# Heavy quarkonium dissociation, regeneration & equilibration in the Quark-Gluon Plasma

Based on S. Zhao & MH, PRD 110, 074040 (2024)
S. Zhao & MH, arXiv:2508.11897, PRD in press

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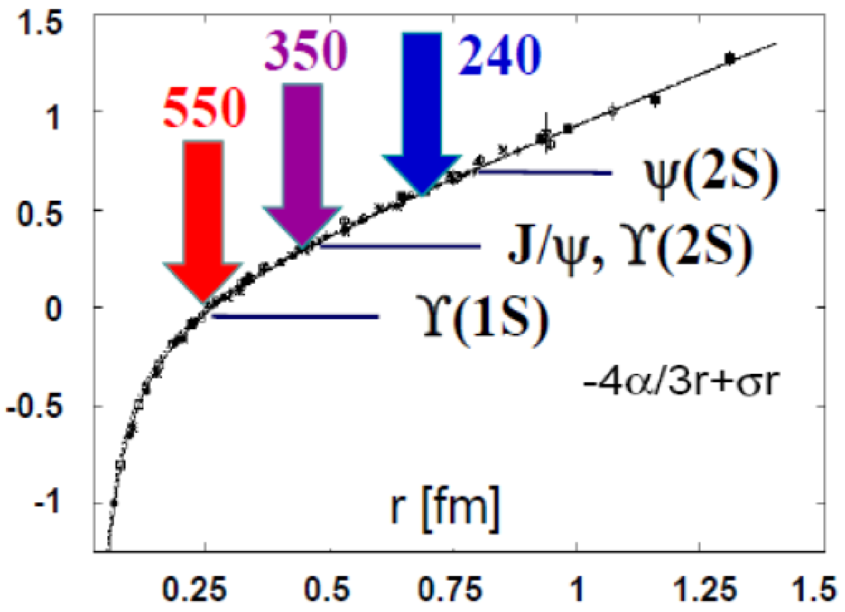
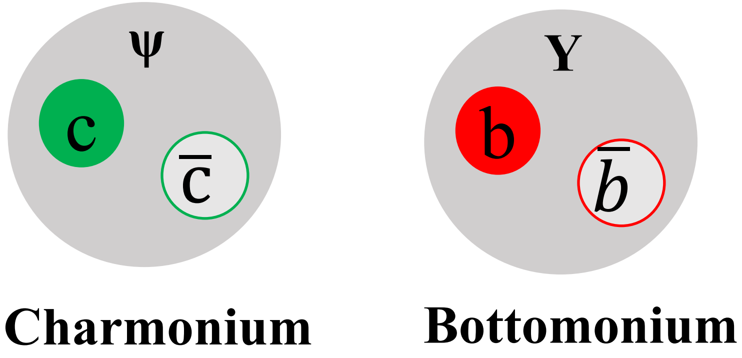
# Heavy quarkonium: in vacuum vs medium

- Heavy quarkonia: non-relativistic bound states of Q-Qbar

$$[2m_Q - \frac{\nabla^2}{m_Q} + V(r)]\psi_i(r) = M_i\psi_i(r)$$

$$V_{Q\bar{Q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$$



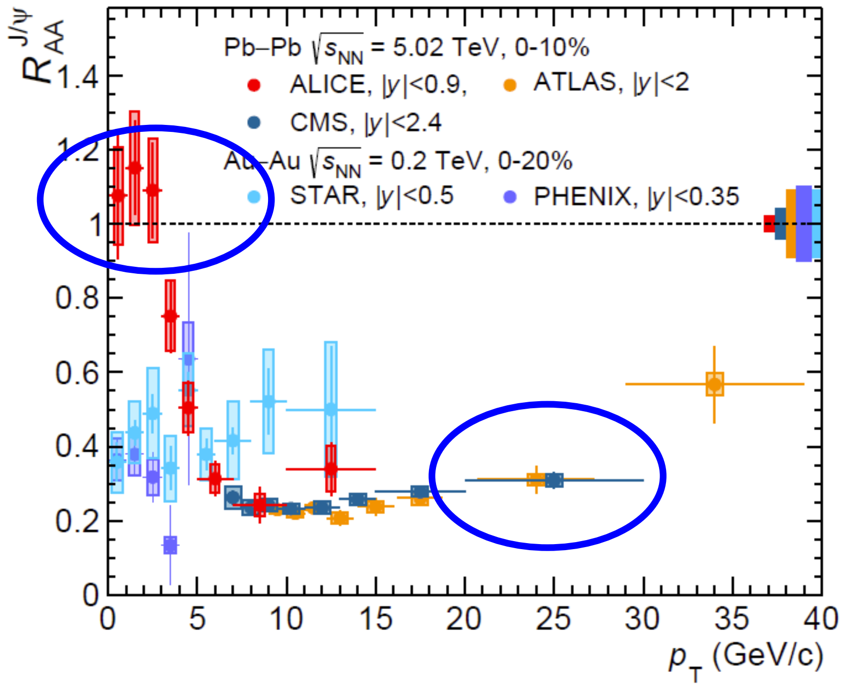




- Finite temperature: static screening/weakening of potential → melting of bound states

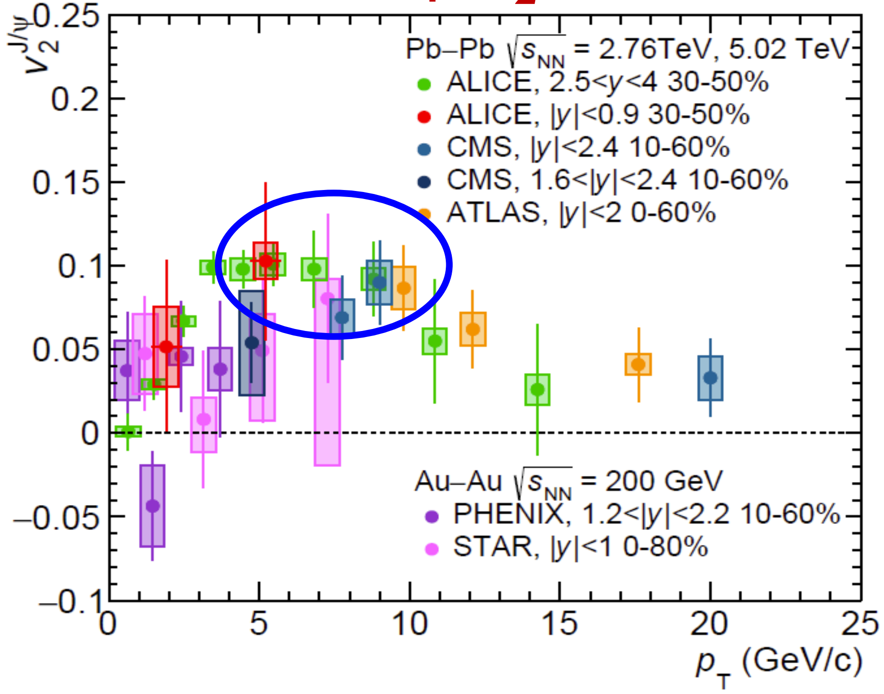
$$V_{Q\bar{Q}}(r,T) = -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r} + \frac{\sigma}{m_D}(1-e^{-m_D r})$$

