The perturbative and non-perturbative interactions between heavy flavor quarks and Quasi-Particle Quark-Gluon Plasma.

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Central China Normal University

2025.10.27

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Contents

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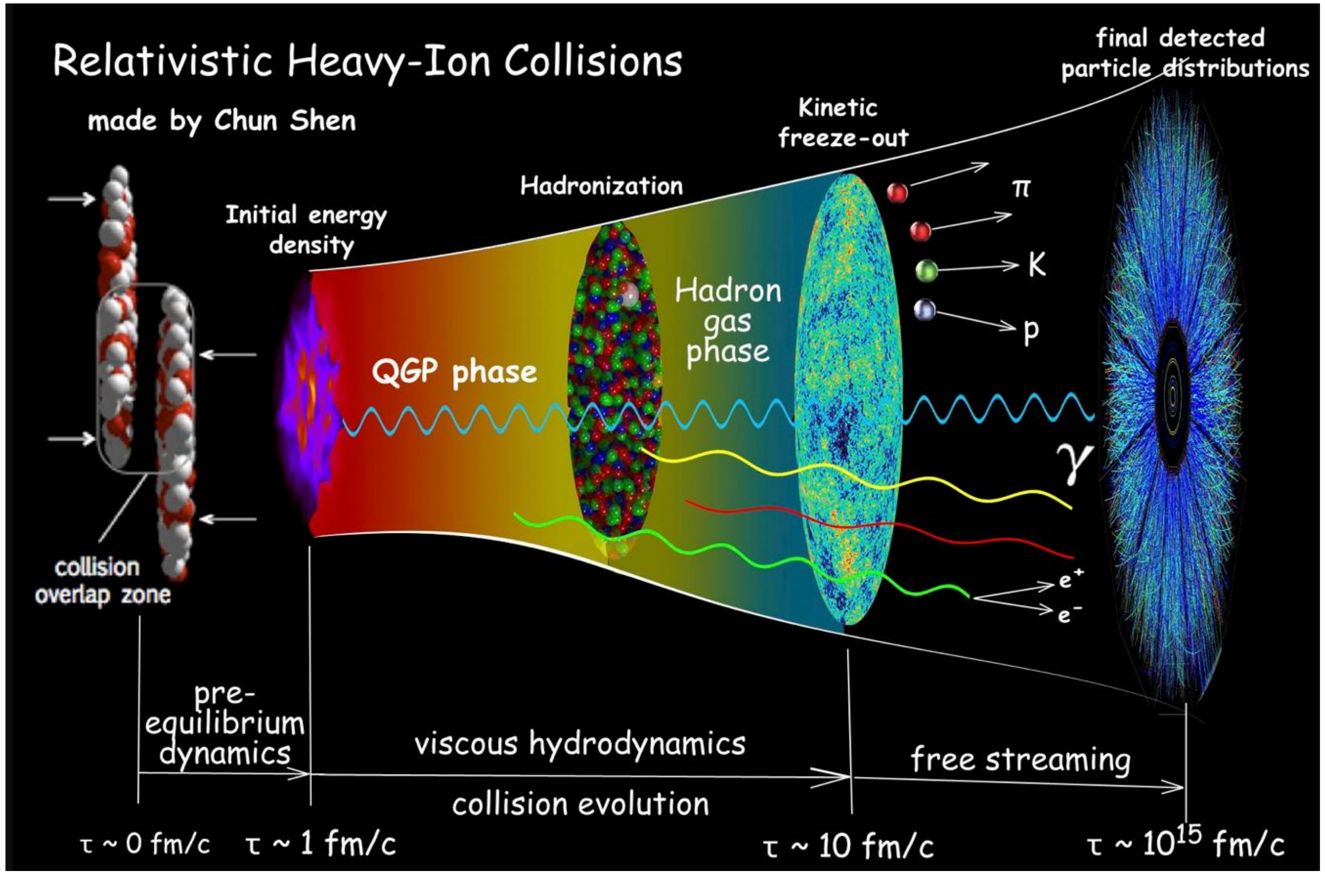
- The hard probe: Heavy flavor

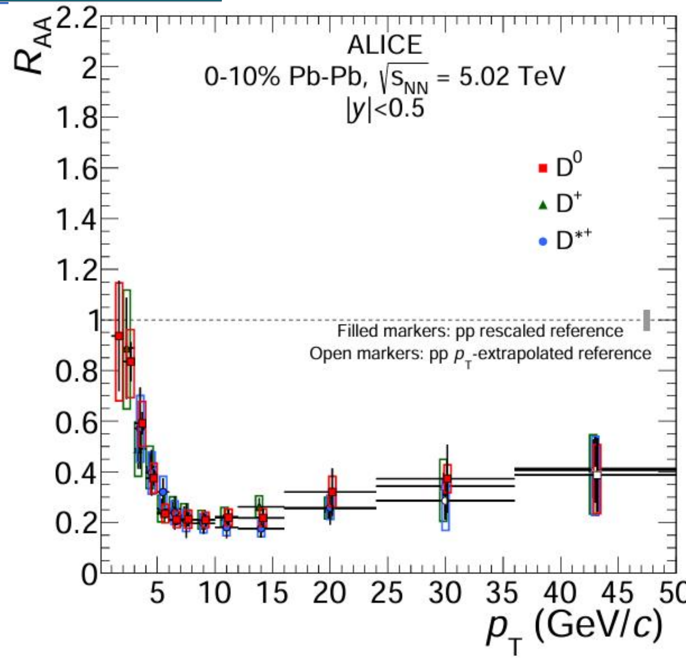
  - Large mass: $m_Q > \Lambda_{QCD}, m_Q > T_{QGP}$.
  - It travels entire evolution of the QGP.
  - Its production is calculable in pQCD.



The probe --- Heavy flavor

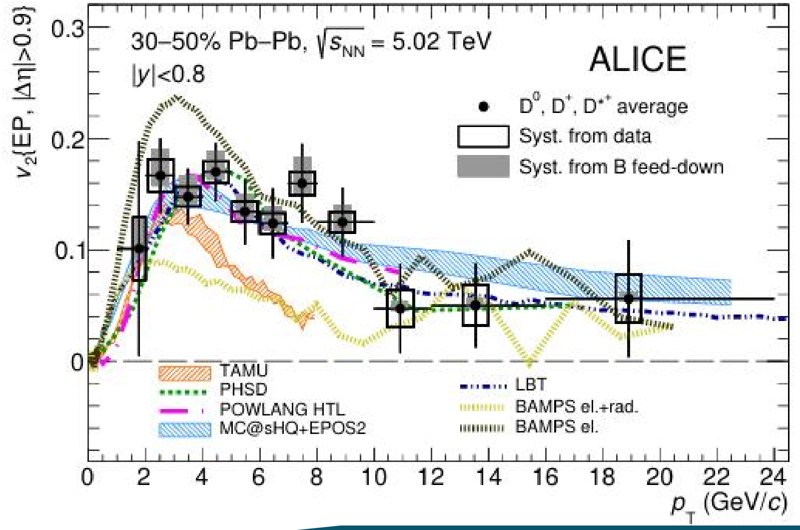
- The R_{AA} - quantify the strength of the energy loss effect:

$$R_{AA} = \frac{1}{N_{\mathrm{coll}}} \frac{\mathrm{d}N_{\mathrm{AA}}/\mathrm{d}y\mathrm{d}^2 p_T}{\mathrm{d}N_{\mathrm{pp}}/\mathrm{d}y\mathrm{d}^2 p_T}.$$



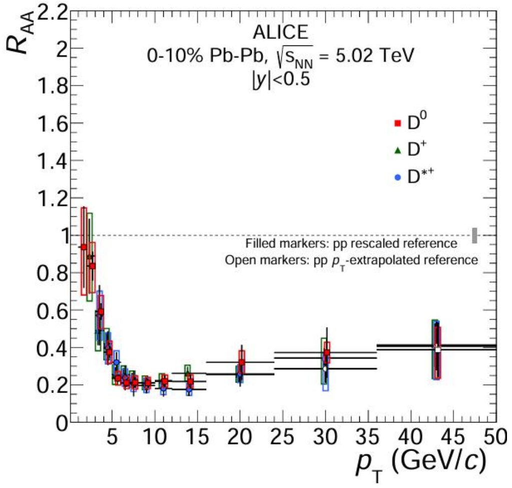
- The v_2 - reveal momentum-space anisotropy:

$$v_2(p_T) = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle.$$



Acharya S, et al. arXiv:2110.09420v3

The probe --- Heavy flavor



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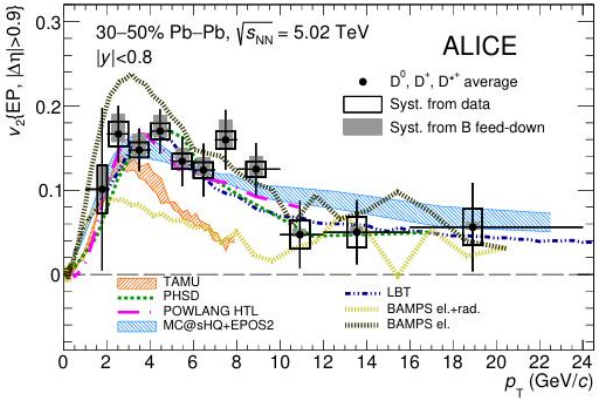

- The v_2 - reveal momentum-space anisotropy:

$$v_2(p_T)=\langle\cos\left(2\phi\right)\rangle=\left\langle\frac{p_x^2-p_y^2}{p_x^2+p_y^2}\right\rangle.$$

- Can we better incorporate the non-perturbative nature of the QGP background?

Acharya S, et al. arXiv:2110.09420v3

Contents

The Quasi-particle Model of the QGP

- The QGP is a strongly coupled system.
- Quasi-particle model of the QGP.

$$m_q = m_q(\mathrm{T})$$

$$m_g = m_g(T)$$

- Partition function:

$$\ln Z_g(T) = -\frac{16V}{2\pi^2}\int_0^\infty p^2 \mathrm{d}p \ln\left[1 - \exp(-\frac{1}{T}\sqrt{p^2 + m_g^2(T)})\right];$$
$$\ln Z_{q_i}(T) = \frac{12V}{2\pi^2}\int_0^\infty p^2 \mathrm{d}p \ln\left[1 + \exp(-\frac{1}{T}\sqrt{p^2 + m_{q_i}^2(T)})\right].$$

Chandra V, Ravishankar V. Phys. Rev. D 84, 074013.
Li F P, Lü H L, Pang L G, et al. arXiv:2211.07994 .

