Quarkonium Spectroscopy in the Quark-Gluon Plasma Based on T-Matrix Approach

Zhanduo Tang

Institute of Modern Physics
Fudan University, Shanghai, China

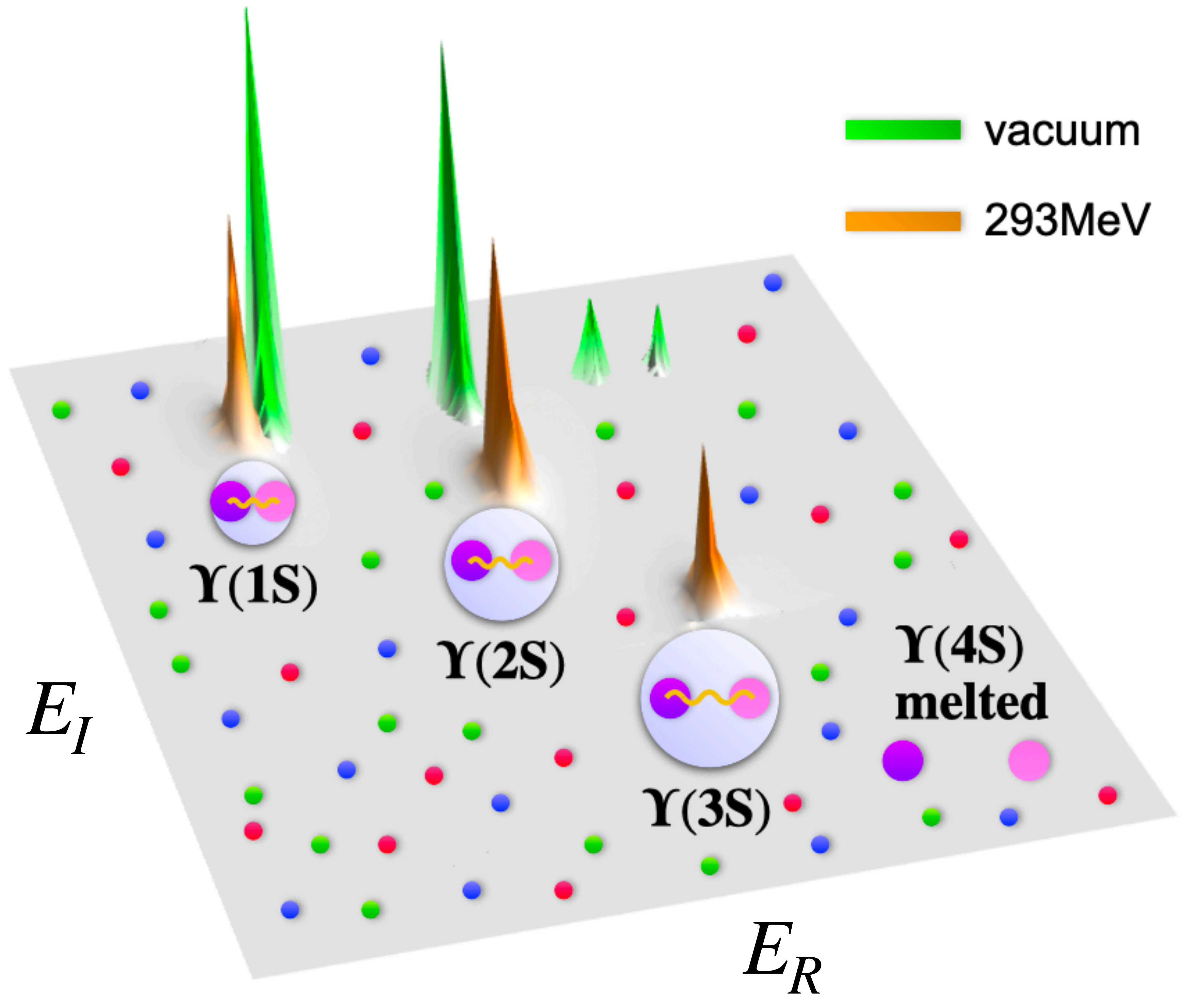
Outline

1. Theoretical Framework: T-Matrix Approach to QGP

2. Constraints on Input Potential by Lattice QCD

3. Quarkonium Spectroscopy in the QGP

4. Summary



1. T-MATRIX APPROACH TO QGP

1.1 T-Matrix Framework

◆ **Research Object**

- Microscopic interactions between partons

◆ **Theoretical Tool**

- **Non-perturbative quantum** many-body theory—T-matrix approach (with input potential V)

◆ **Physical Quantities Computed via T-matrix**

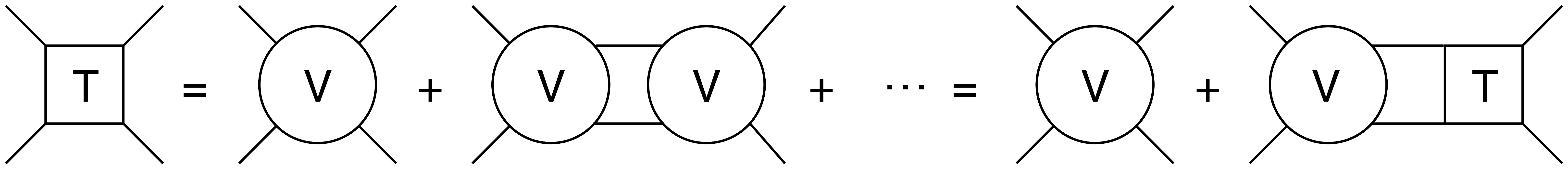
- spectral & transport properties of parton/meson $\xrightarrow{\text{inputs}}$ phenomenological studies ($R_{AA}, v_2 \ldots$)

1.2 T-Matrix Equation [Riek+Rapp '10, Liu+Rapp '18]

◆ 2-body scattering equation: covariant Bethe-Salpeter (BS) equation

resummation

$i = Q, q, g$



$j = Q, q, g$

- T-matrix equation: $T_{ij} = V_{ij} + \int V_{ij} G_i G_j V_{ij} + \ldots = V_{ij} + \int V_{ij} G_i G_j T_{ij}$

Born term | non-perturbative parts

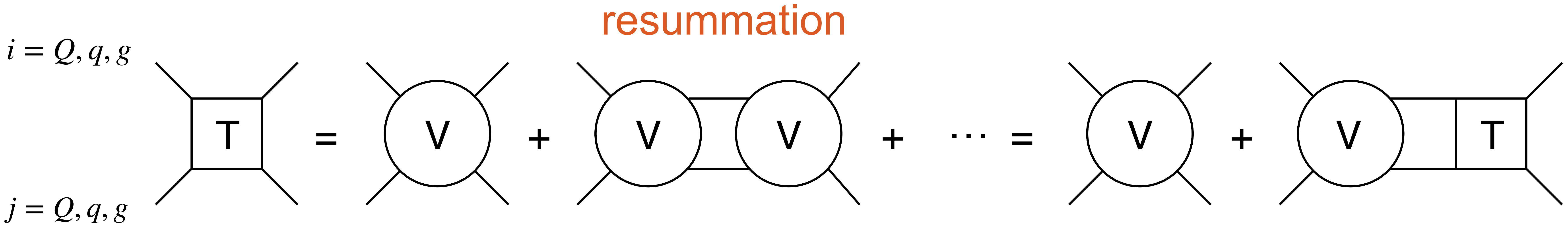
$$\longrightarrow T = \frac{V}{1 - \int GGV}$$

◆ Approximations

- Quark propagator (positive-energy projection): $G_i = 1/(\omega - \omega_k - \Sigma_i)$

1.2 T-Matrix Equation [Riek+Rapp ’10, Liu+Rapp ’18]

◆ **2-body scattering equation: covariant Bethe-Salpeter (BS) equation**



• T-matrix equation: $T_{ij} = \underbrace{V_{ij}}_{\text{Born term}} + \underbrace{\int V_{ij} G_i G_j V_{ij} + \ldots}_{\text{non-perturbative parts}} = V_{ij} + \int V_{ij} G_i G_j T_{ij}$

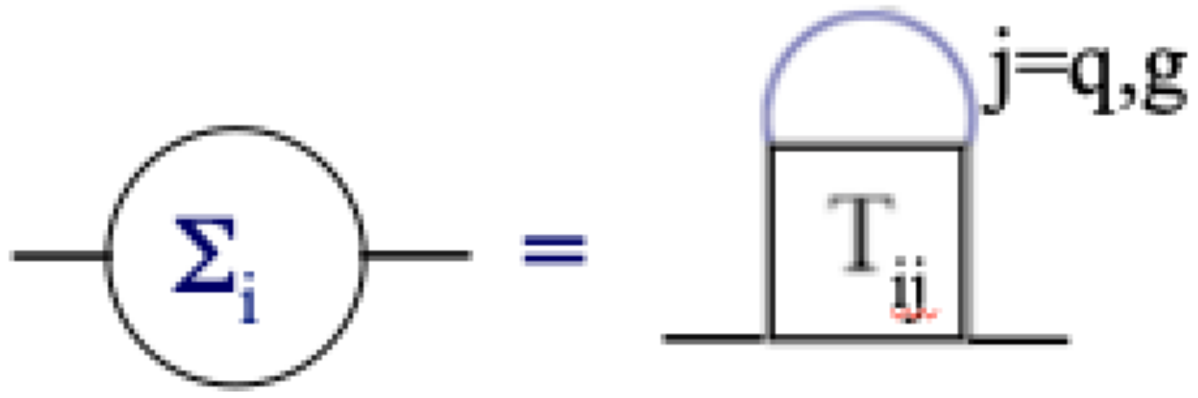
$$T = \frac{V}{1 - \int GGV}$$ ⟶ **pole indicates bound state**

◆ **Approximations**

• Quark propagator (positive-energy projection): $G_i = 1/(\omega - \omega_k - \Sigma_i)$

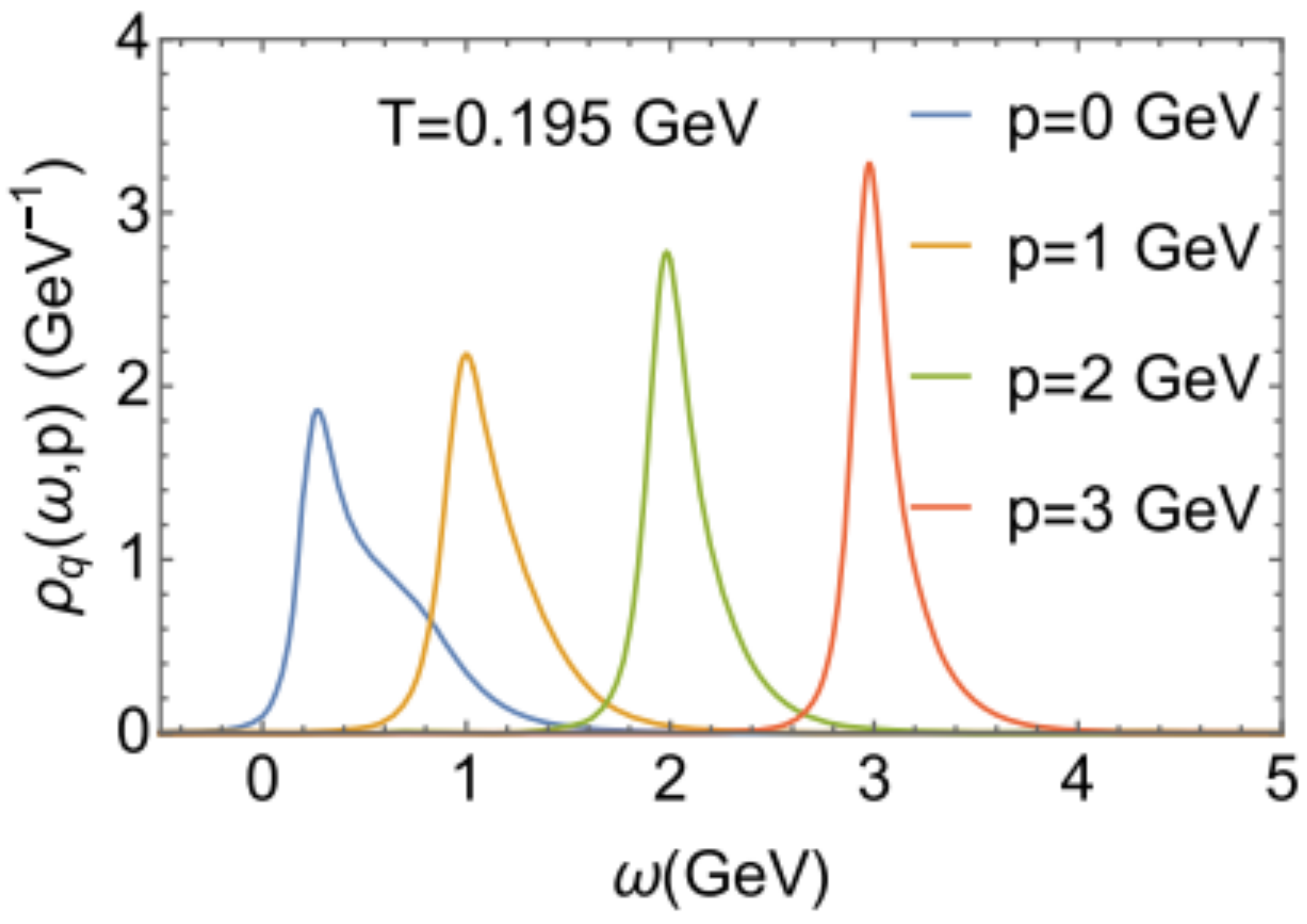
1.3 Strongly Coupled QGP [Riek+Rapp '10, Liu+Rapp '18]

◆ **Finite parton selfenergy:**



$\Sigma = T(\Sigma)G(\Sigma)$ Self-consistent many-body theory

◆ **Spectral function:** $\rho_i \sim \mathrm{Im}G_i = \mathrm{Im}[1/(\omega - \omega_k - \Sigma_i)]$



- Finite parton widths (off-shell) ✔ vs. quasiparticle (on-shell) ✘
- Large collision rate → strong interaction with medium

→ strongly coupled QGP

Quantum effect

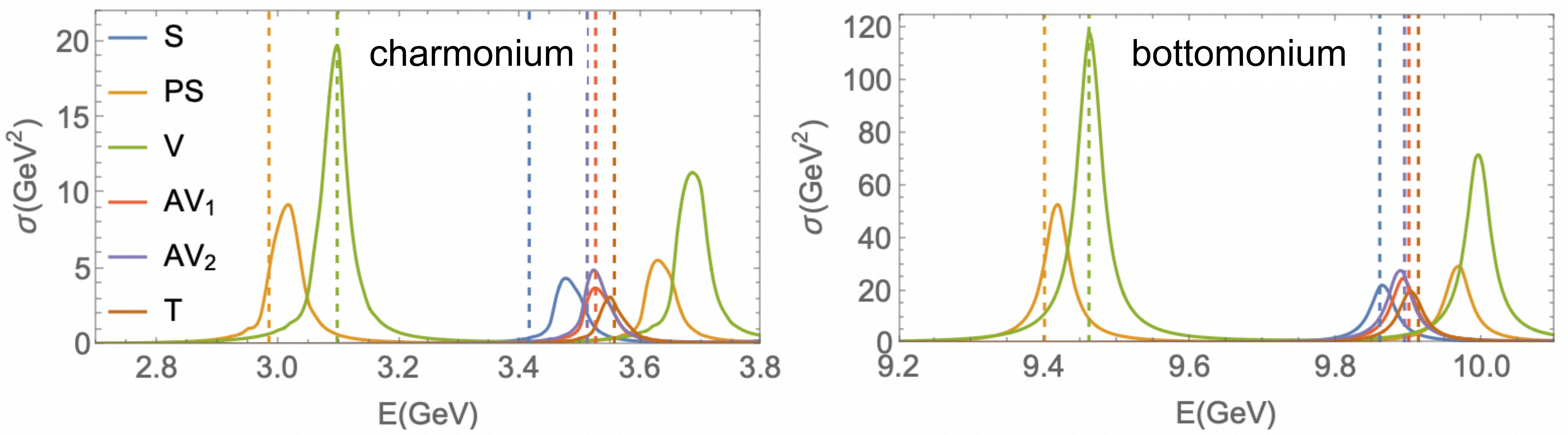
1.4 Input Potential to T-Matrix

◆ **Vacuum**

- Cornell potential: $V = -\underbrace{\frac{4}{3}\alpha_s \frac{1}{r}}_{\text{Coulomb}} + \underbrace{\sigma r}_{\text{confining}} \xrightarrow{\text{LO } 1/M_Q \text{ expansion}}$ effective field theory (potential non-relativistic QCD)