# Dynamical reactions beyond static screening

**J/ψ $R_{AA}$**




**J/ψ $v_2$**



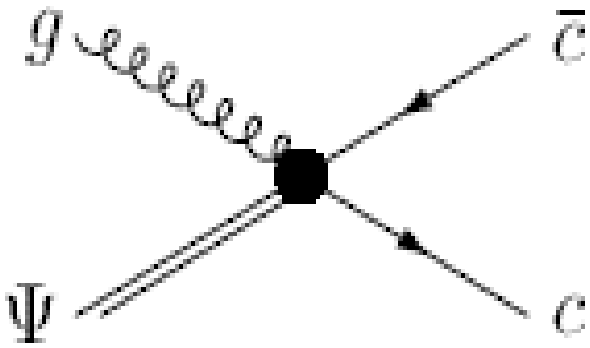

A. Andronic, R. Arnaldi, 2501.08290

- Suppression at high $p_T$ ← dissociation of bound states
- Enhanced production at low $p_T$ & large $v_2$ ← regeneration from near-thermalized $c + \bar{c}$

# Part I: Reaction cross sections & rates

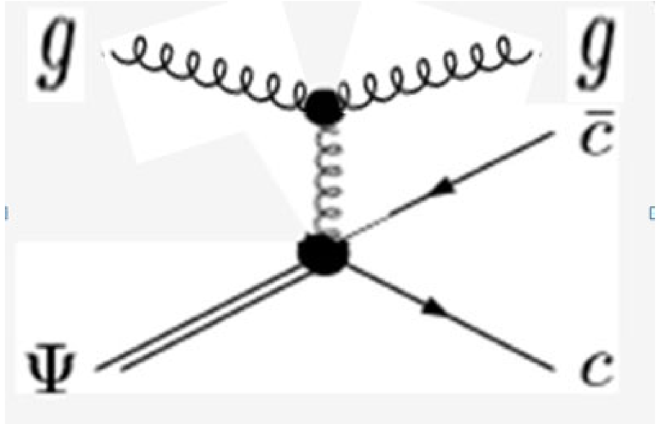
- LO – gluo-dissociation:

$$g + J/\psi \leftrightarrow c + \bar{c}$$




- NLO – inelastic partonic scattering:

$$g/q + J/\psi \leftrightarrow g/q + c + \bar{c}$$



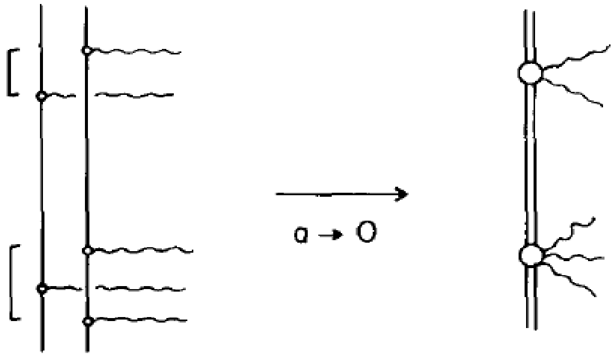
# Heavy Quarkonium coupling to thermal gluons

- Peskin's OPE analysis: Coupling of quarkonium to external gluons = a short-distance process

Peskin et al., Nucl. Phys. B (1979)

➔ color-octet QQbar can only persist over short space-time range $\Delta t \sim \frac{1}{V_8 - V_1} \sim \frac{a}{g_s^2} \sim \frac{1}{\epsilon_B}$,

➔ gluon emissions assemble into small singlet clusters: OPE local operators




- soft gluon wavelength λ >> bound state size r → multipole expansion → pNRQCD color-electric dipole (E1) coupling

$$H_{E1} = -\vec{d}^a \cdot \vec{E}^a(t,\vec{0}) = \frac{g_s}{2}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\vec{r} \cdot \frac{\partial \vec{A}^a}{\partial t}$$

Brambilla et al., RMP (2005)

$Q\bar{Q}$ dipole moment

$$\vec{A}^a(t,\vec{x}) = \sum_{\vec{k},\lambda} N_{\vec{k}} \vec{\epsilon}_{\vec{k}\lambda} [a^a_{\vec{k}\lambda} e^{i\vec{k}\cdot\vec{x} - i\omega_{\vec{k}} t} + h.c.]$$

➔ Selection rules: ΔL = 1, ΔS = 0

# Effective vertices & Feynman rules

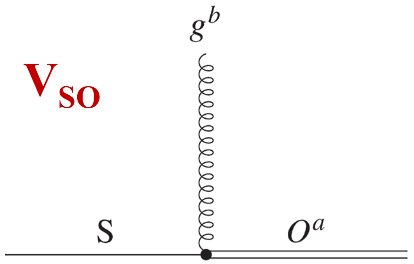
- Effective Hamiltonian

$$H_{\text{eff}} = H_0 + H_I, \begin{cases} H_0 = \dfrac{\vec{p}^2}{m_Q} + V_1(|\vec{r}|) + \sum_a \dfrac{\lambda^a}{2}\dfrac{\bar{\lambda}^a}{2} V_2(|\vec{r}|) \\ H_I = V_{SO} + V_{OO} + V_{3g} + V_{q\bar{q}g} \end{cases}$$

heavy quarkonium system

quarkonium coupling to external gluons + light quark/gluon system

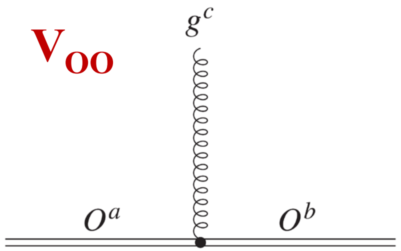
**$V_{SO}$**




$$\langle O, a|V_{SO}|S\rangle = \frac{g_s}{\sqrt{2N_c}} \vec{E}^a(t,\vec{x}) \cdot \langle O|\vec{r}|S\rangle$$

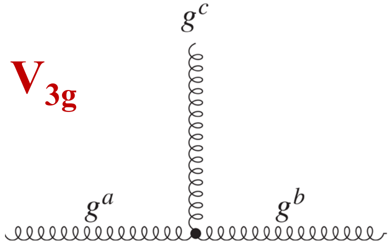
$Q\bar{Q}$ singlet → octet

**$V_{OO}$**



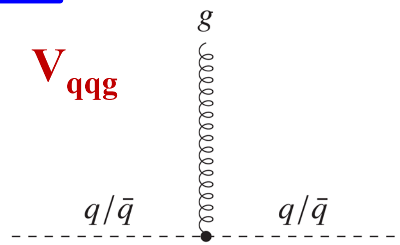

$$\langle O, a|V_{OO}|O, b\rangle = \frac{ig_s}{2} d^{abc} \vec{E}^c(t,\vec{x}) \cdot \langle O|\vec{r}|O\rangle$$

**$V_{3g}$**




$$V_{3g} \sim \int d^3\vec{x} \frac{1}{2} \vec{B}^a \cdot \vec{B}^a = \left(-\frac{g_s}{2}\right) f^{abc} \int d^3\vec{x} (\nabla \times \vec{A}^a) \cdot (\vec{A}^b \times \vec{A}^c)$$