The Quasi-particle Model of the QGP

- Quasi-particle quark and gluon masses m_q *and* m_g:

$$\boxed{\begin{aligned} m_g^2 &= \frac{1}{2} m_D^2, \\ m_{u,d}^2 &= \frac{2}{9} m_D^2, \\ m_s^2 - m_{0s}^2 &= \frac{2}{9} m_D^2, \end{aligned}}$$

$$m_{\mathrm{D}} = m_D(\mathrm{T}) \longleftarrow \begin{aligned} m_q &= m_q(\mathrm{T}) \\ m_g &= m_g(T) \end{aligned}$$

Biro T S, Levai P, Muller B. Phys. Rev. D 42, 3078

The Quasi-particle Model of the QGP

■ Quasi-particle quark and gluon masses: m_q *and* m_g

$$m_g^2 = \frac{1}{2} m_D^2,$$

$$m_{u,d}^2 = \frac{2}{9} m_D^2,$$

$$m_s^2 - m_{0s}^2 = \frac{2}{9} m_D^2,$$

Neglect m_{0s}^2

$m_{\mathrm{D}} = m_D(\mathrm{T})$

Parameterize $m_D = \mathrm{a}\frac{1}{\mathrm{T}} + \mathrm{c} + \mathrm{bT}.$

$m_q = m_q(\mathrm{T})$

$m_g = m_g(T)$

Biro T S, Levai P, Muller B. Phys. Rev. D 42, 3078

The Quasi-particle Model of the QGP

- Quasi-particle quark and gluon masses m_q *and* m_g:

$$m_g^2 = \frac{1}{2} m_D^2,$$

$$m_{u,d,s}^2 = \frac{2}{9} m_D^2,$$

- Simplifications to the quasiparticle effective masses:

  (1) Neglect the strange quark current mass.

  (2) Employ a parameterization for the Debye screening mass $m_D = \mathrm{a}\frac{1}{\mathrm{T}} + \mathrm{bT} + \mathrm{c}.$

Biro T S, Levai P, Muller B. Phys. Rev. D 42, 3078

The Quasi-particle Model of the QGP

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(2) Employ a parameterization for the Debye screening mass $m_D = \mathrm{a}\frac{1}{\mathrm{T}} + \mathrm{c} + \mathrm{bT}.$

- For a system in thermal equilibrium:

$$P(T) = T \cdot \frac{\partial lnZ(T)}{\partial V};$$

$$\epsilon(T) = \frac{T^2}{V} \cdot \frac{\partial lnZ(T)}{\partial T};$$

$$\boxed{\begin{aligned} s(T) &= \frac{\epsilon + p}{T}; \\ \Delta(T) &= \frac{\epsilon - 3p}{T^4}; \end{aligned}}$$

Biro T S, Levai P, Muller B. Phys. Rev. D 42, 3078

The Bayesian analysis

Physics Model results: y_i

Experiment data: y_{exp}

Bayes' theorem

$$P(\theta \mid \text{data}) \propto P(\theta)P(\text{data} \mid \theta)$$

Prior distribution: uniform .

$[a, b, c]$

Likelihood function: Gaussian distribution function.

$$P(\text{data} \mid \theta) = \prod_i \frac{1}{\sqrt{2\pi\sigma_i}} e^{-\frac{(y_i - y_{exp})^2}{2\sigma_i^2}}$$

Markov Chain Monte Carlo (MCMC)

Random walk in the parameter space.

The Bayesian analysis

Physics Model results: y_i
Quasi-particle Model of QGP;
s/T^3 and Δ/T^4

Experiment data: y_{exp}

Lattice data	Parameters	Prior
Hot QCD	a	(0,1)
Hot QCD	b	(0,2)
Hot QCD	c	(-0.2,0.8)

Bayes' theorem

$$P(\theta \mid \text{data}) \propto P(\theta) P(\text{data} \mid \theta)$$

Prior distribution: uniform .

$$[a, b, c] \in [(0, 1), (0, 2), (-0.2, 0.8)].$$

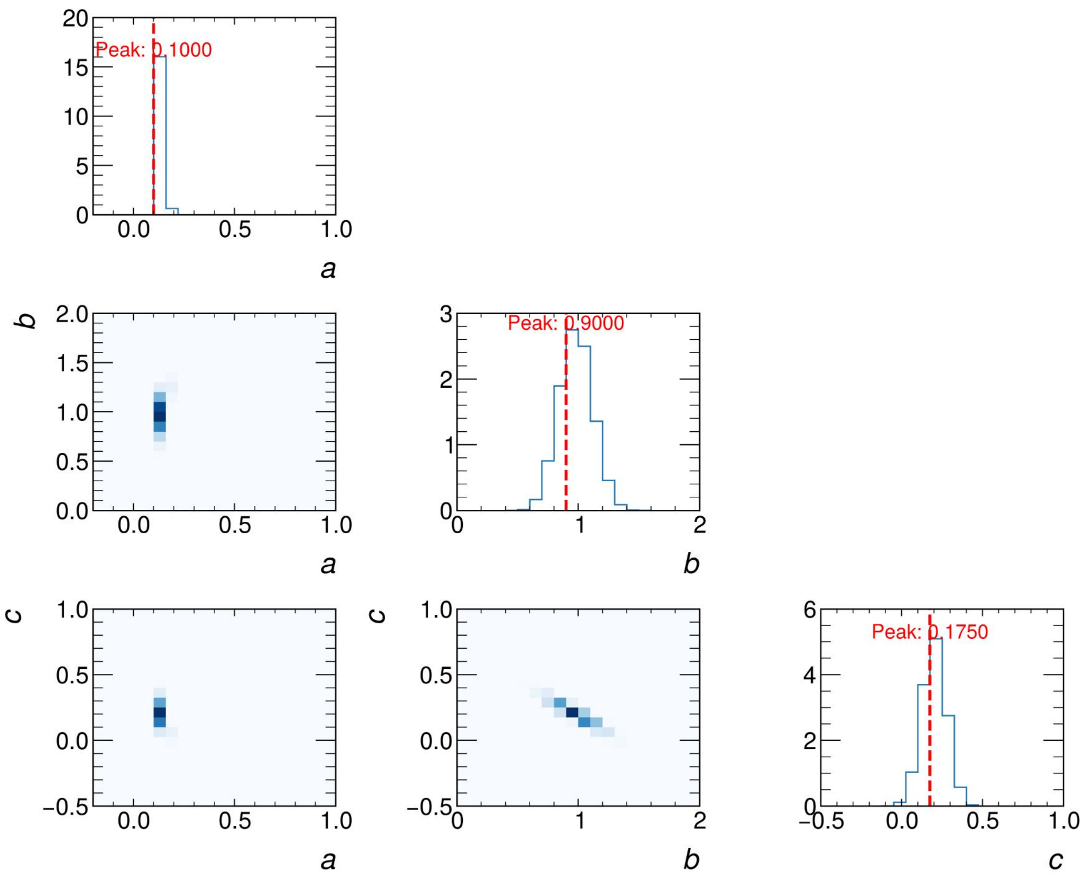
Likelihood function: Gaussian distribution function.

$$P(\text{data} \mid \theta) = \prod_i \frac{1}{\sqrt{2\pi\sigma_i}} e^{-\frac{(y_\mathrm{i} - y_\mathrm{exp})^2}{2\sigma_i^2}}$$

Markov Chain Monte Carlo (MCMC)
Random walk in the parameter space.

The Bayesian analysis

■ Bayesian analysis results.



Lattice data	Parameters	Prior	Mean
Hot QCD	a	(0,1)	0.1410
Hot QCD	b	(0,2)	0.9853
Hot QCD	c	(-0.2,0.8)	0.1979

The Bayesian analysis results

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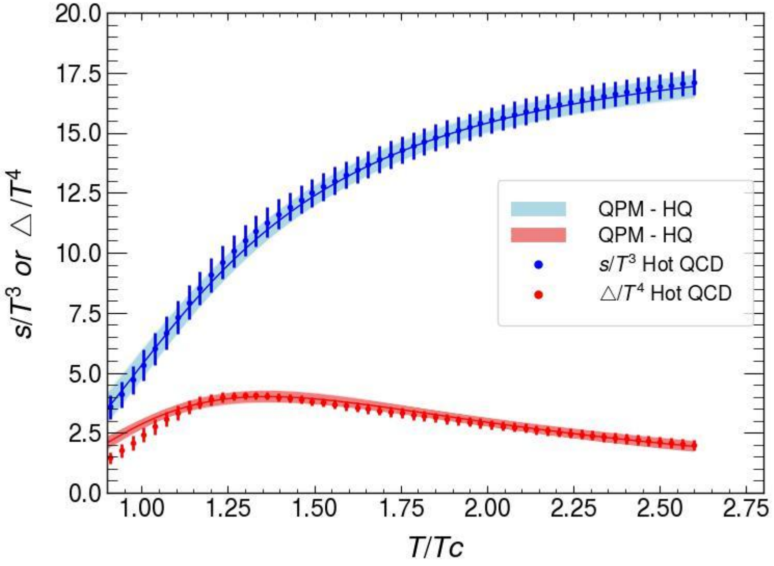
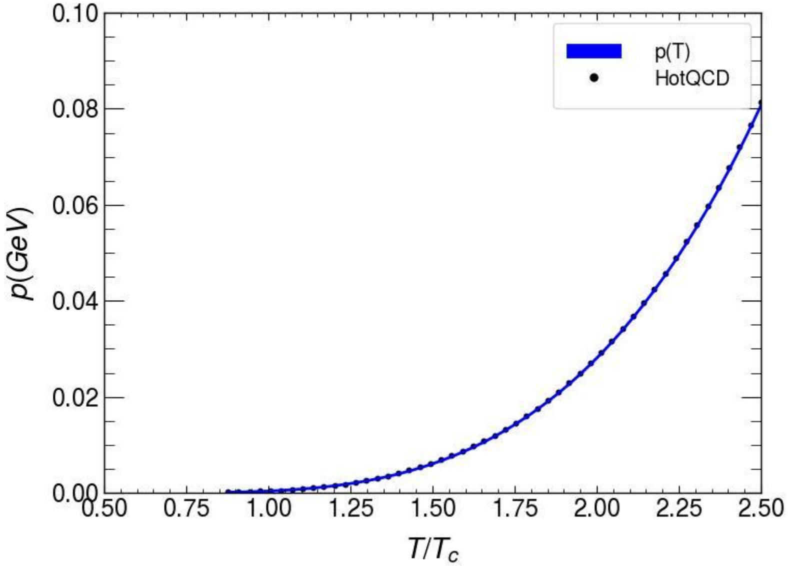
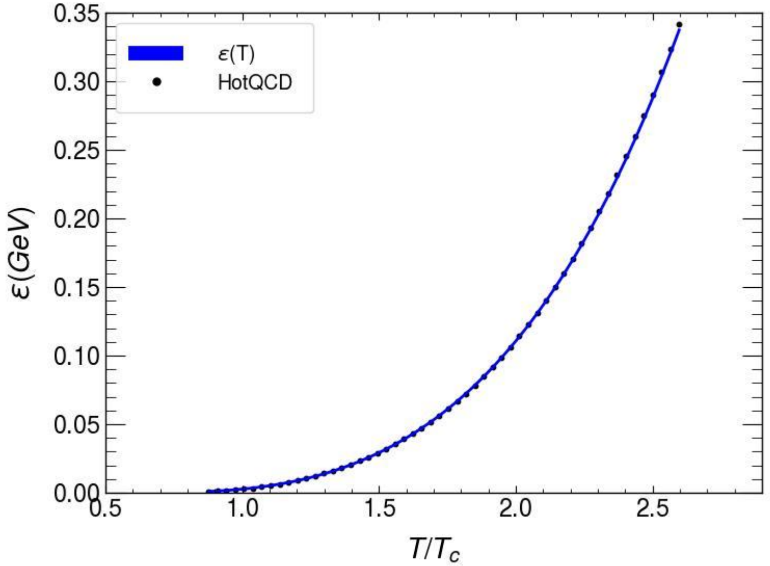
Entropy density & Trace anomaly



