Hydrodynamization Time Hierarchies Across n-Point Functions

Navid Abbasi

26 Oct, 2025

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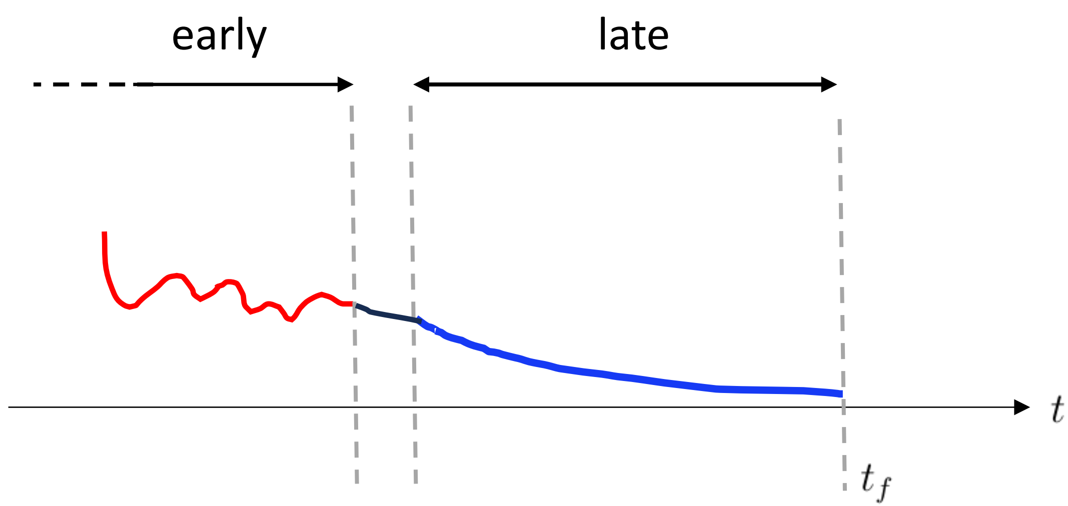

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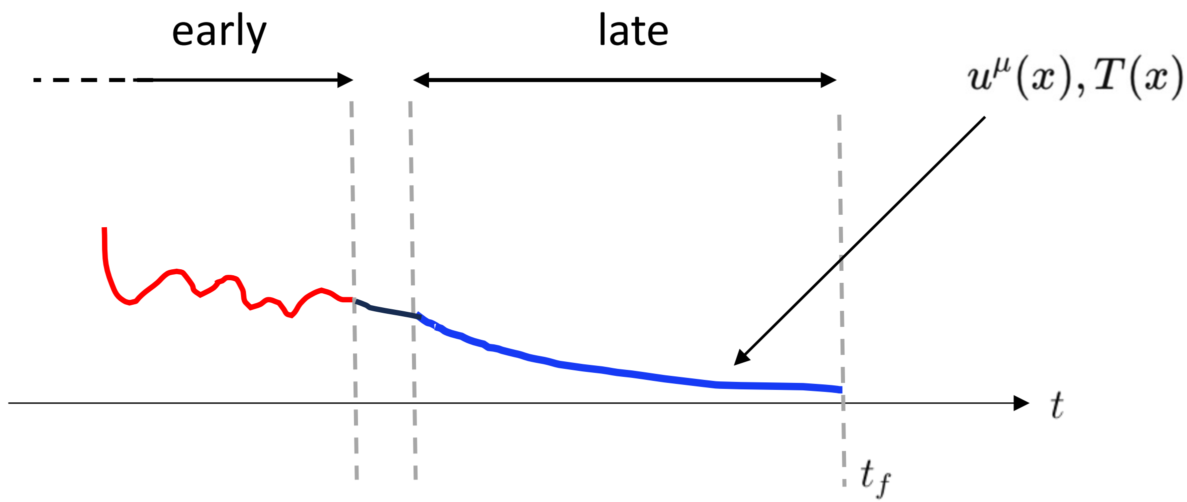

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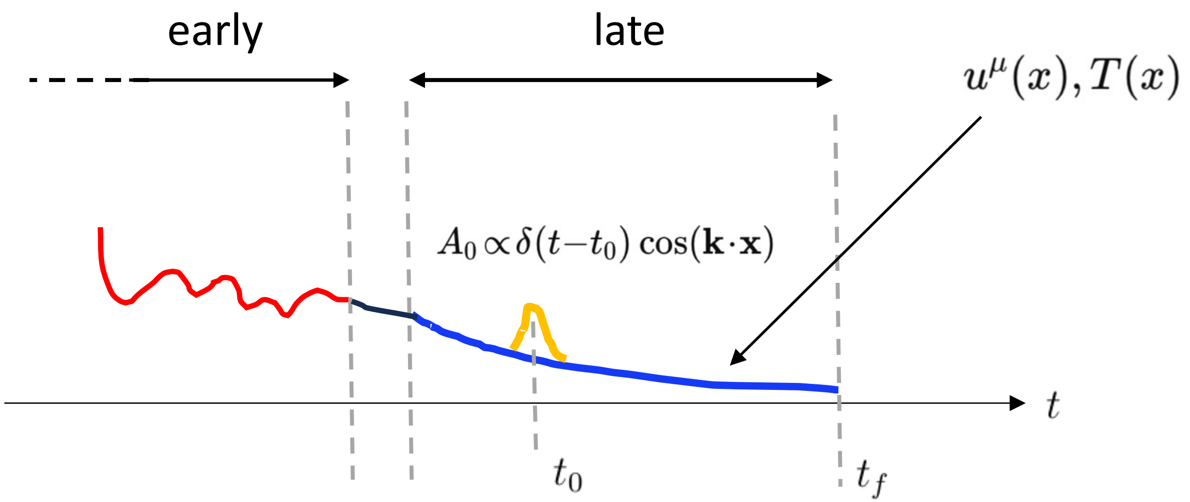

