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Perturbative and non-perturbative properties of heavy quark transport in a thermal QCD medium

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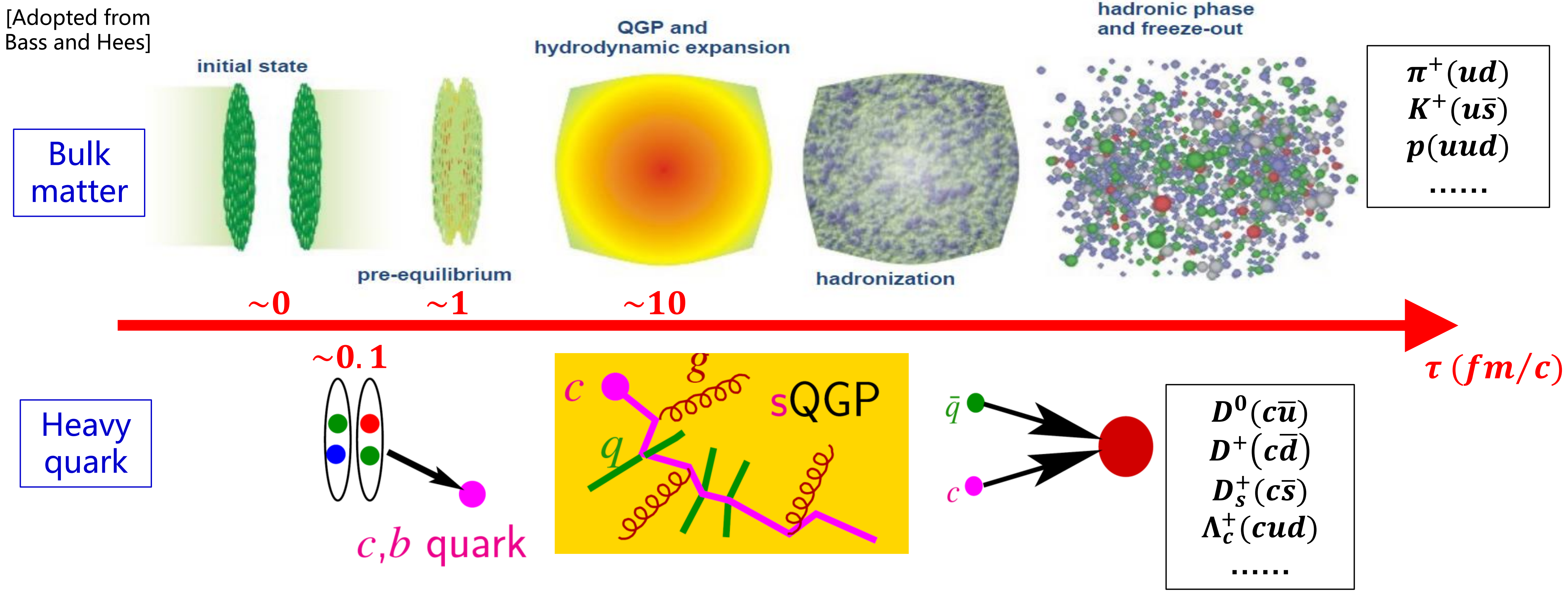
+ based on: arXiv: 2509.23872; 2510.10294; PRD 109, 096028 (2024); EPJC 81, 536 (2021)

Outline

- Introduction
- HQ transport in perturbation theory: the soft-hard factorized approach
- Non-perturbation approach: the background field effective theory
- Numerical results
- Summary and outlook

Heavy quarks (HQ) as probes of QGP

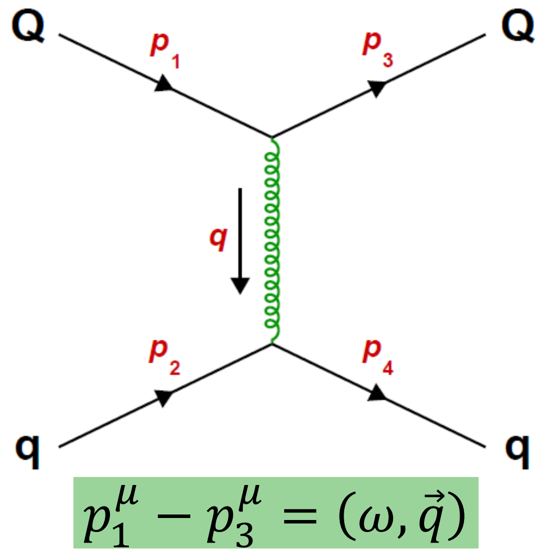


- $m_Q \gg \Lambda_{QCD}$: their initial production can be well described by pQCD
- $m_Q \gg T$: thermal abundance in QGP is negligible ~ final multiplicity set by the initial hard production
- $m_Q \gg gT$: many soft scatterings necessary to change significantly the momentum of HQ ~ Brownian motion

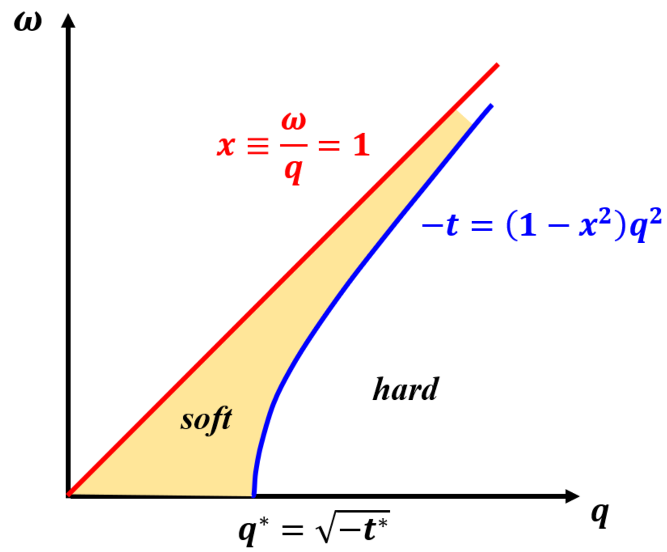
Soft-hard factorization model

- Divergence from t-channel contribution $\frac{d\sigma}{dt} \propto \overline{|\mathcal{M}^2|} \propto \frac{1}{t^2}$

  ~ infrared divergence when $|t| \to 0$



- Infrared regulator can be well determined on first principles: soft-hard factorization approach [Braaten and Yuan, PRL PRL 66, 2183 (1991)]
  - ✓ hard collision: $|t| > |t^*|$, where the pQCD Born approximation is valid
  - ✓ soft collision: $|t| < |t^*|$, where the t-channel long wavelength gluons are screened by the mediums ~ they feel the presence of the medium and require the resummation ~ HTL approximation



- The intermediate scale t^* is formally chosen as

$$m_D^2 \ll -t^* \ll T^2$$

implying weak-coupling or high-temperature limit

[Braaten and Yuan, PRL 66, 2183 (1991)]

Calculation strategy

- The energy loss per traveling distance

$$-\frac{dE}{dz} = \int d^3\vec{q}\,\frac{d\Gamma}{d^3\vec{q}}\frac{\omega}{v_1}$$