Pressure



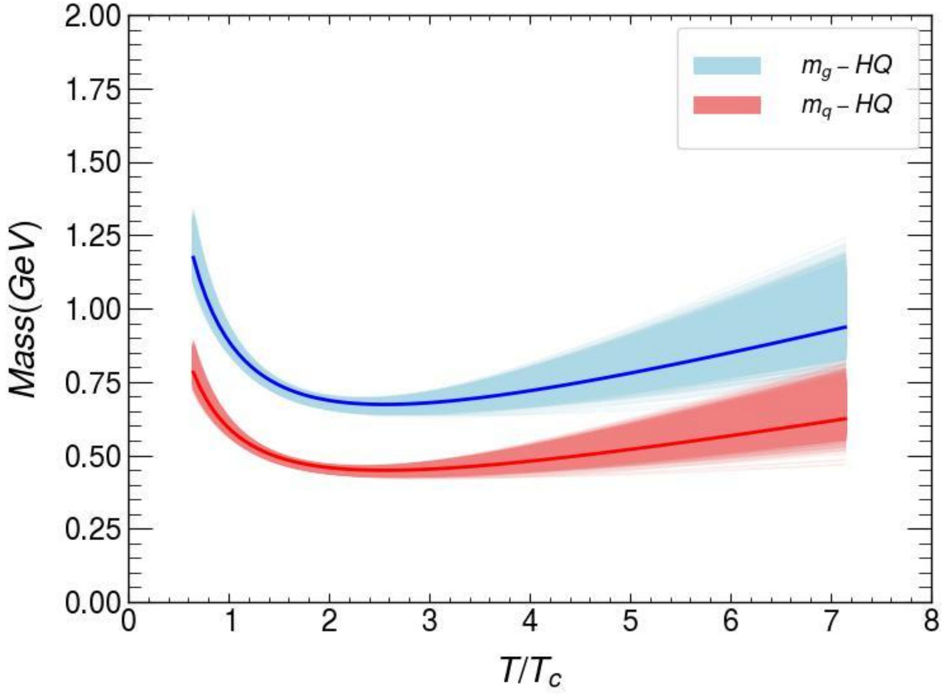
Energy density



- The results obtained from the Bayesian analysis are consistent with the Hot QCD collaboration's data.

The Bayesian analysis results

Quasi-particle gluon & quark mass



$$m_D = 0.1410\ \mathrm{GeV}^2 \frac{1}{T} + 0.1979\ \mathrm{GeV} + 0.9853T;$$

$$m_q = \sqrt{\frac{2}{9}} m_D;$$

$$m_g = \sqrt{\frac{1}{2}} m_D;$$

- $1 < T/T_c < 2$, quasi-particle's masses decrease with temperature increasing.
- $T/T_c > 2$, quasi-particle's masses increase with temperature increasing.

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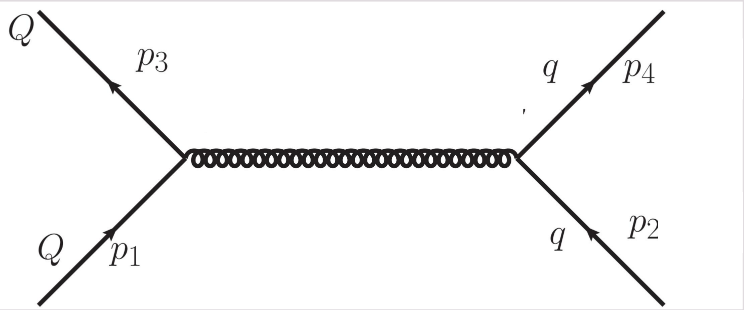
- The Linear Boltzmann Transport Model, LBT Model.

- Boltzmann Equation:

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1(C_{\text{inel}} + C_{\text{el}}).$$

- For elastic process, the scattering rate:

$$\Gamma_{12\to34}(\vec{p}_1, T) = \frac{\gamma_2}{2E_1} \int \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2, T) \times [1 \pm f_3(\vec{p_3}, T)][1 \pm f_4(\vec{p_4}, T)]$$
$$\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \times S_2$$
$$\times \boxed{|M_{12\to34}|^2}.$$



Cao S, Luo T, Qin G Y, et al , Phys. Rev. C 94, 014909

- Cornell potential as an effective gluon propagator:

$$V(r) = V_Y + V_S = -\frac{4}{3}\alpha_s \frac{e^{-m_D r}}{r} - \frac{\sigma e^{-m_s r}}{m_s}$$

Cornell potential in momentum-space:

$$V(\vec{q}) = V_Y + V_S = -\frac{4\pi C_F \alpha_s}{m_D^2 + |\vec{q}|^2} - \frac{8\pi\sigma}{(m_s^2 + |\vec{q}|^2)^2}$$

- Elastic scattering amplitude:

$$iM = iM_Y + iM_S$$

Xing W J, Qin G Y, Cao S, arXiv:2112.15062

The Improved LBT-PNP Model

Quasi-particle QGP.

LBT-PNP Model.

Background

Transport

The Improved LBT-PNP Model.

Xing W J, Qin G Y, Cao S, arXiv:2112.15062

The Improved LBT-PNP Model

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Xing W J, Qin G Y, Cao S, arXiv:2112.15062

A plasma of quasi quarks and gluons.

LBT-PNP Model.

Background

Transport

The Improved LBT-PNP Model.

Elastic scattering amplitude: iM.

Screening mass: $m_D = a\frac{1}{T} + c + bT$.

Factor: $S_2 = \Theta[s \geq (M + m_{q/g})^2]\Theta[-\frac{(s + M^2 - m_{q/g})^2}{s} + 4M^2 \leq t \leq 0]$

Re-calculation the scattering amplitude:

Heavy flavor with the massless gluon:

$$
\begin{aligned}
|M_{Qg}|^2 &= |M_Y|^2 + |M_S|^2 \\
&= \frac{64\pi^2\alpha_s^2}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2} \\
&+ \frac{64\pi^2\alpha_s^2}{9}\frac{(s-M^2)(M^2-u)+2M^2(u+M^2)}{(u-M^2)^2} \\
&+ 8\pi^2\alpha_s^2\frac{5M^4+3M^2t-10M^2u+4t^2+5tu+5u^2}{(t-m_D^2)^2} \\
&+ 8\pi^2\alpha_s^2\frac{(M^2-s)(M^2-u)}{(t-m_D^2)^2} \\
&+ 16\pi^2\alpha_s^2\frac{3M^4-3M^2s-M^2u+s^2}{(s-M^2)(t-m_D^2)} \\
&+ \frac{16\pi^2\alpha_s^2}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)} \\
&+ 16\pi^2\alpha_s^2\frac{3M^2-M^2s-3Mu+u^2}{(t-m_D^2)(u-M^2)} \\
&+ \frac{C_A}{C_F}\frac{(8\pi\sigma)^2}{N_c^2-1}\frac{t^2-4M^2t}{(t-m_s^2)^2}.
\end{aligned}
$$

Heavy flavor with the massive quasi-particle gluon:

$$
\begin{aligned}
|M_{Qg}|^2 &= |M_Y|^2 + |M_S|^2 \\
&= \frac{64\pi^2\alpha_s^2}{9}\frac{(s-(M^2+m_g^2))((M^2+m_g^2)-u)+2M^2(s+M^2)}{(s-M^2)^2} \\
&+ \boxed{\frac{64\pi^2\alpha_s^2}{9}\frac{[-m_g^2(s+u)+2m_g^4+6M^2m_g^2]}{(s-M^2)^2}} \\
&+ \frac{64\pi^2\alpha_s^2}{9}\frac{(s-(M^2+m_g^2))((M^2+m_g^2)-u)+2M^2(u+M^2)}{(u-M^2)^2} \\
&\boxed{+\frac{64\pi^2\alpha_s^2}{9}\frac{[-m_g^2(s+u)+2m_g^4+6M^2m_g^2]}{(u-M^2)^2}} \\
&+ 8\pi^2\alpha_s^2\frac{5(M^2+m_g^2)^2+3M^2t-10(M^2+m_g^2)u+4t^2+5tu+5u^2}{(t-m_D^2)^2} \\
&\boxed{+ 8\pi^2\alpha_s^2\frac{4M^2m_g^2+2m_g^2t}{(t-m_D^2)^2}} \\
&+ 8\pi^2\alpha_s^2\frac{(M^2-s)(M^2-u)\boxed{+m_g^4}}{(t-m_D^2)^2} \\
&+ 16\pi^2\alpha_s^2\frac{3M^4-3M^2s-M^2u+s^2+\boxed{5m_g^4+2m_g^2(5M^2-2s-u)}}{(s-M^2)(t-m_D^2)} \\
&+ \frac{16\pi^2\alpha_s^2}{9}\frac{M^2(4M^2-t)\boxed{-2m_g^2t}}{(s-M^2)(M^2-u)} \\
&+ 16\pi^2\alpha_s^2\frac{3M^2-M^2s-3Mu+u^2\boxed{+5m_g^4+2m_g^2(5M^2-2u-s)}}{(t-m_D^2)(u-M^2)} \\
&+ \frac{C_A}{C_F}\frac{(8\pi\sigma)^2}{N_c^2-1}\frac{t^2-4M^2t\boxed{-4m_g^2t+16M^2m_g^2}}{(t-m_s^2)^4}.
\end{aligned}
$$