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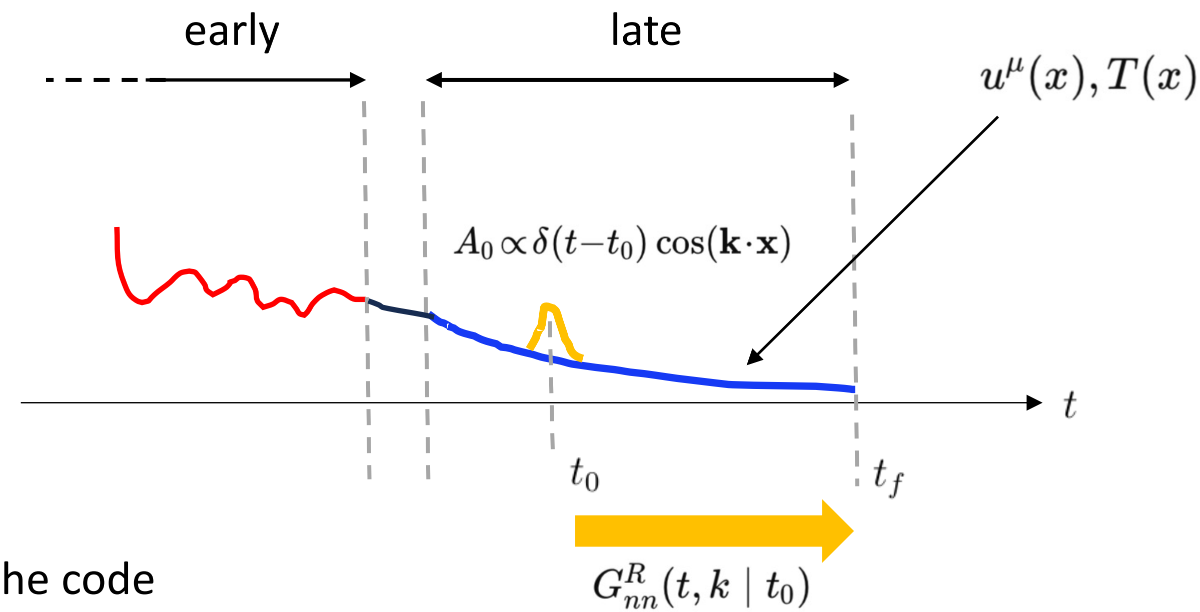

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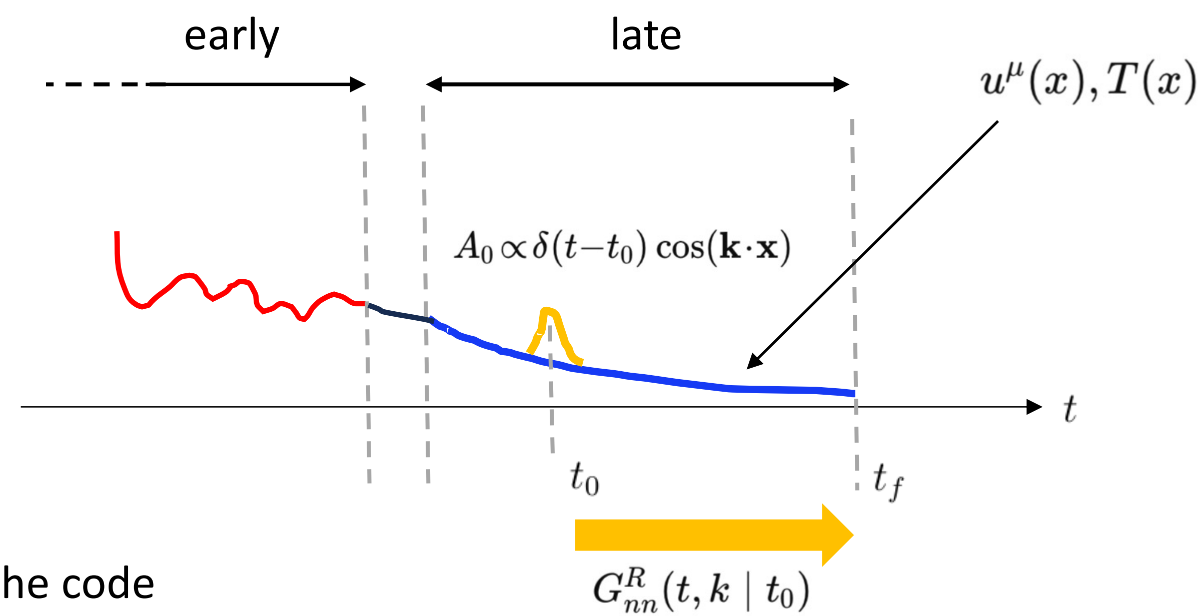