- Momentum diffusion coefficients

$$\kappa_T = \frac{1}{2}\int d^3\vec{q}\,\frac{d\Gamma}{d^3\vec{q}}\vec{q}_T^{\,2} \qquad \kappa_L = \int d^3\vec{q}\,\frac{d\Gamma}{d^3\vec{q}}\vec{q}_L^{\,2}$$

where, Γ is the interaction rate between HQ and medium partons,

key variable

$$\Gamma = \Gamma_{(t)}^{soft} + \Gamma_{(t)}^{hard} + \Gamma_{(su)}$$

Collisional energy loss: hard component

- The interaction rate for a given elastic process $Q + i \to Q + i \quad (i = q, g)$

$$\Gamma^{Qi}(E_1, T) = \frac{1}{2E_1}\int_{p_2} \frac{\mathcal{N}(E_2)}{2E_2}\int_{p_3} \frac{1}{2E_3}\int_{p_4} \frac{1}{2E_4}\overline{|\mathcal{M}^2|}^{Qi}(2\pi)^4\delta^{(4)}(p_1 + p_2 - p_3 - p_4)$$

and the total energy loss for the hard collisions in t-channel

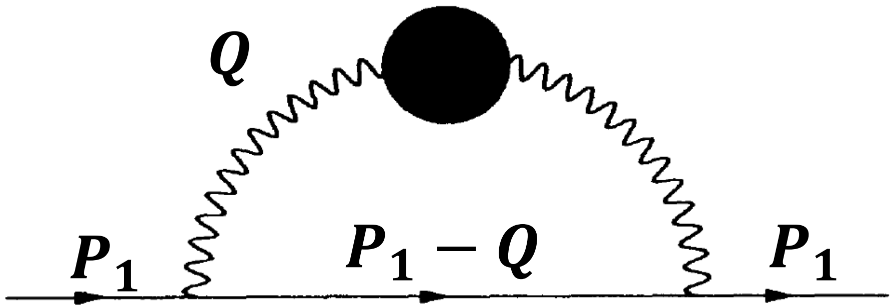
$$\boxed{\left[-\frac{dE}{dz}\right]_{(t)}^{hard} = \frac{1}{256\pi^3\vec{p}_1^{\,2}}\sum_{i=q,g}\int_{|\vec{p}_2|_{min}}^{\infty} d|\vec{p}_2|E_2\mathcal{N}(E_2)\int_{-1}^{cos\psi|_{max}} d(cos\psi)\int_{t_{min}}^{t^*} dt\,\frac{b}{a^3}\overline{|\mathcal{M}^2|}_{Qi(t)}}$$

- The contributions from s- and u-channels are not divergent for small momentum transfers → no need to introduce the intermediate cutoff

$$\boxed{(i = g) \qquad (|\vec{p}_2|_{min} \Rightarrow 0) \qquad (cos\psi|_{max} \Rightarrow 1) \qquad (t^* \Rightarrow 0)}$$

Collisional energy loss: soft component

- Basic formulas [Weldon, PRD 28, 2007 (1983)]



$$\Gamma(E_1,T) = -\frac{1}{2E_1}\overline{n}_F(E_1)Tr[(P_1\cdot\gamma + m_1)Im\Sigma(P_1)]$$

with the HQ self-energy in Minkowski space

$$\Sigma(P_1) = iC_F g^2 \int \frac{d^4Q}{(2\pi)^4}\Delta^{\mu\nu}(Q)\gamma_\mu \frac{1}{(P_1 - Q)\cdot\gamma - m_1}\gamma_\nu$$

and the HTL gluon propagator in Coulomb gauge

$$\Delta^{\mu\nu}(Q) = -(\delta^{\mu 0}\delta^{\nu 0})\Delta_L - (\delta^{ij} - \hat{q}^i\hat{q}^j)\delta^{\mu i}\delta^{\nu j}\Delta_T$$

The longitudinal and transverse effective propagators are

$$(\Delta_L)^{-1} = \vec{q}^2 + \Pi_L \qquad (\Delta_T)^{-1} = q_0^2 - \vec{q}^2 - \Pi_T$$

[Blaizot and Iancu, Phys. Rep. 359, 355 (2002); Alberico et. al., EPJC 71, 1666 (2011)]

Collisional energy loss: soft component

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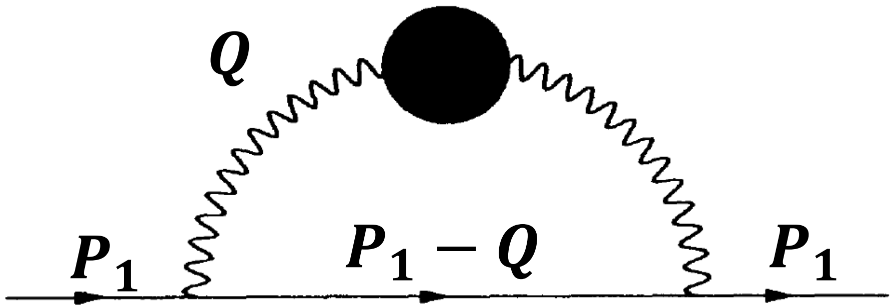


$$\Gamma(E_1,T) = C_F g^2\int_q\int d\omega\bar{n}_B(\omega)\delta(\omega-\vec{v}_1\cdot\vec{q})\{\rho_L(\omega,q)+\vec{v}_1^2[1-(\hat{v}_1\cdot\hat{q})^2]\rho_T(\omega,q)\}$$

The total energy loss in soft collisions reads

$$\boxed{\left[-\frac{dE}{dz}\right]_{(t)}^{soft} = \frac{C_F g^2}{8\pi^2 v_1^2}\int_{t^*}^{0}dt(-t)\int_0^{v_1}dx\frac{x}{(1-x^2)^2}[\rho_L(t,x)+(v_1^2-x^2)\rho_T(t,x)]}$$

Collisional energy loss: hard+soft

$$-\frac{dE}{dz} = \left[-\frac{dE}{dz}\right]_{(t)}^{soft} + \left[-\frac{dE}{dz}\right]_{(t)}^{hard} + \left[-\frac{dE}{dz}\right]_{(su)}$$

Full results

$$\left[-\frac{dE}{dz}\right]_{(t)}^{soft} = \frac{C_F g^2}{8\pi^2 v_1^2}\int_{t^*}^{0} dt(-t)\int_0^{v_1} dx\,\frac{x}{(1-x^2)^2}\left[\rho_L(t,x) + (v_1^2 - x^2)\rho_T(t,x)\right]$$

$$\left[-\frac{dE}{dz}\right]_{(t)}^{hard} = \frac{1}{256\pi^3\vec{p}_1^{\,2}}\sum_{i=q,g}\int_{|\vec{p}_2|_{min}}^{\infty} d|\vec{p}_2|E_2\mathcal{N}(E_2)\int_{-1}^{cos\psi|_{max}} d(cos\psi)\int_{t_{min}}^{t^*} dt\,\frac{b}{a^3}\overline{|\mathcal{M}^2|}_{Qi(t)}$$

$$\left[-\frac{dE}{dz}\right]_{(su)} = \frac{1}{256\pi^3\vec{p}_1^{\,2}}\int_0^{\infty} d|\vec{p}_2|E_2\mathcal{N}(E_2)\int_{-1}^{1} d(cos\psi)\int_{t_{min}}^{0} dt\,\frac{b}{a^3}\overline{|\mathcal{M}^2|}_{Qg(su)}$$

Toward an analytical form

① High-energy approach (HEA): $E \gg m_Q^2/T$ ② Weak-coupling approximation: $m_D^2 \ll -t^* \ll T^2$

③ Large momentum transfer: $-t \sim s \gg m_1^2$

HEA results

finite-μ correction

$$\left[-\frac{dE}{dz}\right]_{(t)}^{soft} \Rightarrow \left[-\frac{dE}{dz}\right]_{(t)}^{soft-HEA}(E_1,T,\mu) \approx \frac{C_F}{16\pi}g^2 m_D^2 \ln\frac{-2t^*}{m_D^2} + \mathcal{F}_1$$

$$\left[-\frac{dE}{dz}\right]_{(t)}^{hard} \Rightarrow \left[-\frac{dE}{dz}\right]_{Qq(t)}^{hard-HEA}(E_1,T,\mu) \approx \frac{N_f N_c}{216\pi}g^4T^2\left[\ln\frac{8E_1T}{-t^*} - \frac{3}{4} - \gamma + \frac{\zeta'(2)}{\zeta(2)}\right] + \mathcal{F}_3$$

$$\left[-\frac{dE}{dz}\right]_{Qg(t)}^{hard-HEA} = \frac{N_c^2-1}{96\pi}g^4T^2\left(ln\frac{4E_1T}{-t^*} - \frac{3}{4} + c\right)$$