- α_s: coupling constant
- σ: string tension

constrained by vacuum spectroscopy

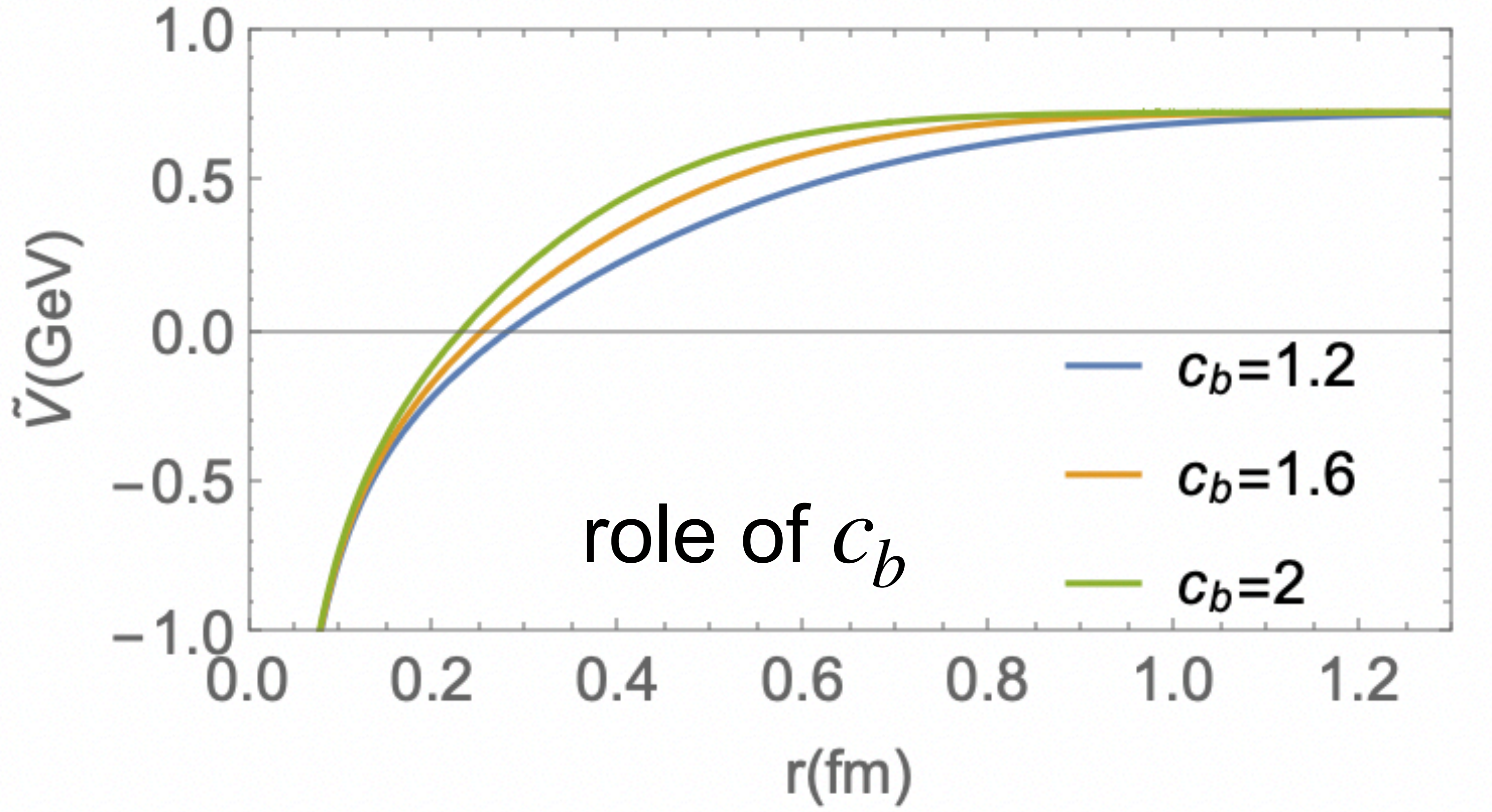


◆ **In-medium**

- Screened Cornell potential: $V = -\frac{4}{3}\alpha_s \left(\frac{e^{-m_d r}}{r} + m_d\right) - \frac{\sigma}{m_s}\left(e^{-m_s r - (c_b m_s r)^2} - 1\right)$

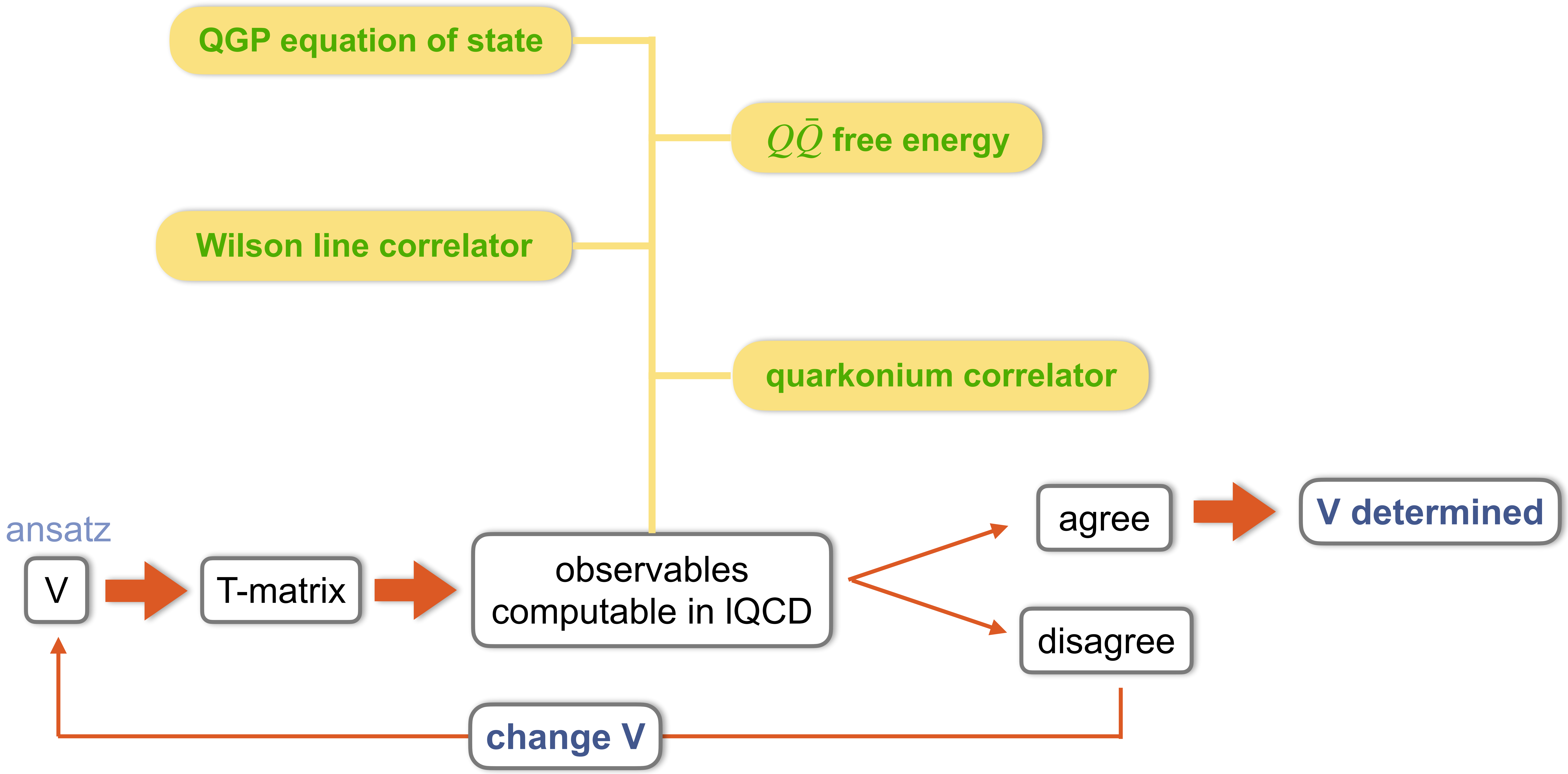
- $m_{d/s}(T)$: Debye screening masses
- $c_b(T)$: string breaking

constrained by thermal lQCD data



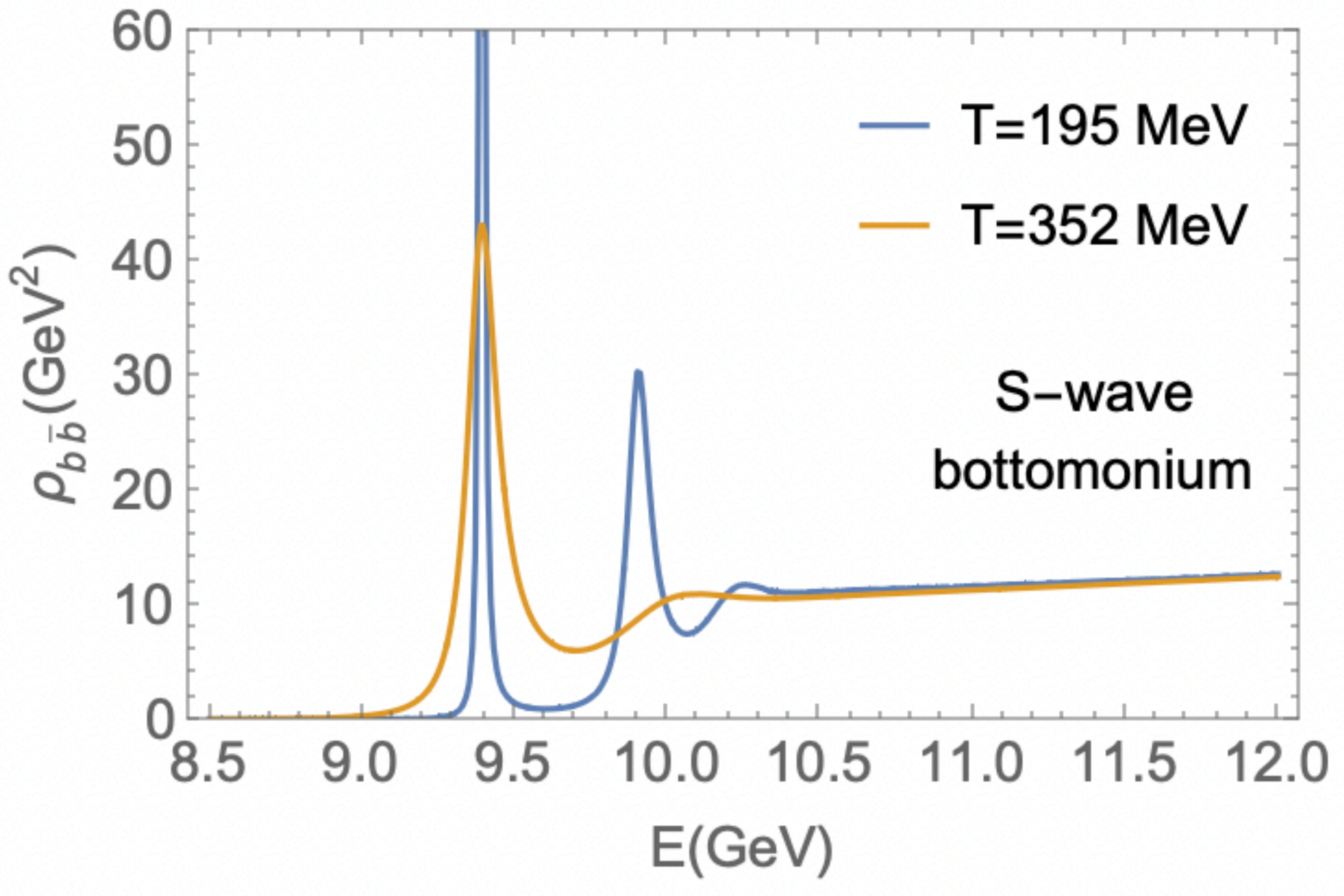
2. CONSTRAINTS ON INPUT POTENTIAL

2. Constraints on Input Potential



3. PREDICTED QUARKONIUM SPECTRAL PROPERTIES

3.1 T-matrix in Complex Energy Plane



- **Common (schematic) prescriptions to quarkonium melting temperature:**
  - $\Gamma_{Q\bar{Q}}$ (width)~E_b (binding energy)
  - $E_b = 0$

→ ambiguous at high T with broad spectral peak

[Tang+Wu et al., PRL 135 (2025) 142302]

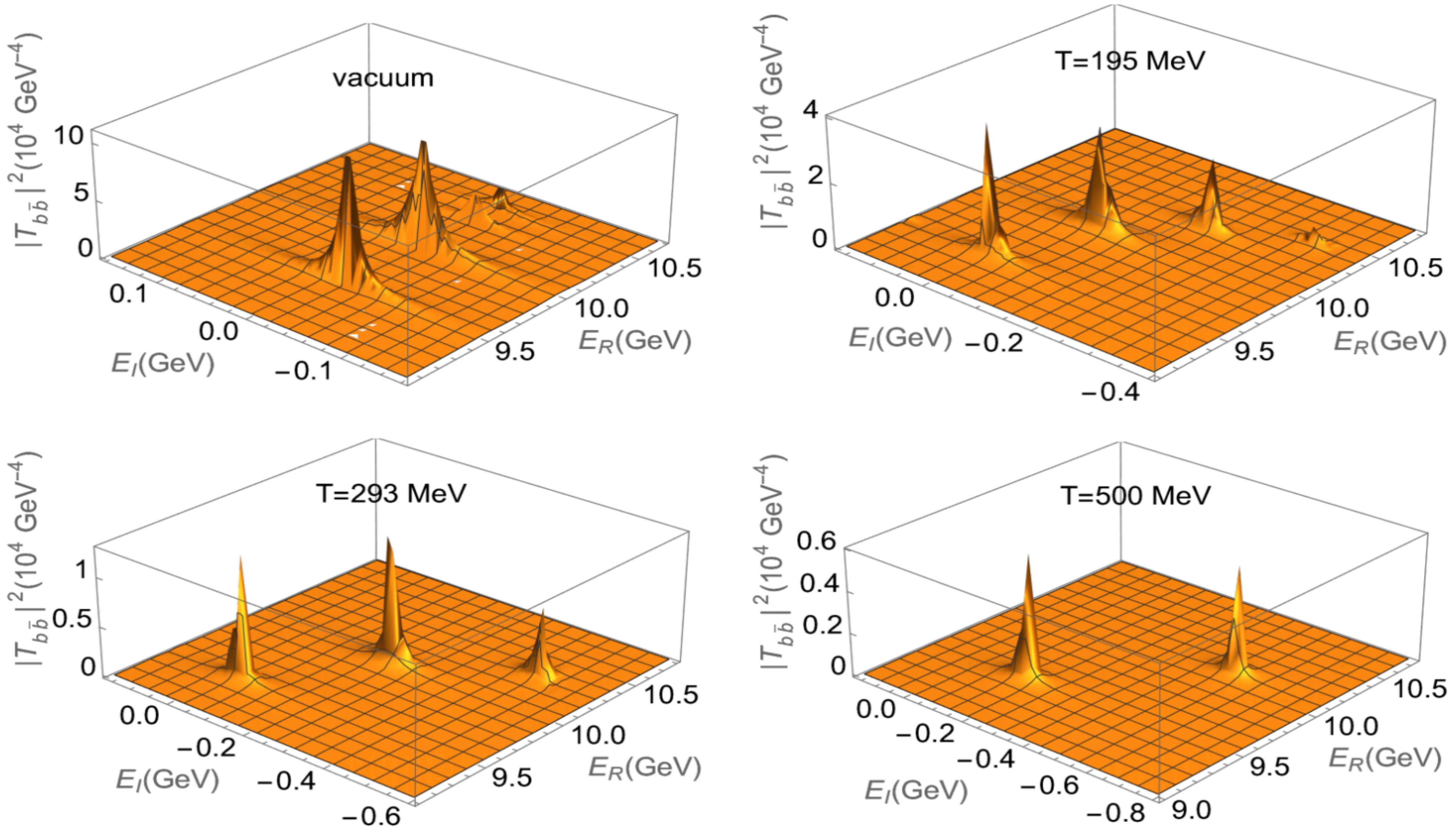
- **Rigorous criterion: pole analysis of T-matrix in complex-energy plane**
  - Widely used in bound-state analysis in vacuum
  - **First in-medium study** using fully off-shell propagators (recovers vacuum at zero width)

