The 16th Workshop on QCD Phase Transition and Relativistic Heavy-Ion Physics (QPT 2025)

Functional renormalization group study of anomalous magnetic moment in a low energy effective theory

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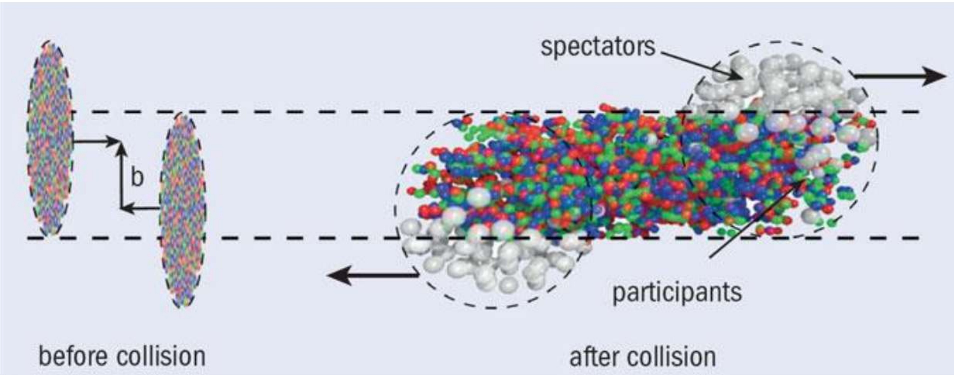
2025.10.26

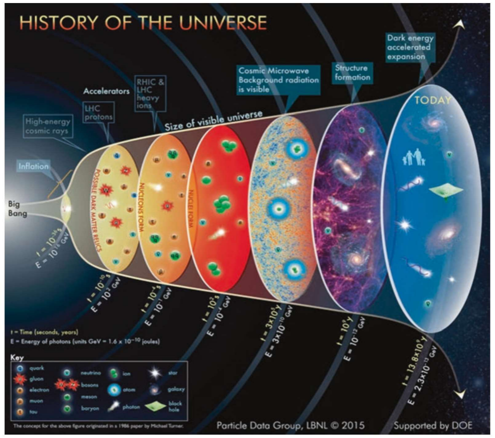
Based on: Rui Wen, Chuang Huang and Mei Huang (2025) arXiv:2506.20246

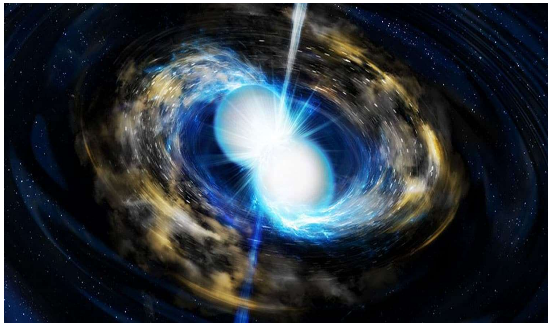
Outline

- Introduction
- low energy effective theory within FRG
  - The Nambu-Jona-Lasinio type effective action
  - Anomalous magnetic moment
- Numerical results
- Summary

Introduction







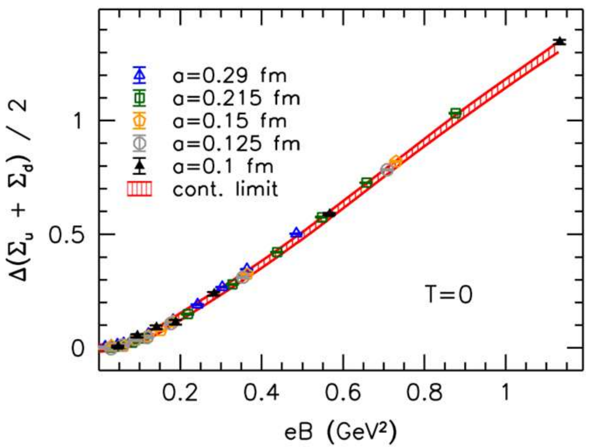
non-central heavy-ion collisions $B \sim 10^{18}$ Gauss

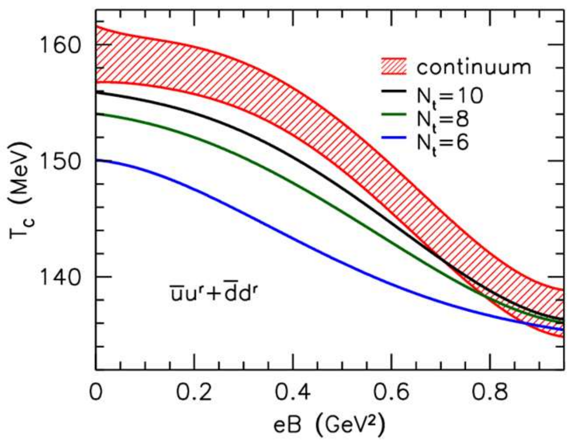
Toia A 2013 CERN Courier April 31

Early universe

Magnetars $10^{10} - 10^{15}$ Gauss

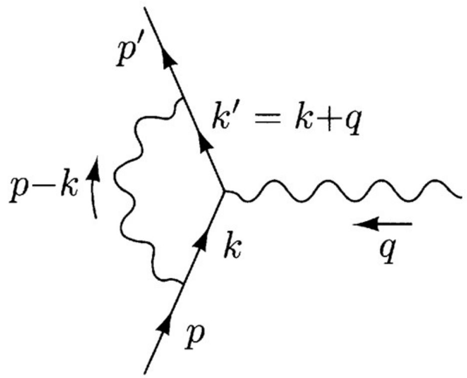
G. S. Bali, F. Bruckmann et. al. , (2012) Phys. Rev. D 86, 071502 arXiv:1206.4205