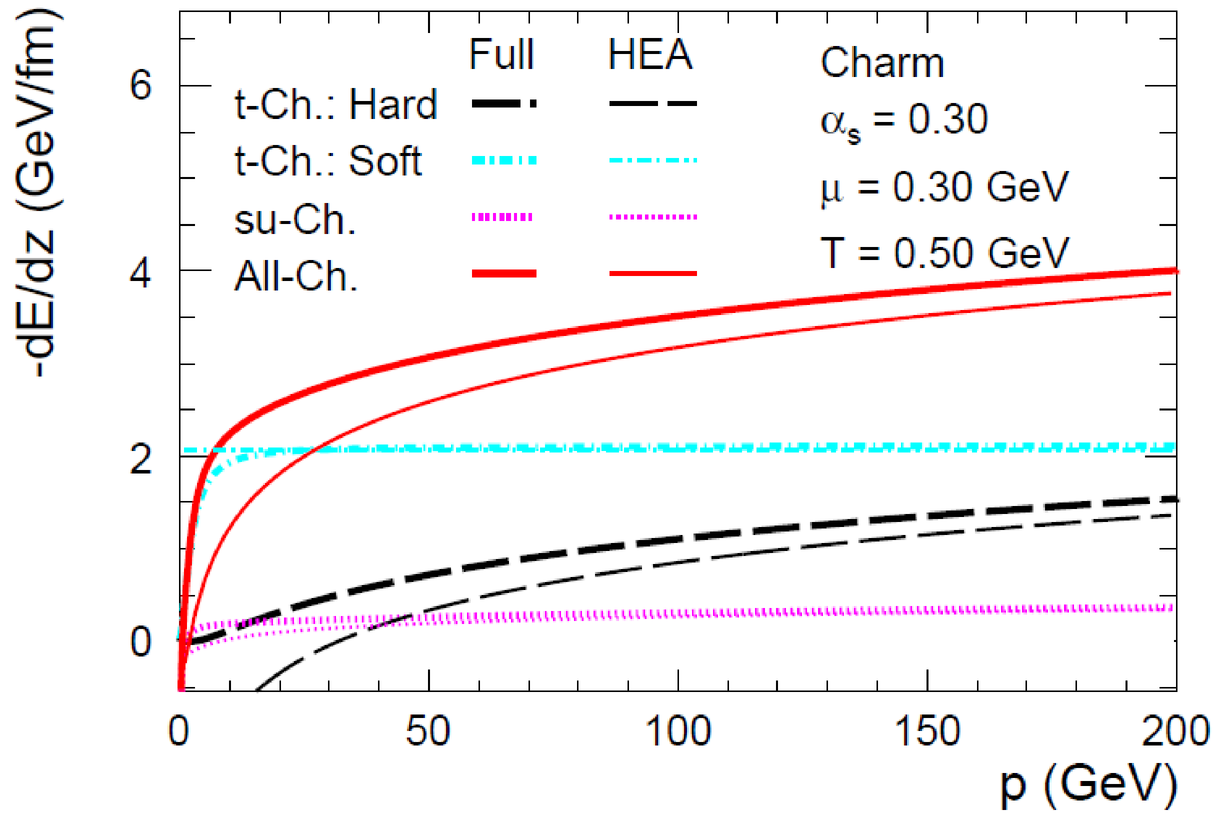
$$\left[-\frac{dE}{dz}\right]_{(s+u)} \Rightarrow \left[-\frac{dE}{dz}\right]_{Qg(s+u)}^{hard-HEA} = \frac{N_c^2-1}{432\pi}g^4T^2\left(ln\frac{4E_1T}{m_1^2} - \frac{5}{6} + c\right)$$

$$\left[-\frac{dE}{dz}\right]_{Qq+Qg}^{HEA}(E_1,T,\mu) \approx \frac{4}{3}\pi\alpha_s^2T^2\left[\left(1+\frac{N_f}{6}\right)\ln\frac{E_1T}{m_D^2} + \frac{2}{9}\ln\frac{E_1T}{m_1^2} + d(N_f)\right] + \mathcal{G}$$

- The T- and E-dependencies are similar to the results for the scattering off a light hard parton off a light soft parton

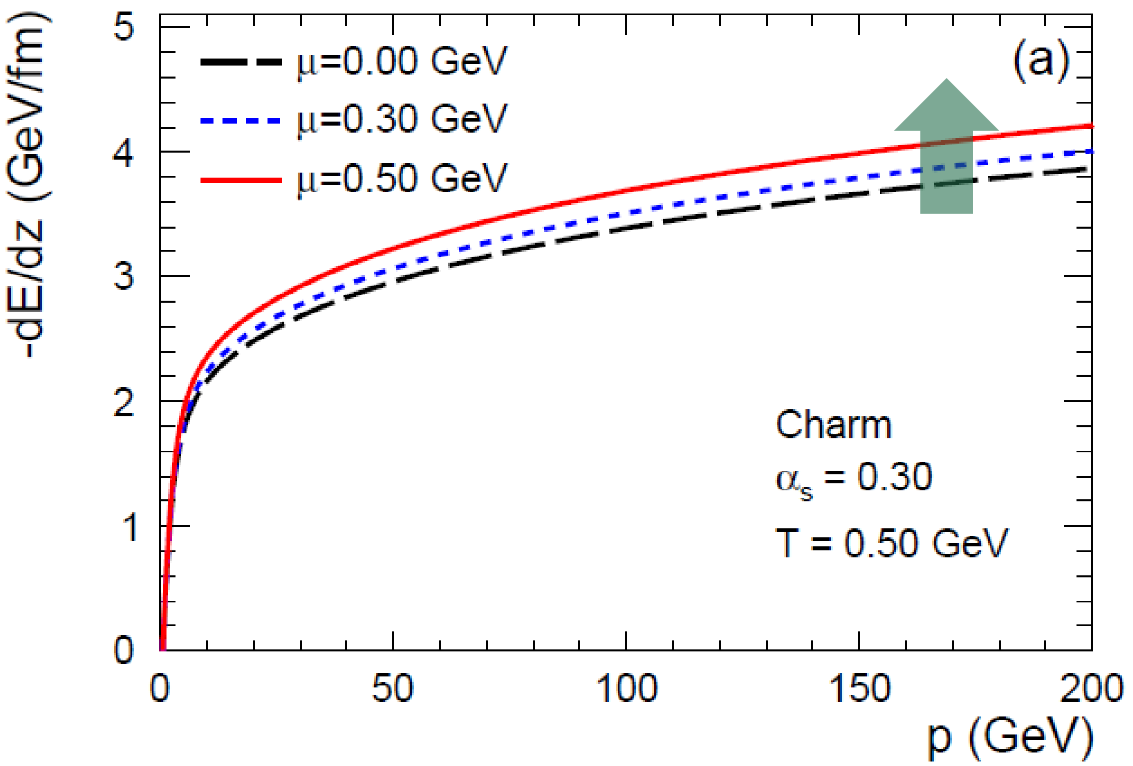
[Qin et. al., PRL 100, 072301 (2008)]

“Full” vs. “HEA”



- As expected, the **asymptotic behavior** is presented toward high energy, while a considerable variation is found at low and moderate energy for each channel