- **Formalism**
  - Extend 2-body propagator to complex-energy plane:

$\int dk_0 G_Q(E-k_0,k) G_{\bar{Q}}(k_0,k)$

$$G_2(E,k) = \frac{1}{E - 2\varepsilon_Q(k) - \Sigma_{Q\bar{Q}}(E,k)}, \quad E \to z = E_R + iE_I$$

  - $\Sigma_{Q\bar{Q}}$: 2-body selfenergy (computed self-consistently)
  - T-Matrix in complex plane: $T(z) = \dfrac{V}{1 - G_2(z)V}$



E_R^{pole}: mass
$2E_I^{pole}$: width

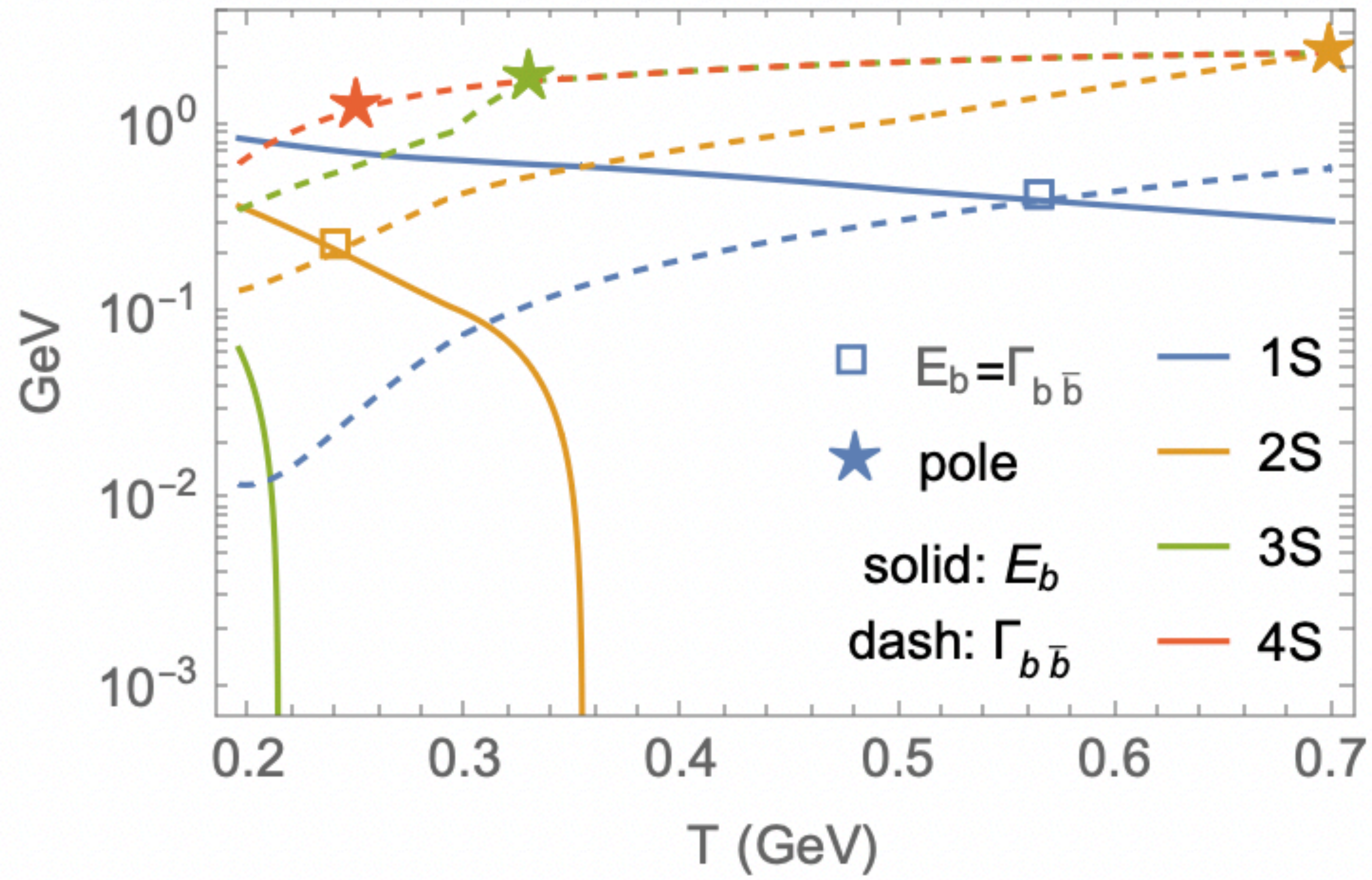
3.2 Quarkonium Melting Temperature

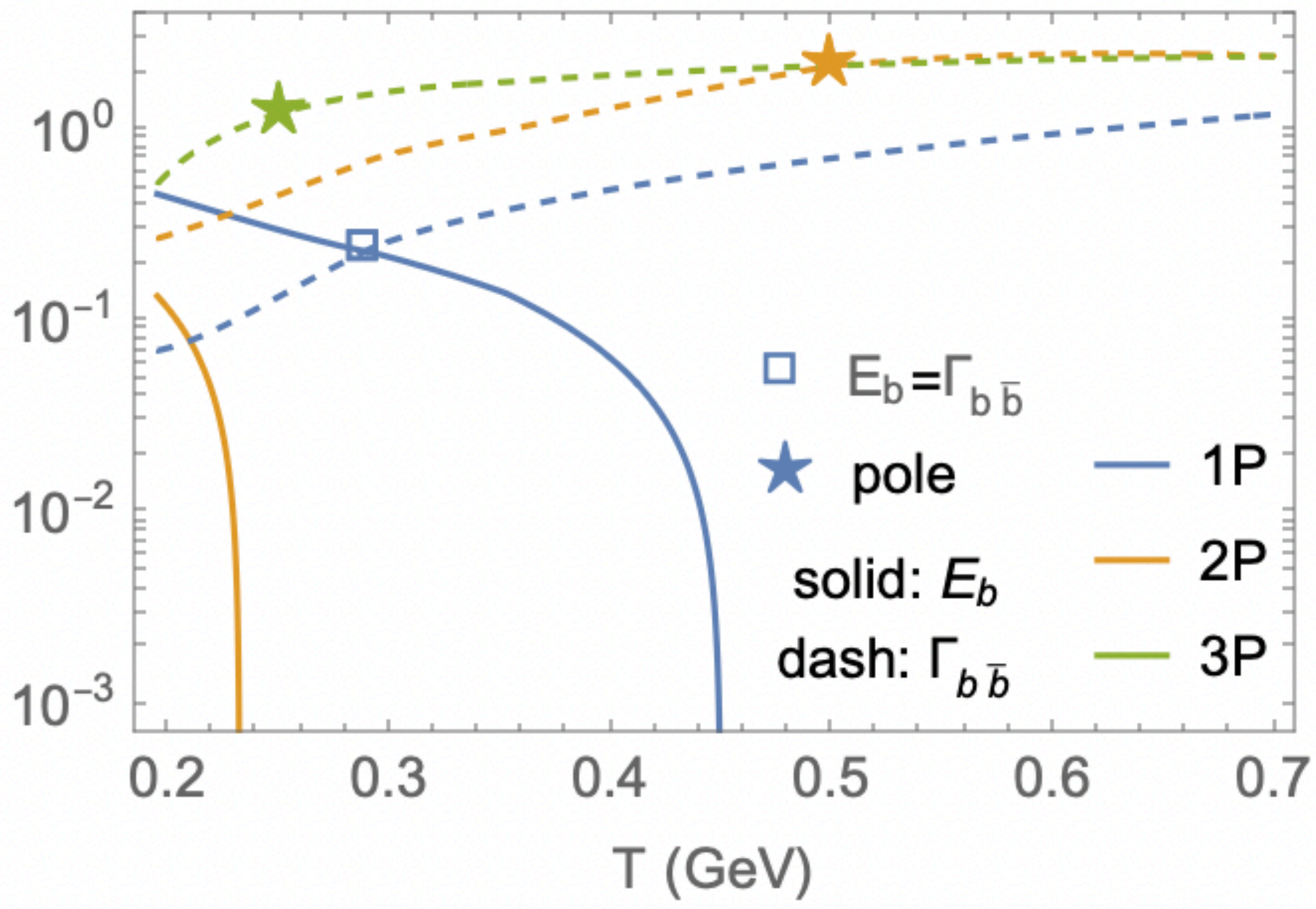
◆ Comparison between different criterions:

(1) $\Gamma_{Q\bar{Q}}$ (width)=E_b (binding energy)

(2) $E_b = 0$

(3) Disappearance of T-matrix pole in complex plane





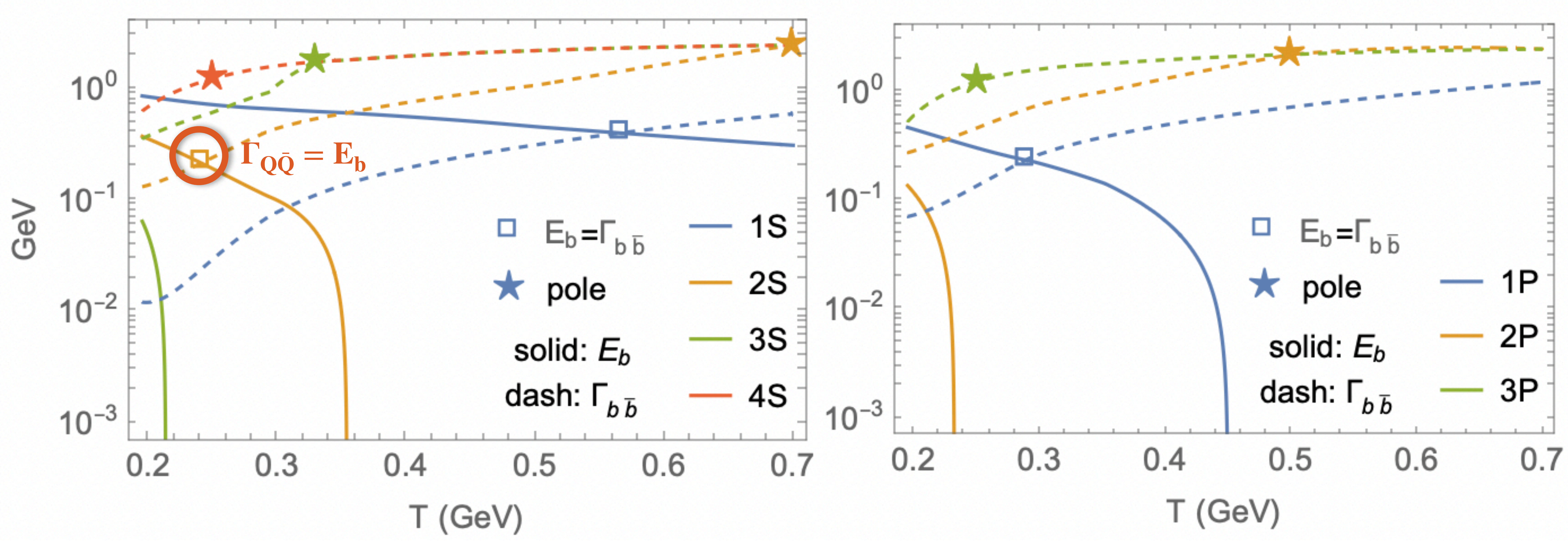
3.2 Quarkonium Melting Temperature

- Comparison between different criterions:

  (1) $\Gamma_{Q\bar{Q}}$ (width)=E_b (binding energy)

  (2) $E_b = 0$

  (3) Disappearance of T-matrix pole in complex plane



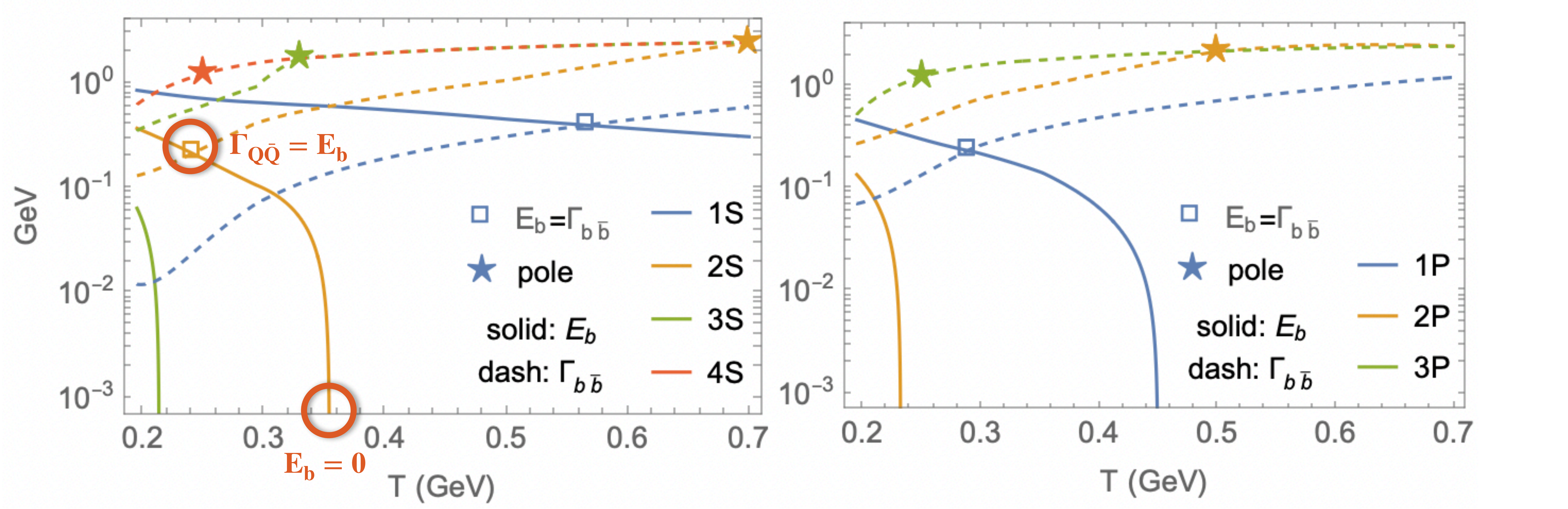
3.2 Quarkonium Melting Temperature

◆ Comparison between different criterions:

(1) $\Gamma_{Q\bar{Q}}$ (width)=E_b (binding energy)

(2) $E_b = 0$

(3) Disappearance of T-matrix pole in complex plane



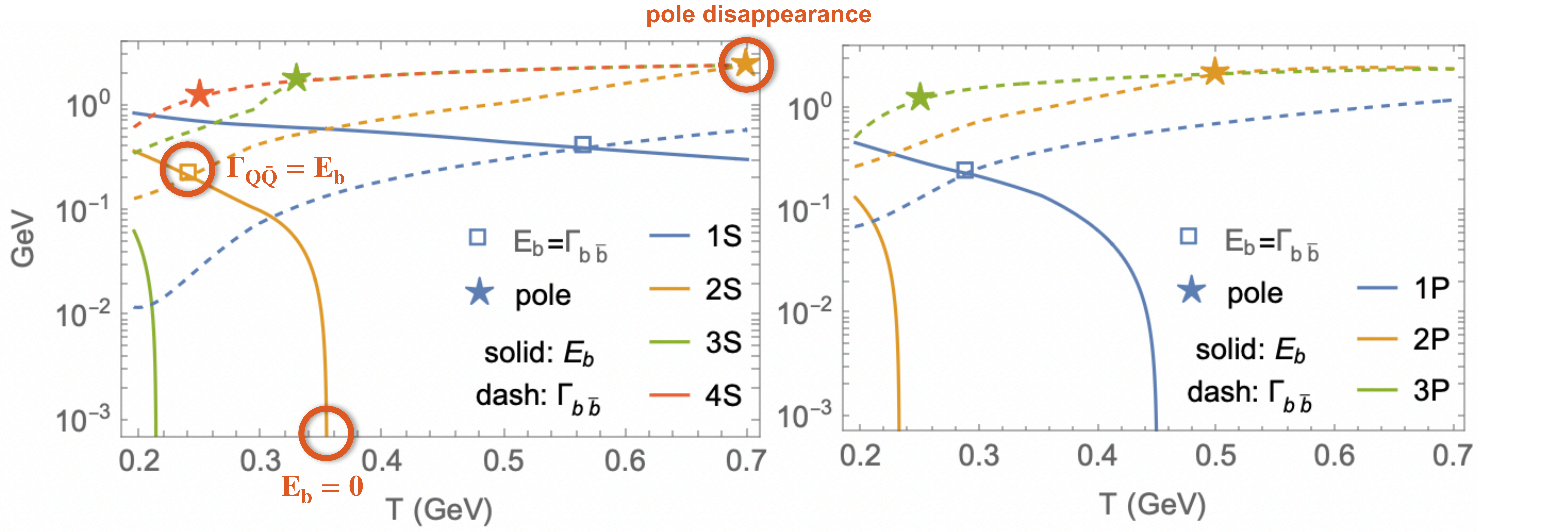
3.2 Quarkonium Melting Temperature

◆ Comparison between different criterions:

(1) $\Gamma_{Q\bar{Q}}$ (width)=E_b (binding energy)

(2) $E_b = 0$

(3) Disappearance of T-matrix pole in complex plane



3.2 Quarkonium Melting Temperature

[Wu+Tang+Rapp, JHEP 07,162 (2025)]

◆ Comparison between different criterions:

(1) $\Gamma_{Q\bar{Q}}$ (width)=E_b (binding energy)

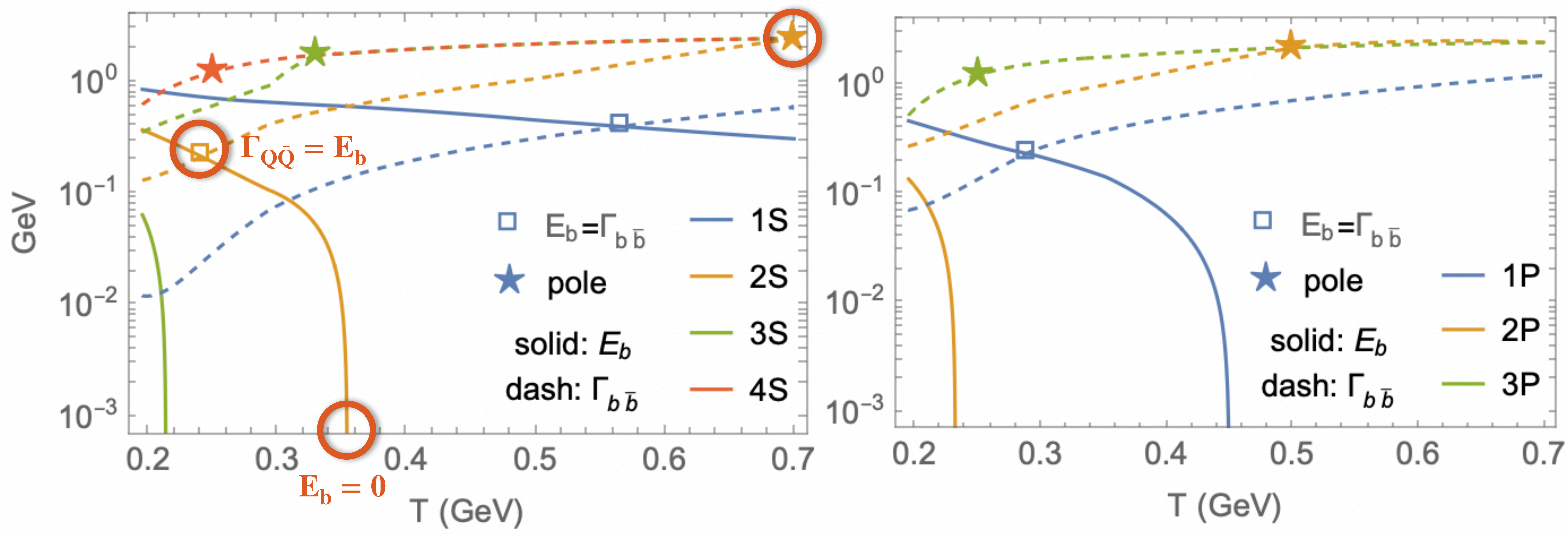
(2) $E_b = 0$

(3) Disappearance of T-matrix pole in complex plane

$\Gamma_{Q\bar{Q}}$ & E_b → quarkonium dissociation rate at finite momenta

→ phenomenological applications in heavy-ion collisions (ongoing)

pole disappearance



4. SUMMARY & OUTLOOK

4.1 Summary

◆ Constrained in-medium input potential to T-matrix through thermal lattice QCD data

◆ Studied bottomonium spectroscopy through T-matrix pole analysis in complex plane & Proposed a rigorous criterion to define quarkonium melting temperature

4.2 Outlook

◆ Impact of high melting temperature on heavy-ion collision phenomenology

◆ Extending pole analysis in complex plane to hadronic phase to study the existence of e.g., deuteron & triton

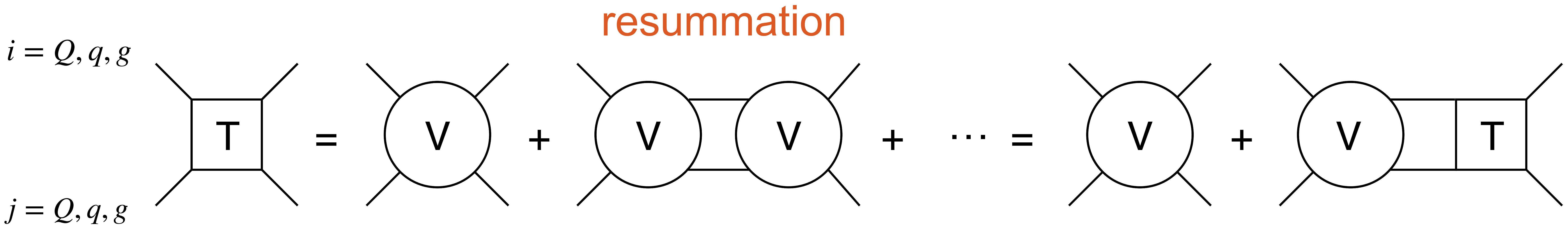
Thanks for Your Attention!

Zhanduo Tang

BACKUP

1.2 T-Matrix Equation [Riek+Rapp '10, Liu+Rapp '18]

◆ **2-body scattering equation: covariant Bethe-Salpeter (BS) equation**



- T-matrix equation: $T_{ij} = V_{ij} + \int V_{ij} G_i G_j V_{ij} + \ldots = V_{ij} + \int V_{ij} G_i G_j T_{ij}$

Born term | **non-perturbative parts**

$\rightarrow$ $T = \dfrac{V}{1 - \int GGV}$ $\longrightarrow$ **pole indicates bound state**

◆ **Approximations**

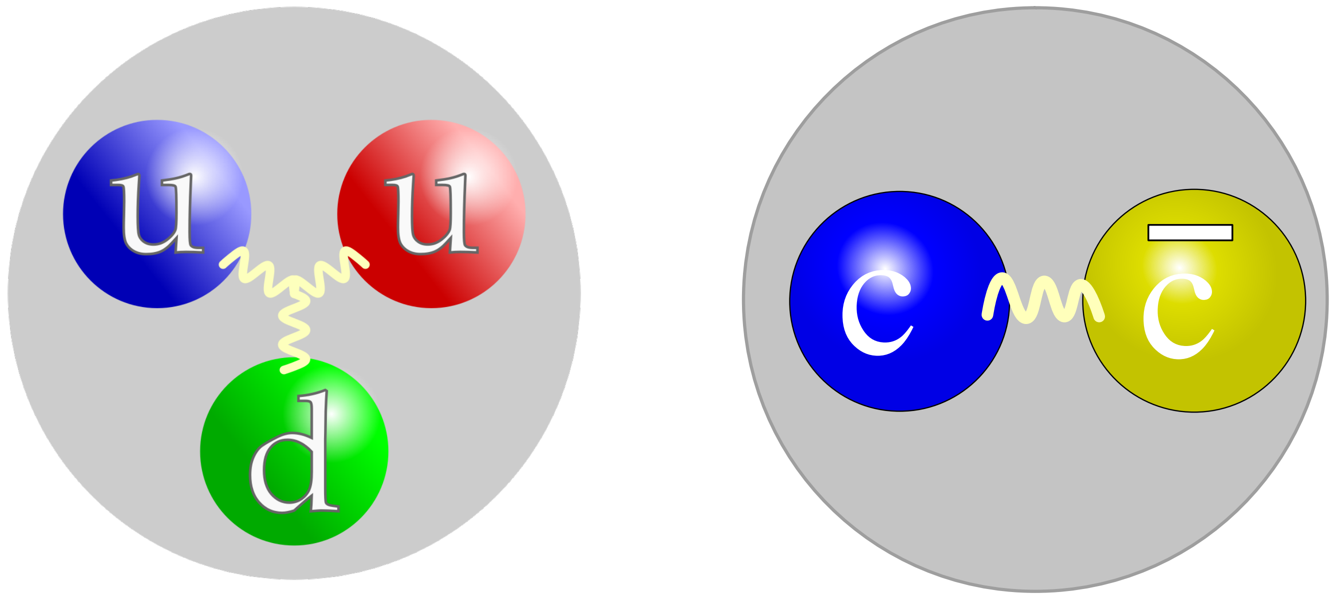
- Small energy transfer: $q_0 \sim \mathbf{q}^2/m_Q \ll |\mathbf{q}| \sim T$ $\rightarrow$ $1/t = 1/(q_0^2 - \mathbf{q}^2) \sim -1/\mathbf{q}^2$

  $\rightarrow$ potential approximation (4D BS eq.→3D T-matrix)

- Quark propagator (positive-energy projection): $G_i = 1/(\omega - \omega_k - \Sigma_i)$

Confinement & String Breaking

◆Confinement:

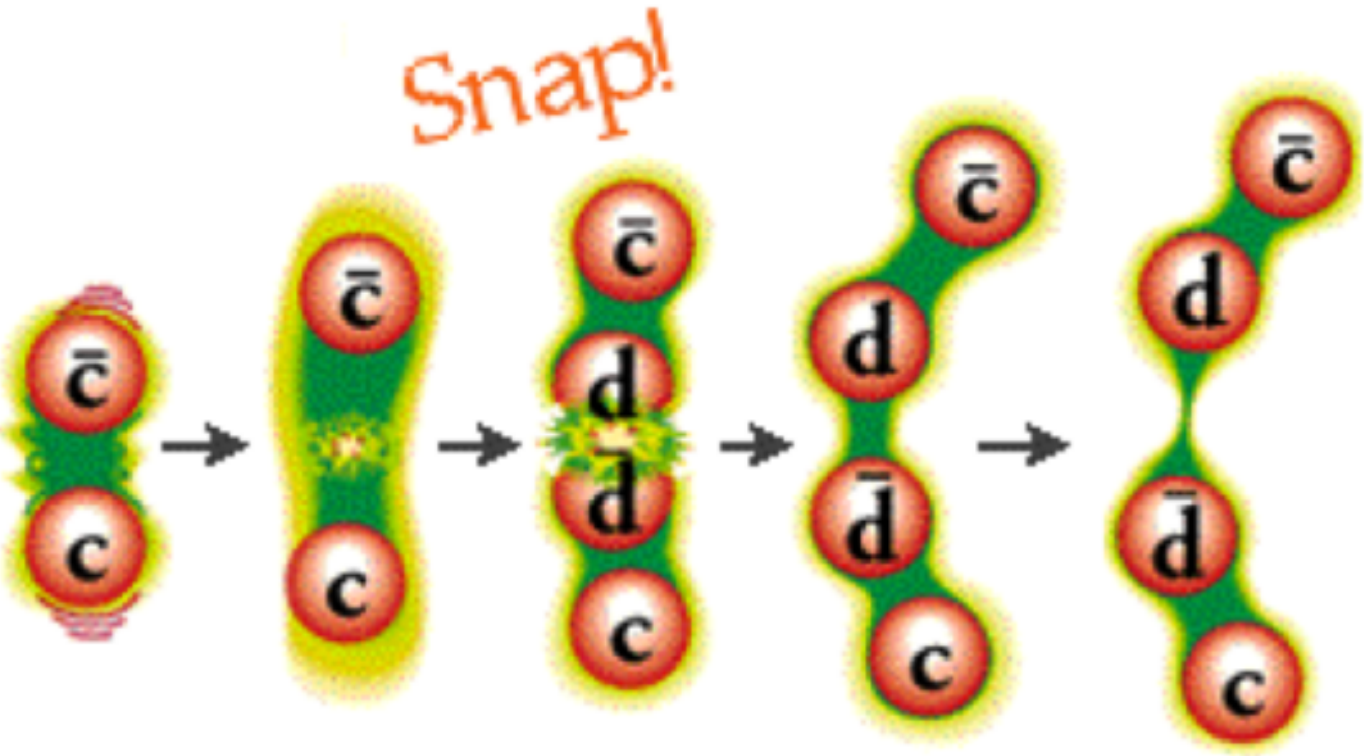


meson baryon

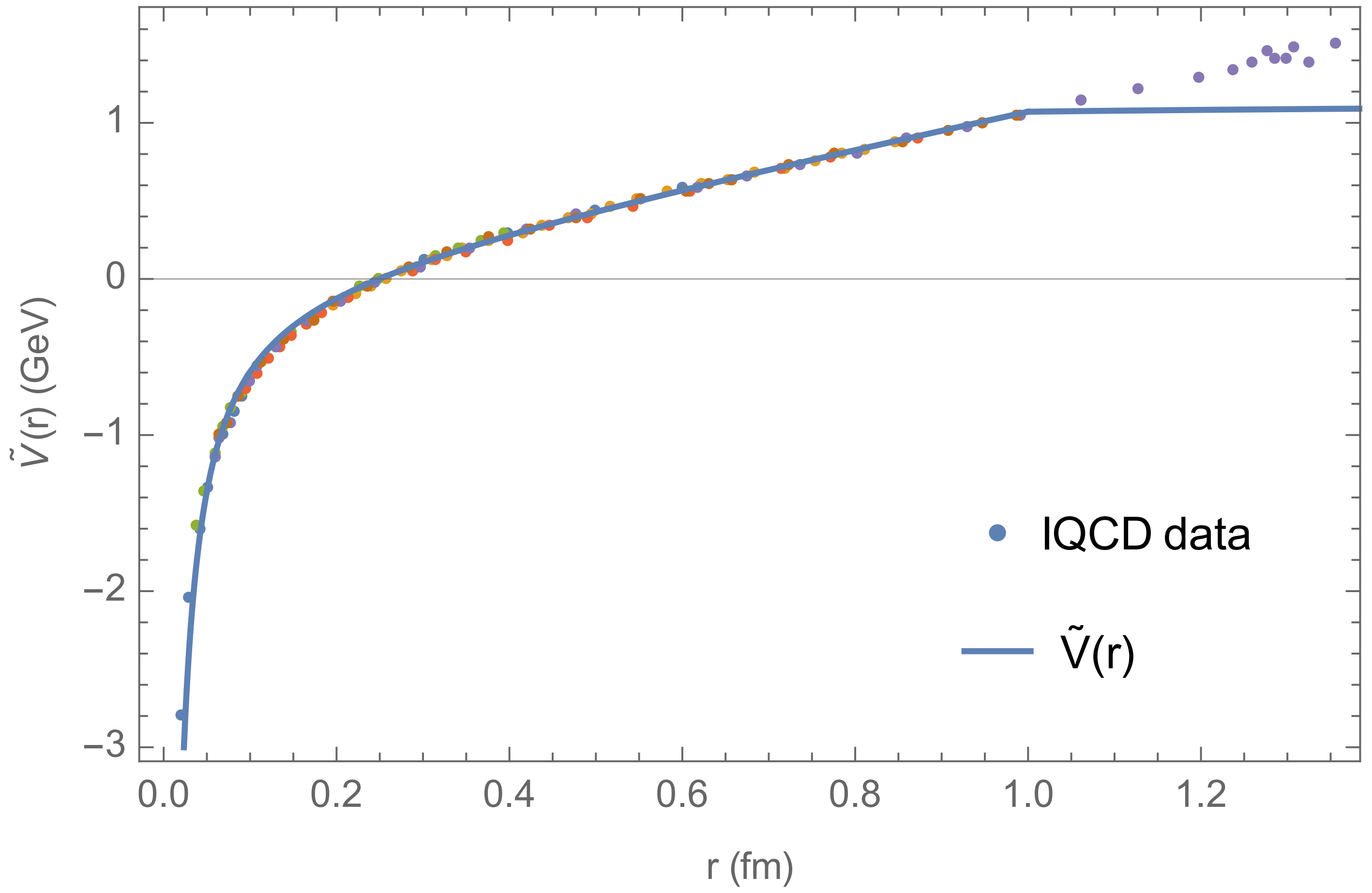
Quarks & gluons confined within hadrons

◆String breaking:

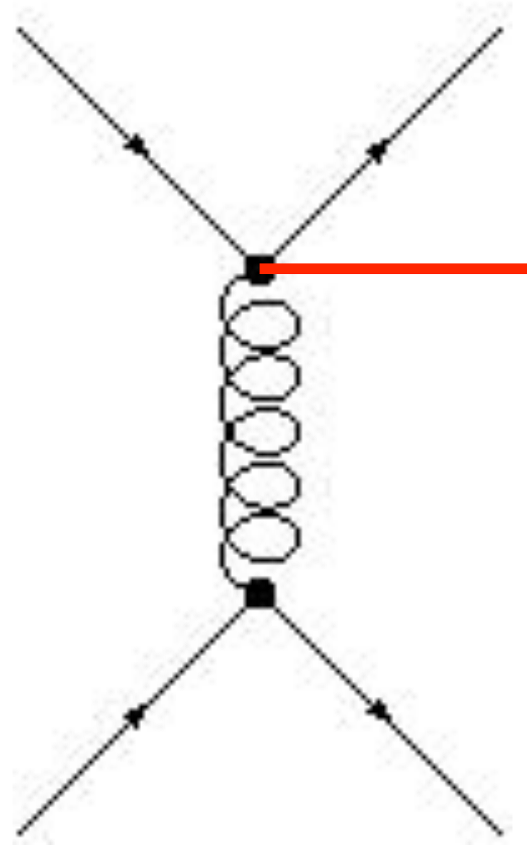
- The strong interaction between two quarks creates a new quark-antiquark pair as the quarks are pulled apart.