**$V_{qqg}$**




$$V_{q\bar{q}g} = g_s \int d^3x \psi^{i\dagger}(x) \vec{\alpha} \cdot \vec{A}^a(x) \left(\frac{\lambda^a}{2}\right)^{ij} \psi^j(x)$$

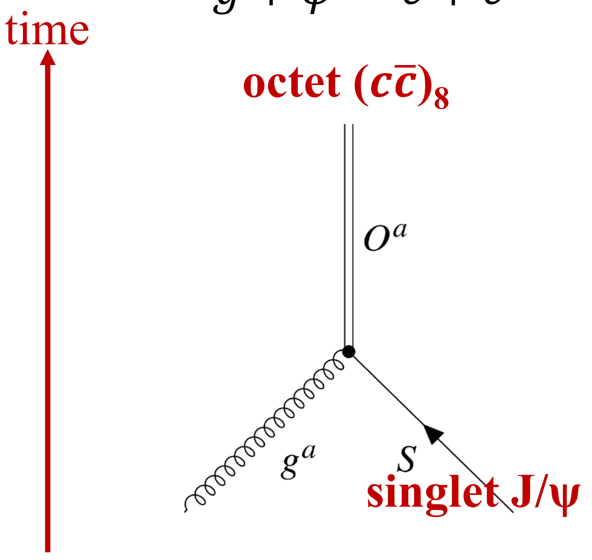
# LO & NLO dissociation amplitudes

- Transition matrix elements (OFPT): LO + NLO

$$T_{fi} = \langle f|H_{\rm int}|i\rangle + \sum_m \frac{\langle f|H_{\rm int}|m\rangle\langle m|H_{\rm int}|i\rangle}{E_i - E_m + i\epsilon} = T_{fi}^{\rm LO} + T_{fi}^{\rm NLO}.$$

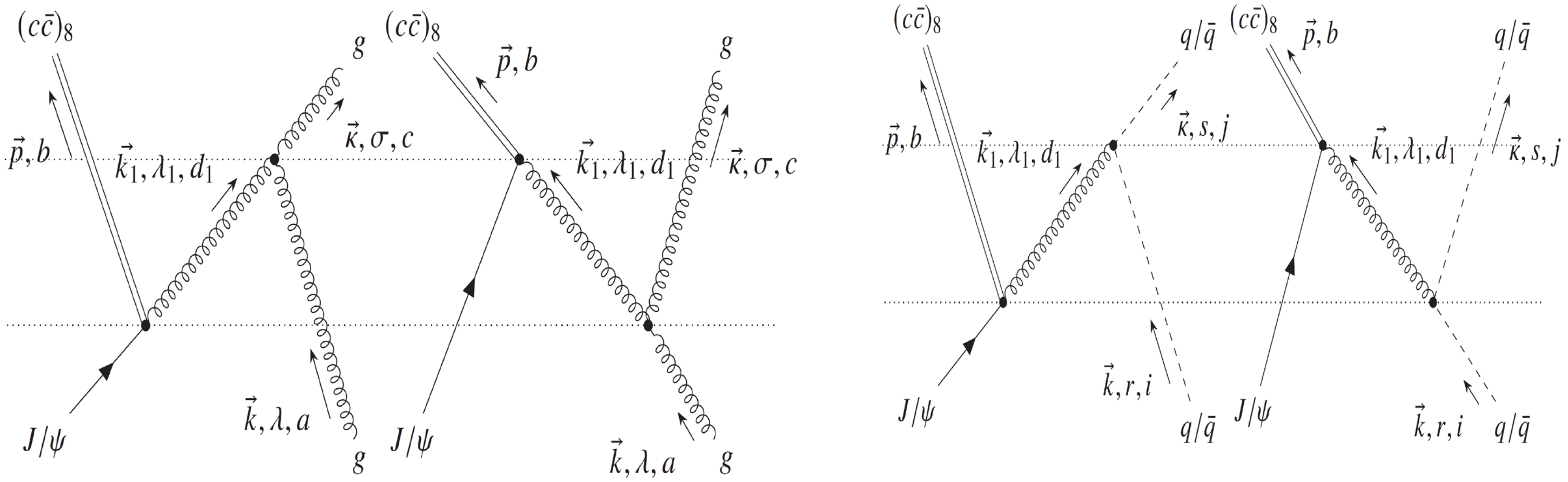
- Time-ordered Feynman diagrams

$g + \psi \to c + \bar{c}$




LO: $O(g_s r)$

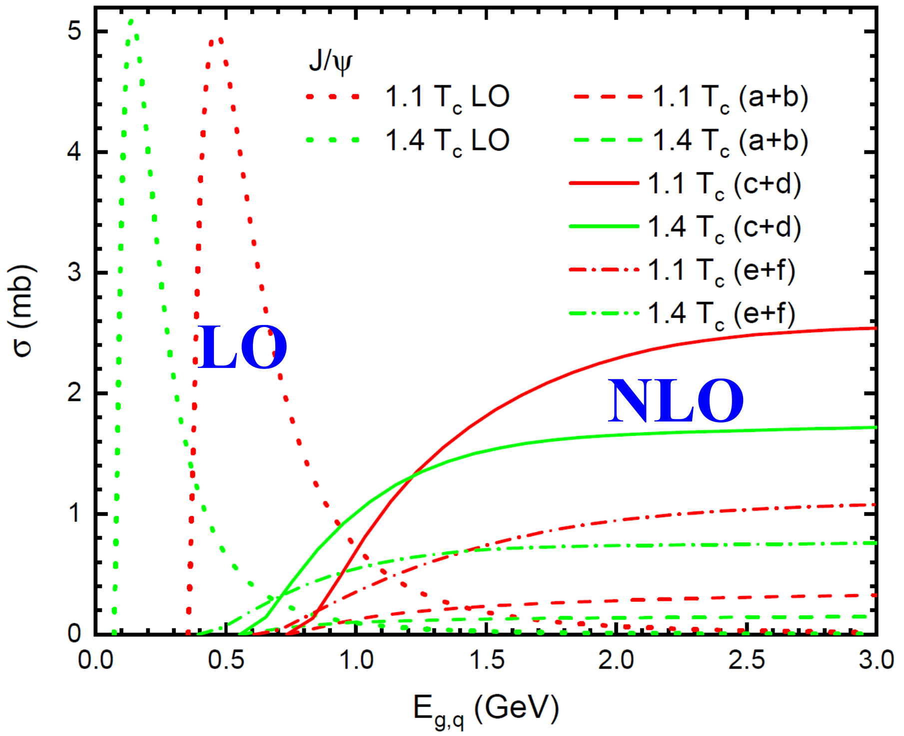
$g/q(\bar{q}) + \psi \to g/q(\bar{q}) + c + \bar{c}$



