



Nuclear Science
Computing Center at CCNU



Baryon Electric Charge Correlation as a Magnetometer of QCD

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Central China Normal University

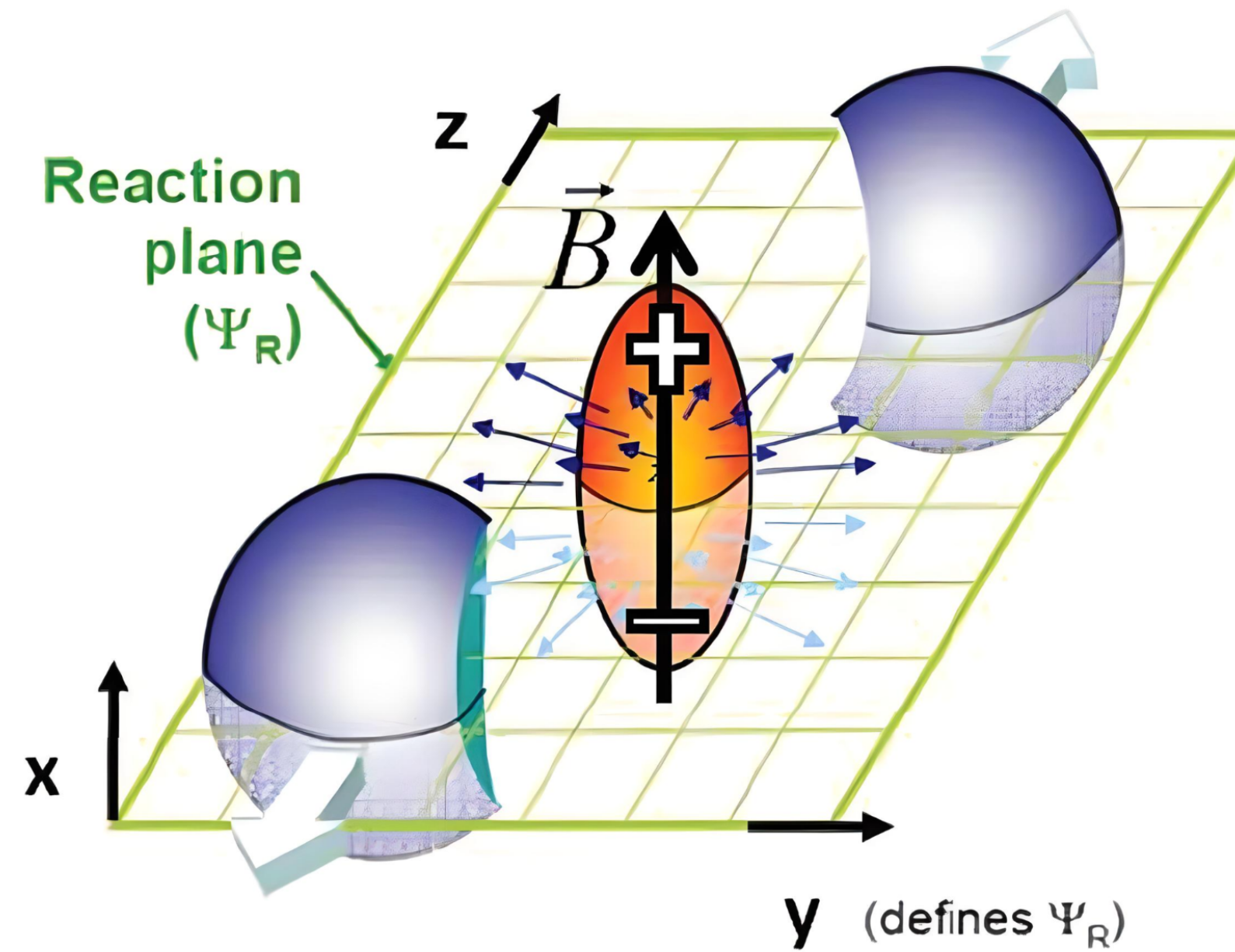
Based on *Phys. Rev. Lett.* **132**, 201903 (2024), *Phys. Rev. D* **111**, 114522 (2025)

and *arXiv*:2508.07532 (accepted by *PRD*)

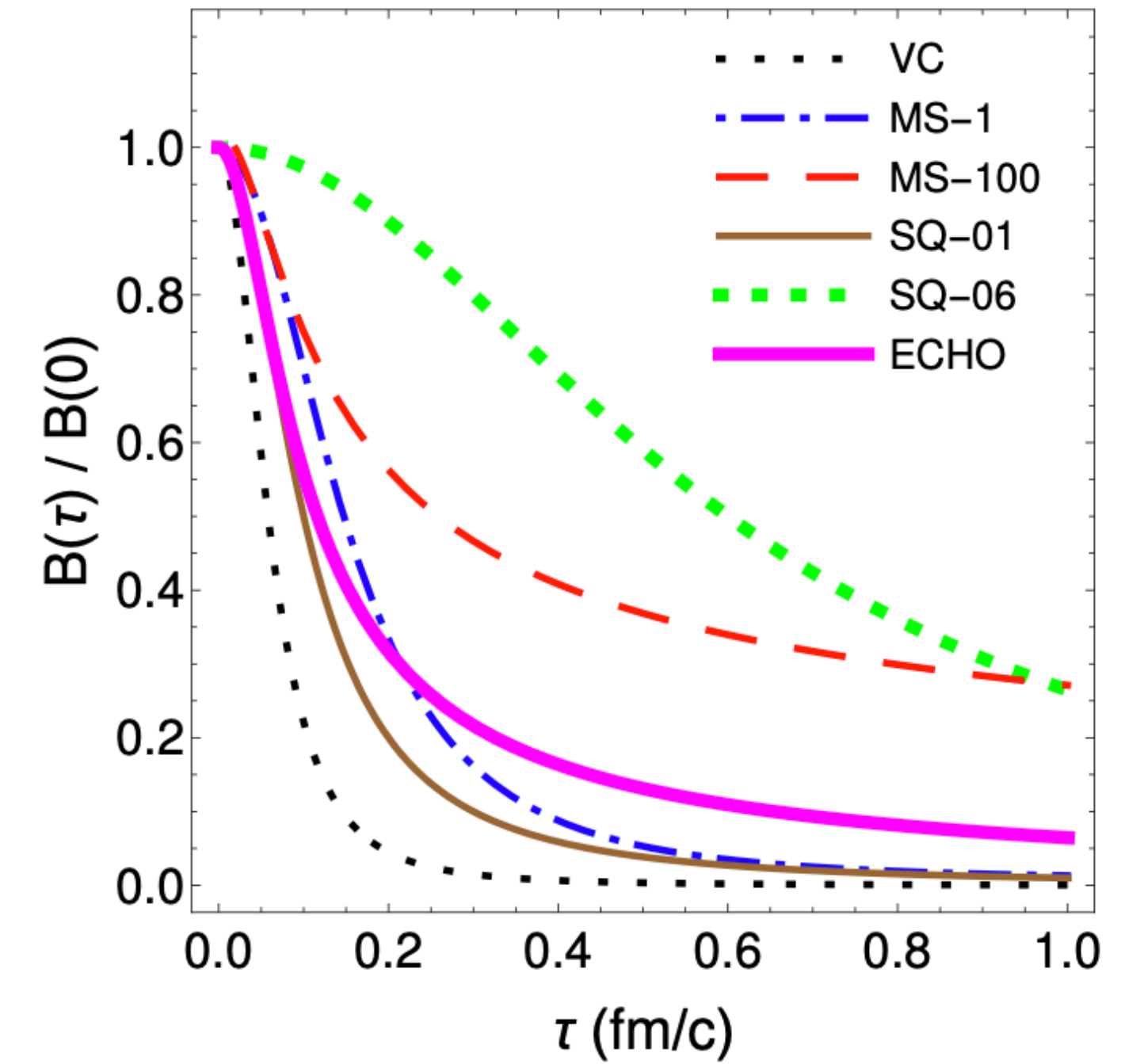
In collaboration with H.-T. Ding, A. Kumar and S.-T. Li

The 16th Workshop on QCD Phase Transition and Relativistic Heavy-Ion Physics,
Oct. 24th - 28th, 2025 @Guilin, Guangxi

Strong magnetic fields in heavy-ion collisions



J. Zhao and F. Wang, Prog.Part.Nucl.Phys. 107 (2019) 200-236



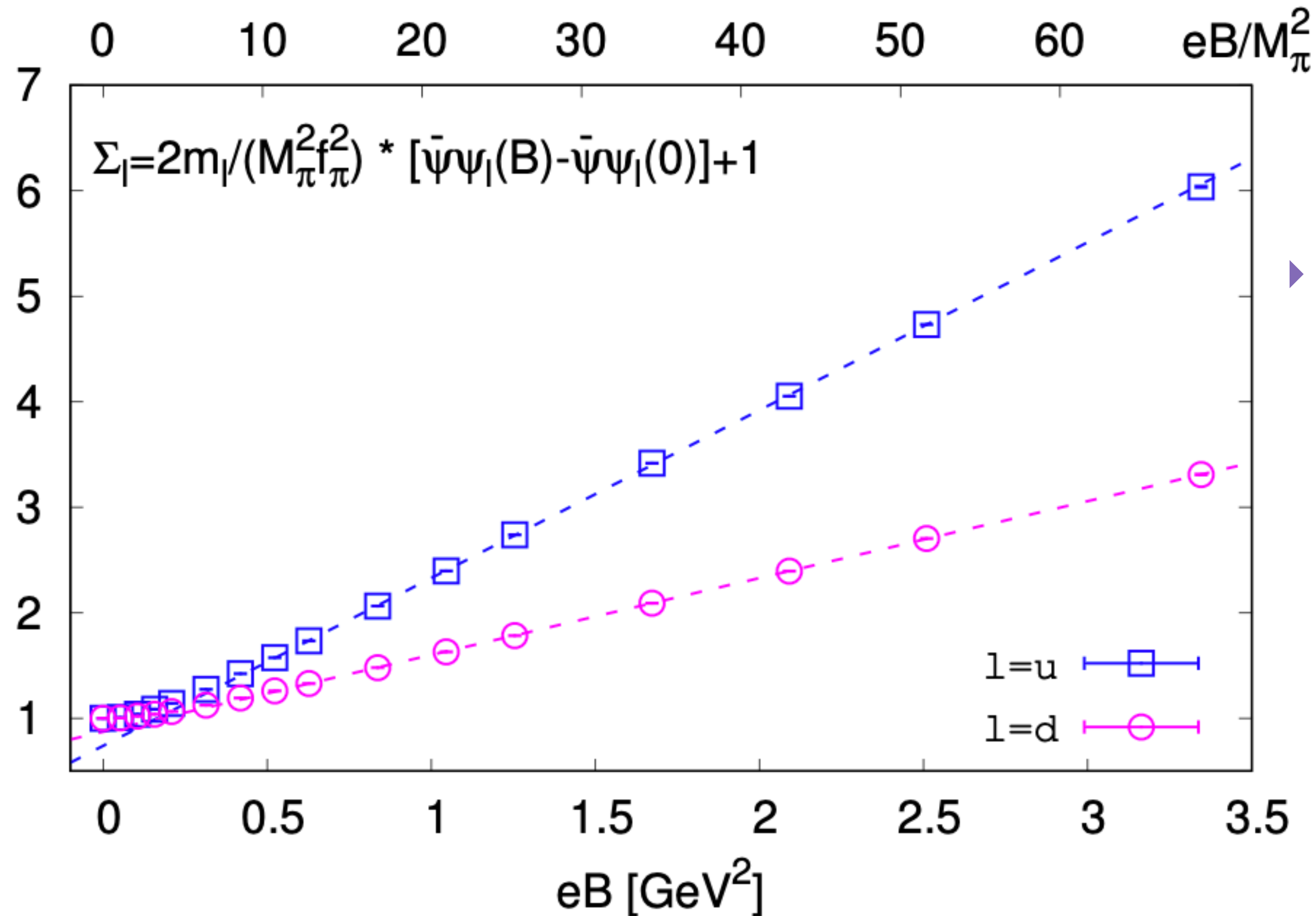
A. Huang et al. Phys.Lett.B 777 (2018) 177-183

$$eB_{\tau=0} \sim 5 M_\pi^2 \text{ in RHIC, } eB_{\tau=0} \sim 70 M_\pi^2 \text{ in LHC}$$

W.-T.Deng, X.-G. Huang, Phys.Rev.C 85, 044907 (2012)

Does this strongly decaying magnetic field manifest itself in the final stage of HIC?

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



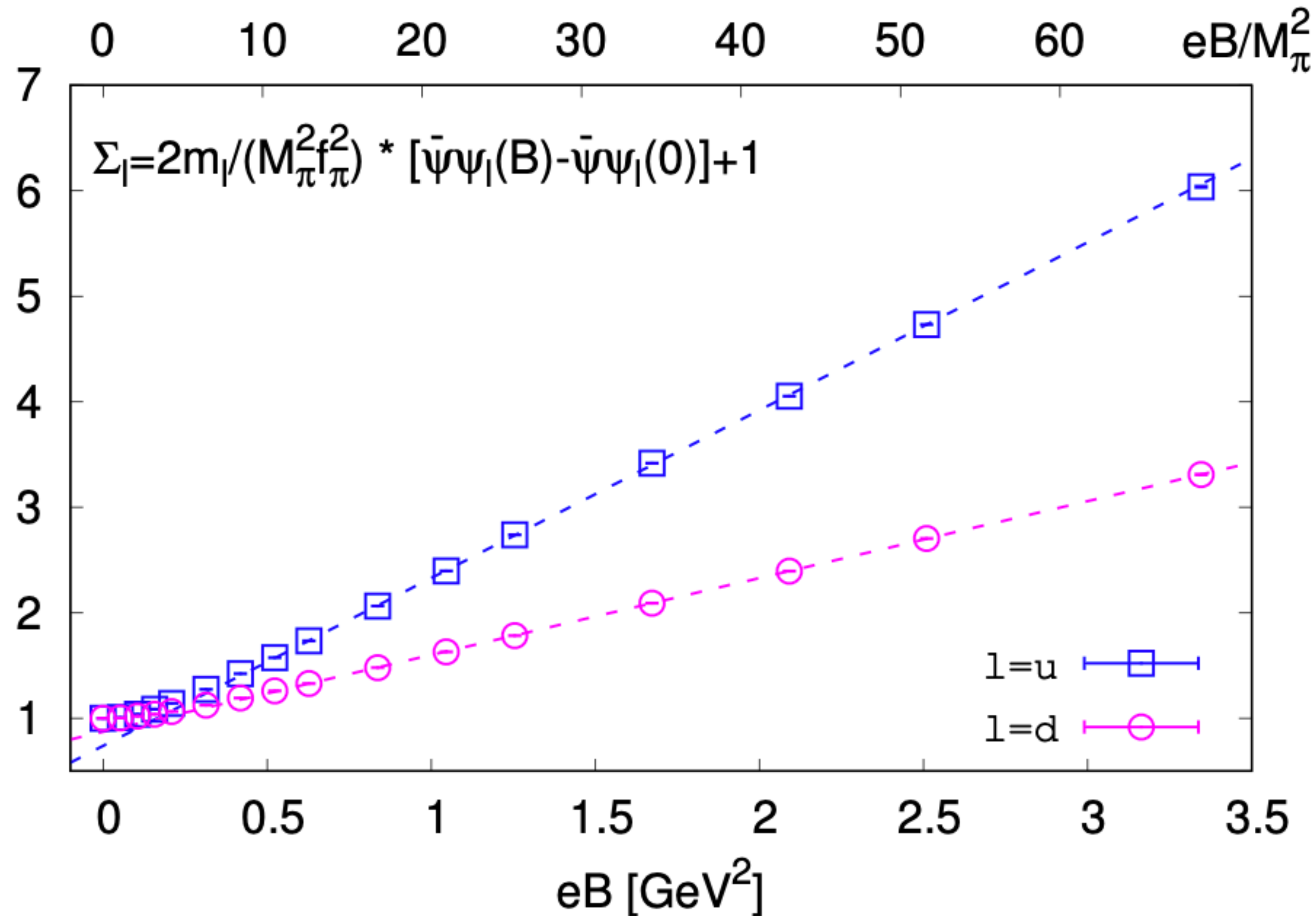
► Sign of isospin symmetry breaking:

Non-degeneracy of u and d condensates at strong magnetic fields

H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104, 014505 (2021)

See also in e.g. Bali et al., Phys.Rev.D 86, 071502(2012)

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



A clear effect but Not
accessible in HIC
experiments!

H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104, 014505 (2021)

See also in e.g. Bali et al., Phys.Rev.D 86, 071502(2012)

Fluctuations of net baryon number (B), electric charge (Q) and strangeness (S)

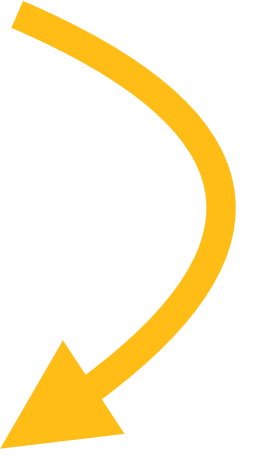
Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} \left(T, V, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S \right) = \sum_{i,j,k=0}^{\infty} \boxed{\frac{\chi_{ijk}^{\text{BQS}}}{i!j!k!}} \left(\frac{\mu_B}{T} \right)^i \left(\frac{\mu_Q}{T} \right)^j \left(\frac{\mu_S}{T} \right)^k$$

C. Allton et al., Phys.Rev. D 66, 074507 (2002)

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\chi_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_u/T)^i \partial (\mu_d/T)^j \partial (\mu_s/T)^k} \right|_{\mu_{u,d,s}=0}$$

$$\chi_{ijk}^{\text{BQS}} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_B/T)^i \partial (\mu_Q/T)^j \partial (\mu_S/T)^k} \right|_{\mu_{B,Q,S}=0}$$


$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \end{aligned}$$

Lattice computable & Experimentally measurable

LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, *Int. J. Mod. Phys. E* 24, no.10, 1530007 (2015)

Exp.: X.-F. Luo & N. Xu, *Nucl. Sci. Tech.* 28 (2017) 112

Effective model investigation at $eB \neq 0$

HRG: G. Kadam et al., *JPG* 47, 125106 (2020); M. Ferreira et al., *Phys. Rev. D* 98, 034003 (2018); K. Fukushima and Y. Hidaka, *Phys. Rev. Lett.* 117, 102301 (2016); A. Bhattacharyya et al., *EPL* 115, 62003 (2016); M. Marczenko et al., *Phys.Rev.C* 110, 065203, (2024)

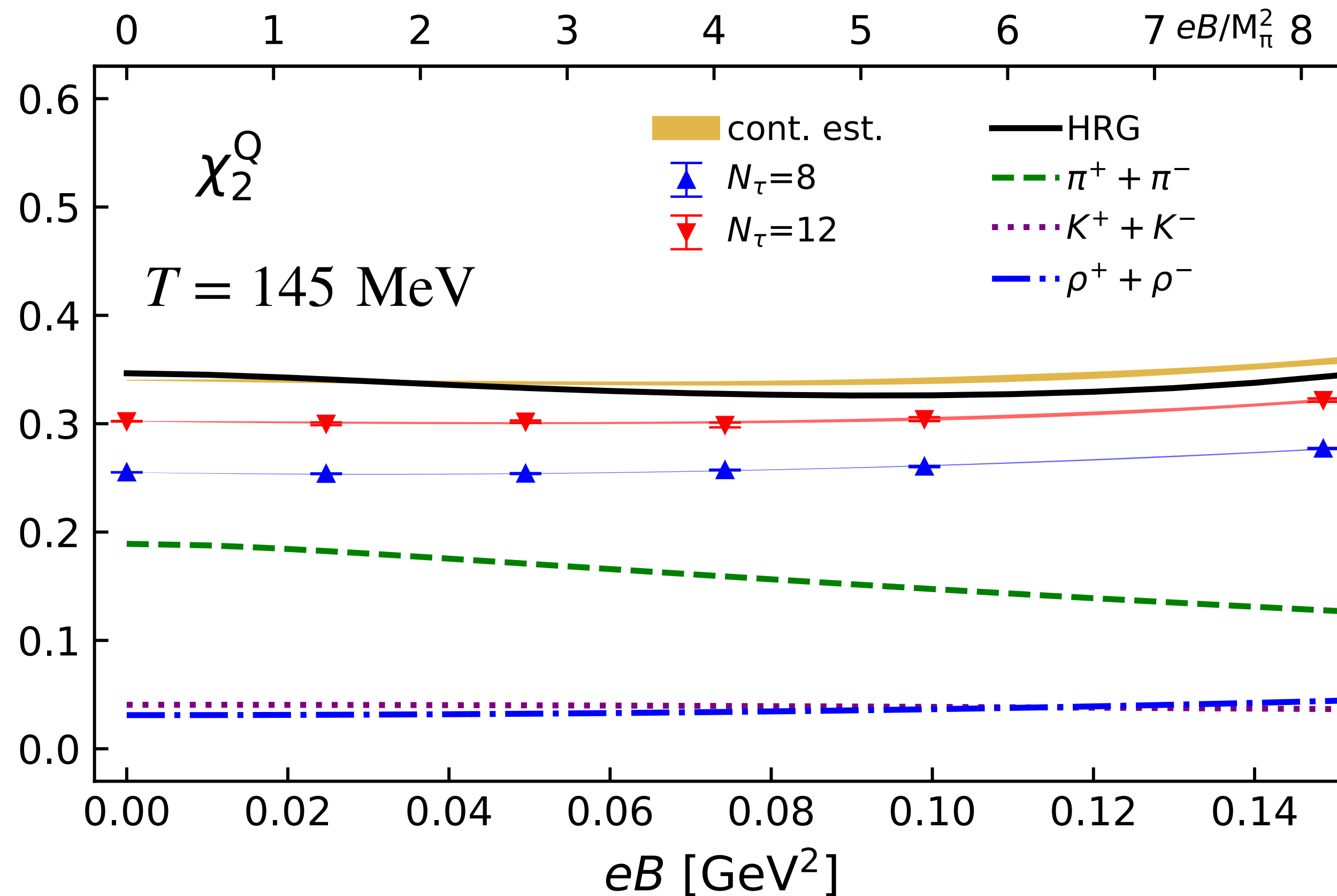
PNJL: W.-J. Fu, *Phys. Rev. D* 88, 014009 (2013); S.-J. Mao, *Chin.Phys.C* 49, 063106, (2025)

Lattice Setup

- ◆ Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- ◆ $N_f = 2 + 1$
- ◆ Lattice sizes : $32^3 \times 8$, $48^3 \times 12$; $64^3 \times 16$
- ◆ $m_s^{\text{phy}}/m_l = 27$, $M_\pi(eB = 0) \approx 135 \text{ MeV}$
- ◆ T window : (144 MeV, 166 MeV), i.e. $(0.9T_{pc}, 1.1T_{pc})$
- ◆ eB window: $eB \lesssim 8M_\pi^2 \sim 0.15 \text{ GeV}^2$ — Magnetometer
 $eB \lesssim 45M_\pi^2 \sim 0.8 \text{ GeV}^2$ — Equation of State

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = 1, 2, 3, 4, 6, 12, 16, 24, 32$$

Electric charge fluctuations at $T = 145$ MeV



H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, Phys. Rev. Lett. 132, 201903 (2024)

❖ χ_2^Q almost independent of eB

❖ Hadron Resonance Gas model (HRG):
Pressure arising from charged hadrons
($eB \neq 0$):

$$\frac{p_i^{M/B}}{T^4} = \frac{|q_i| B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \epsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1 \left(\frac{n\epsilon_0}{T} \right)$$

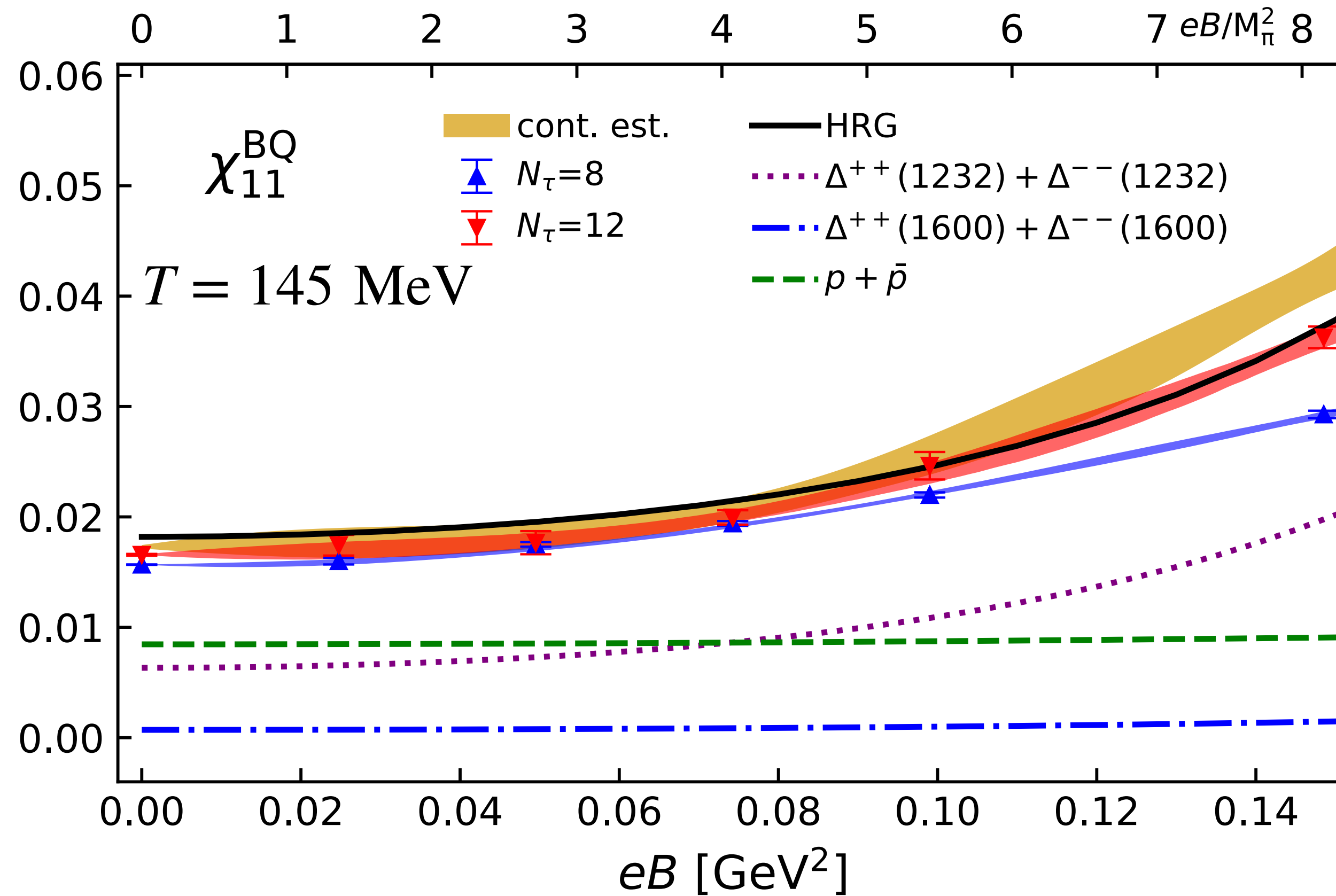
Quantized energy level:

$$\epsilon_0 = \sqrt{m_i^2 + 2 |q_i| B (l + 1/2 - s_z)}$$

$(l + 1/2 - s_z) > 0$: ϵ_0 increases
 $(l + 1/2 - s_z) < 0$: ϵ_0 decreases
 with eB