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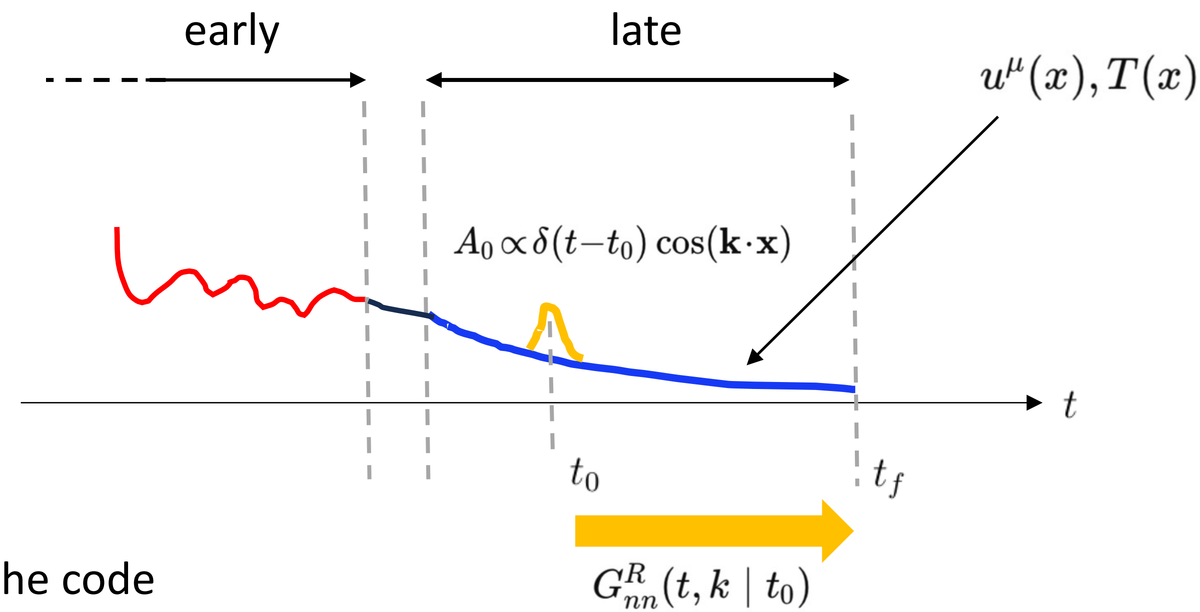

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memory

hydro build-up

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- We propose **two conjectures** about the late-time dynamics of correlators in

  large-N, strongly coupling quantum field theories

- We test our conjectures in a **SK-EFT** which describes a class of such theories.

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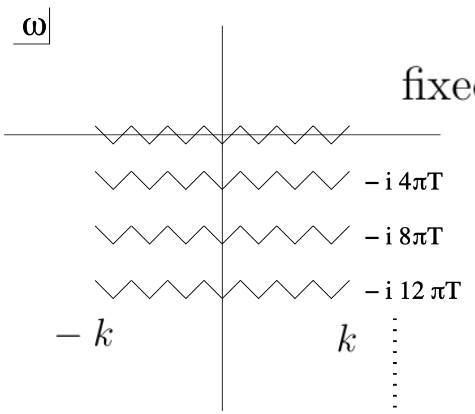
$\lambda = g_{\mathrm{YM}}^2 N \qquad \lambda = 0$



$$G(t,\mathbf{k}) \to \frac{\sin\left(2|\mathbf{k}|t\right)}{t^{\gamma}}$$

[Hartnoll, Kumar JHEP (2005)]
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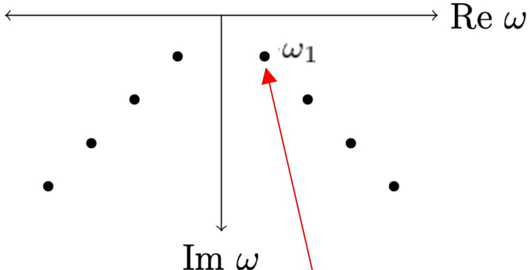
2) N=4 SYM at Strong coupling

$\lambda \to +\infty$



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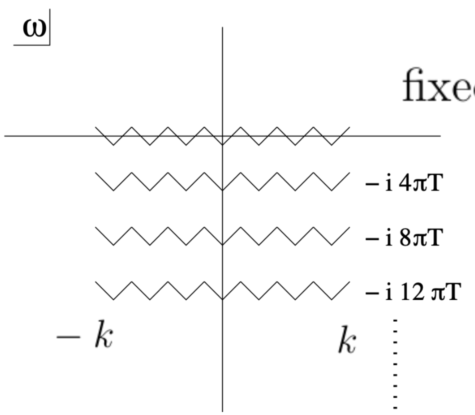

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No decay → no thermalization!

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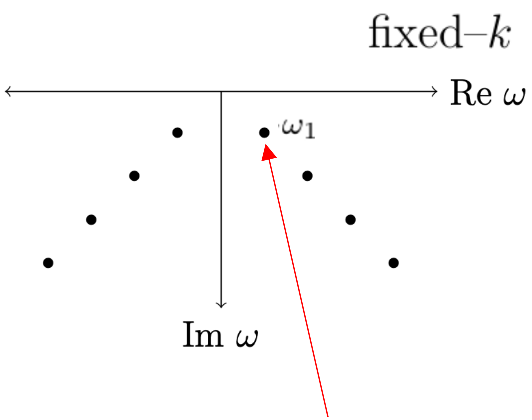

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Exponential decay → fast thermalization!

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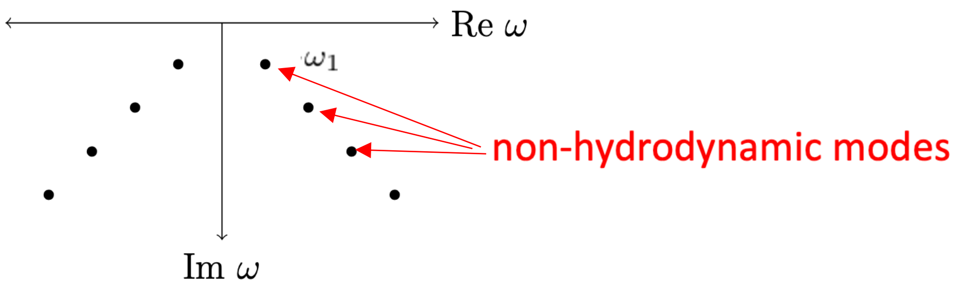

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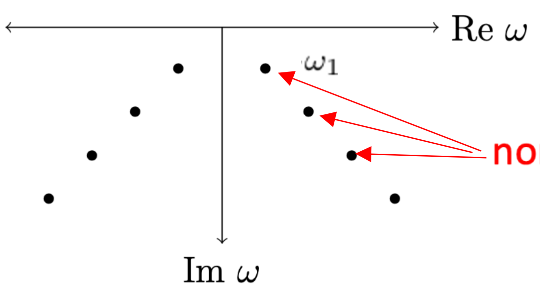

