上海交通大學
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Calculation of Charmed Baryon Decay Constants on the Lattice

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Li Lei, Ran Jie

2025. 10. 26 **Guilin**

Outline

1. Background
2. Decay Matrix Element
3. Renormalization Constants
4. Summary

Background: The Puzzle of Matter-Antimatter Asymmetry

Particle Data Group,LBNL© 2000

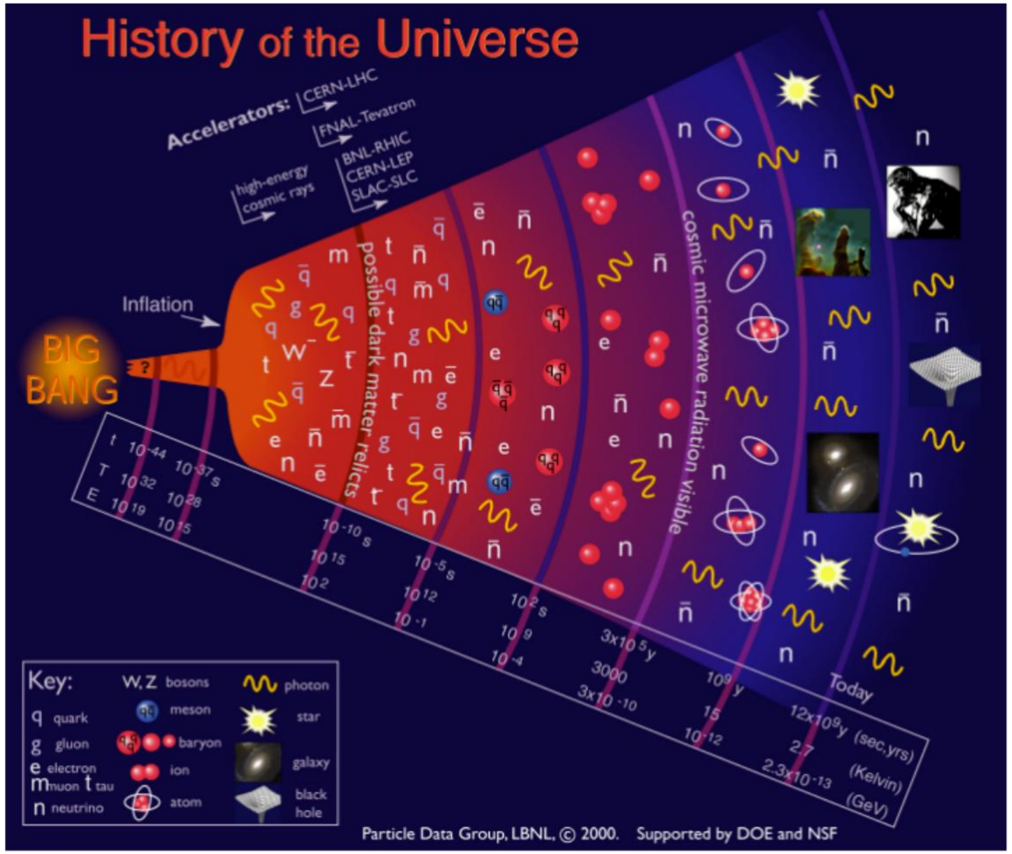


Figure 1. History of the Universe (Particle Data Group, LBNL © 2000)

Sakharov Conditions:

1. C and CP Violation
2. Baryon Number Violation
3. Departure from Thermal Equilibrium

A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967)

1.1 C and CP Violation

Semileptonic decay processes can be used to extract CKM matrix elements V_{cb}:

$$\mathrm{Br}\left(\Lambda_\mathrm{b}^0 \to \Lambda_c^+ l\bar{\nu}\right) = 6.2^{+1.4}_{-1.3}\%$$

Non-leptonic decay processes can be used to search for new sources of CP violation:

$$\mathrm{Br}\left(\Lambda_\mathrm{b}^0 \to \Lambda_\mathrm{c}^+ \pi^-\right) = 6.7^{+3.2+0.3+2.5}_{-2.2-2.8-1.3}\ \%$$

Factorization of form factors:

$$\langle \Lambda_c | iA | \Lambda_b \rangle \sim C \otimes T \otimes \underbrace{\langle \Lambda_c | O_{\Lambda_c} | 0 \rangle}_{f_{\Lambda_c} \phi_{\Lambda_c}} \otimes \langle 0 | O_{\Lambda_b} | \Lambda_b \rangle$$

Definition of Charmed Baryon Decay Constants:

$$\langle 0 | \epsilon_{ijk} (q_{1,i}^T C \Gamma_1 q_{2,j}) \Gamma_2 q_{3,k} \, |\mathcal{B}_c \rangle = m_{\mathcal{B}_c} f_{\mathcal{B}_c} u_{\mathcal{B}_c}$$

