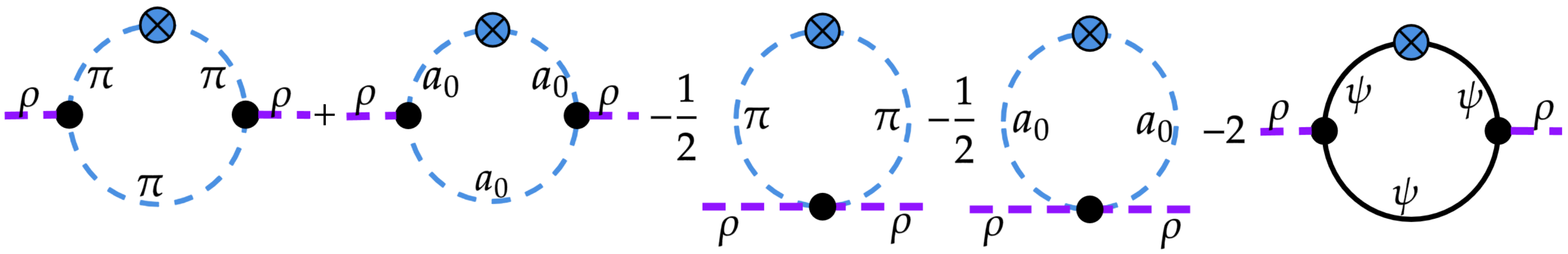
Electric Component: $$\Gamma^{(2),\parallel}_{\rho_\mu\rho_\nu}(p) = \Gamma^{(2)}_{\rho_\mu\rho_\nu}(p)\Pi^{T,\parallel}_{\mu\nu}(p)$$

Projector:

$$\Pi^{T,\perp}_{\mu\nu}(p) = \begin{cases} 0 & \text{if } \mu = 0 \text{ or } \nu = 0 \\ \delta_{\mu\nu} - \frac{p_\mu p_\nu}{\vec{p}^2} & \text{else} \end{cases},$$

$$\Pi^{T,\parallel}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} - \Pi^T_{\mu\nu}(p)\,,$$

$$\partial_t\Gamma^{(2)}_{\rho\rho} =$$



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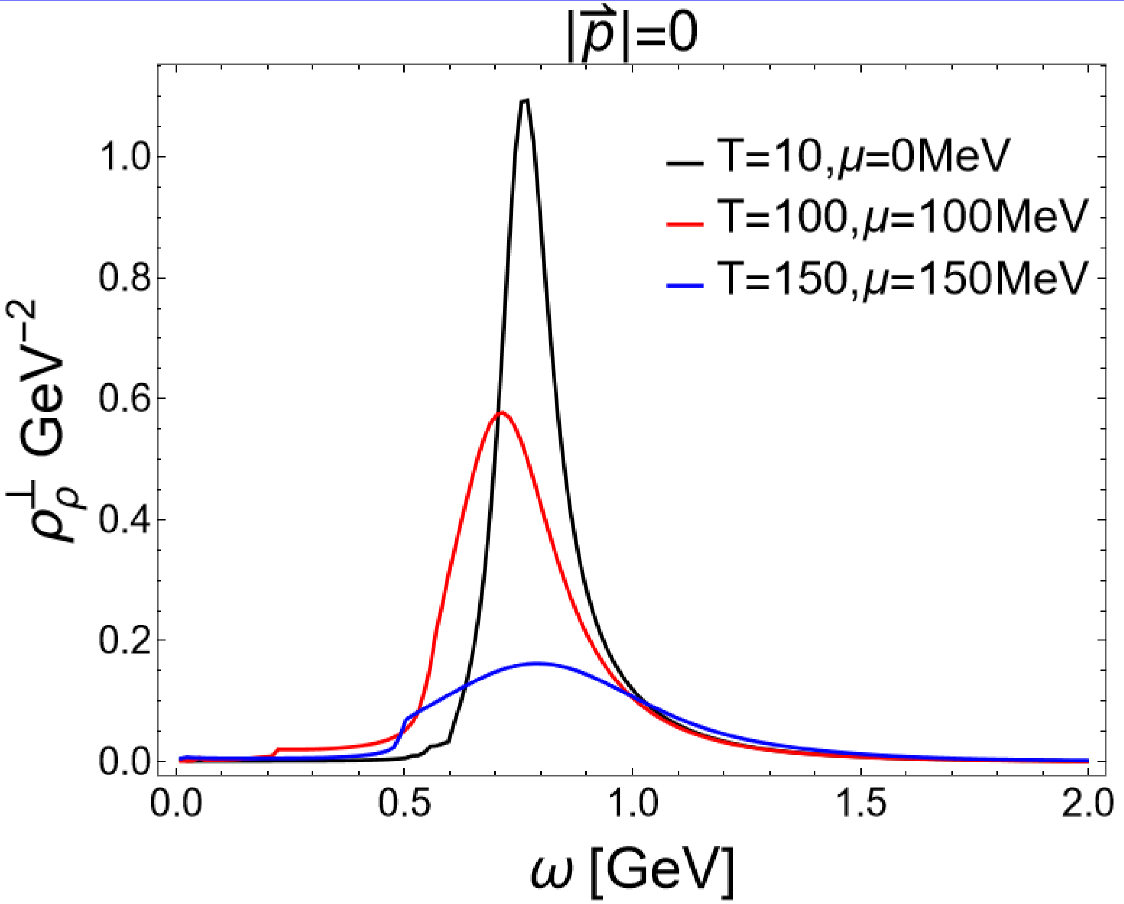

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The spectral function is **significantly broadened** with increasing temperature and chemical potential.

Summary&outlook

- We have developed a new FRG framework for the Quark-Meson model that includes vector mesons.

- We calculated ρ Meson Spectral Function in the Two-Flavor Case. The spectral function exhibits significant broadening with increasing temperature and chemical potential.

- In future work, we plan to apply this framework to study dilepton and Vector Meson spin alignment.

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Thank you very much for your attentions!

Back up

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