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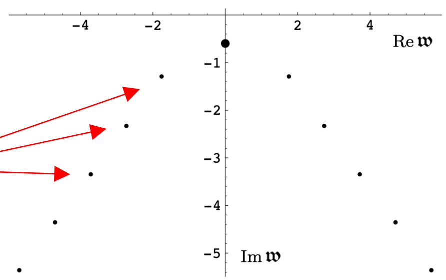

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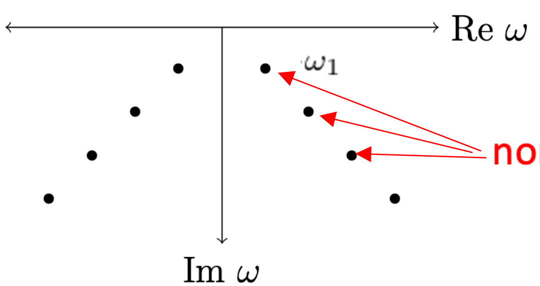

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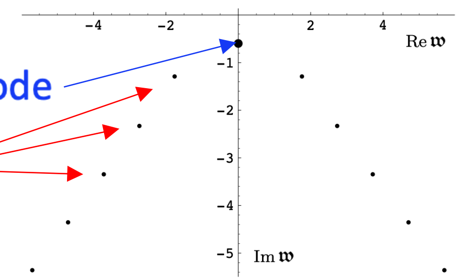

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How long does it take?

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The time at which:
correlation function from micro → hydrodynamic correlation function:

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- More precisely:

$$\boxed{t_h(k) = \min\left\{t:\ \forall t' \geq t,\ \frac{|D_{\text{inst}}(t',k) - D|}{D} < \eta \ \text{ and } \ \left|\frac{G(t',k)}{e^{-Dk^2 t'}} - 1\right| < \delta\right\}}$$

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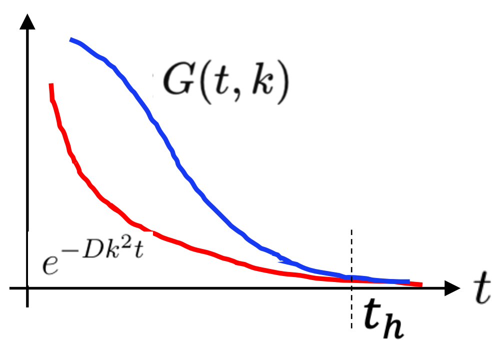
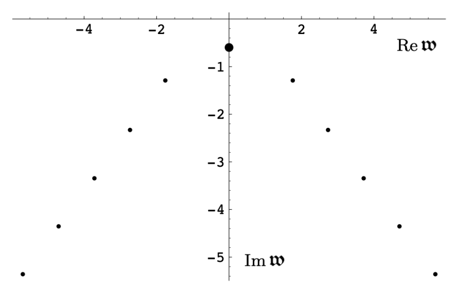
- For 2pt functions in holography

  t_h can be found numerically





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There is no clear intuition!

This question sets the stage for our main conjectures.

7. What we explore

Symmetrized *n*-point function in momentum space

$$\boxed{G_{\underbrace{r,\cdots,r}_{n}}(t_{n-1},\ldots,t_1,0;\mathbf{k}_1,\cdots,\mathbf{k}_n) \equiv \int d\omega_1\ldots d\omega_{n-1}\; G_{\underbrace{r,\cdots,r}_{n}}(\omega_1,\cdots,\omega_n;\mathbf{k}_1,\cdots,\mathbf{k}_n)\; e^{-i(\omega_1 t_1+\cdots+\omega_n t_n)}}$$

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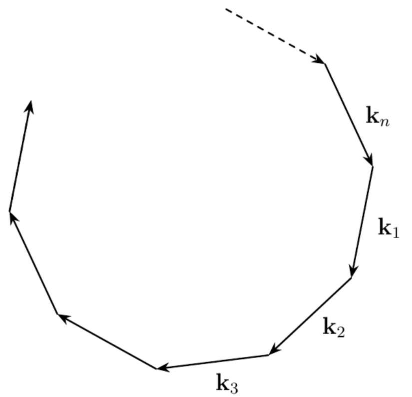

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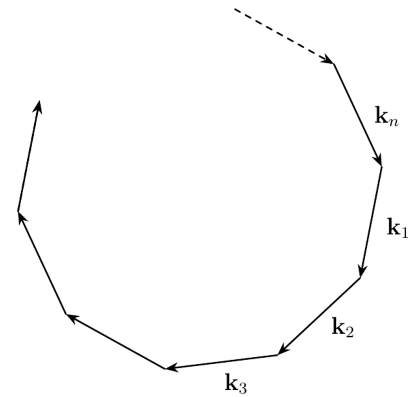

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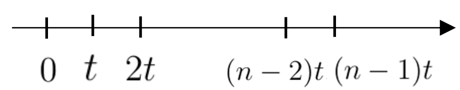

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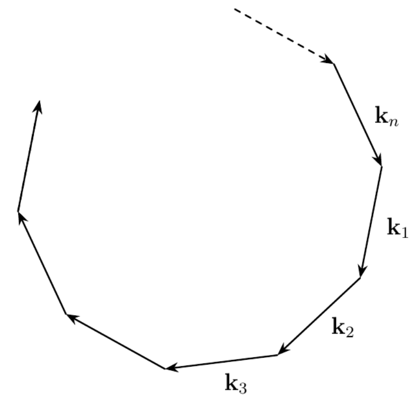

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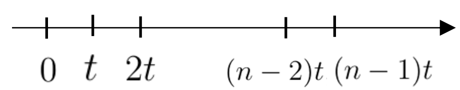

- The object we study:

$$\boxed{G_{\underbrace{r,\cdots,r}_{n}}\Big((n-1)t,(n-2)t,\cdots,0,k\Big)}$$

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Claim: In a holographic system, each leg contributes a factor