H.-T. Ding et al., Eur. Phys. J. A 57 (2021) 6, 202

Baryon electric charge correlation at $T = 145$ MeV



H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, *Phys. Rev. Lett.* 132, 201903 (2024)

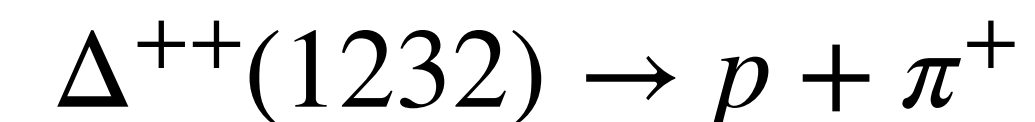
◆ χ_{11}^{BQ} increases $\sim 140\%$ at $eB \sim 8M_\pi^2$,

Magnetometer of QCD

◆ The results of HRG model are consistent with LQCD up to $eB \sim 5M_\pi^2$

◆ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give **most of the contributions** of magnetic field dependence of χ_{11}^{BQ}

◆ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ are **not measurable** in HIC experiments



Proxy for $R(\chi_{11}^{\text{BQ}})$ along the transition line

◆ R_{cp} –like observables:

$$R(\mathcal{O}) = \frac{\mathcal{O}(eB, T_{pc}(eB))}{\mathcal{O}(eB = 0, T_{pc}(0))}$$

◆ At $eB \simeq 8M_\pi^2$, $R(\chi_{11}^{\text{BQ}}) \sim 2.1$

◆ Proxies within HRG framework:

$$\text{net-B} \rightarrow \tilde{p} : \omega_{R \rightarrow \tilde{p}}$$

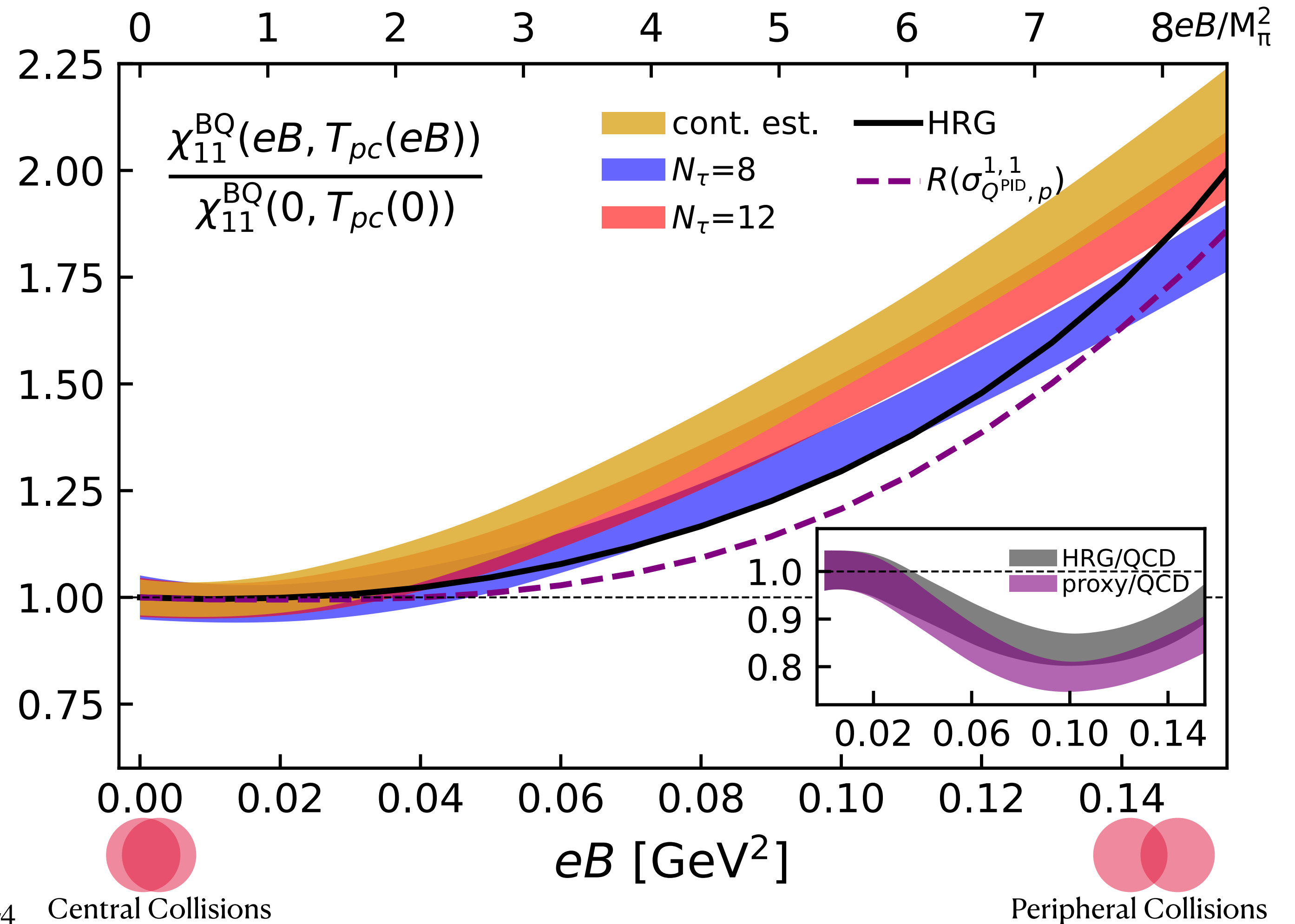
$$\text{net-Q} \rightarrow \tilde{Q}^{\text{PID}} : \omega_{R \rightarrow \tilde{\pi}^+} + \omega_{R \rightarrow \tilde{K}^+} + \omega_{R \rightarrow \tilde{p}}$$

$$\text{net-S} \rightarrow \tilde{K}^+ : \omega_{R \rightarrow \tilde{K}^+}$$

Where $\omega_{R \rightarrow i}$ represents number of **stable** particle i produced by particle R after the **entire decay chain**

$$\sum_R \boxed{\text{B}_R \text{Q}_R} \frac{\partial^2 p_R / T^4}{\partial \mu_R^2} \rightarrow \sum_R \boxed{(\omega_{R \rightarrow \tilde{p}})(\omega_{R \rightarrow \tilde{Q}^{\text{PID}}})} \frac{\partial^2 p_R / T^4}{\partial \mu_R^2}$$

R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)



H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, Phys. Rev. Lett. 132, 201903 (2024)

Proxy for $R(\chi_{11}^{\text{BQ}})$ with kinematic cuts along the transition line

◆ Proxies with kinematic cuts on p_T and η within HRG framework:

$$\int d^3p \times \Theta(p_{T_{\min}}, p_{T_{\max}}, \eta_{\min}, \eta_{\max})$$

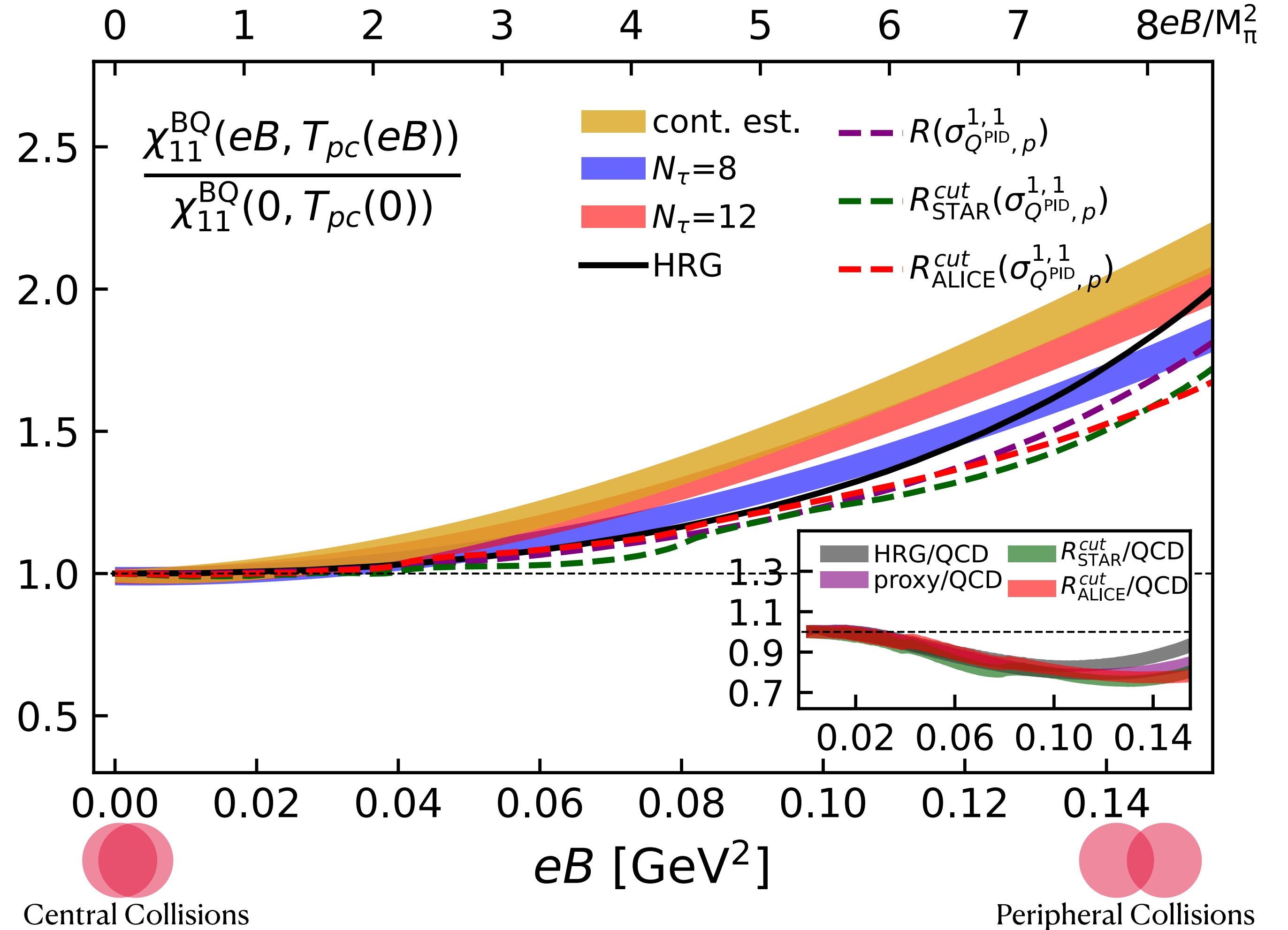
$$R_{\text{STAR}}^{\text{cut}} : |\eta| < 0.5,$$

$$\pi^\pm, K^\pm \text{ and } p(\bar{p}) : 0.4 < p_T < 1.6 \text{ GeV}/c$$

$$R_{\text{ALICE}}^{\text{cut}} : |\eta| < 0.8,$$

$$\pi^\pm \text{ and } K^\pm : 0.2 < p_T < 2 \text{ GeV}/c$$

$$p(\bar{p}) : 0.4 < p_T < 2 \text{ GeV}/c$$

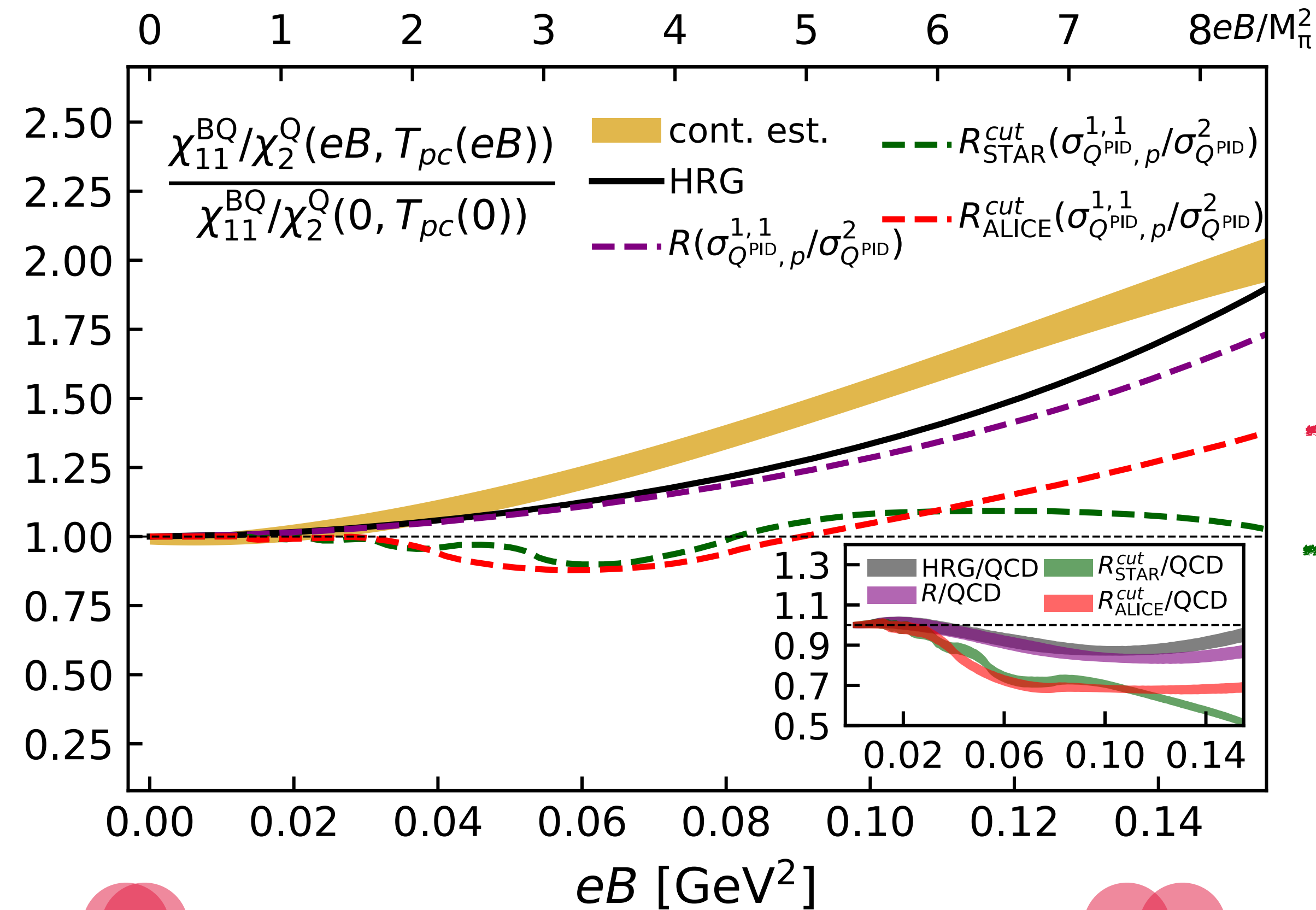


H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, *Phys.Rev.D* 111, 114522 (2025)

STAR collaboration, *Phys.Rev.C* 100 (2019) 1, 014902

ALICE collaboration, *arXiv:2503.18743*

Double ratio $R(\sigma_{Q^{\text{PID}},p}^{1,1}/\sigma_{Q^{\text{PID}}}^2)$ in Lattice QCD



Central Collisions

Peripheral Collisions

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, Phys.Rev.D 111, 114522 (2025)

◆ R_{cp} –like observables:

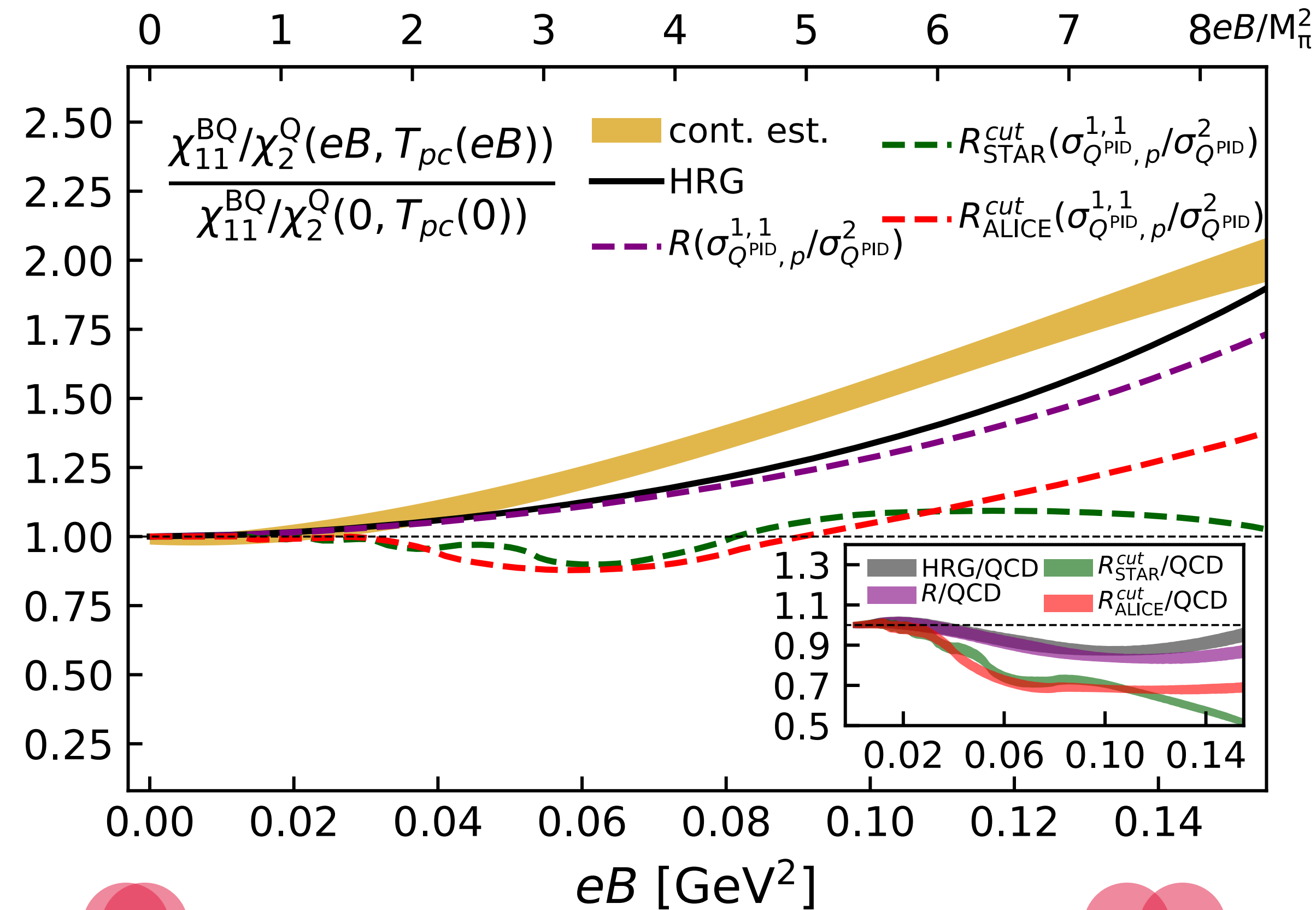
$$R(\mathcal{O}) \equiv \frac{\mathcal{O}(eB, T_{pc}(eB))}{\mathcal{O}(eB = 0, T_{pc}(0))}$$

◆ At $eB \simeq 8M_\pi^2$, $R(\chi_{11}^{\text{BQ}}/\chi_2^{\text{Q}}) \sim 2$

ALICE cut: $\sim 25\%$ enhancement

No clear dependence on eB

Double ratio $R(\sigma_{Q^{\text{PID}},p}^{1,1}/\sigma_{Q^{\text{PID}}}^2)$ in Lattice QCD and HIC

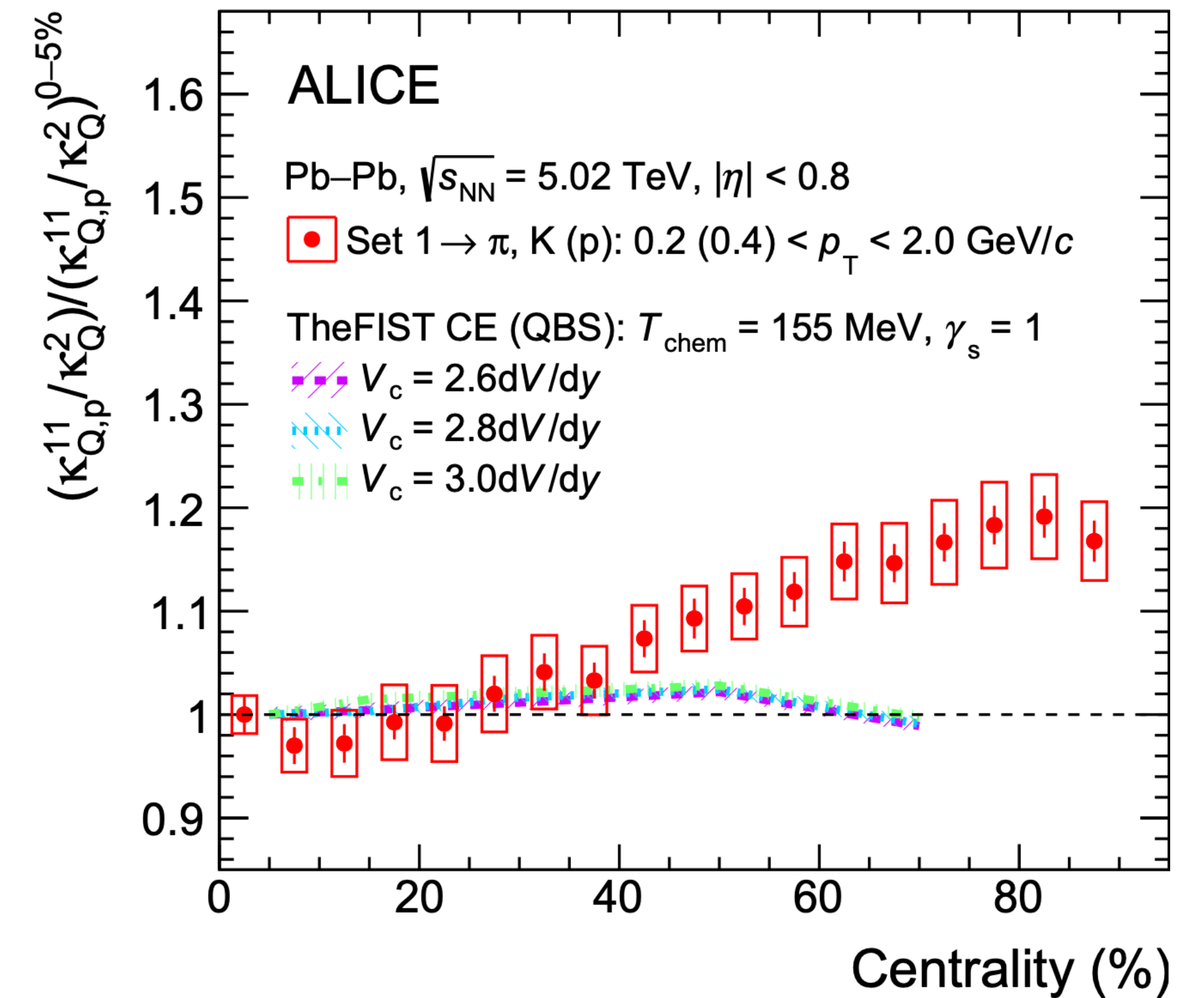


Central Collisions

Peripheral Collisions

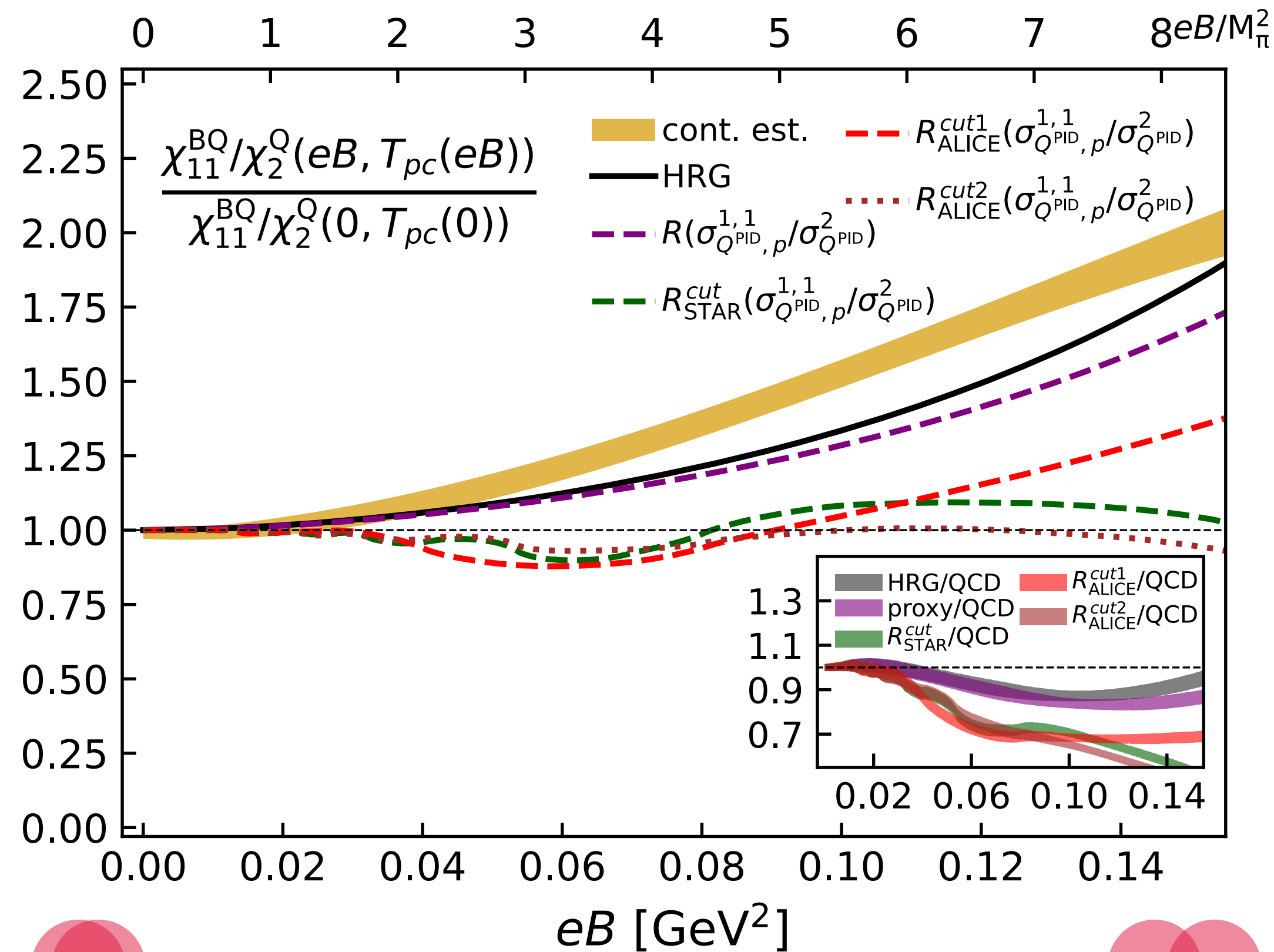
H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, Phys.Rev.D 111, 114522 (2025)