G. S. Bali, F. Bruckmann et. al. (2011) JHEP 02, 044, arXiv:1111.4956

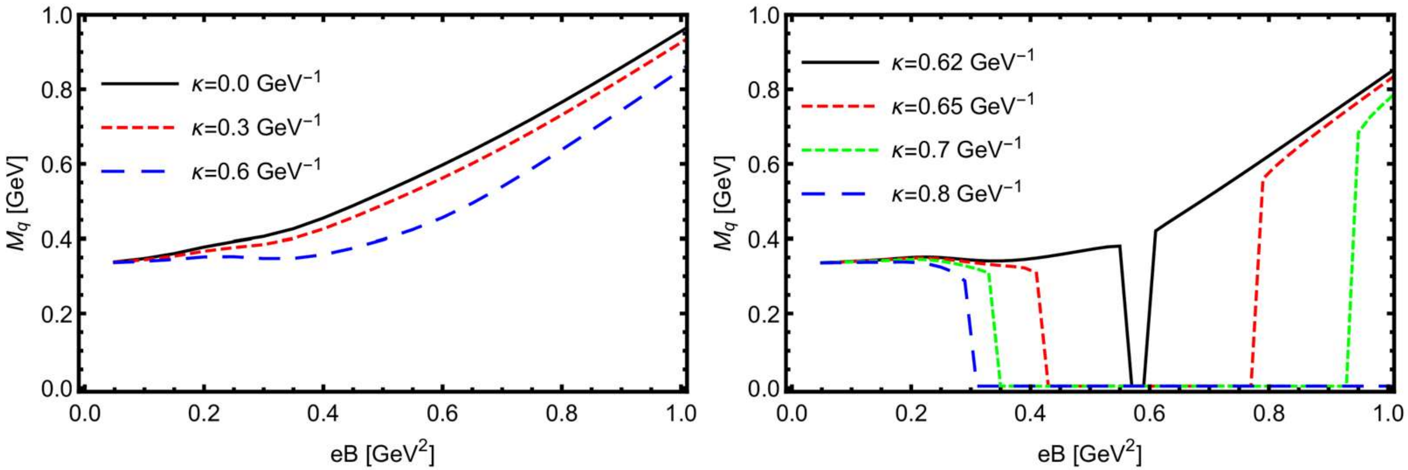
Introduction



$$\Gamma^{\mu}(p',p) = \gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2),$$

An Introduction To Quantum Field Theory Michael E. Peskin Daniel V. Schroeder

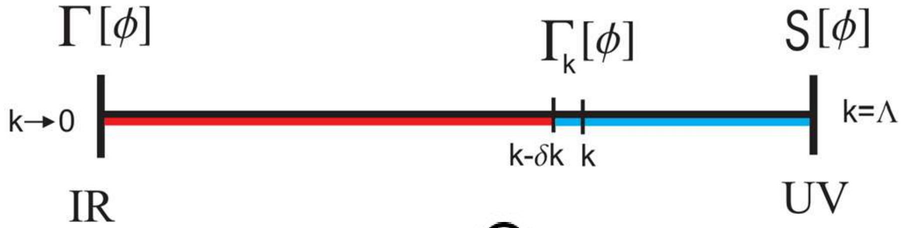
$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m_0 + \kappa_f q_f F_{\mu\nu}\sigma^{\mu\nu})\psi + G_S\left\{(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \vec{\tau}\psi)^2\right\}$$



Phys. Rev. D 103.076015 K. Xu, J. Chao, and M. Huang

Functional renormalization group

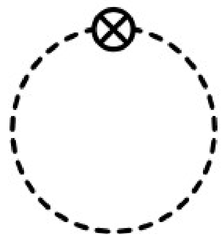
$$\Gamma_k[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\phi+\hat{\varphi}] + \frac{1}{2}\int_p \hat{\varphi}(p) R_k(p^2)\hat{\varphi}(-p) + \int_x \hat{\varphi}(x)\frac{\delta\Gamma_k[\phi]}{\delta\phi(x)}}$$