C. Q. Zhang, Z. Rui, et al. Phys. Rev. D 105, 073005 (2022)

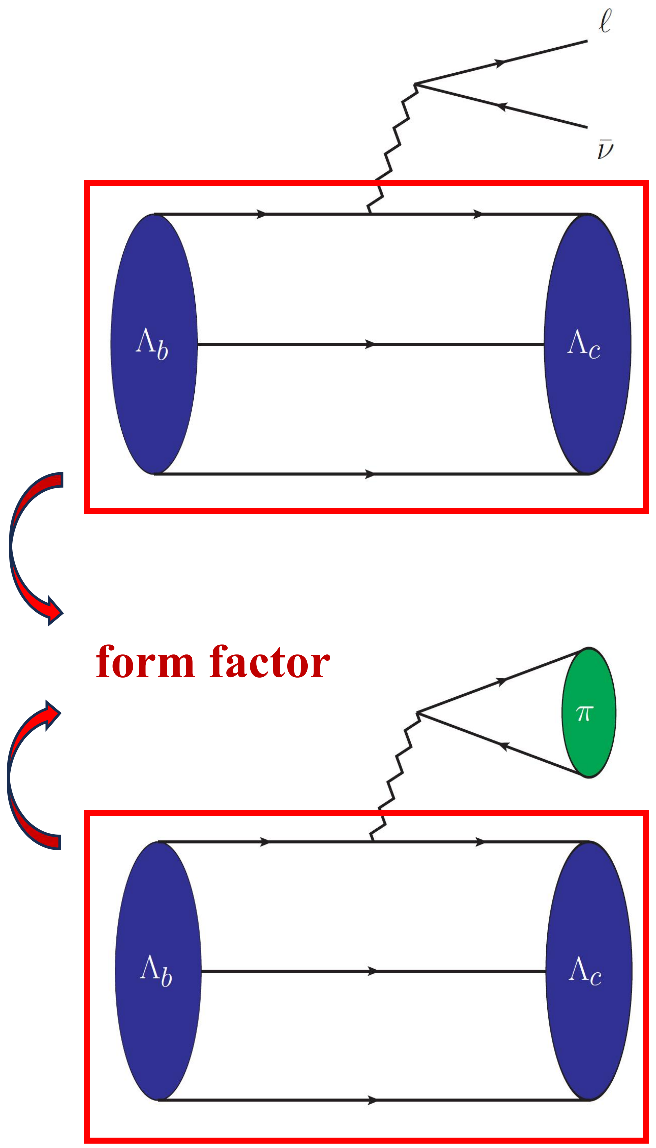


Figure 2. Non-leptonic and Semileptonic Decays

1.2 Baryon Number Violation

P. del Amo Sanchez, et al. [BaBar], Phys. Rev. D 83, 091101 (2011)

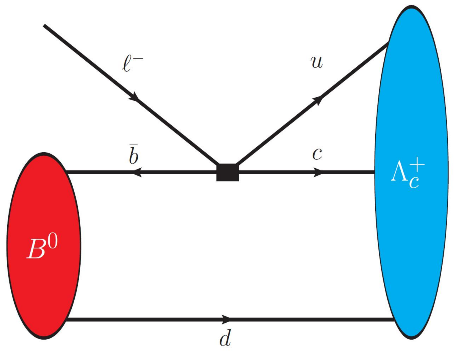


Figure 3. Grand Unification:

$B^0 \to \Lambda_c^+ l^-$

C.O.Dib, et al. JHEP 02, 224 (2023)

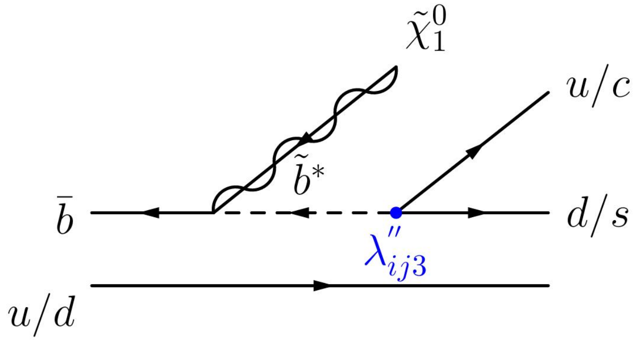


Figure 4. Supersymmetry:
$B^+ \to \Lambda_c^+ \tilde{\chi}_1^0$

J.P.Lees, et al. [BaBar], Phys. Rev. D 111, L031101 (2025)

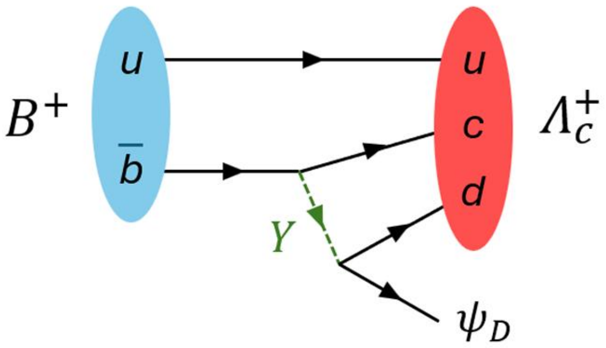


Figure 5. Models: $B^+ \to \Lambda_c^+ \psi_D$

Factorization of Form Factors in New Physics:

$$\langle \Lambda_c | iA | B \rangle \sim C_{new} \otimes T \otimes \underbrace{\langle \Lambda_c | O_{\Lambda_c} | 0 \rangle}_{f_{\Lambda_c} \phi_{\Lambda_c}} \otimes \langle 0 | O_B | B \rangle$$

Experiment + $\underbrace{\text{Form Factors}}_{\text{Proportional to decay constant}}$ → Constrain parameters of New Physics models

1.3 Current Research Status of charmed decay constant

	Decay constants ($10^{-2}\mathrm{GeV}^2$)				
Baryon	Λ_c^+	Ξ_c	Ξ_c'	Σ_c	Ω_c^0
QCD sum rule 【1】	-	-	2.13(62)	1.83(61)	3.45(85)
QCD sum rule 【2】	0.96(35)	1.09(32)	-	-	-
QCD sum rule 【3】	$1.19^{+0.19}_{-0.28}$	-	-	$3.08^{+0.49}_{-0.74}$	-
QCD sum rule 【4】	-	-	-	-	3.30(48)

【1】 Z.G.Wang, Phys. Lett. B 685, 59 (2010)

【2】 Z.G.Wang, Eur. Phys. J. C 68, 479 (2010)

【3】 A. Khodjamirian, Y. M. Wang, et al. JHEP 09, 106

【4】 Y.J. Shi, W. Wang, and Z. X. Zhao, Eur. Phys. J. C 80,568 (2020)

Precision at 15%-20%

First-principles calculations are necessary.

2. Bare decay constant of charm baryons

1. Construction of Charmed Baryon Operators

SU(3) symmetry of di-quarks:·

$3 \otimes 3 = \bar{3} \oplus 6$

$$\bar{3}: \begin{cases} O_{\Lambda_c^+} = \epsilon_{ijk}(u_i^T C\gamma_5 d_j)c_k, \\ O_{\Xi_c^+} = \epsilon_{ijk}(u_i^T C\gamma_5 s_j)c_k, \\ O_{\Xi_c^0} = \epsilon_{ijk}(d_i^T C\gamma_5 s_j)c_k, \end{cases}$$

Pseudoscalar current

$$6: \begin{cases} O_{\Xi_c'^+} = \epsilon_{ijk}(u_i^T C\gamma_\mu s_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Xi_c'^0} = \epsilon_{ijk}(d_i^T C\gamma_\mu s_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Sigma_c^+} = \epsilon_{ijk}(u_i^T C\gamma_\mu d_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Sigma_c^0} = \frac{1}{\sqrt{2}}\epsilon_{ijk}(d_i^T C\gamma_\mu d_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Sigma_c^{++}} = \frac{1}{\sqrt{2}}\epsilon_{ijk}(u_i^T C\gamma_\mu u_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Omega_c^0} = \frac{1}{\sqrt{2}}\epsilon_{ijk}(s_i^T C\gamma_\mu s_j)\gamma^\mu\gamma_5 c_k. \end{cases}$$