Proxy for $R(\chi_{11}^{\text{BQ}}/\chi_2^{\text{Q}})$ from ALICE

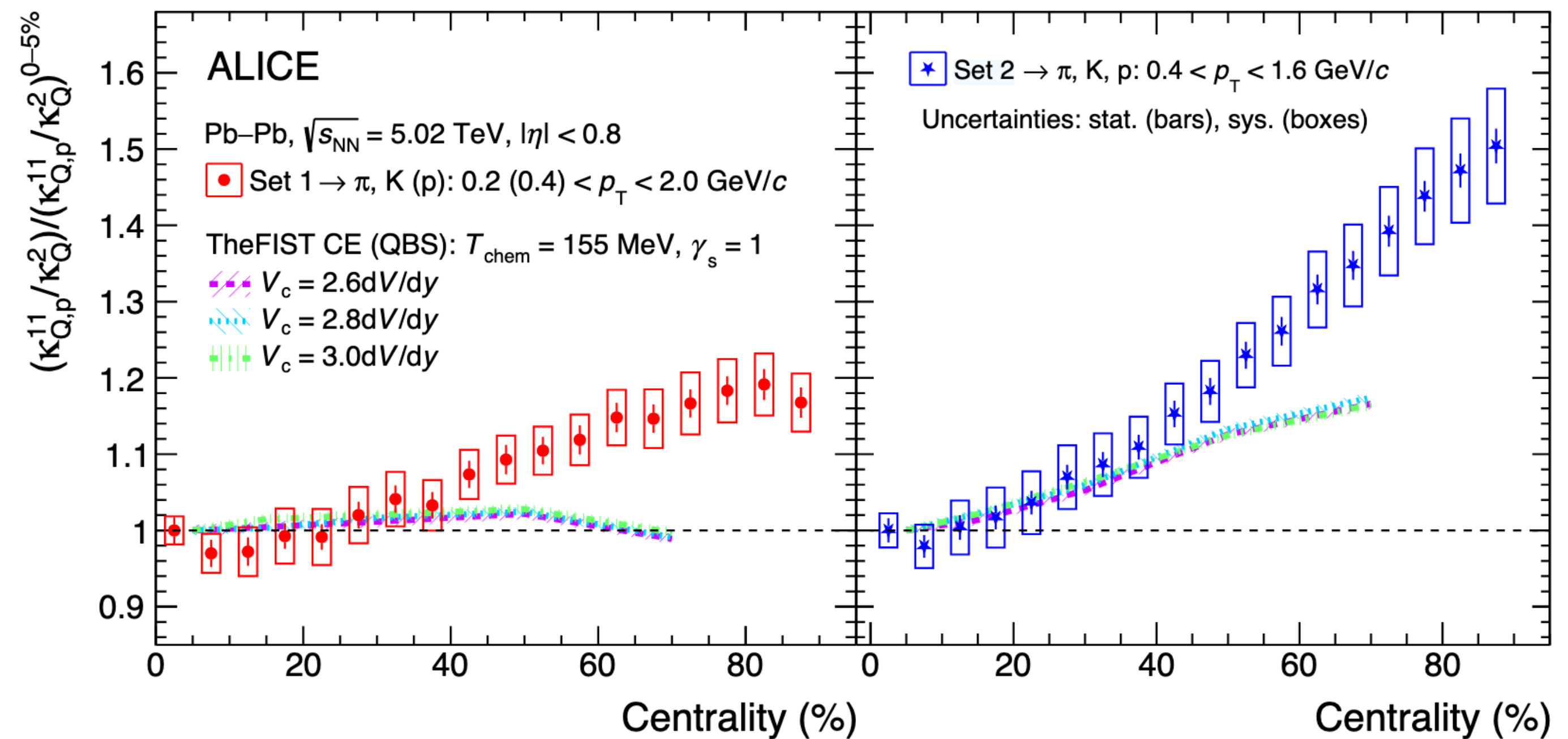


ALICE collaboration, JHEP 08 (2025) 210

Double ratio $R(\sigma_{Q^{PID},p}^{1,1}/\sigma_{Q^{PID}}^2)$ in Lattice QCD and HIC



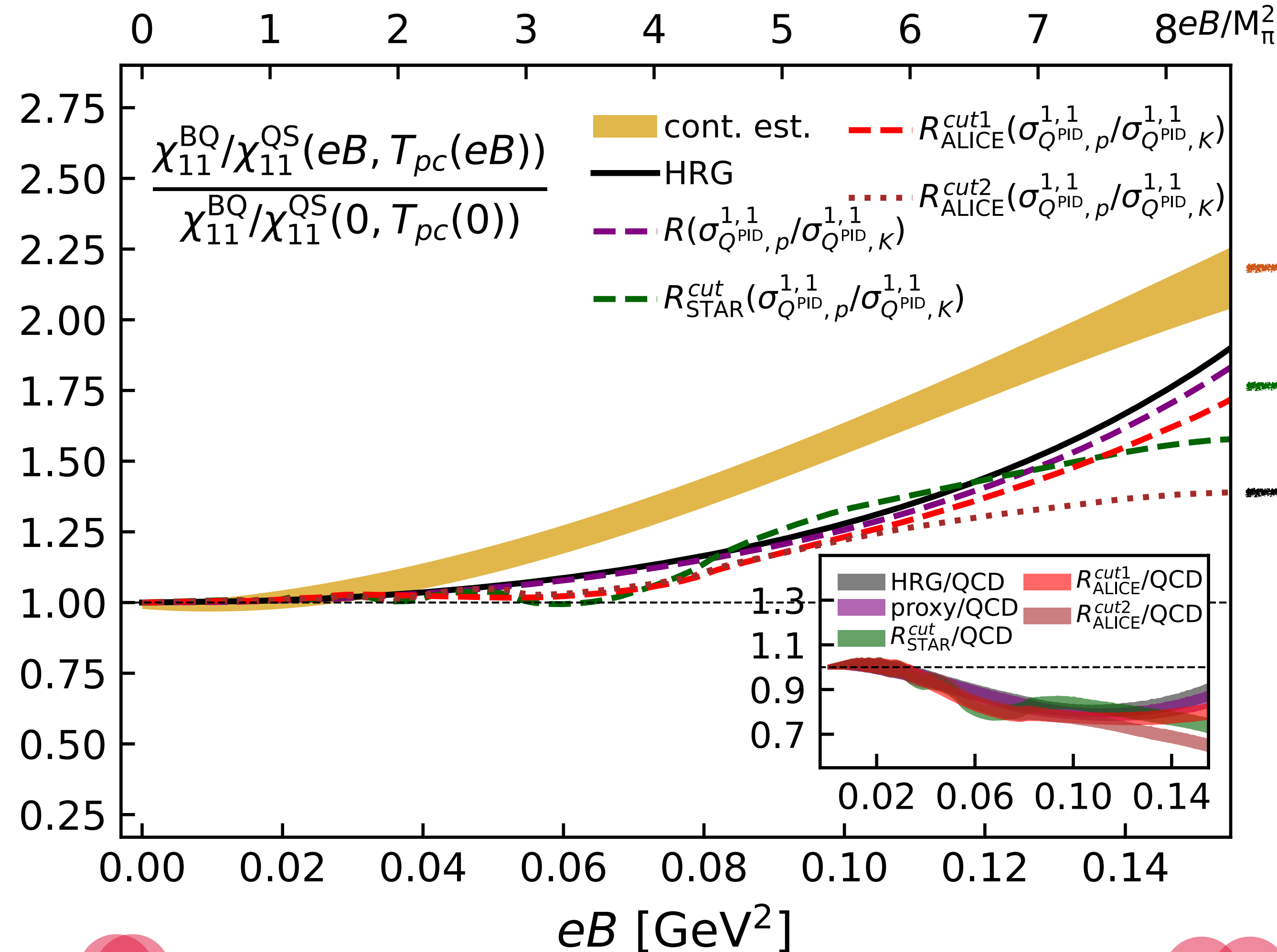
Proxy for $R(\chi_{11}^{BQ}/\chi_2^Q)$ from ALICE



ALICE collaboration, JHEP 08 (2025) 210

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, Phys.Rev.D 111, 114522 (2025)

$R(\chi_{11}^{\text{BQ}}/\chi_{11}^{\text{QS}})$ from Lattice QCD and Proxies



$$R(\chi_{11}^{\text{BQ}}/\chi_{11}^{\text{QS}}) = \frac{\chi_{11}^{\text{BQ}}/\chi_{11}^{\text{QS}}(eB, T_{pc}(eB))}{\chi_{11}^{\text{BQ}}/\chi_{11}^{\text{QS}}(eB = 0, T_{pc}(0))}$$

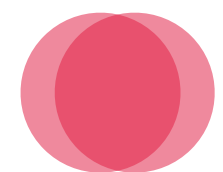
→ ~ 2.1

→ ~ 1.5 – 1.75

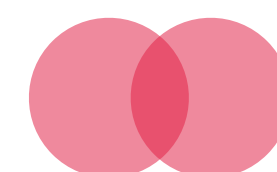
→ ~ 1.3

◆ Weaker dependence on the kinematic cuts

◆ More effective probe!



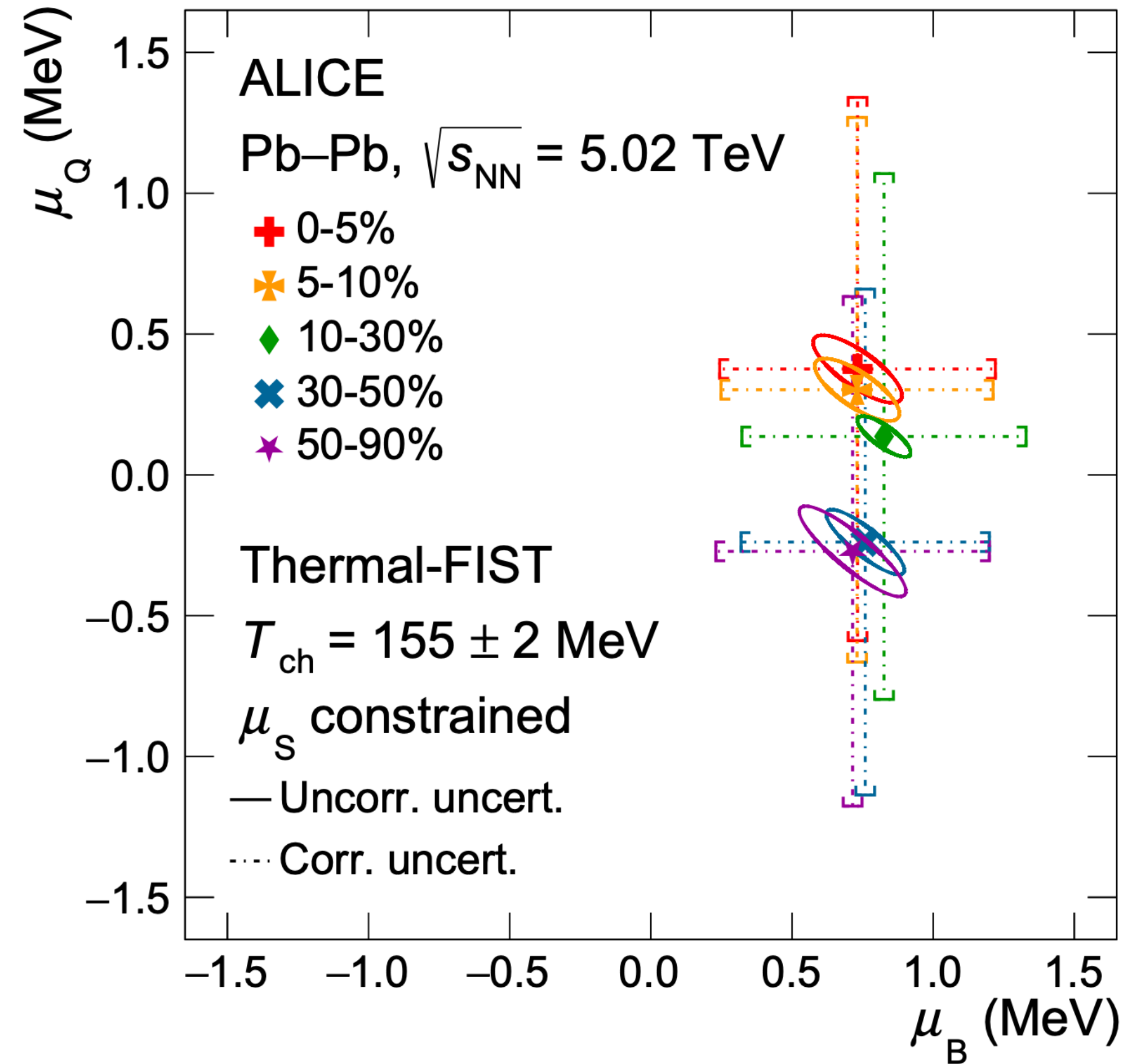
Central Collisions



Peripheral Collisions

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, Phys.Rev.D 111, 114522 (2025)

Electric charged chemical potential over baryon chemical potential



ALICE, *Phys. Rev. Lett.* 133, 092301 (2024)

- μ_Q/μ_B can be obtained from the thermal statistics fits to particle yields
- μ_Q/μ_B also can be obtained from fluctuations of B, Q, S

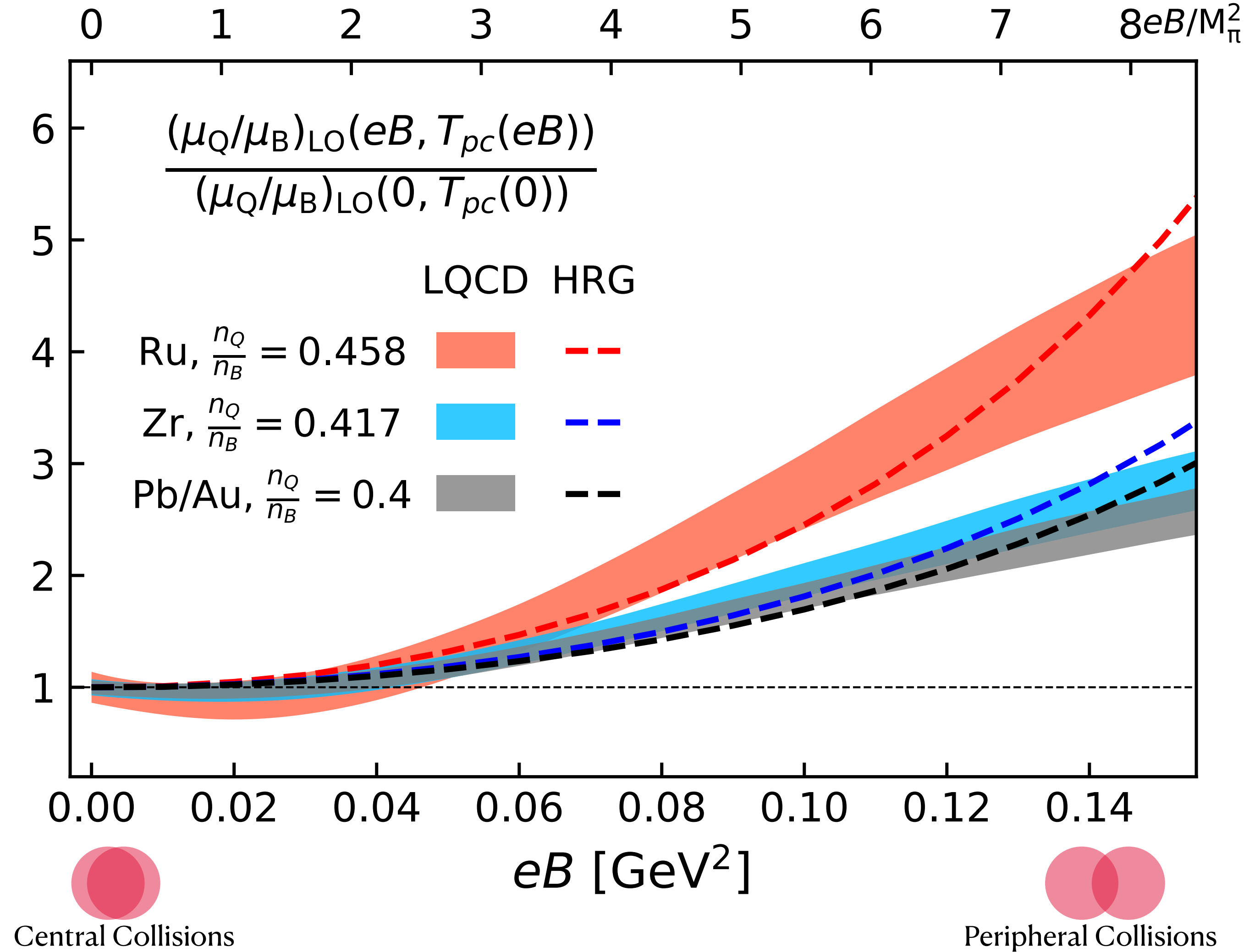
$$\mu_Q/\mu_B = q_1 + q_3 \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$q_1 = \frac{r (\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$

with constraints: $r = n_Q/n_B$, $n_S = 0$

HotQCD, *Phys. Rev. Lett.* 109 (2012) 192302

$(\mu_Q/\mu_B)_{\text{LO}}$ in different collision system



$$\mu_Q/\mu_B = q_1 + q_3 \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$${}^{96}_{44}\text{Ru} + {}^{96}_{44}\text{Ru} : r = 0.458$$

$${}^{96}_{40}\text{Zr} + {}^{96}_{40}\text{Zr} : r = 0.417$$

$${}^{208}_{82}\text{Pb} + {}^{208}_{82}\text{Pb} : r = 0.4$$

◆ At $eB \simeq 8M_\pi^2$,

Ratio of $(\mu_Q/\mu_B)_{\text{LO}}$ for Pb, Au, Zr ~ 2.4

Ratio of $(\mu_Q/\mu_B)_{\text{LO}}$ for **Ru** ~ 4

H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, Phys. Rev. Lett. 132, 201903 (2024)

QCD Equation of State in extremely strong magnetic field at nonzero density

Equation of State(EoS): Equilibrium properties of strong interacting matter

$p, \epsilon, s, \dots$
Thermodynamic observables

 $\equiv$

$f(T, \mu, eB, \dots)$
Controllable parameters

$0 \leq eB \lesssim 0.8 \text{ GeV}^2$
 $145 \leq T \leq 165 \text{ MeV}$

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{\text{BQS}}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

Initial nuclei constraint
 $r = n_Q/n_B, n_S = 0$

$$\hat{\mu}_Q = q_1 \hat{\mu}_B + q_3 \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$\hat{\mu}_S = s_1 \hat{\mu}_B + s_3 \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$p, \epsilon, s, \dots = f(\chi_{ijk}^{\text{BQS}}, T, eB, \mu_B, q_1, s_1 \dots)$$

$r = 0.4$

$$q_1 = \frac{r(\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r(\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$

$(\hat{\mu}_Q/\hat{\mu}_B)_{\text{LO}}$

$$s_1 = - \frac{(\chi_{11}^{BS} + q_1 \chi_{11}^{QS})}{\chi_2^S}$$

$(\hat{\mu}_S/\hat{\mu}_B)_{\text{LO}}$

$\hat{\mu}_X \equiv \mu_X/T$

HotQCD, Phys. Rev. Lett. 109 (2012) 192302

Pressure in magnetic fields

QCD pressure at nonzero μ_B :

$$\Delta\hat{p} = \frac{P(T, eB, \hat{\mu}_B) - P(T, eB, 0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T) \hat{\mu}_B^{2k}$$

Leading order:

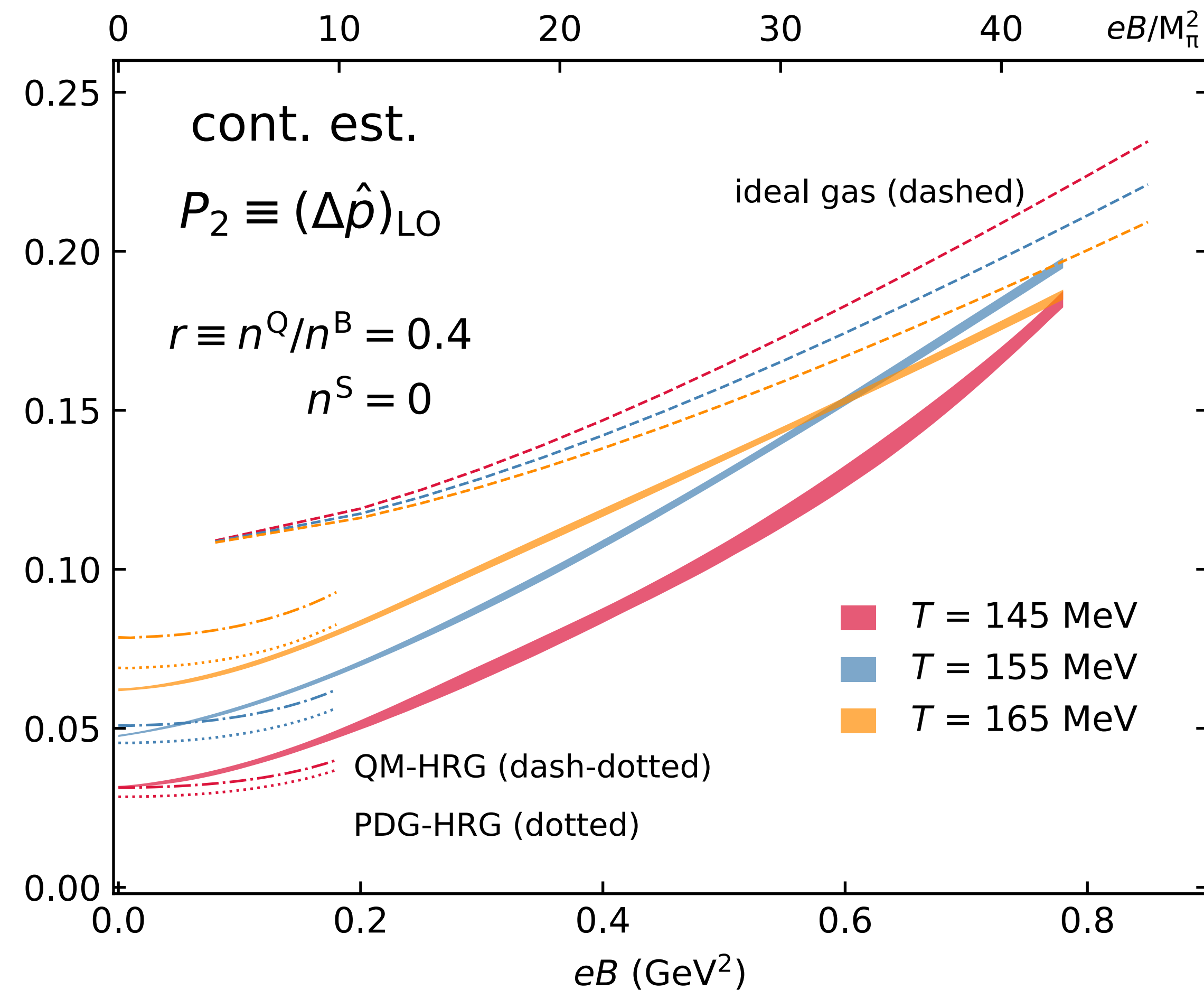
$$P_2 = \frac{1}{2} \left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2 \right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$

◆ P_2 grows with eB

◆ Strong- eB : Lattice results approach ideal gas but no saturation

$$\frac{P_2(T, eB \simeq 0.8 \text{ GeV}^2)}{P_2(T, eB = 0)} = \begin{cases} \sim 6, & T = 145 \text{ MeV} \\ \sim 4, & T = 155 \text{ MeV} \\ \sim 3, & T = 165 \text{ MeV} \end{cases}$$