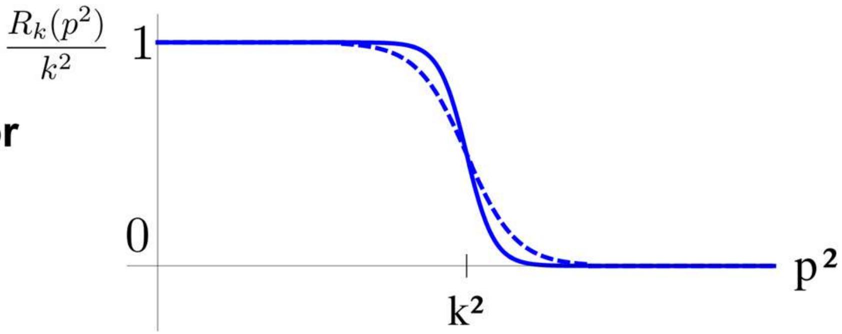
Wetterich equations

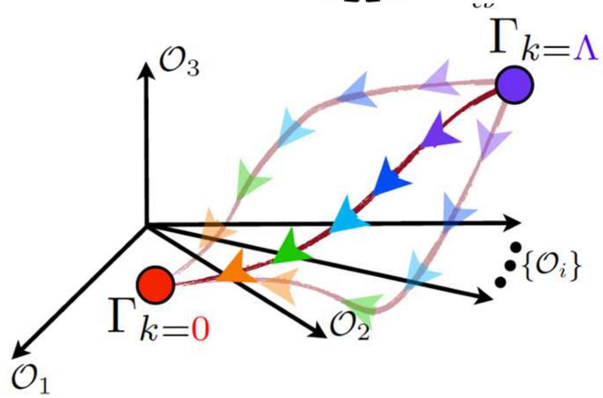
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathbf{STr}[G_k \cdot \partial_t R_k]$$

$$\partial_t \Gamma_k = \frac{1}{2}$$

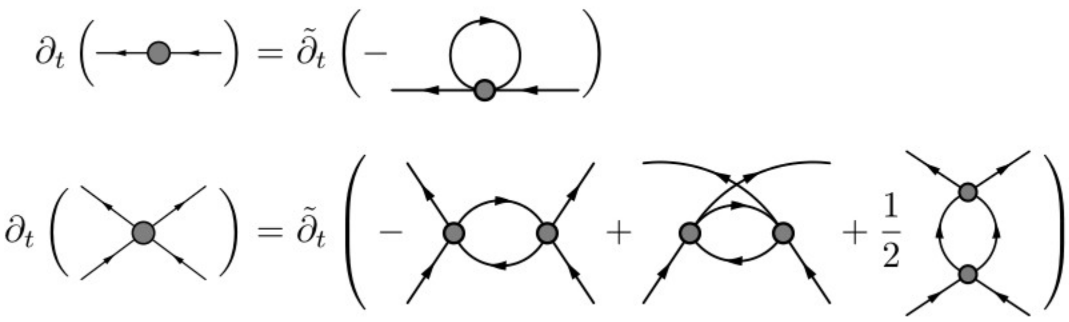


Regulator





The effective action



The effective action of 2-flavor Nambu-Jona-Lasinio type low energy effective theory in Euclidean space

$$\Gamma_k[\bar{q},q] = \int_x \bar{q}(\gamma_\mu D_\mu + m)q + \sum_\alpha \lambda^{(\alpha)} \mathcal{T}^{(\alpha)}_{4q,ijlm} \bar{q}_i q_j \bar{q}_l q_m$$

The quark fields $\quad q = (u,d)^T \qquad Q = diag(2/3, -1/3)e$

The covariant derivative of quark fields $\quad D_\mu \equiv \partial_\mu - iQA_{\mu,0}$

A background homogeneous magnetic field along z-direction

$$A_{\mu,0} = (0,0,xB,0)$$

Four-quark scatterings.

$$\alpha \in \{\pi, \sigma, \eta, a, (V \pm A), (V-A)^{adj}, (S \pm P)^{adj}_{-}, (S+P)^{adj}_{+}\}.$$

The Schwinger formalism of quark propagator

$$G(x,y;q_f) = \Phi(x,y)\int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}\tilde{G}(p;q_f).$$

Translation invariance part

$$\begin{aligned}\tilde{G}(p;q_f) = &\int_0^\infty ds \exp\left[-s\left((1+r_{f,s})^2\left(p_\parallel^2 + p_\perp^2\frac{\tanh(q_fBs)}{q_fBs}\right)+m^2\right)\right]\\ &\cdot\left[(-ip_\parallel\gamma_\parallel(1+r_{f,s})+m)(1+i\gamma_1\gamma_2\tanh(q_fBs))\right.\\ &\left.-ip_\perp\gamma_\perp(1+r_{f,s})(1-\tanh^2(q_fBs))\right].\end{aligned}$$

4d regulator

$$r_{f,s} = \frac{e^{-x}}{x}, \qquad x = \frac{p_\parallel^2 + p_\perp^2\tanh(q_fBs)/(q_fBs)}{k^2}$$

W.-j. Fu, C. Huang, J. M. Pawlowski, and Y.-y. Tan, SciPost Phys.14.4.069(2022)

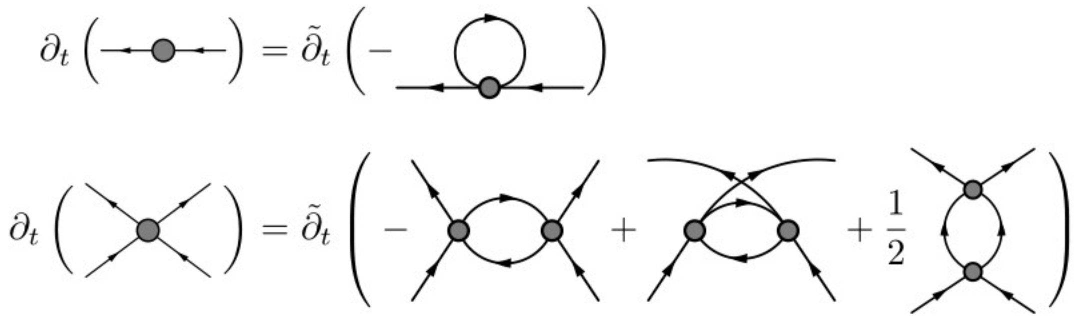
The quark masses as functions of magnetic fields