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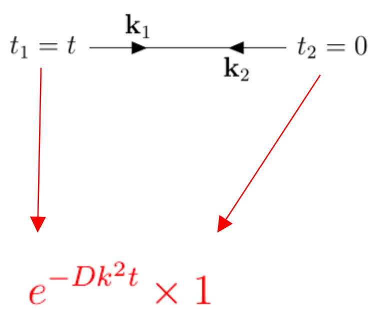

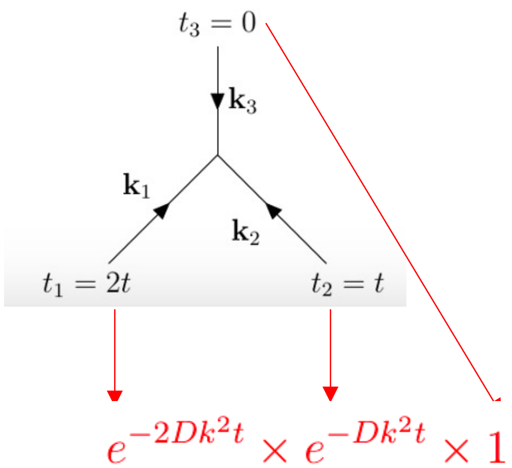

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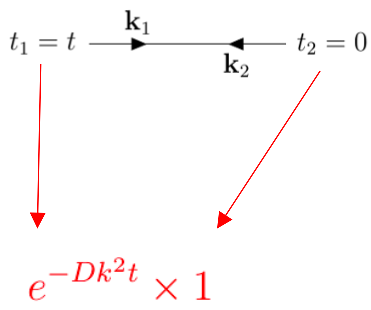

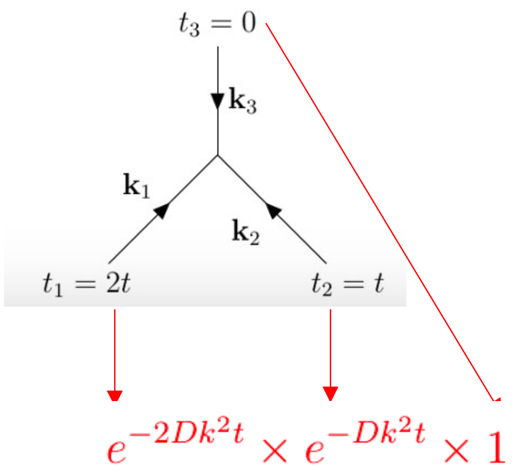

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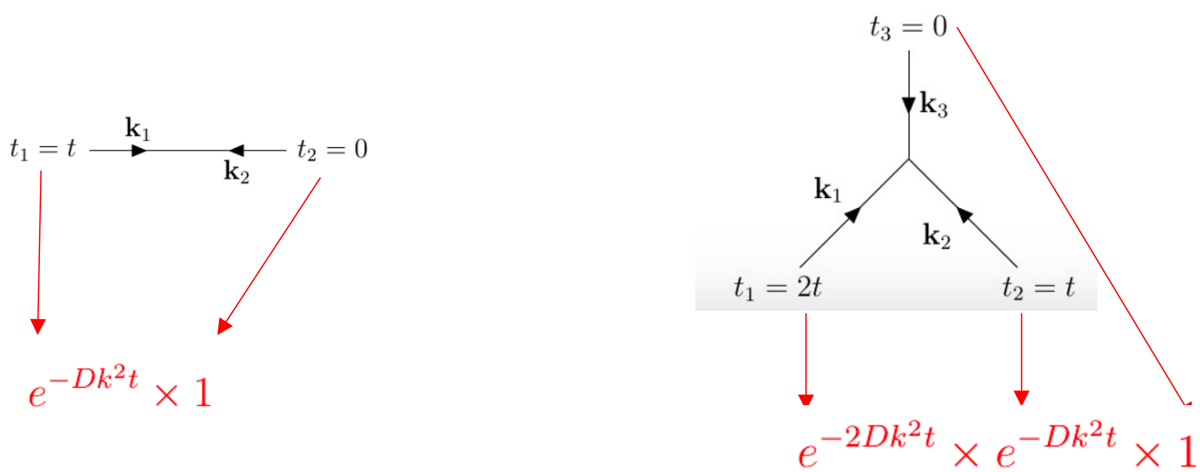

- In general, we sum the ladder: $(0 + t + 2t + \cdots + (n-1)t)$
- In large-N strongly coupled systems, the late time envelope is

$$\boxed{G_{\underbrace{r,\ldots,r}_{n}}\big((n-1)t,\ (n-2)t, \ldots, 0;\ k\big) \ \sim\ \exp\Big[-\ S_n\, Dk^2\, t\Big], \qquad S_n = \frac{n(n-1)}{2}}$$

$S_n :\ 1, 3, 6, 10, 15\ (n = 2, 3, 4, 5, 6)$

Envelope only; model details enter subleading prefactors.

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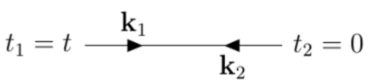
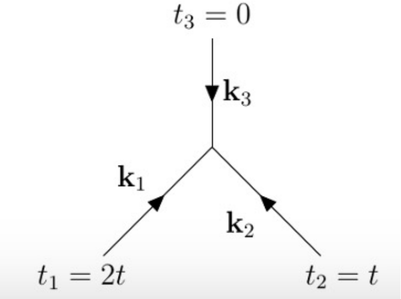
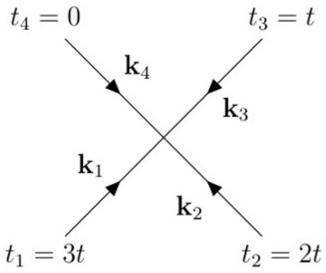
Hydrodynamization happens once the **shortest propagation time** t is hydro.