◆ For $eB \gtrsim 0.6 \text{ GeV}^2$: Crossing among T -bands

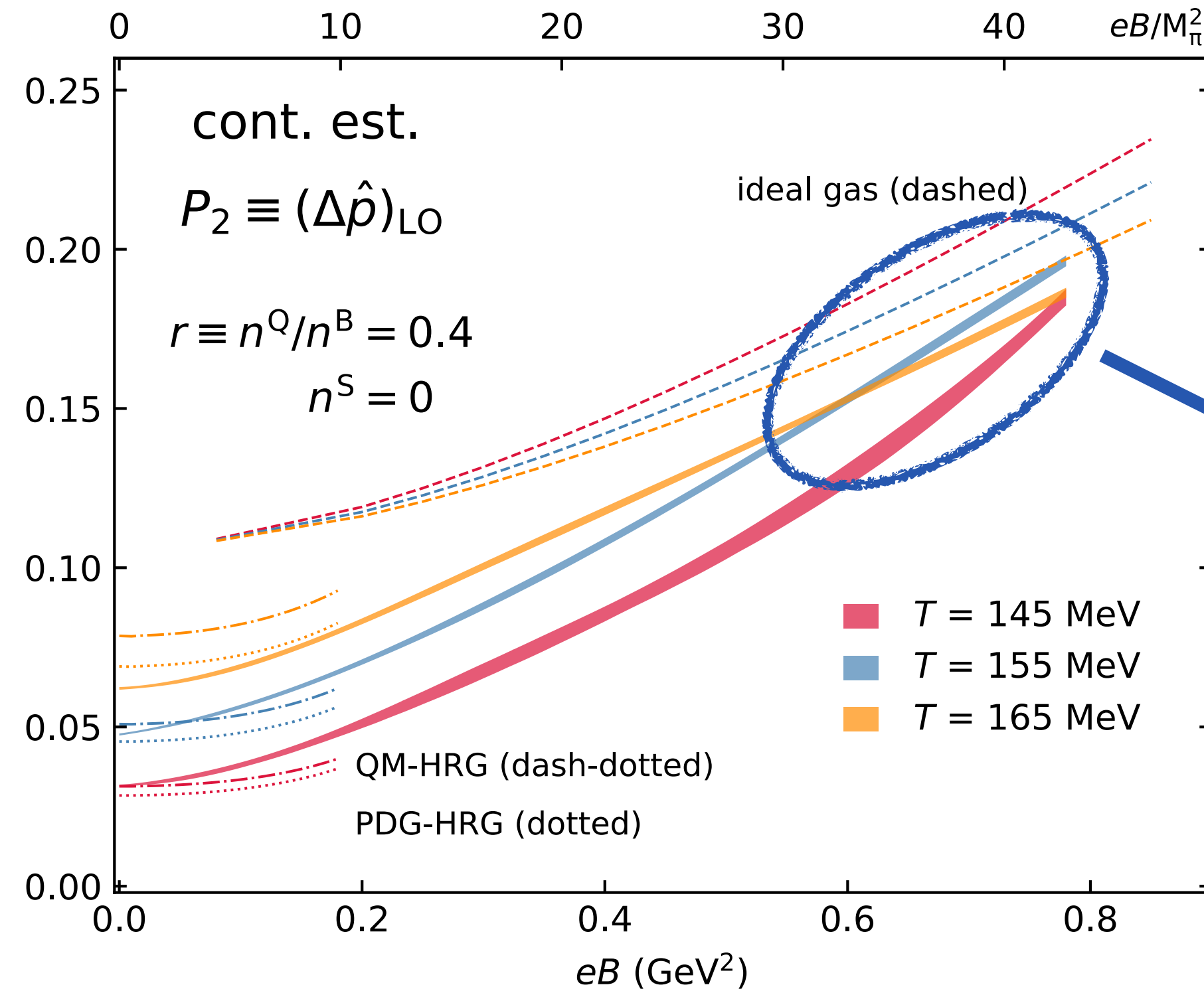


H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, arXiv:2508.07532

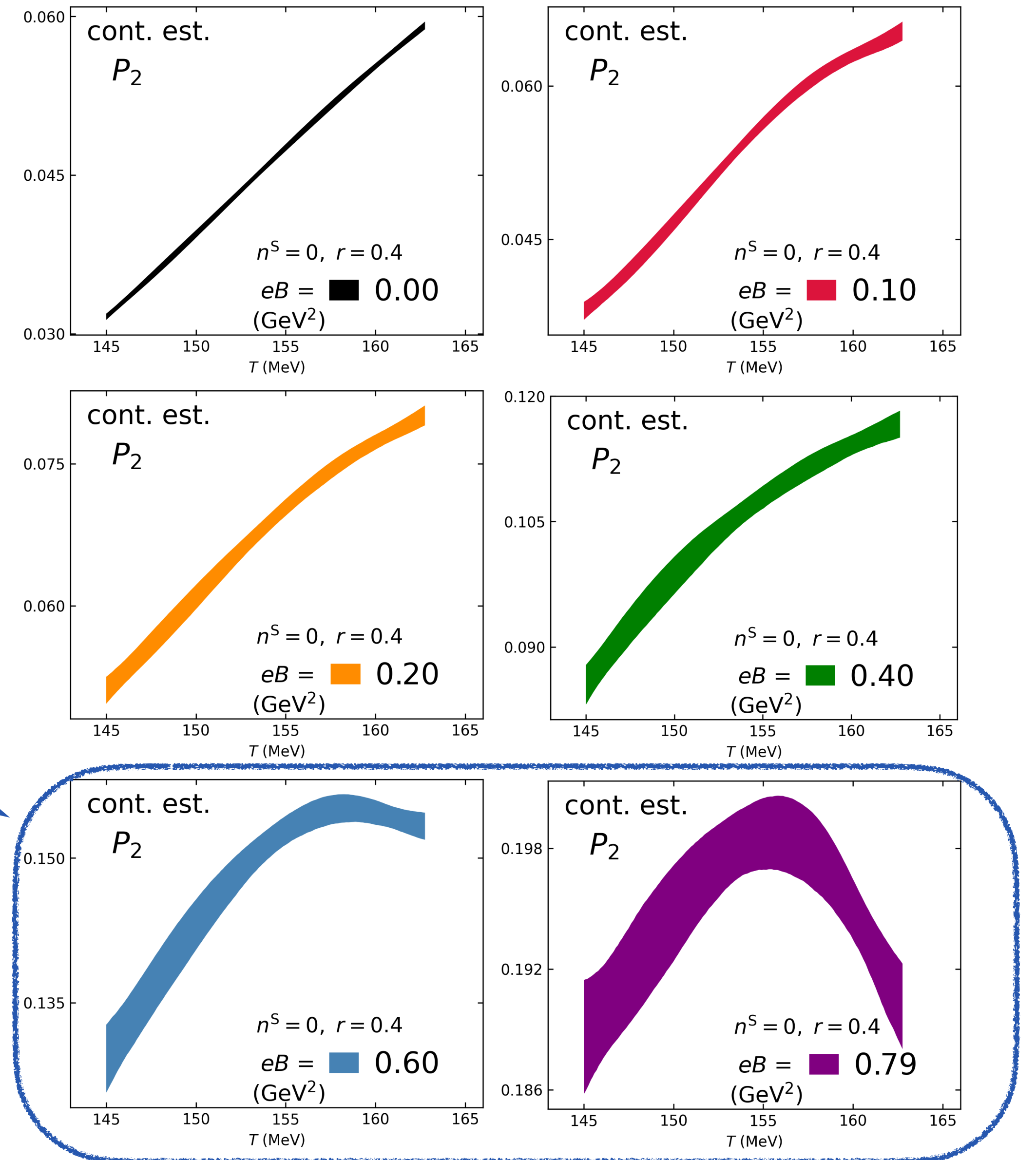
Pressure in magnetic fields

◆ For $eB \gtrsim 0.6 \text{ GeV}^2$

Crossing among T -bands $\longleftrightarrow$ Mild peak formation
Peak shift towards low- T



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, arXiv:2508.07532

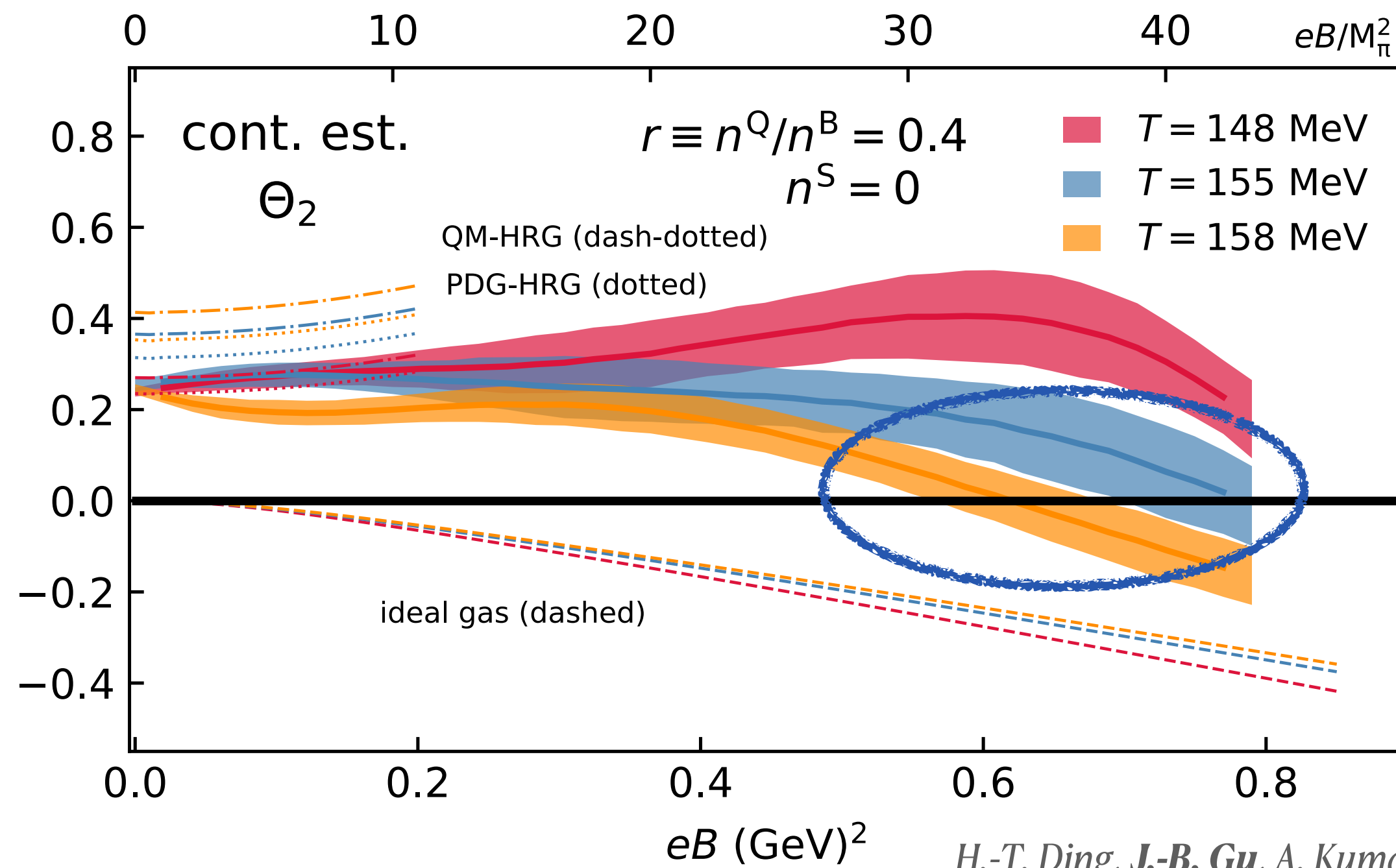


Trace anomaly density Θ_2

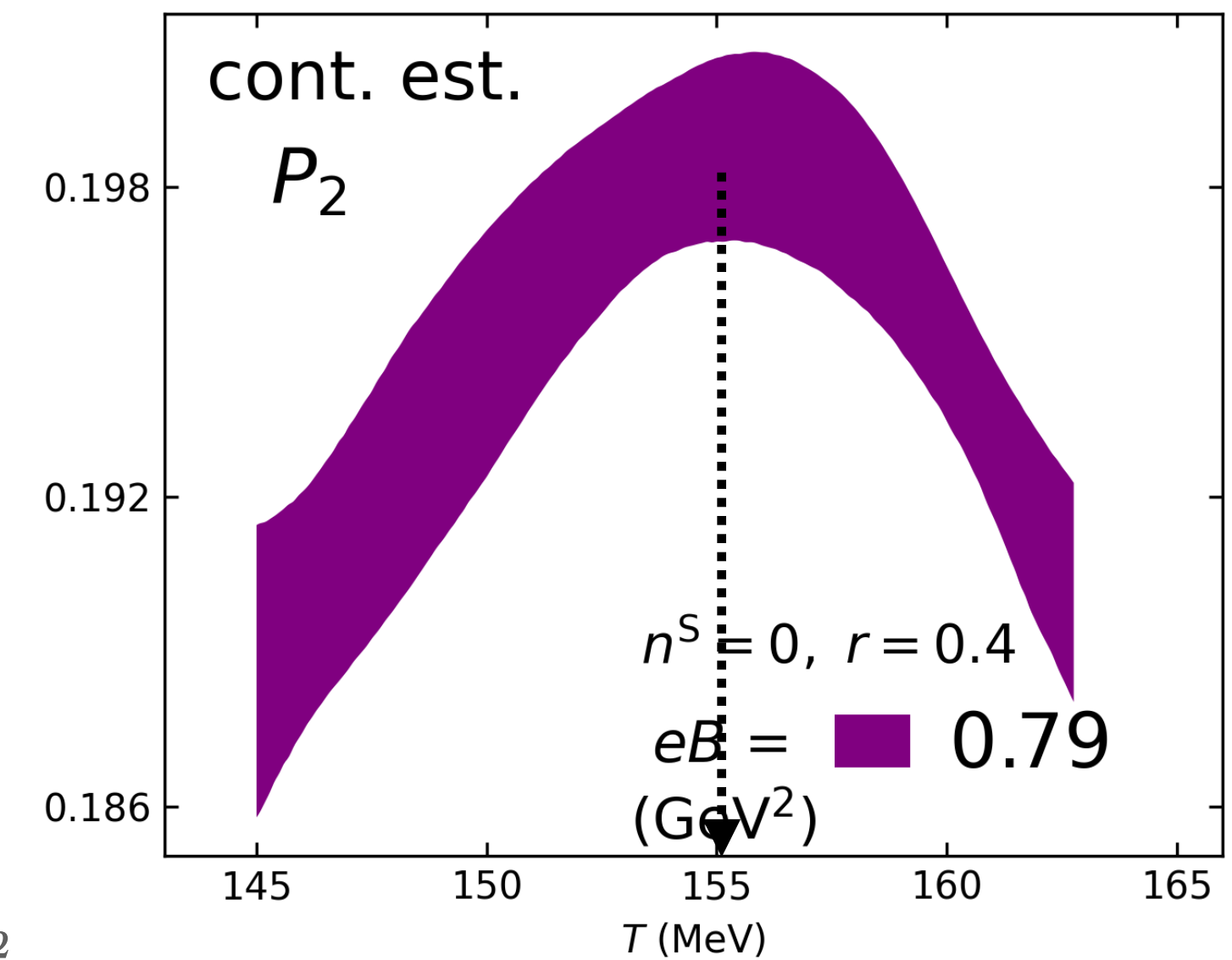
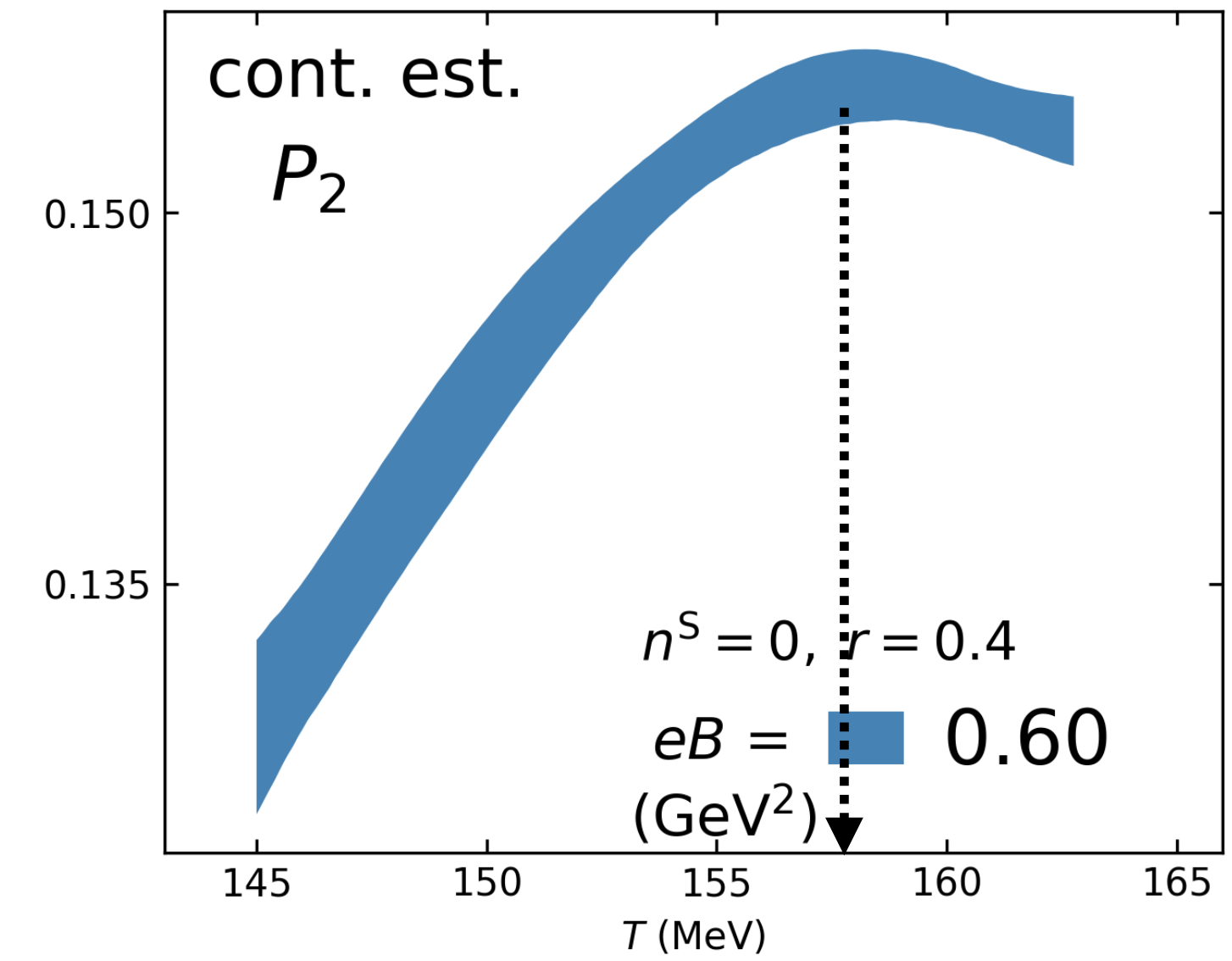
LO coefficient of the trace anomaly and energy density:

$$\Theta_2 = -rT(\partial_T q_1) N_1^B + T(\partial_T P_2)$$

Peak structure of P_2 $\longleftrightarrow$ Sign change of Θ_2



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, arXiv:2508.07532

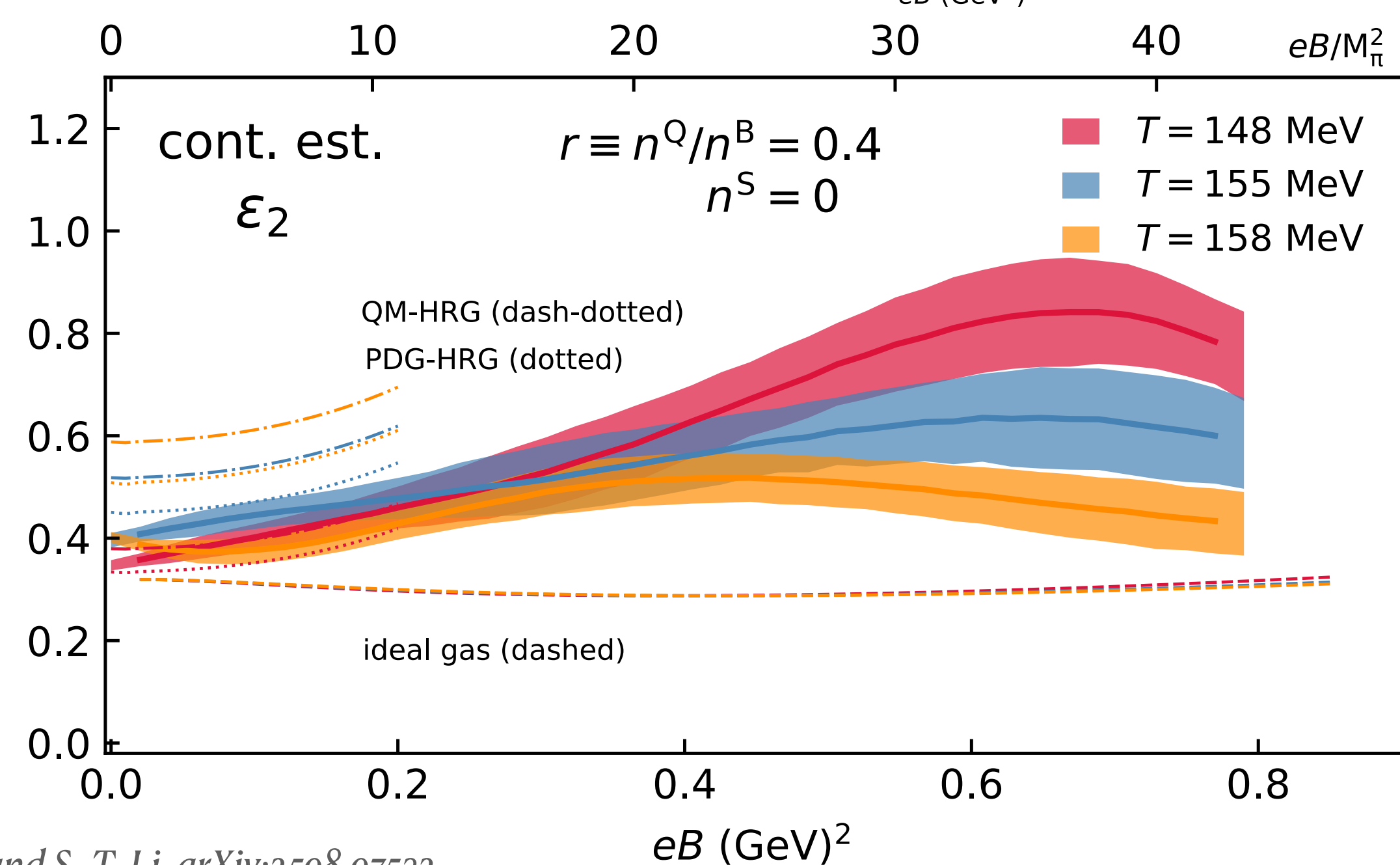
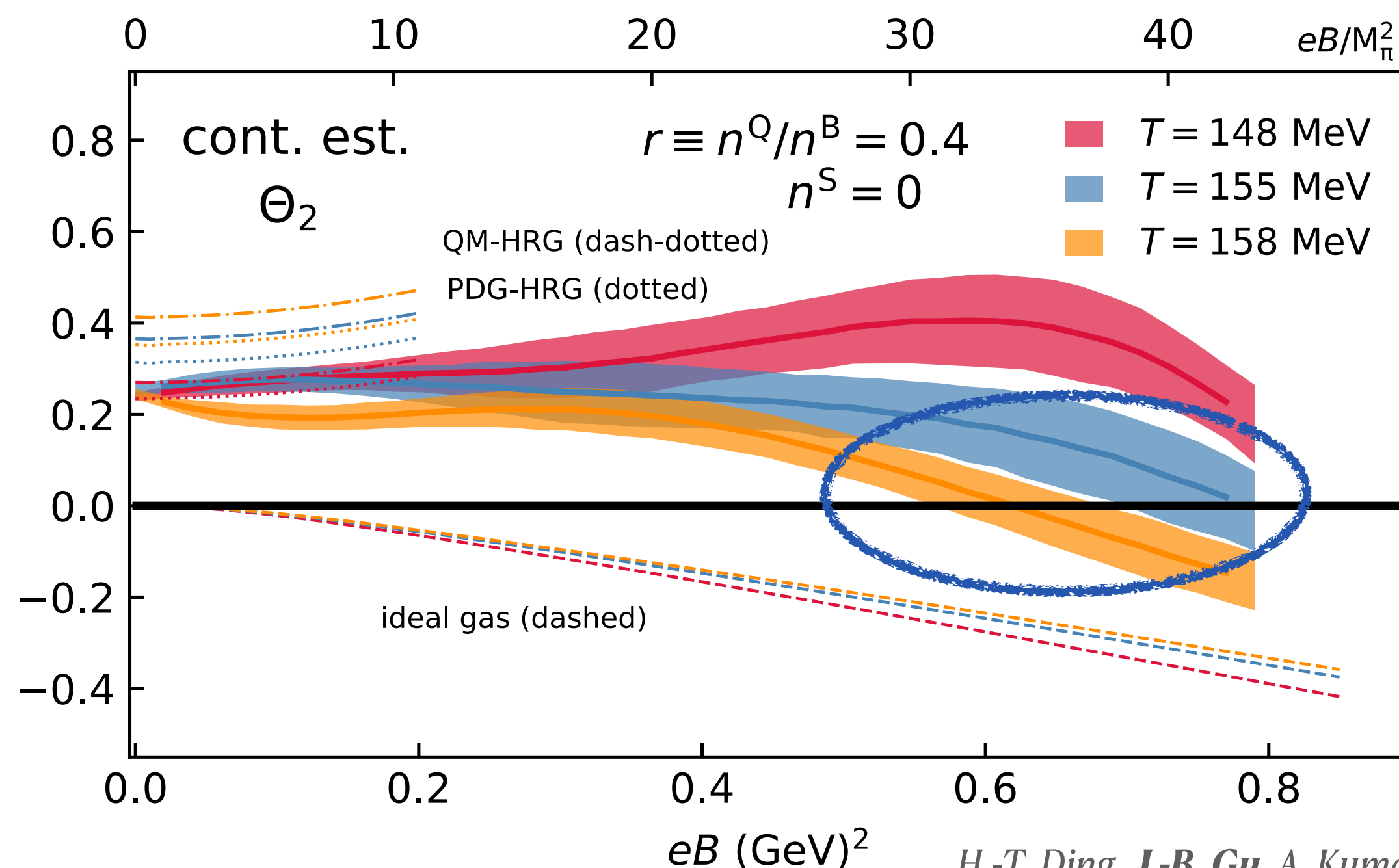
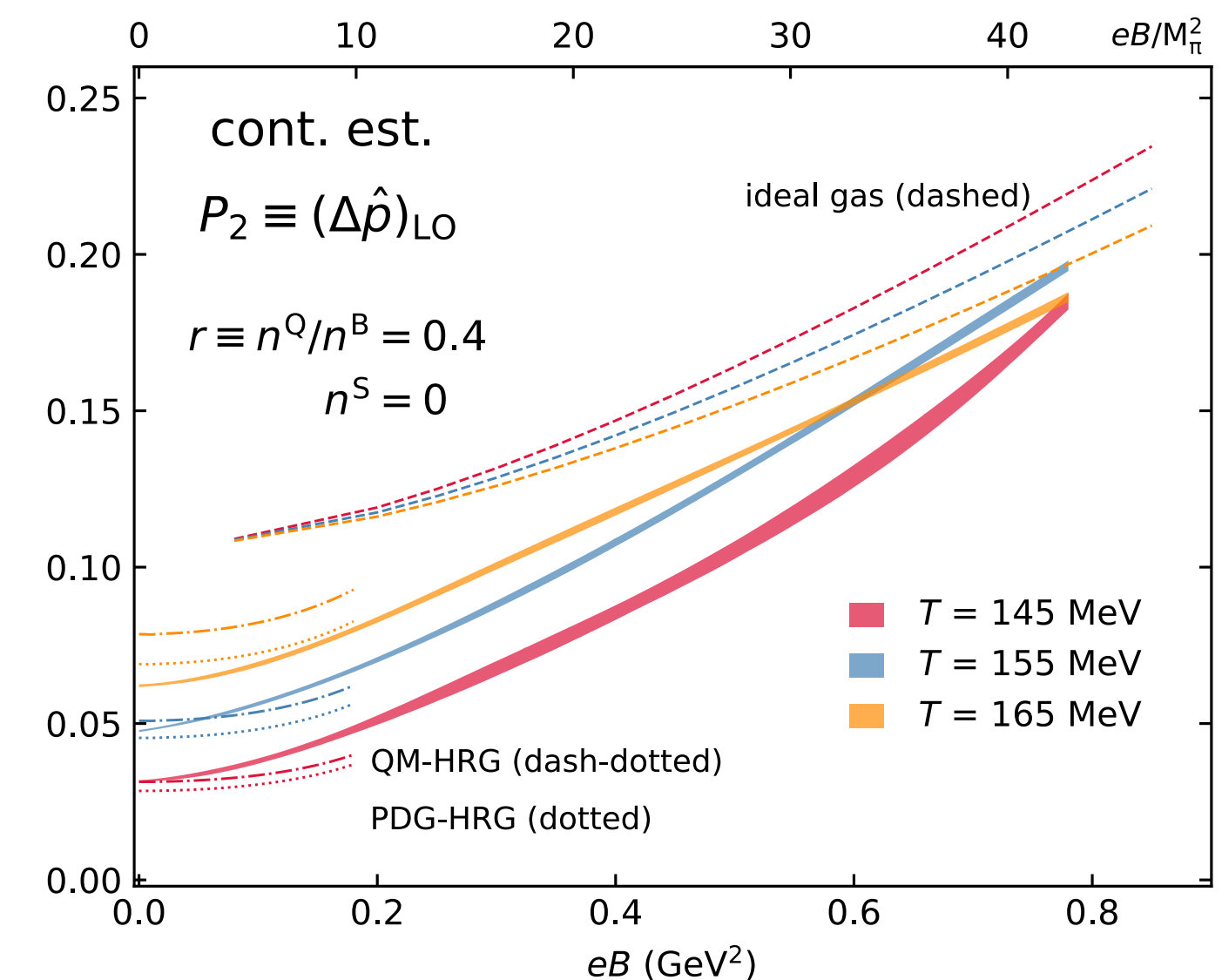