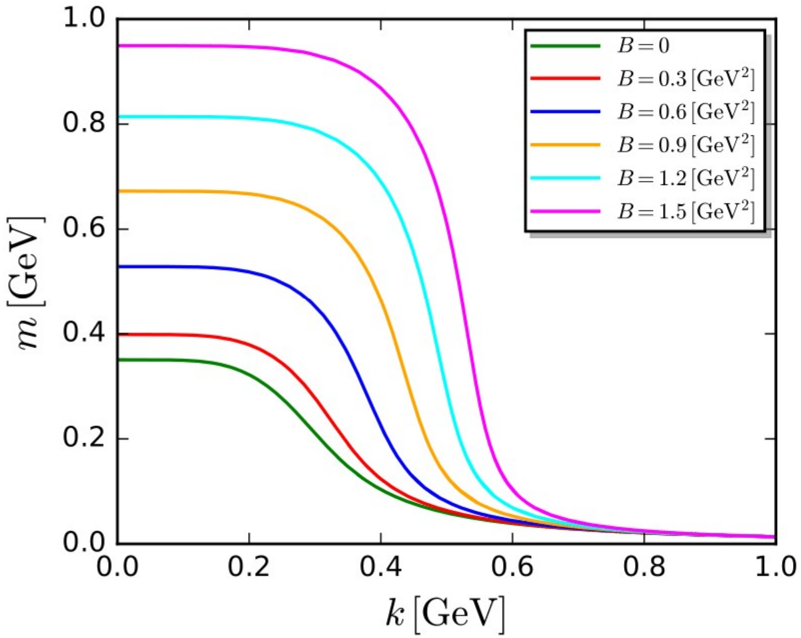
$\Lambda = 1\text{GeV},\ m_\Lambda = 13.5\text{MeV}$

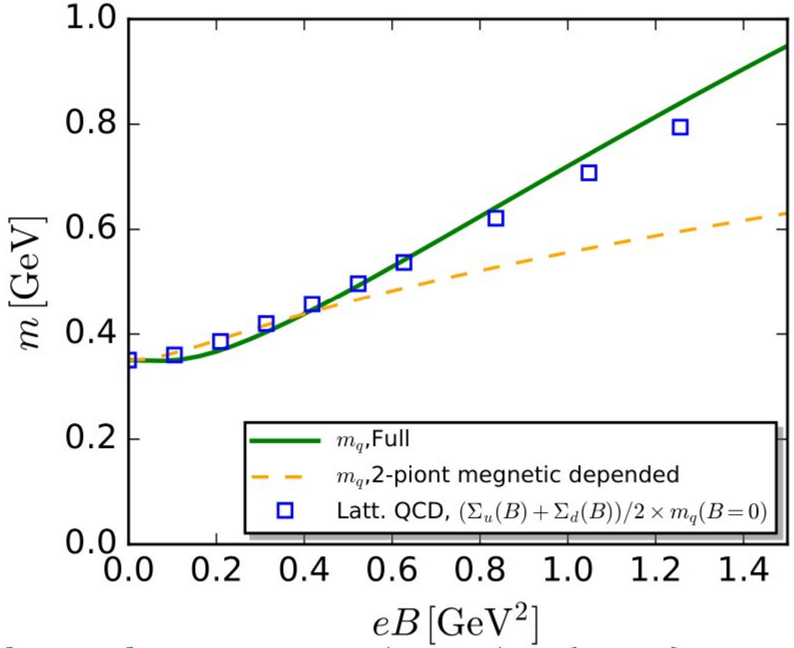
$\lambda_{\pi,\Lambda} = \lambda_{\sigma,\Lambda} = 10.6\Lambda^{-2}$