Vector current

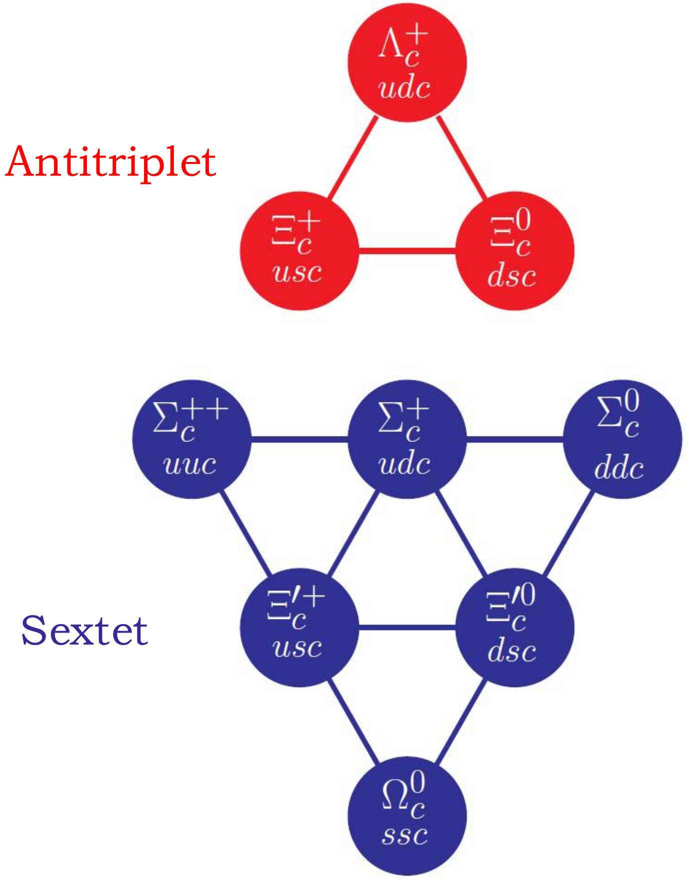


Figure 6. Classification of charmed baryon operators according to SU(3) symmetry

A. Khodjamirian and Y.M.Wang, et al. JHEP 09, 106 (2011)

H. Liu, W. Wang, Y. B. Yang, and Q.A. Zhang et al. Phys. Lett. B 841, 137941(2023)

2.2 Two-point correlation function:

According to the definition of the decay constant:

$$\langle 0|O_{\mathcal{B}_c}|\mathcal{B}_c\rangle = m_{\mathcal{B}_c} f_{\mathcal{B}_c} u_{\mathcal{B}_c}$$

Two-point correlation function:

$$C_{2pt}(t) = \sum_x e^{-ip\cdot x}\langle 0|O_{\mathcal{B}_c}(x), \bar{O}_{\mathcal{B}_c}(0)|0\rangle$$

Positive parity projection:

$$J^P = \left(\frac{1}{2}\right)^+ : \quad C_{2pt}^+(t) = \mathrm{Tr}\{C_{2pt}(t)P_+\}$$

Parameterization of the two-point correlation function at the hadronic level:

$$C_{2pt}^+(t) = 2m_{\mathcal{B}_c}^2 f_{\mathcal{B}_c}^{(0)2} e^{-m_{\mathcal{B}_c}\cdot t}(1 + \Delta C \cdot e^{\Delta m \cdot t})$$

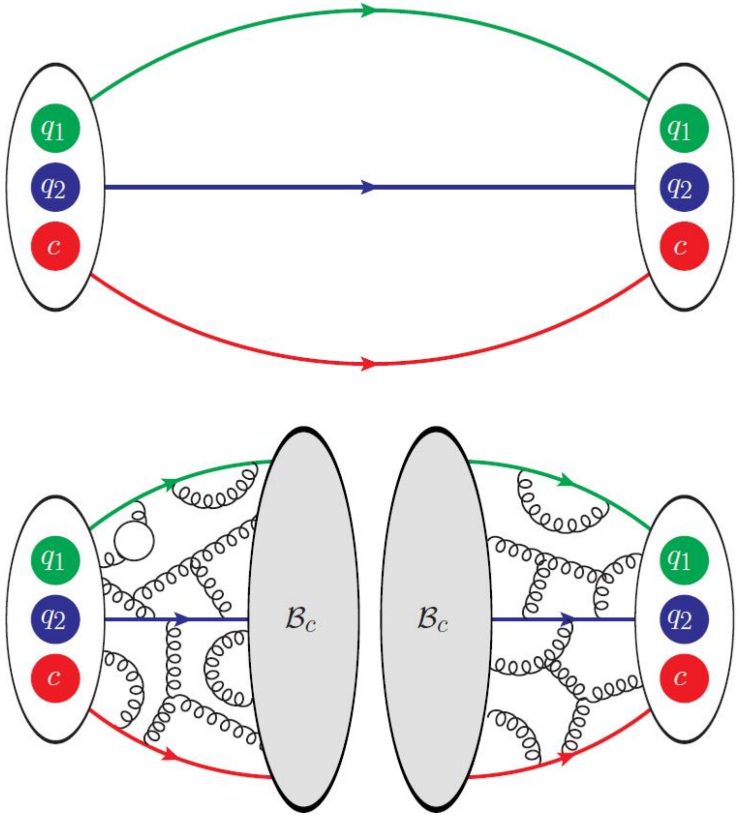


Figure 7. Two-point correlation functions at the quark and hadronic levels

2.3 Lattice set-up

Ensembles generated by the CLQCD collaboration:

Table 1. CLQCD Ensemble Information

Ensemble	$L^3 \times T$	a (fm)	Pion (MeV)	Nconf	Nsrc
C24P29	$24^3 \times 72$	0.105	290	864	2×10
C32P29	$32^3 \times 64$	0.105	290	984	2×10
C48P14	$48^3 \times 96$	0.105	135	187	2×20
C32P23	$32^3 \times 64$	0.105	230	451	2×10
F32P30	$32^3 \times 96$	0.077	300	777	2×10
H48P32	$48^3 \times 144$	0.052	320	550	2×6

Point source to point sink

Extrapolate to: 1.Physical pion mass

2. Zero lattice spacing

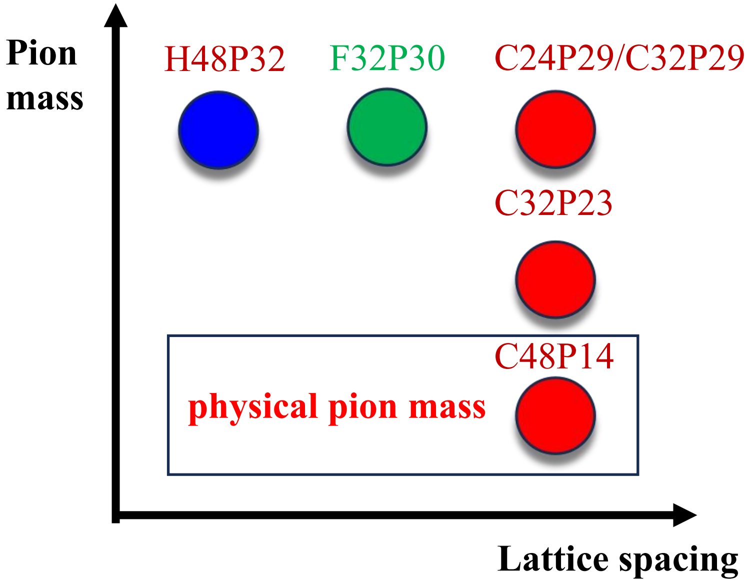


Figure 8. CLQCD ensembles

2.4 Ξ_c ,Ξ_c' as examples

The difference between Ξ_c and Ξ_c' lies in the Lorentz structure:

$$O_{\Xi_c} = \epsilon_{ijk}\left(l_i^T C\gamma_5 s_j\right)c_k$$

$$O_{\Xi_c'} = \epsilon_{ijk}\left(l_i^T C\gamma_\mu s_j\right)\gamma_5\gamma^\mu c_k$$