Finite-μ effects

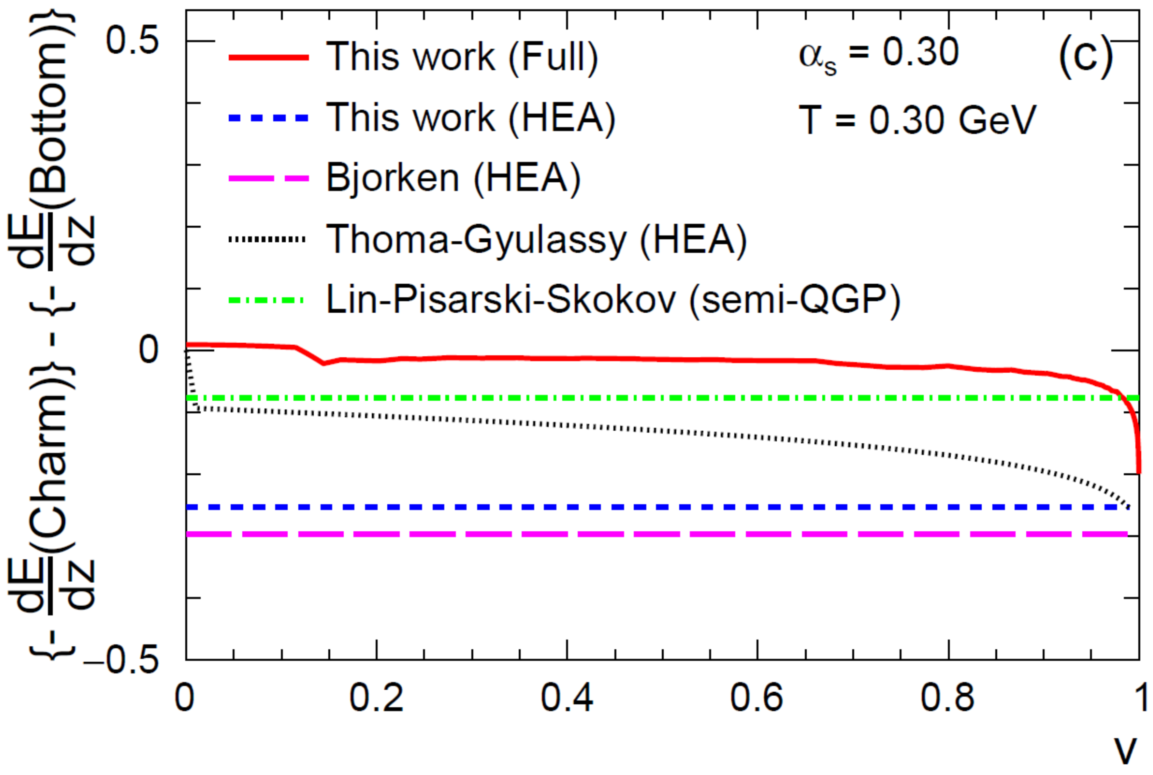


- $-dE/dz$ **increases with** μ
  - soft part: m_D increases with $\mu \rightarrow$ more long-wavelength interactions screened $\rightarrow$ larger Γ and $-dE/dz$
  - hard part: thermal distribution of fermion enhanced accordingly

$$\mathcal{N}_F(E,T,\mu) = n_F + \frac{\mu^2}{2T^2} n_F(1-n_F)\tanh\frac{E}{2T} + \mathcal{O}(\mu^4)$$

$$n_F(E,T) = \frac{1}{e^{E/T}+1} = \mathcal{N}_F(E,T,\mu=0),$$

Charm vs. Bottom $(\mu = 0)$



- **Mass hierarchy**: for a given velocity, quark with larger mass will lose more its initial energy

$$-\frac{dE}{dz} = \frac{4}{3}\pi\alpha_s^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) \ln \frac{q_{\max}^2}{q_{\min}^2}. \qquad (41)$$

Bjorken (HEA)

[Bjorken, FERMILAB-Pub-82/59-THY (1982)]

$$\left[-\frac{dE}{dz}\right]^{\rm HEA}_{Qq+Qg} = \frac{4}{3}\pi\alpha_s^2 T^2 \left[\left(1 + \frac{N_f}{6}\right) \ln \frac{E_1 T}{m_D^2} + \frac{2}{9}\ln\frac{E_1 T}{m_1^2} + d(N_f)\right],$$

This work (HEA)

Thoma-Gyulassy (HEA)

$$-\frac{dE}{dz} = \frac{4}{3}\pi\alpha_s^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) \ln \frac{4T|\vec{p}_1|}{(E_1 - |\vec{p}_1| + 4T)m_D^2},$$

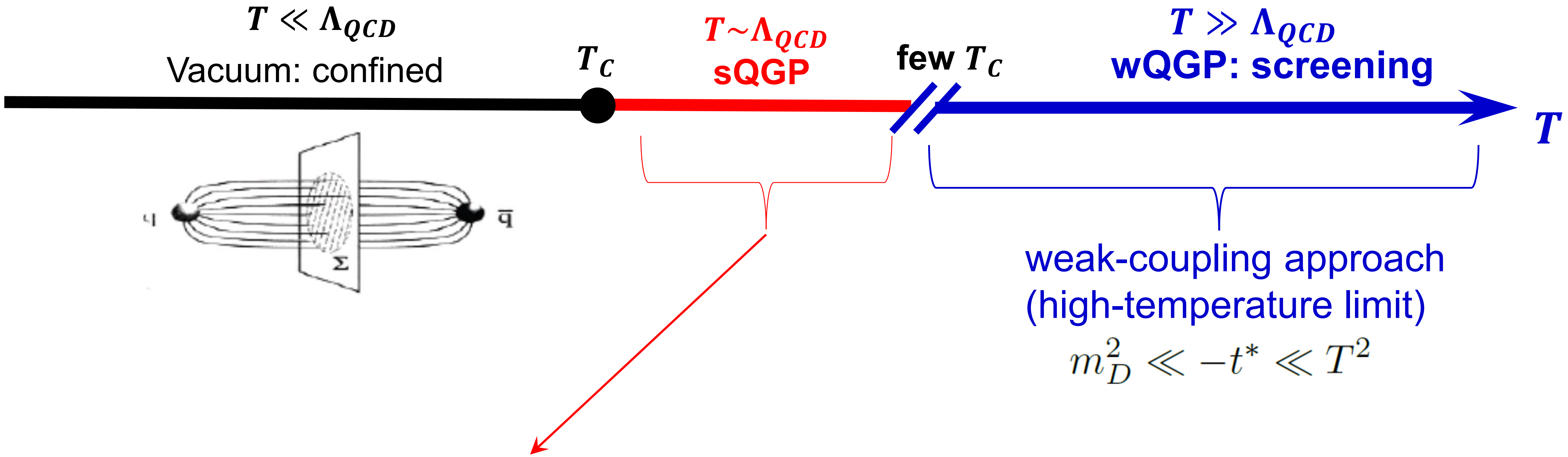
[Thoma and Gyulassy, NPB 351, 491 (1991)]

Lin (HEA)

$$-\frac{dE}{dz} = \pi\alpha_s^2 T^2 \left\{ S^{qk}\frac{N_f(N_c^2-1)}{12N_c}\ln\left(\frac{E_1 T}{m_D^2}\right) + S^{gl}\left[\frac{N_c^2-1}{6}\ln\left(\frac{E_1 T}{m_D^2}\right) + \frac{C_F^2}{6}\ln\left(\frac{E_1 T}{m_1^2}\right)\right]\right\}$$

[Lin, et. al., PLB 730, 236 (2014)]

Semi-QGP near T_c



- Strong-coupling behavior in the "semi-QGP" ($T_C < T \lesssim 3-4\,T_C$), may have an important influence on the HQ energy loss which should be reconsidered in an effective theory
- Non-perturbative contribution included

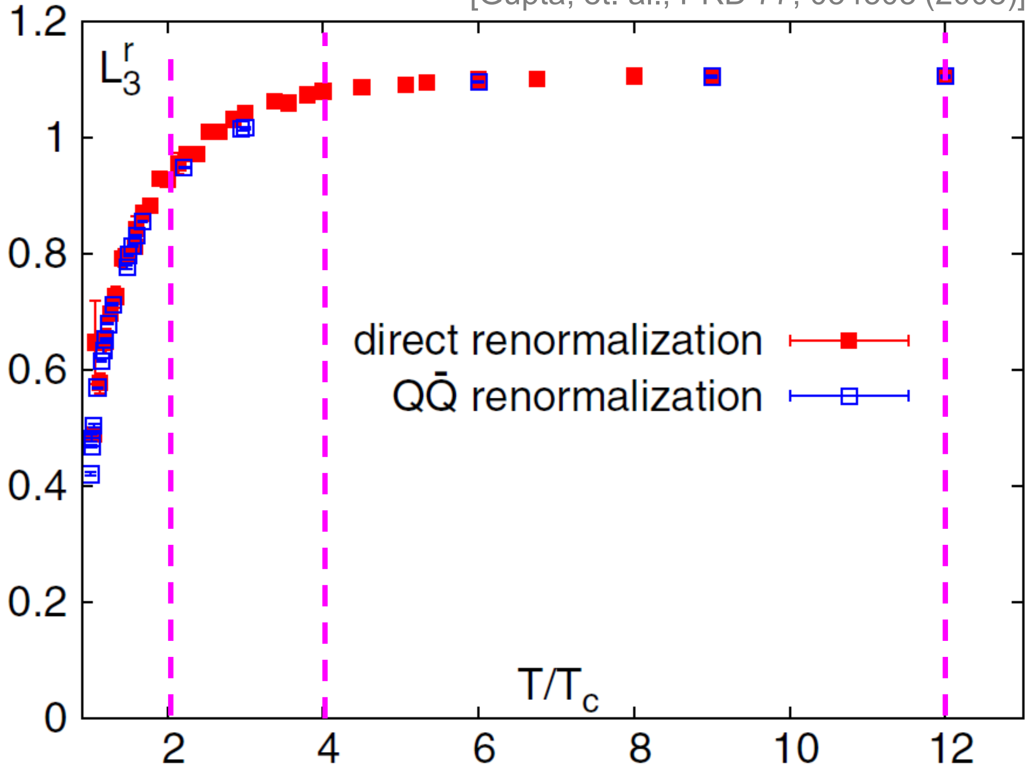
[Hidaka and Pisarski, PRD 78, 071501 (2008)]

How to describe the semi-QGP ?