$m(T=0, B=0) = 350\ \text{MeV}$

$m_\pi = 140\ \text{MeV}$

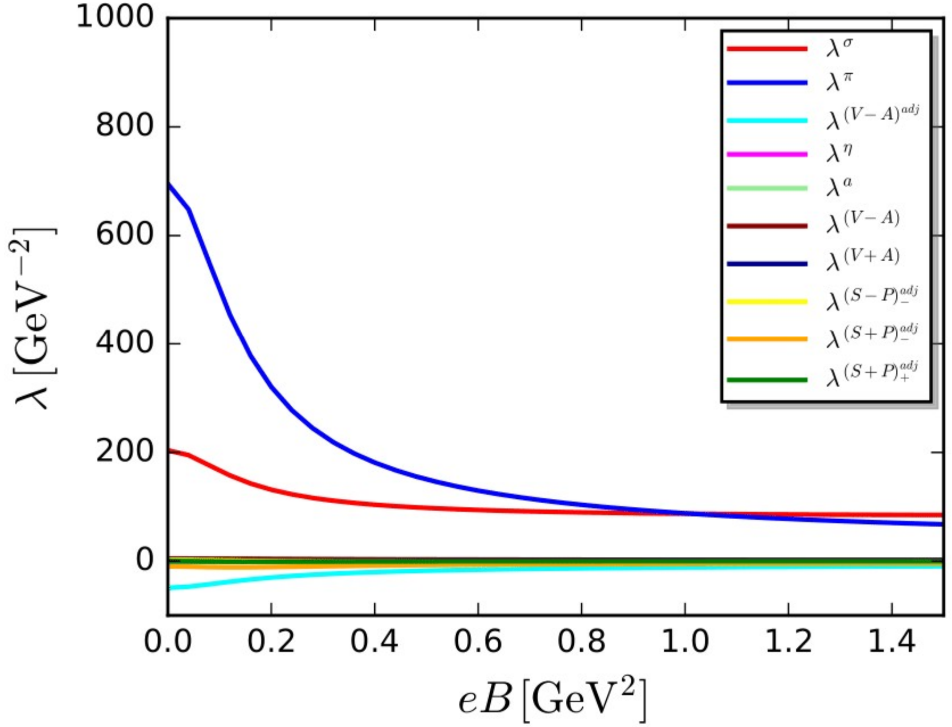


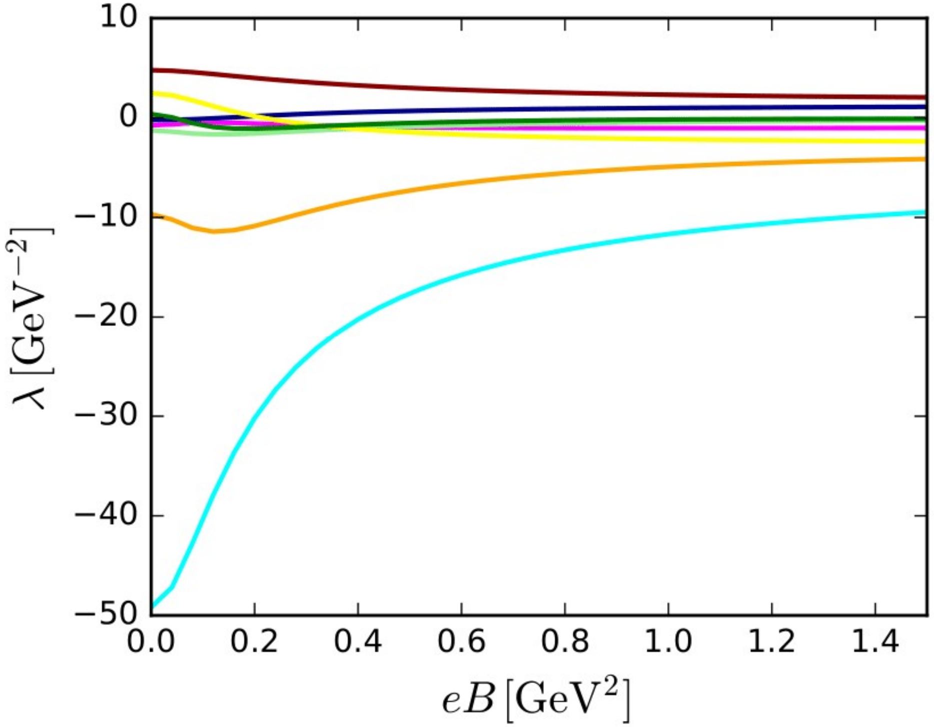




H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang, and Y. Zhang Phys. Rev. D 105, 034514 (2022)

Four-point quark correlation functions





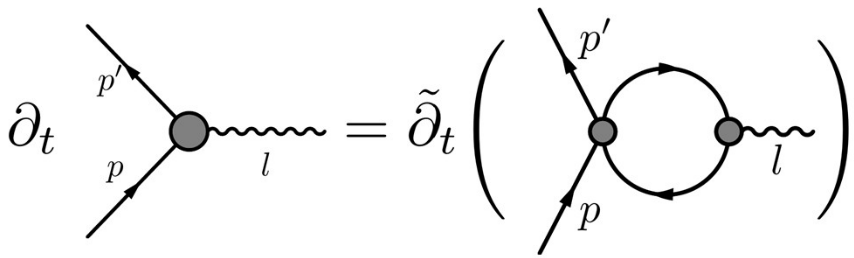
$$\lambda_{\pi,\Lambda} = \lambda_{\sigma,\Lambda} = 10.6\Lambda^{-2}$$

Anomalous magnetic moment

Full quark-photon vertex

$$\Gamma_{A\bar{q}q,\mu} = \delta(p+p'+l) Q \otimes \sum_{i=1}^{8} \Gamma^{(i)}_{A\bar{q}q,\mu}$$

$$\Gamma^{(i)}_{A\bar{q}q,\mu} = \lambda^{(i)}_{A\bar{q}q} [\mathcal{T}^{(i)}]_\mu (p,p',l)$$



Friederike Ihssen, Jan M. Pawlowski, Franz R. Sattler, Nicolas Wink arXiv:2408.08413

The classical tensor structure

$$[\mathcal{T}^{(1)}]_\mu (p,p',l) = \Pi^{\perp}_{\mu\nu}(l) i\gamma_\nu ,$$

tensor structure related to the anomalous magnetic moment

$$[\mathcal{T}^{(4)}]_\mu (p,p',l) = i\sigma_{\mu\nu}(l)_\nu .$$

We define the form factor

$$F_{2,f}(p,p',l) \equiv 2m\lambda^{(4)}_{A\bar{q}_f q_f}(p,p',l) , \quad f \in \{u,d\},$$

And quark AMM

$$\kappa_f \equiv \lim_{l\to 0} \lambda^{(4)}_{A\bar{q}_f q_f}(p,p',l) , \quad f \in \{u,d\},$$