Trace anomaly density Θ_2 and energy density ϵ_2

LO coefficient of the trace anomaly and energy density:

$$\Theta_2 = -rT(\partial_T q_1) N_1^B + T(\partial_T P_2) \quad \epsilon_2 = 3P_2 + \Theta_2$$

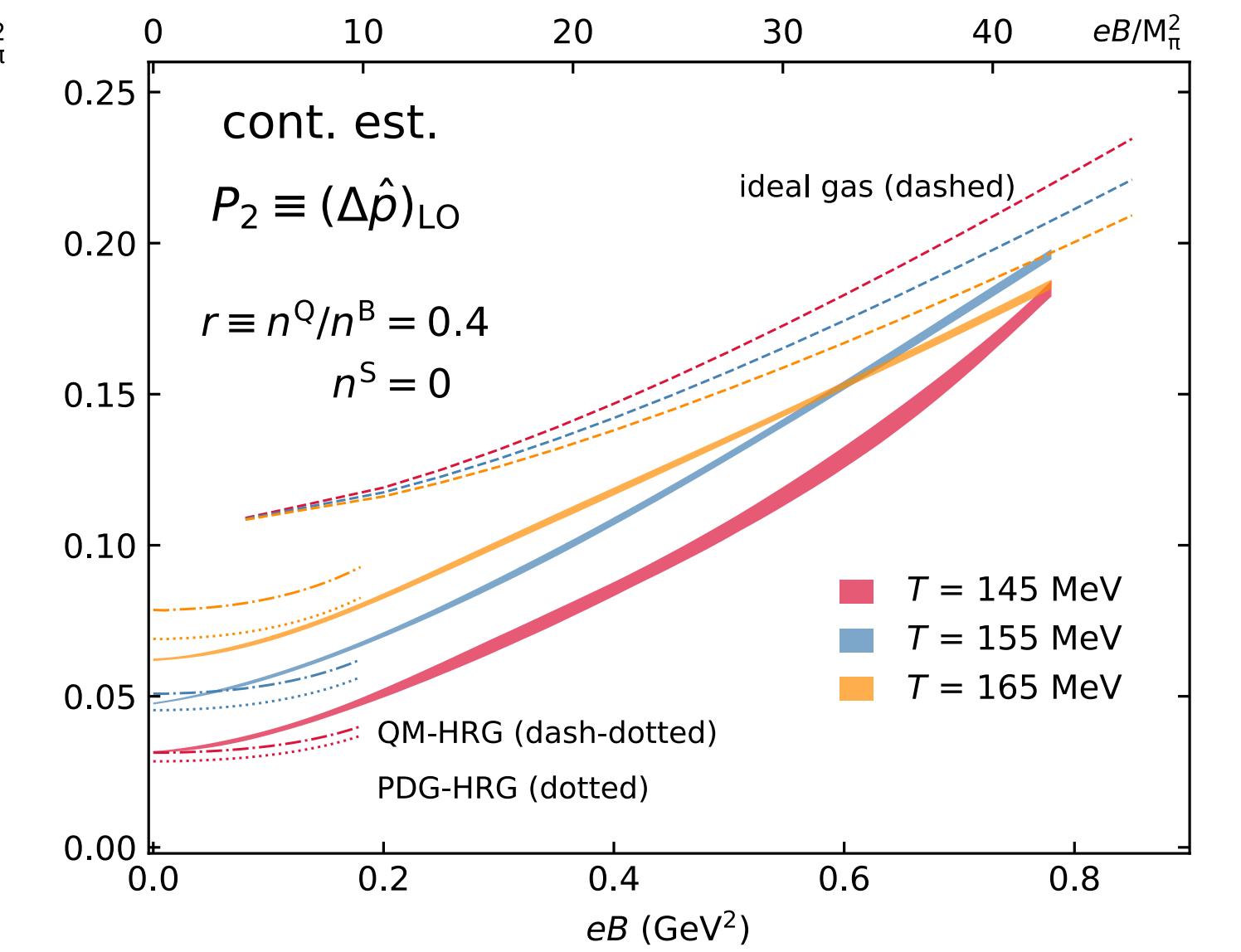
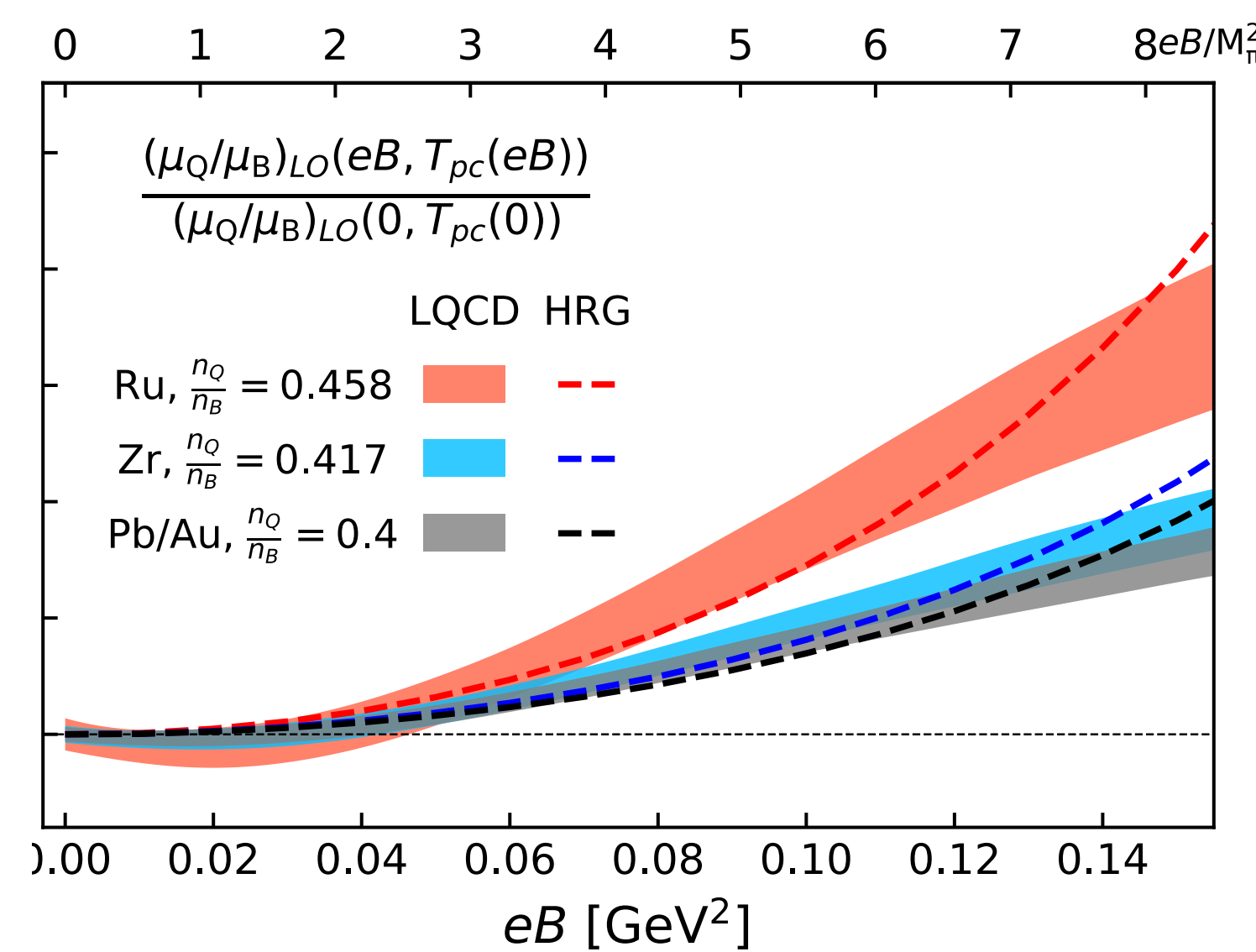
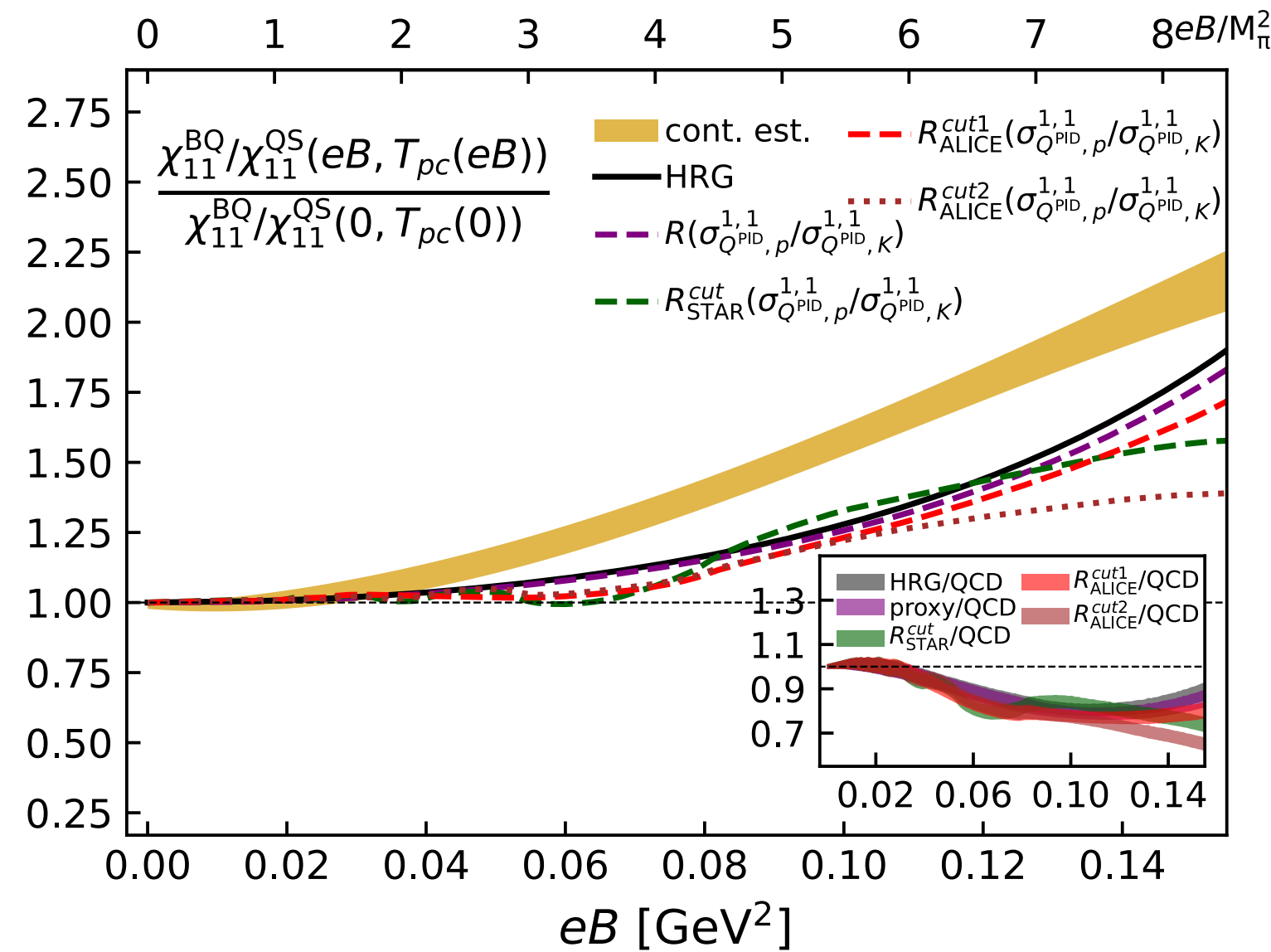
Peak structure of P_2 $\longleftrightarrow$ Sign change of Θ_2
Reduction of ϵ_2



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, arXiv:2508.07532

Summary

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges and LO EoS at nonzero μ_B based on LQCD computation on $N_\tau=8$ and 12 lattices
- χ_{11}^{BQ} and μ_Q/μ_B is strongly affected by eB , and a promising magnetometer $\chi_{11}^{\text{BQ}}/\chi_{11}^{\text{QS}}$ is provided for measurement in HIC
- P_2 : Crossing among fixed T -bands $\rightarrow$ Non-monotonic and peak structure



Thank you!

Backup

B pointing along the z direction

$$u_x(n_x, n_y, n_z, n_\tau) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$u_y(n_x, n_y, n_z, n_\tau) = \exp[iqa^2Bn_x]$$

$$u_z(n_x, n_y, n_z, n_\tau) = u_t(n_x, n_y, n_z, n_\tau) = 1$$

No sign problem !

Quantization of the magnetic field

$$\begin{aligned} q_u &= 2/3 \, e \\ q_d &= -1/3 \, e \\ q_s &= -1/3 \, e \end{aligned} \longrightarrow eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

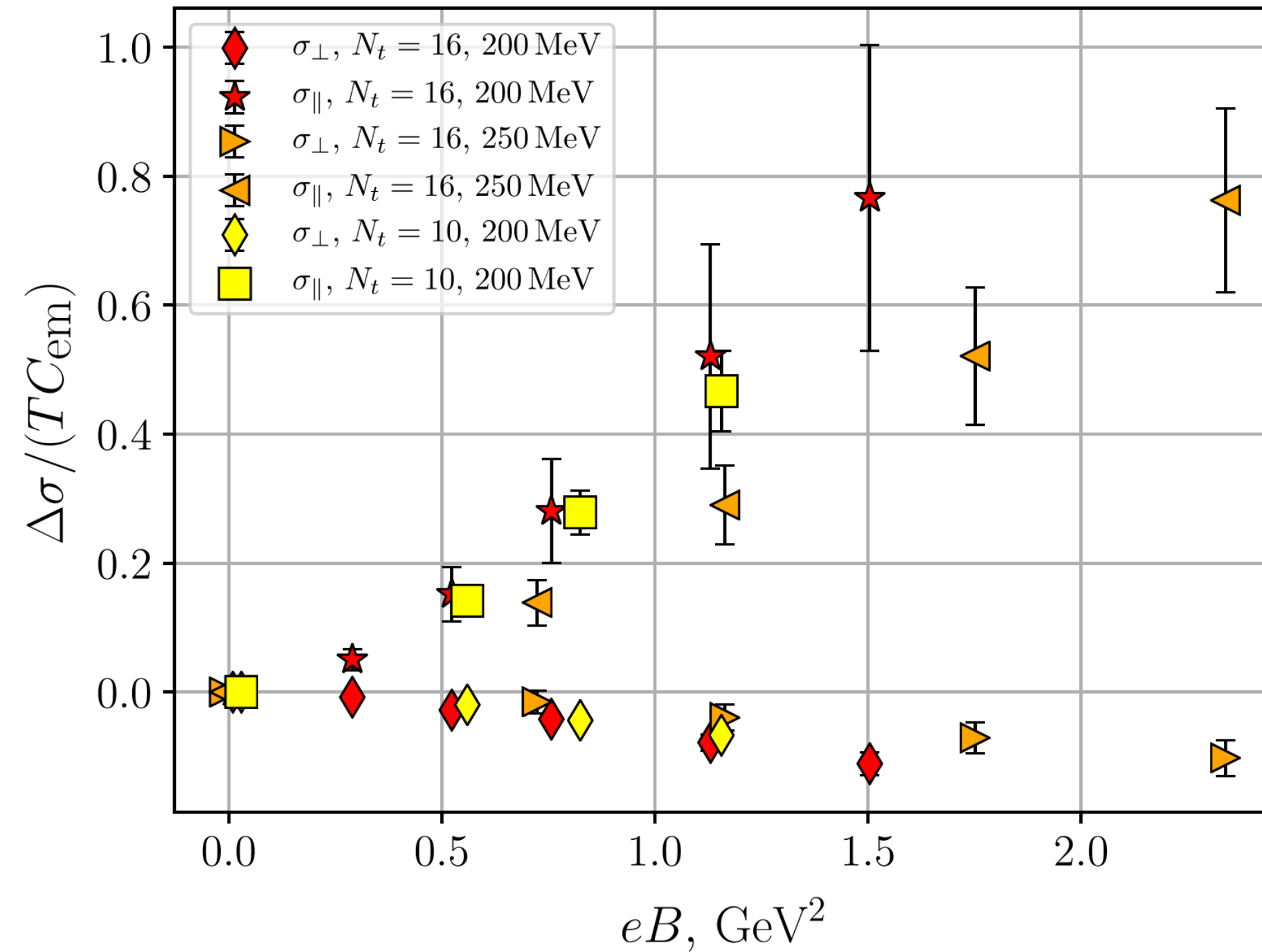
a is changed to get the targeted T , $T = \frac{1}{aN_\tau}$

- Statistics($eB \neq 0$): $N_\tau=8$: ~ 40000 (N_{rv} : 603)
 $N_\tau=12$: ~ 5000 (N_{rv} : 102 $\sim$ 705)
 $N_\tau=16$: ~ 3000 (N_{rv} : 603)

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz,
S. Krieg et al., JHEP 02 (2012) 044.

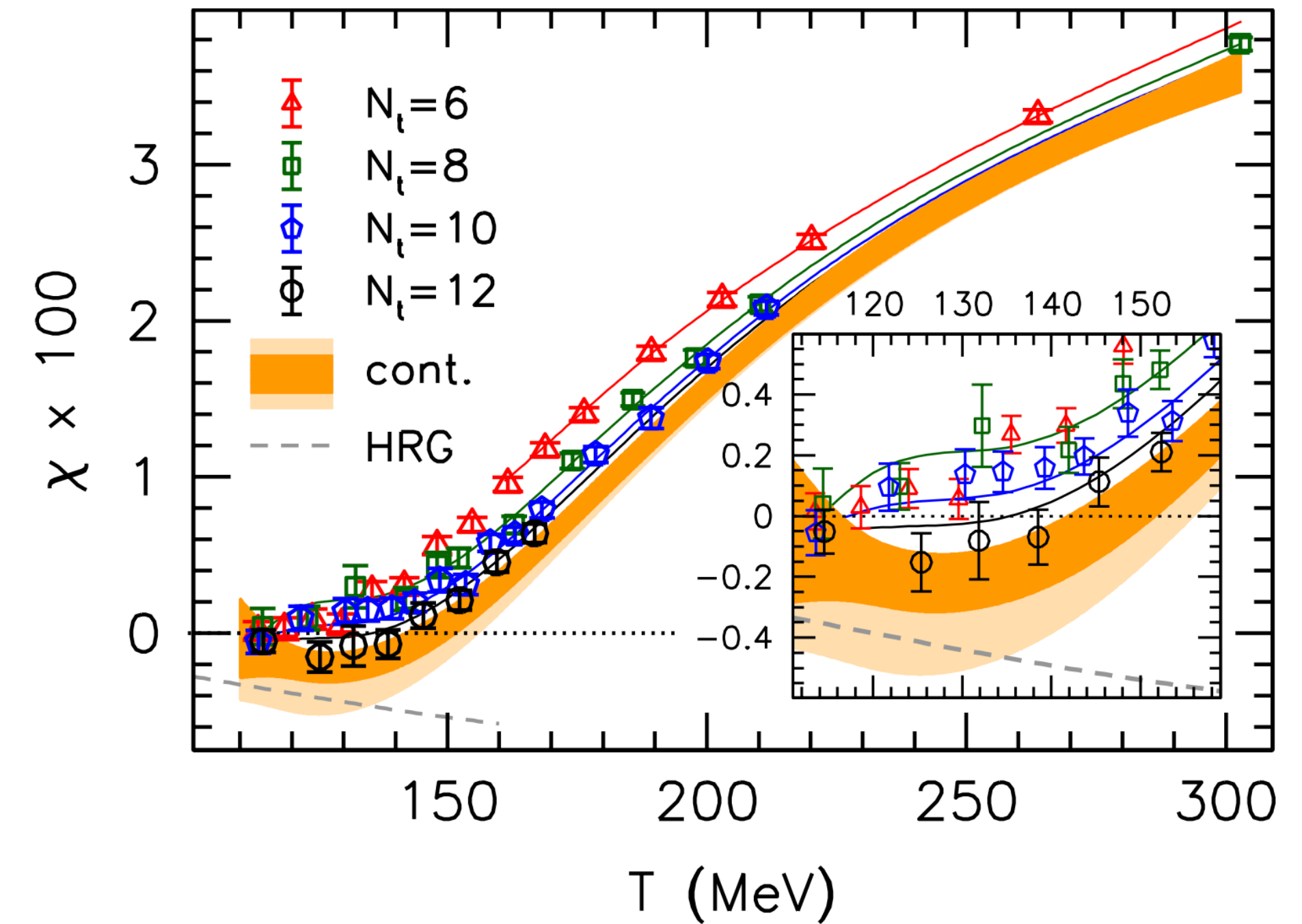
Electromagnetic conductivity and type of magnetism of QGP