- For pure gauge theory without quarks, the order parameter of the deconfining phase transition is the Polyakov loop which has a nontrivial dependence on the temperature

$T < T_c$: $\ell \approx 0$	$T \sim T_c^+$: $\ell \approx 0.5$
$T \approx 2T_c$: $\ell \approx 0.9$	$T \approx 4T_c$: $\ell \approx 1.1$
$4T_c \lesssim T \lesssim 12T_c$: $\ell \approx const.$	

[Gupta, et. al., PRD 77, 034503 (2008)]



Phase	Temperature	ℓ from LQCD	Method
QGP	$T \gtrsim 3-4T_c$	$\ell \approx 1$	Perturbation theory (pQCD + HTL)
semi-QGP	$T_c < T \lesssim 3-4T_c$	$0 < \ell < 1$	Background field effective theory
Hadronic	$T < T_c$	$\ell \approx 0$	Effective theory (HRG)

The background field effective theory

- Introducing a classical background field to describe the nontrivial Polyakov loop in the deconfining phase transition for $SU(N)$

$$(A_0^{cl})_{ab} = \frac{1}{g} \mathcal{Q}^a \delta_{ab}; \quad \boldsymbol{L} = \mathcal{P} \exp(ig \int_0^\beta A_0^{cl}\, d\tau); \quad \ell = \frac{1}{N} Tr\boldsymbol{L}$$

$q^a \equiv \mathcal{Q}^a/(2\pi T) = (q, 0, -q)$

$q^{ab} \equiv q^a - q^b$

- The effective potential reads

$$\mathcal{V} = \mathcal{V}_{pt} + \mathcal{V}_{npt} = \frac{2\pi^2 T^4}{3} \sum_{ab} \mathcal{P}^{ab,ba} B_4(|q^{ab}|) + \frac{M^2 T^2}{2} \sum_{ab} \mathcal{P}^{ab,ba} B_2(|q^{ab}|)$$

- The T-dependent background field obtained from the relevant EoM for the background field ($N = 3$)

$$q_{conf} = \frac{1}{3} \qquad q_{deconf} = \frac{1}{36}\left(9 - \sqrt{81 - 80\frac{T_c^2}{T^2}}\right)$$

[Hidaka and Pisarski, PRD 80, 036004 (2008); Guo and Kuang, PRD 104, 014015 (2021)]

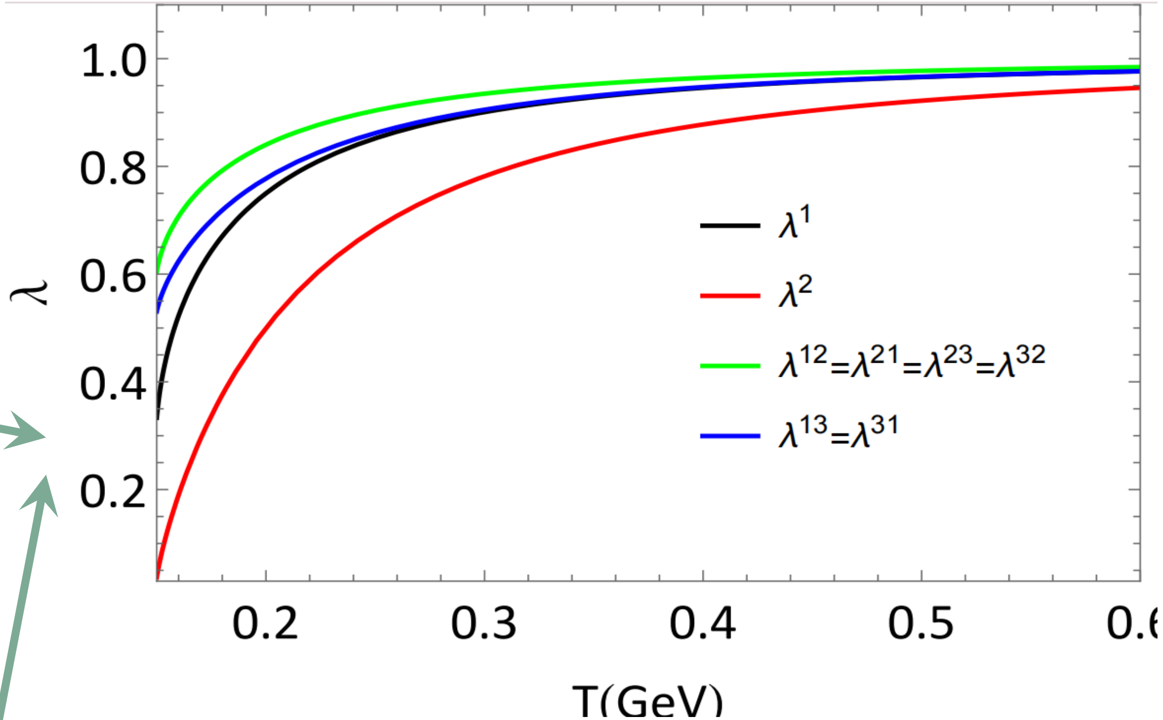
Resummed gluon propagator

- The off-diagonal components

$$D_{\mu\nu}^{de.fg}(Q^{de}) = \Delta_L^{de,fg} B_{ij} + \Delta_T^{de,fg} A_{ij}$$

$$\Delta_T^{de,fg} = \delta^{dg}\delta^{ef}\frac{1}{\omega^2 - q^2 - \lambda^{de}\Pi_T}$$

$$\Delta_L^{de,fg} = \delta^{dg}\delta^{ef}\frac{1}{q^2 + \lambda^{de}\Pi_L}$$



- The diagonal components

$$\sum_{color} P^{dd.ff} D_{\mu\nu}^{dd.ff}(Q) = \sum_{\alpha=1}^{N-1}\left[\frac{1}{q^{o2} - q^2 - \lambda^{\alpha}\Pi_T} A_{ij} + \frac{1}{q^2 + \lambda^{\alpha}\Pi_L} B_{ij}\right]$$

[Guo and Kuang, PRD 104, 014015 (2021)]

Interaction rate: hard component

BF effective theory

$$\Gamma^{hard-BF}_{Qi(t)}(E_1,T) = \frac{1}{2E_1}\int_{p_2} \boxed{\frac{n_B^+(E_2,q^{de}) + n_B^-(E_2,q^{de})}{4E_2}} \int_{p_3} \frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4}$$

$$\boxed{\overline{|\mathcal{M}^2|}^{BF}_{Qi(t)}}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4),$$

$$\boxed{\begin{aligned} n_B^+(k_0,q^{de}) &= \frac{1}{e^{|k_0|/T-2\pi i q^{de}}-1} \\ n_B^-(k_0,q^{de}) &= \frac{1}{e^{|k_0|/T+2\pi i q^{de}}-1}\end{aligned}}$$

(BF enters like an imaginary chemical potential)

$$\boxed{\overline{|\mathcal{M}^2|}^{BF} = \frac{1}{8}\sum_{de}(1-\frac{1}{3}\delta^{de})\overline{|\mathcal{M}^2|}}$$

$$\boxed{n_B(k) = \frac{1}{e^{k/T}-1}}$$

Perturbative

$$\Gamma^{hard}_{Qi}(E_1,T) = \frac{1}{2E_1}\int_{p_2}\boxed{\frac{n(E_2)}{2E_2}}\int_{p_3}\frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4}$$

$$\times\boxed{\overline{|\mathcal{M}^2|}^{Qi}}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