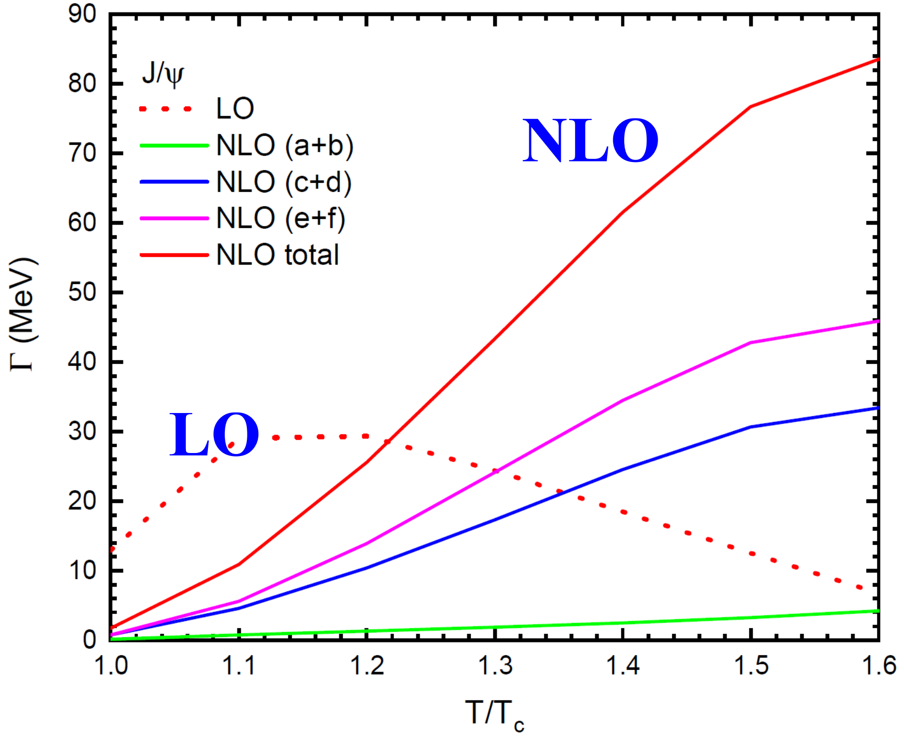

NLO: $O(g_s^2 r)$

# J/ψ dissociation cross sections & rates

## Cross section




## Diss. rate $\Gamma(T) = d_p \int \frac{d^3k}{(2\pi)^3} f_p(E_p, T) v_{rel} \sigma(k_p, T)$




- LO peaking at low incident gluon energy ($\lambda_g \sim r_\psi$) & falling off thereafter
- NLO growing with $E_g$ and saturating eventually: outgoing g/q carrying away the excess energy

- LO diss. rate dominating at lower T
- NLO diss. rate increasing with T & taking over at higher T ← new phase space opening up.

# Part II: Quarkonium transport & equilibration

- Do these reactions lead to the quarkonium equilibration (both chemical & kinetic), should they be given sufficiently long time for transport in the QGP?

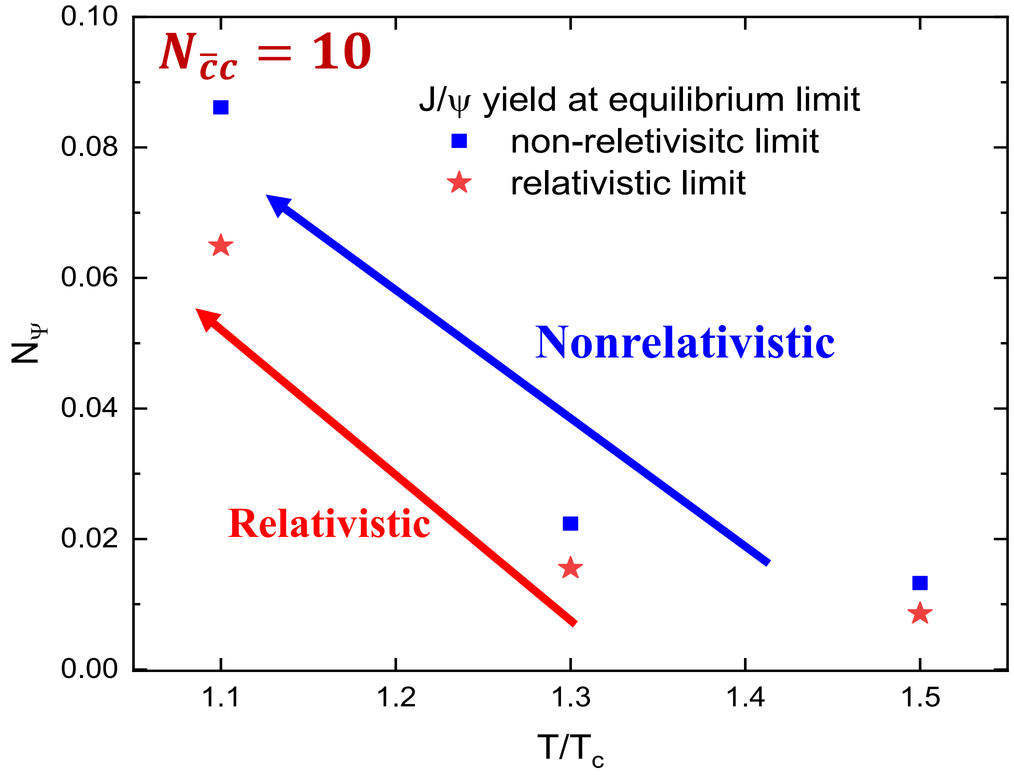
➔ Simulate $J/\psi$ dissociation & regeneration in a static & uniform QGP box, with 10 pairs of $c\bar{c}$

➔ Provide a dynamical way to understand the rationale & success of Statistical Hadronization Model (SHM) for charmonia production in HIC (see, **P. B.-Munzinger et al., Nature, 561 (2018)**)

# Equilibrium limit for heavy quarkonium

- *Relative* chemical equilibrium between open vs hidden heavy flavor (constrained by HQ number conservation) → balance equation

$$N_{Q\bar{Q}} = N_Q^{\rm open} + N_\Psi \quad \begin{cases} N_Q^{\rm open} = d_Q \gamma_Q V \dfrac{m_Q^2 T}{2\pi^2} K_2\left(\dfrac{m_Q}{T}\right), \\ N_\Psi = d_\Psi \gamma_Q^2 V \dfrac{m_\Psi^2 T}{2\pi^2} K_2\left(\dfrac{m_\Psi}{T}\right) \end{cases}$$




- HQ fugacity $\gamma_Q$ – characterizing deviation from *absolute* chemical equilibrium

- < 1% of $c\bar{c}$ existing in J/ψ & at lower T, more $c\bar{c}$ existing in the form of bound states having lower energy than open ones

# J/ψ transport in QGP: Boltzmann equation

- Time evolution of J/ψ phase space distributions:

$$\frac{\mathrm{d}}{\mathrm{d}t}f_\Psi(\vec{x},\vec{p},t)=(\frac{\partial}{\partial t}+\frac{\partial\vec{x}}{\partial t}\cdot\nabla_{\vec{x}})f_\Psi(\vec{x},\vec{p},t)=C_{22}(\Psi+g\rightleftharpoons Q+\bar{Q})+C_{23}(\Psi+p\rightleftharpoons Q+\bar{Q}+p)$$