SU(3) symmetry effects are reflected

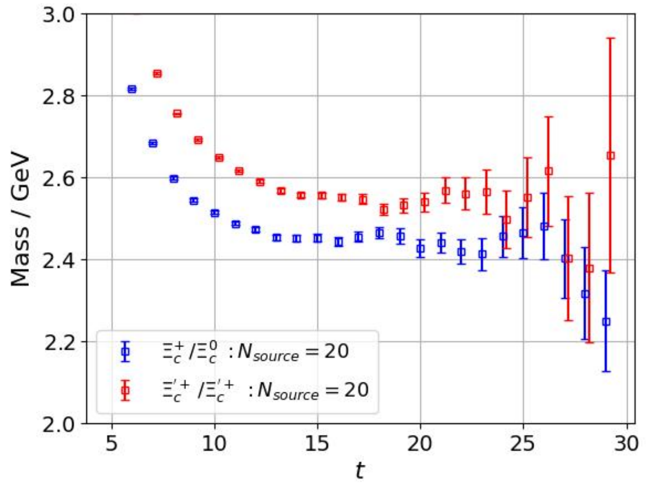


Figure. Effective mass plots for Ξ_c and Ξ_c'

First, perform a two-state fit to extract the effective mass and bare decay constants:

$$C_{2pt}^+(t) = 2m_{B_c}^2 f_{B_c}^{(0)2} e^{-m_{B_c}\cdot t}\left(1 + \Delta C \cdot e^{\Delta m \cdot t}\right)$$

Next, perform chiral and continuum extrapolations:

$$m_{B_c}(m_\pi, a) = m_{B_c,phy} + c_1\left(m_\pi^2 - m_{\pi,phy}^2\right) + c_2 a^2$$

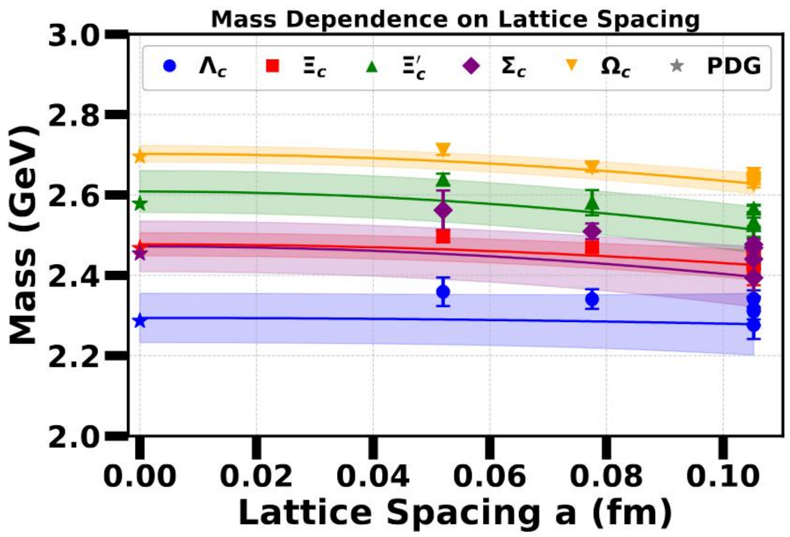


Figure 10. Mass extrapolation plot

2.5 Bare Decay Constants

Table 2. Summary of Bare Charmed Baryon Decay Constants

	Decay constant (bare) ($10^{-2}\mathrm{GeV}^2$)					
Baryon	**C24P29**	**C32P23**	**C32P29**	**C48P14**	**F32P30**	**H48P32**
$\Lambda_c^+(udc)$	1.831(74)	1.831(88)	1.735(83)	1.75(11)	2.006(84)	2.15(11)
$\Xi_c^+(usc)$	1.70(10)	1.743(40)	1.740(76)	1.795(63)	2.000(35)	2.21(10)
$\Xi_c^0(dsc)$	1.70(10)	1.743(40)	1.740(76)	1.795(63)	2.000(35)	2.21(10)
$\Xi_c'^+(usc)$	2.569(94)	2.393(68)	2.504(48)	2.45(16)	2.991(64)	3.34(13)
$\Xi_c'^0(dsc)$	2.569(94)	2.393(68)	2.504(48)	2.45(16)	2.991(64)	3.34(13)
$\Sigma_c^+(udc)$	2.41(10)	2.390(93)	2.507(52)	2.40(17)	3.065(57)	3.620(85)
$\Sigma_c^0(ddc)$	2.41(10)	2.390(93)	2.507(52)	2.40(17)	3.065(57)	3.620(85)
$\Sigma_c^{++}(uuc)$	2.41(10)	2.390(93)	2.507(52)	2.40(17)	3.065(57)	3.620(85)
$\Omega_c^0(ssc)$	2.830(55)	2.72(11)	2.699(43)	2.724(78)	3.307(32)	3.76(11)

C24P29： 3~5%　C32P29： 3~5%　C32P23： 4~5%

F32P30： 4~6%　H48P32： 2~5%

3. Non-Perturbative Renormalization

The earliest study of baryon renormalization on the lattice was for proton decay::

J. S. Yoo, et al, Phys. Rev. D 105, 074501 (2022).

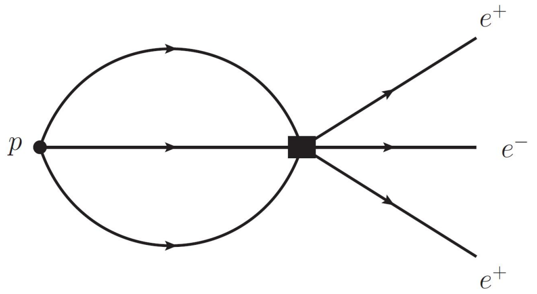


Figure 11. Two-point correlation function $p \to e^+e^-e^+$

Extended to the study of charmed baryon decay ants:

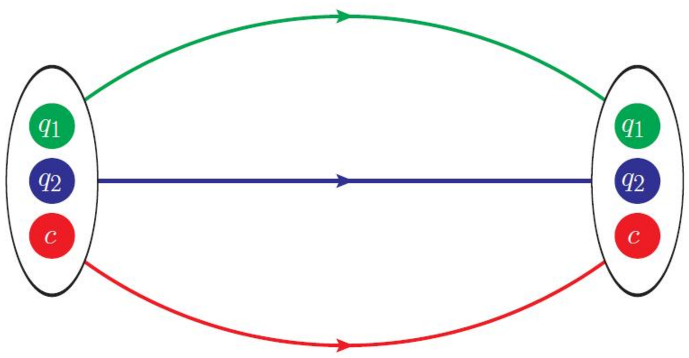


Figure 12. Two-point correlation function

NPR(Non-Perturbative Renormalization):

From the renormalization constants of charmed baryon operators:

$$O = Z_{O_i} O^{(0)}$$

Obtain non-perturbative renormalization conditions:

$$Z_{O_i} Z_{q1}^{-1/2} Z_{q2}^{-1/2} Z_{q3}^{-1/2} \Gamma_O |_{\text{SMOM}} = 1$$

Calculation uses the Symmetric Momentum Subtraction scheme (SMOM)

Advantage: NLO convergence of conversion factors

3.1 Non-Perturbative Renormalization Scheme

Symmetric Momentum Subtraction scheme (SMOM)

Meson: SMOM$_{\gamma_\mu}$

Momentum choice: $p_1 = \frac{2\pi}{L}$ (n, n, 0, 0), $p_1 = \frac{2\pi}{L}$ (n, 0, n, 0)

Renormalization condition: $p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$

Baryon: SYM3q

Momentum choice: $ap_1 = \frac{2\pi}{L}$ (n, n, 0, 0), $ap_2 = \frac{2\pi}{L}$ (-n, 0, n, 0)

$ap_3 = \frac{2\pi}{L}$ (-n, 0, -n, 0)

Renormalization condition:

$p_1^2 = p_2^2 = p_3^2 = \mu^2, \; p_1 + p_2 + p_3 = 0$

Quark mass is neglected in renormalization

Corresponding lattice calculation:

$$O_1 = \epsilon_{\mathrm{ijk}}(l_1^T C\gamma_5 l_2)\, l_3$$
$$O_2 = \epsilon_{\mathrm{ijk}}(l_1^T C\gamma_\mu l_2)\, \gamma_\mu\gamma_5 l_3$$
$$O_3 = \epsilon_{\mathrm{ijk}}(l_1^T C\gamma_\mu l_1)\, \gamma_\mu\gamma_5 l_3$$

Charmed baryon operator nonet degenerates into three types:

$$O_1 = Z_{O1} O_1^{(0)}$$
$$O_2 = Z_{O2} O_2^{(0)}$$
$$O_3 = Z_{O3} O_3^{(0)}$$

3.2 Baryon Operator Vertex Function

Calculation

$$Z_{O_i} Z_{q1}^{-1/2} Z_{q2}^{-1/2} Z_{q3}^{-1/2} \Gamma_O|_{\mathrm{SMOM}} = 1$$

Define momentum-space Green's functions:

$$G(x, y_1, y_2, y_3) = \langle O(x), \bar{q}_1(y_1)\bar{q}_2(y_2)\bar{q}_3(y_3)\rangle$$

Construct momentum-space truncated Green's functions:

$$\Lambda(p_1, p_2, p_3) = G(p_1, p_2, p_3) S^{-1}(p_1) S^{-1}(p_2) S^{-1}(p_3)$$