Interaction rate: soft component

- The off-diagonal components

$$\Gamma^{soft}_{offdiag}(E_1,T)=\frac{1}{2N}g^2\sum_{d,e=1}^{N}\int_q\int d\omega\,\bar{n}_B(\omega)\delta(\omega-\vec{v}_1\cdot\vec{q}\,)\left\{\boxed{\rho_L^{de}(\omega,q)}+\vec{v}_1^{\,2}\left[1-(\hat{v}_1\cdot\hat{q})^2\right]\boxed{\rho_T^{de}(\omega,q)}\right\}$$

- The diagonal components

$$\Gamma^{soft}_{diag}(E_1,T)=\frac{1}{2N}g^2\sum_{\alpha=1}^{N-1}\int_q\int d\omega\,\bar{n}_B(\omega)\delta(\omega-\vec{v}_1\cdot\vec{q}\,)\left\{\boxed{\rho_L^{\alpha}(\omega,q)}+\vec{v}_1^{\,2}\left[1-(\hat{v}_1\cdot\hat{q})^2\right]\boxed{\rho_T^{\alpha}(\omega,q)}\right\}$$

cross check

$$\Gamma^{soft}_{offdiag}(E_1,T)+\Gamma^{soft}_{diag}(E_1,T)\xrightarrow[\text{high temperature}]{\text{sum over color}}\Gamma^{soft}(E_1,T)$$

BF effective theory | **Perturbative**

Collisional energy loss with $\mathcal{Q} \neq 0$

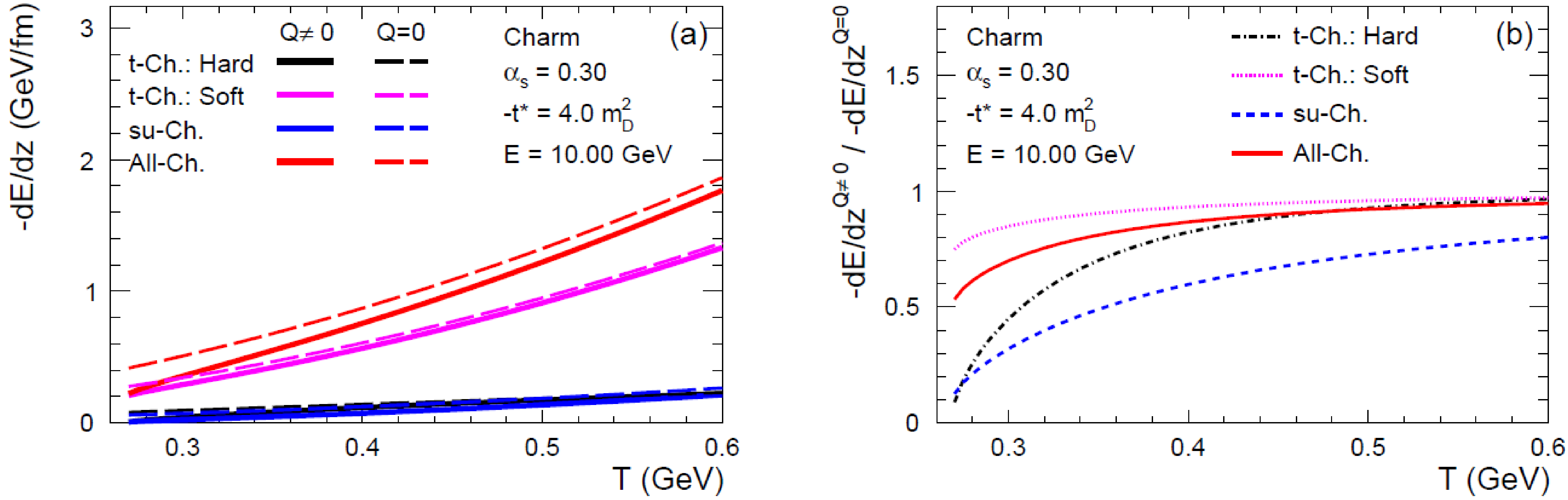
$$-\frac{dE}{dz} = \left[-\frac{dE}{dz}\right]_{(t)}^{soft} + \left[-\frac{dE}{dz}\right]_{(t)}^{hard} + \left[-\frac{dE}{dz}\right]_{(su)}$$

$$\left[-\frac{dE}{dz}\right]_{(t)}^{\mathcal{Q}\neq 0;\ soft} = \frac{g^2}{16\pi^2 N_c v_1^2}\int_{t^*}^{0} dt\,(-t)\int_0^{v_1} dx \frac{x}{(1-x^2)^2}\boxed{[\tilde{\mathcal{B}}_{L-} + (v_1^2 - x^2)\tilde{\mathcal{B}}_{T-}]}$$

$$\left[-\frac{dE}{dz}\right]_{(t)}^{\mathcal{Q}\neq 0;\ hard} = \frac{1}{256\pi^3 p_1^2}\boxed{\left[\frac{1}{8}\sum_{j,k=1}^{3}\left(1-\frac{1}{3}\delta^{jk}\right)\right]}\int_{p_{2,min}}^{\infty} dp_2 E_2 \boxed{\frac{n_B(E_2 - i\mathcal{Q}^{jk}) + n_B(E_2 + i\mathcal{Q}^{jk})}{2}}$$
$$\int_{-1}^{cos\psi|_{max}} d(cos\psi)\int_{t_{min}}^{t^*} dt \frac{b}{a^3}\,\overline{|\mathcal{M}^2|}_{(t)}^{\mathcal{Q}=0},$$

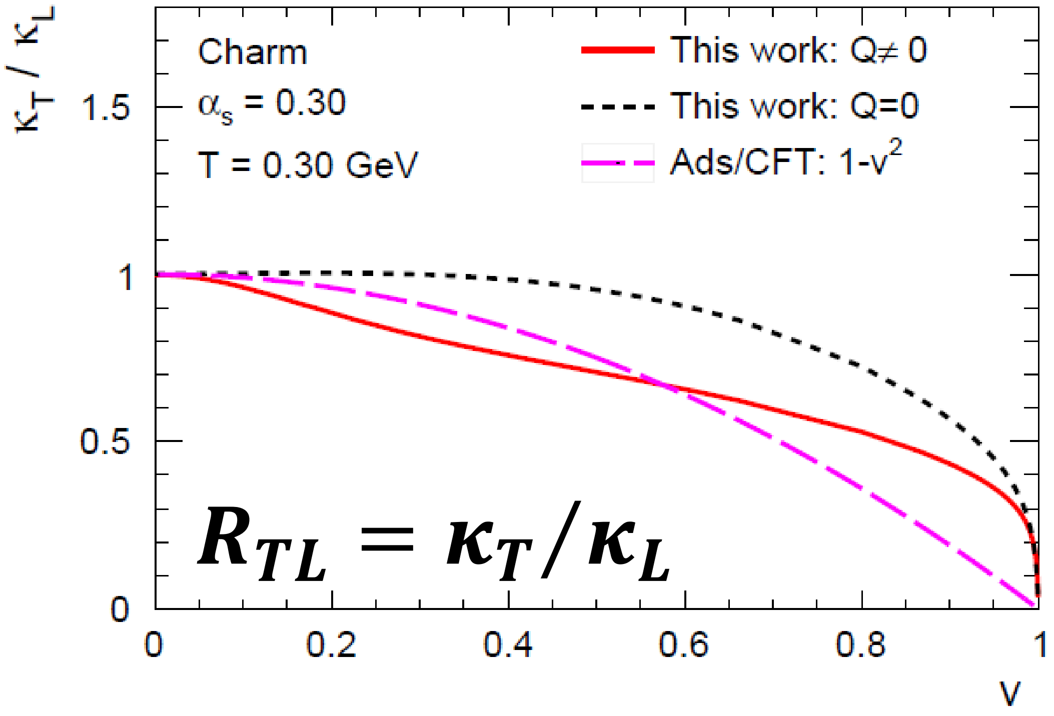
$$\left[-\frac{dE}{dz}\right]_{(su)}^{\mathcal{Q}\neq 0} = \frac{1}{256\pi^3 p_1^2}\boxed{\left[\frac{1}{8}\sum_{j,k=1}^{3}\left(1-\frac{1}{3}\delta^{jk}\right)\right]}\int_{0}^{\infty} dp_2 E_2 \boxed{\frac{n_B(E_2 - i\mathcal{Q}^{jk}) + n_B(E_2 + i\mathcal{Q}^{jk})}{2}}$$
$$\int_{-1}^{1} d(cos\psi)\int_{t_{min}}^{0} dt \frac{b}{a^3}\,\overline{|\mathcal{M}^2|}_{(su)}^{\mathcal{Q}=0},$$

Collisional energy loss



- $-dE/dz$ **in the** $Q \neq 0$ **scenario suppressed compared to the** $Q = 0$ **baseline**
  - hard t-channel result more affected: mainly arises from the reduced color degrees of freedom rather than complete color screening
  - soft t-channel result less affected: long-range chromoelectric interactions remain partially active in the semi-QGP

Momentum diffusion coefficients



[Ads/CFT: J. Casalderrey-Solana, D. Teaney, JHEP 0704, 039 (2007); S.S. Gubser, Nucl. Phys. B 790, 175 (2008); H. Liu, K. Rajagopal, Y. Shi, JHEP 0808, 048 (2008).]