- LO 2→2: $g+J/\psi\leftrightarrow c+\bar{c}$

Reaction amplitude

$$\begin{aligned}C_{22}&=C_{22}[\mathrm{gain}]-C_{22}[\mathrm{loss}]\\&=\frac{1}{2E_\Psi(\vec{p})}\int\frac{\mathrm{d}^3p_2}{2E_g(2\pi)^3}\frac{\mathrm{d}^3p_3}{2E_Q(2\pi)^3}\frac{\mathrm{d}^3p_4}{2E_{\bar{Q}}(2\pi)^3}\sum|\mathcal{M}^{\mathrm{LO}}|^2(2\pi)^4\delta^4(p+p_2-p_3-p_4)\\&\times[\frac{d_\Psi}{d_Qd_{\bar{Q}}}f_Q(\vec{x},\vec{p}_3,t)f_{\bar{Q}}(\vec{x},\vec{p}_4,t)\,(1+f_g(\vec{x},\vec{p}_2,t))-f_\Psi(\vec{x},\vec{p},t)f_g(\vec{x},\vec{p}_2,t)].\end{aligned}$$

- NLO 2→3: $p+J/\psi\leftrightarrow p+c+\bar{c}\ ,\ \ p=g,q/\bar{q}$

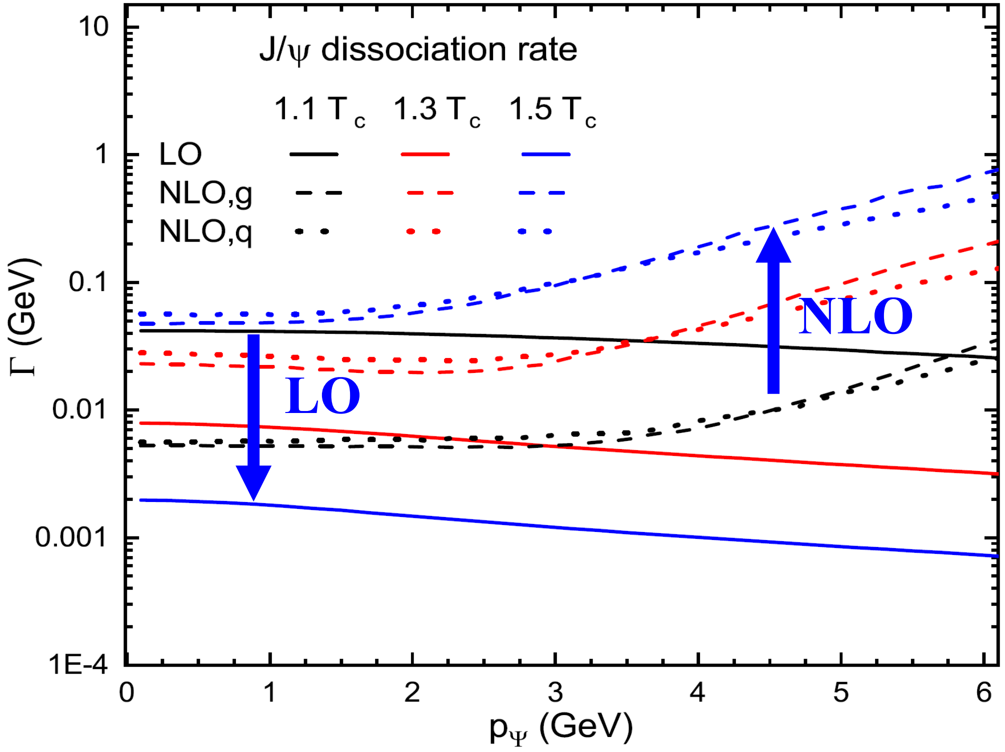
detailed balance ➔
Same for disso. & regener.

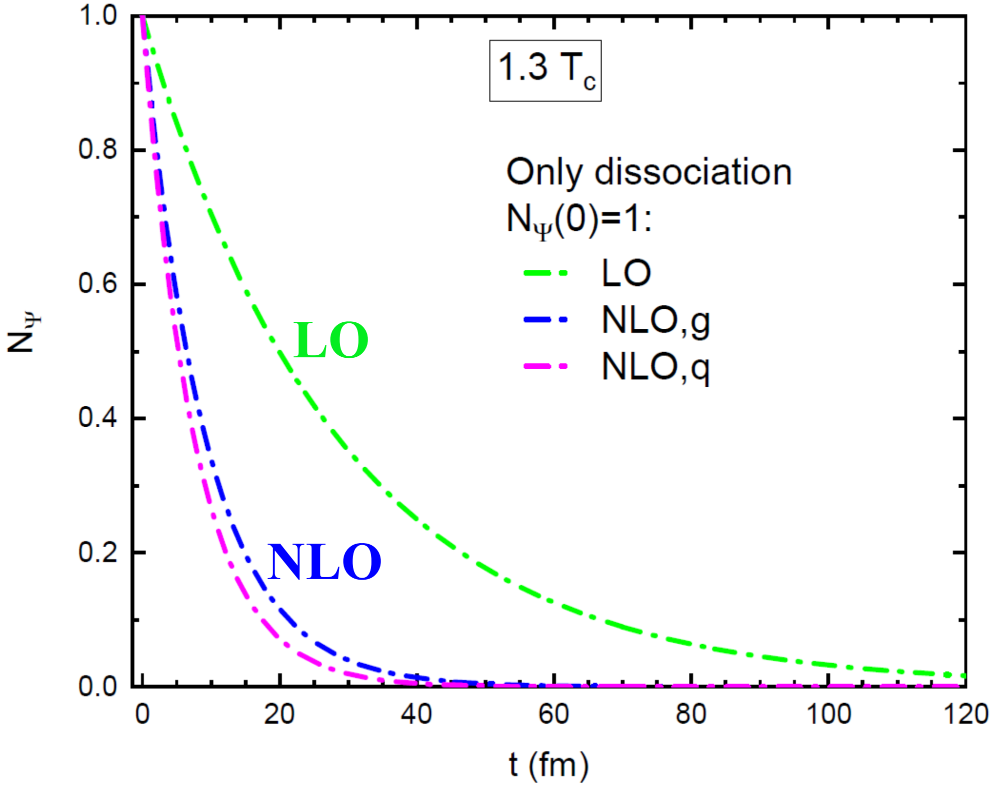
$$\begin{aligned}C_{23}&=C_{23}[\mathrm{gain}]-C_{23}[\mathrm{loss}]\\&=\frac{1}{2E_\Psi(\vec{p})}\int\frac{\mathrm{d}^3p_2}{2E_p(\vec{p}_2)(2\pi)^3}\frac{\mathrm{d}^3p_3}{2E_Q(2\pi)^3}\frac{\mathrm{d}^3p_4}{2E_{\bar{Q}}(2\pi)^3}\frac{\mathrm{d}^3p_5}{2E_p(\vec{p}_5)(2\pi)^3}\sum|\mathcal{M}^{\mathrm{NLO}}|^2(2\pi)^4\delta^4(p+p_2-p_3-p_4-p_5)\\&\times[\frac{d_\Psi}{d_Qd_{\bar{Q}}}f_Q(\vec{x},\vec{p}_3,t)f_{\bar{Q}}(\vec{x},\vec{p}_4,t)f_p(\vec{x},\vec{p}_5,t)\,(1\pm f_p(\vec{x},\vec{p}_2,t))-f_\Psi(\vec{x},\vec{p},t)f_p(\vec{x},\vec{p}_2,t)\,(1\pm f_p(\vec{x},\vec{p}_5,t))]\end{aligned}$$