Vertex function: $\Gamma_O(p_1, p_2, p_3) = P * \Lambda(p_1, p_2, p_3)$

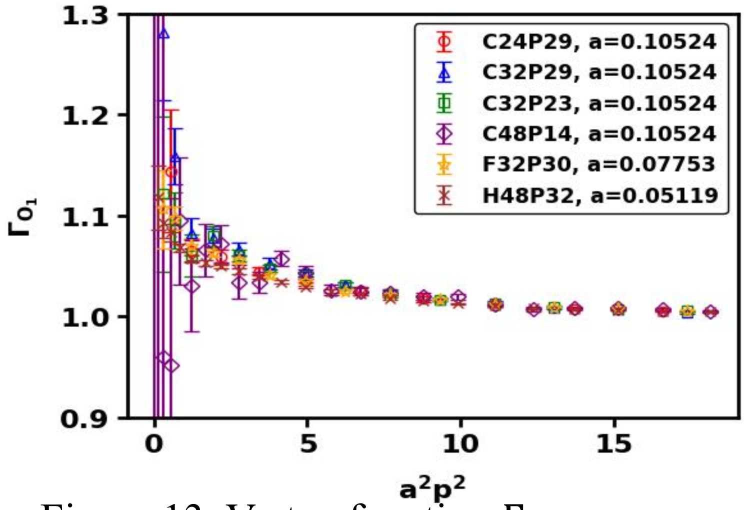


Figure 13. Vertex function: Γ_{O_i}

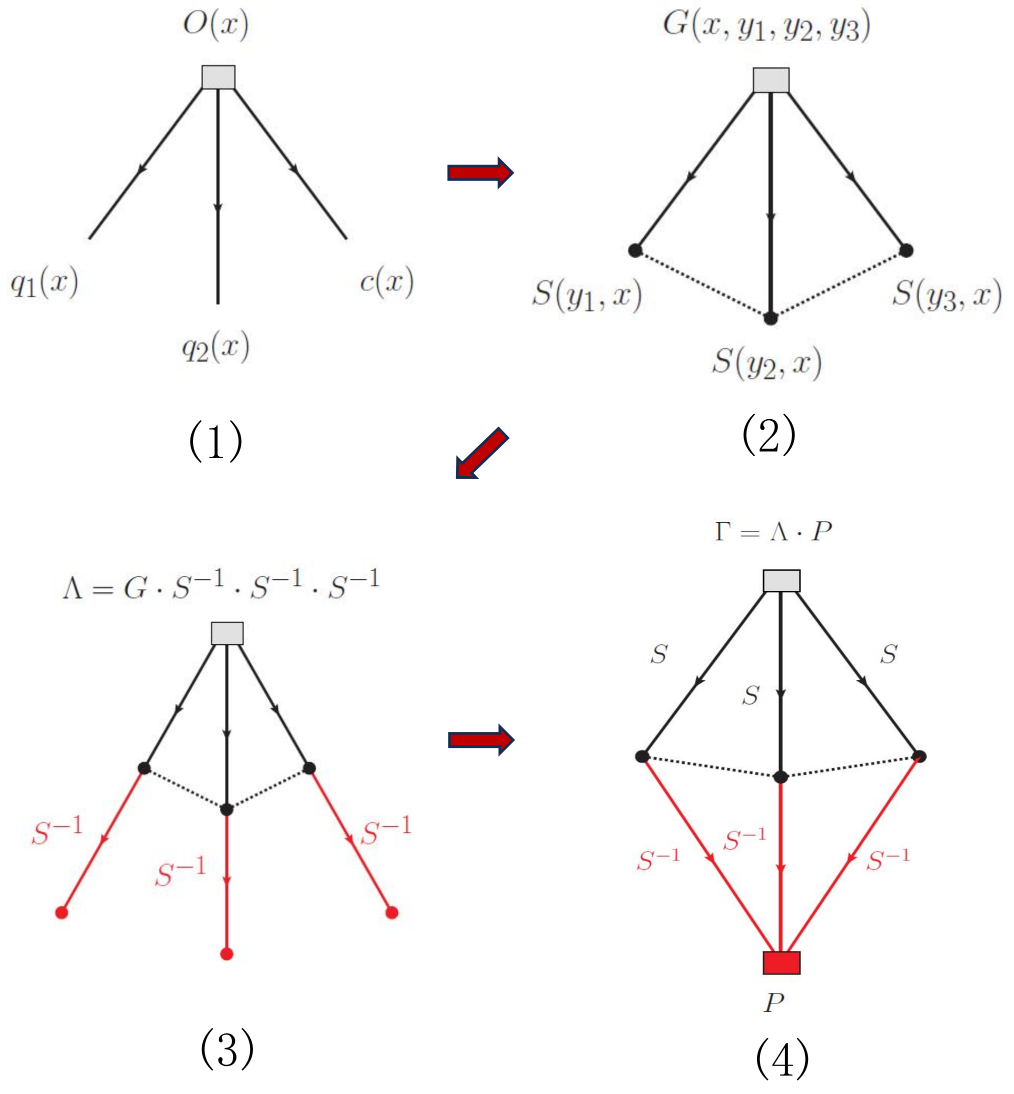


Figure 14. Renormalization flow

3.3 Bilinear Operator Vertex Function

Bilinear operator:

$$O_V^{\mu} = \overline{\psi}_1 \gamma^{\mu} \psi_2$$

Momentum-space Green's function:

$$G_V^{\mu}(p_1, p_2) = \langle \psi_1(p_1)\, O_V^{\mu}\, \overline{\psi}_2(p_1) \rangle = \frac{1}{N} S_{out}(p_1) \gamma^{\mu} S_{in}(p_2)$$

Where:

$$S_{out}(p_1) = \frac{1}{N} \sum_x e^{-ip_1 \cdot x} \langle \psi(x)\, \overline{\psi}(0) \rangle$$

$$S_{in}(p_2) = \frac{1}{N} \left(\gamma_5 \sum_x e^{-ip_2 \cdot x} \langle \psi(x)\, \overline{\psi}(0) \rangle\, \gamma_5 \right)^{\dagger}$$

Vector current vertex function：

$$\Gamma_V = \frac{1}{48} \mathrm{Tr}\{ S_{in}^{-1}(p_2) G_V^{\mu}(p_1, p_2) S_{out}^{-1}(p_1) \gamma_{\mu} \}$$

Z_q J. S. Yoo, et al. Phys. Rev. D 105, 074501 (2022):

$$Z_q = Z_V * \Gamma_V$$

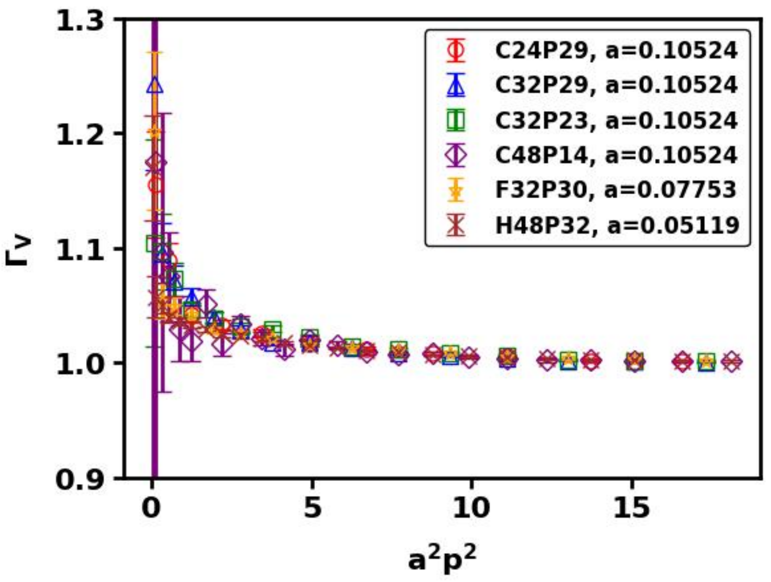


Figure 15. Vector current vertex function Γ_V

3.4 Quark Mass Extrapolation

Definition of operator renormalization constant

$$Z_{O_i} Z_{q1}^{-1/2} Z_{q2}^{-1/2} Z_{q3}^{-1/2} \Gamma_{O_i}|_{\mathrm{SMOM}} = 1$$

Where

$$Z_q = Z_V * \Gamma_V$$

Obtain

$$\frac{Z_{Oi}}{Z_V^{3/2}}(m_q, a^2p^2) = \frac{\Gamma_V^{3/2}}{\Gamma_{O_i}}(m_q, a^2p^2)$$

Zero-mass limit extrapolation formula:

$$\frac{Z_{Oi}}{Z_V^{3/2}}(m_q, a^2p^2) = \frac{\Gamma_V^{3/2}}{\Gamma_{O_i}}(m_q, a^2p^2) = \boldsymbol{A} + Ba\, m_q$$