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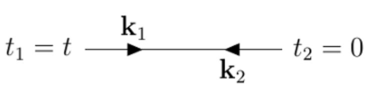

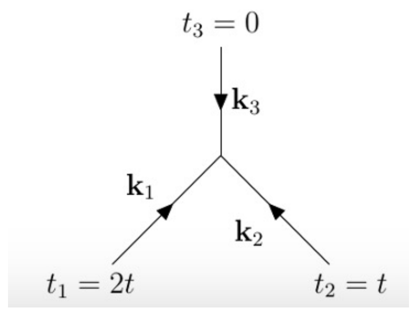

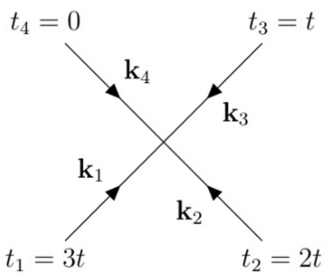

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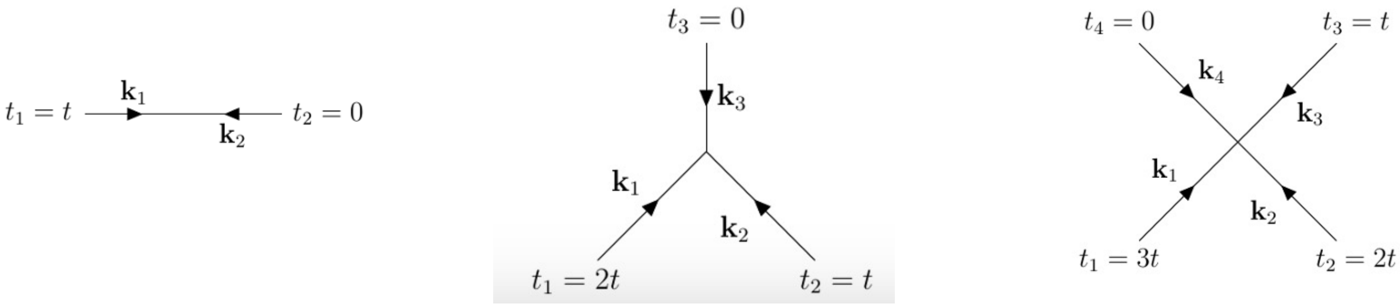

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This suggests:

- In large-N strongly coupled systems, hydro-dynamization time for $G_{\underbrace{r,\cdots,r}_{n}}\big((n-1)t,(n-2)t,\cdots,t,0,k\big)$ doesn't depend on n

$$\boxed{t_h^{(n)}(k) \approx t_h^{(2)}(k) \quad (\text{large } N)}$$

Assumptions: large-N, pole-dominated spectrum, equilateral k

10. Testing pipeline

Goal: measure full time-domain 2-pt and 3-pt in a microscopic theory.

Status in holography: time-domain connected $\langle nnn \rangle$ has not yet been computed

→ we use a controllable kinetic model first.

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Our microscopic model:

Interacting Effective field theory derived from kinetic theory in RTA

- RTA → SK-EFT
- Compute G_{rr}, G_{rrr}
- Verify KMS
- Extract $t_h^{(2)}, t_h^{(n)}$

11. Analytic laboratory: RTA with constant τ

- Kinetic theory in RTA with a constant tau

$$\boxed{p^\nu \partial_\nu f(t,\mathbf{x},\mathbf{p}) + F^\alpha \nabla^{(p)}_\alpha f(t,\mathbf{x},\mathbf{p}) = \frac{p^\alpha u_\alpha}{\tau_D}\left(f - f_{\mathrm{eq}}\right)}$$

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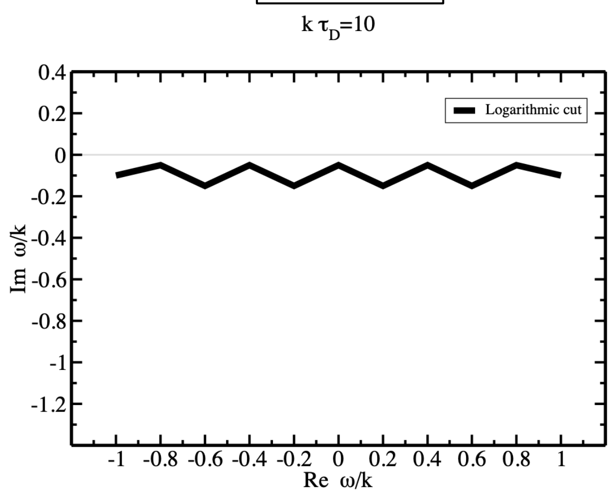
- Calculating retarded Green's function

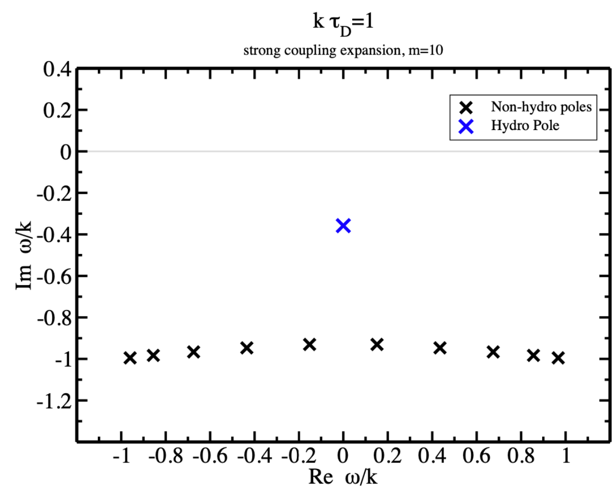
$$G_J^{0,0}(\omega,k) = -\chi \frac{1 + \left(i\omega - \frac{1}{\tau_D}\right)\frac{1}{2ik}\ln\left(\frac{\omega - k + \frac{i}{\tau_D}}{\omega + k + \frac{i}{\tau_D}}\right)}{1 - \frac{1}{2ik\tau_D}\ln\left(\frac{\omega - k + \frac{i}{\tau_D}}{\omega + k + \frac{i}{\tau_D}}\right)}$$

$$\boxed{\tau_D T \gg 1}$$



Weak coupling

$$\boxed{\tau_D T \to 0}$$



Strong coupling

[Romatschke Eur.Phys.J.C (2016)]