Re-calculation the scattering amplitude:

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Heavy flavor with the massless quark:

$$\begin{aligned}|M_{Qq}|^2 &= |M_Y|^2 + |M_S|^2 \\ &= \frac{64\pi^2\alpha_s^2}{9}\frac{\left[s-m_q^2\right]^2+\left[M^2-u\right]^2+2M^2t}{(t-m_d^2)^2} \\ &+ \frac{(8\pi\sigma)^2}{N_c^2-1}\frac{t^2-4M^2t}{(t-m_s^2)^4}.\end{aligned}$$

Heavy flavor with the massive quasi-particle quark:

$$\begin{aligned}|M_{Qq}|^2 &= |M_Y|^2 + |M_S|^2 \\ &= \frac{64\pi^2\alpha_s^2}{9}\frac{\left[s-(M^2+m_q^2)\right]^2+\left[(M^2+m_q^2)-u\right]^2+2M^2t+\boxed{2m_q^2t}}{(t-m_d^2)^2} \\ &+ \frac{(8\pi\sigma)^2}{N_c^2-1}\frac{t^2-4M^2t\boxed{-4m_q^2t+16M^2m_q^2}}{(t-m_s^2)^4}\end{aligned}$$

Xing W J, Qin G Y, Cao S, arXiv:2112.15062

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Heavy flavor with the massive quasi-particle quark:

$$\begin{aligned} |M_{Qq}|^2 &= |M_Y|^2 + |M_S|^2 \\ &= \frac{64\pi^2\alpha_s^2}{9} \frac{\left[s - (M^2 + m_q^2)\right]^2 + \left[(M^2 + m_q^2) - u\right]^2 + 2M^2 t + \boxed{2m_q^2 t}}{(t - m_d^2)^2} \\ &+ \frac{(8\pi\sigma)^2}{N_c^2 - 1} \frac{t^2 - 4M^2 t \boxed{- 4m_q^2 t + 16M^2 m_q^2}}{(t - m_s^2)^4} \end{aligned}$$

The Improved LBT-PNP Model.

Xing W J, Qin G Y, Cao S, arXiv:2112.15062
Wu X Y, Qin G Y, Pang L G, Phys. Rev. C, 2022, 105(3): 034909.

Contents

Result I: PbPb 0 - 10%

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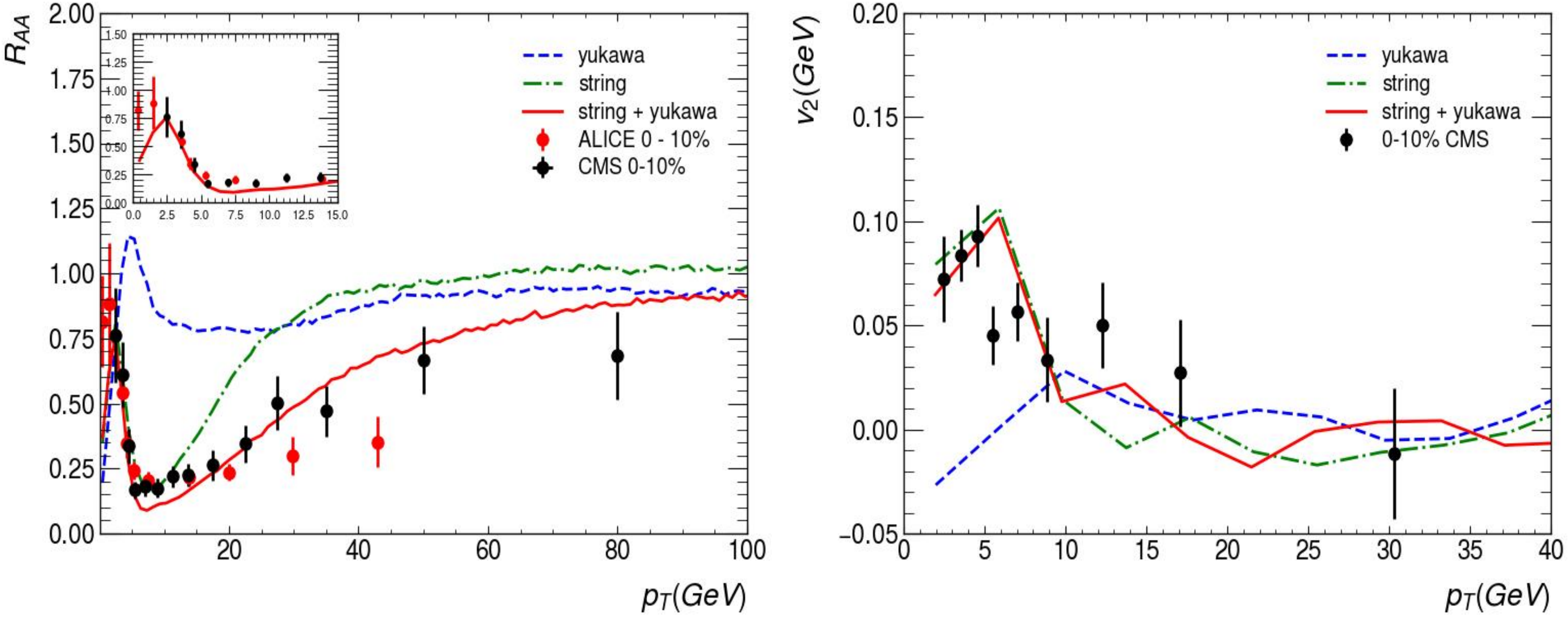
Phys. Rev. Lett., 2018, 120(20).
JHEP, 2018, 10: 174.

☑ D_0 R_{AA} and v_2.

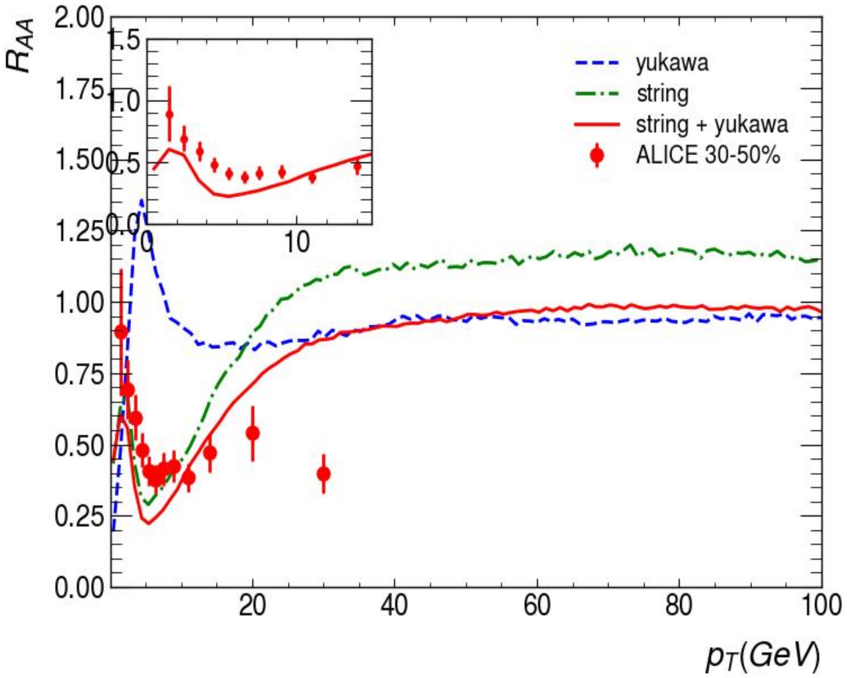


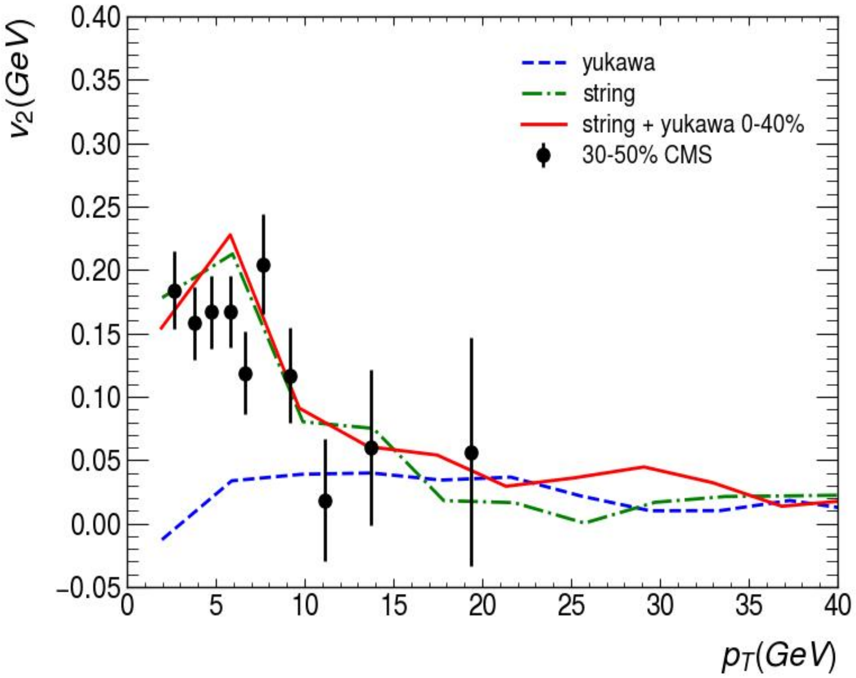
➢ The non-perturbative term dominates in the low transverse momentum region, while the perturbative Yukawa term dominates in the high transverse momentum region.

Result I: PbPb 30 - 50%

Phys. Rev. Lett., 2018, 120(20).
JHEP, 2018, 10: 174.

☑ R_{AA} and v_2.