- $R_{TL} \to 1$ in the static limit $v \to 0$: indistinguishable between transverse and longitudinal fluctuations
- R_{TL} decreases monotonically in both cases: kinematic factors and the redistribution of energy-momentum transfers make longitudinal broadening increasingly efficient relative to transverse broadening for a fast-moving probe

Summary and outlook

- **We have extended the recently developed soft-hard factorization model for heavy-quark transport to finite chemical potential.** Our results show that
  - ✓ both the dE/dz and the $\kappa_{T/L}$ are found to increase with μ, with the enhancement being most pronounced at low T where μ effects dominate the medium response
  - ✓ mass hierarchy $-dE/dz(charm) < -dE/dz(bottom)$ observed at fixed velocity

- **We have extended the above perturbative framework by incorporating a T-dependent background field that accounts for non-perturbative QCD effects near T_c.** This unified approach
  - ✓ allows for a continuous interpolation between the perturbative and quasi-confining regimes and is valid in both the small and large momentum transfer limits
  - ✓ shows a distinct suppression of both dE/dz and the $\kappa_{T/L}$ relative to conventional perturbative estimates, especially near T_c

- **New paths forward for future work**
  - ✓ coupling the present results to our LGR framework and performing the model-data comparisons at RHIC and LHC energies

Thank you all for the attention !

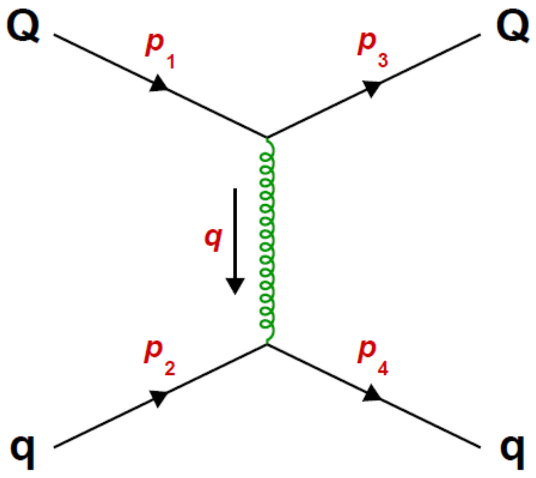
Three Gorges Dam

Backup

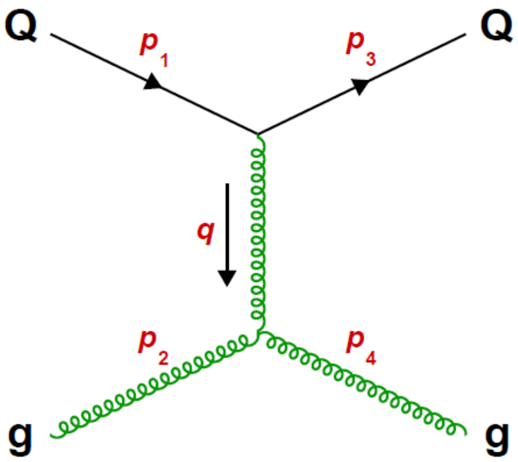
Tree-level Feynman diagrams in vacuum

- The elastic scattering processes between heavy quark (Q) and the quark-gluon plasma constituents ($i = q, g$)

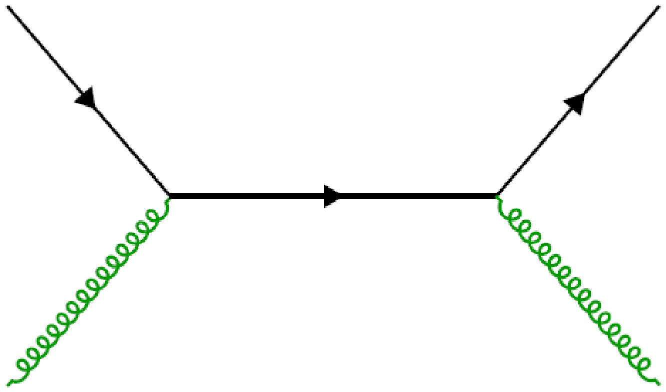
$$\boldsymbol{Q(P_1) + i(P_2) \to Q(P_3) + i(P_4)}$$



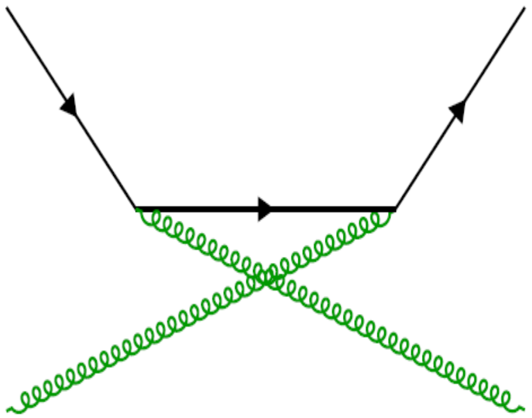
Qq: $t-channel$



Qg: $t-channel$



Qg: $s-channel$



Qg: $u-channel$

Spectral functions

$$\rho_T(\omega,q,\mu) = \frac{\pi\omega M_D^2}{2q^3}(q^2-\omega^2)\left\{\left[q^2-\omega^2+\frac{\omega^2 M_D^2}{2q^2}\right]\times\left(1+\frac{q^2-\omega^2}{2\omega q}\ln\frac{q+\omega}{q-\omega}\right)^2+\left(\frac{\pi\omega M_D^2}{4q^3}(q^2-\omega^2)\right)^2\right\}^{-1}$$

$$\rho_L(\omega,q,\mu) = \frac{\pi\omega M_D^2}{q}\left\{\left[q^2+M_D^2\left(1-\frac{\omega}{2q}\ln\frac{q+\omega}{q-\omega}\right)\right]^2+\left(\frac{\pi\omega M_D^2}{2q}\right)^2\right\}^{-1}.$$

$$M_D^2(T,\mu) = -\pi\alpha_s d_g\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\partial}{\partial|\vec{p}\,|}(N_c\mathcal{N}_B+N_f\mathcal{N}_F) = m_D^2+\frac{N_f g^2\mu^2}{2\pi^2}$$

$$m_D^2(T) = \left(\frac{N_c}{3}+\frac{N_f}{6}\right)g^2T^2$$

Finite-μ corrections

$$\left[-\frac{dE}{dz}\right]^{soft-HEA}_{(t)}(E_1,T,\mu) \approx \frac{C_F}{16\pi}g^2 m_D^2 \ln\frac{-2t^*}{m_D^2} + \mathcal{F}_1,$$