- When the potential energy in the string exceeds the energy required to create a new quark-antiquark pair (~1 GeV), the string “breaks."



3.1.1 Relativistic Corrections to Potential



◆ **Breit correction**: $V = RV^{vec} + V^{sca}$

with $R = \sqrt{\frac{\omega_i(p)\omega_j(p)}{M_iM_j}}\sqrt{1+\frac{p^2}{\omega_i(p)\omega_j(p)}}\sqrt{\frac{\omega_i(p')\,\omega_j(p')}{M_iM_j}}\sqrt{1+\frac{p'^2}{\omega_i(p')\,\omega_j(p')}}$

◆ **Lorentz structures**

- Color-Coulomb: $V^{vec} = V_{Coul}$

- Confining:
  - Common assumption: $V^{sca} = V_{conf}$
  - **Improved assumption**: $V^{vec} = V_{Coul} + (1-\chi)V_{conf}$, $V^{sca} = \chi V_{conf}$

[Brambilla+Vairo '97]
[Szczepaniak et al. '96, '97]
[Ebert et al. '98, '03]

◆ **Higher order** $1/M_Q$: $V = RV^{vec} + V^{sca} + V^{LS} + V^{SS} + V^{T}$

- spin-orbit: $V^{LS} = \frac{1}{2M_Q^2 r}\langle L\cdot S\rangle\left(3\frac{d}{dr}V^{vec} - \frac{d}{dr}V^{sca}\right)$
- tensor: $V^{T} = \frac{1}{12M_Q^2}S_{12}\left(\frac{1}{r}\frac{d}{dr}V^{vec} - \frac{d^2}{dr^2}V^{vec}\right)$
- spin-spin: $V^{ss} = \frac{3}{3M_Q^2}\left\langle S_1\cdot S_2\right\rangle \Delta V^{vec}$

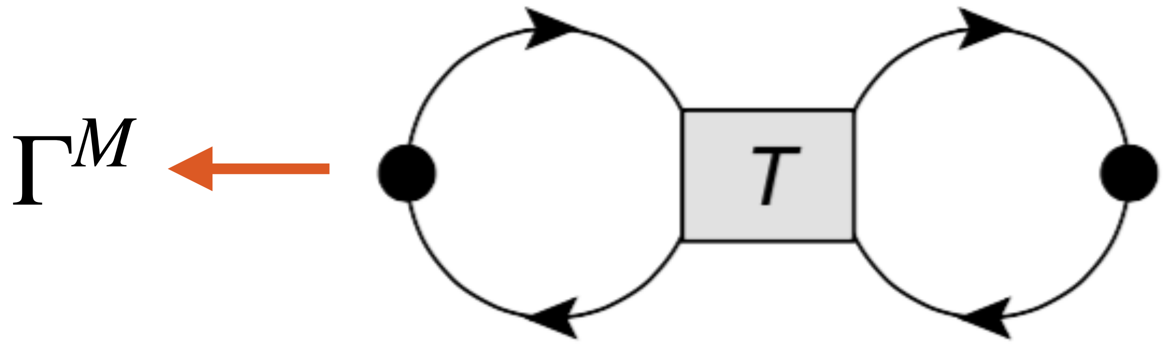
Previously constructed T-matrix:
- w/o vector component in V_{conf}
- w/o spin-related interactions

3.1.2 Correlation & Spectral Functions

- Quakonium mass read from (peak value of) **spectral functions**

$$\sigma(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[G_{Q\bar{Q}}(E)\right]$$

- **Correlation functions** (probability of a $Q\bar{Q}$ pair propagating from one space-time to another):



$$G_{Q\bar{Q}}(E) \sim \Gamma^M G^0_{Q\bar{Q}}(E,p) G^0_{Q\bar{Q}}\left(E,p'\right) \Gamma^M T_{Q\bar{Q}}\left(E,p,p'\right)$$

	scalar (S)	pseudoscalar (PS)	vecor (V)	axial-vector (AV)	tensor (T)
Γ^M	1	$i\gamma_5$	γ^μ	$\gamma^\mu\gamma_5$	$i[\gamma^\mu,\gamma^\nu]/2$

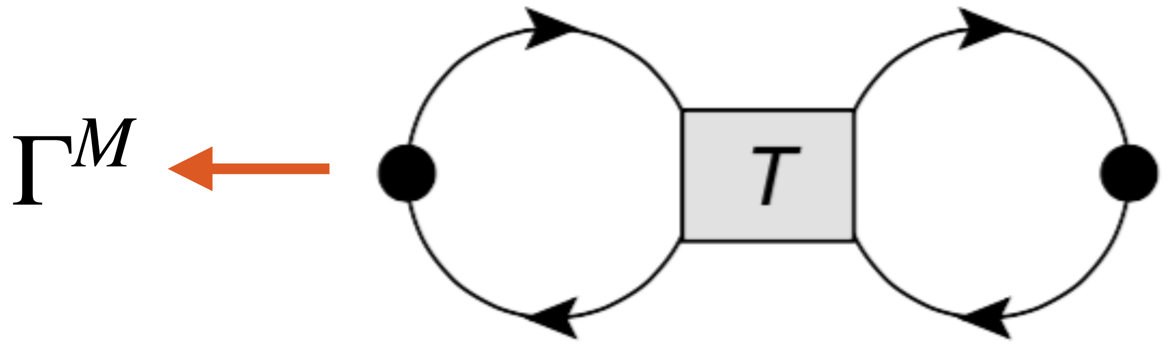
- S-wave (PS, V) and P-wave (S, AV, T) states degenerate without spin-related ($1/M_Q$) corrections

3.1.2 Correlation & Spectral Functions

- Quakonium mass read from (peak value of) **spectral functions**

$$\sigma(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[G_{Q\bar{Q}}(E)\right]$$

- **Correlation functions** (probability of a $Q\bar{Q}$ pair propagating from one space-time to another):



$$G_{Q\bar{Q}}(E) \sim \Gamma^M G^0_{Q\bar{Q}}(E,p) G^0_{Q\bar{Q}}(E,p')\,\Gamma^M T_{Q\bar{Q}}(E,p,p')$$

	scalar (S)	pseudoscalar (PS)	vecor (V)	axial-vector (AV)	tensor (T)
Γ^M	1	$i\gamma_5$	γ^μ	$\gamma^\mu\gamma_5$	$i[\gamma^\mu,\gamma^\nu]/2$

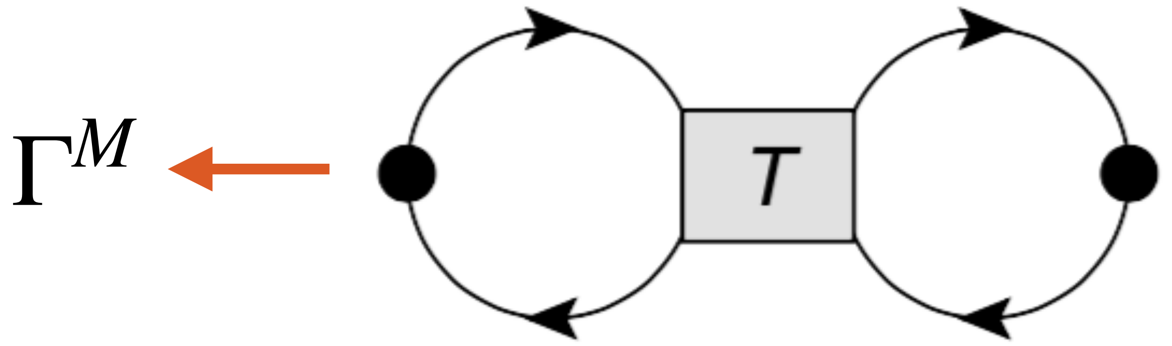
- S-wave (PS, V) and P-wave (S, AV, T) states degenerate without spin-related ($1/M_Q$) corrections

3.1.2 Correlation & Spectral Functions

- Quakonium mass read from (peak value of) **spectral functions**

$$\sigma(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[G_{Q\bar{Q}}(E)\right]$$

- **Correlation functions** (probability of a $Q\bar{Q}$ pair propagating from one space-time to another):



$$G_{Q\bar{Q}}(E) \sim \Gamma^M G^0_{Q\bar{Q}}(E,p) G^0_{Q\bar{Q}}\left(E,p'\right) \Gamma^M T_{Q\bar{Q}}\left(E,p,p'\right)$$

	scalar (S)	pseudoscalar (PS)	vecor (V)	axial-vector (AV)	tensor (T)
Γ^M	1	$i\gamma_5$	γ^μ	$\gamma^\mu\gamma_5$	$i[\gamma^\mu,\gamma^\nu]/2$

- S-wave (PS, V) and P-wave (S, AV, T) states degenerate without spin-related $(1/M_Q)$ corrections

In-Medium Constraints from Lattice QCD

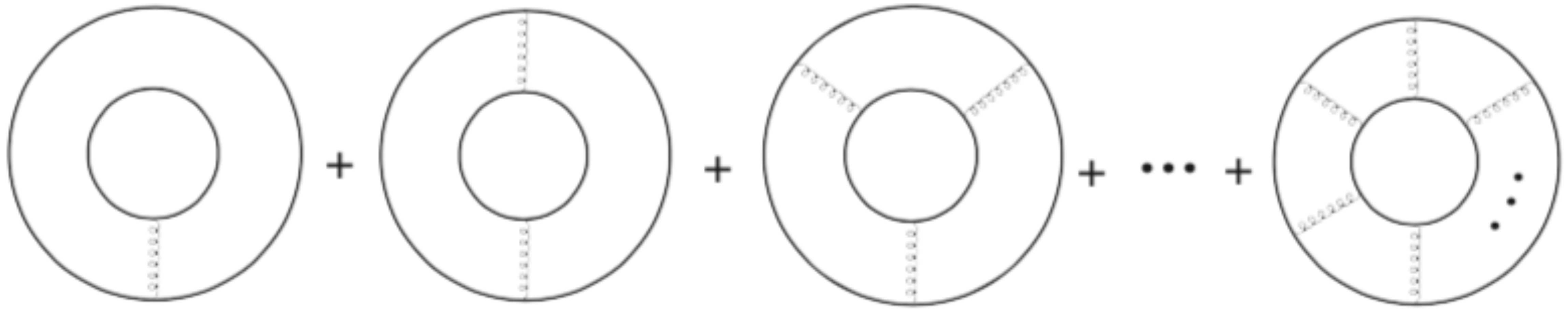
— Equation of State (EoS)

2-body interaction contribution:~ T_{ij}

- $$P = \pm \frac{-1}{\beta} \sum_n \mathrm{Tr}\left\{ \ln\left(-G^{-1}\right) + \left[\left(G^0\right)^{-1} - G^{-1}\right] G \right\} \pm \Phi$$

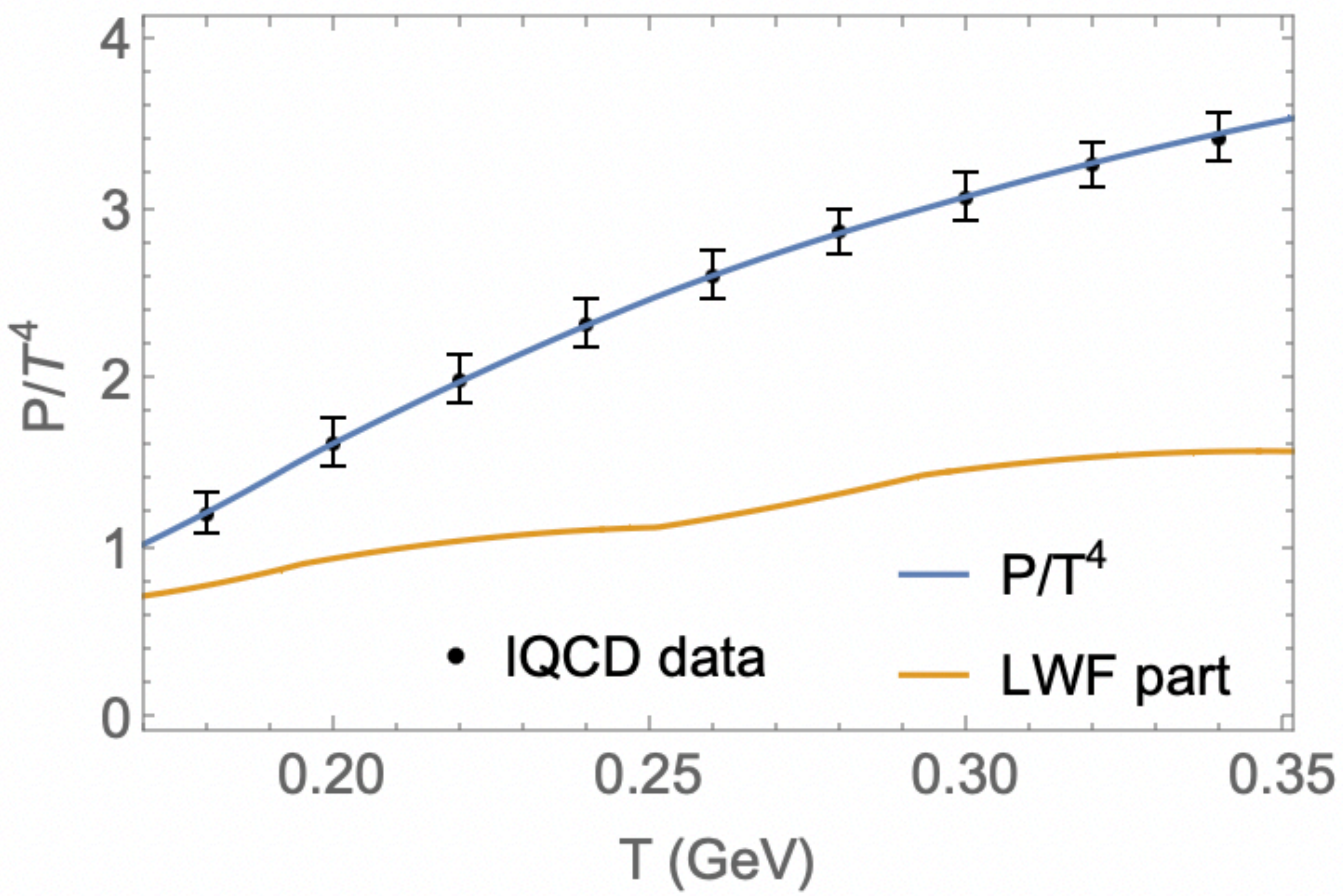
quasi-particle selfenergy Luttinger-Ward functional (LWF)