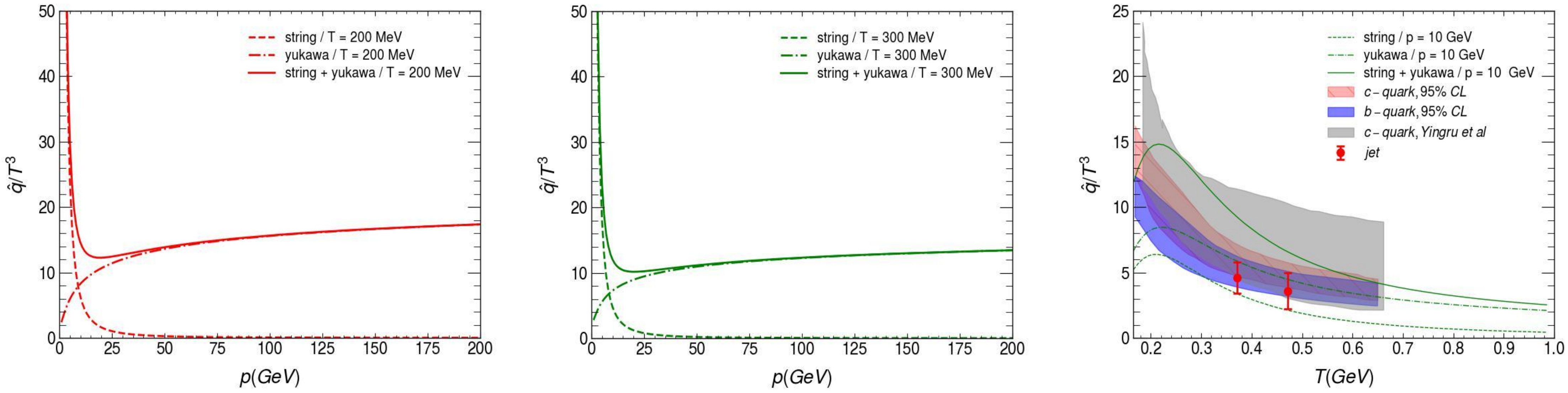


- ➢ The improved LBT-PNP model can describe the experimental data better, especially low transverse momentum region.

Result II: Jet quenching coefficient

$$\hat{q} = \left\langle (\vec{p}_3 - \hat{p}_1 \cdot \vec{p}_3)^2 \right\rangle ;$$

Shu H T. DOI: arXiv:2401.08040.
Ke W, Xu Y, Bass S A. Phys. Rev. C 98, 064901.



➢ At a fixed momentum, the $\hat{q}/T^3$ is slightly higher than the result from the JET Collaboration, and its trend is consistent with the Bayesian statistical analysis.

Result II: Diffusion coefficient

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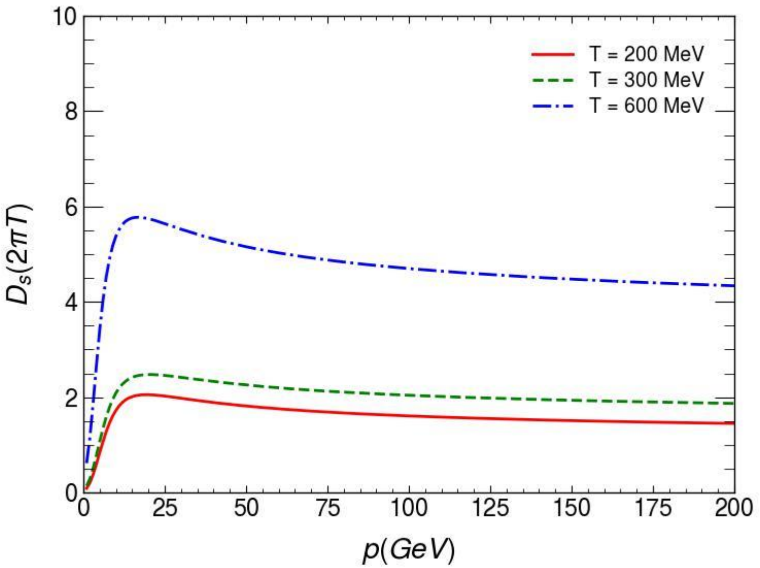
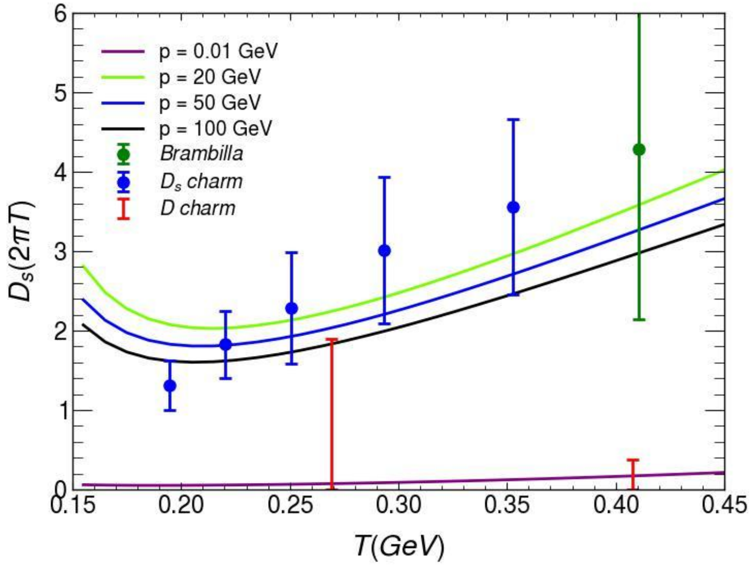
Shu H T. DOI: arXiv:2401.08040.
Ke W, Xu Y, Bass S A. Phys. Rev. C 98, 064901.

$$\boxed{D_s(2\pi T) = 8\pi/(\hat{q}/T^3)}$$





- At fixed temperature, the $D_s(2\pi T)$ initially increases, then slightly decreases, and finally saturates with momentum increasing.
- At fixed momentum, the $D_s(2\pi T)$ first decreases and then increases linearly with temperature rising.

Contents

Summary and Outlook

Summary

- ☑ The quasi-particle model is added to the LBT framework, improving the treatment of non-perturbative effects in heavy quark scattering.
- ☑ The improved LBT-PNP model describes the R_{AA} and v_2 of heavy-flavor mesons in Pb-Pb collisions simultaneously, especially at low-to-medium transverse momentum.
- ☑ The transport coefficients $\hat{q}$ and $D_s(2\pi T)$ consistent with results from the JET Collaboration and lattice QCD.

Outlook

- ➢ Other observables: higher-order flow, baryon-to-meson ratios, and azimuthal correlations.

Thanks for your attention !

back up

The LBT Model

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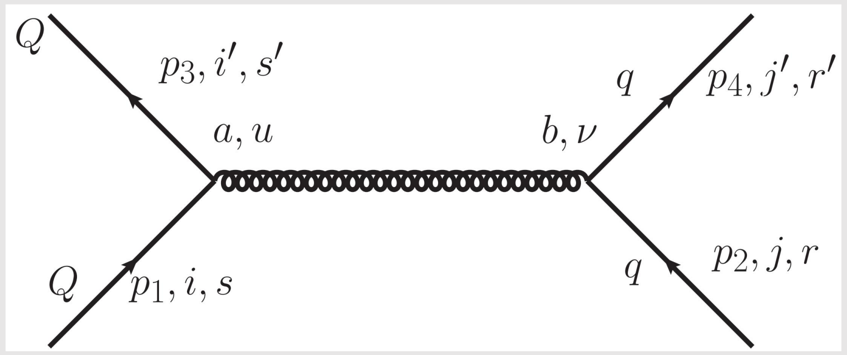
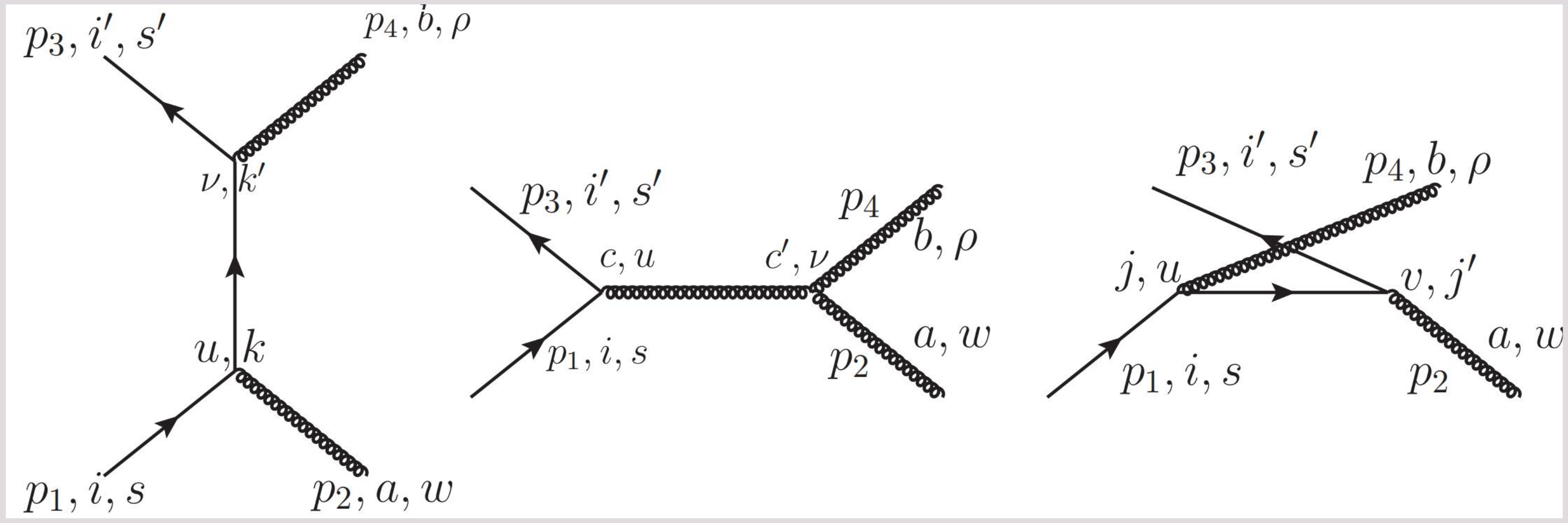
- Leading order：

Qq → Qq



Qg → Qg



$$
\begin{aligned}
iM &= M_Y + M_S \\
&= \bar{u}^{s'}(p_3)\gamma^{\mu}u^{s}(p_1)V_Y\bar{u}^{r'}(p_4)\gamma^{\nu}u^{r}(p_2) \\
&+ \bar{u}^{s'}(p_3)u^{s}(p_1)V_S\bar{u}^{r'}(p_4)u^{r}(p_2).
\end{aligned}
$$