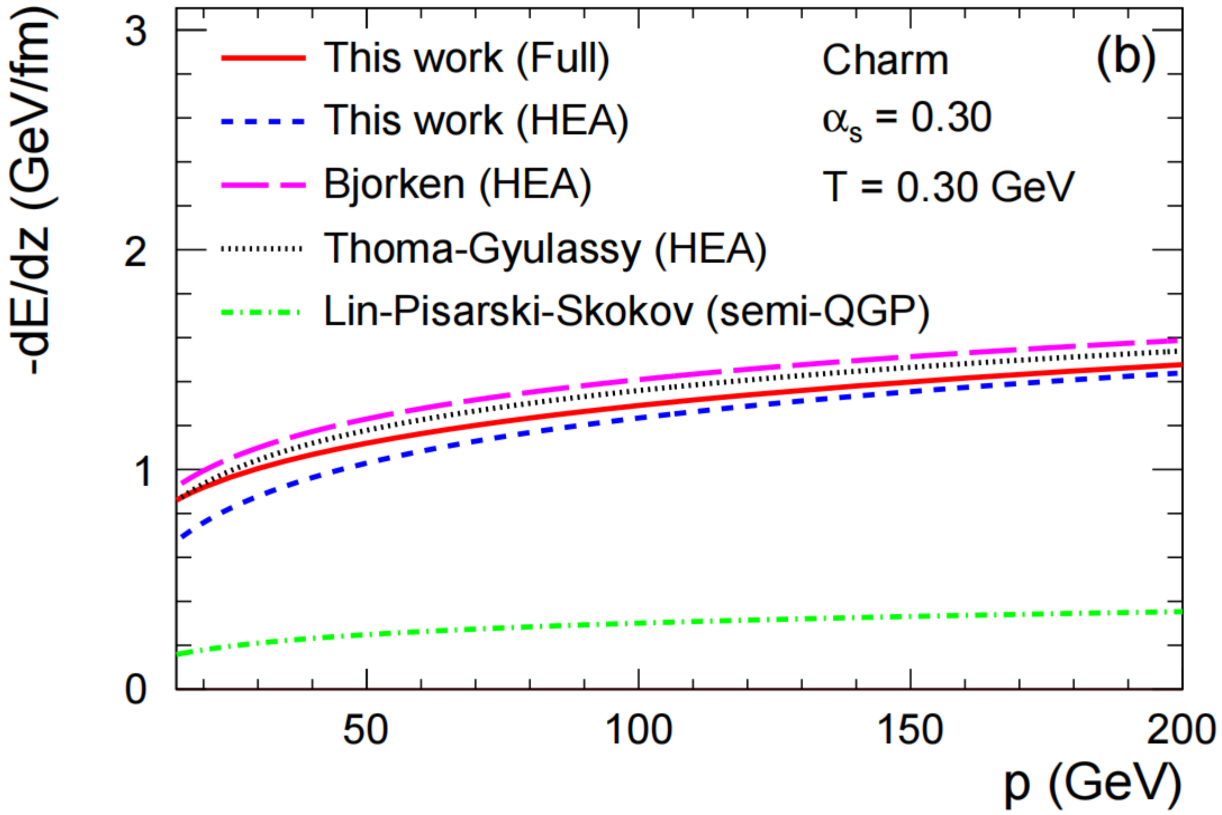
$$\mathcal{F}_1(T,\mu) = \frac{C_F g^2}{16\pi}\left(\frac{N_f g^2\mu^2}{2\pi^2}\ln\frac{-2t^*}{M_D^2} - m_D^2 \ln\frac{M_D^2}{m_D^2}\right).$$

$$\left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qq(t)}(E_1,T,\mu) \approx \frac{N_f N_c}{216\pi}g^4 T^2\left[\ln\frac{8E_1 T}{-t^*} - \frac{3}{4} - \gamma + \frac{\zeta'(2)}{\zeta(2)}\right] + \mathcal{F}_3,$$

$$\mathcal{F}_3(E_1,T,\mu) = \frac{N_f N_c}{18\pi^3}g^4\left[\mathcal{F}_2\big|_{\mathcal{A}=-t^*/(4E_1)} - \frac{7\mu^2}{16}\right].$$

$$\mathcal{F}_2(T,\mu) = \frac{\mu^2}{4}\left(\ln\frac{T}{\mathcal{A}} + 1 - \gamma_E\right) - \frac{T^2}{2}\left[\frac{\partial}{\partial s}Li_s(-e^{-\mu/T})\Big|_{s=2} + \frac{\partial}{\partial s}Li_s(-e^{\mu/T})\Big|_{s=2}\right] - \frac{\pi^2 T^2}{12}\left[\ln 2 + \frac{\zeta'(2)}{\zeta(2)}\right]$$

Comparison with other models（ $\mu = 0$ ）



- ***Bjorken*** : keep only the logarithmically divergent integral over momentum transfer; imposing physically reasonable upper and lower limits to regulate the divergences

[Bjorken, FERMILAB-Pub-82/59-THY]

- ***Thoma-Gyulassy*** : update the Bjorken approach by including a more careful treatment of the infrared divergences

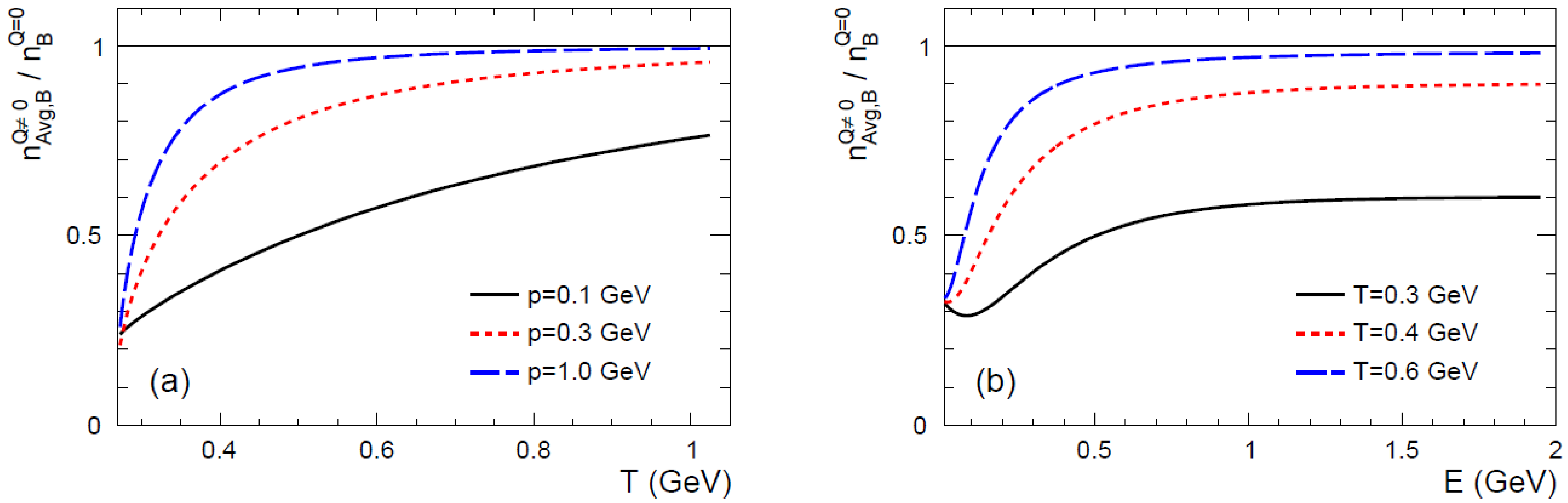
[Thoma and Gyulassy, NPB 1991]

- ***Lin-Pisarski-Skokov*** : incorporate partially confinement effect through purely imaginary background color charge determined by Polyakov loop from lattice studies, leading reduced quark and gluon degrees of freedom

[Lin, Pisarski and Skokov, PLB 2014]

- A common behavior is observed for all the models

Comparison of the bosonic distribution



$$n_{Avg,B}(E,T) = \frac{1}{N_c^2}\sum_{a,b=1}^{N_c} n_B(E - i\mathcal{Q}^{ab}) = \frac{1}{N_c^2}\sum_{a,b=1}^{N_c} n_B(E + i\mathcal{Q}^{ab})$$

$$= \frac{1}{9}\left[\frac{3}{e^{\beta E}-1} + \frac{e^{\beta E}(6\ell - 2) - 4}{1 + e^{2\beta E} + e^{\beta E}(1-3\ell)} + \frac{e^{\beta E}(9\ell^2 - 6\ell - 1) - 2}{1 + e^{2\beta E} + e^{\beta E}(1 + 6\ell - 9\ell^2)}\right]$$

$$n_B(E,T) = \frac{1}{e^{E/T} - 1}$$