# Simulation of J/ψ dissociation in a static box

- Iteration scheme: differential rate equation for the loss/dissociation of J/ψ

$$\frac{\mathrm{d}}{\mathrm{d}t} f_\Psi(\vec{p},t)[\mathrm{loss}] = -\Gamma(p,T) f_\Psi(\vec{p},t).$$




- NLO prevailing over LO at higher T & larger p
- At 1.3$T_c$, much faster dissociation by NLO than LO

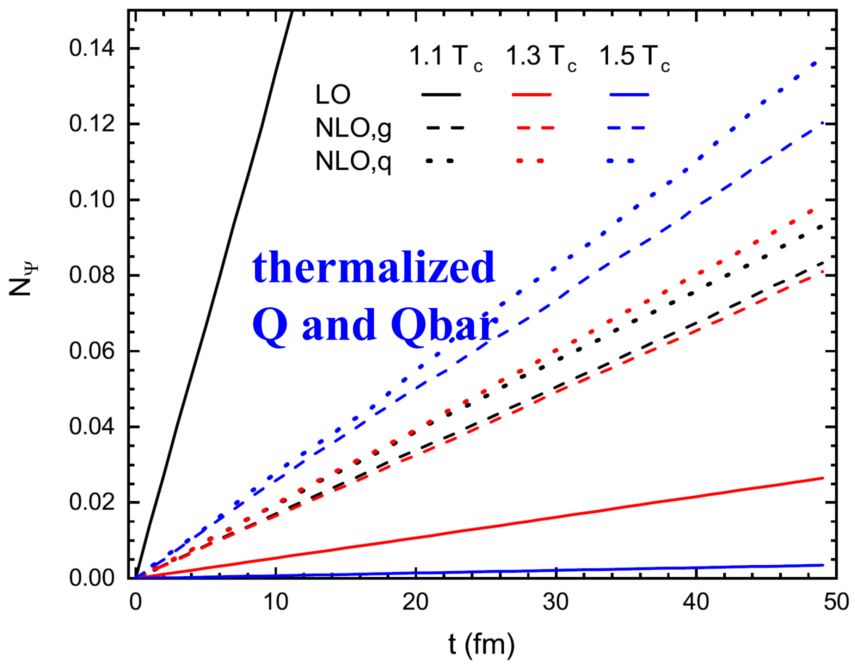
# Simulation of J/ψ regeneration in a static box

- Test particle representation for Q/Qbar distributions

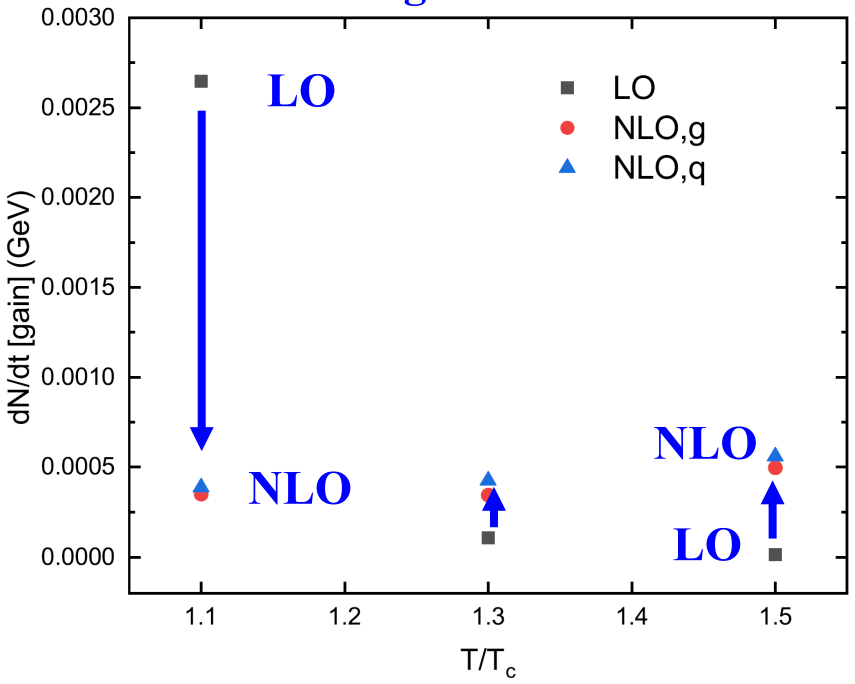
$$f_{Q/\bar{Q}}(\vec{x},\vec{p}) = (2\pi)^3 \sum_{n_{Q/\bar{Q}}=1}^{10} \delta^3(\vec{x}-\vec{x}_{nQ/\bar{Q}})\delta^3(\vec{p}-\vec{p}_{nQ/\bar{Q}})$$

➔ off-diagonal $c\bar{c}$ recombination into J/ψ

$$\frac{dN}{dp_\Psi dt} \propto \sum_{n_Q=1}^{10}\sum_{n_{\bar{Q}}=1}^{10} |\mathcal{M}|^2 \delta^3(\vec{p}_Q-\vec{p}_{nQ})\delta^3(\vec{p}_{\bar{Q}}-\vec{p}_{n\bar{Q}})$$