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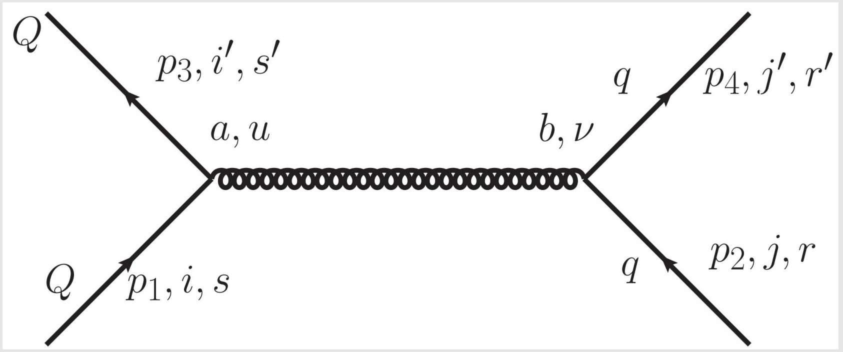
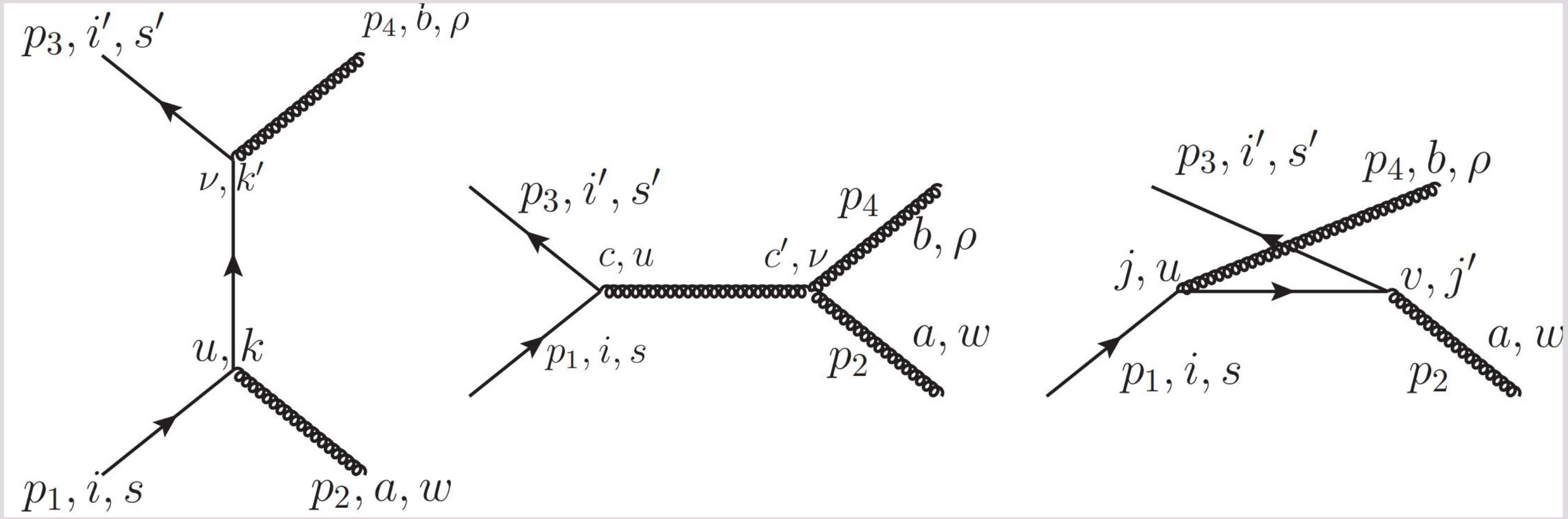
求实创新　立德树人

■ Leading order:

Qq → Qq



Qg → Qg



■ time step Δt， the possibility of the elastice scattering:

$$\Delta N_{\text{el}} = \Gamma_{12\to34} \cdot \Delta t$$

$$P_{el} = \frac{\Delta N_{\text{el}}^n}{n!} e^{-\Gamma_{12\to34}\cdot\Delta t}.$$

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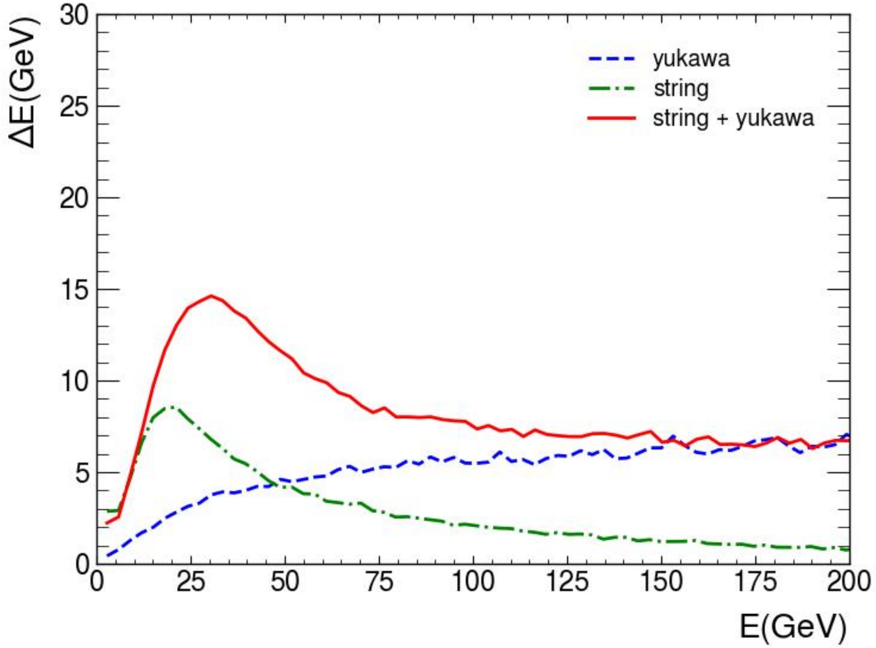
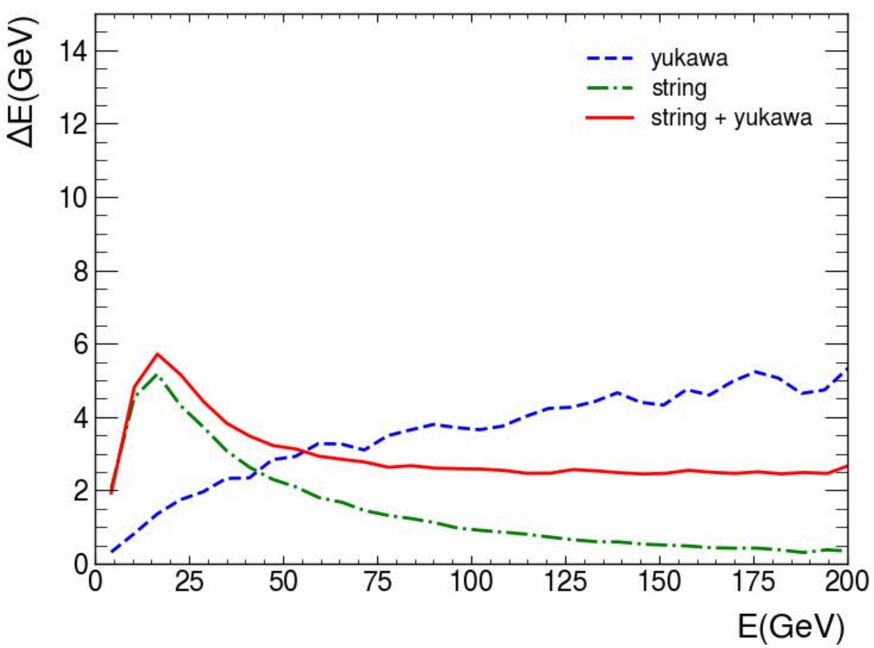
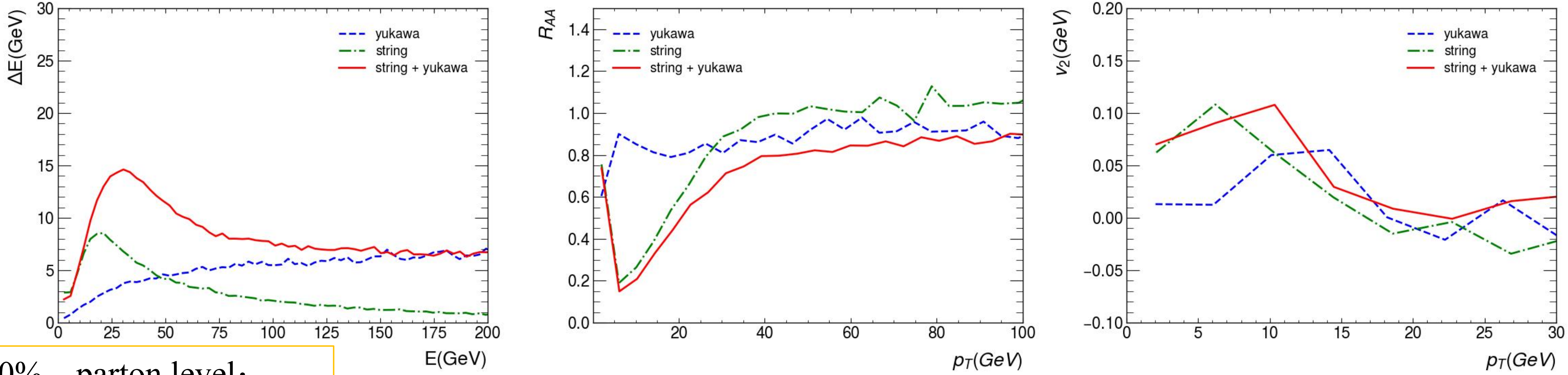
PbPb，0-10%，parton level：