- **LWF resummation**:



[Luttinger+Ward '60, Baym+Kadanoff '61, Liu+Rapp '18]

- **Pressure**



- LWF becomes leading contribution at low temperatures
  → transition in degree of freedom (QGP to hadronic)

2.1 Vacuum Constraints from Quakonium Spectroscopy

◆ **Breit (relativistic) correction**: $V = RV^{vec} + V^{sca}$

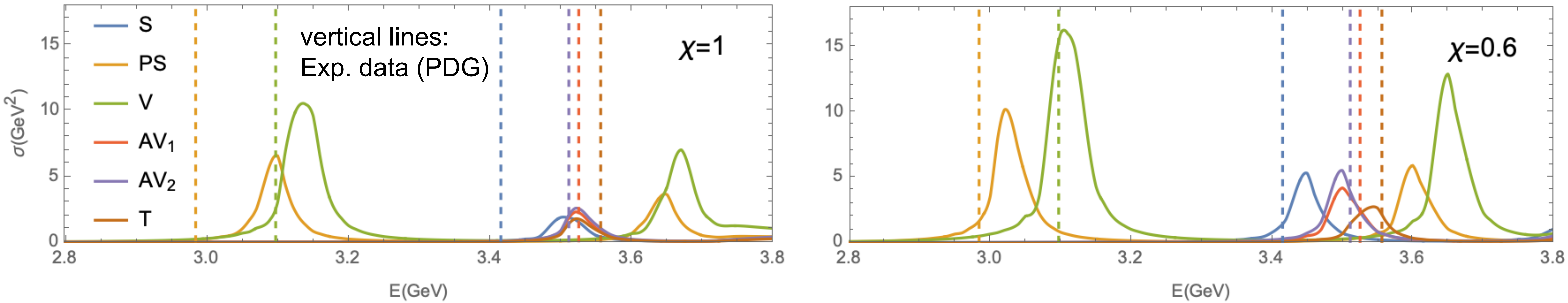
vector $\sim \bar{q}\gamma^{\mu}q$
scalar $\sim \bar{q}q$

◆ **Lorentz structures:**

- Color-Coulomb: $V^{vec} = V_{Coul}$
- Confining:
  - Common assumption: $V^{sca} = V_{conf}$
  - **Improved assumption**: $V^{vec} = V_{Coul} + (1-\chi)V_{conf}\,,$ $V^{sca} = \chi V_{conf}$

[Brambilla+Vairo '97]
[Szczepaniak et al. '96, '97]
[Ebert et al. '98, '03]

◆ **Charmonium ($c\bar{c}$) spectral functions:** [Tang and Rapp, PRC **108**, 9044906 (2023)]



2.2 In-Medium Constraints from Lattice QCD
— *Static Wilson Line Correlators (WLCs)*

◆ **Definition** (static limit of $Q\bar{Q}$ meson correlator)

- $W(r,\tau,T) = \frac{1}{3}\left\langle \mathrm{Tr}\left(L(0,\tau)L^{\dagger}(r,\tau)\right)\right\rangle_T$

  computable in T-matrix framework

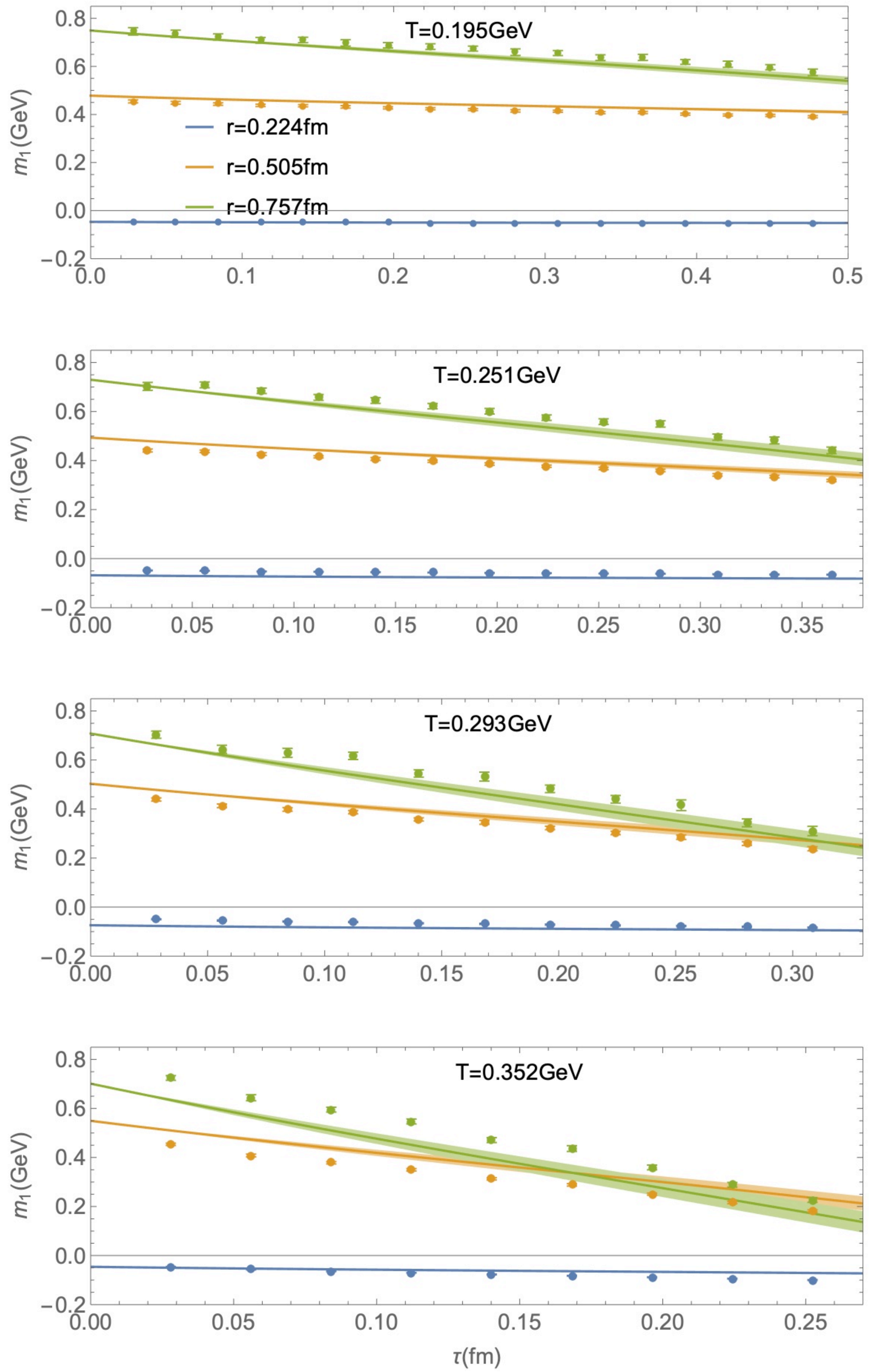
- 1st cumulant: $m_1(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T)$

Physical meaning:

- Intercept: $Q\bar{Q}$ effective mass - Slope: interacting strength with medium

◆ **Motivations**

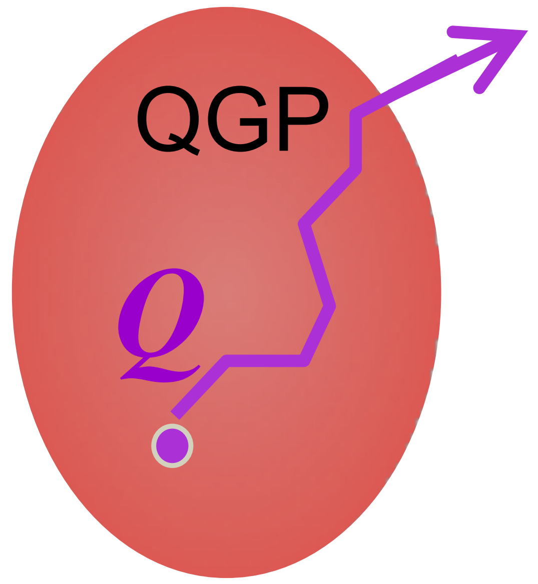
- Hard-thermal loop perturbation theory cannot describe lQCD data; motivates non-perturbative method—T-matrix approach
- WLCs provide more constraints on input potential



[Tang, Mukherjee, Petreczky, and Rapp, EPJA **60**, 92 (2024)]

3. Heavy-Quark Transport in QGP

Brownian motion
for heavy quarks through the QGP



- **Fokker-Planck equation**

$$\frac{\partial}{\partial t} f(p,t) = \frac{\partial}{\partial p_i}\left\{ A(p) p_i f(p,t) + \frac{\partial}{\partial p_j}\left[B_{ij}(p) f(p,t)\right]\right\}$$

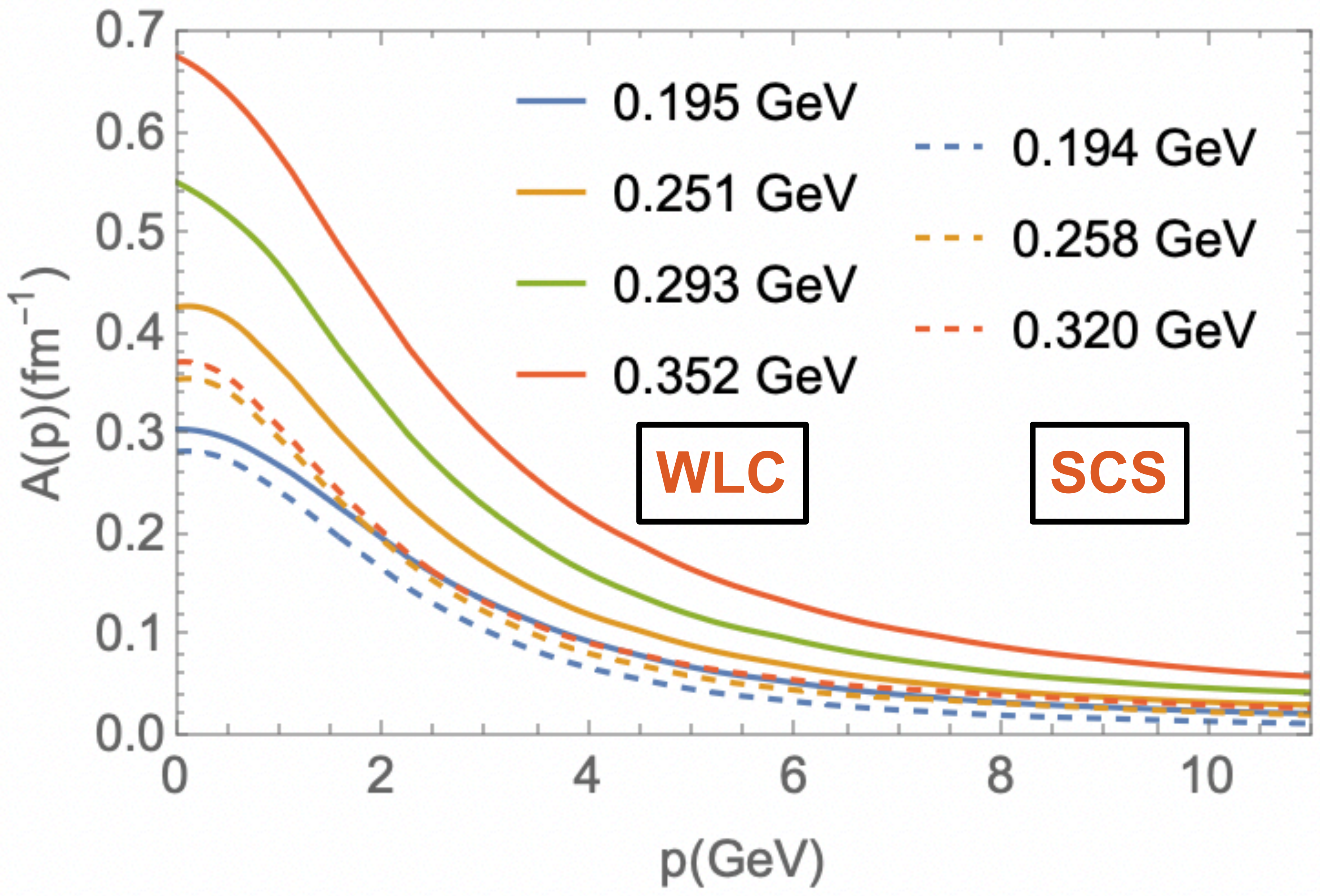
- Thermal relaxation rate (friction coefficient)

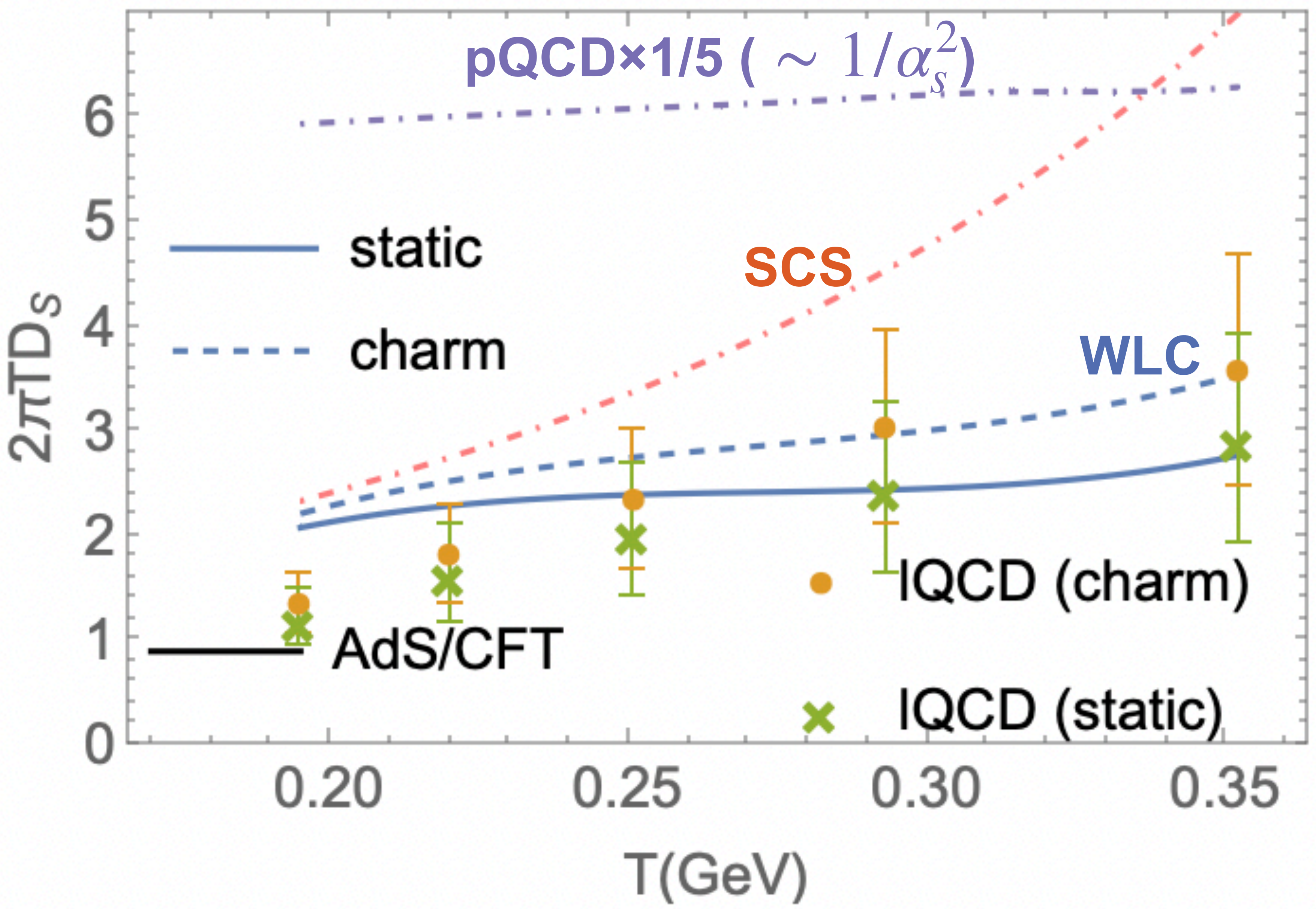
$$A(p) \sim \sum_i \int d^4p' d^4q d^4q' \left|T_{Qi}\right|^2 \left(1 - \frac{\boldsymbol{p}\cdot\boldsymbol{p}'}{\boldsymbol{p}^2}\right)$$