Electromagnetic conductivities at non-zero magnetic fields



N. Astrakhantsev et al., PRD 102 (2020) 054516

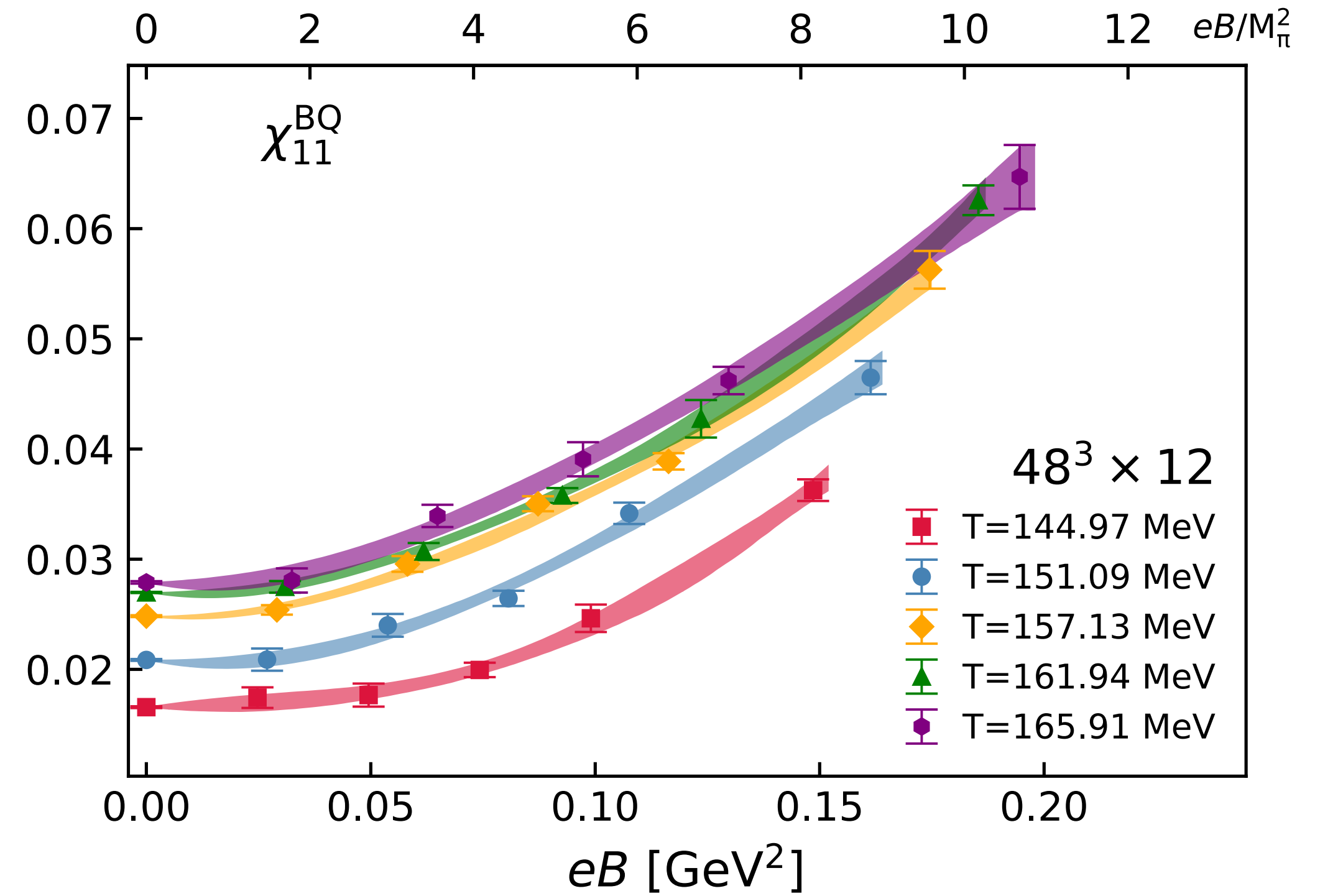
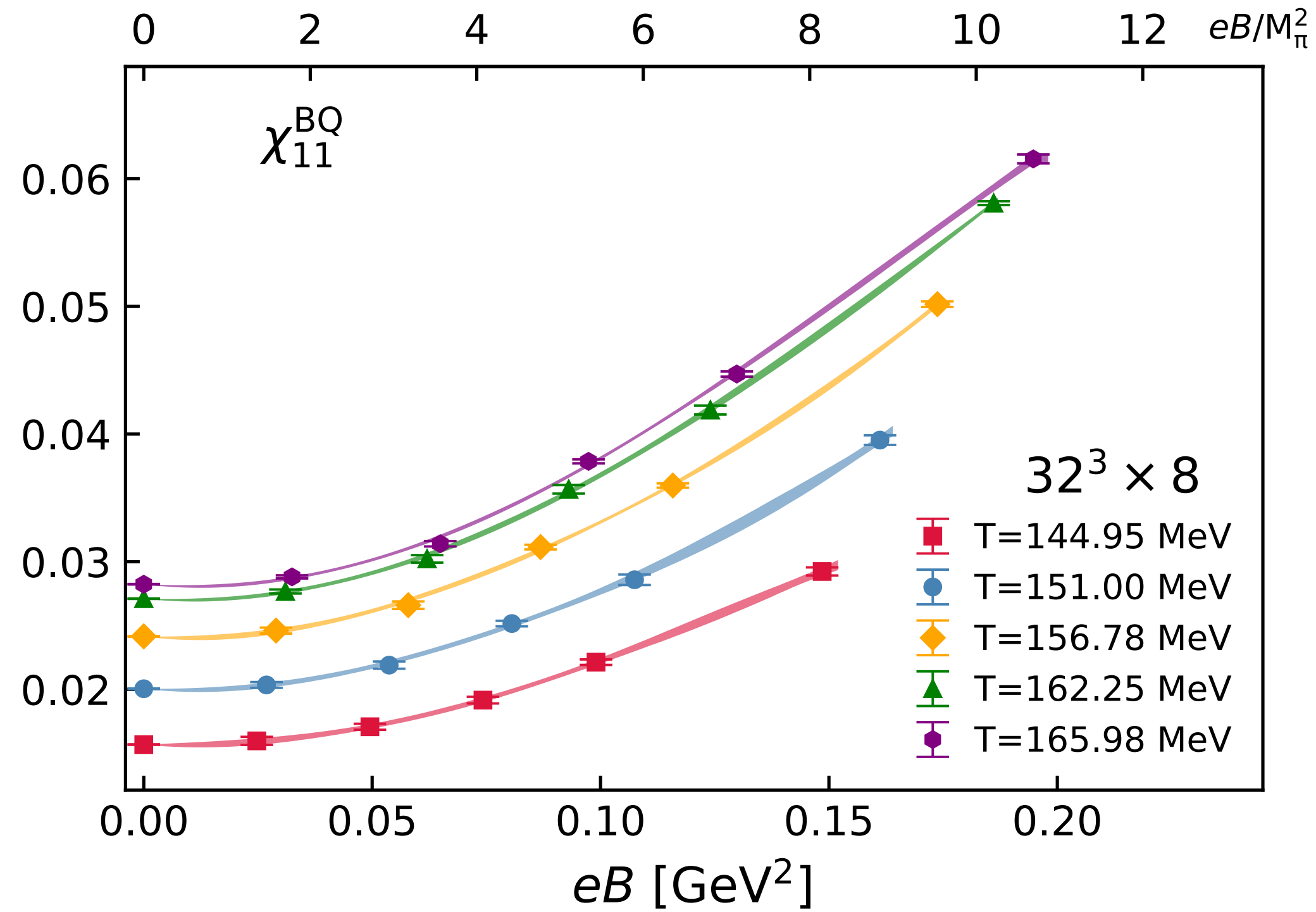
Magnetic susceptibility at non-zero magnetic fields



G. Bali et al., JHEP 07 (2020) 183

The magnetic field could live **longer** in the evolution of HICs than in the vacuum.

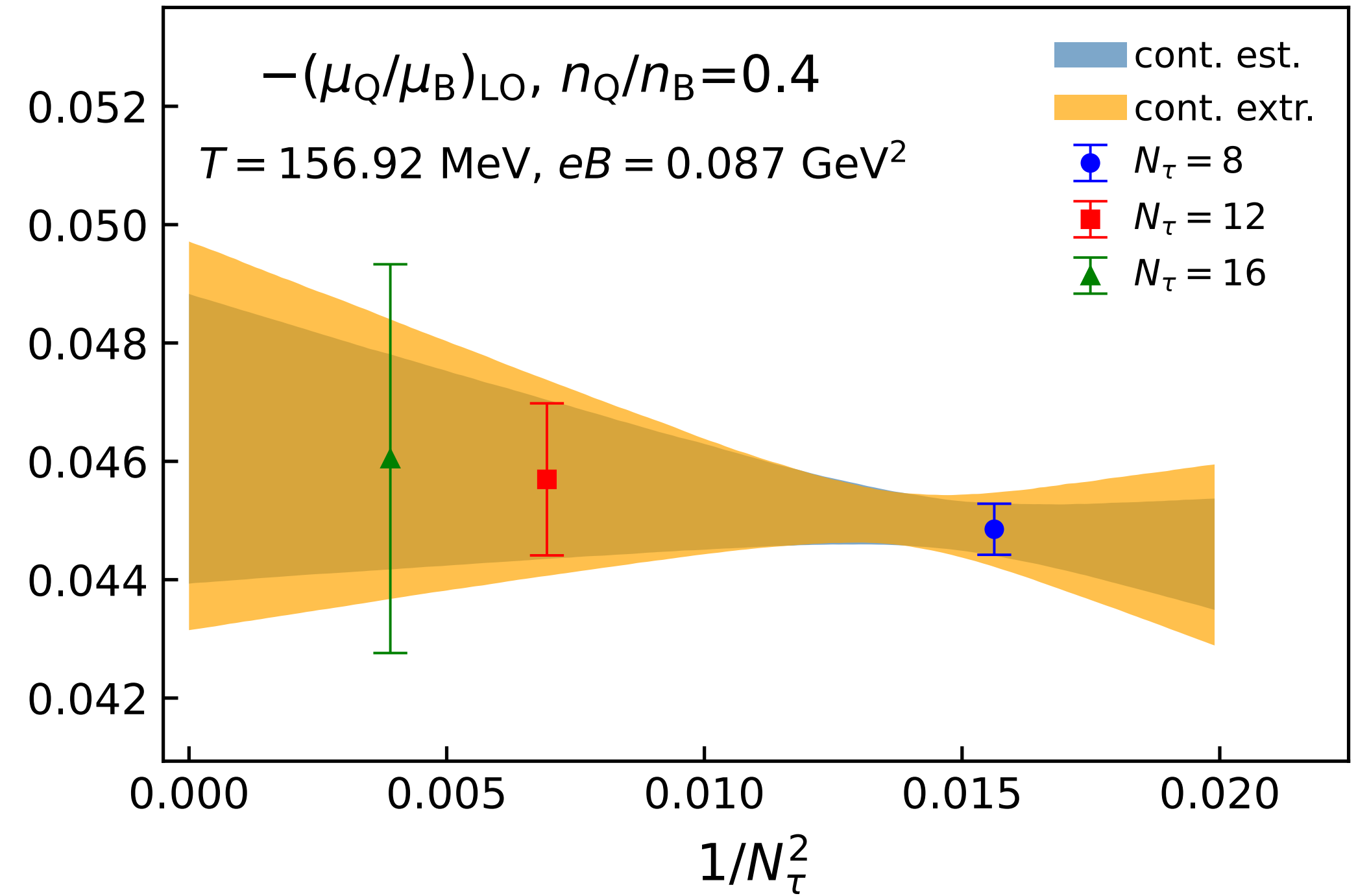
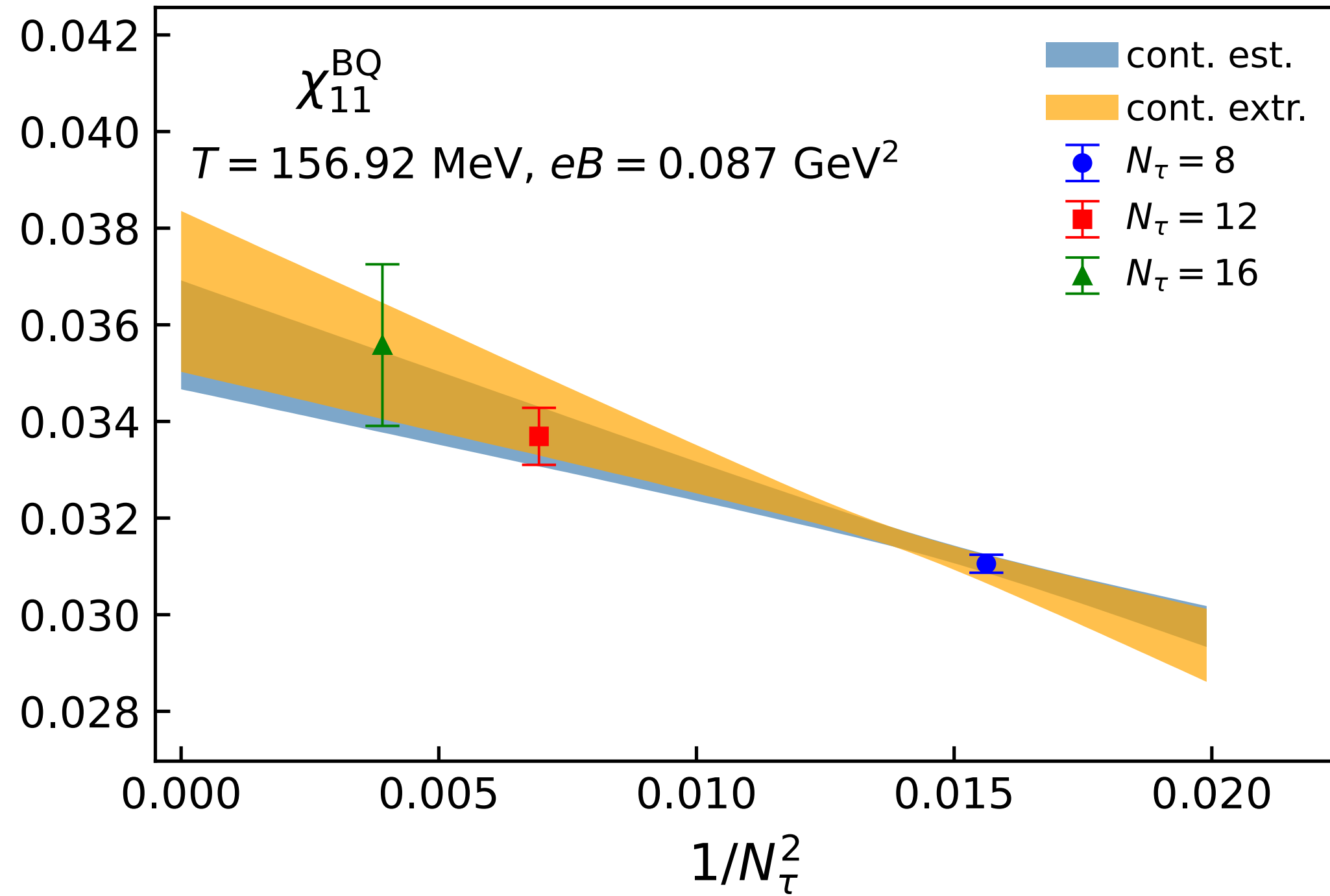
Lattice data on $N_\tau = 8$ and 12 lattices



H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li, Phys. Rev. Lett. 132, 201903 (2024)

The zero magnetic field data comes from D. Bollweg et al., Phys. Rev. D 104, 074512 (2021)

Continuum estimate and extrapolation

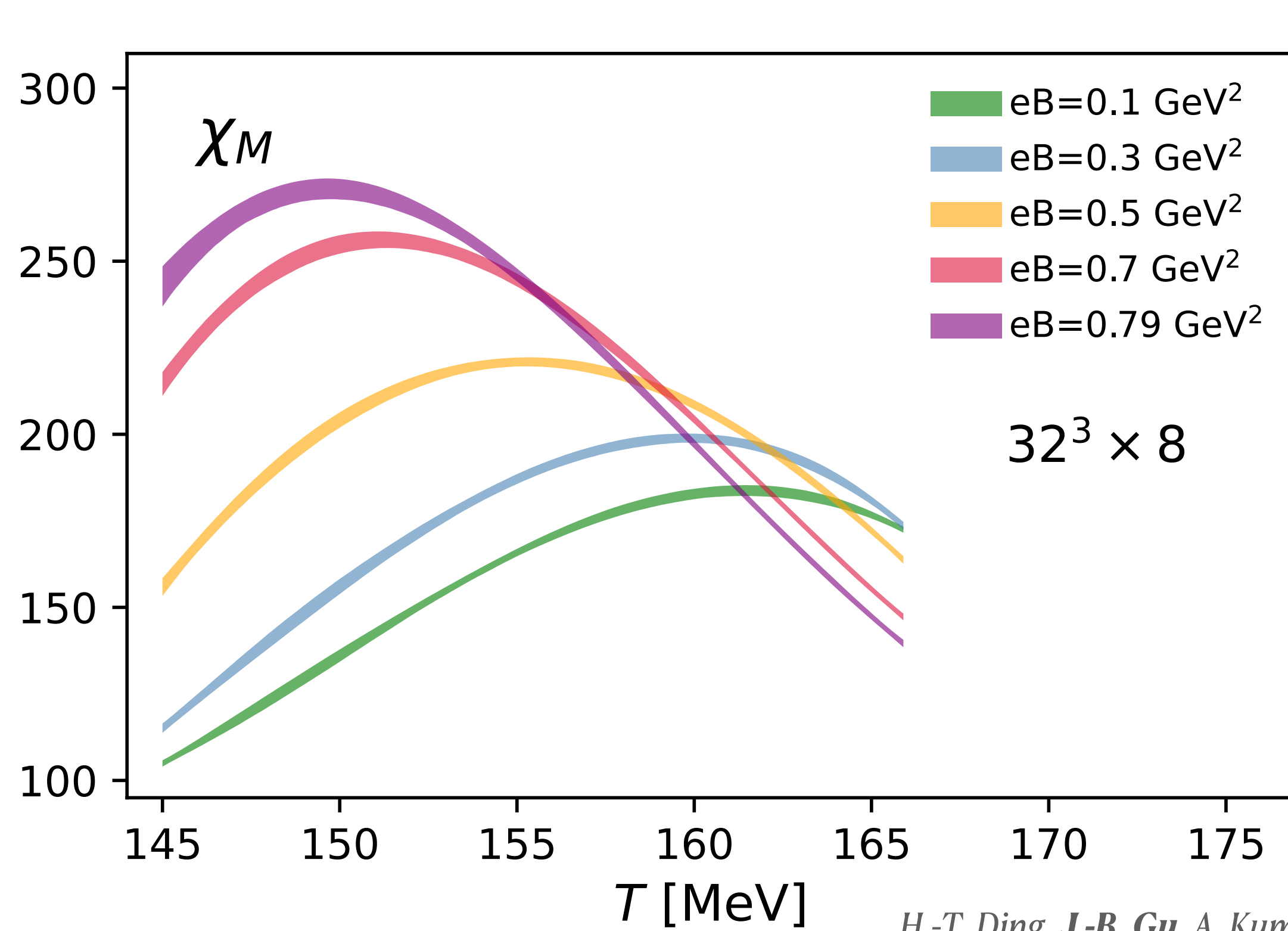


H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, Phys. Rev. Lett. 132, 201903 (2024)

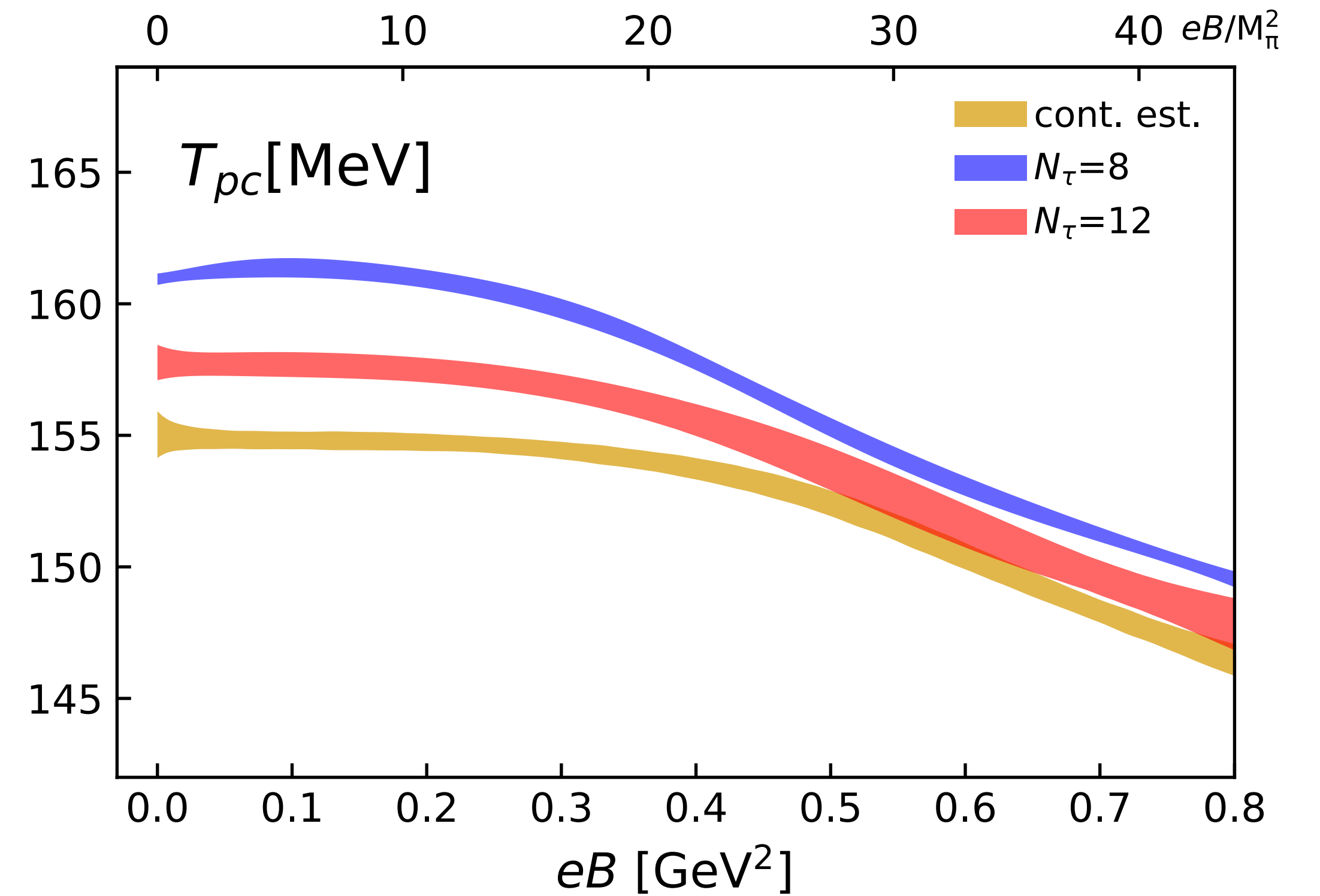
$$\mathcal{O}(T, eB, N_\tau) = \mathcal{O}(T, eB) + \frac{c}{N_\tau^2} \begin{cases} \text{Continuum estimate} \\ \text{Continuum extrapolation} \end{cases} \quad \begin{matrix} \text{obtained from} \\ N_\tau = 8 \text{ and } 12 \\ N_\tau = 8, 12, \text{ and } 16 \end{matrix}$$

Continuum estimate and continuum extrapolation are consistent within uncertainty

QCD transition temperature in nonzero magnetic field



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, Phys.Rev.D 111, 114522 (2025)



$$\chi_M(eB) = \frac{m_s}{f_K^4} \left[m_s \chi_l(eB) - 2 \langle \bar{\psi} \psi \rangle_s(eB=0) - 4 m_l \chi_{sl}(eB=0) \right]$$

$T_{pc}(eB)$ determined as the peak location of chiral susceptibility (χ_M) at each eB value

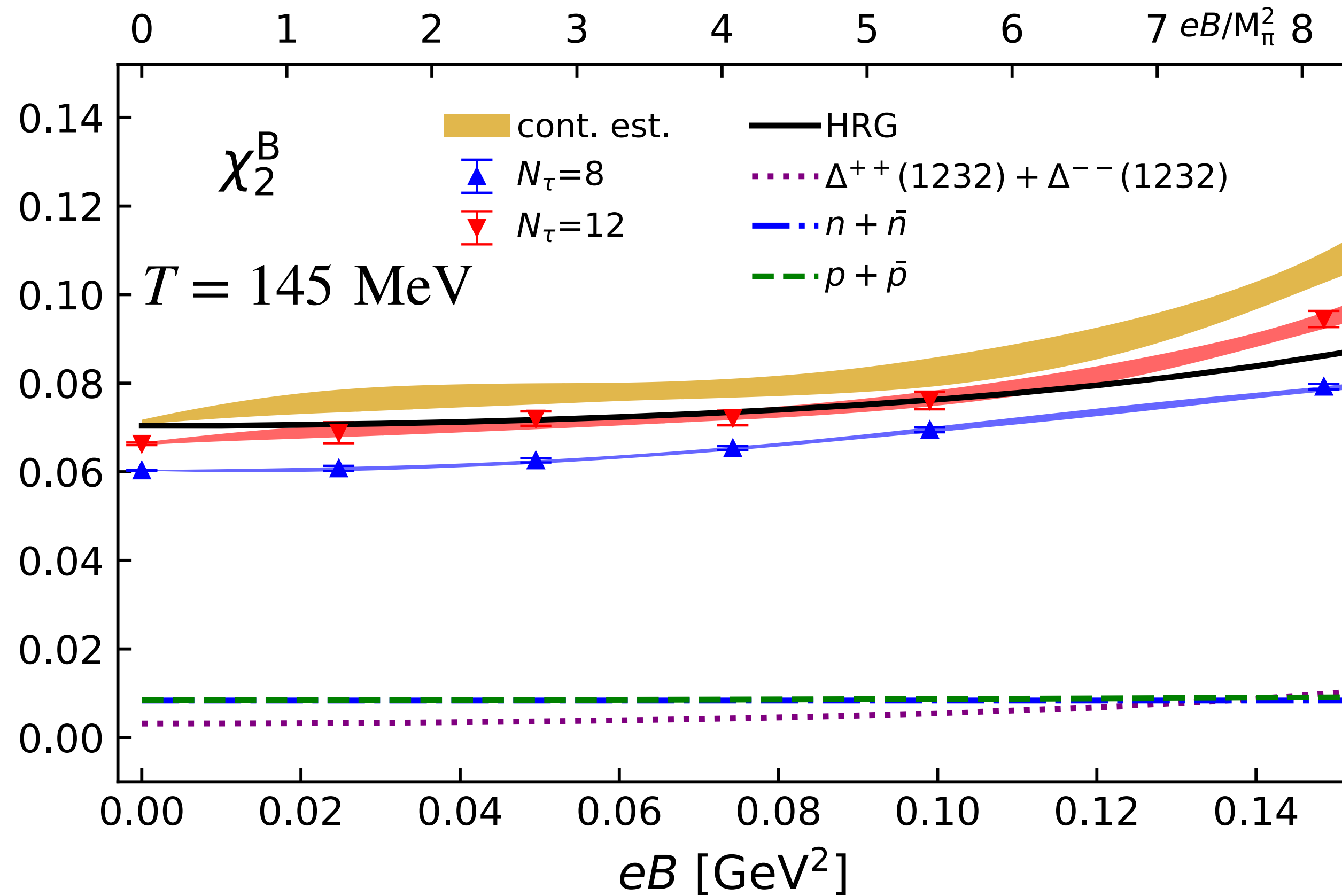
H.-T. Ding et al., Phys. Rev. Lett. 123, 062002 (2019)

T_{pc} : ▶ almost **independent** of eB , $eB \lesssim 0.3 \text{ GeV}^2$

▶ **decreases** with eB , $eB \gtrsim 0.3 \text{ GeV}^2$

H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, Phys. Rev. Lett. 132, 201903 (2024)