12. RTA -> SK-EFT

- Boltzmann eq. in RTA:

$$\mathbf{D}f = -\frac{f - f^{(0)}}{\tau}$$

$$\frac{\mathbf{D}}{1+\tau\mathbf{D}} f^{(0)} = -\frac{f - f^{(0)}}{\tau}$$

$$\boxed{\mathbf{D} = \partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla}}$$

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[NA, Rischke JHEP (2025)]

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- All-order diffusion eq.: going beyond Fick's law

$$\left[\underbrace{\left(\tilde{\partial}_t - \frac{1}{3}\tilde{\boldsymbol{\nabla}}^2\right)}_{\text{Fick's law}} + \left(-\tilde{\partial}_t^2 + \tilde{\partial}_t\tilde{\boldsymbol{\nabla}}^2 - \frac{1}{5}\tilde{\boldsymbol{\nabla}}^4\right) + \left(\tilde{\partial}_t^3 - 2\tilde{\partial}_t^2\tilde{\boldsymbol{\nabla}}^2 + \tilde{\partial}_t\tilde{\boldsymbol{\nabla}}^4 - \frac{1}{7}\tilde{\boldsymbol{\nabla}}^6\right) + \dots\right] n(x) = 0$$

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- Connection to SK-EFT:

$$\boxed{\mathcal{L}_{\text{EFT}}[\varphi_r,\varphi_a]_{A=0} = \underbrace{E[\varphi_r]}_{\text{deterministic hydro}}\varphi_a + \varphi_a \underbrace{F[\varphi_r,\partial]}_{\text{fixed by KMS}}\varphi_a + \mathcal{O}(\varphi_a^3)}$$

$$E[\varphi_r] \equiv \partial_\mu J^\mu[\varphi_r] = 0$$

[Crossley, Glorioso, Liu]
[Jensen, Pinzani-Fokeeva, Yarom]
[Heahl, Loganayagam, Rangamani]
[Grozdanov, Polonyi PRD (2015)]

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13. Correlation functions

- KMS fixed Lagrangian

$$\boxed{\mathcal{L}^{\text{free}}[n,\phi_a] = \int_\Omega \left[(\mathbf{D}\phi_a) \frac{1}{1+\tau\mathbf{D}} n + iT\sigma\ (\mathbf{D}\phi_a) \left(\frac{1}{1+\tau\mathbf{D}} \right)_\Theta \mathbf{D}\phi_a \right]}$$

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- Standard techniques:

$$\begin{aligned} G^S_{J^0 J^0}(p,-p) &= \frac{1}{i^2}\frac{\delta^2 \ln Z}{\delta A_{a0}(-p)\delta A_{a0}(p)} \\ &= -2\chi T\tau + \frac{\chi T}{2k}\frac{4\tau k - i\ln\left(\frac{\frac{i}{\tau}-\omega+k}{\frac{i}{\tau}-\omega-k}\right) + i\ln\left(\frac{\frac{i}{\tau}+\omega-k}{\frac{i}{\tau}+\omega+k}\right)}{\left[1+\frac{1}{2i\tau k}\ln\left(\frac{\frac{i}{\tau}-\omega+k}{\frac{i}{\tau}-\omega-k}\right)\right]\left[1-\frac{1}{2i\tau k}\ln\left(\frac{\frac{i}{\tau}+\omega-k}{\frac{i}{\tau}+\omega+k}\right)\right]} \end{aligned}$$

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[NA, Rischke JHEP (2025)]

- FD theorem $G^S_{J^0J^0}(\omega,\mathbf{k}) = -\frac{2T}{\omega}\,\mathrm{Im}\,G^R_{J^0J^0}(\omega,\mathbf{k}) \rightarrow$ reproduces Romatschke

[Romatschke Eur.Phys.J.C (2016)]

- Next step: 3-pt!

14. Beyond linear

- Wilson idea: all coefficients depend on , expand:

$$\delta n(\mu) = \chi\,\delta\mu + \chi'\frac{\delta\mu^2}{2} + \ldots\,, \qquad \tau(\mu) = \tau + \tau'\,\delta\mu + \ldots\,, \quad \sigma(\mu) = \sigma + \sigma'\,\delta\mu + \ldots$$

- KMS:

$$\boxed{\sigma = \chi T, \qquad \sigma' = \chi' T + \chi\,\tau'}$$

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- Result: **Cubic SK-EFT enables 3-pt**

$$\boxed{\mathcal{L}^{(3)} = \int_\Omega \left[-\frac{\tau'}{\chi} n \left(\frac{\mathbf{D}}{1+\tau\mathbf{D}} n \right) \frac{\mathbf{D}}{1-\tau\mathbf{D}}\phi_a + iT\frac{\sigma'}{\chi}\, n \left(\frac{\mathbf{D}}{1+\tau\mathbf{D}}\phi_a \right) \frac{\mathbf{D}}{1-\tau\mathbf{D}}\phi_a \right]}$$

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- Symmetrized 3pt

  (lengthy result!)

$$G_{rrr}(p_1, p_2) = \frac{1}{i^3} \frac{\delta^3 \ln Z}{\delta A_{a0}(-p_1)\delta A_{a0}(-p_2)\delta A_{a0}(-p_3)}$$

[NA, Rischke JHEP (2025)]

[Wang , Heinz PRD (2002)]