- Spatial diffusion coefficient: $2\pi T D_s = \dfrac{2\pi T^2}{M_Q A(p \to 0)}$

$\langle x^2 \rangle - \langle x \rangle^2 = 6 D_S t$

Small D_s indicates stronger interaction with medium



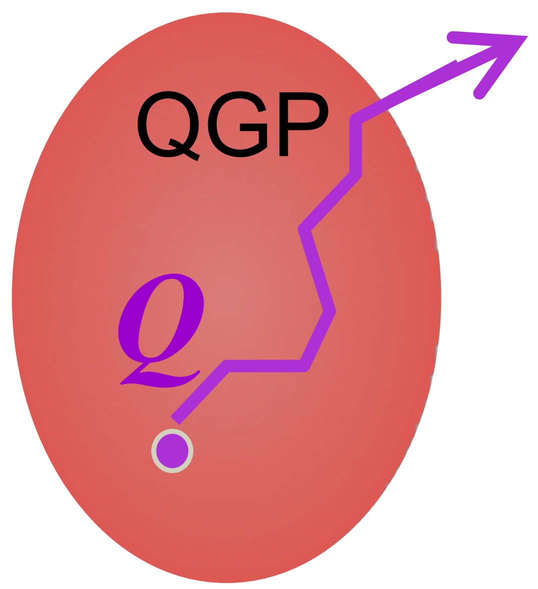


$A(p)\&D_S$

D^0 meson v_2

4. Heavy-Quark Transport in QGP

Brownian motion
for heavy quarks through the QGP



- **Fokker-Planck equation**

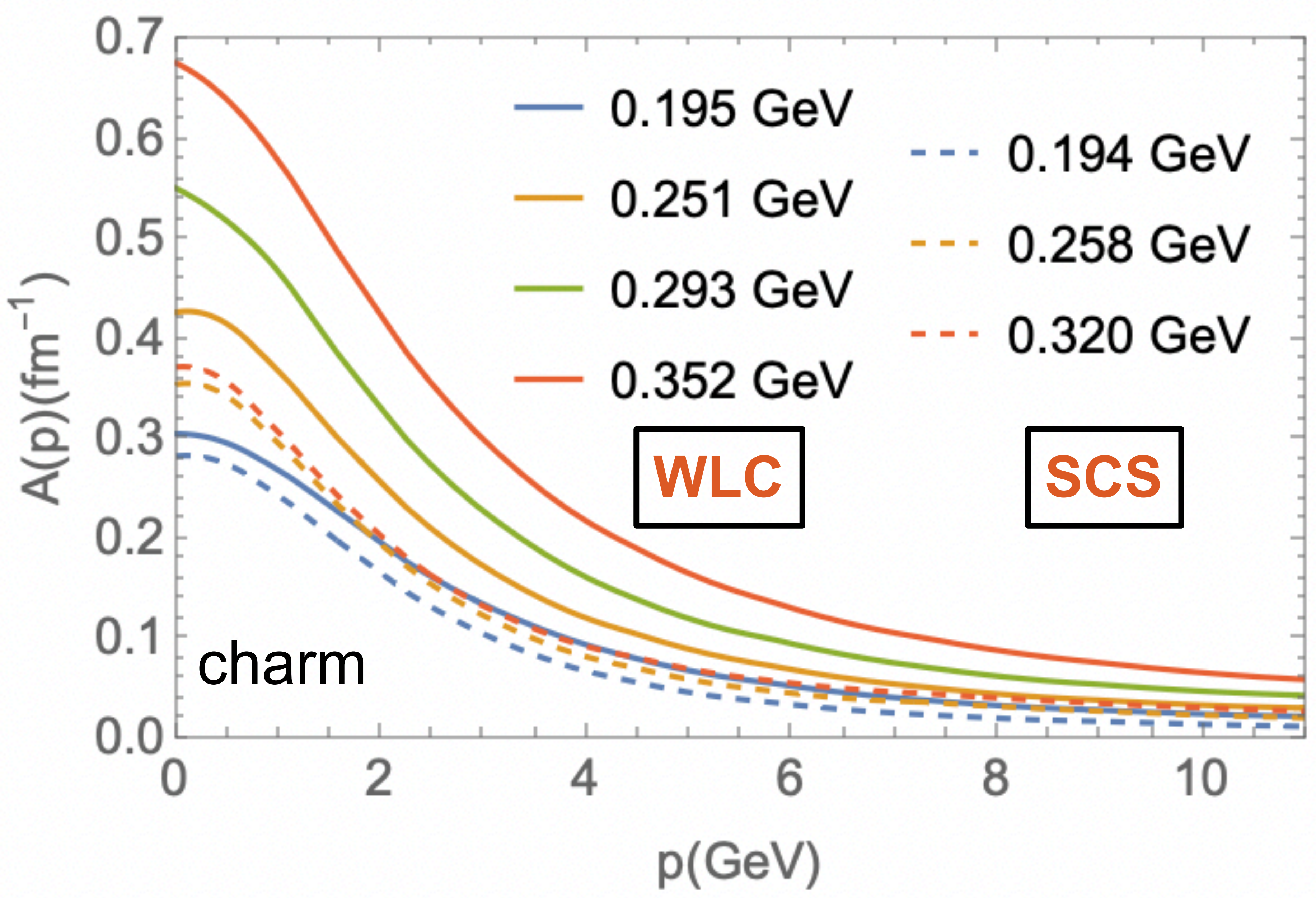
$$\frac{\partial}{\partial t} f(p,t) = \frac{\partial}{\partial p_i}\left\{ A(p) p_i f(p,t) + \frac{\partial}{\partial p_j}\left[B_{ij}(p) f(p,t) \right] \right\}$$

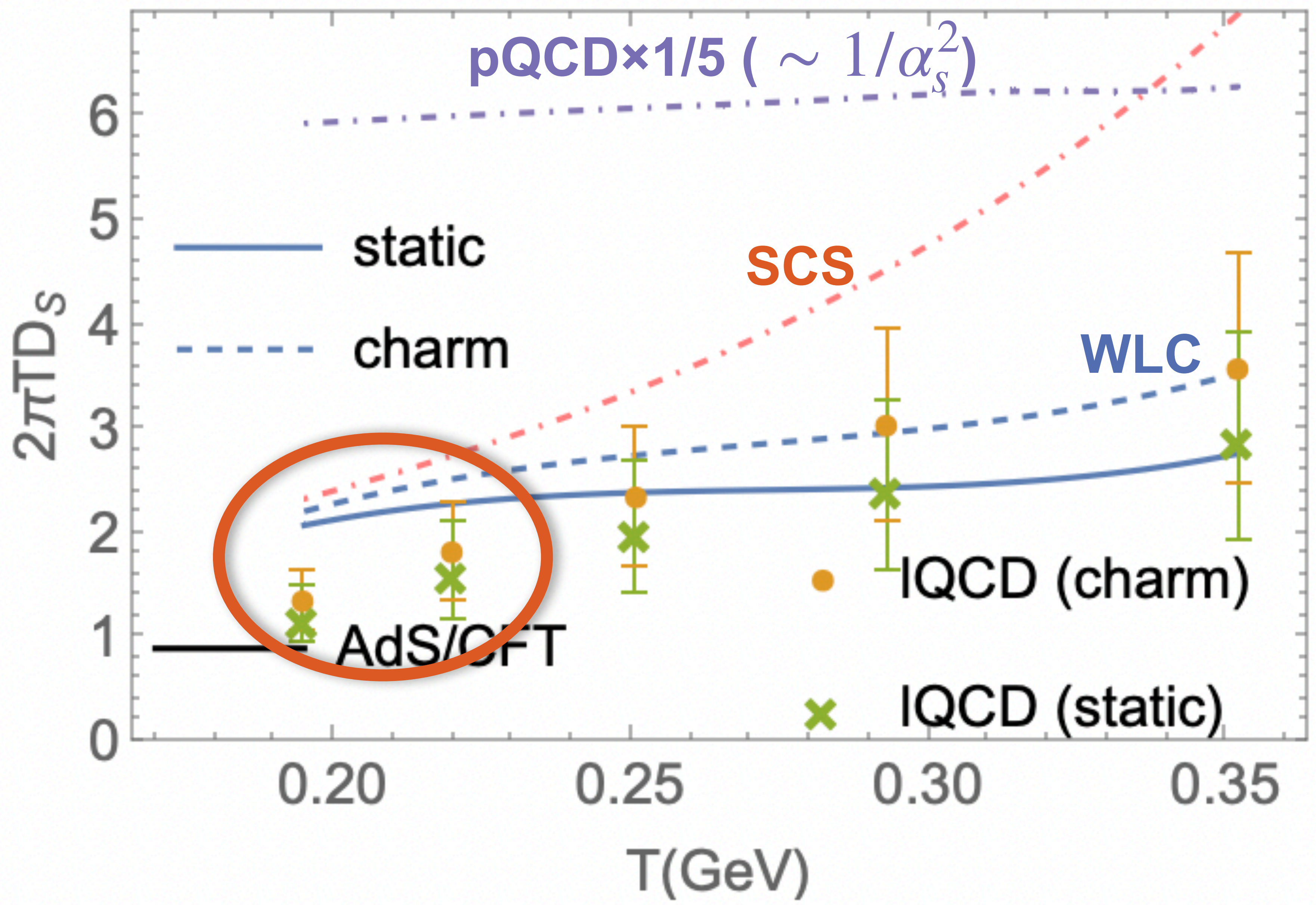
- Thermal relaxation rate (friction coefficient)

$$A(p) \sim \sum_i \int d^4p' d^4q d^4q' \left| T_{Qi} \right|^2 \left(1 - \frac{\boldsymbol{p} \cdot \boldsymbol{p}'}{\boldsymbol{p}^2} \right)$$

- Spatial diffusion coefficient: $2\pi T D_s = \dfrac{2\pi T^2}{M_Q A(p \to 0)}$

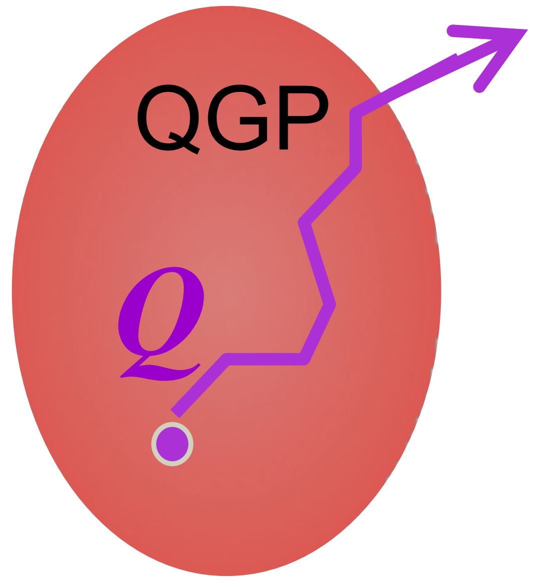
$\langle x^2 \rangle - \langle x \rangle^2 = 6 D_S t$





4. Heavy-Quark Transport in QGP

Brownian motion
for heavy quarks through the QGP



- **Fokker-Planck equation**

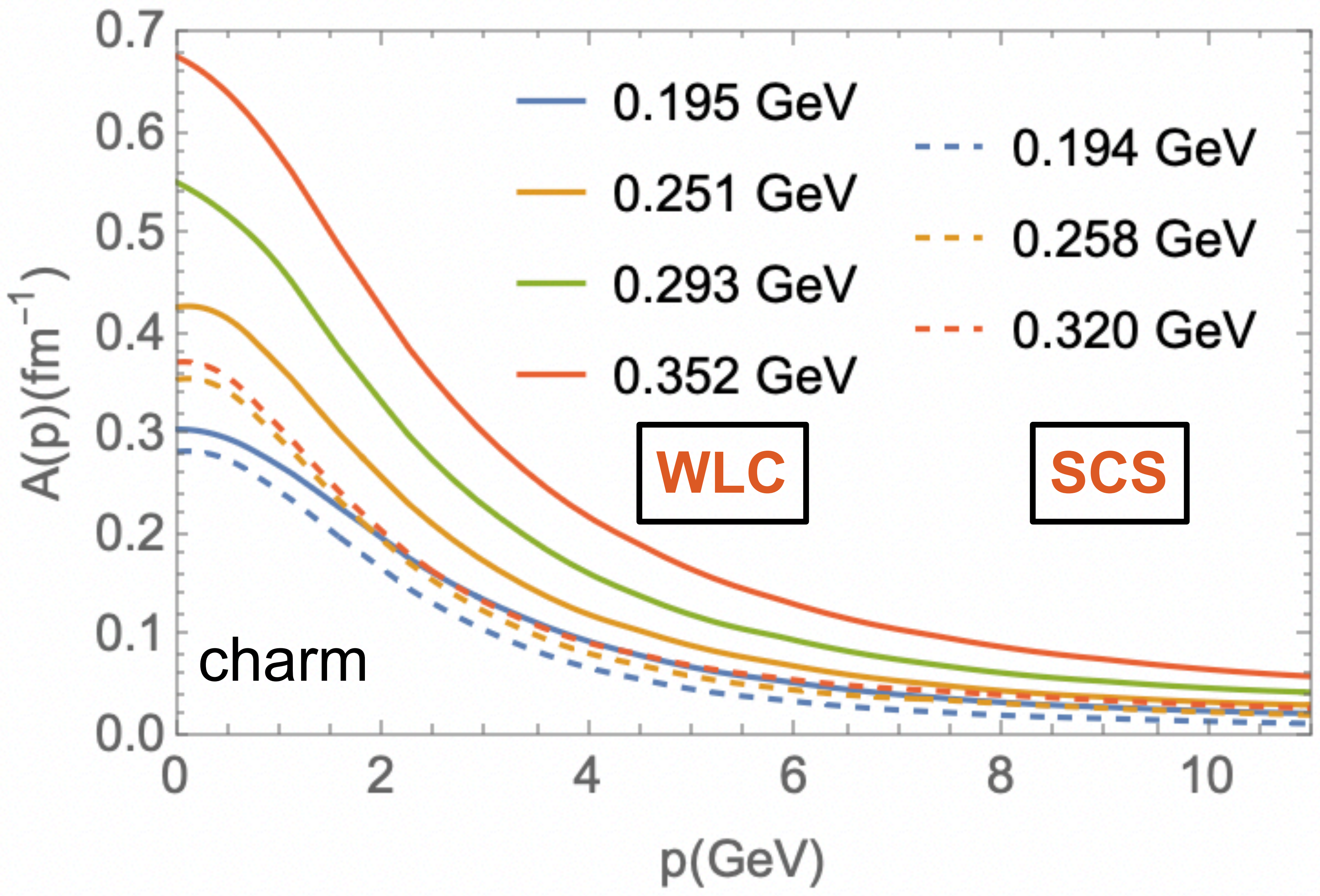
$$\frac{\partial}{\partial t} f(p,t) = \frac{\partial}{\partial p_i}\left\{ A(p) p_i f(p,t) + \frac{\partial}{\partial p_j}\left[B_{ij}(p) f(p,t) \right] \right\}$$