Form factors

The magnetic moments of quarks

$$\mu_f = q_f(1 + F_2(0))\frac{m_N}{m}\mu_N\,, \quad f \in \{u, d\}$$

The constituent quark model

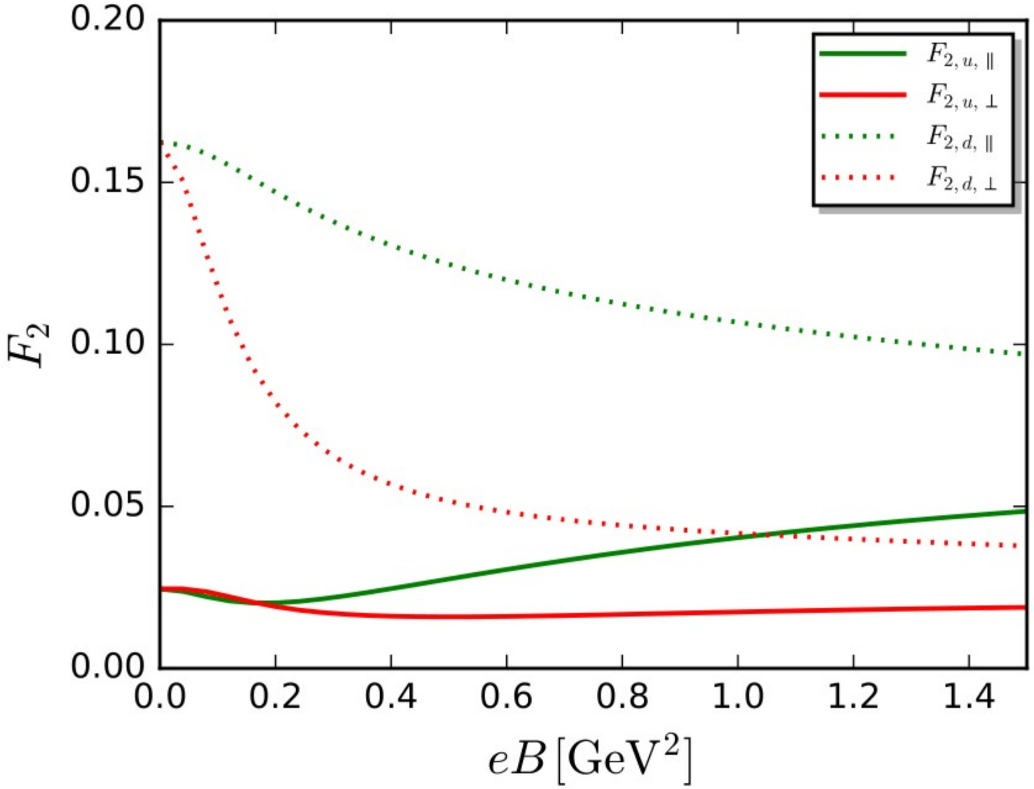
$$\mu_{\text{proton}} = \frac{1}{3}(4\mu_u - \mu_d),$$
$$\mu_{\text{neutron}} = \frac{1}{3}(4\mu_d - \mu_u).$$

Our results with the constituent quark model

$$\mu_{\text{proton}}/\mu_N = 2.7877$$
$$\mu_{\text{neutron}}/\mu_N = -1.9967$$

Committee on Data of the International Science Council (CODATA) 2022   arXiv:2409.03787