15. Final results: testing the conjectures

- We perform invers Fourier analysis on our EFT results:

$G_{rr}(t,k)$

$G_{rrr}(2t,t,k)$

4

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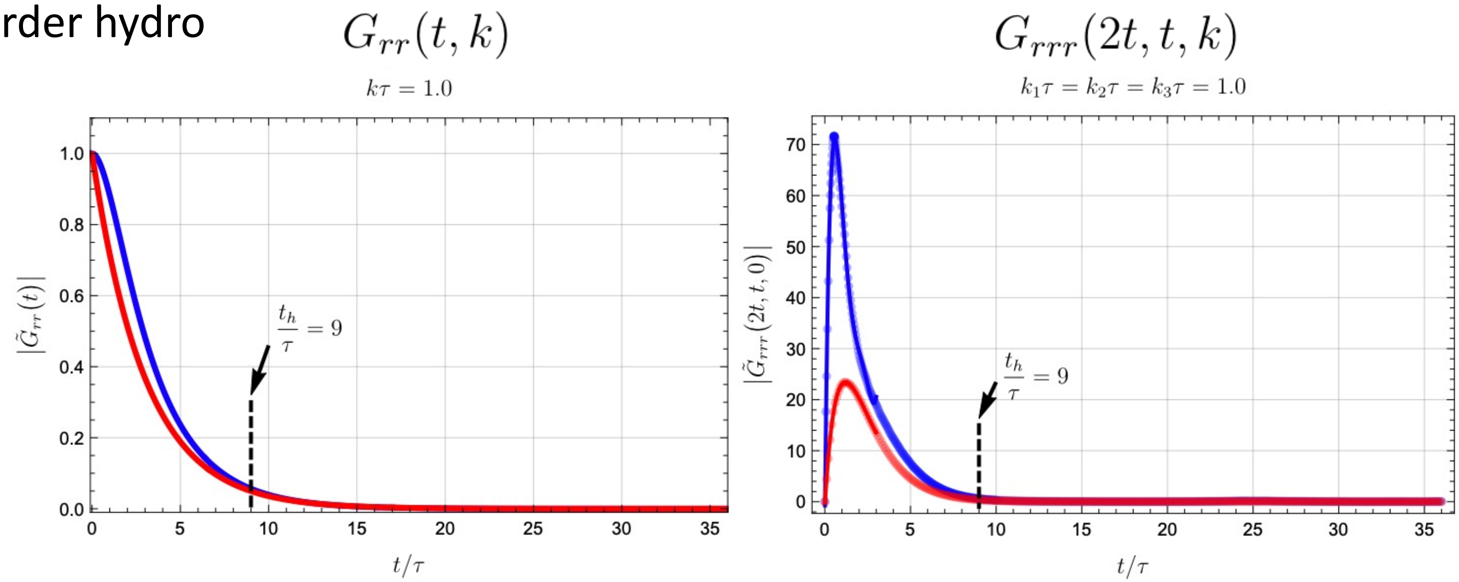
- We perform invers Fourier analysis on our EFT results:

Blue: All order EFT

Red: first-order hydro



- <u>TDL</u>:

  Holds!

[ongoing]

4

- <u>HTI</u>: $\boxed{t_h^{(2)}(k) \;=\; t_h^{(3)}(k) \qquad \text{(within numerical error)}}$

  hydrodynamization time is universal

16. Conclusion

In large-N systems whose late-time dynamics are dominated by a small number of long-lived modes:

- **TDL:** $S_n = \frac{n(n-1)}{2}$ controls the late-time envelope.

- **HTI:** $t_h^{(n)} \approx t_h^{(2)}$ ⇒ simulate **2-pt** to calibrate the **hierarchy**.

17. What about expanding backgrounds

Can these conjectures be generalized to expanding backgrounds?

[Heller, Janik, Witaszczyk PRL (2013)]
[Akamatsu, Mazeliauskas, Teaney PRC (2017)]

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In Bjorken flow, we naturally have **two competing time scales**:

1. $\tau_{\text{exp}} \sim \tau$ the expansion time-scale → $\theta \sim \frac{1}{\tau}$
2. $\tau_{\text{diff}} \sim \frac{1}{Dk^2}$ Diffusion time → $\Gamma_k \sim Dk^2$

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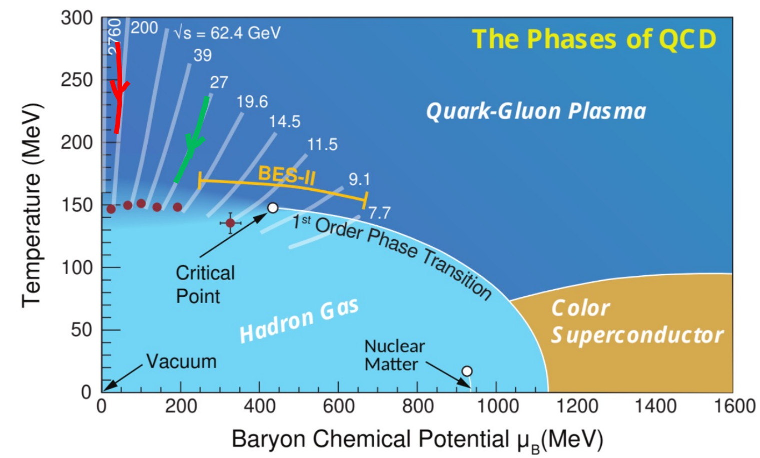
- Away from the Critical Point: $\Gamma_k \gg \theta$,

  Conjectures hold

- Near the QCD CEP: $\Gamma_k \ll \theta$,

  Need Hydro+ or quasi-diffusion

[Grozdanov, Lucas, Poovuttikul PRD (2019)]
[Stephanov, Yin PRD (2019)]
[NA, Kaminski, Rischke 2506.20500]



[BEST, Nucl.Phys.A (2022)]

Thank you for your attention!