Baryon number fluctuations at $T = 145$ MeV



H.-T. Ding, **J.-B. Gu**, A. Kumar, S.-T. Li and J.-H. Liu, *Phys. Rev. Lett.* 132, 201903 (2024)

❖ χ_2^B increases $\sim 45\%$ at $eB \sim 8M_\pi^2$

❖ Hadron Resonance Gas model (HRG):
Pressure arising from charged hadrons
($eB \neq 0$):

$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i| B}{2\pi^2 T^3} \sum_{s_z = -s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1 \left(\frac{n\varepsilon_0}{T} \right)$$

❖ χ_2^B receives contributions also from
neutral baryons

H.-T. Ding et al., *Eur. Phys. J. A* 57 (2021) 6, 202

Proxy in experiment and kinematic cuts

◆ Conserved charges susceptibilities in experiment:

$$\chi_\alpha^2 = \frac{1}{VT^3} \kappa_\alpha^2, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3} \kappa_{\alpha,\beta}^{1,1}$$

the second-order cumulants(κ) are the variance or covariance(σ) of the net-multiplicity N :

$$\kappa_\alpha^2 = \sigma_\alpha^2 = \langle (\delta N_\alpha - \langle \delta N_\alpha \rangle)^2 \rangle$$

$$\kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle (\delta N_\alpha - \langle \delta N_\alpha \rangle)(\delta N_\beta - \langle \delta N_\beta \rangle) \rangle$$

with $\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}$ and $\alpha, \beta = p, Q^{PID}, k$

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID} : identified π, k and p

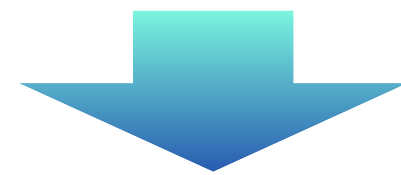
STAR, Phys.Rev.C 100 (2019) 1, 014902

◆ Proxies with kinematic cuts on p_T and η within HRG framework:

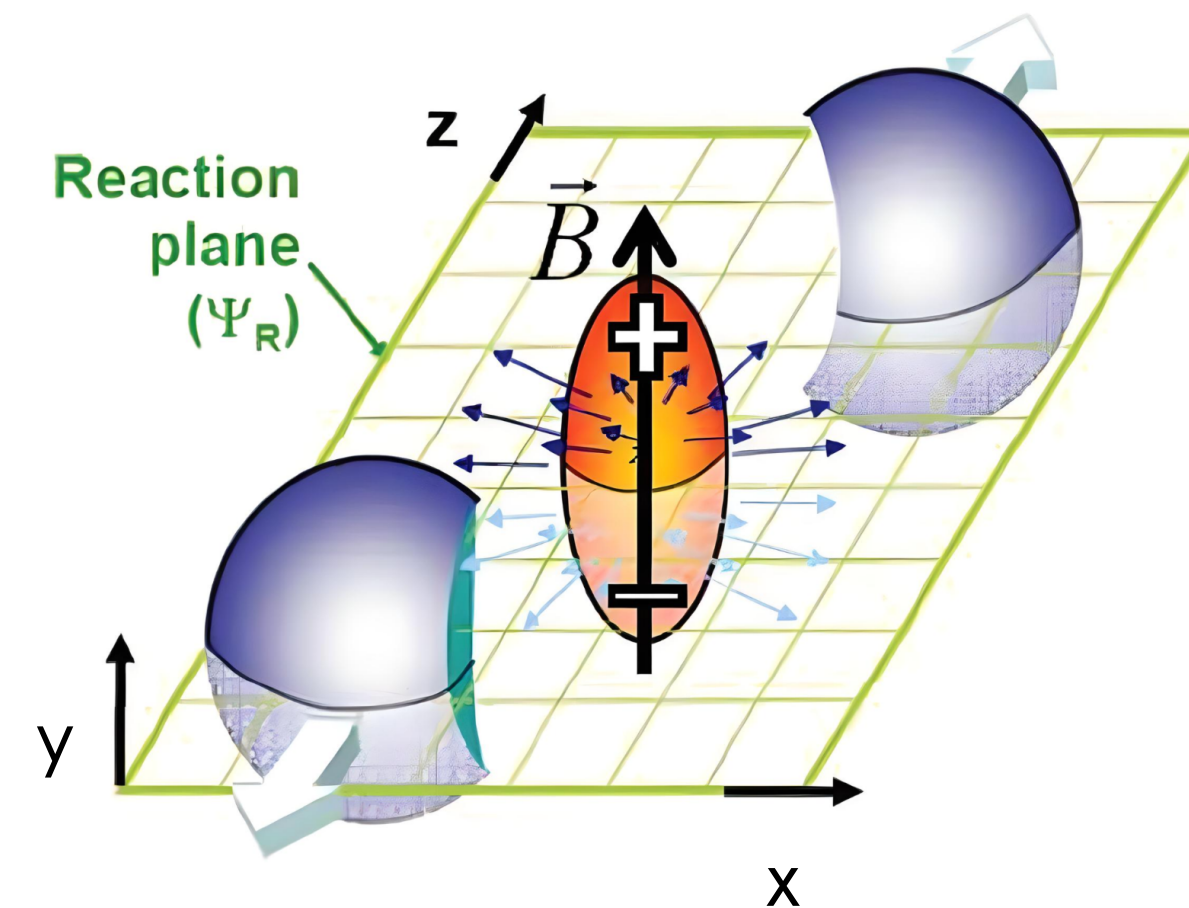
$$\int d^3p \times \Theta(p_{T_{\min}}, p_{T_{\max}}, \eta_{\min}, \eta_{\max})$$

Quantized momentum in reaction plane:

$$p_{xz}(l, s_y) = \sqrt{2|q|B(l + 1/2 + s_y)}$$



$$\eta = \operatorname{arcsinh}(p_{xz} \cos \phi_{xz} / p_T) \quad p_T = \sqrt{(p_{xz} \sin \phi_{xz})^2 + p_y^2}$$



Proxy construction based on the HRG

$\Delta^{++}(1232) \rightarrow p + \pi^+$: branching ratio almost **100%** !

HRG: Fluctuations expressed in terms of stable hadronic states:

$$\chi_{ijk}^{\text{BQS}} \left(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S \right) = \sum_R B_R^i Q_R^j S_R^k \frac{\partial^l p_R / T^4}{\partial \hat{\mu}_R^l}$$

net- B : $\tilde{p} + \tilde{n} + \tilde{\Lambda} + \tilde{\Sigma}^+ + \tilde{\Sigma}^- + \tilde{\Xi}^0 + \tilde{\Xi}^- + \tilde{\Omega}^-$
 net- Q : $\tilde{\pi}^+ + \tilde{K}^+ + \tilde{p} + \tilde{\Sigma}^+ - \tilde{\Sigma}^- - \tilde{\Xi}^- - \tilde{\Omega}^-$
 net- S : $\tilde{K}^+ + \tilde{K}^0 - \tilde{\Lambda} - \tilde{\Sigma}^+ - \tilde{\Sigma}^- - 2\tilde{\Xi}^0 - 2\tilde{\Xi}^- - 3\tilde{\Omega}^-$

B_R, Q_R, S_R are the baryon number, electric charge and strangeness of the species R

R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)

HIC: fluctuations are related to the variance or covariance of net-multiplicity for identified π, K, p

STAR, Phys.Rev.C 100, 014902 (2019); STAR, Phys.Rev.C 105, 029901 (2019)

$\sigma_{Q^{\text{PID}},p}^{1,1}$ as proxy for χ_{11}^{BQ}

$$\sum_R B_R Q_R \frac{\partial^2 p_R / T^4}{\partial \mu_R^2} \rightarrow \sum_R (\omega_{R \rightarrow \tilde{p}})(\omega_{R \rightarrow \tilde{Q}^{\text{PID}}}) \frac{\partial^2 p_R / T^4}{\partial \mu_R^2}$$

net-B $\rightarrow \tilde{p} : \omega_{R \rightarrow \tilde{p}}$

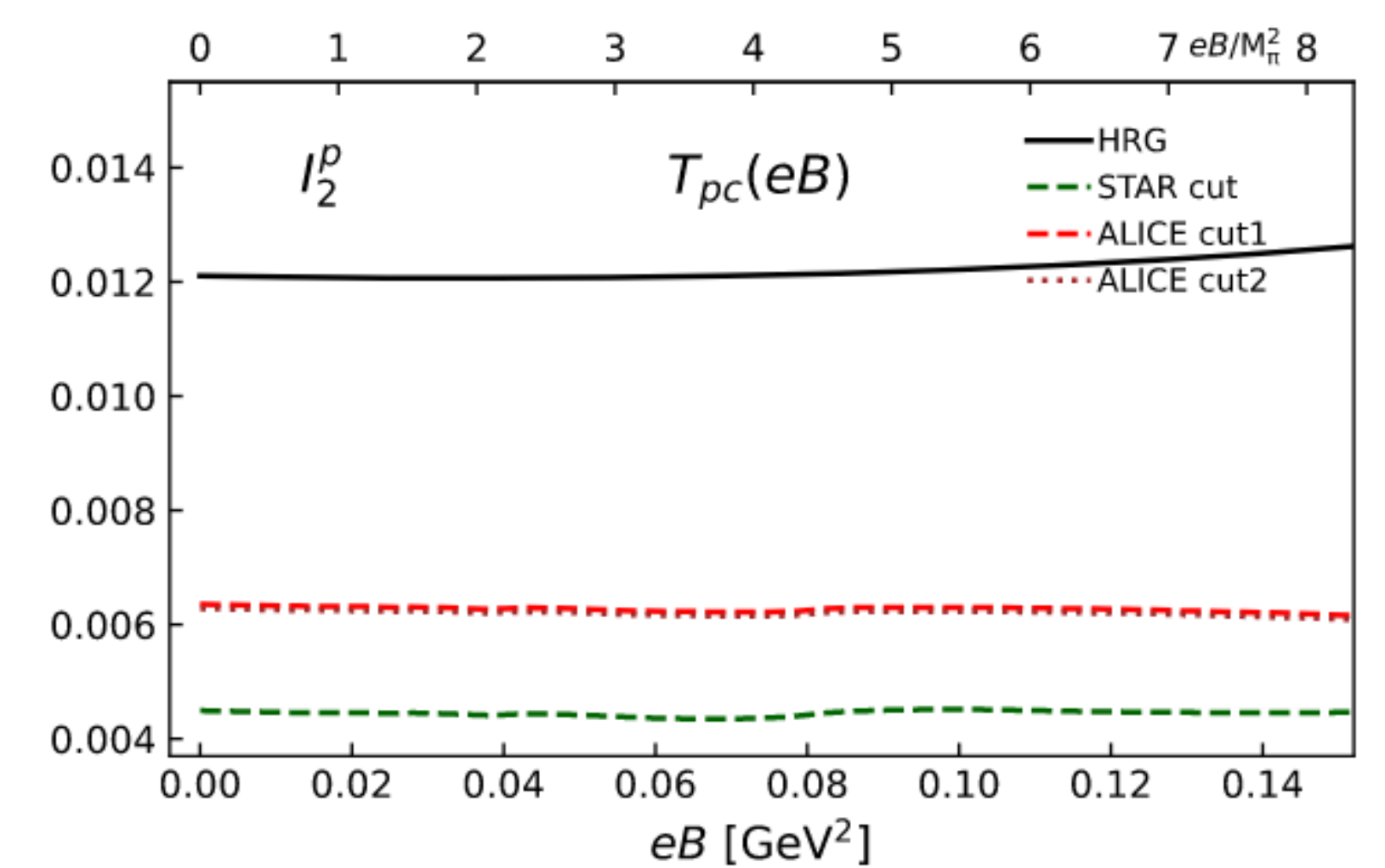
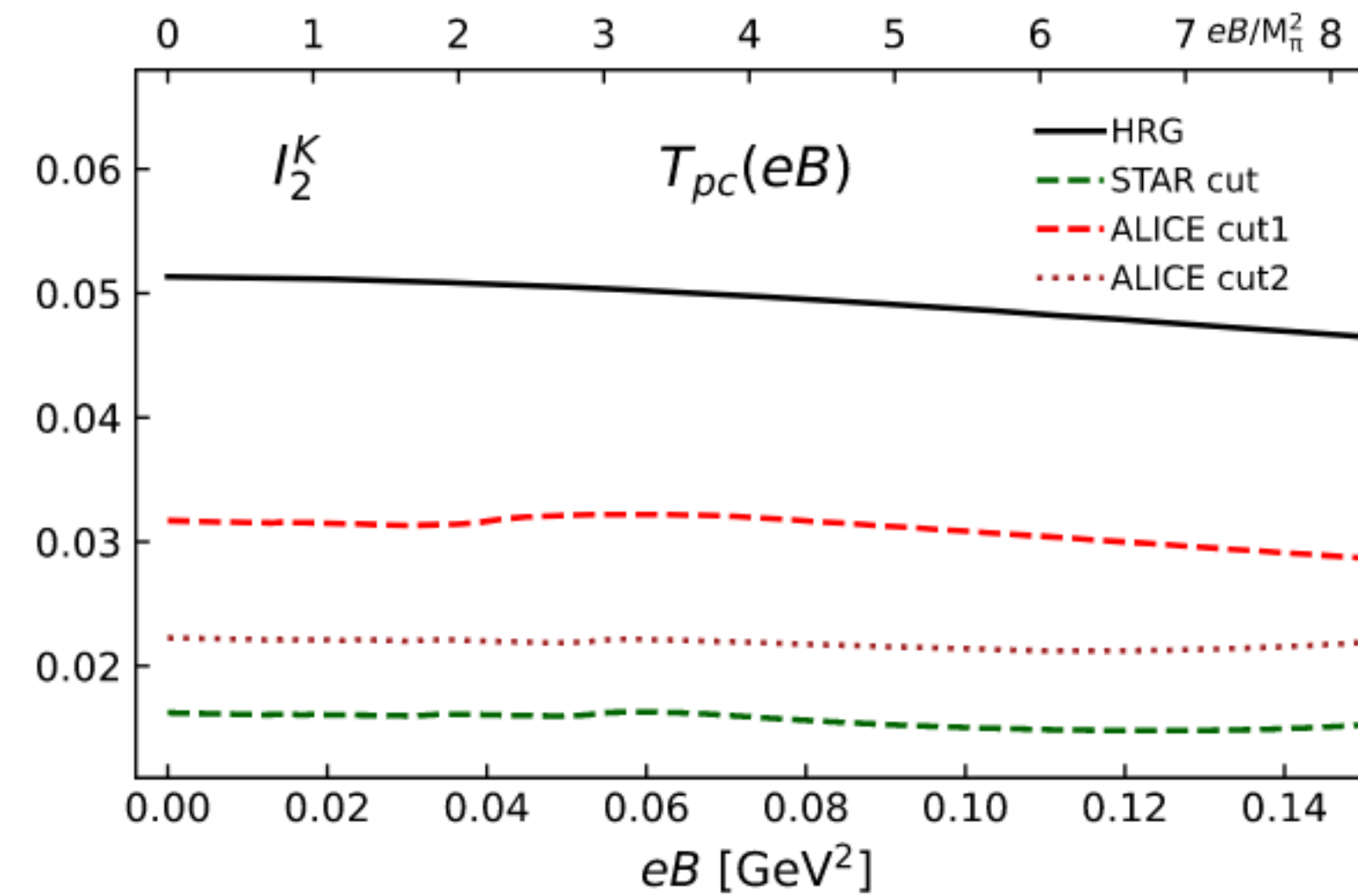
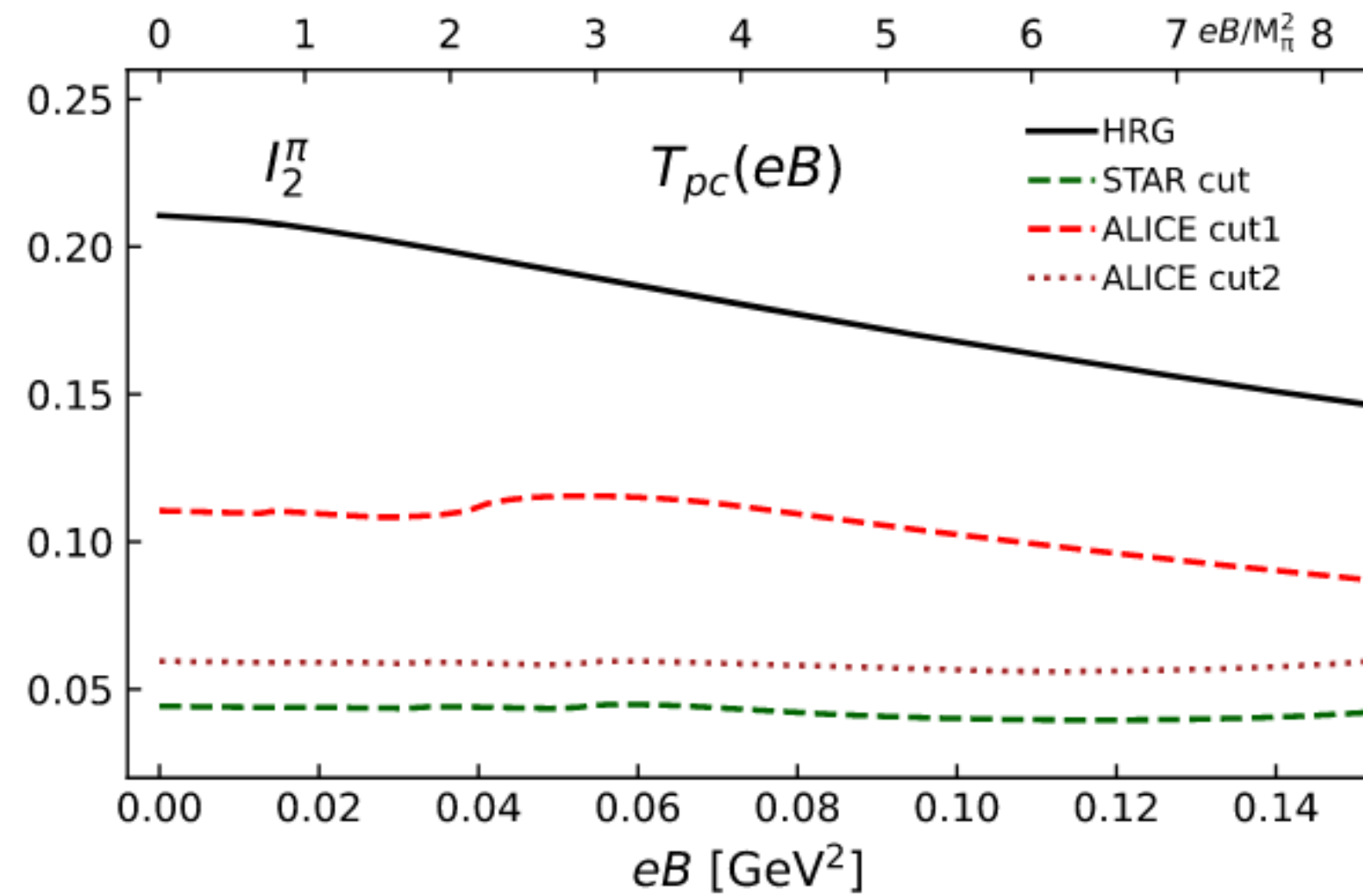
net-Q $\rightarrow \tilde{Q}^{\text{PID}} : \omega_{R \rightarrow \tilde{\pi}^+} + \omega_{R \rightarrow \tilde{K}^+} + \omega_{R \rightarrow \tilde{p}}$

Where $\omega_{R \rightarrow \tilde{i}}$ represents net number of **stable** particle i produced by particle R after the **entire decay chain**

In proxy, contributions from **all resonance decays** are considered!

Phase space integral for π , K and p

$$I_2^i = \frac{\partial^2 p_i / T^4}{\partial \mu_i^2} \text{ with different kinematic cuts. } i \in \{\pi, K, p\}$$