$$\mu_{\text{proton}}/\mu_N = 2.7928$$
$$\mu_{\text{neutron}}/\mu_N = -1.9130$$



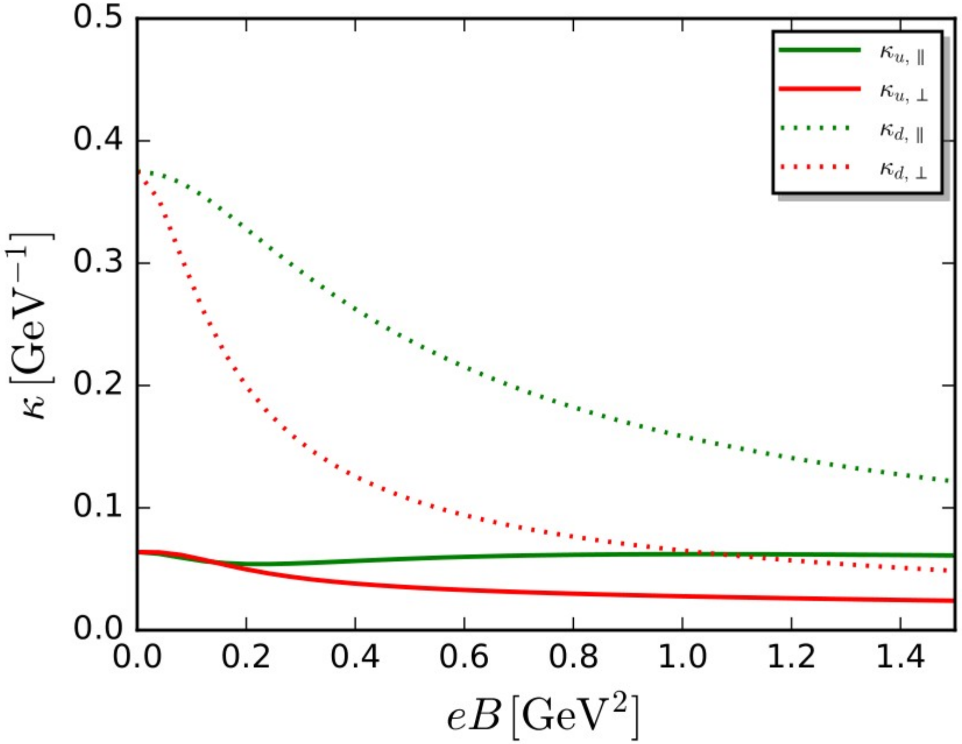
Quarks AMM

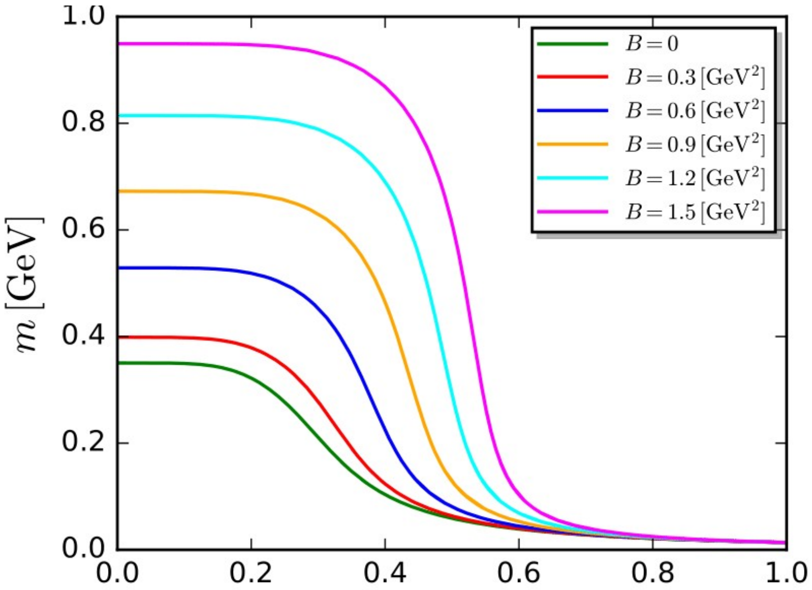
$$\partial_t \kappa_{u,\|/\perp} = -\frac{1}{6q_u}$$

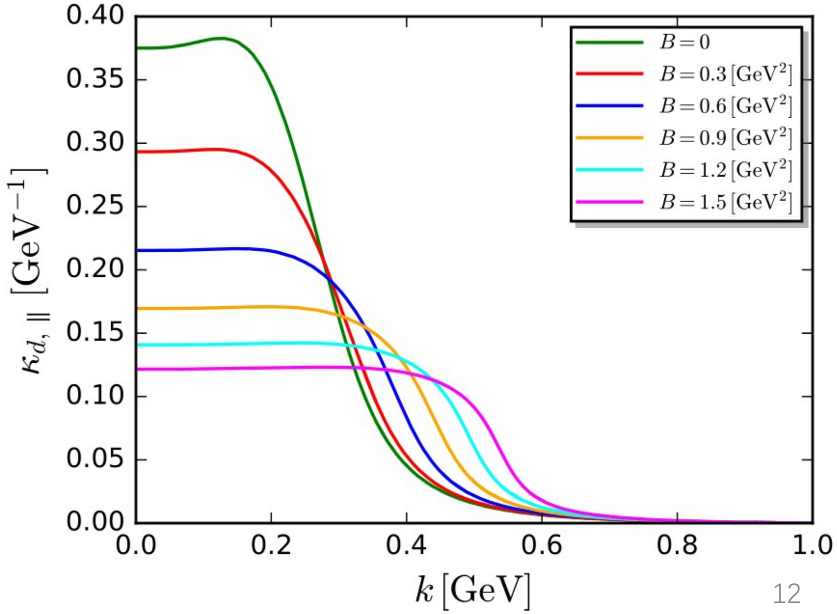
$$\Big[2(3\lambda^a - 3\lambda^\pi - 8\lambda^{(S+P)^{adj}_-} + 8\lambda^{(S+P)^{adj}_+})q_d \mathscr{L}_{\kappa,\|/\perp}(d)$$

$$+ (3\lambda^a - 3\lambda^\eta - 3\lambda^\pi + 3\lambda^\sigma + 16\lambda^{(S+P)^{adj}_+})q_u \mathscr{L}_{\kappa,\|/\perp}(u)\Big].$$

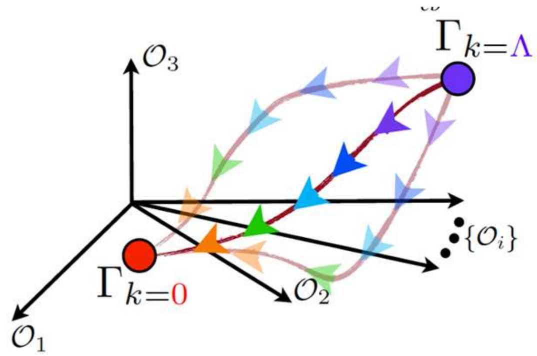
$$\kappa_d/\kappa_u = (q_u^2/q_d^2)\cdot(\mathscr{L}\kappa(u)/\mathscr{L}\kappa(d)) = 4\mathscr{L}\kappa(u)/\mathscr{L}\kappa(d)$$