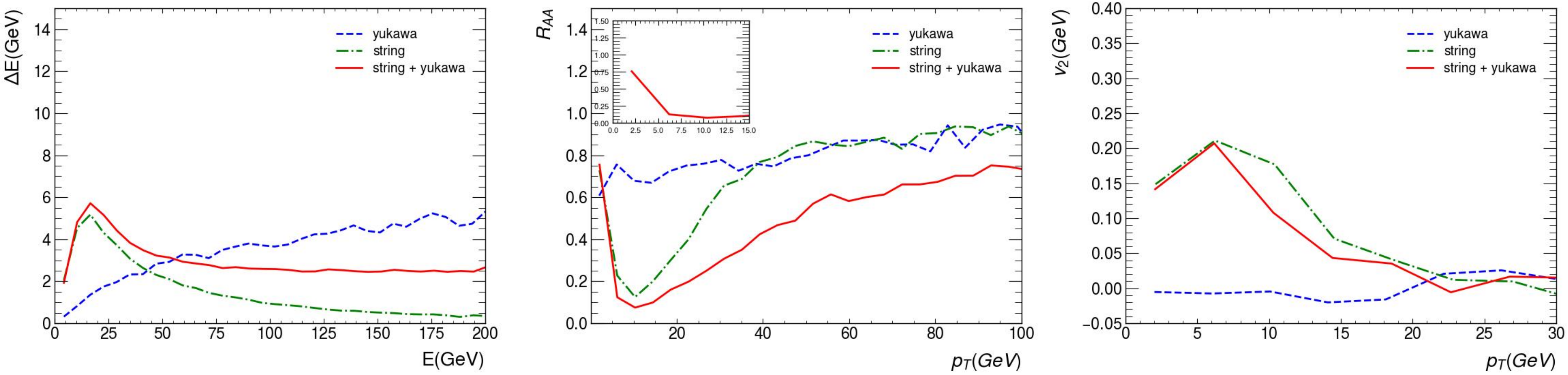
PbPb，30-50%，parton level：



PbPb，0-10%，parton level：



PbPb，30-50%，parton level：

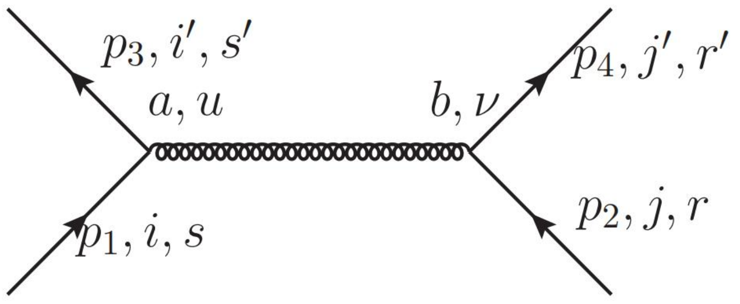


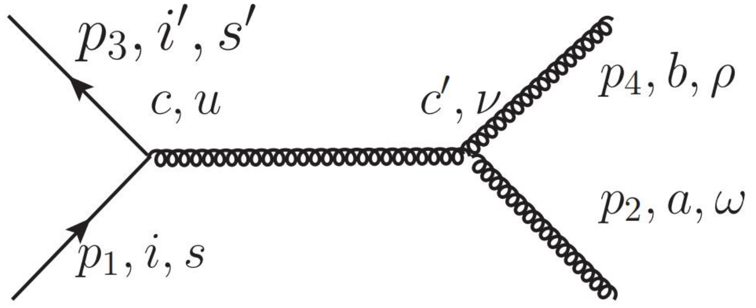
- Cornel lpotential：

$$V(\vec{q}) = V_Y + V_S = -\frac{4\pi C_F \alpha_s}{m_D^2 + |\vec{q}|^2} - \frac{8\pi\sigma}{(m_s^2 + |\vec{q}|^2)^2}.$$

- elastic：

$$\begin{aligned} iM &= M_Y + M_S \\ &= \bar{u}^{s'}(p_3)\gamma^{\mu}u^{s}(p_1)V_Y\bar{u}^{r'}(p_4)\gamma_{\mu}u^{r}(p_2) \\ &+ \bar{u}^{s'}(p_3)u^{s}(p_1)V_S\bar{u}^{r'}(p_4)u^{r}(p_2). \end{aligned}$$





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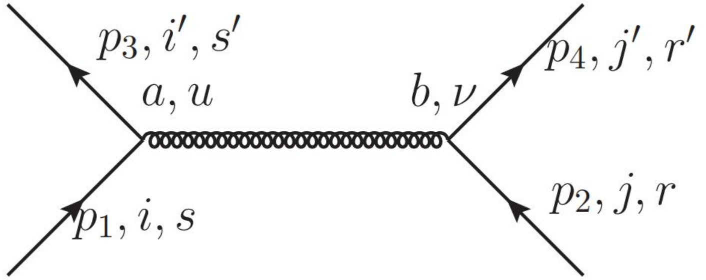
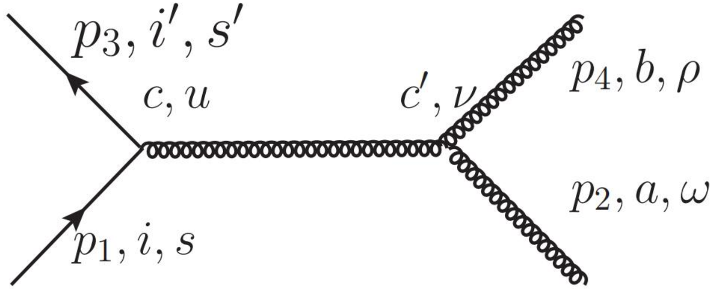
- $cq \rightarrow cq$ color factor：



$$\frac{1}{N_c}\frac{1}{N_c}\cdot\sum(t^a_{i'i}t^b_{j'j})\delta^{ab}(t^a_{i'i}t^b_{j'j})^*\delta^{ab}$$
$$=\frac{1}{N_c}\frac{1}{N_c}\cdot\sum_{i',i,j,j'}(t^a_{i'i}t^a_{j'j})(t^b_{ii'}t^b_{jj'})$$
$$=\frac{1}{N_c}\frac{1}{N_c}\cdot Tr\left[t^at^b\right]\cdot Tr\left[t^at^b\right]$$
$$=\frac{1}{N_c}\frac{1}{N_c}\cdot(C(r)\delta^{ab})(C(r)\delta^{ab})$$
$$=\frac{1}{N_c}\frac{1}{N_c}\cdot\frac{N_c^2C_F^2}{(N_c^2-1)^2}\cdot(N_c^2-1)$$
$$=\frac{C_F}{N_c^2-1}\cdot C_F.$$

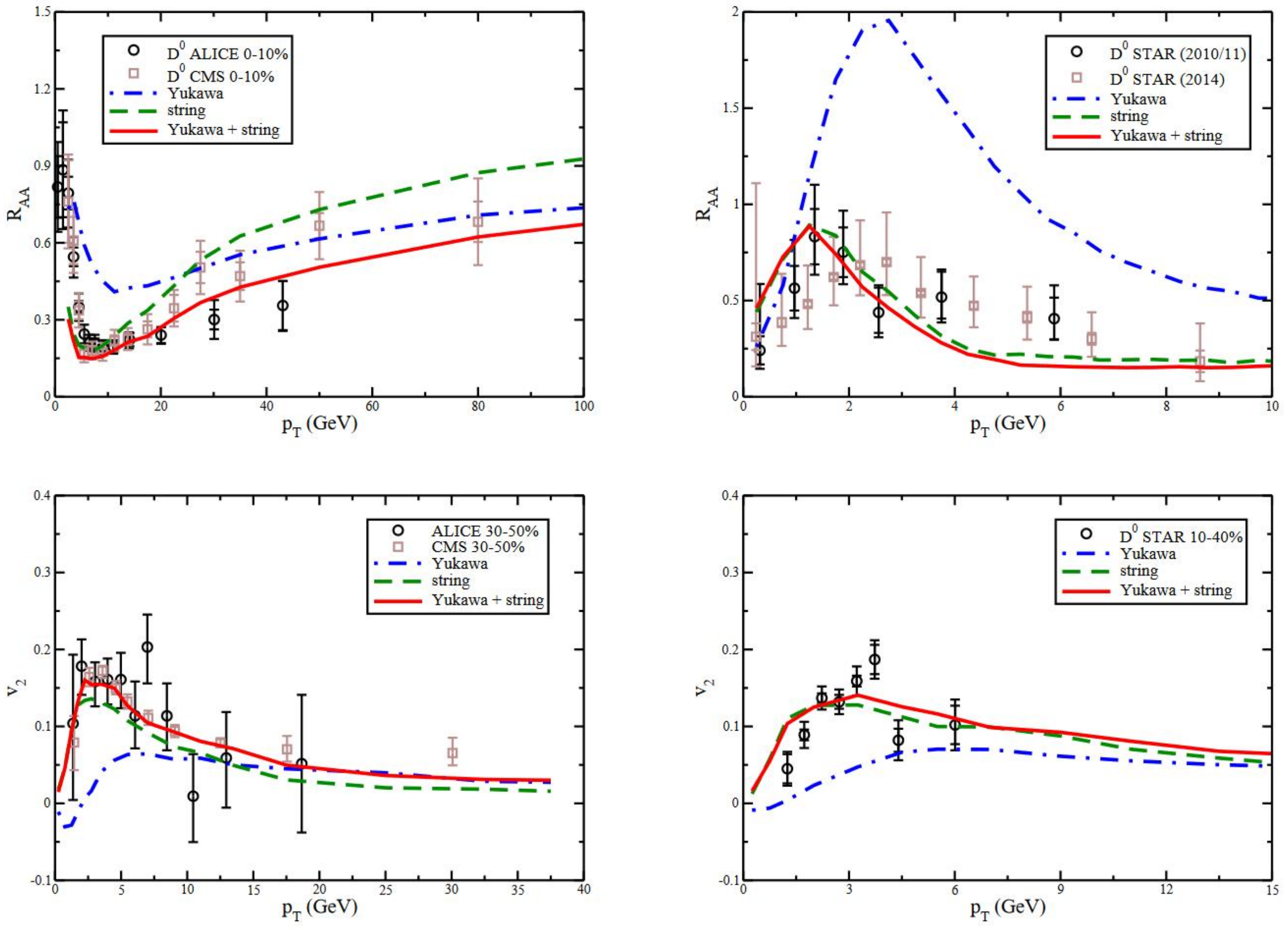
- $cg \rightarrow cg$ color factor：



$$\frac{1}{N_c}\frac{1}{N_c^2-1}\cdot\sum(t^c_{i'i}\delta^{cc'}f^{c'ab})(t^c_{i'i}\delta^{cc'}f^{c'ab})^*$$
$$=\frac{1}{N_c}\frac{1}{N_c^2-1}\cdot\sum(t^c_{i'i}f^{cab})(t^c_{i'i}f^{cab})^*$$
$$=\frac{1}{N_c}\frac{1}{N_c^2-1}\cdot\frac{N_cC_F}{N_c^2-1}C_A(N_c^2-1)$$
$$=\frac{C_A}{N_c^2-1}\cdot C_F.$$

$$iM = M_Y + M_S$$
$$= \bar{u}^{s'}(p_3)\gamma^\mu u^s(p_1)V_Y\bar{u}^{r'}(p_4)\gamma^\nu u^r(p_2)$$
$$+ \bar{u}^{s'}(p_3)u^s(p_1)V_S\bar{u}^{r'}(p_4)u^r(p_2).$$

Xing W J, Qin G Y, Cao S DOI: 10.1016/j.physletb.2023.137733.