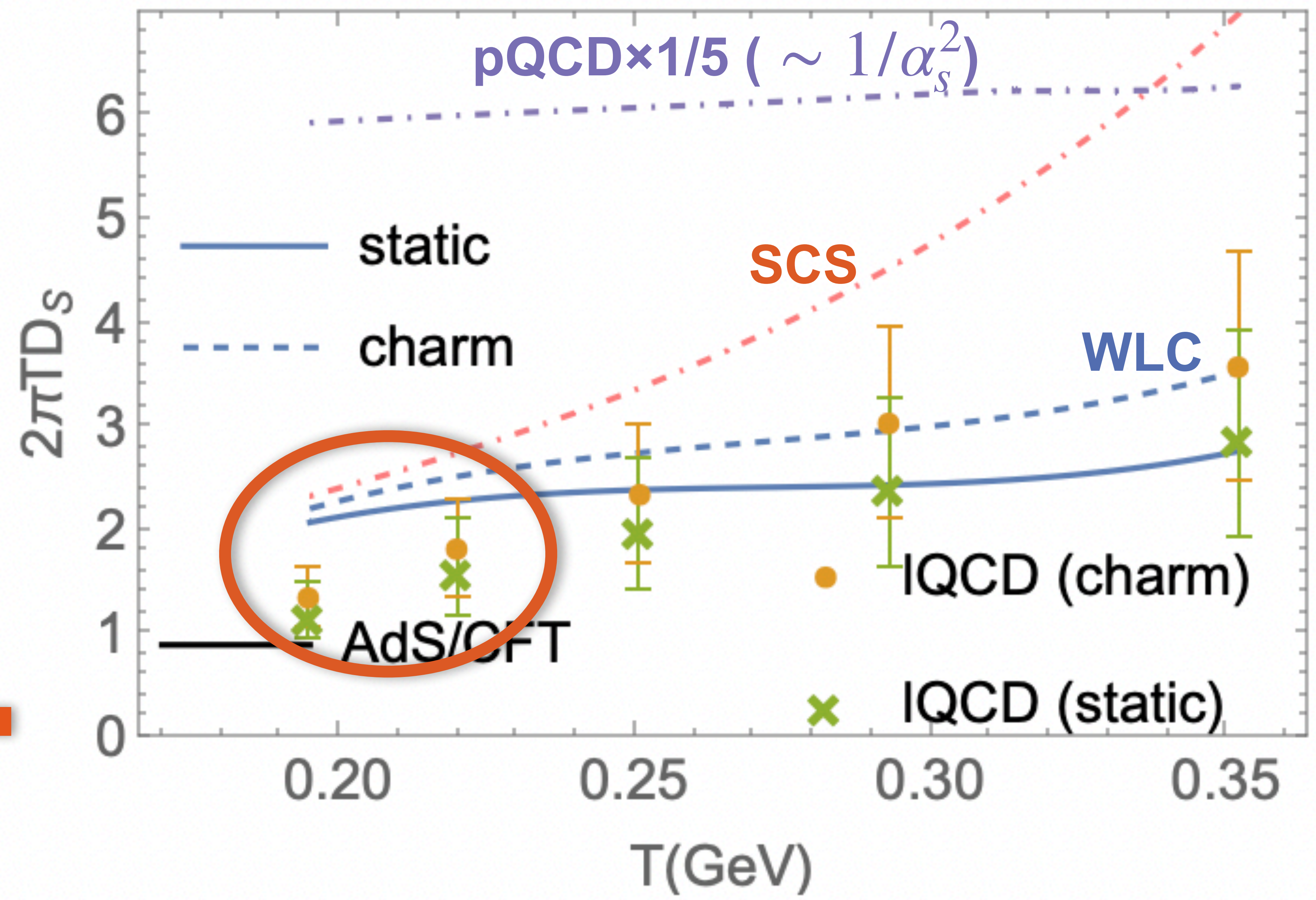
- Thermal relaxation rate (friction coefficient)

$$A(p) \sim \sum_i \int d^4p' d^4q d^4q' \left| T_{Qi} \right|^2 \left(1 - \frac{\boldsymbol{p}\cdot\boldsymbol{p}'}{\boldsymbol{p}^2}\right)$$

- Spatial diffusion coefficient: $2\pi T D_s = \dfrac{2\pi T^2}{M_Q A(p\to 0)}$

$$\langle x^2 \rangle - \langle x \rangle^2 = 6 D_S t$$





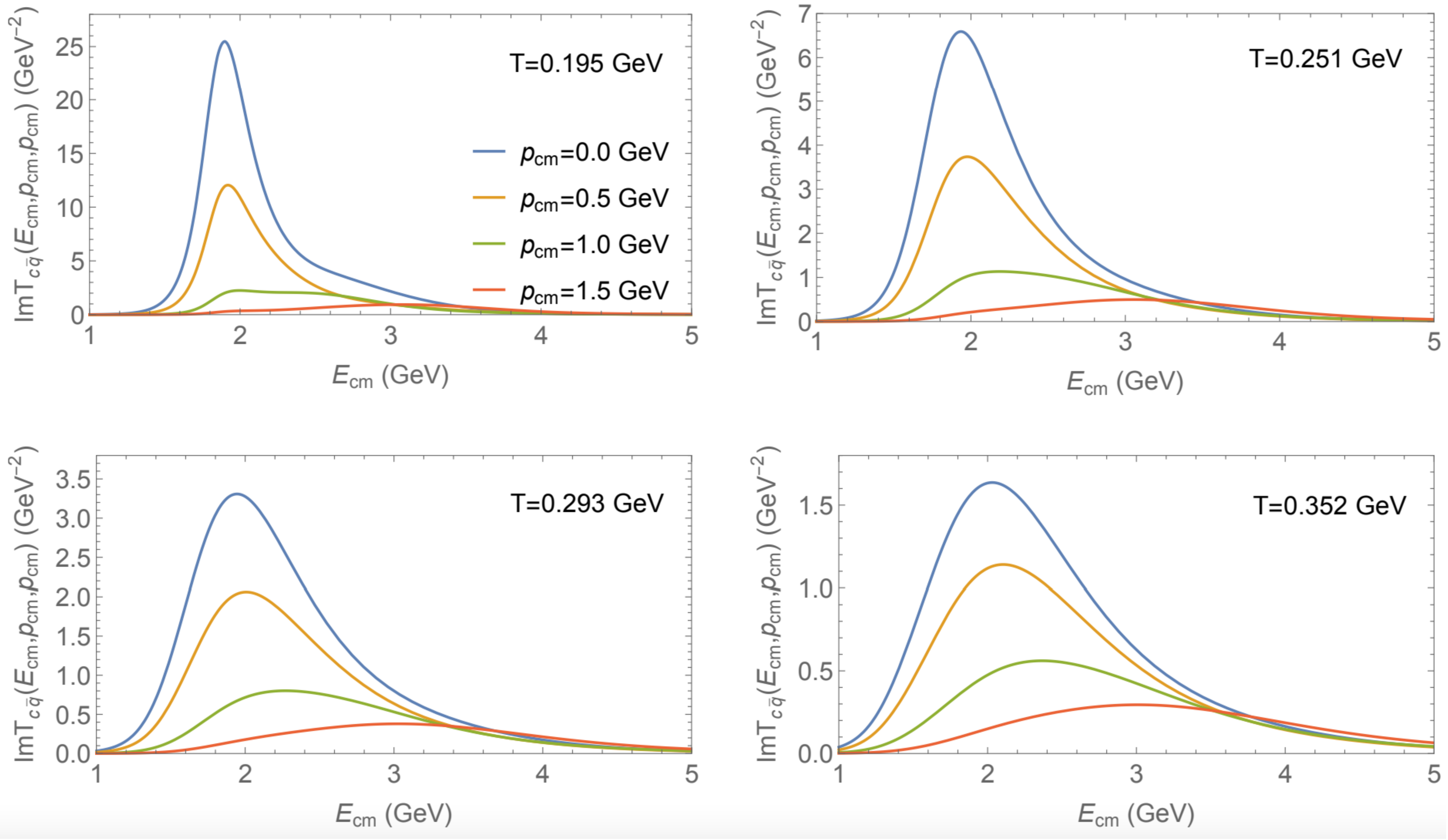
phenomenological applications in heavy-ion collisions (ongoing)

$A(p)\&D_S$

indicating missing charm-diquark interaction (ongoing)

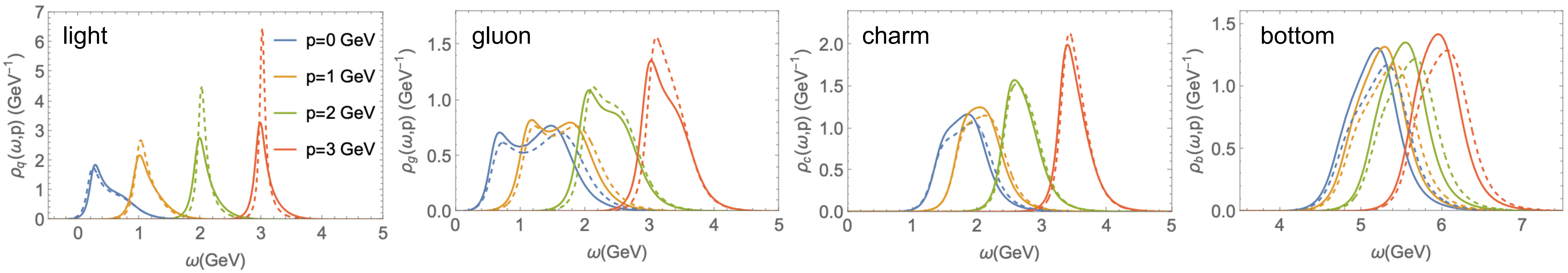
Heavy-Light T-Matrices

[Tang+Mukherjee et al, EPJA **60**, 92 (2024)]

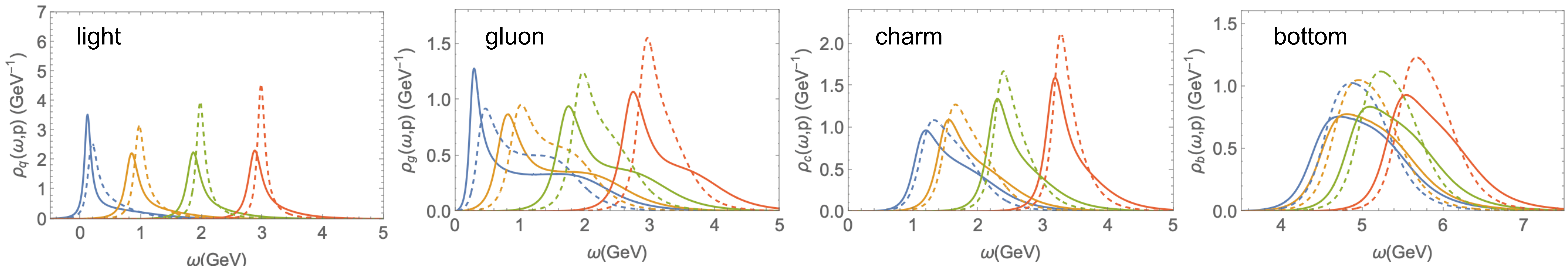


Parton Spectral Functions: $\rho_i(\omega,p) = -\frac{1}{\pi}\,\mathrm{Im}\left[G_i(\omega,p)\right]$

Solid: WLC (T=195 MeV) & Dash: SCS (T=194 MeV):

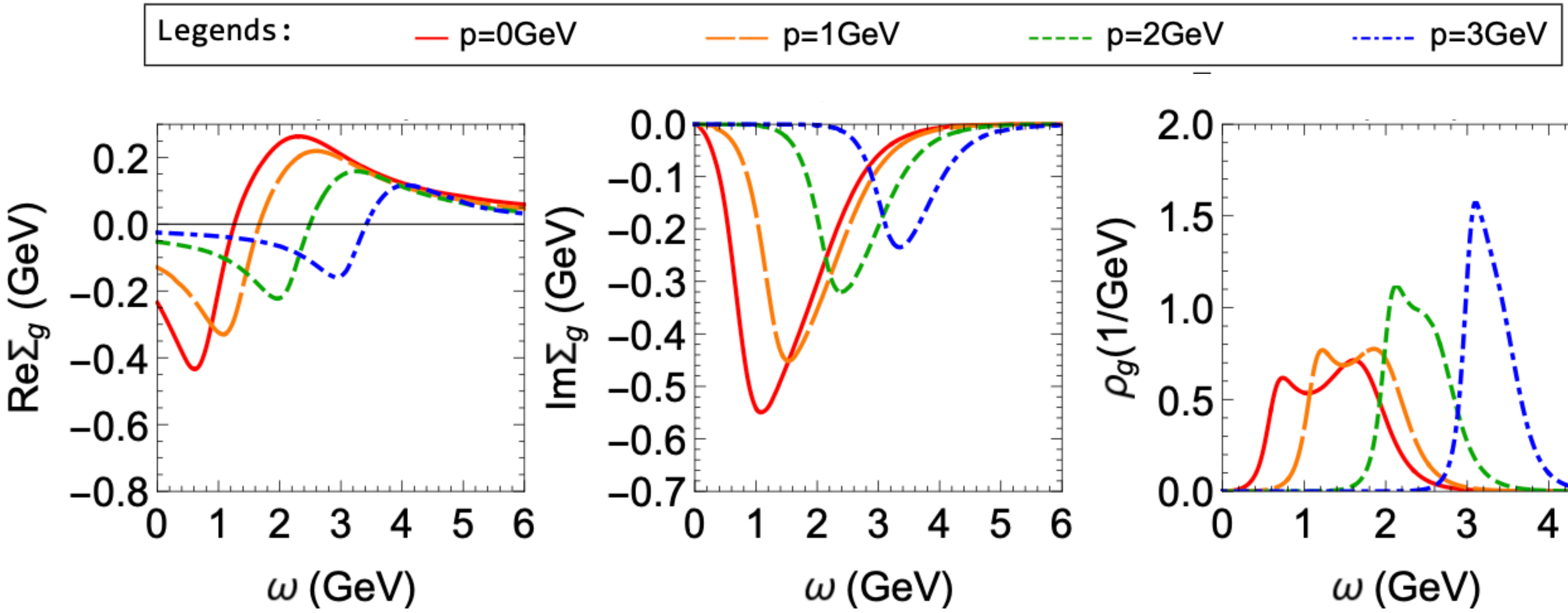


Solid: WLC (T=352 MeV) & Dash: SCS (T=320 MeV):



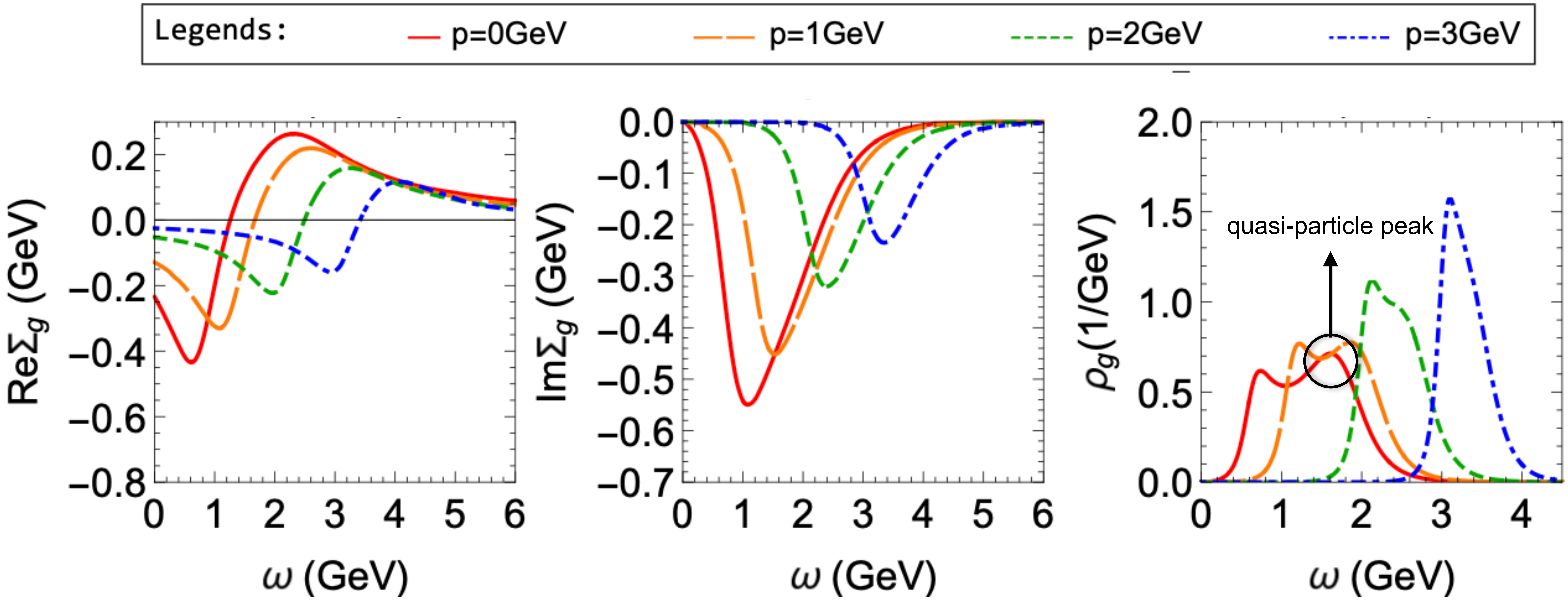
Collective Mode

[Liu+Rapp '18]



Collective Mode

[Liu+Rapp '18]



Collective Mode

[Liu+Rapp '18]