$$R_{\text{STAR}}^{\text{cut}} : |\eta| < 0.5,$$

$$R_{\text{ALICE}}^{\text{cut1}} : |\eta| < 0.8,$$

$$R_{\text{ALICE}}^{\text{cut2}} : |\eta| < 0.8,$$

$$\pi^\pm, K^\pm \text{ and } p(\bar{p}) : 0.4 < p_T < 1.6 \text{ GeV}/c$$

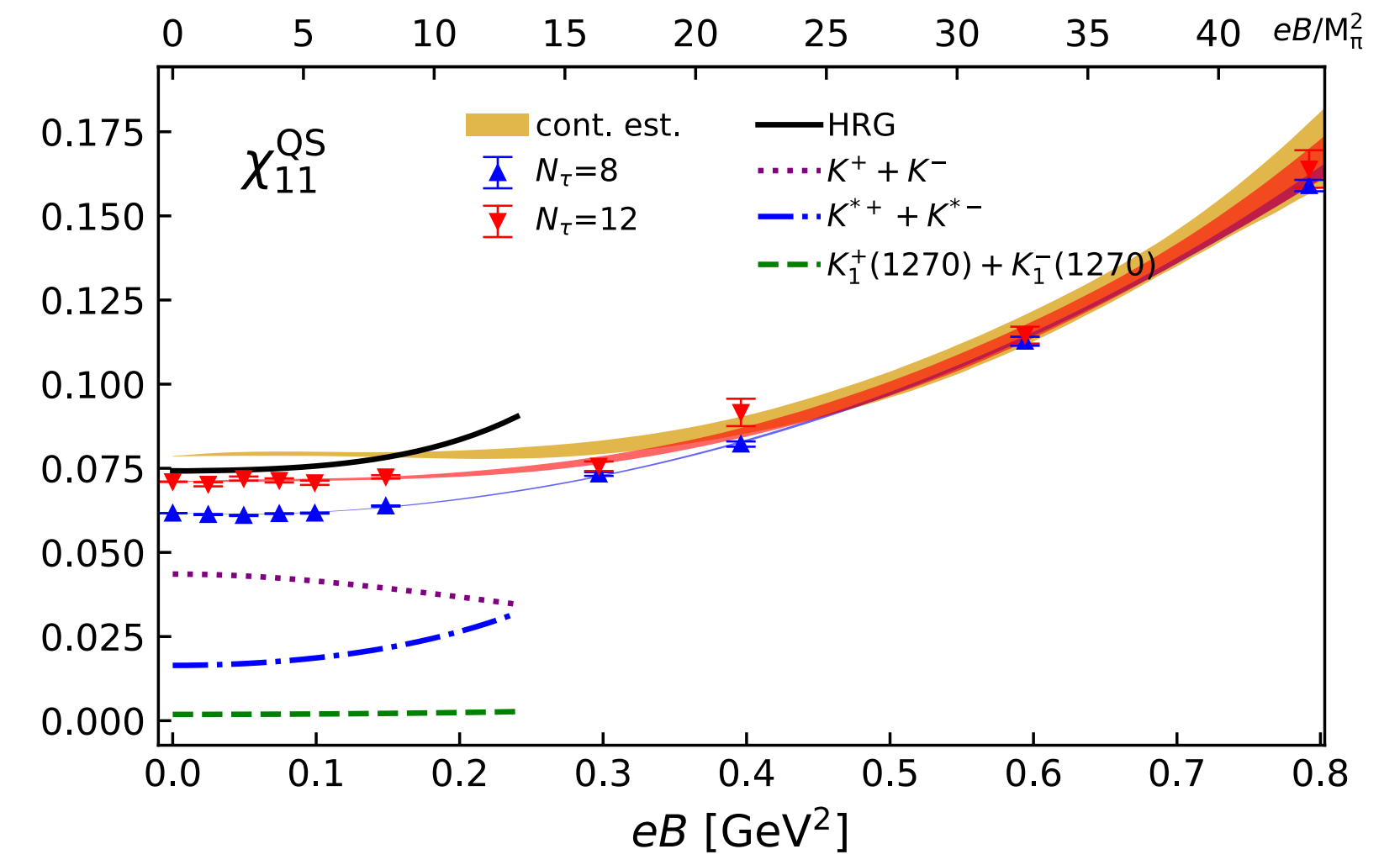
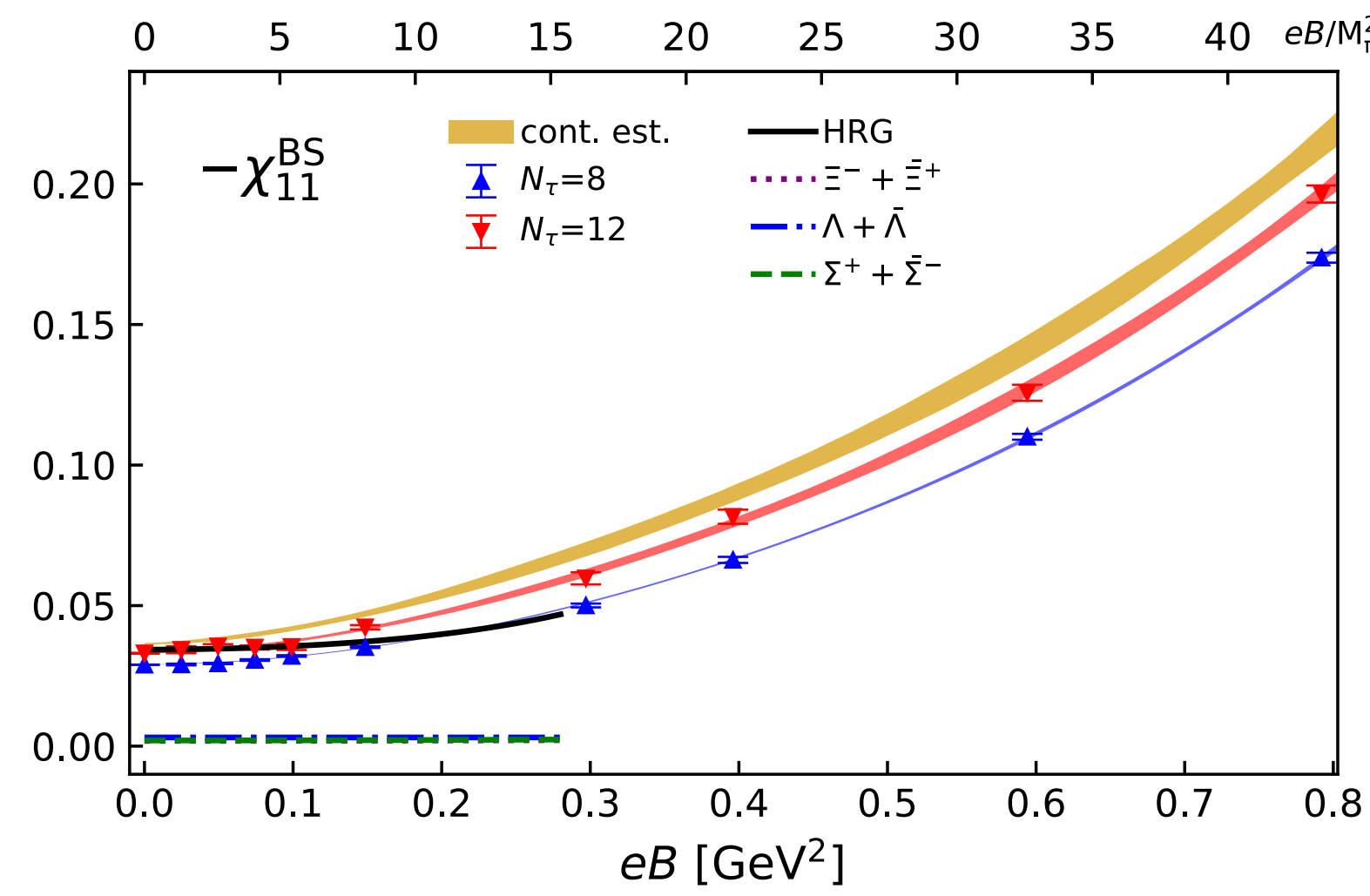
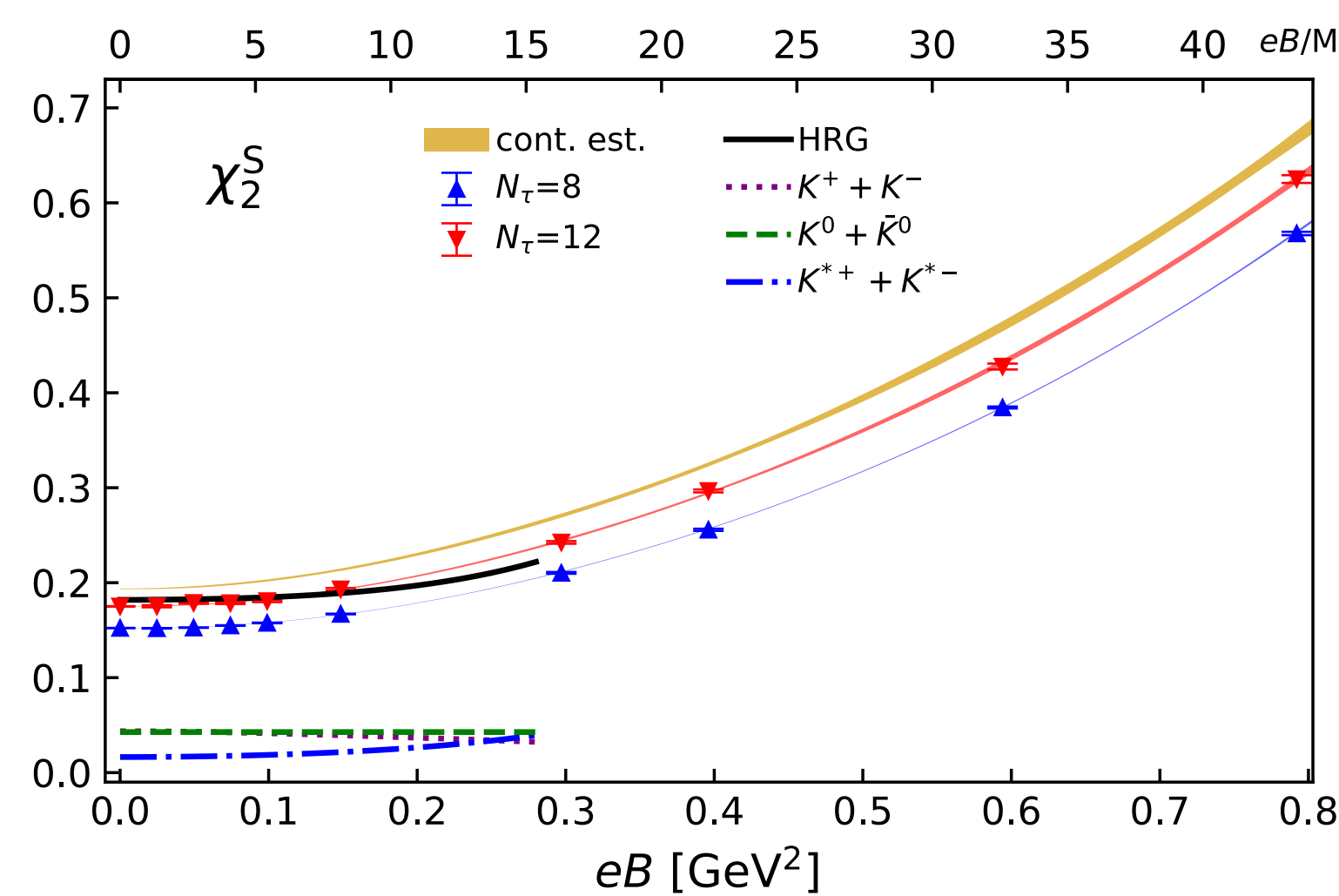
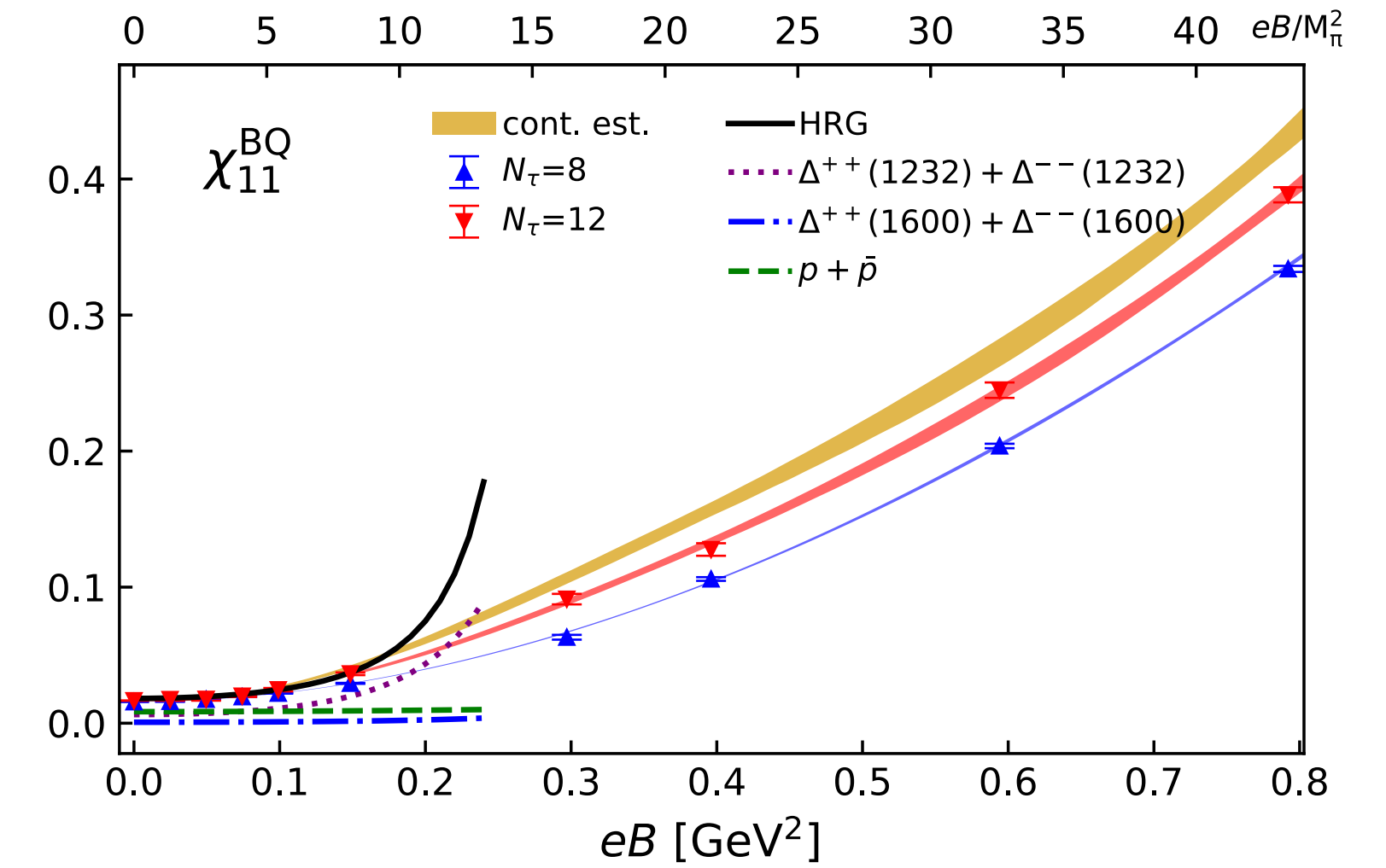
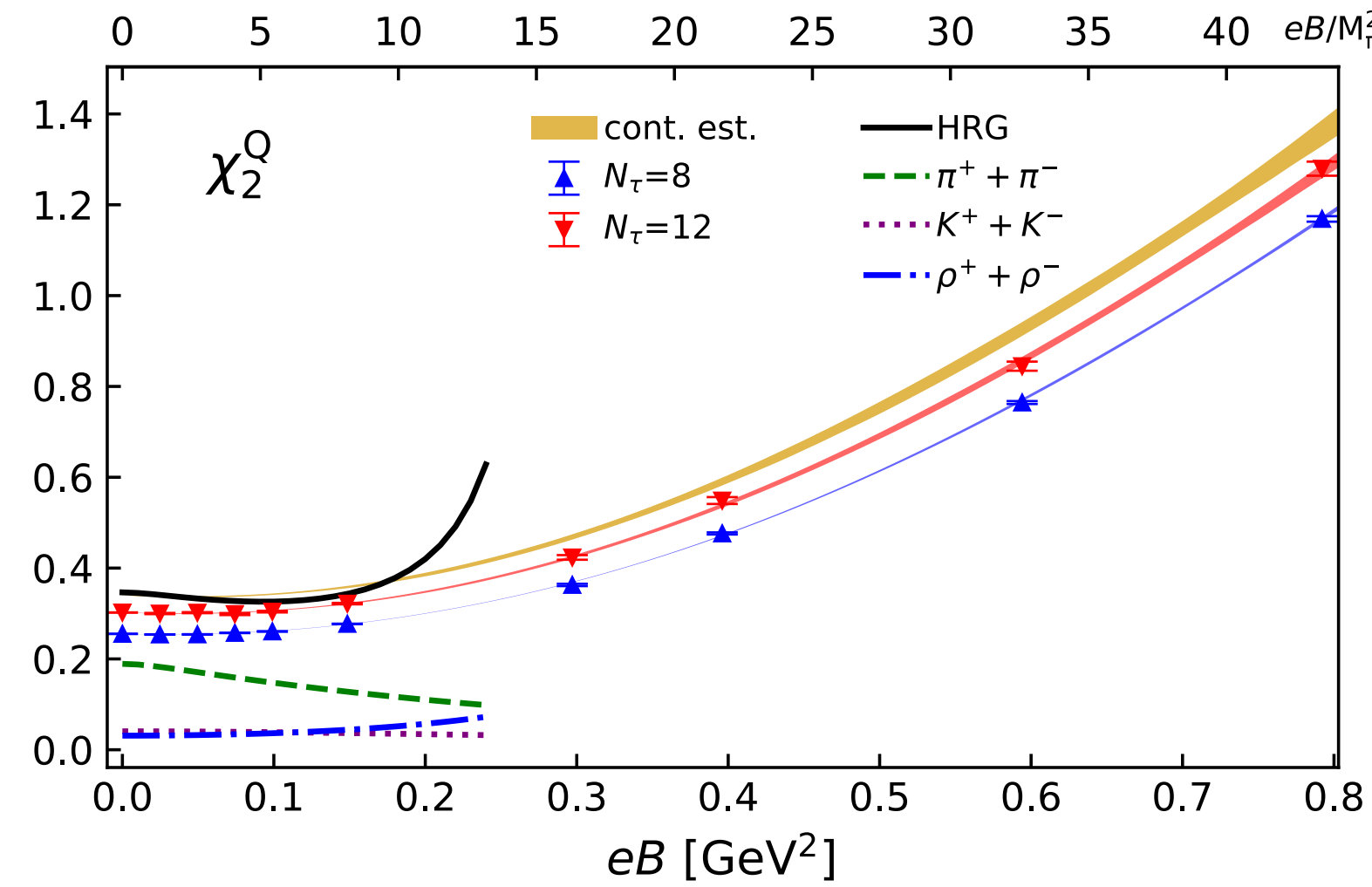
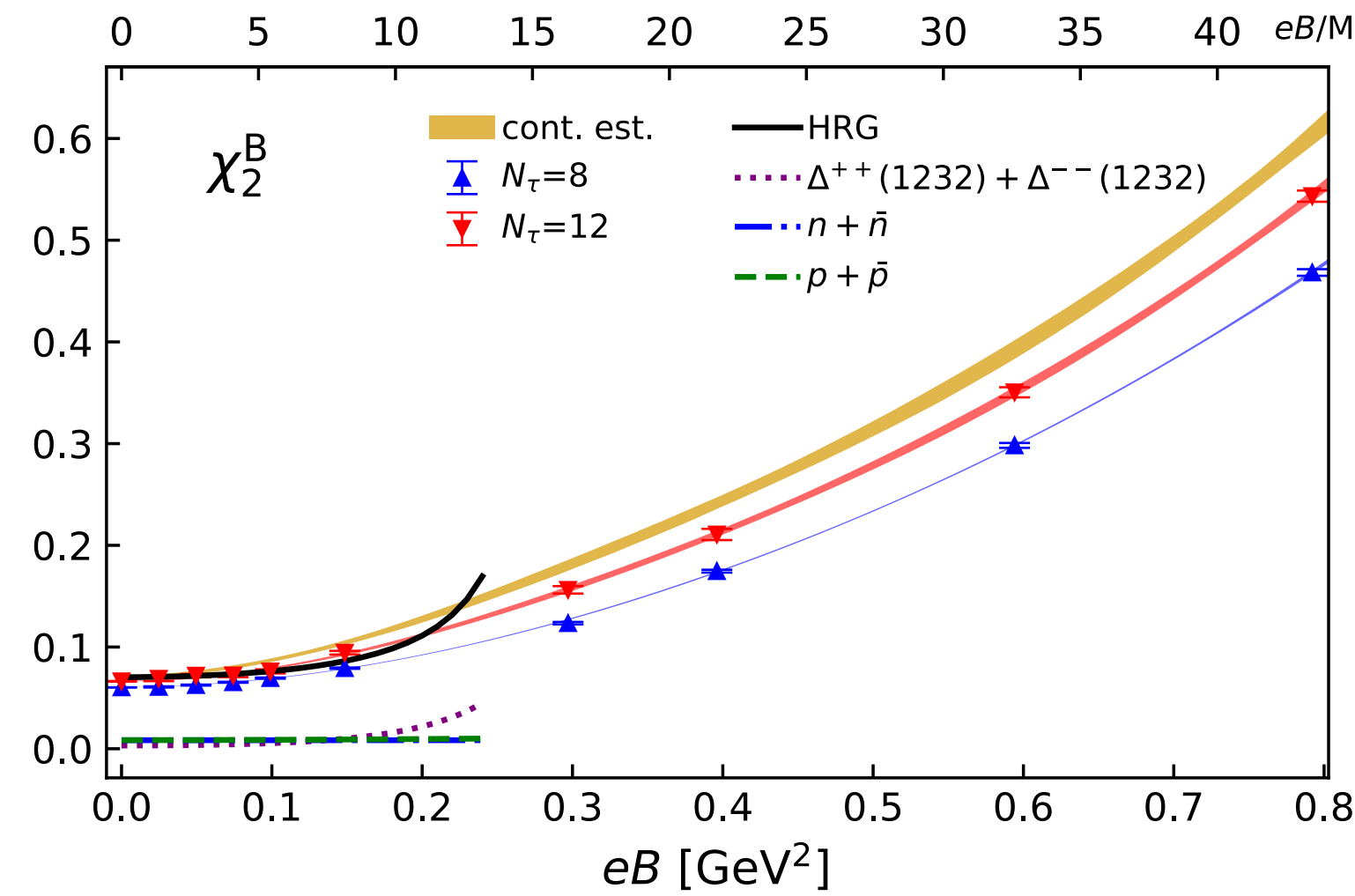
$$\pi^\pm \text{ and } K^\pm : 0.2 < p_T < 2 \text{ GeV}/c$$

$$\pi^\pm, K^\pm \text{ and } p(\bar{p}) : 0.4 < p_T < 1.6 \text{ GeV}/c$$

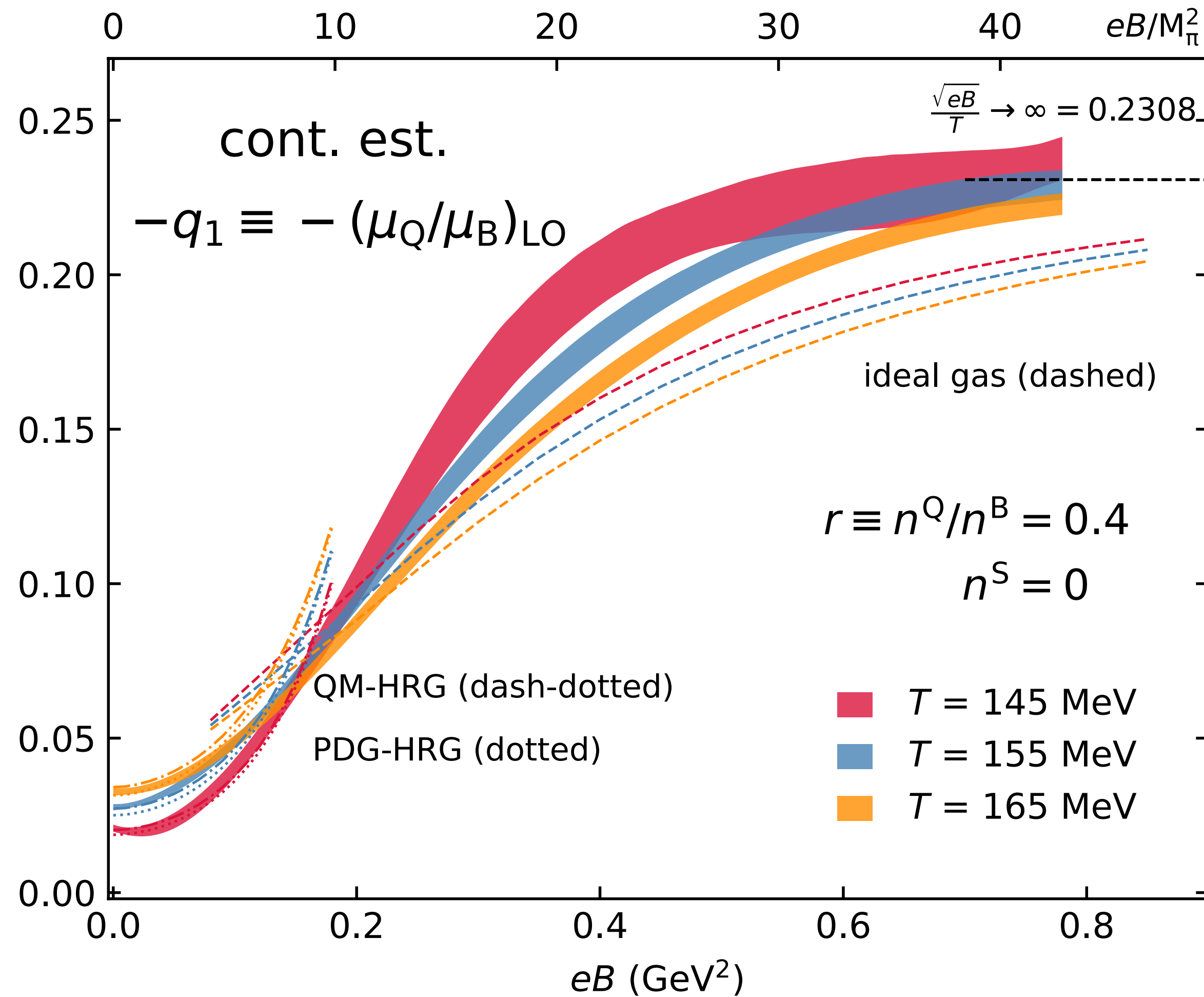
$$p(\bar{p}) : 0.4 < p_T < 2 \text{ GeV}/c$$

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, Phys.Rev.D 111, 114522 (2025)

Fluctuation of conserved charge at extremely strong eB and breaking down of HRG



$(\mu_Q/\mu_B)_{\text{LO}}$ in extremely strong magnetic field



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, arXiv:2508.07532

◆ Negative q_1

$$r = 0.4$$

Isospin asymmetry : $n^p < n^n \longrightarrow \mu_Q < 0$

◆ Strong eB : saturation to free limit

$$\sqrt{eB}/T \rightarrow \infty, T \rightarrow \infty$$

$$\frac{p_{\text{ideal}}}{T^4} = \frac{8\pi^2}{45} + \sum_f \frac{3|q_f|B}{\pi^2 T^2} \left[\frac{\pi^2}{12} + \frac{\mu_f^2}{4} + p_f^{\text{ll}}(B) \right]$$

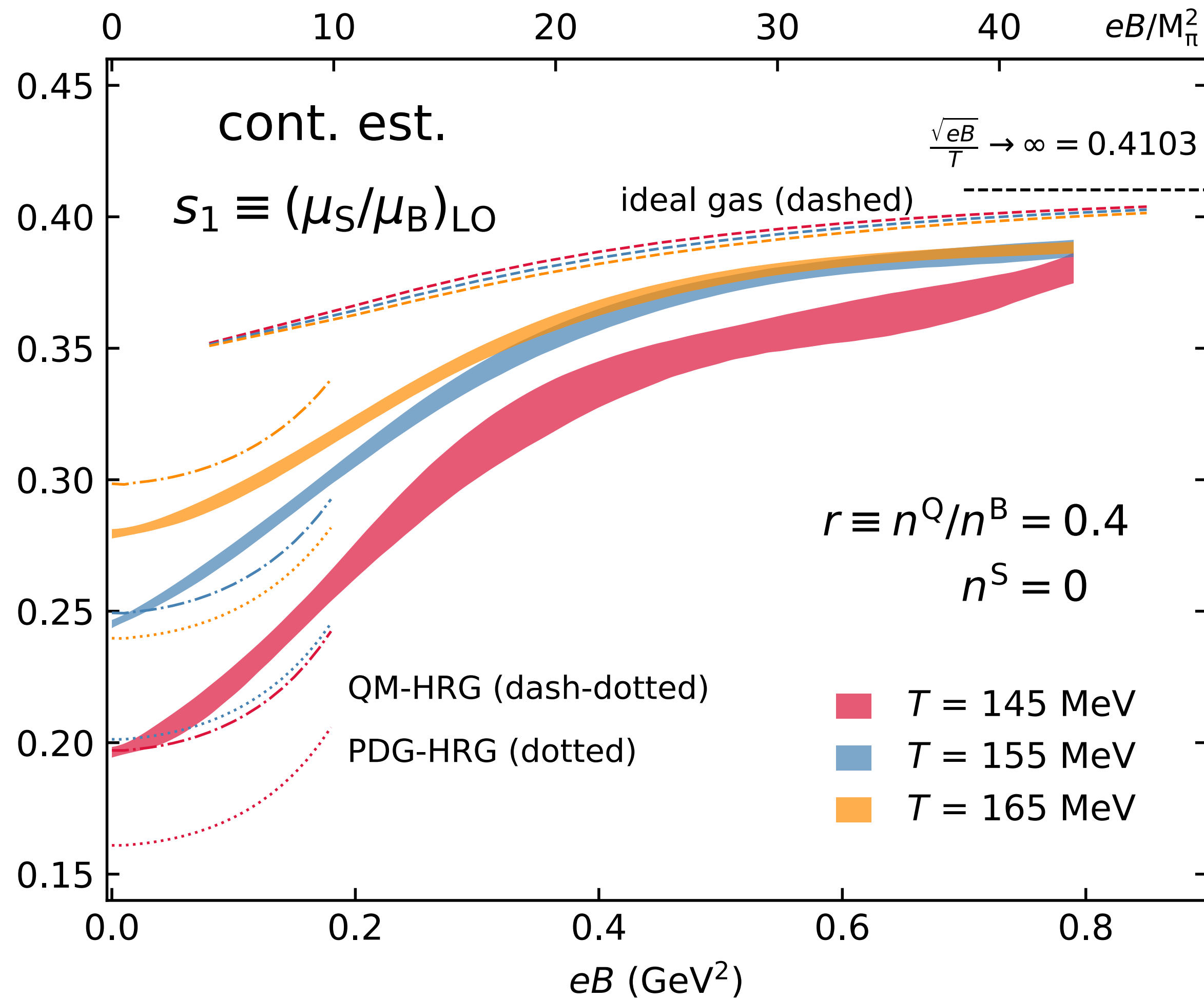
$f \in \{u, d, s\}$

◆ Saturation early in lower temperature

◆ Overlap among T -bands at $eB \sim 0.16 \text{ GeV}^2$

H.-T. Ding et al., Eur. Phys. J. A 57 (2021) 6, 202

$(\mu_S/\mu_B)_{LO}$ in extremely strong magnetic field



$$\hat{\mu}_S = s_1 \hat{\mu}_B + s_3 \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$s_1 = - \frac{(\chi_{11}^{BS} + q_1 \chi_{11}^{QS})}{\chi_2^S}$$

- ◆ s_1 grows with eB in all T
- ◆ Weak eB : QM-HRG agrees better with lattice results than PDG-HRG
- ◆ s_1 close to free limit at large eB range

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li, arXiv:2508.07532

Baryon number density

$$N_1^B = \chi_2^B + q_1 \chi_{11}^{BQ} + s_1 \chi_{11}^{BS}$$

$$\frac{N_1^B}{2P_2} = \frac{1}{1 + rq_1}$$

◆ Similar eB -dependence to pressure

◆ $N_1^B/2P_2$ deviation from unity: isospin symmetry breaking