m_q Range: Uniformly take six masses between $[l_{mass}, \ s_{mass}]$

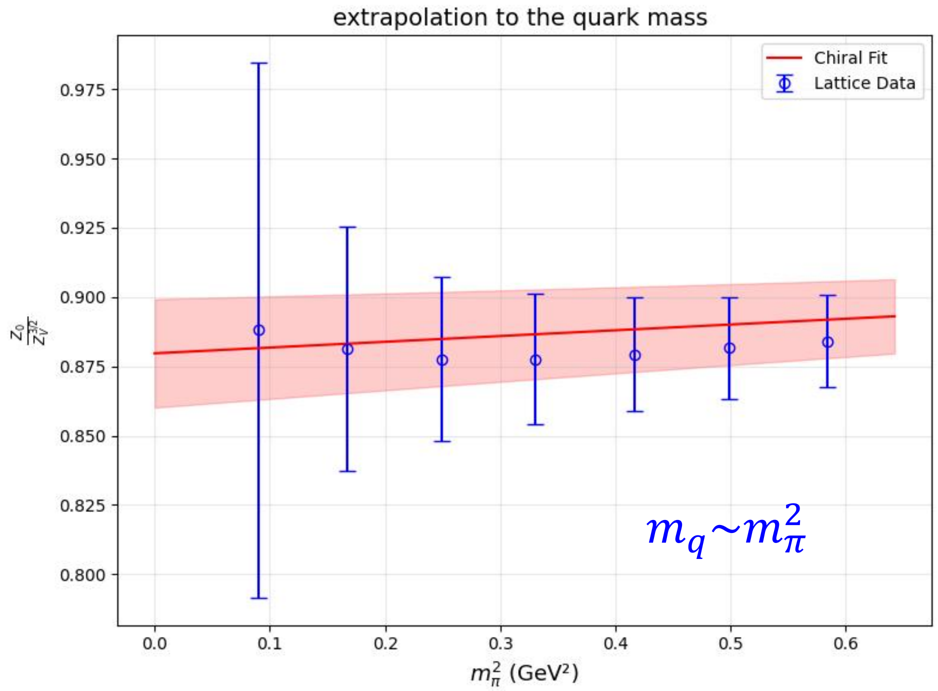


Figure 16. Extrapolation for n=1 in H48P32

3.5 $\mathrm{SMOM}_{\gamma_\mu}$/SYM3q → $\overline{\mathrm{MS}}$ Conversion

After mass extrapolation multiply by conversion factor:

$$\frac{Z_{Oi}}{Z_V^{3/2}}(\mu, a^2p^2) = \lim_{m_q \to 0} C^{\overline{MS} \leftarrow SMOM_{\gamma_\mu}/SYM3q} \frac{Z_{Oi}}{Z_V^{3/2}}$$

Conversion factor:

$$C^{\overline{MS} \leftarrow SMOM_{\gamma_\mu}/SYM3q}(\mu) = [\Lambda^{\overline{\mathrm{MS}}}]_{\mathrm{SYM3q}} \left(C_q^{\overline{\mathrm{MS}} \leftarrow \mathrm{SMOM}_{\gamma_\mu}} \right)^{-3/2}$$

Quark field conversion factor: $C_q^{\overline{MS} \leftarrow SMOM_{\gamma_\mu}} = 1 + \frac{4}{3}\left(\frac{\alpha_s}{4\pi}\right)$

Conversion factor for pseudoscalar current operator J.A.Gracey, JHEP 09, 052 (2012):

$$[\Lambda^{\overline{\mathrm{MS}}}]_{\mathrm{SYM3q}} = 1 + 0.989426\left(\frac{\alpha_s}{4\pi}\right)$$

For vector current operator, using Fierz transformation formula:

$$\epsilon_{ijk}(u_1^T C\gamma^\mu u_2)\gamma_5\gamma_\mu d_3 = 2[\epsilon_{ijk}(u_1^T C u_2)d_3 - \epsilon_{ijk}(u_1^T C\gamma_5 u_2)\gamma_5 d_3]$$

3.6 Momentum Extrapolation

Multiply by conversion factor and evolve to 2 GeV:

$$\frac{Z_{Oi}(2\text{GeV},a^2p^2)}{Z_V^{3/2}} = U(2\text{GeV})C^{\overline{\text{MS}}\leftarrow \text{SMOM}_{\gamma_\mu}/\text{SYM3q}}\frac{Z_{Oi}}{Z_V^{3/2}}(a^2p^2)$$

Extrapolation：

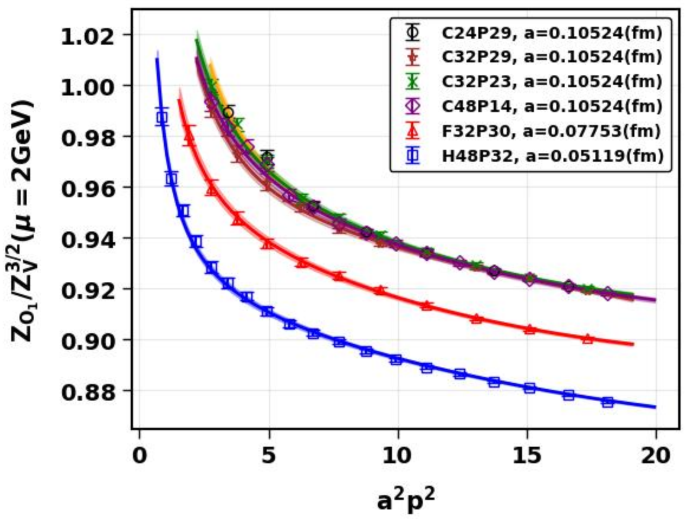
$$\frac{Z_{Oi}(2\text{GeV},a^2p^2)}{Z_V^{3/2}} = \frac{Z_{Oi}(2GeV)}{Z_V^{3/2}} + \frac{c_{-1}}{a^2p^2} + c_1(a^2p^2) + c_2(a^2p^2)^2$$

Renormalization constants for charmed baryon operators:

$$Z_{Oi}(2\text{GeV}) = \frac{Z_{Oi}(2\text{GeV})}{Z_V^{3/2}}\left(Z_V^{q_1}Z_V^{q_2}Z_V^{c}\right)^{1/2}$$

Table 3. Renormalization Constants for Charmed Baryon

Ensemble	$Z_{\Lambda_c}^{\overline{\text{MS}}}$	$Z_{\Xi_c}^{\overline{\text{MS}}}$	$Z_{\Xi_c'}^{\overline{\text{MS}}}$	$Z_{\Sigma_c}^{\overline{\text{MS}}}$	$Z_{\Omega_c}^{\overline{\text{MS}}}$
C24P29	0.9277(82)	0.9584(84)	0.9510(85)	0.9205(82)	0.9824(88)
C32P29	0.9252(91)	0.9557(95)	0.963(10)	0.9319(98)	0.994(10)
C32P23	0.9477(85)	0.9791(88)	0.9701(84)	0.9389(81)	1.0023(86)
C48P14	0.9481(86)	0.9796(89)	0.9783(80)	0.9469(77)	1.0108(83)
F32P30	0.8908(56)	0.9085(57)	0.9048(51)	0.8872(50)	0.9228(52)
H48P32	0.8400(27)	0.8492(28)	0.8513(26)	0.8420(26)	0.8607(27)