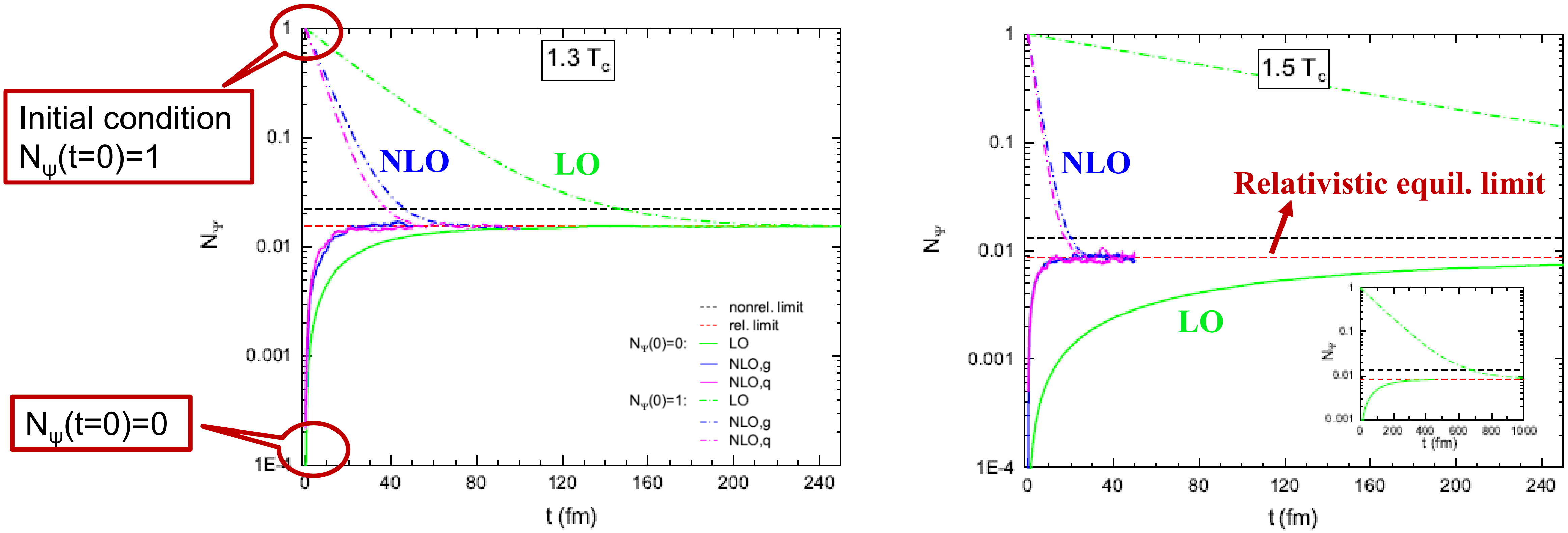



*Absolute* **regeneration rate**



# J/ψ equilibration: with thermalized c quarks

- Both dissociation & regeneration operative ➔ J/ψ yield approaching equilibrium

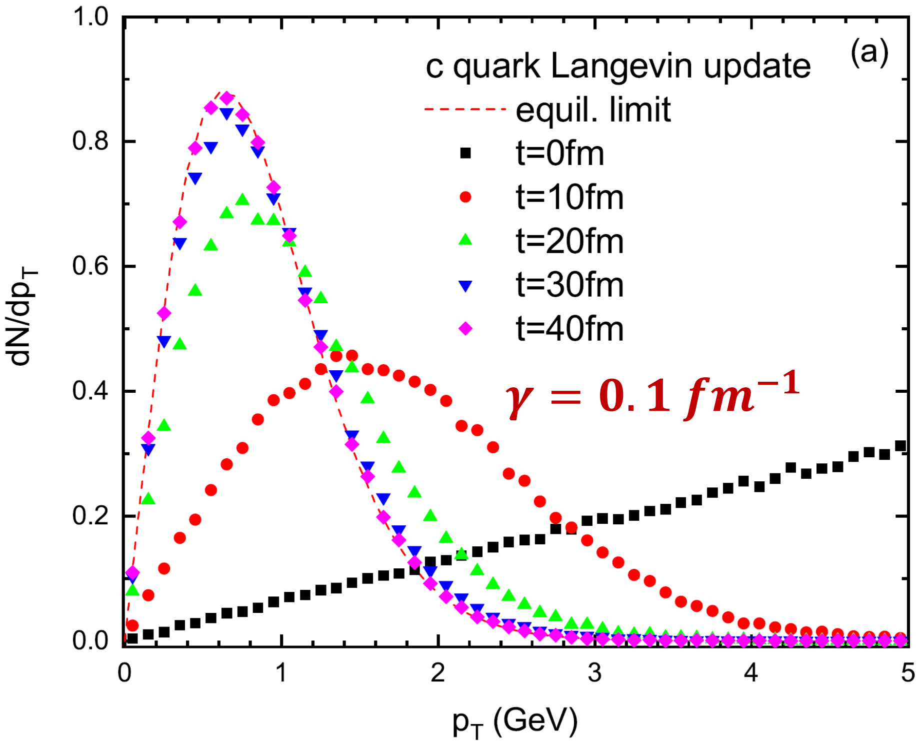


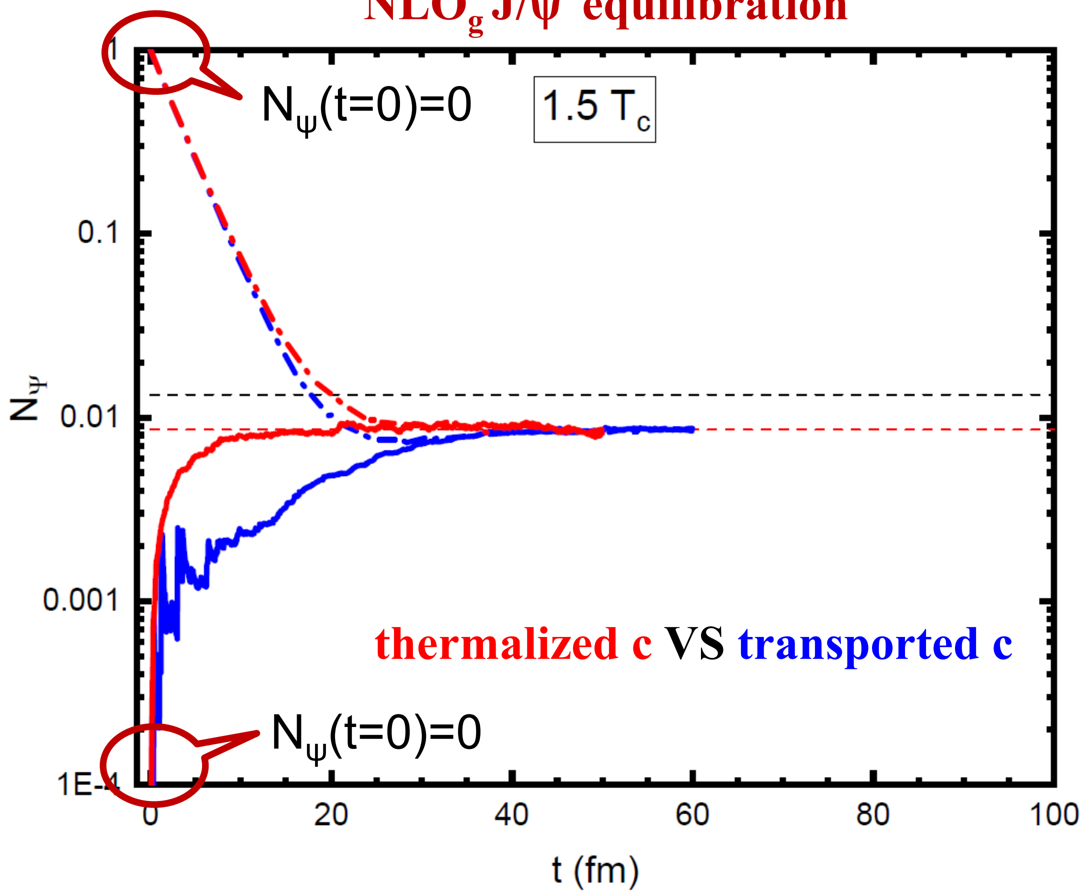

- Much faster equilibration for NLO process than for LO at T= 1.3-1.5$T_c$
- N(t=0)=1 significantly slower equilibration than N(t=0)=0, even if the initial J/ψ being thermal