D^0 ALICE 0-10%
D^0 CMS 0-10%
Yukawa
string
Yukawa + string
R_{AA}
p_T (GeV)
0
0.3
0.6
0.9
1.2
1.5
0
20
40
60
80
100
D^0 STAR (2010/11)
D^0 STAR (2014)
Yukawa
string
Yukawa + string
R_{AA}
p_T (GeV)
0
0.5
1
1.5
2
0
2
4
6
8
10
ALICE 30-50%
CMS 30-50%
Yukawa
string
Yukawa + string
v_2
p_T (GeV)
-0.1
0
0.1
0.2
0.3
0.4
0
5
10
15
20
25
30
35
40
D^0 STAR 10-40%
Yukawa
string
Yukawa + string
v_2
p_T (GeV)
-0.1
0
0.1
0.2
0.3
0.4
0
3
6
9
12
15

- E4：

$$E_4 = \begin{cases} \dfrac{B(E_1+E_2)+\sqrt{A^2\left[B^2+m_{q/g}^2A^2-m_{q/g}^2(E_1+E_2)^2\right]}}{(E_1+E_2)^2-A^2}, A>0; \\ \dfrac{B(E_1+E_2)-\sqrt{A^2\left[B^2+m_{q/g}^2A^2-m_{q/g}^2(E_1+E_2)^2\right]}}{(E_1+E_2)^2-A^2}, A<0. \end{cases}$$

while,

$$A = p_1\cos\theta_4 + p_2\cos\theta_{24};$$

$$B = E_1E_2 + m^2 - p_1p_2\cos\theta_2.$$

- qhat:

$$\langle X(\vec{p}_1,T)\rangle = \frac{\gamma_2}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4} \times f_2(\vec{p}_2,T)\times[1\pm f_3(\vec{p}_3,T)]\,[1\pm f_4(\vec{p}_4,T)]\times S_2 \times (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)\,|M_{12\to34}|^2\cdot\langle X(\vec{p}_1,T)\rangle$$

$$\hat{e} = \langle(E_1 - E_3)\rangle\,.$$

$$\hat{q} = \left\langle(\vec{p}_3 - \hat{p}_1\cdot\vec{p}_3)^2\right\rangle;$$

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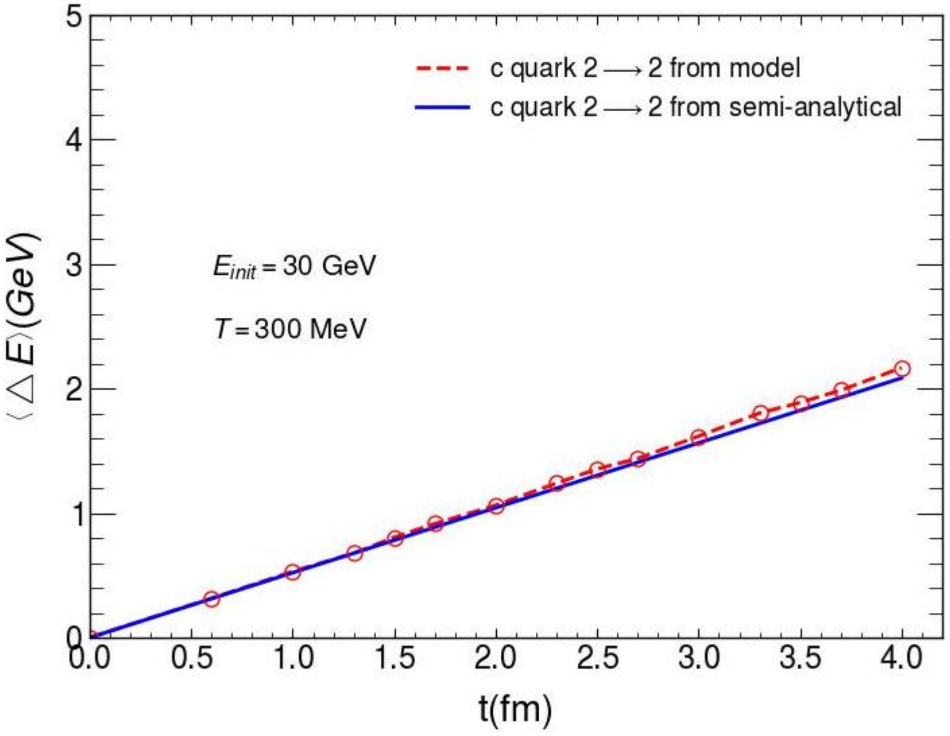
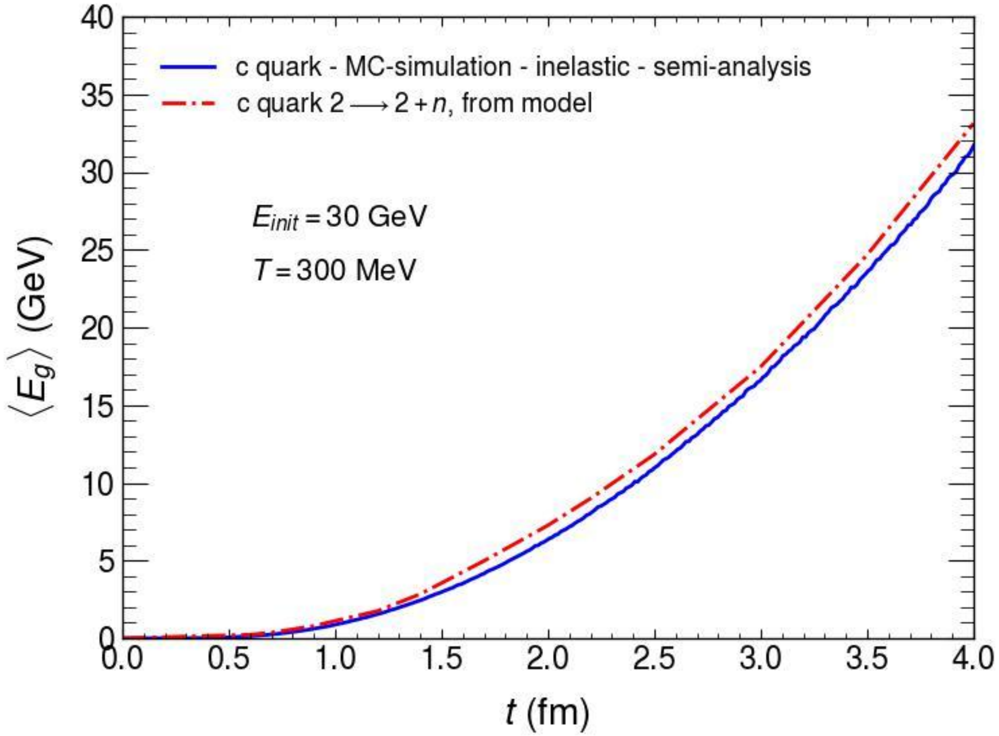
$$\hat{e} = \langle (E_1 - E_3) \rangle \, .$$

$$\langle E_g \rangle (E, T, t) = \int_0^t \mathrm{d}t \int_{\frac{\omega_0}{E}}^1 \mathrm{d}x \int_0^{(xE)^2} \mathrm{d}k_\perp^2 \, xE \frac{\mathrm{d}N_g}{\mathrm{d}x \mathrm{d}k_\perp^2 \mathrm{d}t} .$$





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- inelastic：

$$\langle N_g(t,\Delta t)\rangle = \Delta t \int \mathrm{d}x\mathrm{d}k_\perp^2 \frac{\mathrm{d}N_g}{\mathrm{d}x\mathrm{d}k_\perp^2\mathrm{d}t}.$$

$$P_{\mathrm{inel}}(n) = \frac{\langle N_g\rangle^n}{n!} e^{-\langle N_g\rangle}.$$

- thermal mass：

$$m_g^2(T) = \frac{1}{6}g^2\left[(N_c+\frac{1}{2}N_f)T^2 + \frac{N_c}{2\pi^2}\mu^2\right];$$

$$m_{u,d}^2(T) = \frac{N_c^2-1}{8N_c}g^2(T^2+\frac{\mu^2}{\pi^2});$$

$$m_s^2 - m_{0s}^2 = \frac{N_c^2-1}{8N_c}g^2(T^2+\frac{\mu^2}{\pi^2}).$$

$$\boxed{m_D^2(T) = \frac{1}{3}(N_c+\frac{1}{2}N_f)g^2T^2}$$

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碎裂-重组合强子化模型

- 碎裂强子化：由一个高能部分子碎裂成不同动量分数的多个强子的过程。

- 重组合强子化：部分子与其静止系中的一个或两个临近夸克重组生成一个介子或重子的过程。

- 碎裂函数是反应过程的长程部分，无法用pQCD计算，其关于能标Q^2的依赖可以用DGLAP方程描述：

$$\frac{\mathrm{d}}{\mathrm{d}lnQ^2}D_i^h(z,Q^2)=\sum_{j=-n_f}^{n_f}\int_z^1\frac{\mathrm{d}z'}{z'}P_{ij}(\frac{z}{z'})\frac{\alpha_s}{2\pi}D_j^h(z',Q^2)$$

z，z’，P_{ij}分别是强子相对于部分子i的动量分数；强子相对于部分子j的动量分数以及部分子i劈裂味j的领头阶劈裂函数。为碎裂函数指定一个参数化的形式：

$$D_i^h(z,Q^2)=N_i z^{a_i}(1-z)^{b_i}$$

- LBT-PNP：

$$V(\vec{q}) = V_Y + V_S = -\frac{4\pi C_F \alpha_s}{m_D^2 + |\vec{q}|^2} - \frac{8\pi\sigma}{(m_s^2 + |\vec{q}|^2)^2}.$$

$where, \sigma = 0.45\mathrm{GeV}$， $m_s = \sqrt{0.1\mathrm{T}}$， $m_D = 0.1410\frac{1}{T} + 0.9853T + 0.1979$， $\alpha_s = \frac{g^2}{4\pi} = \frac{{m_g}^2 + {m_q}^2}{T^2\left[{}^1/_6\left(N_c + {}^1/_2 N_f\right) + \frac{N_C^2 - 1}{8N_c}\right]}$。

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Thanks for your attention !