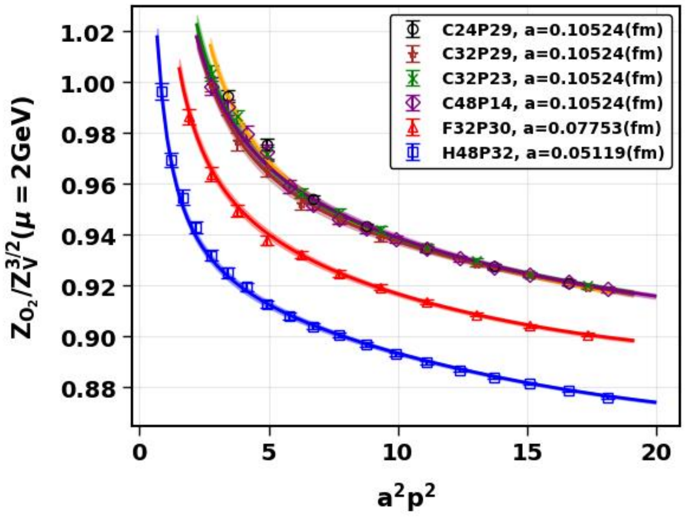


Figure 17. Extrapolation of renormalization constant ratios

3.7 Renormalized Decay Constants

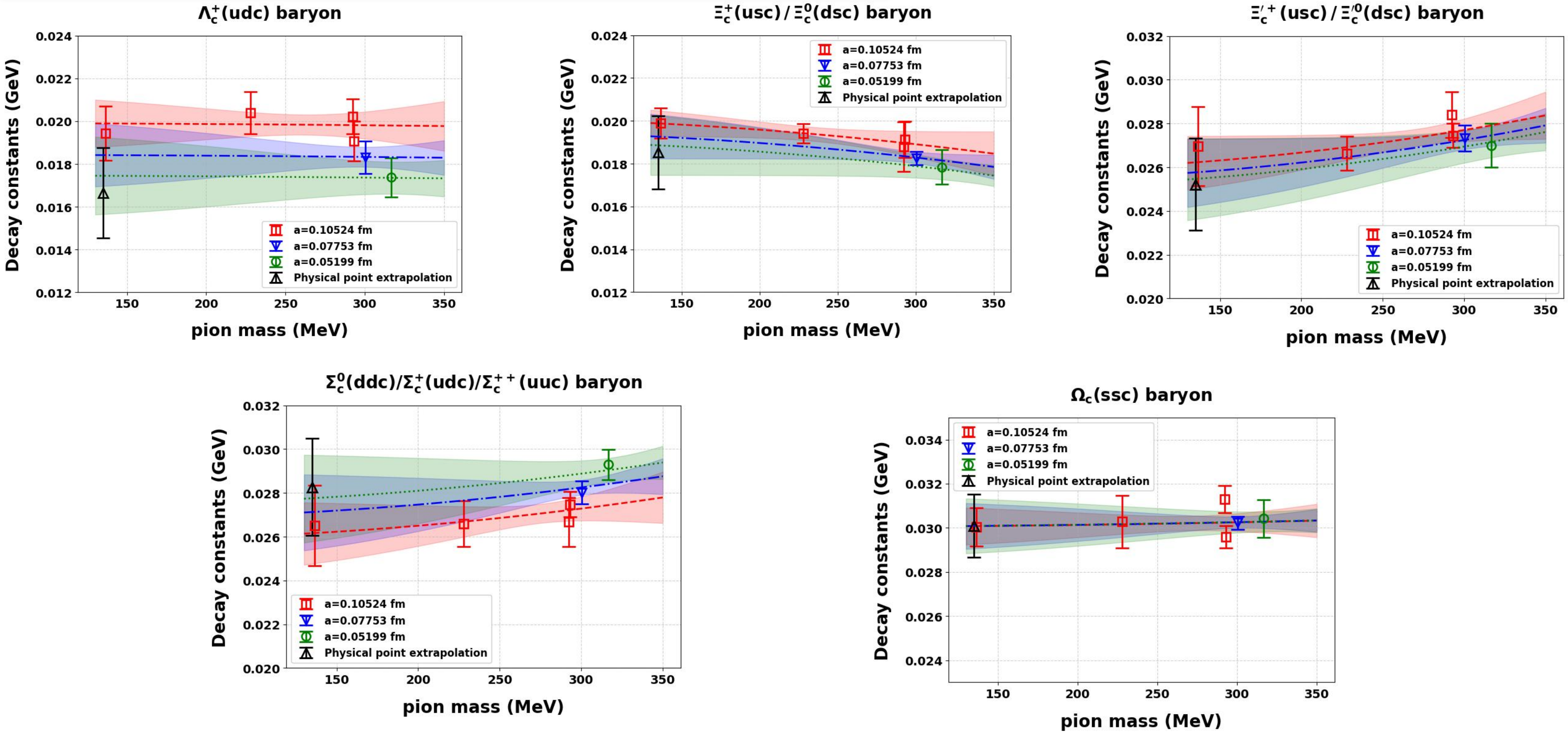


Figure 18. Extrapolation of charmed baryon decay constants

Extrapolation formula:

$$f_{\mathcal{B}}(m_\pi, a) = f_{\mathcal{B},phy} + c_1\left(m_\pi^2 - m_{\pi,phy}^2\right) + c_2 a^2 + c_3 e^{-m_\pi L}$$

Obtain the unique decay constant

3.8 Numerical Results

Table 4. Charmed Baryon Decay Constants at $\mu = 2$ GeV

	Decay constants ($10^{-2}\mathrm{GeV}^2$)				
Baryon	Λ_c^+	Ξ_c	Ξ_c'	Σ_c	Ω_c^0
QCD sum rule 【1】	-	-	2.13(62)	1.83(61)	3.45(85)
QCD sum rule 【2】	0.96(35)	1.09(32)	-	-	-
QCD sum rule 【3】	$1.19^{+0.19}_{-0.28}$	-	-	$3.08^{+0.49}_{-0.74}$	-
QCD sum rule 【4】	-	-	-	-	3.30(48)
This work	1.66(37)	1.98(48)	2.69(50)	2.26(49)	3.22(41)

【1】 Z.G.Wang, Phys. Lett. B 685, 59 (2010)

【2】 Z.G.Wang, Eur. Phys. J. C 68, 479 (2010)

【3】 A. Khodjamirian, Y. M. Wang, et al. JHEP 09, 106

【4】 Y.J. Shi, W. Wang, and Z. X. Zhao, Eur. Phys. J. C 80,568 (2020)

The precision of the lattice calculation for charmed baryon decay constants is 11~22%

4.Summary

Summary

- First calculation of charmed baryon decay constants based on first principles.
- Performed renormalization calculations for charmed baryon operators.
- Performed continuum, physical mass, and infinite volume extrapolations to obtain physical values for charmed baryon decay constants.

Outlook:

- Calculate charmed baryon operators with different Lorentz structures.
- First-principles calculations of weak decays for heavy baryons.

Thank you!