# J/ψ equilibration: with transported c quarks

- Coupling the J/ψ Boltzmann transport to the single HQ diffusion

$$d\vec{p} = -\gamma\vec{p}dt + \sqrt{2Ddt}\vec{\rho}$$



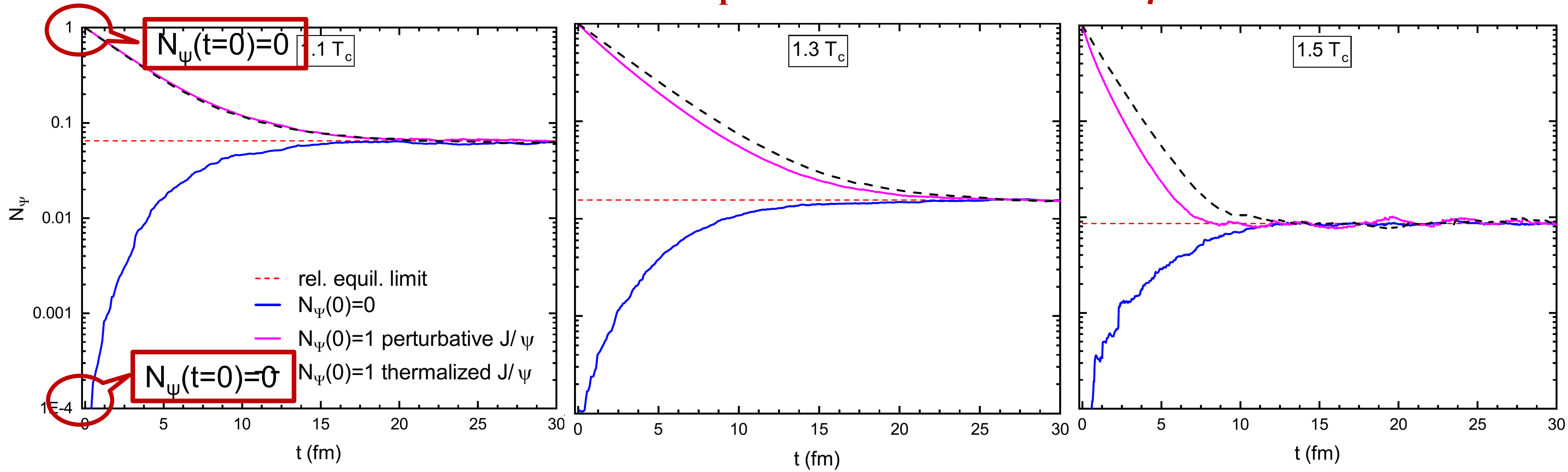




- c-quark thermalization time ~ 30 fm/c for single HQ thermal relaxation rate $\gamma$=0.1 fm$^{-1}$

- Slower J/ψ equilibration with transported c-quarks ➔ J/ψ equilibration lagging behind c-quark thermalization

# J/ψ equilibration: total rates & transported c

**LO + NLO reactions & transported $c$ & $\bar{c}$ w/ more realistic $\gamma = 0.4$ fm$^{-1}$**

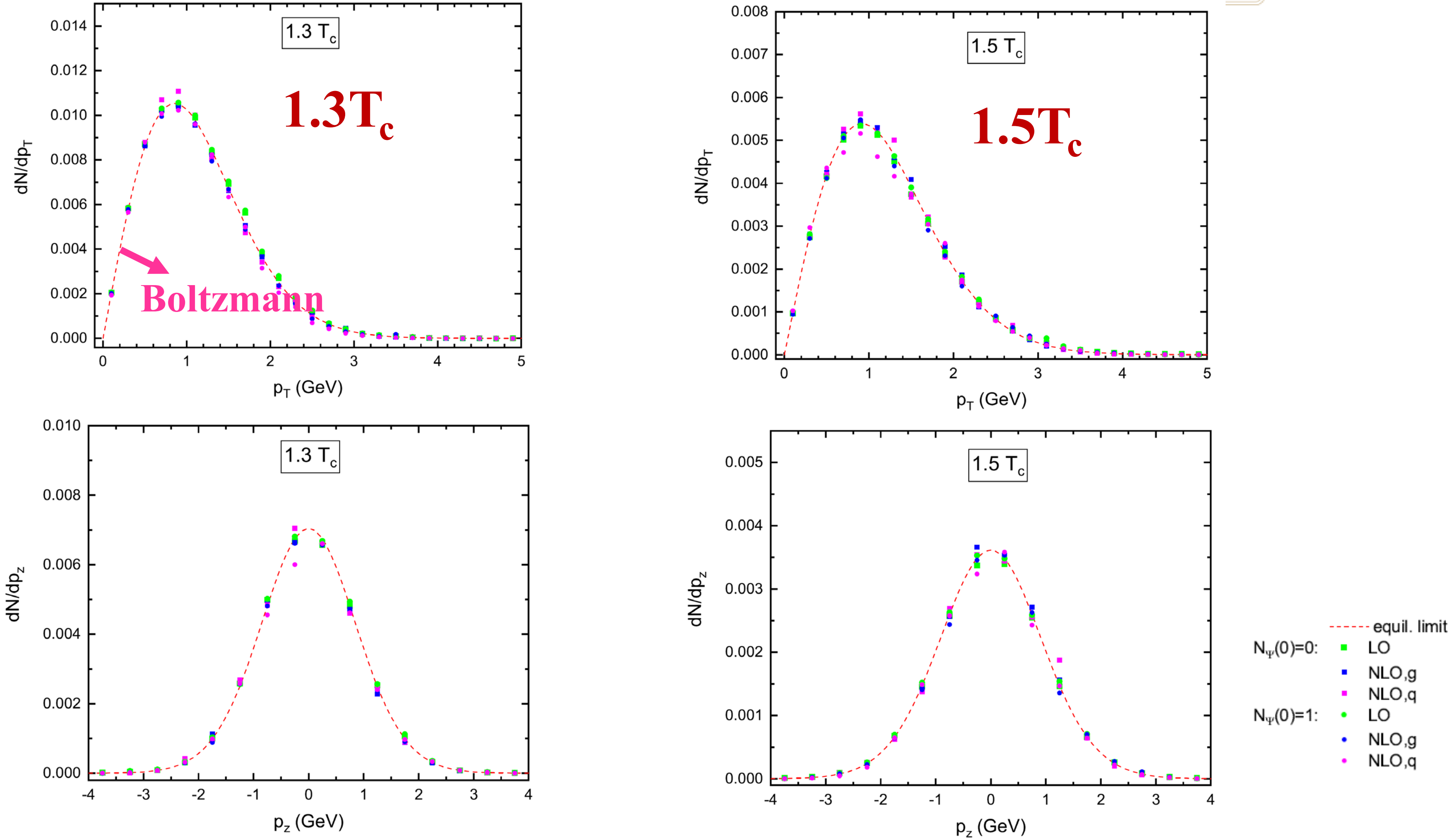



- J/ψ chemical equilibration time reaching 10-15 fm at 1.1-1.5$T_c$ ≈ QGP lifetime in central Pb-Pb collisions
- Equilibration times insensitive to initial conditions (Initially harder J/ψ equilibrating a bit faster)

# J/ψ kinetic equilibration: $p_T$ & $p_z$ spectra

$dN/dp_T$

$dN/dp_z$




- J/ψ $p_T$ and $p_z$ distributions reaching Boltzmann distri. ➔ both chemical & kinetic equilibrium achieved!

# Summary & Outlook

- Heavy quarkonium reaction cross sections & rates were computed, using the color-electric dipole (E1) coupling of quarkonium to external gluons

  - NLO was found to dominate at high incident gluon energy and high T

- J/ψ chemical & kinetic equilibration was demonstrated via simulation of dissociation & regeneration

  - Off-diagonal recombination of thermalizing $c\bar{c}$ was instrumental

  - NLO reactions accelerate the equilibration ➔ final equilibration time ~ 10-15 fm/c

- Our work gives a dynamical way of understanding SHM for charmonia & provides a baseline for realistic phenomenological applications