Regulator Dependence

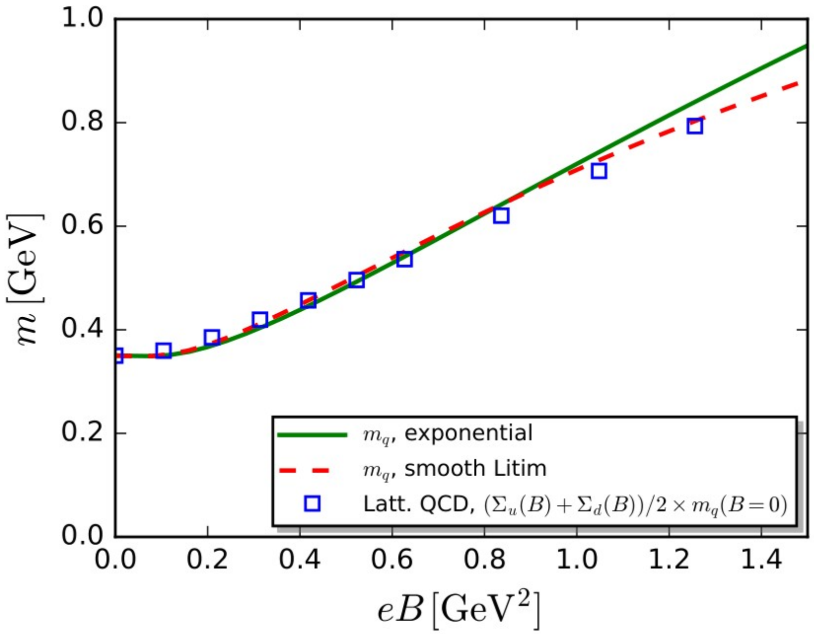


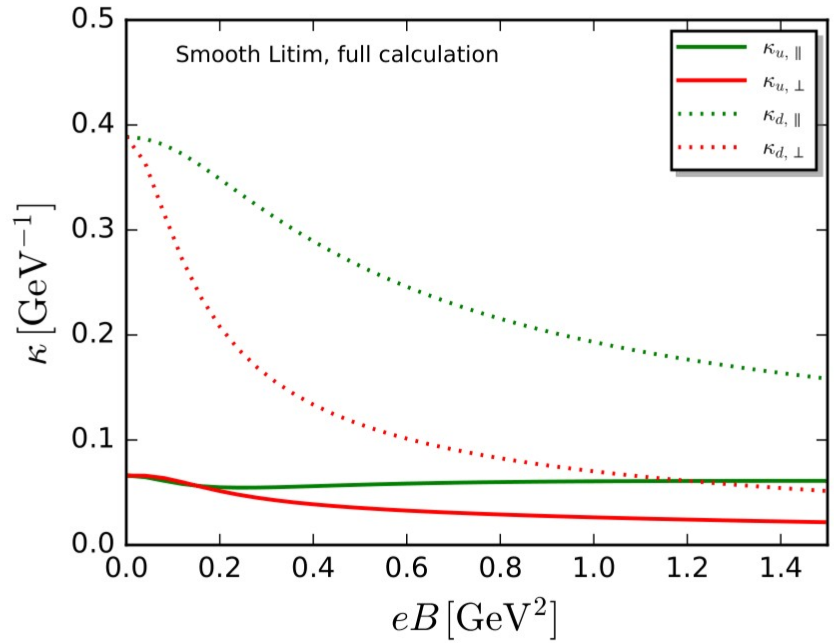
Exponential

$$r_{f,s} = \frac{e^{-x}}{x},$$

A smooth version of the Litim regulator

$$r(x) = \left(\frac{1}{\sqrt{x}} - 1\right) \cdot \frac{1}{1 + e^{\frac{x-1}{a}}}$$





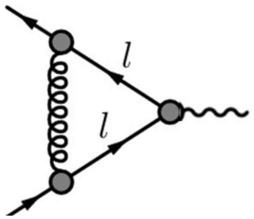
Summary

1.We build a 2-flavor Nambu-Jona-Lasinio type low energy effective theory under magnetic fields, which self-consistently include the Fierz-complete four-quark scatterings through RG flows.

2.The quark mass as a function of the strength of magnetic field are in agreement with the lattice QCD results.

3. The quark AMMs are dynamically generated with the chiral symmetry breaking, and the magnitude of the AMM of the down quark is around 4 times larger than that of the up quark.

We will calculate quark AMM in QCD within in future work.



We hope lattice QCD groups could provide a calculation of the quark AMM.

Thanks for your attention

Backup

Quarks AMM

Expand the loop function

$$\mathscr{L}_{\kappa,\parallel}(f) = \int \frac{q^3 dq}{(2\pi)^2}\tilde{\partial}_t\Big\{ -\frac{m(r_q+1)}{2\left(m^2+q^2(r_q+1)^2\right)^2} + \frac{m(r_q+1)\left(m^2-5q^2(r_q+1)^2\right)}{6\left(m^2+q^2(r_q+1)^2\right)^5}(q_f B)^2 + \mathscr{O}(q_f B)^4\Big\}.$$

$$\mathscr{L}_{\kappa,\perp}(f) = \int \frac{q^3 dq}{(2\pi)^2}\tilde{\partial}_t\Big\{ -\frac{m(r_q+1)}{2\left(m^2+q^2(r_q+1)^2\right)^2} + \frac{m^3(r_q+1)}{\left(m^2+q^2(r_q+1)^2\right)^5}(q_f B)^2 + \mathscr{O}(q_f B)^4\Big\}.$$